The fireworks model for GRB

Guido Barbiellini (University and INFN, Trieste)

Annalisa Celotti (SISSA, Trieste)

Francesco Longo (University and INFN, Trieste)

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ABSTRACT

- The energetics of the long duration GRB phenomenum is compared with models of a rotating Black Hole (BH) in a strong magnetic field generated by an accreting torus. A rough estimate of the energy extracted from a rotating BH with the Blandford-Znajek mechanism is evaluated with a very simple assumption: an inelastic collision between the rotating BH and the torus. The GRB energy emission is attributed to an high magnetic field that breaks down the vacuum around the BH and gives origin to a $e^{\pm}$ fireball. Its following evolution is hypothized, in analogy with the in-flight decay of an elementary particle, to evolve in two distinct phases. The first one occurs close to the engine and is responsible of energizing and collimating the shells. The second one consists of a radiation dominated expansion, which correspondingly accelerates the relativistic photon-particle fluid and ends at the transparency time. This mechanism predicts that the observed Lorentz factor is determined by the product of the Lorentz factor of the shell close to the engine and the Lorentz factor derived by the expansion. The angular distribution of the emitted shells is thus determined by the bulk Lorentz factor at the end of the collimation phase.

Introduction

- The connection of GRB with massive stars and BHs is well established (e.g. Vietri et al. 2001)
- There are indications for jetted emission from GRBs (Frail et al. 2001)
- Jet energy vs angle relationship (Rossi et. al. 2002)
- The opacity problem for GRB is solved with relativistic expansion (e.g. Piran 1999)
- X-ray flashes observations

Available Energy

- Blandford-Znajek mechanism for GRB
- $E_{BZ} = 0.3 \cdot 10^{54} \left(\frac{M_{bh}}{M_{\odot}} \right)$

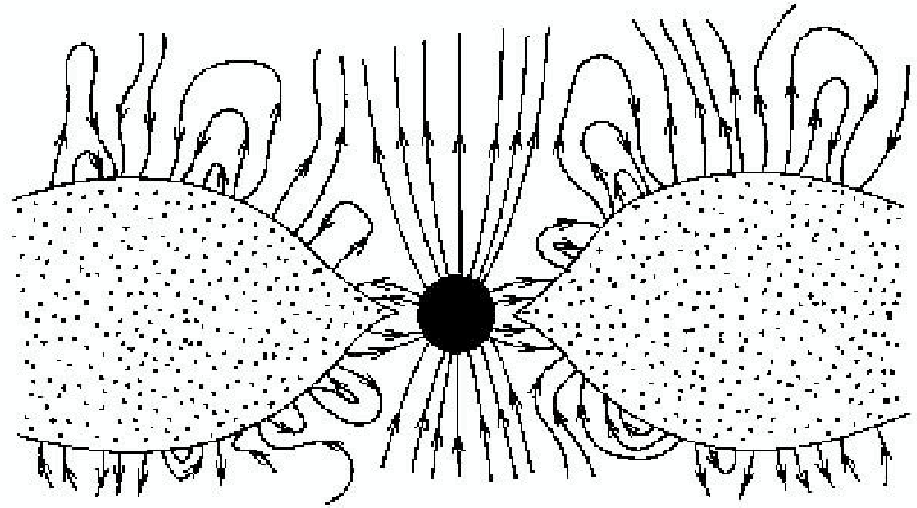


Figure from McDonald, Price and Thorne (1986)

Blandford & Znajek (1977)
Brown et al. (2000)
Barbiellini & Longo (2001)

Available Energy

- Inelastic collision between a rotating BH (10 $M_\odot$)and a massive torus (0.1 $M_\odot$) that falls down onto the BH from the last stable orbit
- Conservation of angular momentum: $\boxed{I_{bh}\Omega_{bh} + I_t\Omega_t = I\Omega}$
- Available rotational energy:

$$\Delta E_{rot} \approx \frac{1}{2} I_{bh}\Omega_{bh}^2\left(1 - \frac{I_{bh}}{I}\right) = 2M_{bh}^3\Omega_{bh}^2\left(1 - \frac{M_{bh}^3}{M^3}\right)$$

$$\Delta E_{rot} \approx 2M_{bh}^3\Omega_{bh}^2\left(3\frac{M_t}{M_{bh}}\right) \approx 3E_{rot,bh}\frac{M_t}{M_{bh}} \approx \frac{3}{8}M_t c^2$$

- Available gravitational energy: $\boxed{\Delta E_{grav} = \frac{GM_tM_{bh}}{R_{bh}} - \frac{GM_tM_{bh}}{3R_{bh}} \approx \frac{1}{3}M_t c^2}$
- Total available energy: $\boxed{\Delta E = \Delta E_{rot} + \Delta E_{grav} \approx 10^{53}\ \text{erg}}$

Vacuum Breakdown

Polar cap BH vacuum breakdown

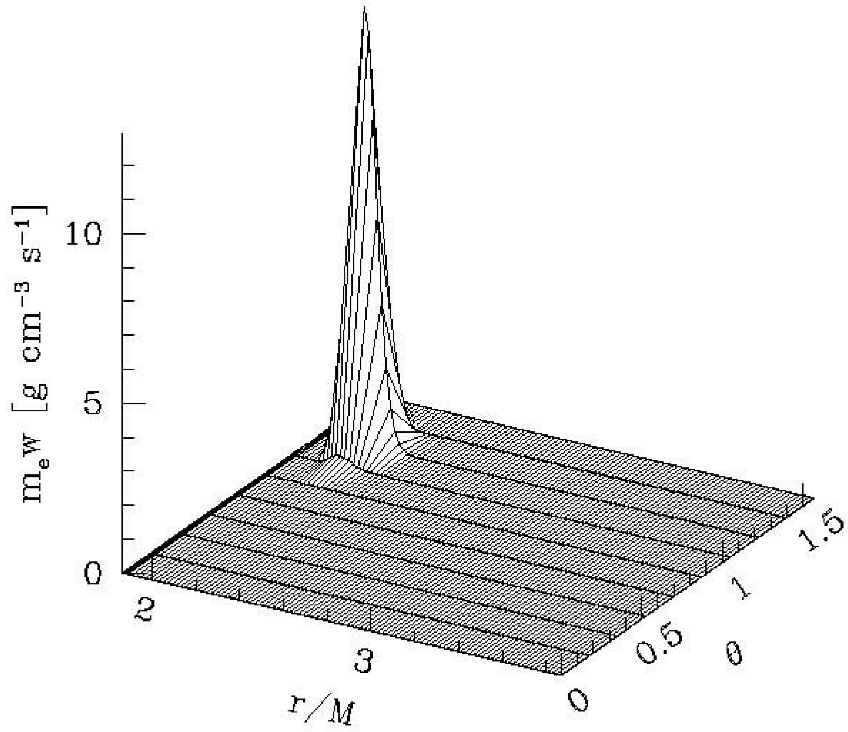


Figure from Heyl 2001

Vacuum Breakdown

- Critical magnetic field: $\boxed{B_c = 4.5 \cdot 10^{13} \text{ Gauss}}$
- Wald mechanism: $\boxed{Q = 2BJ \,\square\, 2 \cdot 10^{16} \text{ C}}$
- Electric field: $\boxed{E \,\square\, 2 \cdot 10^{15} \text{ V/cm}}$
- Pair volume: $\boxed{V_c \,\square\, R_{bh}^3}$

The formation of the fireball

- Pair density (e.g. Fermi 1966): $$n_{e^{\pm}} = 8 \cdot 10^{29}\ \text{cm}^{-3}$$
- Magnetic field density: $$U_B = 8 \cdot 10^{25}\ \text{erg cm}^{-3}$$
- Energy per particle: $$\square_0 \, \square \, \square_{acc} 10^{\square 4}\ \text{erg}$$
- Energy in plasmoid: $$E_{plasmoid} = V_c U_B \, \square \, 10^{45}\ \text{erg}$$
- Number of plasmoids: $$N_{plasmoid} = \square_B \frac{\square E}{E_{plasmoid}} \, \square \, \square_B 10^{8}$$

The formation of the fireball

- Acceleration time scale in E field: $t_{acc} \approx \frac{10^2 \gamma_{acc} m_e c^2}{eEc} \approx 10^{-19} \gamma_{acc}\,\mathrm{s}$
- Particle collimation by B field: $t_{coll} \approx \frac{\rho}{c \sin\theta} \approx \frac{\gamma_{acc}}{\sin\theta} 10^{-19}\,\mathrm{s}$

Curvature radius: $\rho = 10^6 \frac{E(\mathrm{GeV})}{B(\mathrm{Gauss})}\,\mathrm{cm}$

- Randomisation time scale by Compton Scattering in radiation field with temperature T_0: $t_{rand} \approx 10^{-16} \gamma_{acc}\,\mathrm{s}$

$$T_0 = \left(\frac{B^2}{8\pi} \cdot \frac{1}{a} \right)^{1/4} \approx 10^{10}\,\mathrm{K}$$

Two phase expansion

- Phase 1 (acceleration and collimation) ends when:

$$\boxed{t_{rand} = t_{coll}}$$

- Assuming a dependence of the B field: $\boxed{B \propto R^{-3}}$

  this happens at $\boxed{R_1 \approx 10^8\,\mathrm{cm}}$

- Parallel stream with $\boxed{\Gamma_1 = 30\,\sigma_{acc}}$

- Internal "temperature" $\boxed{\gamma_1' \approx 1}$

Two phase expansion

- Phase 2 (adiabatic expansion) ends at the smaller of the 2 radii:
  - Fireball matter dominated: $$R_{\eta} = R_0 \frac{E}{Mc^2}$$
  - Fireball optically thin to pairs: $$R_{pair} = R_0 \left(\frac{3E}{4\pi R_0^3 T_p^4} \right)^{1/4}$$
- R_2 estimation $$R_2 = 50 R_0$$
- Fireball adiabatic expansion $$\frac{\Gamma_2'}{\Gamma_1'} = \frac{R_2}{R_0}$$

Jet Angle estimation

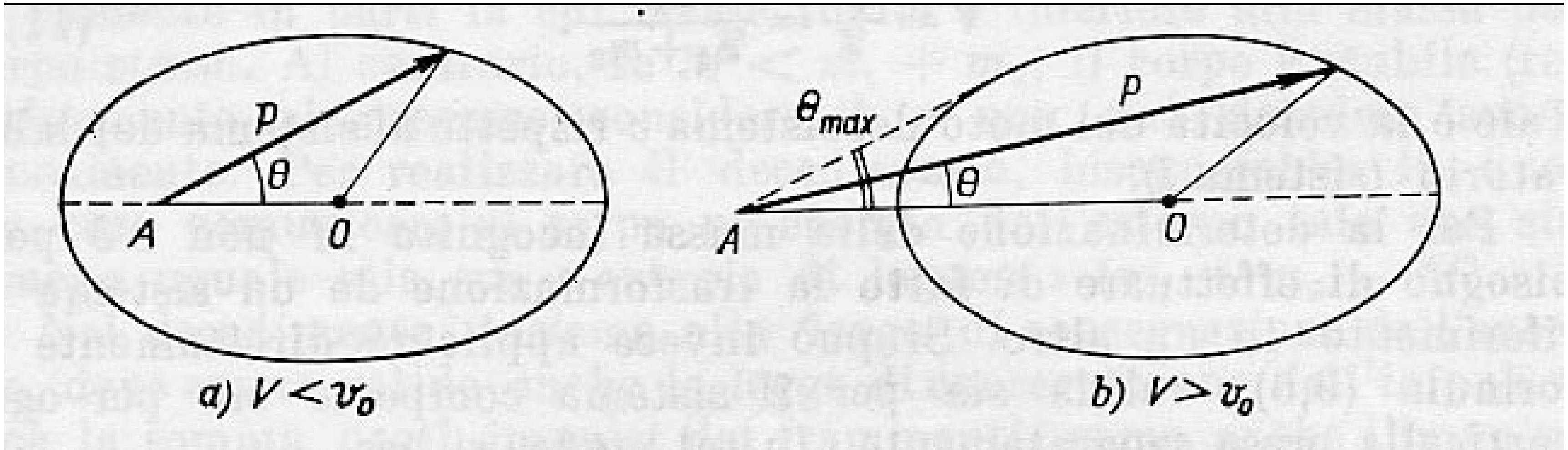


Figure from Landau-Lif_its (1976)

- Analogy with the in-flight particle decay

Jet Angle estimation

- Lorentz factors

$$\Gamma_{||} = 2\Gamma_1\Gamma_2^{'} \qquad \Gamma_{\perp} = \Gamma_2^{'}$$

- Opening angle

$$\theta_c \sim \tan\theta_c = \frac{\Gamma_{\perp}}{\Gamma_1\Gamma_2^{'}} = \frac{\Gamma_2^{'}}{\Gamma_1\Gamma_2^{'}} = \frac{1}{\Gamma_1}$$

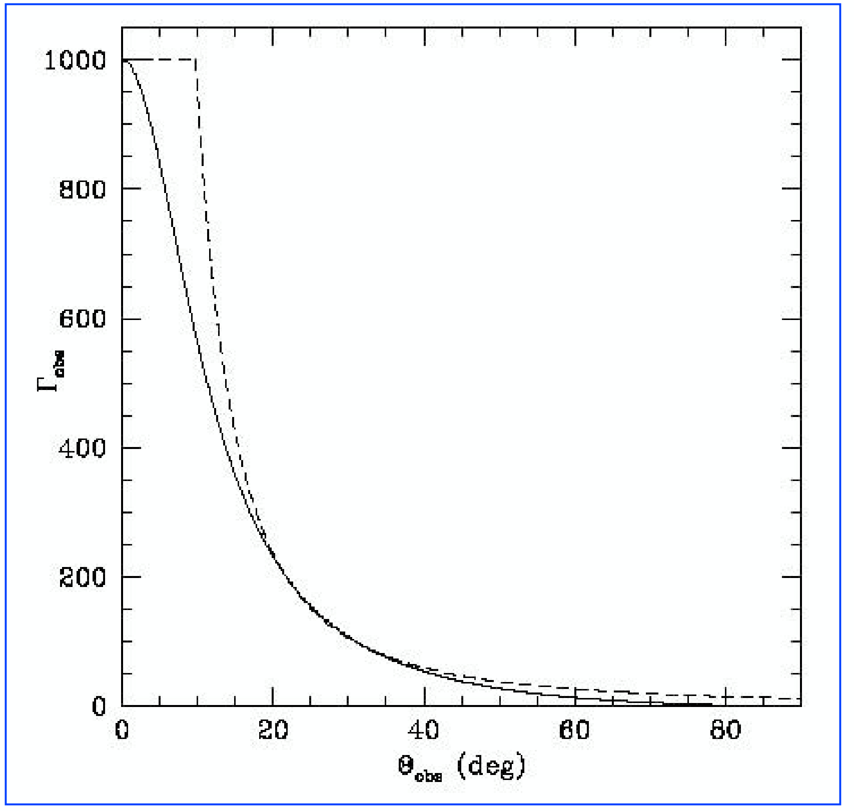
- Result:

$$\theta_c \sim \frac{1}{\Gamma_1} \sim 2 \times 10^{-1}$$

Energy Angle relationship

$$E(\theta') \propto \Gamma_1 \Gamma_2'(1 + \beta_2' \cos\theta')$$

$$\tan(\theta) = \frac{\sin\theta'}{\Gamma_1(\beta_1 + \cos\theta')}$$



Predicted Energy-Angle relation