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TRANSPORT THEORY FOR A LEAF CANOPY OF FINITE-DIMENSIONAL SCATTERING CENTERS

RANGA B. MYNENI,†‡ ALEXANDER L. MARSHAK,§ and YURI V. KNYAZIKHIN¶
‡Hydrological Sciences Branch, Mail Code 974, NASA-Goddard Space Flight Center, Greenbelt, MD 20771, U.S.A., §Institut für Bioklimatalogie, Universität Göttingen, Büsgenweg 1, 3400 Göttingen, Fed. Rep. Germany and ¶Department of Geophysics, Tartu University, Tartu 202400, Estonian SSR, U.S.S.R.

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Abstract—A formalism for photon transport in leaf canopies with finite-dimensional scattering centers that cross shade mutually is developed. Starting from first principles, expressions for the interaction cross sections are derived. The problem of illumination by a monodirectional source is studied in detail using a successive collisions approach. A balance equation is formulated in R^3 and the interaction between a leaf canopy and the adjacent atmosphere is discussed. Although the details are those relating to a leaf canopy, the formalism is equally applicable to other media where the constituents cross shade mutually such as planetary surfaces, rings and ridged-ice in polar regions, i.e., media that exhibit opposition brightening.

1. INTRODUCTION

A fascinating problem in transport theory is that of describing the transfer of photons in a medium of finite-dimensional scattering centers, and one in which the scatterers cross shade mutually. Examples of such media are all rough surfaces and layered media, including vegetation canopies, bare soil surfaces, ridged-ice in polar regions, decks of broken clouds, and planetary surfaces and rings. When these surfaces are illuminated by monodirectional radiation of wavelength much smaller than the size of the constituents, a peak in the reflected radiance distribution along the retro-illumination direction will be noticed, because of the absence of shadows. This phenomenon is known as the opposition effect in astrophysics, Heiligenschein in meterology, and the hot-spot effect in aerial photography and optical remote sensing. In this paper, we propose a new formalism for photon transport in leaf canopies that explains the hot-spot effect; with necessary modifications, the theory can be applied to any layered medium with finite-dimensional scattering centers that cross shade mutually.

The problem of photon transport in plant stands arises in the context of optical remote sensing of vegetated land surfaces, land surface climatology and plant physiology. The classical approach has been to ignore all plant organs other than leaves and treat this leaf canopy as a gas with nondimensional planar scattering centers, i.e., a turbid medium.[1] Such an analogy permits the use of standard transport theory (radiative transfer),[2,3] with minor modifications to account for the angular orientation of the plane-scatterers.[4] In particular, the scattering transfer (phase) functions are not rotationally invariant, thereby precluding the use of polynomial expansion methods for handling the scattering integral.[4,5] Consequently, the parameter mean free path depends on the direction of photon travel. A modified discrete ordinates method that incorporates the exact kernels can be used to solve the resulting transport equation, and this has been done for both one- and three-dimensional problems.[5-7]

If the size of the scatterers is considered in a formulation of the transport problem, it is necessary to include not only the number density of scatterers but also the consequences of introducing finite size gaps (holes or voids) in the medium. Standard transport description based on cross sections derived from elementary volumes is not applicable because of the presence of voids. Moreover, if the far-field assumption is violated, as is the case in a leaf canopy, then the scatterers will cast shadows; hence, information on the spatial distribution of scatterers is required to evaluate cross

†To whom all correspondence should be addressed.

shadowing. A need for such a theory has a strong basis in experimental observations of exiting radiations measured in opposition to a monodirectional source.[8-10] Standard transport theory assumes an infinite number of uncorrelated nondimensional scatterers, thereby affording a continuum description of the configuration space. It is clear that point scatterers cannot cast shadows and thus, the standard theory fails to predict or duplicate experimental observations of exiting radiations about the opposition direction.

The plan of this paper is as follows. In Secs. 2–6, expressions for the interaction cross sections are formulated starting from first principles. The leaf canopy transport problem is discussed in Sec. 7 and generalized in Sec. 9. In Sec. 8, the leaf canopy problem with a reflecting ground surface is considered. The nature of interactions between a leaf canopy and the adjoining atmosphere is discussed in Sec. 10. A balance equation is derived in Sec. 11 to gain insight and to motivate the physical arguments. Finally, in Sec. 12, a detailed synopsis highlighting the important results and relations is given.

2. ABSTRACTION OF THE LEAF CANOPY

Consider a natural community of vegetation. A proper description of a plant in this community should detail its architecture, i.e., shape, size, orientation, number density, etc. of the various components (leaves, twigs, branches and trunk). At the next level, the description should include spatial distribution of plants in the community. It appears that the most elegant way of describing such natural formations is not through the use of statistical measures, as required in transport theory, but by the use of fractals[7] or L-systems.[11] Nevertheless, for purposes of photon transport, the problem is greatly simplified if we consider, as a first approximation, only leaves and ignore other plant organs. The question now is how to best describe a leaf canopy.

Leaves of a plant are generally of finite size, the characteristic linear dimension of which is significantly greater than the wavelength of the interacting beam. They also have a shape that is species-specific. The nature of photon–leaf interactions under study permit us to ignore the thickness of a leaf. Therefore, leaves may be idealized as planar, finite-dimensional scatterers. The angular orientation of a planar element as described by the normal to its surface (say, upper). Thus, it is not the volume occupied by a leaf that is of interest but rather its area and orientation.

The position of leaves in vegetation canopies tends to be spatially correlated because leaves arise on stems, branches, twigs, etc. The position that a particular leaf can take could be excluded by the presence of another leaf in the vicinity because leaves cannot grow through one another, i.e., steric hindrance. Nevertheless, since leaves arise on branches, they are generally clumped in the space around the branch. Consequently, they cast shadows on one another, i.e., the assumption that every scattering center is in the far-field of the radiation scattered from any other scattering center is violated [at wavelengths ($< 4\,\mu$m) of interest]. On the other hand, the branches themselves are quite apart spatially. Thus, the distance along any direction between two adjacent leaf centers is highly variable in a natural plant stand. If we ignore photon interactions with the optically active elements of the atmosphere inside the canopy, the gaps between leaves can be treated as voids. The voids are convolutely shaped and multiply connected three-dimensional structures broken along those regions where leaves are present.

A vegetation canopy can thus be idealized as aggregations or clumps of leaves distributed randomly in free space (vacuum). The intervening free spaces between the clumps constitute the voids. Consequently, a leaf canopy can be abstracted as a binary medium—randomly distributed leaf clumps filled densely with phytoelements and voids.[12] So, the idea of a continuum host material that is central to transport theory is not provided for in this model of a leaf canopy. A question then arises as to the consequences for photon movement in such media where the scattering centers are finite-dimensional oriented plates, spatially distributed in clumps, with large intervening free spaces. We consider an elementary volume $V(x)$, where x is the phase space coordinate of a photon. If V belongs to an aggregation of leaves, the fate of a photon is determined by the standard interaction models,[4-6] or some refinement there of (Sec. 3). On the other hand, if V belongs to the free space, the current disposition of a photon is not altered. Hence, the photon mean free path (the distance between two successive interactions) is the sum of two random values: the length of photon travel in the free space and the length of the photon free path through the leaf aggregation.

As we shall see, it is the former that imbues the cross sections with a correlation property that is equivalent to ascribing memory to photons traversing the binary medium.

3. THE TOTAL INTERACTION CROSS SECTION FOR A LEAF-AGGREGATION

In this section, an expression for the total interaction cross section for an aggregation of leaves is developed. An important assumption here is that the number of leaves in the aggregate is sufficiently great such that a continuum description is permitted and statistical measures relevant to our discussion can be realized to a desired degree of accuracy.

The three basic interactions between photons and matter are absorption, scattering and emission.[3] In the following we shall ignore emission. Let $\tilde{\sigma}$ denote the total interaction cross section, i.e., the sum of the absorption and scattering cross sections. In transport theory, $\tilde{\sigma}$ is defined such that the probability of a photon being captured in traveling a distance ds is given by $\tilde{\sigma}$ ds.[3] Hence, the dimension of $\tilde{\sigma}$ is m^{-1}.

Let $[(2\pi)^{-1} h_L(\mathbf{r}, a_L, \underline{\Omega}_L)]$ be the probability density that a leaf of area $a_L(\mathrm{m}^2)$ has a normal $\underline{\Omega}_L \sim (\theta_L, \phi_L)$, (polar coordinates of the unit vector $\underline{\Omega}_L$), directed away from its upper surface into a unit solid angle about $\underline{\Omega}_L$. Thus,

$$\frac{1}{2\pi}\int_0^\infty \mathrm{d}a_L \int_{2\pi^+} \mathrm{d}\underline{\Omega}_L h_L(\mathbf{r}, a_L, \underline{\Omega}_L) = 1.$$

A key assumption is that all leaf normals are contained in the upper hemisphere ($2\pi^+$). This assumption is always valid since the upper surface of a leaf can be defined as that surface the normal to which is contained in the upper hemisphere. If we assume that the random variables a_L and $\underline{\Omega}_L$ are independently distributed, then

$$\frac{1}{2\pi} h_L(\mathbf{r}, a_L, \underline{\Omega}_L) \equiv \rho_L(\mathbf{r}, a_L)\frac{1}{2\pi} g_L(\mathbf{r}, \underline{\Omega}_L).$$

Models for the probability density of leaf normal orientation g_L are available in literature.[4] If the probability density of the leaf size distribution ρ_L is independent of $\mathbf{r}$, the mean leaf area $\bar{a}_L$ is given by

$$\bar{a}_L = \int_0^\infty \mathrm{d}a_L \rho_L(a_L) a_L$$

and can be used to parameterize the characteristic leaf dimension ℓ_L if some reasonable assumption is made regarding the shape of the leaves, viz. $\ell_L = \sqrt{\bar{a}_L}$ or $\ell_L = 2\sqrt{(\bar{a}_L/\pi)}$, where ℓ_L is the diameter of $\bar{a}_L$. Other details regarding the parameterization of ℓ_L can be found in Nilson and Kuusk.[13]

Let $n_L(\mathbf{r}, a_L)$ be an empirical function (m^{-3}) that relates the number of leaves in the volume element d$\mathbf{r}$ to leaf size a_L. The number of leaves in d$\mathbf{r}$ with sizes in da_L and orientations in d$\underline{\Omega}_L$ is

$$n \equiv n(\mathbf{r}, a_L, \underline{\Omega}_L) = n_L(\mathbf{r}, a_L)\rho_L(\mathbf{r}, a_L)\frac{1}{2\pi} g_L(\mathbf{r}, \underline{\Omega}_L)\, \mathrm{d}a_L\, \mathrm{d}\underline{\Omega}_L\, \mathrm{d}\mathbf{r}.$$

The total number of leaves in d$\mathbf{r}$ is

$$N \equiv N(\mathbf{r}) = \int_0^\infty \mathrm{d}a_L \int_{2\pi^+} \mathrm{d}\underline{\Omega}_L n(\mathbf{r}, a_L, \underline{\Omega}_L)\, \mathrm{d}\mathbf{r}.$$

The position of leaves in plant canopies tends to be spatially correlated because leaves arise on stems, branches, twigs, etc. The position of a leaf is also restricted by the presence of another leaf in the vicinity due to steric hindrance. To account for this fact, the N-leaf distribution function can be introduced.[14,15] However, this problem is far from trivial and we shall leave it for a later analysis.

Let $m(a_L, \underline{\Omega}_L;\ \mathbf{r}, \underline{\Omega})\, \mathrm{d}a_L\, \mathrm{d}\underline{\Omega}_L\, \mathrm{d}\mathbf{r}$ be the area projected by leaves in d$\mathbf{r}$ with size in da_L and orientation in d$\underline{\Omega}_L$ on a plane perpendicular to $\underline{\Omega}$, where

$$m(a_L, \underline{\Omega}_L; \mathbf{r}, \underline{\Omega}) = n(\mathbf{r}, a_L, \underline{\Omega}_L) a_L |\underline{\Omega} \cdot \underline{\Omega}_L| \chi(\mathbf{r}, a_L, \underline{\Omega}_L, \underline{\Omega}). \tag{1}$$

The function χ (dimensionless) is the fraction of nonoverlapped to total projected area of $n(\mathbf{r}, a_L, \Omega_L)$ leaves with respect to Ω. Integrating the above relation over all leaf sizes a_L and orientations Ω_L and dividing by $d\mathbf{r}$ gives the total interaction cross section

$$\tilde{\sigma}(\mathbf{r}, \Omega) = \int_0^\infty da_L \int_{2\pi+} d\Omega_L n(\mathbf{r}, a_L, \Omega_L) a_L |\Omega \cdot \Omega_L| \chi(\mathbf{r}, a_L, \Omega_L, \Omega).$$

In terms of quantities introduced earlier, the total interaction cross section $\tilde{\sigma}$ is

$$\tilde{\sigma}(\mathbf{r}, \Omega) = \int_0^\infty da_L a_L n_L(\mathbf{r}, a_L) \rho_L(\mathbf{r}, a_L) G(\mathbf{r}, \Omega; a_L), \tag{2}$$

where the dimensionless function G is the so-called geometry factor that denotes the nonoverlapped area, per unit leaf area, that is projected on a plane perpendicular to the direction Ω, namely,

$$G(\mathbf{r}, \Omega; a_L) = \frac{1}{2\pi} \int_{2\pi+} d\Omega_L g_L(\mathbf{r}, \Omega_L) |\Omega \cdot \Omega_L| \chi(\mathbf{r}, a_L, \Omega_L, \Omega). \tag{3}$$

The leaf area density function $u_L(\mathbf{r})$ (in m^{-1}) introduced by Ross[4] is equivalent to

$$u_L(\mathbf{r}) = \int_0^\infty da_L a_L n_L(\mathbf{r}, a_L) \rho_L(\mathbf{r}, a_L). \tag{4}$$

If all of the leaves in $d\mathbf{r}$ are assumed to be of size a_0 then $\rho_L(\mathbf{r}, a_L) = \delta(a_L - a_0)$, and

$$\tilde{\sigma}(\mathbf{r}, \Omega) = a_0 n_L(\mathbf{r}, a_0) G(\mathbf{r}, \Omega; a_0) = u_L(\mathbf{r}) G(\mathbf{r}, \Omega; a_0). \tag{5}$$

We now consider the process $a_0 \to 0$. To have the same leaf area density $u_L(\mathbf{r})$, the number of leaves in the elementary volume $N(\mathbf{r}) \to \infty$ at the same rate as $a_0 \to 0$. Therefore, in the limit of nondimensional leaves, there is no mutual shading between leaves and

$$\lim_{a_0 \to 0} G(\mathbf{r}, \Omega; a_0) = \frac{1}{2\pi} \int_{2\pi+} d\Omega_L g_L(\mathbf{r}, \Omega_L) |\Omega \cdot \Omega_L|.$$

This definition of the geometry factor was originally introduced by Ross and his colleagues.[4] The fact that Ross' theory refers to the limit $[N \to \infty, a_0 \to 0]$ is not surprising since the resulting radiative transfer equation is essentially a linearized form of the Boltzmann equation, which describes the evolution of an N-particle system in the limit $N \to \infty$ and $\sigma \to 0$ (σ is size of the particle).[14]

As a matter of fact, Ross' theory of radiative transfer for a leaf canopy contains two contradictory assumptions.[4] On the one hand, it is assumed that the elementary volume is so small that no mutual shading exists along any direction within it. On the other hand, the number of leaves in the elementary volume is assumed to be so great that the functions $u_L(\mathbf{r})$ and $1/2\pi\, g_L(\mathbf{r}, \Omega_L)$ may be defined with adequate accuracy. These assumptions are valid only in case of infinitely small leaves.

The geometric factor G can be accurately evaluated if detailed measurements of leaf spatial coordinates, size, shape, and orientation distribution are available. Accurate collection of such data is tedious and expensive. In practice, one must resort to *ad hoc* models. We propose to approximate χ by the exponential function

$$\chi(\mathbf{r}, a_L, \Omega_L, \Omega) \equiv \exp[-A_{sh}(\mathbf{r}, a_L, \Omega_L, \Omega)],$$

where A_{sh} is a fraction of the total projected area of n leaves with size a_L and orientation Ω_L in a unit volume around $\mathbf{r}$ when illuminated along Ω (adjustable parameter, say). If the leaves are assumed to be nondimensional, then $A_{sh} \equiv 0$, i.e., no cross shading between leaves and $\chi = 1$. On the other hand, if all the leaves are totally shaded by a single leaf of area a_0, then [see Eqs. (4)–(5)]

$$0 < A_{sh}(\mathbf{r}, a_L, \Omega_L, \Omega) = 1 - a_0/u_L(\mathbf{r}),$$

and $\chi(\mathbf{r}, a_0, \Omega_L, \Omega) \approx \exp(-1)$.

A further simplification is to suppose that

$$A_{sh}(\mathbf{r}, a_L, \Omega_L, \Omega) = \bar{A}_{sh}(\mathbf{r}, a_L) |\Omega \cdot \Omega_L|,$$

where $0 \leqslant \bar{A}_{sh} \leqslant 1$ is the mean area projected by n leaves of size a_L and orientation Ω_L on a plane perpendicular to Ω. The evaluation of the geometry factor is considerably simplified now. For instance, if the leaf normals are randomly distributed, then [cf. Eq. (3)]

$$G(\mathbf{r}, \Omega; a_L) = \bar{A}_{sh}(\mathbf{r}, a_L)^{-2}\{1 - \exp[-\bar{A}_{sh}(\mathbf{r}, a_L)] - \bar{A}_{sh}(\mathbf{r}, a_L)\exp[-\bar{A}_{sh}(\mathbf{r}, a_L)]\}$$

and, if $\bar{A}_{sh} \to 0$, then $G \to 0.5$, which is the correct result for nondimensional leaves.[4] For horizontal leaves, the geometry factor is

$$G(\mathbf{r}, \Omega; a_L) = \mu \exp[-\bar{A}_{sh}(\mathbf{r}, a_L)\mu],$$

where $\cos^{-1}\mu$ is the direction of photon travel. Numerical examples of the geometry factor for various leaf normal orientations can be found in a companion article.[12]

4. THE MODIFIED TOTAL INTERACTION CROSS SECTION

In Sec. 2, a leaf canopy was abstracted as a binary medium with aggregates of finite-size leaves interspersed in free space creating convolutely shaped and multiply connected voids. To describe the rules of photon movement in such a medium, one has to know not only the interaction cross sections for the leaf aggregates (Sec. 3) but also the distribution of voids along the path of photon travel.[16] So, we postulate the following picture of photon interactions in a leaf canopy.

We suppose that the event $A = \{$two successive interactions between photons and leaves occurred in the neighborhoods of $\mathbf{r}''$ and $\mathbf{r}'$, where $\mathbf{r}' = \mathbf{r}'' + s'\Omega', s' > 0\}$ is realized (Fig. 1). Then, the total interaction cross section at $\mathbf{r}$ ($\mathbf{r} = \mathbf{r}' + s\Omega; s > 0$) for those photons with previous phase space state $(\mathbf{r}'', \Omega')$ can be represented as the product of the interaction cross section for the leaf aggregate $\tilde{\sigma}(\mathbf{r}, \Omega)$ and the probability $[1 - q(\mathbf{r}, \Omega|\mathbf{r}'', \Omega')]$ of encountering a clump filled densely with leaves at $(\mathbf{r}, \Omega)$ for photons from $(\mathbf{r}'', \Omega')$ with a scattering event at $\mathbf{r}'$ where the direction of travel is changed from Ω' to Ω. The latter clearly depends on the photons' previous phase space state $(\mathbf{r}'', \Omega')$. The realization of the event A means that the interval between the points $\mathbf{r}''$ and $\mathbf{r}'$ is free of interaction centers. In which case, a photon from $\mathbf{r}'$ can unimpededly reach the previous site of interaction $\mathbf{r}''$ along $-\Omega'$. Moreover, this interval is assumed to extend to a small cone depending on the dimensions of the scattering centers.[12] This will provide us the proper mechanism for

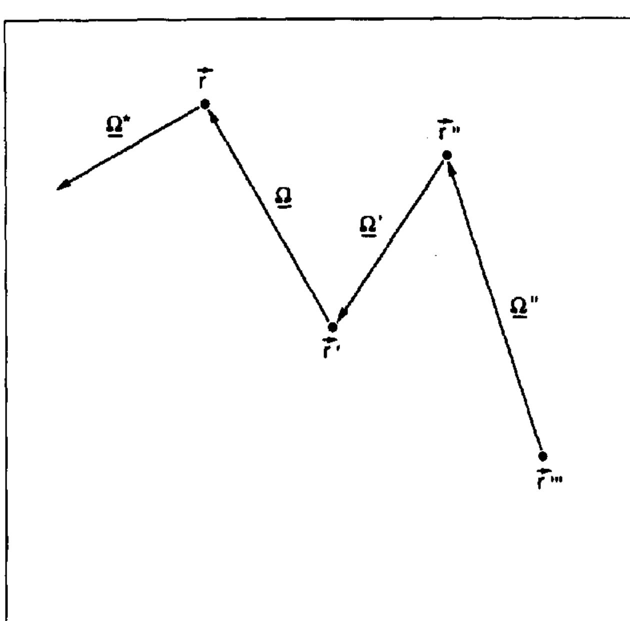

Fig. 1. Radiative transfer process in a medium with finite-dimensional scattering centers. Here, **r**, **r**′, etc. are the points of interaction and Ω, Ω', etc. are the directions of photon travel.

describing the hot spot effect or opposition brightening. These considerations are the foundation for our development of the interaction cross sections and the resulting transport description.

The necessity for introducing the probability of photon arrival $(1 - q)$ at an interaction site arises from our abstraction of the leaf canopy. If size of the scatterers is considered in a formulation of the transport problem, it is necessary to include not only the number density of scatterers but also the consequences of introducing finite size gaps (holes or voids) in the medium. Standard transport description based on cross sections derived from elementary volumes is not applicable because of the presence of voids. Moreover, if the far-field assumption is violated, as is the case in a leaf canopy, then the scatterers will cast shadows; hence, information on the spatial distribution of scatterers is required to evaluate mutual shadowing. A need for such a theory has a strong basis in experimental observations of exiting radiations measured in opposition to a monodirectional source.[8-10] Standard transport theory assumes an infinite number of uncorrelated nondimensional scatterers thereby affording a continuum description of the configuration space. It is clear that point scatterers cannot cast shadows and thus, the standard theory fails to predict or duplicate experimental observations of exiting radiations about the opposition direction.

Kuusk,[16] in an attempt to model the hot-spot effect of leaf canopies, considered the distribution of voids along the path of photon travel with respect to space and angular variables. The probability Q that a point $\mathbf{r}$ inside a leaf canopy can be viewed from two points $\mathbf{r}'$ and $\mathbf{r}''$ was calculated as

$$Q = \exp\left[-\int_0^s dt'\tilde{\sigma}[\mathbf{r}(t'), \Omega]\right]\exp\left[-\int_0^{s'} dt'\tilde{\sigma}[\mathbf{r}(t'), -\Omega']\right]C_{HS}(\mathbf{r}, s, s', \Omega, \Omega'),$$

where $s = |\mathbf{r} - \mathbf{r}''|$, $s' = |\mathbf{r} - \mathbf{r}'|$, $\Omega = (\mathbf{r} - \mathbf{r}'')/s$, $\Omega' = (\mathbf{r}' - \mathbf{r})/s'$, and C_{HS} is a correction factor. The subscript HS is connected with the so-called hot spot effect.[13,16] The product

$$Q'(\mathbf{r}, s, s', \Omega, \Omega') = \exp\left[-\int_0^{s} dt'\tilde{\sigma}[\mathbf{r}(t'), \Omega]\right]C_{HS}(\mathbf{r}, s, s', \Omega, \Omega')$$

can be interpreted as a distribution function of voids from the point $\mathbf{r}$ along the direction Ω of photon travel. The function Q' clearly depends on the point $\mathbf{r}'$ and direction Ω'. If $\mathbf{r}'' = \mathbf{r}'$, $(\Omega = -\Omega')$ then $Q' = 1$, and this provides a model for the hot spot effect [i.e., the elements that are lit by parallel beams incident along Ω' are visible along $-\Omega'$ with unit probability]. It was this idea of Kuusk that led us to imbue the cross sections with the property of dependence on the phase space coordinates of the previous interaction site via the probability q.

A precise definition of the previous phase space state of a photon is as follows. Consider Fig. 1. A photon at $\mathbf{r} = \mathbf{r}' + s\Omega$ $(s > 0)$ traveling along Ω has a previous state $(\mathbf{r}'', \Omega')$, if two successive interactions between the photon and scattering elements occurred in the neighborhoods of $\mathbf{r}''$ and $\mathbf{r}' = \mathbf{r}'' + s'\Omega'$ $(s' > 0)$. The interaction center about $\mathbf{r}'$ at which the direction of photon travel changed from Ω' to Ω due to a scattering event, is still the current (or most-recent) interaction center. Thus the phrase "previous phase space state".

The total interaction cross section $\sigma(\mathbf{r}, \Omega | \mathbf{r}'', \Omega')$ is defined such that the probability of a photon, in an elementary volume about $\mathbf{r}$ with previous state $(\mathbf{r}'', \Omega')$, being captured in traveling a distance ds along Ω is $\sigma\, ds$. Let $[1 - q(\mathbf{r}, \Omega | \mathbf{r}'', \Omega')]$ be the probability of photons encountering an aggregation of leaves at the state $(\mathbf{r}, \Omega)$ provided that the event {two successive interactions between photons and leaves occurred in the neighborhoods of $\mathbf{r}''$ and $\mathbf{r}'$} was realized. Then, the total interaction cross section at $\mathbf{r}$ for photons with previous state $(\mathbf{r}'', \Omega')$ traveling along Ω is

$$\sigma(\mathbf{r}, \Omega | \mathbf{r}'', \Omega') = \tilde{\sigma}(\mathbf{r}, \Omega)[1 - q(\mathbf{r}, \Omega | \mathbf{r}'', \Omega')] \tag{6}$$

where $\tilde{\sigma}$ is the total interaction cross section for the aggregates filled densely by leaves [Eq. (2)].

Recently Verstraete et al[19] described a physically-based model for predicting the bidirectional reflectance of vegetation canopies. They found that "the transmittance of the scattered radiation in a porous medium is not independent of that of the incoming direct radiation: the two optical paths actually share a common volume, free of scatterers, near the scatterer that causes the reflection". Two volumes, defined by the incoming beam of direct solar radiation and the direction of observation, were considered. The optical depth for the scattered beam was represented as the product of a classical optical depth defined as an integral of $\tilde{\sigma}$ and a ratio of the two volumes. Their approach corresponds to ours if the correction factor (the ratio between the two volumes) can be considered as the probability of encountering a leaf aggregation.

The interaction rate for photons traveling along Ω in an elemental volume about $\mathbf{r}$ in a region belonging to an aggregation of leaves is determined by $[\tilde{\sigma}(\mathbf{r}, \Omega)]$. But the probability $[1 - q(\mathbf{r}, \Omega|\mathbf{r}'', \Omega')]$ depends upon the location of their previous interaction center. Thus, the overall outcome of a current interaction for a photon is influenced by its history [cf. Eq. (7)]. In this sense, one may attribute memory to a photon.† Of special importance is the fact that the fundamental element of transfer is no longer a straight line but a broken line (cf. Fig. 1). This is a consequence of considering the distribution of voids along the path of photon travel.

The modified total interaction cross section σ should satisfy the following four criteria:
(1) Convergence

$$\lim_{\ell_L \to 0} \sigma(\mathbf{r}, \Omega|\mathbf{r}'', \Omega') = \tilde{\sigma}(\mathbf{r}, \Omega). \tag{7}$$

(2) Positivity

$$0 \leqslant \sigma(\mathbf{r}, \Omega|\mathbf{r}'', \Omega') \leqslant \tilde{\sigma}(\mathbf{r}, \Omega).$$

(3) Continuity of $\sigma(\mathbf{r}' + s\Omega, \Omega|\mathbf{r}'', \Omega')$ with respect to s and Ω.
(4) No interactions along $(\Omega = -\Omega')$ that explains opposition brightening.
A simple model for the probability q of encountering a void and which imbues the cross sections with the above desired properties can be derived as follows.

We assume that a photon at $(\mathbf{r}, \Omega)$ with previous state $(\mathbf{r}'', \Omega')$ can encounter a void only inside the sphere S of radius $s_0 = |\mathbf{r}'' - \mathbf{r}'|$ centered at $\mathbf{r}'$, i.e.

$$S(\mathbf{r}', s_0) = \{\mathbf{r} : |\mathbf{r} - \mathbf{r}'| \leqslant s_0\}.$$

We also assume that q depends only on the distance d between the point $\mathbf{r}$ and vector Ω', and the angle α between the vectors Ω and Ω' (Fig. 2). We define a continuous function $Z(\alpha)$ such that, it is equal to 0 if $\alpha = 0$ $(\Omega = \Omega')$ and equal to 1 if $\alpha = \pi$ $(\Omega = -\Omega')$. An example is the linear function: $Z(x) = 0.5\ (1 - x)$, where $x = \cos\alpha = (\Omega \cdot \Omega')$. To describe the dependence of q on the distance d we shall use the results of Nilson and Kuusk.[13] They approximated the covariance between two indicator functions for viewing the point $\mathbf{r}'$ along $-\Omega$ and Ω' by an exponential function. The radius of correlation was expressed through the characteristic leaf dimension ℓ_L. The distance

$$d = \frac{|\Omega' \times (\mathbf{r} - \mathbf{r}'')|}{|\Omega|} = s \sin\alpha, \quad \alpha \in (0, \pi),$$

can be used for the covariance shift.

Approximating the covariance by an exponential function results in

$$\Upsilon_{\ell_L}(s, \alpha) = \exp(-d/\ell_L) = \exp(-s \sin\alpha/\ell_L).$$

Thus, a probable model for the probability q of encountering a void is

$$q(s, s_0, \alpha) = Z(\cos\alpha)\Upsilon_{\ell_L}(s, \alpha)H(s_0 - s), \tag{8}$$

where $H(x)$ is the Heaviside function

$$H(x) = \begin{cases} 1, & x \geqslant 0, \\ 0, & x < 0. \end{cases} \tag{9}$$

More details can be found in a compansion paper.[12]

†Recently Lumme et al investigated the problem of light reflection from a stochastically bounded semi-infinite medium.[17] They find that classical transport theory is not applicable in studies of light reflection from planetary regoliths where the scattering media are bounded by a rough particulate surface. These surfaces, like leaf canopies, show opposition brightening, a phenomenon not predicted by standard transport theory. They write that "For reflection from a rough surface, however, the future of a photon does strictly depend on its entire past history within the boundary region". Although in the final analysis they have not considered correlated probability of photon travel, their qualitative discussion of the issues operative in the physical problem is strikingly similar to the discussion here.

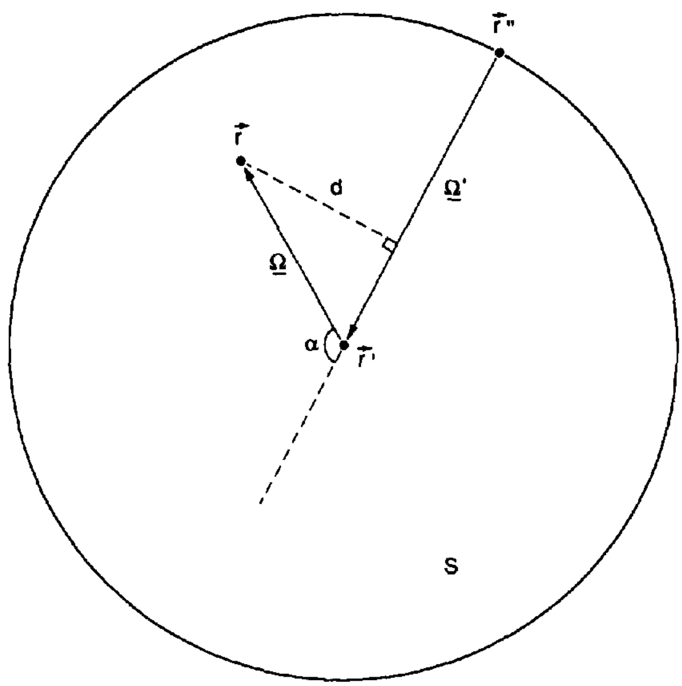

Fig. 2. Schematic illustration for the derivation of the probability q of encountering a void.

5. THE LENGTH OF PHOTON FREE PATH

The process of radiative energy transfer in a medium can be described as a homogeneous Markov chain of interactions between photons and the scatterers. A photon can interact with the medium as a consequence of movement only. This fact is reflected in the definition of the cross sections by using the phrase "in traveling an elementary distance". In this context it is appealing to discuss a related parameter, the length of photon free path. This is the straight line trajectory of photon travel between two successive points of interaction and is denoted here as ℓ. In order to define this length, from the point $\mathbf{r}_0$ along $\underline{\Omega}$, it is convenient to introduce the probability density function of the photon free path

$$f_\ell(t) = \tilde{\sigma}[\mathbf{r}(t)]\exp\left[-\int_0^t dt'\tilde{\sigma}[\mathbf{r}(t')]\right]; \quad \left[\int_0^\infty dt' f_\ell(t') = 1\right], \tag{10}$$

where $\mathbf{r}(t) = \mathbf{r}_0 + t\underline{\Omega}$. The integral term in the exponent is called the optical depth of the interval $[\mathbf{r}_0, \mathbf{r}(t)]$. In slab geometry, it is defined as

$$\tilde{\tau}(z, z_0) = \int_{z_0}^{z} dt'\tilde{\sigma}(t'). \tag{11}$$

Similarly, the probability density f_ℓ in slab geometry is

$$f_\ell(z; z') = \frac{\tilde{\sigma}(z)}{|\mu|}\exp\left[-\frac{\tilde{\tau}(z, z')}{|\mu|}\right],$$

where $\cos^{-1}\mu$ is the direction of photon travel. In a leaf canopy the scatterers generally exhibit a distribution in $\underline{\Omega}_L$ in which case the total interaction cross section depends on the direction of photon travel $\underline{\Omega}$ [viz. Eq. (2)].[4] Thus, $\tilde{\sigma} = \tilde{\sigma}(\mathbf{r}, \underline{\Omega})$, $\tilde{\tau} = \tilde{\tau}(t, \underline{\Omega})$, and $f_\ell = f_\ell(t; \underline{\Omega})$.

The optical depth of the interval $[\mathbf{r}', \mathbf{r}]$ in a canopy of finite-dimensional leaves depends on $(\mathbf{r}'', \underline{\Omega}')$ [cf. Eq. (6)] as follows:

$$\tau(t, \underline{\Omega}) | \mathbf{r}'', \underline{\Omega}') = \int_0^t dt'\sigma[\mathbf{r}(t'), \underline{\Omega} | \mathbf{r}'', \underline{\Omega}'].$$

The probability density f_ℓ in this case is given by [cf. Eq. (10)]

$$f_\ell(t;\Omega|\mathbf{r}'',\Omega') = \sigma[\mathbf{r}(t),\Omega|\mathbf{r}'',\Omega']\exp[-\tau(t,\Omega|\mathbf{r}'',\Omega')],$$

where $\mathbf{r}(t) = \mathbf{r}' + t\Omega$. The point $\mathbf{r}'$ can be uniquely defined as the point of intersection of two vectors, $[(\mathbf{r},-\Omega)$ and $(\mathbf{r}'',\Omega')]$, that belongs to one plane. In a slab geometry the optical depth between z' and z is

$$\tau(z,z',\Omega|z'',\Omega') = \int_{z'}^{z} dt'\sigma(t',z',\Omega|z'',\Omega'), \tag{12}$$

and the probability density f_ℓ is

$$f_\ell(z;z',\Omega|z'',\Omega') = \frac{\sigma(z,z',\Omega|z'',\Omega')}{|\mu|}\exp\left[-\frac{\tau(z,z',\Omega|z'',\Omega')}{|\mu|}\right]. \tag{13}$$

The dependence of τ and f_ℓ on z' in Eqs. (12) and (13) is explicit because it is not possible to uniquely determine it from $[(z,-\Omega)$ and $(z'',\Omega')]$ in slab geometry. Thus, it is not possible to pose the transport problem in a strict slab geometry framework if the scattering centers have a finite size (area). As will be shown later, the independence of the cross sections at $(\mathbf{r},\Omega)$ on the spatial coordinates of the current interaction center ($\mathbf{r}'$ in Fig. 1) in R^3 permits the derivation of an integro-differential balance equation.

It is noteworthy that in media with finite-dimensional scattering centers, radiative energy transfer cannot be described as a Markov chain because the probability density f_ℓ depends on the interaction history of a photon [Eq. (13)]. So, an integral equation describing energy transfer between successive phase space states does not exist. Knyazikhin considered the case where σ depends only on the previous direction of photon travel $[\sigma \equiv \sigma(\mathbf{r},\Omega|\Omega')]$ (Markovian transport) and derived an integral transport equation the corresponding integro-differential equation of which does not exist.[18]

6. THE DIFFERENTIAL SCATTERING CROSS SECTION

A captured photon may be scattered resulting in a change of both photon frequency ν and direction; we shall ignore frequency shifting interactions here. As with photon capture, the scattering interaction can be described by the scattering cross section $\tilde{\sigma}_{s'}$, defined[3] such that the probability that a photon will be scattered in traveling a distance ds is $\tilde{\sigma}_{s'}\,ds$. Thus, the ratio $\tilde{\sigma}_{s'}/\tilde{\sigma}$ is the albedo of single scattering, ω, denoting the probability of scattering given that a collision has occurred. However, since the scattering event serves to change the direction of photon travel, it is convenient to introduce the differential scattering cross section $\tilde{\sigma}_s$. This cross section is defined[3] such that the probability that a photon, in traveling a distance ds, will be scattered from Ω' to a unit solid angle about Ω is $\sigma_s\,ds$. Hence, the dimensions of σ_s are $m^{-1}sr^{-1}$ of Ω. The cross section $\tilde{\sigma}_s$ is related to the scattering cross section $\tilde{\sigma}_{s'}$ as

$$\tilde{\sigma}_{s'}(\mathbf{r}',\Omega') = \int_{4\pi} d\Omega\,\tilde{\sigma}_s(\mathbf{r}';\Omega'\to\Omega). \tag{14}$$

For an aggregation of finite-dimensional leaves the differential scattering cross section can be expressed as

$$\tilde{\sigma}_s(\mathbf{r}';\Omega'\to\Omega) = \int_0^\infty da_L \int_{2\pi^+} d\Omega_L m(a_L,\Omega_L;\mathbf{r}',\Omega')\gamma_L(\mathbf{r}',\Omega_L;\Omega'\to\Omega), \tag{15}$$

where m is given by Eq. (1), γ_L is the single-leaf scattering phase function (in sr^{-1} of Ω). For a leaf about $\mathbf{r}'$ with outward normal Ω_L, this phase function is the fraction of the intercepted energy (from photons initially traveling in direction Ω') that is scattered into a unit solid angle about Ω.[5] Equation (15) can be written in a more appealing form as

$$\tilde{\sigma}_s(\mathbf{r}';\Omega'\to\Omega) = \int_0^\infty da_L a_L n_L(\mathbf{r}',a_L)\rho_L(\mathbf{r}',a_L)\frac{1}{\pi}\Gamma(\mathbf{r}';\Omega'\to\Omega;a_L).$$

The function Γ/π (sr^{-1} of Ω) is the area scattering phase function[4] for an aggregation of finite-size leaves

$$\frac{1}{\pi}\Gamma(\mathbf{r}';\Omega'\to\Omega;a_L)=\frac{1}{2\pi}\int_{2\pi+}d\Omega_L g_L(\mathbf{r}',\Omega_L)|\Omega'\cdot\Omega_L|\chi(\mathbf{r}',a_L,\Omega_L,\Omega')\gamma_L(\mathbf{r}',\Omega_L;\Omega'\to\Omega),$$

which, in general, is not rotationally invariant because of the distribution function g_L. In the case of constant leaf size $\rho_L(\mathbf{r}',a_L)=\delta(a_L-a_0)$, the differential scattering cross section is given by

$$\tilde{\sigma}_s(\mathbf{r}',\Omega'\to\Omega)=u_L(\mathbf{r}')\frac{1}{\pi}\Gamma(\mathbf{r}';\Omega'\to\Omega;a_0).$$

And, if the scattering centers are nondimensional ($\chi\equiv 1$) and $g_L(\mathbf{r}',\Omega_L)\equiv 1$ (spherical orientation), then $\Gamma(\mathbf{r}';\Omega'\to\Omega)\equiv\Gamma(\mathbf{r}';\Omega'\cdot\Omega)$.

The scattering cross section in view of Eqs. (14) and (15) can be written as

$$\tilde{\sigma}_{s'}(\mathbf{r}',\Omega')=\int_{2\pi+}d\Omega_L\omega_L(\mathbf{r}',\Omega';\Omega_L)\int_0^\infty da_L m_L(a_L,\Omega_L;\mathbf{r}',\Omega') \tag{16}$$

where ω_L is the leaf-albedo:

$$\omega_L(\mathbf{r}',\Omega';\Omega_L)=\int_{4\pi}d\Omega\,\gamma_L(\mathbf{r}',\Omega_L;\Omega'\to\Omega).$$

The two important mechanisms of scattering are specular reflection at the leaf surface, in which case,[6]

$$\omega_L(\mathbf{r}',\Omega';\Omega_L)\equiv\omega_L[\mathbf{r}',(\Omega'\cdot\Omega_L)],$$

and multiple reflections inside the leaf due to numerous refractive index discontinuities. For the latter, the bi-Lambertian model[5] is often used and $\omega_L(\mathbf{r}',\Omega';\Omega_L)\equiv\omega_L(\mathbf{r}')$. Within this approximation the scattering cross section can be written as

$$\tilde{\sigma}_s(\mathbf{r}',\Omega')=\omega_L(\mathbf{r}')\tilde{\sigma}(\mathbf{r}',\Omega') \tag{17}$$

and thus, the leaf-albedo ω_L is equivalent to the single scattering albedo ω admitted by the transport equation.

In the discussion leading to Eq. (6) it was emphasized that the spatial distribution of voids along the path of photon travel must be accounted in order to describe its attenuation probabilities. In view of the fact that a collision preceeds a scattering interaction, the modified differential scattering cross section σ_s is (Fig. 1)

$$\sigma_s(\mathbf{r}';\Omega'\to\Omega|\mathbf{r}''',\Omega'')=\tilde{\sigma}_s(\mathbf{r}',\Omega'\to\Omega)[1-q(\mathbf{r}',\Omega'|\mathbf{r}''',\Omega'')] \tag{18}$$

where q is the probability of encountering a void [Eq. (8)]. Thus, if $|\mathbf{r}'-\mathbf{r}''|\leqslant|\mathbf{r}'''-\mathbf{r}''|$ and $\Omega'=-\Omega''$, then $q=1$ and $\sigma_s=0$; which must be since $\sigma(\mathbf{r}',\Omega'|\mathbf{r}''',\Omega'')=0$. The scattering cross section $\sigma_{s'}$ is [cf. Eqs. (14) and (16)]

$$\sigma_{s'}(\mathbf{r}',\Omega'|\mathbf{r}''',\Omega'')=\int_{4\pi}d\Omega\sigma_s(\mathbf{r}';\Omega'\to\Omega|\mathbf{r}''',\Omega'')=\tilde{\sigma}_{s'}(\mathbf{r}',\Omega')[1-q(\mathbf{r}',\Omega'|\mathbf{r}''',\Omega'')].$$

The two-dimensional probability density function f_Ω that describes the distribution of scattering directions is defined by the relation[3]

$$f_\Omega(\Omega;\mathbf{r}',\Omega'|\mathbf{r}''',\Omega'')=\frac{\sigma_s(\mathbf{r}';\Omega'\to\Omega|\mathbf{r}''',\Omega'')}{\sigma_{s'}(\mathbf{r}',\Omega'|\mathbf{r}''',\Omega'')}=\frac{\tilde{\sigma}_s(\mathbf{r}';\Omega'\to\Omega)}{\tilde{\sigma}_{s'}(\mathbf{r}',\Omega')}.$$

Thus, despite dependence of σ_s and $\sigma_{s'}$ on $(\mathbf{r}''',\Omega'')$, the probability density f_Ω is independent of the previous phase space state of the incident photon

$$f_\Omega(\Omega;\mathbf{r}',\Omega'|\mathbf{r}''',\Omega'')\equiv f_\Omega(\Omega;\mathbf{r}',\Omega'),$$

unlike the probability density f_ℓ of the distribution of the length of photon free path [Eq. (13)]. For the case of bi-Lambertian scattering by leaves, the density f_Ω can be written more meaningfully as [Eq. (17)]

$$f_\Omega(\Omega; \mathbf{r}', \Omega') = \frac{1}{\omega_L(\mathbf{r}')} \frac{\bar{\sigma}_s(\mathbf{r}'; \Omega' \to \Omega)}{\bar{\sigma}(\mathbf{r}', \Omega')}$$

since $\omega = \omega_L$.

7. THE SLAB GEOMETRY PROBLEM

The main features of photon–leaf interaction in a leaf canopy idealized as a binary medium have been detailed in the previous sections (3–6). In this section the equation of photon travel from phase space points x_i to x_j is introduced and the "slab geometry" problem is discussed. Here, $x_i \sim (\xi_i, \Omega_i)$ and $x_j \sim (\xi_j, \Omega_j)$ are the points of the phase space X, ξ is the depth-coordinate, and Ω is the direction of photon travel.

Consider a leaf canopy confined between the depth interval $\xi = \xi_0 = 0$ (top of the canopy) and $\xi = \xi_H = H$; where H is the physical depth of the canopy. Assume that the leaf canopy is bounded by an absorbing soil surface at depth ξ_H; this assumption will be relaxed in Sec. 8. Let the leaf canopy be illuminated by a monodirection beam of intensity B_0 along Ω_s ($\mu_s < 0$) at ξ_0. The uncollided intensity at $x \sim (\xi, \Omega)$ is given by

$$I_0(x) = B_0 \exp\left[\frac{1}{\mu} \tau(\xi, \xi_0, \Omega | \xi_{-1}, \Omega_0)\right] \delta(\Omega - \Omega_s)\, \delta(\Omega_0 - \Omega_s), \tag{19}$$

where ξ_{-1} is an arbitrary point above the canopy. A detailed discussion of B_0 in the presence diffuse sky radiation is given in Sec. 10. Let the kernel k_1 denote the interactions as photons travel from x_i to x_{i+1}, provided their previous points of interaction were x_{i-1} and x_{i-2} [Fig. 3(a)], namely

$$k_1(x_i \to x_{i+1} | x_{i-1}, \xi_{i-2}) = \begin{cases} \dfrac{1}{|\mu_{i+1}|} \exp\left[\dfrac{1}{\mu_{i+1}} \tau(\xi_{i+1}, \xi_i, \Omega_{i+1} | \xi_{i-1}, \Omega_i)\right] \\ \quad \times\, \sigma_s(\xi_i; \Omega_i \to \Omega_{i+1} | \xi_{i-1}, \Omega_{i-1}, \xi_{i-2}), \quad \text{for } (\xi_{i+1} - \xi_i)\mu_{i+1} < 0, \\ 0, \quad \text{for } (\xi_{i+1} - \xi_i)\mu_{i+1} > 0. \end{cases} \tag{20}$$

In Eq. (20), the optical depth τ and the differential scattering cross section σ_s are as defined by Eqs. (12) and (18). The description of particle transport as a scattering event followed by the length of photon free path is usual in adjoint transport problems. However, this description is used here for notational ease.

The intensity of first collision photons I_1 can be evaluated by applying the integral operator with kernel k_1 to I_0 as follows:

$$I_1(x) = \int_X \mathrm{d}x_1 k_1(x_1 \to x | x_0, \xi_{-1}) I_0(x_1),$$

where $x_0 \sim (\xi_0, \Omega_0)$, and $X = [0, H] \otimes 4\pi$ is the phase-space. The intensity of the second collision photons at x can be evaluated as

$$I_2(x) = \int_X \mathrm{d}x_1 \int_X \mathrm{d}x_2 k_1(x_2 \to x | x_1, \xi_0) k_1(x_1 \to x_2 | x_0, \xi_{-1}) I_0(x_1).$$

By analogy, the intensity of kth collision photons at x is evaluated as

$$I_k(x) = \int_X \mathrm{d}x_1 \ldots \int_X \mathrm{d}x_k K_k(x_1 \to x_2 \to \cdots \to x_k \to x | x_0, \xi_{-1}) I_0(x_1), \tag{21}$$

where the product-kernel K_k is

$$K_k(x_1 \to x_2 \to \cdots \to x_k \to x_{k+1} | x_0, \xi_{-1}) = \prod_{i=1}^{k} k_1(x_i \to x_{i+1} | x_{i-1}, \xi_{i-2}). \tag{22}$$

Finally, the total intensity at x is given by sum

$$I(x) = \sum_{i=0}^{\infty} I_i(x). \tag{23}$$

Thus, it is seen that the transfer process defined by Eqs. (21) and (22) is not a Markov chain because of the dependence on the previous points of interaction, and hence, it is not possible to derive an integral equation depicting this process.

8. THE LEAF CANOPY PROBLEM WITH SOIL REFLECTION

In this section the leaf canopy problem in slab geometry with a reflecting soil surface at $\xi = H$ is considered. Let $\rho_s(\underline{\Omega}' \to \underline{\Omega})$ be the soil reflection function, i.e.

$$I(H, \underline{\Omega}) = \frac{1}{\pi} \int_{2\pi -} d\underline{\Omega}' \rho_s(\underline{\Omega}' \to \underline{\Omega}) |\mu'| I(H, \underline{\Omega}'), \mu > 0. \tag{24}$$

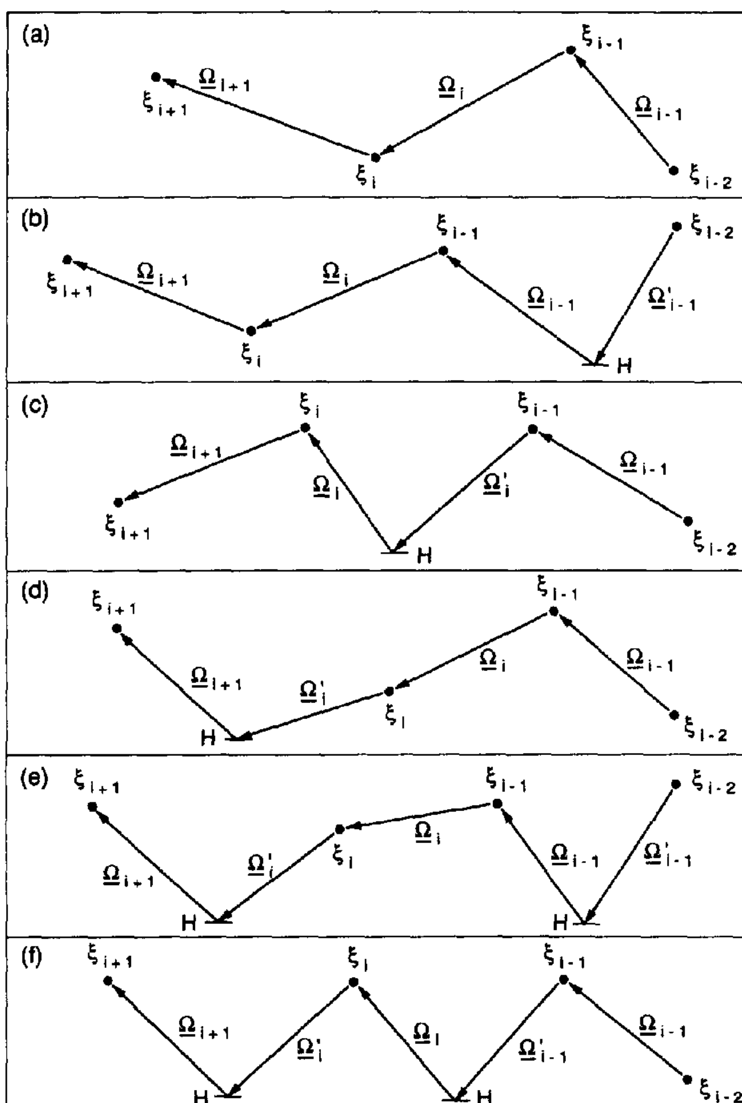

Fig. 3. Possible case of the transfer kernel k in a medium with a reflecting boundary at depth $\xi = H$.

As in Sec. 7 let the kernel $k_1(x_i \to x_{i+1} | x_{i-1}, \xi_{i-2})$ denote the interactions as photons travel from $x_i \sim (\xi_i, \Omega_i)$ to $x_{i+1} \sim (\xi_{i+1}, \Omega_{i+1})$, given that the relevant previous states were x_{i-1} and ξ_{i-2} [Eq. (20); Fig. 3(a)]. Similarly, let the kernel $k_2(x_i \to x_{i+1} | \Omega_i', x_{i-1}, \xi_{i-2})$ denote the interactions as photons travel from x_i to x_{i+1} but with an intermediate interaction (reflection) at the soil surface $[(H, \Omega_i')]$ [Fig. 3(d)], i.e.

$$k_2(x_i \to x_{i+1} | \Omega_i', x_{i-1}, \xi_{i-2})$$

$$= \begin{cases} \dfrac{1}{\mu_{i+1}} \exp\left[\dfrac{1}{\mu_{i+1}} \tau(\xi_{i+1}, H, \Omega_{i+1} | \xi_i, \Omega_i')\right] \dfrac{1}{\pi} \rho_s(\Omega_i' \to \Omega_{i+1}) \exp\left[\dfrac{1}{\mu_i'} \tau(H, \xi_i, \Omega_i' | \xi_{i-1}, \Omega_i)\right] \\ \times \sigma_s(\xi_i; \Omega_i \to \Omega_i' | x_{i-1}, \xi_{i-2},), \quad \text{for } \mu_1' < 0, \mu_{i+1} > 0, \\ 0, \text{ otherwise.} \end{cases} \tag{25}$$

For each of the kernels k_1 and k_2, three unique cases can be distinguished depending upon whether a reflection at the soil has occured or not; Figs. 3(a)–(c) illustrate the kernel k_1 and Figs. 3(d)–(f) the kernel k_2. These cases are designated as A1 [Figs. 3(a) and (d)], A2 [Figs. 3(b) and (e)] and B [Figs. 3(c) and (f)]. Let $\bar{k}$ be the sum of kernels k_1 and k_2; then,

$$\bar{k}(x_i \to x | \Omega_i', x_{i-1,i-1}, \xi_{i-1}') = k_1(x_i \to x | x_{i-1,i-1}, \xi_{i-1}') \delta(\Omega_i' - \Omega^0) + k_2(x_i \to x | \Omega_i', x_{i-1,i-1}, \xi_{i-1}'), \tag{26}$$

where Ω^0 is an arbitrary direction. In Eq. (26), the variables $x_{i-1,i-1}$ and ξ_{i-1}' take the following values:

$$x_{i,i} = \begin{cases} (\xi_i, \Omega_i), & \text{A1, A2}, \\ (H, \Omega_i'), & \text{B}, i = 1, 2, \ldots, \end{cases}$$

$$\xi_i' = \begin{cases} \xi_{i-1}, & \text{A1}, \\ H, & \text{A2}, \\ \xi_i, & \text{B}, i = 2, 3, \ldots, \end{cases}$$

$$\xi_1' = \begin{cases} \xi', & \text{A1, A2} \\ \xi_1, & \text{B}, \end{cases}$$

and $\xi_0' = \xi_0 = 0$, $x_{0,0} = (\xi', \Omega_0)$.

The uncollided radiation field at $x \sim (\xi, \Omega)$ is given by the sum

$$I_0(x) = Q_1(x, \xi') + Q_2(x, \xi'),$$

where

$$Q_1(x, \xi') = B_0 \exp\left[\frac{1}{\mu} \tau(\xi, 0, \Omega | \xi_{-1}, \Omega_0)\right] \delta(\Omega - \Omega_0) \delta(\xi' - 0),$$

$$Q_2(x, \xi') = B_0 \exp\left[\frac{1}{\mu_0} \tau(H, 0, \Omega_0 | \xi_{-1}, \Omega_0)\right] \frac{1}{\pi} \rho_s(\Omega_0 \to \Omega) \times \frac{1}{\mu} \exp\left[\frac{1}{\mu} \tau(\xi, H, \Omega | 0, \Omega_0)\right] \delta(\xi' - H).$$

Here, ξ' is a discrete variable that is either 0 or H.

The intensity of the first collision photons can be evaluated as

$$I_1(x) = \int_0^H d\xi' \int_0^H d\xi_1 \int_{2\pi\mp} d\Omega_1 \int_{2\pi-} d\Omega_1' [Q_1^-(x_1, \xi') + Q_2^+(x_1, \xi')] \bar{k}(x_1 \to x | \Omega_1', x_{0,0}, \xi_0'),$$

where ($\mp$) indicates that Q_1^- should be integrated over $2\pi^-$ and likewise Q_2^+ over $2\pi^+$. The intensity of second collision photons is given by

$$I_2(x) = \int_0^H d\xi' \int_0^H d\xi_1 \int_{2\pi\mp} d\Omega_1 \int_{2\pi^-} d\Omega_1' \int_0^H d\xi_2 \int_{\Omega^*} d\Omega_2 \int_{2\pi^-} d\Omega_2'$$
$$\times [Q_1^-(x_1, \xi') + Q_2^+(x_1, \xi')]$$
$$\times \bar{k}(x_1 \to x_2 | \Omega_1', x_{0,0}, \xi_0')\bar{k}(x_2 \to x | \Omega_2', x_{1,1}, \xi_1'),$$

where $\Omega^* = 4\pi$ (cases A1 and A2) or $2\pi^+$ (case B). By analogy, the intensity of kth collision photons is given by

$$I_k(x_{k+1}) = \int_0^H d\xi' \int_0^H d\xi_1 \ldots \int_0^H d\xi_k \int_{2\pi^-} d\Omega_1' \ldots \int_{2\pi^-} d\Omega_k' \int_{2\pi\mp} d\Omega_1' \int_{\Omega^*} d\Omega_2$$
$$\times \ldots \int_{\Omega^*} d\Omega_k [Q_1^-(x_1, \xi') + Q_2^+(x_1, \xi')] \prod_{i=1}^{k} \bar{k}(x_i \to x_{i+1} | \Omega_i', x_{i-1,i-1}, \xi_{i-1}'). \quad (27)$$

The total intensity is evaluated as in Eq. (23).

The three cases A1, A2 and B can be recognized in the following manner. If the product in Eq. (27) is expanded, we realize a combination, say, $k_1 k_1 k_2 k_1 k_2 k_2 k_1 \bar{k}$. The last k_i ($i = 1, 2$) corresponds either to A1 or A2 if the previous kernel is k_1 or to case B if the previous kernel is k_2. The case A1 can be distinguished from A2 if the pre-previous kernel is k_1.

9. GENERALIZATION: TRANSPORT BETWEEN ASSEMBLIES

We consider the slab geometry problem with vacuum boundary conditions introduced in Sec. 7 in detail. Let $\eta \sim (x_0, \xi_{-1})$ be the initial coordinates of the incident beam. Also, we combine three successive points of interaction and denote the new nine-dimensional ($x \sim \xi, \mu, \phi$) combination as

$$x_{i-1,i,i+1} = (x_{i-1}, x_i, x_{i+1}), \quad i = 2, 3, \ldots.$$

This procedure will facilitate subsequent analysis since the kernel k_1 depends on two previous points of interaction [Eq. (20), Fig. 3(a)]. For notational ease, let $k_1 \equiv k_1(x_i \to x_{i+1} | x_{i-1}, x_{i-2})$, thus including Ω_{i-2} although k_1 is independent of Ω_{i-2}. The functions J_η^i can now be introduced according to the recurrent relation (cf. Fig. 4)

$$J_\eta^{i+1}(x_{i-1,i,i+1}) = \int_X dx_{i-2} k_1(x_i \to x_{i+1} | x_{i-1}, x_{i-2}) J_\eta^i(x_{i-2,i-1,i}), \quad i = 3, 4, \ldots, \quad (28)$$

and

$$J_\eta^3(x_{1,2,3}) = k_1(x_1 \to x_2 | x_0, \xi_{-1}) k_1(x_2 \to x_3 | x_1, x_0) J_0(x_1 | \eta),$$

where $J_0(x_1|\eta)$ is the uncollided radiation field at x_1 [Eq. (19)]. The function $J_\eta^i(x_{i-2,i-1,i}) = J_\eta^i(x_i | x_{i-2}, x_{i-1})$ is the partial intensity of those photons at x_i that had experienced $(i-1)$ collisions and whose previous phase space states were x_{i-1} and x_{i-2}. The full intensity of $(i-1)$-times collided photons ($i = 3, 4, \ldots$) at x can be evaluated via J_η^i as

$$I_i(x) = \int_X dx_{i-1} \int_X dx_{i-2} J_\eta^i(x | x_{i-2}, x_{i-1}). \quad (29)$$

Let y_i be the ith assembly of the triad $(x_{3i-2}, x_{3i-1}, x_{3i})$ starting with $i = 1$. For instance (Fig. 4), $y_1 \sim (x_1, x_2, x_3)$ for the first assembly, $y_2 \sim (x_4, x_5, x_6)$ for the second, and so on. Clearly, photon transport from one assembly to the next does not depend on the previous assembly. Thus, one can construct a Markov chain, but the elements of this chain are now assemblies of three phase space points and not the individual points themselves. So, an integral operator T that relates the partial intensity of the ith assembly to the partial intensity of $(i-1)$-assembly can be defined, namely,

$$(TJ_\eta^3)(y_2) = \int_Y \mathrm{d}y_1 \tau(y_1 \to y_2) J_\eta^3(y_1) = J_\eta^6(y_2),$$

$$(TJ_\eta^6)(y_3) = \int_Y \mathrm{d}y_2 \tau(y_2 \to y_3) J_\eta^6(y_2) = J_\eta^9(y_3),$$

and so on. Here, $Y = X \otimes X \otimes X$ and the new kernel τ of operator T is

$$\tau(y_i \to y_{i+1}) = k_1(x_{3i} \to x_{3i+1} | x_{3i-1}, x_{3i-2}) k_1(x_{3i+1} \to x_{3i+2} | x_{3i}, x_{3i-1}) \times k_1(x_{3i+2} \to x_{3(i+1)} | x_{3i+1}, x_{3i}). \tag{30}$$

The degree of the operator T is defined as

$$(T^m\varphi)(y_{m+1}) = \int_Y \mathrm{d}y_1 \cdots \int_Y \mathrm{d}y_m \tau(y_1 \to y_2)\tau(y_2 \to y_3) \cdots \tau(y_m \to y_{m+1})\varphi(y_1),$$

where φ is some function. From the equality $(T\varphi, \varphi) = (\varphi, T^*\psi)$, where the scalar product (φ, ψ) is

$$\int_Y \mathrm{d}y \varphi(y)\psi(y) = \int_X \int_X \int_X \varphi(x_1, x_2, x_3)\psi(x_1, x_2, x_3)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3$$
$$= \int_0^H \mathrm{d}\xi_1 \int_{-1}^1 \mathrm{d}\mu_1 \int_0^{2\pi} \mathrm{d}\phi_1 \cdots \int_0^H \mathrm{d}\xi_3 \int_{-1}^1 \mathrm{d}\mu_3 \int_0^{2\pi} \mathrm{d}\phi_3 \varphi(\xi_1, \mu_1, \phi_1, \ldots, \xi_3, \mu_3, \phi_3) \times \psi(\xi_1, \mu_1, \phi_1, \ldots, \xi_3, \mu_3, \phi_3),$$

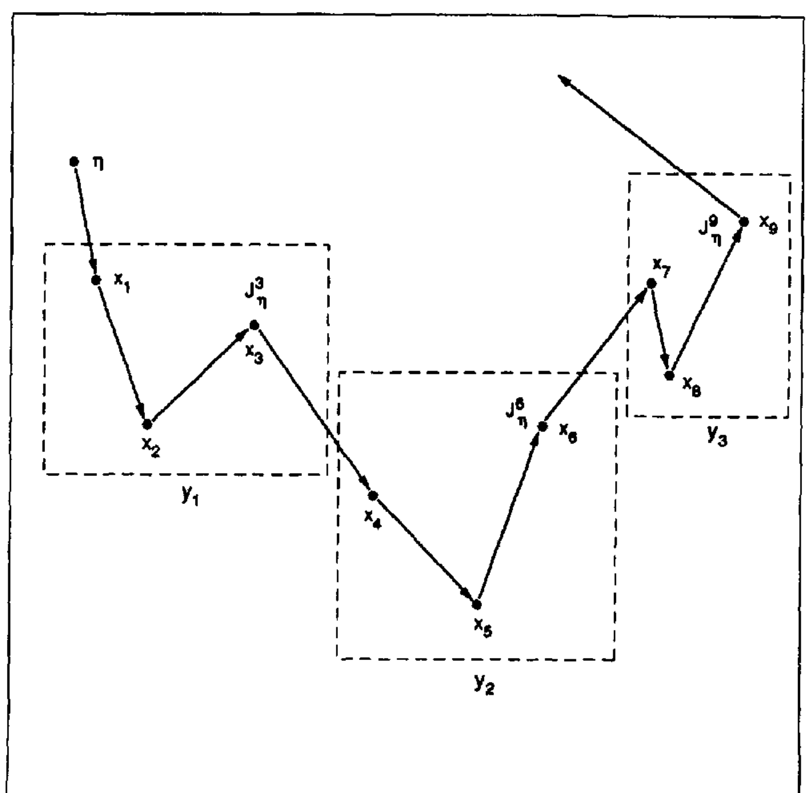

Fig. 4. Markov transfer process between assemblies (y_1, y_2 and y_3) of three successive phase-space points [(x_1, x_2, x_3), etc.; $x \sim (\xi, \Omega)$]. The functions J_η^i $(i = 3, 4, \ldots)$ are the partial intensities of triads [Eq. (28)], with η representing the initial coordinates of the incident beam.

we obtain

$$(T^*\psi)(y_1)=\int_Y dy_2\,\tau(y_1\to y_2)\psi(y_2).$$

Here, T^* is the operator adjoint to T. It can be seen that the kernel τ defined by Eq. (30) is, in general, not symmetric $[\tau(y_1\to y_2)\neq\tau(y_2\to y_1)]$ because of the dependence on the previous phase space points, and thus, the theorem of optical reciprocity is inapplicable.

Since photon transport between phase space triads is a Markov process, an integral equation for the partial intensities J can be written as

$$J_\eta=TJ_\eta+J_\eta^3 \tag{31}$$

where η are the initial coordinate of the incident photon. The solution of this equation can be expressed by the Neumann series

$$J_\eta=J_\eta^3+TJ_\eta^3+T^2J_\eta^3+\cdots$$

which represents the partial intensity $J_\eta(x_i|x_{i-2},x_{i-1})$ at any point x_i provided the previous states of the photons were x_{i-1} and x_{i-2}. To obtain the full intensity $I(x_i)$ it is sufficient to integrate the partial intensity over the states x_{i-1} and x_{i-2} [see Eq. (29); a more precise definition can be found in Eqs. (40)–(42)].

It can be shown that the integral equation (31) collapses to the classical integral transport equation in the case of nondimensional scattering centers. From Sec. 4 it is known that if $\ell_L\to 0$ then $\sigma\to\tilde{\sigma}$ [Eq. (7)], and thus, $\sigma_s\to\tilde{\sigma}_s$. Let T_0 denote the integral operator in the classical sense, i.e.

$$(T_0I)(x)=\int_X dx'k_1(x'\to x)I(x'). \tag{32}$$

and let the initial full intensity be

$$Q(x)=\int_X dx'\int_X dx''J_\eta^3(x|x'',x').$$

Integrating both sides of Eq. (31) over all previous and pre-previous photon states gives

$$I(x)=\int_X dx'\int_X dx''J_\eta(x|x'',x')=\int_X dx'\int_X dx''(TJ_\eta)(x|x'',x')$$
$$+\int_X dx'\int_X dx''J_\eta^3(x|x'',x')=(T_0^3I)(x)+Q(x),$$

which is the standard integral equation of transfer. It should be noted that T_0^3 is the third degree of the operator T_0 defined by Eq. (32).

It must be emphasized that the partial intensity at the point of interaction (immediately after scattering) depends only on the previous point of interaction and its dependence on the pre-previous point disappears and it can be defined as

$$\int_V d\xi_{i-2}\int_{4\pi}d\Omega_{i-1}J_\eta(\xi,\Omega_i|\xi_{i-1},\Omega_{i-1},\xi_{i-2})\sigma_s(\xi;\Omega_i\to\Omega|\xi_{i-1},\Omega_{i-1},\xi_{i-2})=I_p(\xi,\Omega|\xi_{i-1},\Omega_i). \tag{33}$$

Finally, the following two remarks are noteworthy. First, it is not possible to write an integral equation for the full intensity in the same way. Second, it is not difficult to generalize the above point of view—photon trajectories depending on n previous states of the photon; one then has assemblies of n points, and the transport operator T can be applied to the partial intensity of each assembly since photon transport from one assembly to another is Markovian.

10. INTERACTION BETWEEN THE LEAF CANOPY AND ATMOSPHERE

In Sec. 7 we considered the problem of photon transport in a horizontally homogeneous leaf canopy subject to incidence by a stream of "first flight" photons, i.e., photons that have not experienced a collision either in the leaf canopy or in the atmosphere adjacent to it. In the general case, however, a leaf canopy will also be illuminated by photons that have experienced one or more collisions in the atmosphere (diffuse sky light). And, considering the nature of cross sections discussed earlier (Secs. 4 and 6), a leaf canopy subject to an incidence of a diffuse photon field will be re-entrant. This leads to a dynamical coupling of radiative processes between the leaf canopy and the adjacent atmosphere, and this topic is addressed here.

We consider the following boundary-value problem for a plane parallel atmosphere:

$$-\mu\frac{\partial}{\partial\xi}\varphi(\xi,\underline{\Omega})+\tilde{\sigma}(\xi)\varphi(\xi,\underline{\Omega})=\int_{4\pi}d\underline{\Omega}'\tilde{\sigma}_s(\xi;\underline{\Omega}'\cdot\underline{\Omega})\varphi(\xi,\underline{\Omega}'),$$
$$\varphi(-T,\underline{\Omega})=I_s\delta(\underline{\Omega}-\underline{\Omega}_s),\quad \mu>0,$$
$$\varphi(0,\underline{\Omega})=f_\uparrow(\underline{\Omega}),\mu>0, \tag{34}$$

where $f_\uparrow(\underline{\Omega})$ is the intensity reflected from a leaf canopy,

$$f_\uparrow(\underline{\Omega})=\int_X dx'\int_X dx''J(0,\underline{\Omega}|x',x''),\mu>0. \tag{35}$$

Here, J is the partial intensity obtained as a solution of Eq. (31), and the assembly (x'',x',x_Ω), $[x_\Omega\sim(0,\underline{\Omega}),\mu>0]$, is the triad introduced in Sec. 9. In Eq. (34), I_s is the intensity of uncollided solar radiation incident in $\underline{\Omega}_s$, φ is collided intensity in the atmosphere, and T (in m) is the geometrical height of the atmosphere. The cross sections $\tilde{\sigma}$ and $\tilde{\sigma}_s$ are the standard functions encountered in atmospheric radiative transfer.

As a first approximation, assume that the intensity reflected from the leaf canopy $f_\uparrow(\underline{\Omega})$ is known or that some reasonable initial guess about its magnitude and behavior can be made. Then, by solving the boundary-value problem of Eq. (34), the solution φ at any point $\xi_{-2}\in A$ (A being the atmosphere) in any direction $\underline{\Omega}_{-1}\in 4\pi$ is known. The partial intensity at the point $\xi_{-1}\in A$ in the direction $\underline{\Omega}_0$ [the intitial point of $\eta\sim(x_0,\xi_{-1})$, Sec. 9] can be evaluated as (Fig. 5)

$$I_p^A(\xi_{-1},\underline{\Omega}_0|\xi_{-2},\underline{\Omega}_{-1})=k_A[(\xi_{-2},\underline{\Omega}_{-1})\to(\xi_{-1},\underline{\Omega}_0)]\varphi(\xi_{-2},\underline{\Omega}_{-1}),\quad \mu_0<0,$$

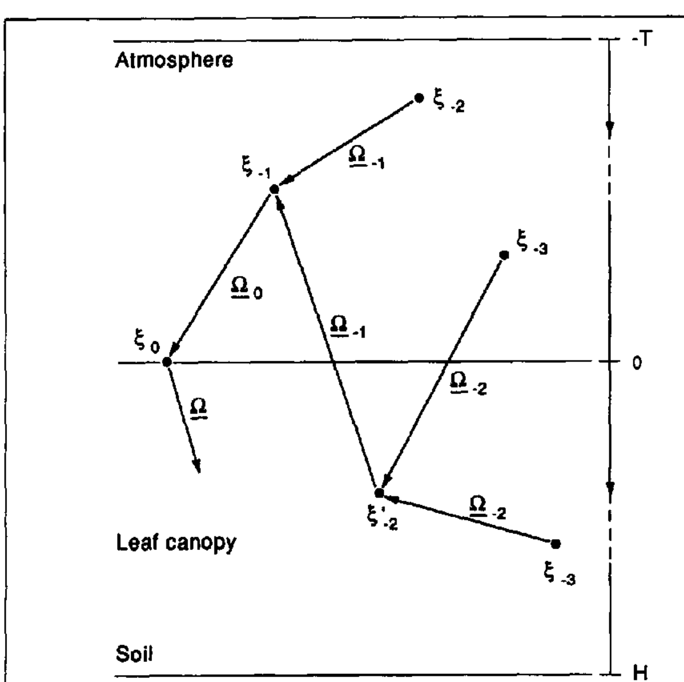

Fig. 5. Interaction between a leaf canopy and the adjacent atmosphere.

where the kernel k_A characterizes the probability density of photon travel from the point x' to x in the atmosphere

$$k_A(x' \to x) = \begin{cases} \dfrac{1}{|\mu'|} \exp\left[\dfrac{1}{\mu'} \tilde{\tau}(\xi, \xi')\right] \tilde{\sigma}_S(\xi; \underline{\Omega}' \cdot \underline{\Omega}), & \text{if } (\xi - \xi')\mu' < 0; \\ 0, & \text{if } (\xi - \xi')\mu' > 0. \end{cases}$$

The optical depth $\tilde{\tau}(\xi, \xi')$ was defined earlier [Eq. (11)].

By extension, photons that have experienced collisions in the leaf canopy can also reach the point ξ_{-1} (Fig. 5). If $I_p(\xi'_{-2}, \underline{\Omega}_{-1} | \xi_{-3}, \underline{\Omega}_{-2})$ is the partial intensity at the point of interaction ξ'_{-2} [immediately after scattering into direction $\underline{\Omega}_{-1}$, $(\mu_{-1} > 0)$ in the leaf canopy; see Eq. (33)], then

$$I_p(\xi_{-1}, \underline{\Omega}_0 | \xi'_{-2}, \underline{\Omega}_{-1}) = \tilde{\sigma}_s(\xi_{-1}; \underline{\Omega}_{-1} \cdot \underline{\Omega}_0) \int_{4\pi} d\underline{\Omega}_{-2} \int_{-T}^{H} d\xi_{-3} I_p(\xi'_{-2}, \underline{\Omega}_{-1} | \xi_{-3}, \underline{\Omega}_{-2})$$
$$\times \frac{1}{\mu_{-1}} \exp\left\{\frac{1}{\mu_{-1}}\left[\int_{\xi'_{-2}}^{0} dt \sigma(t, \underline{\Omega}_{-1} | \bar{\xi}_{-3}, \underline{\Omega}_{-2}) + \tilde{\tau}(\xi_{-1}, 0)\right]\right\}, \quad \mu_0 < 0,$$

where (Fig. 5)

$$\bar{\xi}_{-3} = \begin{cases} \xi_{-3}, & \text{if } \xi_{-3} \in \text{leaf canopy}; \\ 0, & \text{if } \xi_{-3} \in A. \end{cases}$$

The incident partial intensity B_0 [cf. Equation (19)] can be evaluated as

$$B_0(\underline{\Omega}, \underline{\Omega}_0, \xi_{-1}) = B_0(\xi_0, \underline{\Omega} | \xi_{-1}, \underline{\Omega}_0) = k_A[(\xi_{-1}, \underline{\Omega}_0) \to (\xi_0, \underline{\Omega})]$$
$$\times \left[\int_{4\pi} d\underline{\Omega}_{-1} \int_{-T}^{0} d\xi_{-2} I_p^A(\xi_{-1}, \underline{\Omega}_0 | \xi_{-2}, \underline{\Omega}_{-1}) + \int_{2\pi+} d\underline{\Omega}_{-1} \int_{0}^{H} d\xi'_{-2} I_p(\xi_{-1}, \underline{\Omega}_0 | \xi'_{-2}, \underline{\Omega}_{-1})\right]$$

where $\mu_0 < 0$, and $\mu < 0$. The corresponding incident full intensity is simply

$$f_\downarrow(\underline{\Omega}) = \int_{2\pi-} d\underline{\Omega}_0 \int_{-T}^{0} d\xi_{-1} B_0(\underline{\Omega}, \underline{\Omega}_0, \xi_{-1}), \quad \mu < 0. \tag{36}$$

In radiative transfer it is customary to characterize the scattering behavior of a medium by its reflection operator $\mathscr{R}$ that defines the fraction of incident radiation that is reflected [cf. Eq. (24)]:

$$f_\uparrow(\underline{\Omega}) = (\mathscr{R} f_\downarrow)(\underline{\Omega}) = \frac{1}{\pi} \int_{2\pi-} d\underline{\Omega}' \rho_c(\underline{\Omega}', \underline{\Omega}) |\mu'| f_\downarrow(\underline{\Omega}'), \quad \mu > 0, \tag{37}$$

where $\rho_c(\underline{\Omega}', \underline{\Omega})$ is the canopy bidirectional reflection function. To ensure uniqueness of $\mathscr{R}$, it is defined such that it depends only on the characteristics of the medium in question. For instance, in the case of a leaf canopy with nondimensional leaves,

$$\mathscr{R} \equiv \mathscr{R}(\text{canopy parameters}). \tag{38}$$

However, in the presence of finite-dimensional scattering centers; it is not sufficient to specify the full intensity incident on the canopy; one requires partial intensities as well, and these depend on the interaction cross sections $\tilde{\sigma}$ and $\tilde{\sigma}_s$ of the atmosphere. To prove this, we consider two equal incident intensities $f_\downarrow^1(\underline{\Omega}) = f_\downarrow^2(\underline{\Omega})$ that may result from two differnt partial intensities $I_p^1(0, \underline{\Omega}, \underline{\Omega}') \neq I_p^2(0, \underline{\Omega}, \underline{\Omega}')$ and two different differential scattering cross sections $\tilde{\sigma}_s^1(\underline{\Omega}' \cdot \underline{\Omega}) \neq \tilde{\sigma}_s^2(\underline{\Omega}' \cdot \underline{\Omega})$, then [cf. Eq. (36)],

$$f_\downarrow^1(\underline{\Omega}) = \int_{4\pi} d\underline{\Omega}' \tilde{\sigma}_s^1(\underline{\Omega}' \cdot \underline{\Omega}) I_p^1(0, \underline{\Omega}, \underline{\Omega}'),$$
$$f_\downarrow^2(\underline{\Omega}) = \int_{4\pi} d\underline{\Omega}' \tilde{\sigma}_s^2(\underline{\Omega}' \cdot \underline{\Omega}) I_p^2(0, \underline{\Omega}, \underline{\Omega}').$$

Since $I_p^1 \neq I_p^2$, we obviously get two different partial intensities J in the leaf canopy [Eq. (31)], and, thus, different intensities $f_\uparrow$ reflected from the leaf canopy [Eq. (35)]. This means that for the same incident intensity $f_\downarrow$, the reflected intensity from the leaf canopy $f_\uparrow$ can be different, even if all the parameters characterizing the leaf canopy are held constant! It follows from this that the canopy reflection operator $\mathscr{R}_c$ depends not only on the canopy parameters but also on the atmospheric parameters as well, i.e. [cf. Eq. (38)]

$$\mathscr{R}_c \equiv \mathscr{R}_c(\text{canopy and atmospheric parameters}). \tag{39}$$

In this context, the leaf canopy can be considered as a live system, as opposed to the classical black box representation.

11. BALANCE EQUATION IN R^3

In Secs. 7–10, various transport problems in slab geometry were discussed. The cross sections $\sigma(z, z', \underline{\Omega}) | z'', \underline{\Omega}')$ and $\sigma_s(z'; \underline{\Omega}' \to \underline{\Omega} | z'', \underline{\Omega}'', z''')$ lead naturally to the concept of a partial intensity $J_\eta(z, \underline{\Omega} | z'', \underline{\Omega}'', z', \underline{\Omega}')$, with η denoting the incident states. In particular, an integral equality relating partial intensities J_η between successive collision orders [Eq. (28)] was found to exist. And also that photon transport between phase space triads can be expressed by an integral equation (31) involving partial intensities of assemblies of triads. Unlike the slab geometry problem, in R^n, $n = 2, 3$ the interaction cross sections do not depend on the current site of interaction, i.e., $\sigma = \sigma(\mathbf{r}, \underline{\Omega} | \mathbf{r}'', \underline{\Omega}')$ and $\sigma_s = \sigma_s(\mathbf{r}'; \underline{\Omega}' \to \underline{\Omega} | \mathbf{r}''', \underline{\Omega}'')$; (Fig. 1). This is so since the point $\mathbf{r}'$ can be uniquely defined as the point of intersection of two vectors $[(\mathbf{r}, -\underline{\Omega})$ and $(\mathbf{r}'', \underline{\Omega}')]$ that belong to one plane. In this section an integro-differential balance equation for partial intensity in R^3 will be formulated.

To derive a formal statement of balance for the partial intensity $I_p(\mathbf{r}, \underline{\Omega} | \mathbf{r}', \underline{\Omega}')$ in R^3 we begin with the relation between the partial intensity I_p and the full intensity I. Let the function $I_p(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}', \underline{\Omega}')$, $\xi > 0$, be the partial intensity of those photons at $(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega})$ whose previous phase space states were $(\mathbf{r}', \underline{\Omega}')$ and $(\mathbf{r}, \underline{\Omega})$. The full intensity I and the partial intensity I_p are connected by the following relationship

$$I(\mathbf{r}, \underline{\Omega}) = \lim_{\xi \to 0+} I_p(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}, \underline{\Omega}), \tag{40}$$

where

$$I_p(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}, \underline{\Omega}) = \int_{4\pi} \mathrm{d}\underline{\Omega}' \int_0^{s(\underline{\Omega}')} \mathrm{d}s' \, \mathrm{I}_p(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}' - s'\underline{\Omega}', \underline{\Omega}'). \tag{41}$$

Here, $s(\underline{\Omega}')$ is the distance between r and the boundary of V along the direction $-\underline{\Omega}'$. In the case $\underline{\Omega} = \underline{\Omega}'$, we need an additional definition

$$I(\mathbf{r}-, \underline{\Omega}') = \int_{V(\underline{\Omega}')} \mathrm{d}\mathbf{r}' I_p(\mathbf{r}-, \underline{\Omega}' | \mathbf{r}', \underline{\Omega}'), \tag{42}$$

where $\mathbf{r}- = \mathbf{r}' + |\mathbf{r} - \mathbf{r}'|\underline{\Omega}'$.

A balance equation for partial intensity I_p can be written, provided $\underline{\Omega} \neq \underline{\Omega}'$ and $\mathbf{r} \neq \mathbf{r}'$ (Fig. 6)

$$\frac{\partial}{\partial \xi} I_p(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}', \underline{\Omega}') + \sigma(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}', \underline{\Omega}') I_p(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}', \underline{\Omega}') = S(\mathbf{r}+, \underline{\Omega} | \mathbf{r}', \underline{\Omega}')\delta(\xi), \tag{43}$$

where

$$S(\mathbf{r}+, \underline{\Omega} | \mathbf{r}', \underline{\Omega}') = \int_V \mathrm{d}\mathbf{r}'' \int_{4\pi} \mathrm{d}\underline{\Omega}'' \sigma_s(\mathbf{r}-; \underline{\Omega}' \to \underline{\Omega} | \mathbf{r}'', \underline{\Omega}'') I_p(\mathbf{r}-, \underline{\Omega}' | \mathbf{r}'', \underline{\Omega}''). \tag{44}$$

Here, $\mathbf{r}+ = \lim_{\xi \to 0+} \mathbf{r} + \xi\underline{\Omega}$. The Dirac delta-function on the right-hand part of Eq. (43) means that the integral term in Eq. (44) has an impulse nature, i.e., valid at the point of interaction $\mathbf{r}$ only.

On the assumption that the canopy is non re-entrant, the following boundary condition to Eq. (43) can be written:

$$I_p(\mathbf{r}_s, \underline{\Omega} | \mathbf{r}_0, \underline{\Omega}_0) = B_0(\mathbf{r}_s, \underline{\Omega})\delta(\mathbf{r}_s - \mathbf{r}_0)\delta(\underline{\Omega} - \underline{\Omega}_0), \ \mathbf{n} \cdot \underline{\Omega} < 0,$$

where B_0 is the incident full intensity, $\mathbf{r}_s$ is a point on the boundary of the leaf canopy of volume V, $\mathbf{n}$ is an outward normal vector at the point $\mathbf{r}_s$ and $(\mathbf{r}_0, \underline{\Omega}_0)$ is the initial phase space point.

To reduce Eqs. (43)–(44) to standard expressions, the independence of σ and σ_s on the previous phase state $(\mathbf{r}', \underline{\Omega}')$ should be supposed, i.e., $\sigma = \tilde{\sigma}$ and $\sigma_s = \tilde{\sigma}_s$. Then [cf. Eq. (41)],

$$\begin{aligned} S(\mathbf{r}+, \underline{\Omega} | \mathbf{r}', \underline{\Omega}') &= \tilde{\sigma}_s(\mathbf{r}; \underline{\Omega}' \rightarrow \underline{\Omega}) \int_{4\pi} d\underline{\Omega}'' \int_V d\mathbf{r}'' I_p(\mathbf{r}-, \underline{\Omega}' | \mathbf{r}'', \underline{\Omega}'') \\ &= \tilde{\sigma}_s(\mathbf{r}; \underline{\Omega}' \rightarrow \underline{\Omega}) \int_{4\pi} d\underline{\Omega}'' \int_0^{s(\Omega'')} ds' I_p(\mathbf{r}-, \underline{\Omega}' | \mathbf{r}' - s'\underline{\Omega}'', \underline{\Omega}'') \\ &= \tilde{\sigma}_s(\mathbf{r}; \underline{\Omega}' \rightarrow \underline{\Omega}) I_p(\mathbf{r}-, \underline{\Omega}' | \mathbf{r}', \underline{\Omega}). \end{aligned}$$

We now integrate Eq. (43) with respect to $\mathbf{r}'$ and $\underline{\Omega}'$ to obtain [cf. (42)]

$$\begin{aligned} \int_{4\pi} d\underline{\Omega}' \int_V d\mathbf{r}' S(\mathbf{r}+, \underline{\Omega} | \mathbf{r}', \underline{\Omega}') &= \int_{4\pi} d\underline{\Omega}' \sigma_s(\mathbf{r}; \underline{\Omega}' \rightarrow \underline{\Omega}) \int_V d\mathbf{r}' I_p(\mathbf{r}-, \underline{\Omega}' | \mathbf{r}', \underline{\Omega}') \\ &= \int_{4\pi} d\underline{\Omega}' \tilde{\sigma}_s(\mathbf{r}; \underline{\Omega}' \rightarrow \underline{\Omega}) I(\mathbf{r}-, \underline{\Omega}') = \int_{4\pi} d\underline{\Omega}' \tilde{\sigma}_s(\mathbf{r}; \underline{\Omega}' \rightarrow \underline{\Omega}) I(\mathbf{r}, \underline{\Omega}'). \end{aligned}$$

By analogy,

$$\begin{aligned} &\int_{4\pi} d\underline{\Omega} \int_V d\mathbf{r}' \tilde{\sigma}(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}', \underline{\Omega}') I_p(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}', \underline{\Omega}') \\ &= \tilde{\sigma}(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega}) \int_{4\pi} d\underline{\Omega}' \int_V d\mathbf{r} I_p(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}', \underline{\Omega}') = \tilde{\sigma}(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega}) I_p(\mathbf{r} + \xi\underline{\Omega}, \underline{\Omega} | \mathbf{r}, \underline{\Omega}). \end{aligned}$$

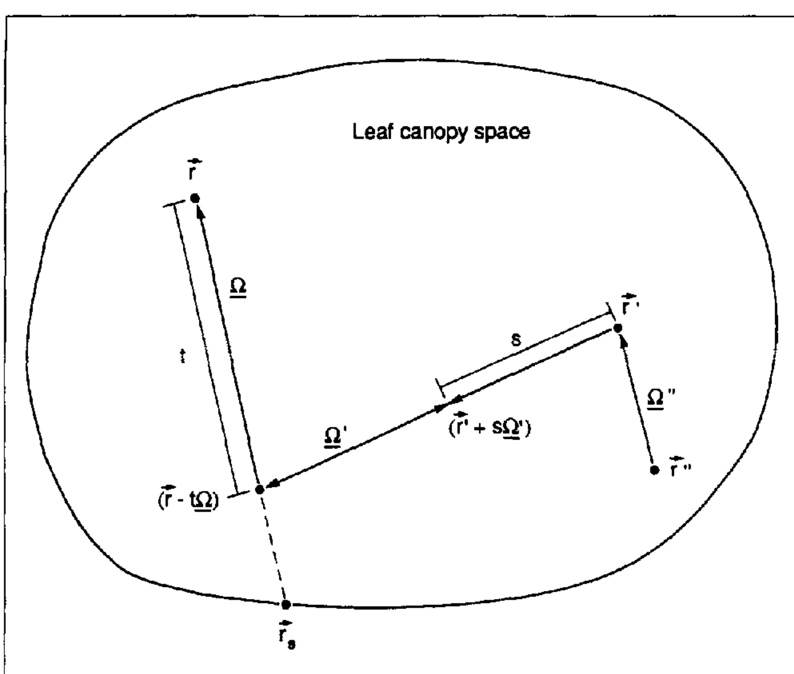

Fig. 6. Schematic illustration of photon transport in R^3 [cf. Eq. (40)].

We next consider the last expression at the point $\xi = 0+$. Using Eq. (40) gives

$$\frac{\partial}{\partial \xi} I(\mathbf{r} + \xi \Omega, \Omega)\bigg|_{\xi=0} + \tilde{\sigma}(\mathbf{r}, \Omega) I(\mathbf{r}, \Omega) = \int_{2\pi} d\Omega' \sigma_s(\mathbf{r}; \Omega' \to \Omega) I(\mathbf{r}, \Omega'),$$

which is a standard integro-differential equation of transfer in R^3.

It can be seen that the equality

$$I_p(\mathbf{r}+, \Omega | \mathbf{r}', \Omega') = S(\mathbf{r}+, \Omega | \mathbf{r}', \Omega') \tag{45}$$

is valid as an analogue of Eq. (33) in R^3. Then, solving formally Eq. (43) with Eq. (45), we obtain

$$I_p(\mathbf{r} + \xi\Omega, \Omega | \mathbf{r}', \Omega') = I_p(\mathbf{r}+, \Omega | \mathbf{r}', \Omega') \exp\left[-\int_0^{\xi} dt \sigma(\mathbf{r} + t\Omega, \Omega | \mathbf{r}', \Omega') \right] H(\xi), \tag{46}$$

where $H(\xi)$ is a Heaviside function defined in Eq. (9). Finally, substituting Eq. (46) in Eq. (44), we have

$$S(\mathbf{r}+, \Omega | \mathbf{r}', \Omega') = \int_V d\mathbf{r}'' \int_{4\pi} d\Omega'' \sigma_s(\mathbf{r}-; \Omega' \to \Omega | \mathbf{r}'', \Omega'')$$
$$\times I_p(\mathbf{r}'+, \Omega' | \mathbf{r}'', \Omega'') \exp\left[-\int_0^{|\mathbf{r}-\mathbf{r}'|} ds' \tilde{\sigma}_s(\mathbf{r}' + s'\Omega', \Omega' | \mathbf{r}'', \Omega'') \right].$$

12. SYNOPSIS

In this paper an attempt at developing a formalism for photon transport in media with finite-dimensional scattering centers has been made. Although the details are those relating to a leaf canopy, it appears that the principles developed here are also applicable in studies on light scattering from rough surfaces that show opposition brightening.[17]

A vegetation canopy can be idealized as aggregations or clumps of leaves distributed randomly in free space (vacuum). The intervening free spaces between the clumps constitute the voids. The voids are convolutely shaped and multiply connected three-dimensional structures broken along those regions where leaves are present. Consequently, a leaf canopy can be abstracted as a binary medium of randomly distributed leaf clumps and voids.[12]

The transport of energy by radiation can be visualized as consisting of two events; the mean length of photon free path (along this length a photon streams without a change in its direction of flight) and the scattering event (where the direction of photon travel is altered). These two events are characterized by the total interaction cross section σ and the differential scattering cross section σ_s. Starting with leaf size, orientation and spatial distribution functions, the interaction cross sections are derived for an aggregation of finite dimensional leaves [Eqs. (2) and (15)].

In order to describe the rules of photon movement in media with finite-size leaves and voids, one has to know not only the interaction cross sections for the leaf aggregates (Sec. 3) but also the distribution of voids along the path of photon travel.[16] So, we proposed the following picture of photon interactions in a leaf canopy. Suppose that the event $A \equiv \{$two successive interactions between photons and leaves occurred in the neighborhoods of $\mathbf{r}''$ and $\mathbf{r}'$, where $\mathbf{r}' = \mathbf{r}'' + s'\Omega', s' > 0\}$ is realized (Fig. 1). Then, the total interaction cross section at $\mathbf{r}$ $(\mathbf{r} = \mathbf{r}' + s\Omega; s > 0)$ for those photons with previous phase space state $(\mathbf{r}'', \Omega')$ can be represented as the product of the interaction cross section for the leaf aggregate $\tilde{\sigma}(\mathbf{r}, \Omega)$ and the probability $[1 - q(\mathbf{r}, \Omega | \mathbf{r}'', \Omega')]$ of encountering a leaf aggregate at $(\mathbf{r}, \Omega)$ traveling from $(\mathbf{r}'', \Omega')$ [Eq. (6)]. Since a collision proceeds a scattering event, the scattering cross section must be similarly modified [Eq. (18)]. The realization of the event A means that there are no interaction centers between $\mathbf{r}''$ and $\mathbf{r}'$. In which case, a photon from $\mathbf{r}'$ can unimpededly reach the previous site of interaction $\mathbf{r}''$ along $-\Omega'$. This will provide us the proper mechanism for describing the hot spot effect or opposition brightening.

The two cross sections can be succiently written as a transfer kernel k_t that denotes the interactions as photons travel from phase space point x_i to x_{i+1}, provided their previous points of interaction were x_{i-1} and x_{i-2} [Eq. (20); Fig. 3(a)]. Thus, photon transport between successive points of interaction cannot be described as a Markov chain. Conceptually, this means that the transfer kernel cannot be visualized as consisting of two jointed straight lines, as in classical transport theory, but as two jointed broken lines (Fig. 3).

Yet another interesting feature is that the slab geometry problem does not exist if one considers scattering centers of finite area. It is, however, possible to introduce the parameter characteristic length of the scatterer and assume horizontal homogeneity in a statistical sense, and consider dependence on the depth variable ξ only. Nevertheless, in this "slab geometry", one must retain explicitly the depth coordinate of the most recent point of interaction in the argument lists [as in Eq. (13)]. In R^n, $n = 2, 3$, however, the coordinates of this point can be uniquely defined as the intersection of two vectors. The standard problem of illumination by a monodirectional source in slab geometry (Sec. 7) with a reflecting boundary (Sec. 8) can be formulated as a successive collisions approach [Eqs. (21) and (27), respectively]. In the latter case, it is necessary to define the transfer kernel $\bar{k}$ [Eq. (26)] as the sum of two kernels: k_1—between two points of interaction in the canopy [Eq. (20); Figs. 3(a)–(c)], and k_2—between the same two points but through an intermediary reflection event at the soil surface [Eq. (25); Figs. 3(d)–(f)].

Although photon travel between successive points of interaction is not a Markov process, transport between successive assemblies of three points is Markovian because such transport is independent of the previous triad (Fig. 4). This recognition requires us to introduce the concept of a partial or a conditional intensity J [Eq. (28)] or I_p [Eq. (33)] that describes the energy fluence at any phase space point comprising of photons that have had a common history; to obtain the full intensity I it is only needed to summarize over all photon histories [Eq. (29)]. One can now relate partial intensities of successive triads by an integral equation (31), the kernel of which is the superposition of transfer from one point of interaction to the next.

A topic of some interest is that dealing with the interactions between a leaf canopy and the adjacent atmosphere. To solve the leaf canopy problem it is necessary to specify as initial data the partial intensity incident from the atmosphere. One important result is that the reflection operator $\mathscr{R}$ of the canopy, as defined by Eq. (37), depends not only on the leaf canopy characteristics [cf. Eq. (38)], but also on the atmospheric parameters as well [Eq. (39)].

Finally, to gain physical insight, we derived in R^3 the integro-differential balance equation [Eqs. (43)–(44)] supposing that Eq. (40)–(42) are valid. However, we cannot derive the one-dimensional analogue of this balance equation because it is not possible to pose the strict slab geometry problem for a canopy with finite area scatterers. With respect to full intensity there exist neither an integro-differential nor even an integral transport equation. An ancilliary point is that, in the event the size of the scatterers tends to zero, the proposed formalism collapses to the standard transport theory.[3]

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