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Assessing the accuracy of land surface characteristics estimated from multi-angular remotely sensed data

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Abstract. New missions and technically advanced sensors are being developed by researchers to monitor our planet from space at different spatial and spectral resolutions. Characterizing the terrestrial biomes on a global scale is a key issue in understanding climate change and the evolution of the Earth's atmosphere system. New advanced models on radiation in plant stands and more heterogeneous biomes are used to interpret satellite- and airborne sensor data and identify information on the Earth's surface. This paper presents an investigation on estimation of canopy and leaf level quantities by multi-directional remotely sensed data in three spectral bands. The purpose is to evaluate the goodness of a model in a controlled environment, using artificial input data. The results of the experiment indicate that the required information on leaf optical properties can be derived with a good accuracy within the constraints of the experiment. Estimated stand structure characteristics are more prone to error. Scaling issues, including temporal, spectral and spatial resolution, and surface heterogeneity are not addressed in this experiment.

1. Introduction

The directional reflectance of land surfaces has been a topic of scientific interest in the remote sensing branch for over two decades. The phenomenon was first studied by astronomers in connection with planetary science and backscattering from planetary surfaces. Related phenomena are also encountered in atmospheric sciences and climatology, concerning scattering of light by the Earth's atmosphere. The consequences of bidirectional reflectance distribution function (BRDF) in remote sensing are twofold: (i) brightness variations caused by anisotropic reflectance have to be accounted for in analysis and interpretation of images acquired by space- or airborne sensors; and (ii) BRDF can be exploited to identify information on the terrestrial ecosystem both on a local and a global scale. The latter aspect is becoming increasingly important along with the advances in modelling techniques.

The latest developments in radiation transfer modelling research, along with the new generation of space-borne platforms and sensors that are recently launched, or scheduled to be launched in the near future, are expected to be particularly beneficial for the understanding of climatic and ecological processes. Inventory of biomass and the world's forest resources is of interest in the context of global

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warming, since it has been proposed that forest biomass acts as a carbon sink in the global ecosystem, together with phytoplankton in the oceans (Myneni *et al.* 2001). Remote monitoring of forest cover change as well as biomass burning over large inaccesible areas is a target of current research interests, since previously cited results imply that a reduction of forest cover will lead to increasing concentrations of the greenhouse gas carbon dioxide in the atmosphere. The study of vegetation photosynthetic activity relies mainly on radiation transfer methods to estimate the fraction of absorbed photosynthetically active radiation (fAPAR) by multi-angular remotely sensed data. In large-scale studies of the coupled surface–atmosphere system, the quantitative land-cover information provided by the new advanced methods sets initial and boundary conditions for global climate models.

This paper is a study of the information content of spectral BRDF over vegetation, and accuracy assessment of canopy characteristics estimated using remotely sensed data. First, the information content of remotely sensed optical and near-infrared spectral reflectance and BRDF data is discussed. Section 3 is an overview of vegetation modelling using radiation transfer methods, focusing on validation and model intercomparison issues. An experimental approach to analyse the inversion of a radiative transfer model is outlined in §4, and some results of the experiment are presented.

2. Overview of image interpretation methods

2.1. *Interpretation using spectral information*

The most basic methods to interpret and identify information by multispectral images are based on analysing the spectral signature of individual pixels of the image through different types of band ratios and vegetation indices (Gobron *et al.* 2000). The spectral signature of vegetation captures the characteristic features of vegetation condition and biotic activity through spectral absorptive properties of plants. Vegetation dynamics, represented by long-term data series, acts as a quantitative indicator of vegetation response to climate activity, as well as human activities during the period of study. Interpretation of images and identification of information from images involves development of procedures and algorithms to decouple the different sources contributing to the incoming radiance at the sensor. Many different vegetation and non-vegetation factors are involved in forming the detected compound signal; the platform, measuring instrument, the atmosphere and the surface together form a coupled system consisting of the individual contributions of each component. Moreover, the sensitivity of a vegetation index is generally not confined to a single characteristic of the surface, and variations inferred by the joint actions of multiple different sources can easily be misinterpreted.

Some results, presented in the literature (Gobron *et al.* 2002), indicate that vegetation indices delivered by space-borne instruments are sensor specific, and fusion of data by the increasing number of different sensors and platforms delivering data is not always straightforward. The problem arises from different bandwidths, different central wavelengths, and spectral sensitivites of different sensors, as well as complex relations to atmospheric scattering and absorption, viewing geometry, and pixel size. Simple Normalized Difference Vegetation Index (NDVI) based methods used for estimation of Leaf Area Index (LAI) and fAPAR, which is commonly used as a substitute for chlorophyll abundance in vegetation, exploit the empirical correlation between NDVI and LAI/fAPAR. This type of site- and sensor-specific empirical methods is computationally inexpensive, but lacks the general applicability provided by radiation transfer models. Gobron *et al.* (2000) have studied sensor-specific Bidirectional reflectance factor (BRF) and NDVI of three space-borne sensors by simulations, in order to design optimization procedures for estimation of the same biogeophysical information by remotely sensed data delivered by different instruments in such a way that discrepancies between sensors are minimized. Their approach is to derive formulas for new sensor-specific vegetation indices which are insensitive to atmospheric effects, viewing geometry, bandwidth, and spectral response of the sensor, and which generate equivalent values of the requested quantity. The objective is to facilitate vegetation related studies on a global scale, including monitoring of plant productivity and photosynthetic activity, which are important indicators of the status of the terrestrial ecosystem. The disadvantage with these sensor-specific vegetation indices is the re-optimization procedure required for each new sensor.

Individual pixels of remotely sensed data are often treated as if they were acquired over a spatially and spectrally homogeneous surface. In §2.3 of this paper, some problems related to spatial resolution of sensors, such as increases in subpixel heterogeneity with decreasing spatial resolution, are discussed.

2.2. *BRDF as indicator of surface structure and heterogeneity*

In the early years of space science, the potential of the directional signature of surface reflectance as a source of information on state variables of the terrestrial surface was poorly known and exploited. The research on BRDF during the past two centuries, and development of new space platforms and sensors specifically to exploit directional anisotropy of surface reflectance have created opportunities to estimate more exact and detailed measurable characteristics related to vegetation structure and optical properties. Radiation transfer algorithms provide generic and flexible interpretation methods for identification of both leaf and canopy level quantities. Issues of bandwith and viewing geometry, as well as surface heterogeneity, can be addressed with the more advanced methods, and the requirements of *a priori* knowledge are reduced. The wavelength dependence of BRDF reflects spectral heterogeneity of a biome. The problem is best understood over incomplete covering or sparse shrubs and grasslands, and croplands where varying fractions of bare soil are visible within the sensor's field of view in different geometrical configurations. Qualitatively, a larger fraction of the ground surface is visible over a plant stand for a vertically downwards viewing sensor than for a tilted sensor. Gap fraction as seen by the sensor tends to decrease at slant viewing angles, which gives an impression of a more homogeneous surface. The rate of change of reflectance factor with viewing angle thus depends on wavelength. Spectral BRDF analysis of multiple samples within the same biome or land-use class is proposed as a tool for detecting and quantifying surface heterogeneity experimentally.

2.3. *On the role of scale*

The ongoing product development in space technology and increasing use of data fusion have created new demands for better understanding of the relation between land surface characteristics estimated from remotely sensed data, and scale. In optical remote sensing, surface heterogeneity arises both by textural effects, such as fractional coverage and spatial clumping of leaves and branches, as well as the presence of multiple types of surfaces. Scaling issues in land surface monitoring are related to spectral and spatial resolution of different sensors. The study of surface heterogeneity on subpixel scale means abandoning the assumption of each pixel representing a locally homogeneous surface. From the spectral point of view, the problem is characterized by sensor bandwidth and spectral heterogeneity of land surfaces. The spectral signal of each pixel is treated as a composite of multiple homogeneous end-members, arising from the presence of multiple cover types within the same pixel. Calibration of vegetation indices and information derived there from requires a strategy to extract subpixel scale information in terms of types and abundances of spectral end-members. A first approximation to calibration of spectrally heterogeneous pixels is linear combination of homogeneous spectra. This strategy to decontamination of vegetation indices relies on knowledge of the number and species of subpixel cover types, which is usually unavailable. Use of multidirectional data may provide a better constrained problem, with additional information on stand structure given by the angular signature available (Widlowski *et al.* 2001). Current operational or pre-operational algorithms to estimate vegetation characteristics from space-borne data rely on look-up tables (LUT) for solving the inverse problem. To account for surface heterogeneity, different LUT can be generated for various types of biomes, characterized by the level of heterogeneity. The incorporation of scaling algorithms directly in radiation transfer models is so far an almost unexploited subject.

3. Modelling vegetation

3.1. *General*

As stated earlier in this paper, extraction and quantification of higher level information from remote sensing data requires radiation transfer models and sufficient amounts of high-quality data. The model development activities in remote sensing have now reached a point where a large number of sophisticated models are available for data interpretation, and a demand for a coordinated evaluation action has arisen among model developers (Pinty *et al.* 2001). An evaluation action requires the existence of a standard, or a norm which defines the 'truth', in other words the correct result. In the absence of such a standard, the evaluation is best carried out by examining the performance of a chosen model in relation to all other participating models. An evaluation experiment is designed in such a way that it examines how the selected model quantifies the transfer of radiation in a modelled canopy in terms of absorption and scattering processes at the surface and within the canopy. This approach is supporting model development activities by identifying the underlying reasons to disagreements between the outputs of different models, as well as ensuring that no violations of fundamental laws, such as the law of energy conservation, are allowed to happen.

The second requisite of an evaluation exercise is assessment of a model's capability to reproduce a measured BRDF as accurately as possible given the structure and scattering properties of the target, in other words validation of the model using *in situ* measurements. This problem raises the question: how accurately and to what extent is it possible to quantify the structure and characteristics of a vegetation canopy? Obviously, a certain degree of deficiency is unavoidable. Both canopy structure, and leaf-scattering properties can, at best, only be quantified with distribution laws and averages. Scattering properties of individual leaves are best determined by measuring a limited number of samples, and the characteristics of the whole population of simulated scatterers can be assigned values based on the measured samples, in practice a constant value representing the statistical average of measurements. Leaf inclination angles/leaf normal distributions are often described with distribution laws, or random distributions, the most common approximations being beta and gamma distributions, or simple trigonometric functions. Naturally, the implementation of radiation transfer laws in the investigated model sets the level of requirements on validation. A simple representation of a homogeneous scene, which is only a crude approximation of a real plant stand, cannot be validated in the same sense as a three-dimensional heterogeneous scene attempting to describe in detail the structure of the surface and vegetation.

The third aspect of model evaluation is the inverse problem. A model is required to produce its best estimate of plant stand characteristics, given a top of canopy BRDF, corresponding to the smallest distance between calculated BRDF and the given BRDF. The inverse problem is closely linked to the validation task described in the previous paragraph. Validation against *in situ* measurements is carried out to confirm that the model input describing specific canopy attributes truly are sensitive to the corresponding measurable quantities. The absence of a 'true' solution, arising from the inherent inaccuracy and incompleteness of experiments, together with the limited capability of a computer model to exactly represent reality, set restrictions on what information realistically is possible to attempt to estimate, and what level of accuracy is feasible.

One of the problems to be addressed related to the inverse problem is the possible occurrence of multiple solutions. Such a situation may arise due to insufficient quality of input data, model faults, or the choice of inversion method. Synthetic data, generated with a radiation transfer model, provide an inverse problem with a known solution. The use of this approach is further exploited in §4.

3.2. *Model intercomparison activities*

The radiation transfer model intercomparison (RAMI) exercise (Pinty *et al.* 2001) is an initiative by the European network for the development of advanced models to interpret optical remote sensing data over terrestrial environments (ENAMORS). The project started in 1999 with phase 1, and was continued with phase 2ab, which is a repetition of phase 1, between February and June 2002. Phase 2 provides an opportunity for participants to update results due to model improvements after the first phase. The objective of the first phase is to carry out a model intercomparison exercise in direct mode, to assess to what extent the participating models, representing a range of different approaches, are able to generate the same bidirectional reflectance function given very specific instructions on the measuring conditions and target structure. A number of predefined homogeneous and heterogeneous scenes provide a framework for the initiative. Some more detailed sections include intercomparison of single- and multiply scattered contributions, as well as absorption. Some results of the intercomparison initiative are presented in the paper by Pinty *et al.* (2001). They conclude that outputs of eight participating models—DART (Discrete Anisotropic Radiative Transfer) (Gastellu-Etchegorry *et al.* 1996), FLIGHT (Forest Light Interaction Model) (North 1996), RAYTRAN (Govaerts and Verstraete 1998), RGM (Radiosity Graphics Combined Model) (Qin and Gerstl 2000), SPRINT (Thompson and Goel 1998), ProKuusk (Jacquemoud and Baret 1990, Kuusk 1995), ProSAIL (Scattering by arbitrarily inclined leaves) (Verhoef 1984, Jacquemoud and Baret 1990), and semi-discrete model (Gobron *et al.* 1997)—are in agreement in the sense that each of the curves belonging to the swarm of simulated reflectance functions over the same virtual canopy is close to the common average at each point, except some local deviations by one model. The results of the initiative confirm that the generation of existing models are in agreement despite the different implementations. A higher level of agreement is achieved for heterogeneous scenes, modelled by the four three-dimensional models RAYTRAN, SPRINT, FLIGHT and DART. The first phase of the exercise is intended to provide a standard against which to test future developments of models.

4. Assessing the accuracy of estimation

4.1. *Description of the experiment*

This experiment has been designed to explore the estimation of quantitative information on terrestrial vegetation using multidirectional remote sensing data in narrow spectral bands. It is generally known that the information content of monodirectional data in the context of quantitative investigation of terrestrial surfaces is limited. Monodirectional data are best suited to identify different types of terrestrial biomes and estimate vegetation characteristics using simple empirical relationships. The requirements on data analysis methods are dictated by the information level of output quantities, as well as the allowed computational cost.

The input data of this experiment are generated using a radiative transfer model with soil background. The model assumes a horizontal leaf canopy which is illuminated spatially uniformly over the top of the surface. The steady-state radiance distribution function at a given wavelength is determined by the radiative transfer equation, and the transport equation of this model is solved with a modified discrete-ordinates method, as outlined by Shultis and Myneni (1988). Leaf hemispherical transmittance and leaf hemispherical reflectance quantify the bi-Lambertian leaf-scattering model. Stand structure is described by LAI and horizontal and vertical leaf angle distribution. A soil background characterized by single scattering albedo of soil particles is incorporated in the program to better account for sparse canopies. The solar zenith angle is set to 40° and fraction of direct solar radiation 0.9.

Experimental data are always contaminated by systematic or random measuring errors or background noise caused by technical or environmental factors. The synthetic input data have been modified by adding random noise drawn from a probability distribution to create a meaningful experiment. A Lorentz function is suitable for an experiment which involves multidirectional measurements. The function is defined as

$$p(y)dy = \frac{1}{\pi} dy \Big/ \left(1 + y^2\right) \tag{1}$$

and its inverse indefinite integral is the tangent function. Random deviates drawn from the Lorentz distribution can easily be generated with the transformation method as outlined by Press *et al.* (1992). The function is scaled to describe an error of approximately δ in such a way that the generated random deviates are in the range ($-\delta$, $+\delta$), with outliers outside the range (figure 1). The corresponding Gaussian distribution with $3\sigma = \delta$ is also plotted. In this experiment the upper limit of measuring error δ for each measuring geometry is set to 20% of the generated BRF value.

A repeated experiment with approximately 1000 samples each in red, green and near-infrared spectral ranges is carried out to investigate the properties of a radiative transfer model and assess the accuracy of estimation. The nature of required output information and computational cost dictates the choice of spectral

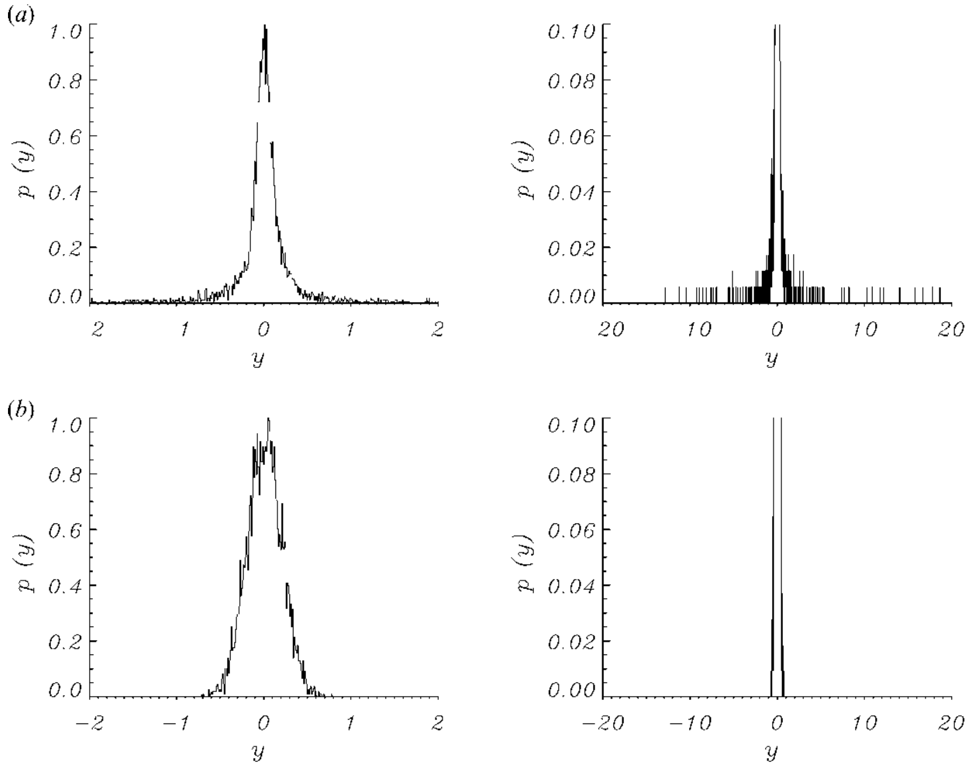

Figure 1. Random deviates drawn from (*a*) Lorentz distribution and (*b*) normal distribution.

bands for inversion. The bulk of information on canopy and leaf optical properties lies in the visible and near-infrared range. Major absorption bands by pigments (chlorophyll and carotenoids) are centred in the blue (450 nm) and red (670 nm), with low absorption in the green (550 nm) and near the red edge (transition between red and near-infrared). Leaf water absorption bands are mainly in the infrared range (1450 nm and 1950 nm). Weakly reflecting high absorption bands which are computationally inexpensive compared with the strongly reflecting near-infrared plateau provide quantitative information on leaf pigment concentrations.

4.2. *Results*

In this paper, leaf reflectance and leaf transmittance are jointly represented by leaf absorptance. Normalized distributions of estimated LAI, leaf absorptance and soil single scattering albedo are plotted in figures 2–4. The abscissa y is defined as output – true value, in other words the difference between estimated quantity and its true value, and $p(y)$ is a frequency normalized to the central maximum.

The positions of centres of distribution are of particular interest, in view of the uniqueness of the solution. Interdependences between two or more variables can be identified through displacements of the central maxima, as well as the existence of secondary maxima (bimodal distribution). A certain complementarity between vegetation and background is possible, in that the model to some extent can compensate variations in absorptance level of the canopy by a corresponding adjustment of soil reflectance.

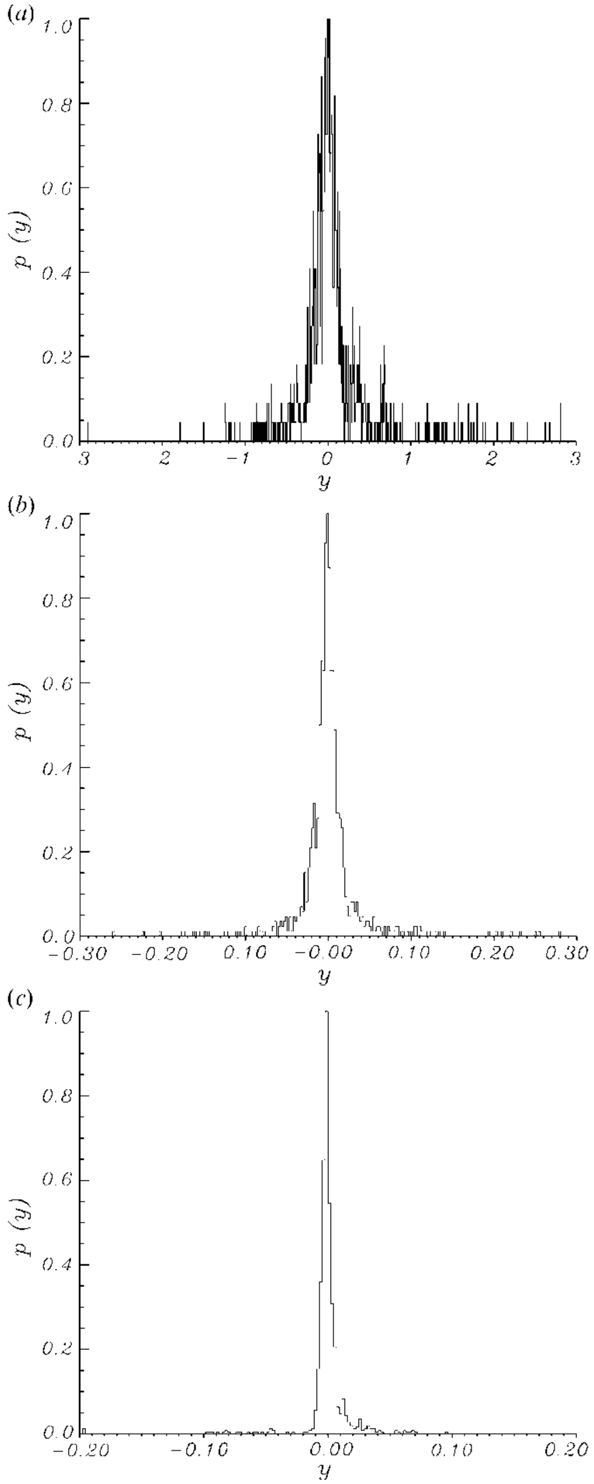

Figure 2. Estimated vegetation and soil characteristics, $\lambda = 550\,\mathrm{nm}$. The abscissa y is defined as difference between estimation and true value; $p(y)$ is a normalized frequency. (*a*) LAI; (*b*) leaf absorptance and (*c*) soil albedo.

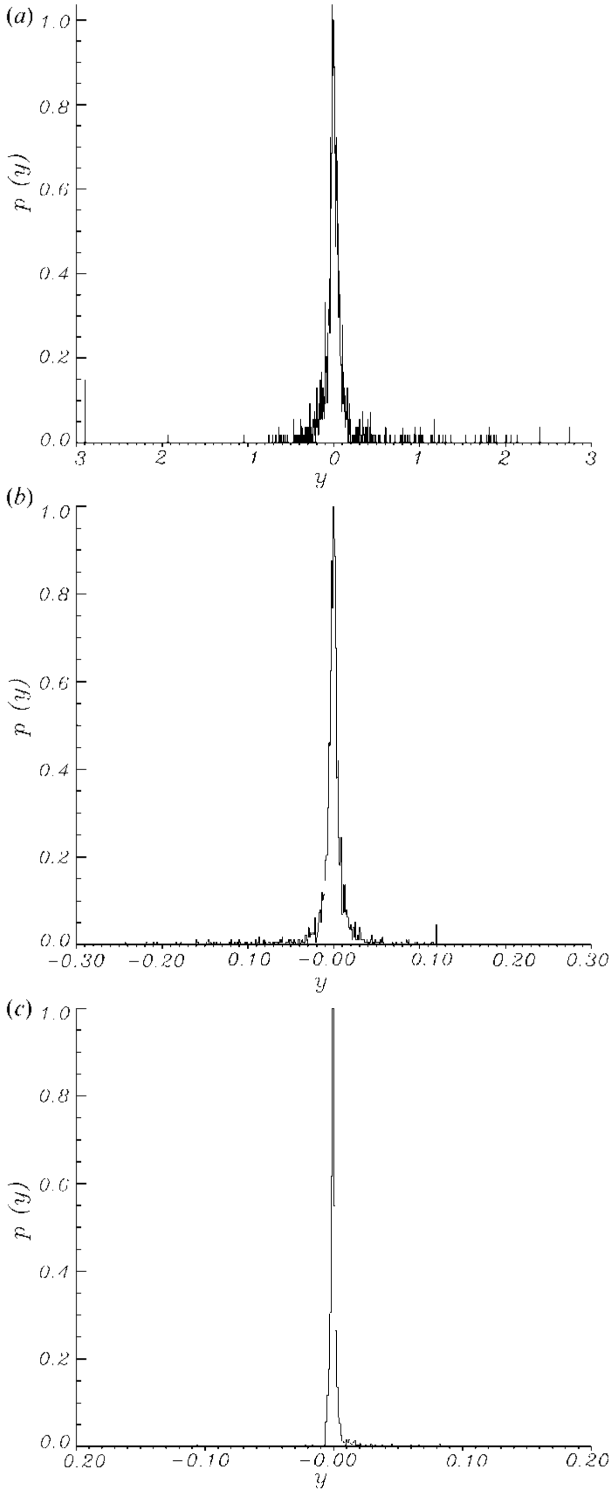

Figure 3. Estimated vegetation and soil characteristics, $\lambda = 670\,\mathrm{nm}$. (*a*) LAI; (*b*) leaf absorptance and (*c*) soil albedo.

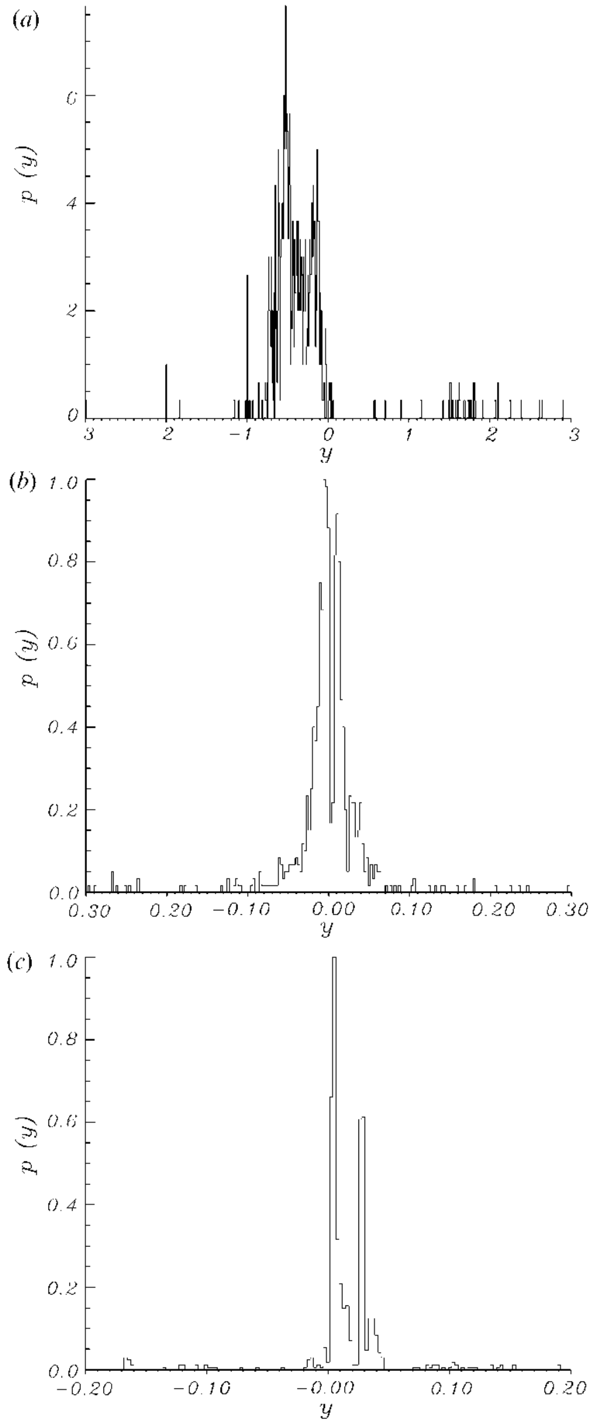

Figure 4. Estimated vegetation and soil characteristics, $\lambda = 850$ nm. (*a*) LAI; (*b*) leaf absorptance and (*c*) soil albedo.

Leaf absorptance distributions are centred at their true values in each figure. The least accurate result, corresponding to largest width of distribution, is estimated in the strongly reflecting near-infrared. The near-infrared wavelength required the largest computing time of the three experiments. True values are, respectively, 0.70 (550 nm), 0.86 (670 nm) and 0.20 (850 nm).

Distributions of soil particle single scattering albedo and LAI, shown in figure 4, are shifted from the centre of the coordinate axis. Soil single scattering albedo is systematically overestimated, and LAI is underestimated. The shift indicates a failure of the program to accurately reproduce the initial values of the investigated quantities. A reduced LAI is compensated by higher background reflectance to produce an equally probable solution. The bimodal character indicates two preferred values; one very near the centre, and one shifted by 0.5 units (LAI) or 0.03 units (soil single scattering albedo). The frequency of outliers is low in each experiment, and the number of saturated results ($\mathrm{LAI} > 6$), which are left out from the figure, is small.

Evidently, the best constraints for an inverse problem are provided by a combination of near-infrared and visible wavelengths with multiple viewing angles, to account for leaf optical properties as well as stand structure. Highly absorptive wavelengths provide information on leaf absorption properties and vegetation condition. Limitations on modelling of remotely sensed data are posed by the model used for estimation of vegetation characteristics—its capability to provide the best estimate of the required quantities corresponding to the smallest distance between simulated and measured bidirectional reflectances. Naturally, the interpretation of estimated values as physically measurable quantities relies on validation of model simulations, in other words assessing the model's ability to accurately simulate well-documented *in situ* measurements of targets spanning the whole range of biomes allowed by the model's capacity.

Some main conclusions of this study are: (1) the model is able to correctly estimate leaf absorptance using any of the three investigated wavelengths in the visible and near-infrared spectral range, which accounts for the major information on leaf optical properties; (2) estimation of canopy structural characteristics is more prone to error. The model used in this study failed to estimate correctly LAI and soil particle single scattering albedo. LAI is slightly underestimated, and soil single scattering albedo is overestimated. The shifts are small, compared to the true values of both quantities; and (3) the quality of estimation is relatively good, despite a comparatively high noise level in the input data.

5. Summary

The objective of this experiment was to evaluate the estimation of vegetation stand characteristics from BRF data in three different visible and near-infrared spectral bands using a radiative transfer model. The main focus is on testing the model itself; its capacity to provide a unique solution to this inverse problem, and the restrictions posed by required accuracy of input data. An idealized experiment has been designed with synthetic input data including a controlled amount of random noise. The choice of artificial data ensures that the true value of each involved quantity is known, and all different aspects of the experiment can be controlled. The distribution of noise has been chosen to describe the nature of multi-angular measurements as closely as possible. The experiment is carried out using three different wavelengths in the visible and near-infrared spectral range, which represents the bulk of information on optical properties of vegetation and stand structure.

Despite a high maximum noise level of 20%, the quality of estimated plant stand and leaf characteristics is fairly good. In inverse problems related to vegetation modelling, an ambiguity between background and vegetation may arise in certain circumstances. A radiation transfer model can compensate variations in the amount of radiation absorbed by the canopy with a corresponding adjustment of soil brightness. In other words, model input variables are not truly independent, and multiple equally probable solutions to the inverse problem are estimated. The set of equally probable solutions provides an approximation of the true values of estimated quantities. In this study, the model failed to estimate LAI and soil particle single scattering albedo correctly. Both quantities are shifted from the true values, but the shifts are small relative to initial values. Leaf absorptance was correctly estimated in each experiment. The use of radiation transfer models to estimate physically measurable quantities from remotely sensed data relies on the capacity of the model to accurately reproduce the characteristic spectral bidirectional reflectance signature of a chosen surface as a function of a relatively small number of characteristics describing the stand structure and leaf optical properties. Accuracy assessment against experimental top-of-canopy data in each individual biome ensures that the model can correctly describe the interaction of the radiation field with the Earth's surface. The validation of existing radiation transfer models is still incomplete, and remains as a subject for future research.

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