Utilization of Normal Mode Initial Conditions for Detecting Errors in the Dynamics Part of Primitive Equation Global Models[1]

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ABSTRACT

When a global atmospheric basic state has constant angular velocity and its temperature varies with altitude only, there exist normal mode solutions to the linearized global primitive equations. The use of these normal modes, which have known behavior in time, is superior to the use of the Rossby-Haurwitz wave as initial conditions for detecting errors in the dynamics part of primitive equation global models. With these initial conditions, integration through only one time step is sufficient to detect many formulation and coding errors. Other tests are still required for detecting problems of nonlinear instability and conservation of integral properties, however.

1. Introduction

Since Phillips' work (1959), Rossby-Haurwitz waves have been used commonly as initial conditions for testing the formulation and coding of the dynamics part of primitive equation global models (grid or spectral) (e.g., Hoskins, 1973; Monaco and Williams, 1975). These Rossby-Haurwitz waves are the eigensolutions for the oscillations of a nondivergent, barotropic, thin fluid layer on the surface of a sphere. A test with these initial conditions is sufficient to detect many model errors, since after a few days of running, the waves should retain their symmetry, or antisymmetry, with respect to the equator and the wave shape should remain smooth with the wave evolution in time being regular. However, because primitive equation models allow divergence, the initial Rossby-Haurwitz wave changes character with its shape varying and its trough and ridge lines becoming distorted as the model continues running. Since there is no simple and objective method to determine the change in wave shape even when the results pass the symmetry and smoothness test, there is still uncertainty concerning the correctness of the model.

We have found that a more sensitive and powerful test is to use the eigensolutions for the free oscillations of the linearized primitive equations for the adiabatic atmosphere (referred to here as normal modes) as initial conditions. The properties and the method of construction of these normal modes have been studied extensively by many authors (e.g., Longuet-Higgins, 1968; Kasahara, 1976). These solu-

tions are commonly used in tidal studies (e.g., Siebert, 1961; Chapman and Lindzen, 1970); in objective analysis schemes (e.g., Flattery, 1971); in spectral analysis of global data (e.g., Kasahara, 1976); in nonlinear normal model initialization (e.g., Machenhauer, 1977); in the design of schemes for numerical time integration (Daley, 1980); and in spectral modeling (Kasahara, 1977, 1978).

Since these normal modes have known behavior in time and are exact solutions of the linearized primitive equations, the normal modes should serve as better initial conditions than the Rossby-Haurwitz wave solutions in the testing for errors in the formulation and the coding of the dynamics part of primitive equation global models. Slightly less obvious is that once the existence of any errors is revealed, the equations that contain these errors can be easily identified. This will be detailed in the following sections.

2. Normal mode initial conditions for testing

The horizontal structure of the normal modes is the Hough function, which can be specified by its equivalent depth h_0, zonal wavenumber, meridional mode and wave class. Having specified these parameters, the latitudinal structure of the perturbation velocities and height (u', v', h'), along with the nondimensional eigenfrequency $\bar{\sigma}$, can be obtained by using computational methods given in the literature on Hough functions (e.g., Kasahara, 1976).

The simplest vertical structure for the normal modes is for the case of an isentropic basic state stratification. The equivalent depth in this case is the scale height at the surface (Siebert, 1961). Since the isentropic atmosphere and "shallow water" are phys-

ically and mathematically analogous (Siebert, 1961), it is apparent that horizontal velocities are uniform in height and can be specified as $u = cu' \cos(k\lambda)$ and $v = cv' \sin(k\lambda)$, where c is an arbitrary constant, λ longitude and k the zonal wavenumber. In addition, the surface pressure p_s can be specified by $p_s = p_0 + p_s' = p_0 + c\rho_0 g h' \cos(k\lambda)$, where p_0 and ρ_0 are the surface pressure and density of the basic state and g is the acceleration due to gravity. The temperature at any level can be found immediately, since the potential temperature and the pressure are already known. The pressure at the top of the model should be zero. If the model formulation does not permit this, the solution constructed in this way is nearly exact if a very small value for the pressure at the top of the model is used.

Depending on the model formulation, there are some situations for which the isentropic normal mode initial conditions cannot detect errors associated with the $\partial \mathbf{v}/\partial s$ terms in the equations, where s is the vertical coordinate. For instance, in an advective form (as opposed to the flux form) model, an incorrect factor or an incorrect sign multiplied by the $\partial \mathbf{v}/\partial s$ terms will escape detection (since $\partial \mathbf{v}/\partial s = 0$). Moreover, in a sigma (σ) coordinate system, since $\dot{\sigma}$ equals zero; errors in the $\dot{\sigma}$ terms may escape detection. Thus, as a supplementary test, one may use the isothermal atmosphere, $T = T_0$, as the basic state for the initial conditions. In this case, the equivalent depth is γH, where γ is the ratio of the specific heats ($\gamma = c_p/c_v$) and H ($=RT_0/g$) is the scale height. Furthermore, the vertical structure equations are only slightly more complicated than those in the isentropic case (Siebert, 1961; Geller, 1970). The initial conditions are given as follows[2]:

$$u = cu'e^{z/2}(dY/dz - \tfrac{1}{2}Y)\cos(k\lambda),$$

$$v = cv'e^{z/2}(dY/dz - \tfrac{1}{2}Y)\sin(k\lambda),$$

$$T = T_0 - ch'T_0\kappa e^{z/2}Y\cos(k\lambda),$$

$$p_s = p_0 + c\rho_0 g h'\cos(k\lambda),$$

where $z = \tilde{z}/H = \ln(p_0/p)$, $\kappa = R/c_p$, $Y = \exp[-(\tfrac{1}{4} - H\kappa h_0^{-1})z^{1/2}]$, and $\tilde{z}$ is altitude.

Also, it is known that the angular speed ν of the normal mode is $\nu = 2\tilde{\sigma}\Omega$, where Ω is the earth's angular speed (when $\tilde{\sigma}$ is positive, the wave moves eastward). Moreover, if there is a basic flow of solid rotation with angular speed ω, then $\nu = 2\tilde{\sigma}(\Omega + \omega) + \omega$. Somewhat arbitrarily, we elected to use the mixed Rossby-gravity mode with wavenumber 1 for our tests. We also specified ω as zero and an arbitrary nonzero value in separate tests. The value of c was set such that the maximum v velocity was 15 m s^{-1}.

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[2] Note that since we are concerned with free modes, these solutions are for zero vertical velocity at $p = p_s$ and $p = 0$, i.e., we are dealing with a single external mode.

3. Discussion

In the previous section we chose to use the normal modes of the primitive equations instead of those of the model for two reasons. The construction of the model normal modes is a major undertaking and must be performed for an individual model (Williamson and Dickinson, 1976). Moreover, if the model with model normal mode initial conditions produces incorrect results, it may not be easy to tell if the error(s) exist(s) in the model or in the program that generates the initial conditions.

Since normal modes of the primitive equations are used, it is necessary to have a sufficient number of vertical levels to simulate them properly. However, isentropic initial conditions are good for any number of levels.

For short-term tests (e.g., 12 h) and for small wave amplitude, it is expected that the nonlinear effects and the differences between normal modes of the primitive equations and those of the model are negligible. Thus, in the runs starting from the normal mode initial conditions, the wave shape should change very little and the phase change should be very close to the theoretical estimate. In this sense, the normal model initial conditions are superior to the Rossby-Haurwitz wave initial conditions.

The model integration for one time step starting from the normal mode initial condition is sufficient to detect most errors. If, after one time step, any of the prognostic variables deviates from its analytical solution, it is immediately apparent that there is something wrong in the formulation or the coding of the equation governing that variable. This information can greatly facilitate the eventual locating of the error(s).

It should be stressed that although we have found these normal mode initial condition tests very useful, this is only one step in the testing of such models. The correct formulation and coding of the nonlinear terms need to be tested as do the nonlinear stability and conservation properties of the model. Our experience has shown that normal mode initial condition testing is a very simple and efficient way to quickly locate and correct many of the errors in formulating and coding the dynamics part of global primitive equation models, however.

REFERENCES

Chapman, S., and R. S. Lindzen, 1970: *Atmospheric Tides—Thermal and Gravitational.* Gordon and Breach, 200 pp.

Daley, R., 1980: The development of efficient time integration schemes using model normal modes. *Mon. Wea. Rev.,* **108,** 100–110.

Flattery, T. W., 1971: Spectral models for global analysis and forecasting. *Proc. Sixth AWS Technical Exchange Conf.,* U.S. Naval Academy, Air Weather Service Tech. Rep. 242, 42–53 [NTIS AD724093].

Geller, M. A., 1970: An investigation of the lunar semidiurnal tide in the atmosphere. *J. Atmos. Sci.,* **27,** 202–218.

Hoskins, B. J., 1973: Stability of the Rossby-Haurwitz waves. *Quart. J. Roy. Meteor. Soc.,* **99,** 723–765.

Kasahara, A., 1976: Normal modes of ultralong waves in the atmosphere. *Mon. Wea. Rev.,* **104,** 669–690.

——, 1977: Numerical integration of the global barotropic primitive equations with Hough harmonic expansions. *J. Atmos. Sci.,* **34,** 687–701.

——, 1978: Further studies on a spectral model of the global

barotropic primitive equations with Hough harmonic expansions. *J. Atmos. Sci.,* **35,** 2043–2051.

Longuet-Higgins, M. S., 1968: The eigenfunctions of Laplace's tidal equations over a sphere. *Phil. Trans. Roy. Soc. London,* **A262,** 511–607.

Machenhauer, B., 1977: On the dynamics of gravity oscillations in a shallow water model, with applications to normal model initialization. *Beit. Phys. Atmos.,* **50,** 253–271.

Monaco, A. V., and R. T. Williams, 1975: An atmospheric global prediction model using a modified Arakawa differencing scheme. Naval Postgraduate School, Rep. NPS-51 Wu75041 [Available from Naval Postgraduate School Library, Monterey, CA 93940].

Phillips, N. A., 1959: Numerical integration of the primitive equations on the hemisphere. *Mon. Wea. Rev.,* **87,** 333–345.

Siebert, M., 1961: Atmospheric tides. *Advances in Geophysics,* Vol. 7, Academic Press, 105–182.

Williamson, D. L., and R. E. Dickinson, 1976: Free oscillations of the NCAR global circulation model. *Mon. Wea. Rev.,* **104,** 1372–1391.

A Differential Advection Model of Orographic Rain

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ABSTRACT

The history of the theory of orographic rain, and recent evidence against the "stable upglide" model, are briefly reviewed. A new model is proposed in which the blocking of low level air by a mountain causes approaching cold air to override the warm air, producing an unstable layer upstream of the mountain. This model is compared with recent observations in the Cascades and San Juan Mountains. The suggestion is that under some conditions it is the blocking action of the mountain, rather than forced ascent, which causes enhanced precipitation.

1. Introduction

The influence of mountains on the global distribution of precipitation, particularly the upslope rain-rain shadow contrast across major mountain ranges, is well documented. The idea that upslope rain is caused by the adiabatic cooling of moist air forced to rise following the topography, was put forward in the 19th century as one of the first applications of modern thermodynamics to the atmosphere. An especially clear description of this process was given in John Tyndall's *The Forms of Water* published in 1872. Certainly this theory was well established by 1920 when the Norwegian physicist Vilhelm Bjerknes used this "simplest" of rainfall situations as a reference point in his discussion of the more complex rain systems in moving frontal cyclones.

The theory of smooth orographic lifting seems to have been first questioned by Mr. L. C. W. Bonacina in a meeting of the Royal Meteorological Society in

1945. He pointed out that orographic rain does not occur every time a moisture laden wind approaches a mountain slope. Heavy upslope rain seems to require a combination of orography and some sort of meteorological preconditioning associated with an existing weather disturbance. Bonacina further suggested that the "preconditioning" is most likely the establishment of near instability in the column of air approaching the mountain. In the same forum two years later, Douglas and Glasspoole (1947) showed that the only special condition needed to account for orographic rains in Wales and Scotland was a deep layer of nearly saturated air—thus promoting the smooth ascent idea. It is evident from the discussion which followed their presentation, that the audience was not fully convinced as to the correctness or the generality of their conclusions. The observation that orographic rain often begins well upstream of the mountain was mentioned as counter evidence. Yet,