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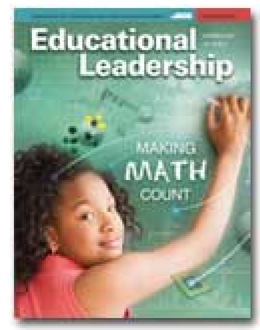

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How Mathematics Counts

Lynn Arthur Steen

Fractions and algebra represent the most subtle, powerful, and mind-twisting elements of school mathematics. But how can we teach them so students understand?

Much to the surprise of those who care about such things, mathematics has become the 600-pound gorilla in U.S. schools. High-stakes testing has forced schools to push aside subjects like history, science, music, and art in a scramble to avoid the embarrassing consequences of not making "adequate yearly progress" in mathematics. Reverberations of the math wars of the 1990s roil parents and teachers as they seek firm footing in today's turbulent debates about mathematics education.

Much contention occurs near the ends of elementary and secondary education, where students encounter topics that many find difficult and some find incomprehensible. In earlier decades, schools simply left students in the latter category behind. Today, that option is neither politically nor legally acceptable. Two topics—fractions and algebra, especially Algebra II—are particularly troublesome. Many adults, including some teachers, live their entire lives flummoxed by problems requiring any but the simplest of fractions or algebraic formulas. It is easy to see why these topics are especially nettlesome in today's school environment. They are exemplars of why mathematics counts and why the subject is so controversial.

Confounded by Fractions

What is the approximate value, to the nearest whole number, of the sum 19/20 + 23/25? Given the choices of 1, 2, 42, or 45 on an international test, more than half of U.S. 8th graders chose 42 or 45. Those responses are akin to decoding and pronouncing the word *elephant* but having no idea what animal the word represents. These students had no idea that 19/20 is a number close to 1, as is 23/25.

Neither, it is likely, did their parents. Few adults understand fractions well enough to use them fluently. Because people avoid fractions in their own lives, some question why schools (and now entire states) should insist that all students know, for instance, how to add uncommon combinations like 2/7 + 9/13 or how to divide 1 3/4 by 2/3. When, skeptics ask, is the last time any typical adult encountered problems of this sort? Even mathematics teachers have a hard time imagining authentic problems that require these exotic calculations (Ma, 1999).

Moreover, many people cannot properly express in correct English the fractions and proportions that *do* commonly occur, for instance, in ordinary tables of data. A simple example illustrates this difficulty (Schield, 2002). Even though most people know that 20 percent means 1/5 of something, many cannot figure out what the something is when confronted with an actual example, such as the table in Figure 1. Although calculators can help the innumerate cope with such exotica as 2/7 + 9/13 and 1 3/4 ÷ 2/3, they are of no help to someone who has trouble reading tables and expressing those relationships in clear English.

Figure 1. The Challenge of Expressing Numerical Data in Ordinary Language

Percentage Who Are Runners

	Non Smokers	Smokers	Total
Female	50%	20%	40%
Male	25%	10%	20%
Total	37%	15%	30%

Which of the following correctly describes the 20% circled above:

1. 20% of runners are female smokers
2. 20% of females are runners who smoke
3. 20% of female smokers are runners
4. 20% of smokers are females who run

These examples illustrate two very different aspects of mathematics that apply throughout the discipline. On the one hand is calculation; on the other, interpretation. The one reasons *with* numbers to produce an answer; the other reasons *about* numbers to produce understanding. Generally, school mathematics focuses on the former, natural and social sciences on the latter. For lots of reasons—psychological, pedagogical, logical, motivational—students will learn best when teachers combine these two approaches.

There may be good reasons that so many children and adults have difficulty with fractions. It turns out that even mathematicians cannot agree on a single proper definition. One camp argues that fractions are just names for certain points on the number line (Wu, 2005), whereas others say that it's better to think of them as multiples of basic unit fractions such as 1/3, 1/4, and 1/5 (Tucker, 2006). Textbooks for prospective elementary school teachers exhibit an even broader and more confusing array of approaches (McCrory, 2006).

Instead of beginning with formal definitions, when ordinary people speak of fractions they tend to emphasize contextual meaning. Fractions (like all numbers) are human constructs that arise in particular social and scientific contexts. They represent the magnitude of social problems (for example, the percentage of drug addiction in a given population); the strength of public opinion (for example, the percentage of the population that supports school vouchers); and the consequences of government policies (for example, the unemployment rate). Every number is the product of human activity and is selected to serve human purposes (Best, 2001, 2007).

Fractions, ratios, proportions, and other numbers convey quantity; words convey meaning. For mathematics to make sense to students as something other than a purely mental exercise, teachers need to focus on the interplay of numbers and words, especially on expressing quantitative relationships in meaningful sentences. For users of mathematics, calculation takes a backseat to meaning. And to make mathematics meaningful, the three *R*s must be well blended in each student's mind.

Algebra for All?

Conventional wisdom holds that in Thomas Friedman's metaphorically flat world, all students, no matter their talents or proclivities, should leave high school prepared for both college and high-tech work (American Diploma Project, 2004). This implies, for example, that all students should master Algebra II, a course originally designed as an elective for the mathematically inclined. Indeed, more than half of U.S. states now require Algebra II for almost all high school graduates (Zinth, 2006).

Advocates of algebra advance several arguments for this dramatic change in education policy:

- Workforce projections suggest a growing shortage of U.S. citizens having the kinds of technical skills that build on such courses as Algebra II (Committee on Science, Engineering, and Public Policy, 2007).
- Employment and education data show that Algebra II is a “threshold course” for high-paying jobs. In particular, five in six young people in the top quarter of the income distribution have completed Algebra II (Carnevale & Desrochers, 2003).
- Algebra II is a prerequisite for College Algebra, the mathematics course most commonly required for postsecondary degrees. Virtually all college students who have not taken Algebra II will need to take remedial mathematics.
- Students most likely to opt out of algebra when it is not required are those whose parents are least engaged in their children's education. The result is an education system that magnifies inequities and perpetuates socioeconomic differences from one generation to the next (Haycock, 2007).

Skeptics of Algebra II requirements note that other areas of mathematics, such as data analysis, statistics, and probability, are in equally short supply among high school graduates and are generally more useful for employment and daily life. They point out that the historic association of Algebra II with economic success may say more about common causes (for example, family background and peer support) than about the usefulness of Algebra II skills. And they note that many students who complete Algebra II also wind up taking remedial mathematics in college.

Indeed, difficulties quickly surfaced as soon as schools tried to implement this new agenda for mathematics education. Shortly after standards, courses, and tests were developed to enforce a protocol of “Algebra II for all,” it became clear that many schools were unable to achieve this goal. The reasons included, in varying degrees, inadequacies in preparation, funding, motivation, ability, and instructional quality. The result has been a proliferation of “fake” mathematics courses and lowered proficiency standards that enable districts and states to pay lip service to this goal without making the extraordinary investment of resources required to actually accomplish it (Noddings, 2007).

Several strands of evidence question the unarticulated assumption that additional instruction in algebra would necessarily yield increased learning. Although this may be true in some subjects, it is far less clear for subjects such as Algebra II that are beset by student indifference, teacher shortages, and unclear purpose. For many of the reasons given, enrollments in Algebra II have approximately doubled during the last two decades (National Center for Education Statistics [NCES], 2005a). Yet during that same period, college enrollments in remedial mathematics and mathematics scores on the 12th grade National Assessment of Educational Progress (NAEP) have hardly changed at all (NCES, 2005b; Lutzer, Maxwell, & Rodi, 2007). Something is clearly wrong.

Although we cannot conduct a randomized controlled study of school mathematics, with some students receiving a treatment and others a placebo, we can examine the effects of the current curriculum on those who go through it. Here we find more disturbing evidence:

- One in three students who enter 9th grade fails to graduate with his or her class, leaving the United States with the highest secondary school dropout rate among industrialized nations (Barton, 2005). Moreover, approximately half of all blacks, Hispanics, and American Indians fail to graduate with their class (Swanson, 2004). Although mathematics is not uniquely to blame for this shameful record, it is the academic subject that students most often fail.
- One in three students who enter college must remediate major parts of high school mathematics as a prerequisite to taking such courses as College Algebra or Elementary Statistics (Greene & Winters, 2005).
- In one study of student writing, one in three students at a highly selective college failed to use any quantitative reasoning when writing about subjects in which quantitative evidence should have played a central role (Lutsky, 2006).
- College students in the natural and social sciences consistently have trouble expressing in precise English the meaning of data presented in tables or graphs (Schield, 2006).

One explanation for these discouraging results is that the trajectory of school mathematics moves from the concrete and functional (for example, measuring and counting) in lower grades to the abstract and apparently nonfunctional (for example, factoring and simplifying) in high school. As many observers have noted ruefully, high school mathematics is the ultimate exercise in deferred gratification. Its payoff comes years later, and then only for the minority who struggle through it.

In the past, schools offered this abstract and ultimately powerful mainstream mathematics curriculum to approximately half their students—those headed for college—and little if anything worthwhile to the other half. The conviction that has emerged in the last two decades that all students should be offered useful and powerful mathematics is long overdue. However, it is not yet clear whether the best option for all is the historic algebra-based mainstream that is animated primarily by the power of increasing abstraction.

Mastering Mathematics

Fractions and algebra may be among the most difficult parts of school mathematics, but they are not the only areas to cause students trouble. Experience shows that many students fail to master important mathematical topics. What's missing from traditional instruction is sufficient emphasis on three important ingredients: communication, connections, and contexts.

Communication

Colleges expect students to communicate effectively with people from different backgrounds and with different expertise and to synthesize skills from multiple areas. Employers seek the same things. They emphasize that formal knowledge is not, by itself, sufficient to deal with today's challenges. Instead of looking primarily for technical skills, today's business leaders talk more about teamwork and adaptability. Interviewers examine candidates' ability to synthesize information, make sound assumptions, capitalize on ambiguity, and explain their reasoning. They seek graduates who can interpret data as well as calculate with it and who can communicate effectively about quantitative topics (Taylor, 2007).

To meet these demands of college and work, K–12 students need extensive practice expressing verbally the quantitative meanings of both problems and solutions. They need to be able to write fluently in complete sentences and coherent paragraphs; to explain the meaning of data, tables, graphs, and formulas; and to express the relationships among these different representations. For example, science students could use data on global warming to write a letter to the editor about carbon taxes; civics students could use data from a recent election to write op-ed columns advocating for or against an alternative voting system; economics students could examine tables of data concerning the national debt and write letters to their representatives about limiting the debt being transferred to the next generation.

We used to believe that if mathematics teachers taught students how to calculate and English teachers taught students how to write, then students would naturally blend these skills to write clearly about quantitative ideas. Data and years of frustrating experience show just how naïve this belief is. If we want students to be able to communicate mathematically, we need to ensure that they both practice this skill in mathematics class and regularly use quantitative arguments in subjects where writing is taught and critiqued.

Connections

One reason that students think mathematics is useless is that the only people they see who use it are mathematics teachers. Unless teachers of all subjects—both academic and vocational—use mathematics regularly and significantly in their courses, students will treat mathematics teachers' exhortations about its usefulness as self-serving rhetoric.

To make mathematics count in the eyes of students, schools need to make mathematics pervasive, as writing now is. This can best be done by cross-disciplinary planning built on a commitment from teachers and administrators to make the goal of numeracy as important as literacy. Virtually every subject taught in school is amenable to some use of quantitative or logical arguments that tie evidence to conclusions. Measurement and calculation are part of all vocational subjects; tables, data, and graphs abound in the social and natural sciences; business requires financial mathematics; equations are common in economics and chemistry; logical inference is fundamental to history and civics. If each content-area teacher identifies just a few units where quantitative thinking can enhance understanding, students will get the message.

The example of many otherwise well-prepared college students refraining from using even simple quantitative reasoning to buttress their arguments shows that students in high school need much more practice using the mathematical resources introduced in the elementary and middle grades. Much of this practice should take place across the curriculum. Mathematics is too important to leave to mathematics teachers alone.

Contexts

One of the common criticisms of school mathematics is that it focuses too narrowly on procedures (algorithms) at the expense of understanding. This is a special problem in relation to fractions and algebra because both represent a level of abstraction that is significantly higher than simple integer arithmetic. Without reliable contexts to anchor meaning, many students see only a meaningless cloud of abstract symbols.

As the level of abstraction increases, algorithms proliferate and their links to meaning fade. Why do you invert and multiply? Why is $(a + b)^2 \neq a^2 + b^2$? The reasons are obvious if you understand what the symbols mean, but they are mysterious if you do not. Understandably, this apparent disjuncture of procedures from meaning leaves many students thoroughly confused. The recent increase in standardized testing has aggravated this problem because even those teachers who want to avoid this trap find that they cannot. So long as procedures predominate on high-stakes tests, procedures will preoccupy both teachers and students.

There is, however, an alternative to meaningless abstraction. Most applications of mathematical reasoning in daily life and typical jobs involve sophisticated thinking with elementary skills (for example, arithmetic, percentages, ratios), whereas the mainstream of mathematics in high school (algebra, geometry, trigonometry) introduces students to increasingly abstract concepts that are then illustrated with oversimplified template exercises (for example, trains meeting in the night). By enriching this diet of simple abstract problems with sophisticated realistic problems that require only simple skills, teachers can help students see that mathematics is really helpful for understanding things they care about (Steen, 2001). Global warming, college tuition, and gas prices are examples of data-rich topics that interest students but that can also challenge them with surprising complications. Such a focus can also help combat student boredom, a primary cause of dropping out of school (Bridgeland, DiIulio, & Morison, 2006).

Most important, the pedagogical activity of connecting meaning to numbers needs to take place in authentic contexts, such as in history, geography, economics, or biology—wherever things are counted, measured, inferred, or analyzed. Contexts in which mathematical reasoning is used are best introduced in natural situations across the curriculum. Otherwise, despite mathematics teachers' best efforts, students will see mathematics as something that is useful only in mathematics class. The best way to make mathematics count in the eyes of students is for them to see their teachers using it widely in many different contexts.

My "Aha!" Moment

Douglas Hofstadter, Distinguished Professor of Cognitive Science, Indiana University, Bloomington.

I first realized the deep lure of mathematics when, at about age 3, I thought up the "great idea" of generalizing the concept of 2×2 to what seemed to me to be the inconceivably fancier concept of $3 \times 3 \times 3$. My inspiration was that since 2×2 uses the concept of two-ness *twice*, I wanted to use the concept of three-ness *thrice*! It wasn't finding out the actual value of this expression (27, obviously) that thrilled me—it was the idea of the fluid conceptual structures that I could play with in my imagination that turned me on to math at that early age.

Another "aha" moment came a few years later, when I noticed that $3^2 \times 5^2$ is equal to $(3 \times 5)^2$. Once again I was playing around with structures, not trying to prove anything. (I didn't even know that proofs existed!) It thrilled me to discover this pattern, which of course I verified for other values and found mystically exciting.

I believe that teachers should encourage playfulness with mathematical concepts and should encourage the discoveries of patterns of whatever sort. Any time a child recognizes an unexpected pattern, it may evoke a sense of wonder.

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