Interaction of Photons in a Canopy of Finite-Dimensional Leaves

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The physics of neutral particle interaction for photons traveling in media consisting of finite-dimensional scattering centers that cross-shade mutually is investigated. A leaf canopy is a typical example of such media. The leaf canopy is idealized as a binary medium consisting of gaps (voids) and regions with phytoelements (turbid phytomedium). Gaps through which photons travel unimpededly are assumed to be randomly distributed. The mathematical approach for characterizing the structure of the host medium is considered in detail. In this approach, the leaf canopy is represented by a combination of all possible open oriented spheres. With rigorous definitions and notations, dependence of the extinction coefficient on the phase-space coordinates of the previous interaction center is shown to be a logical consequence. Specifically, the extinction coefficient at any phase-space location in a leaf canopy is the product of the extinction coefficient in the turbid phytomedium and the probability of absence gaps at that location. Using a similar approach, an expression for the differential scattering coefficient is derived. Numerical results are presented to illustrate the influence of canopy parameters and direction of photon travel on the extinction coefficient.

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INTRODUCTION

The problem of photon transport in plant stands arises in the context of optical remote sensing of vegetated land surfaces, land surface climatology, and plant physiology. For instance, in the application of remote sensing from satellite-based sensors to vegetated land surfaces, an understanding of the spectral response resulting from the aggregation of leaves in a canopy and the intervening atmosphere is required. The physics of this problem is most conveniently posed as a photon transport equation, the solution of which is the remote spectral measurement. This sets the context for our presentation; we now begin with a precise statement of the problem.

Consider a region D of three-dimensional space filled with finite-dimensional planar leaves of given optical properties. For purposes of photon transport, it is sufficient to characterize the leaves by probability distributions of their location, size, shape, and orientation of normals. It is supposed that photons interact with leaves only. Therefore, we ignore photon interactions with the optically active elements of the atmosphere inside the region D. A photon traversing an elementary path in region D can be assigned some probability of either being absorbed or scattered into another direction. To describe these events quantitatively in transport theory, the concept of extinction and scattering coefficients are required. In this paper we derive expressions for these coefficients.

Ross (1981) defined these coefficients on the basis of ideas extensively used in atmospheric optics and nuclear reactor physics—the so-called turbid plate medium theory (Shifrin, 1953). In many cases this approach leads to acceptable results (Shultis and Myneni, 1988). Nevertheless, the turbid plate medium analogy for transfer processes in leaf canopies is a rather crude, and even incorrect, model. To confirm this, one only has to compare solutions of the transport equation for a plant canopy with Monte Carlo models (Ross and Marshak, 1989; Antyufeev and Marshak, 1990). What, then, is the main difference between transport problems in an atmosphere and a leaf canopy?

As a rule, an atmosphere (nuclear reactor) is a multilayer (multizone) medium. The geometry (or structure) of layers is strictly determined. Inside any layer, other than vacuum, the probability of scatter or absorption for a photon traveling an elementary path is greater than zero. This property is violated in a leaf canopy. For example, in a leaf canopy there are gaps of finite size (voids) through which a photon can travel without hindrance; that is, the probability of an interaction is zero. In other words, a leaf canopy can be considered as a binary medium—voids and regions filled with phytoelements. The distribution of gaps can be assumed to have a random character. It is noteworthy that a similar problem arises in radiative transfer studies in decks of broken clouds (Avaste and Vainikko, 1973; Vainikko, 1973).

Thus, we begin with the proposition that D consists of two regions. The first is a turbid medium filled densely with phytoelements. We shall call it turbid phytomedium or, for short, phytomedium. The second is a region of randomly distributed free spaces or voids without phytoelements. The region D comprising the phytomedium and voids is our leaf canopy. We introduce an elementary volume $V(\vec{r},\mathbf{\Omega})$ where $\vec{r}$ is the three-dimensional spatial coordinate of a photon and $\mathbf{\Omega}$ is its direction of travel. Clearly, the fate of a photon depends on the availability of phytoelements inside V. If V belongs to the phytomedium, the fate of a photon is determined by the coefficients of extinction and scattering (Ross, 1981; Shultis and Myneni, 1988; Marshak, 1989). If, on the other hand, V belongs to the free space, a photon does not interact with a phytomedium. It follows from this that the mean free path of a photon (the distance between two successive interactions) is the sum of two random values—the first is the length of photon travel in the free space, and the second is the length of photon free path in the phytomedium. Thus, in order to describe the rules of photon movement in a leaf canopy, one has to know not only the coefficients of extinction and scattering of turbid phytomedium but also the distribution of voids along the path of photon travel.

Kuusk (1985) considered dependence of the distribution of free spaces (along the path of photon travel) with respect to space and angular variables (also see Nilson and Kuusk, 1989). The probability Q that a point $\vec{r}$ inside a leaf canopy can be viewed from two points $\vec{r}'$ and $\vec{r}''$ was calculated as

$$Q = P(\vec{r}, \xi, \mathbf{\Omega})P(\vec{r}, \xi', -\mathbf{\Omega}')C_{\mathrm{HS}}(\vec{r}, \xi, \xi', \mathbf{\Omega}, \mathbf{\Omega}'),$$

where $P(\vec{r},\xi,\mathbf{\Omega})$ is a permitivity function, $\xi = |\vec{r} - \vec{r}''|$, $\xi' = |\vec{r} - \vec{r}'|$, $\mathbf{\Omega} = (\vec{r} - \vec{r}'')/\xi$, $\mathbf{\Omega}' = (\vec{r}' - \vec{r})/\xi'$, and $C_{\mathrm{HS}}(\vec{r},\xi,\xi',\mathbf{\Omega},\mathbf{\Omega}')$ is a correction factor. The subscript HS is connected with the so-called hot spot effect. The product

$$Q'(\vec{r}, \xi, \xi', \mathbf{\Omega}, \mathbf{\Omega}') = P(\vec{r}, \xi, \Omega)C_{\mathrm{HS}}(\vec{r}, \xi, \xi', \mathbf{\Omega}, \mathbf{\Omega}')$$

can be interpreted as a distribution function of voids from the point $\vec{r}$ along the direction $\mathbf{\Omega}$ of photon travel. The function Q' clearly depends on the point $\vec{r}'$ and direction $\mathbf{\Omega}'$. Therefore, the extinction coefficient in a leaf canopy depends not only on the phase-space location of photon travel $(\vec{r},\mathbf{\Omega})$ but also on the previous point of interaction $(\vec{r}',\mathbf{\Omega}')$(Myneni et al., 1991).

Strictly stated, the outcome of a current interaction for a photon is influenced by its history. In this fashion, one may attribute "memory" to a photon. Nevertheless, it must be clearly understood that the correlation of photon fates is a direct consequence of the binary nature of the host medium. From a probabilistic point of view, the fate of photons at $(\vec{r},\mathbf{\Omega})$ can be evaluated provided the event $A(\vec{r}',\xi',\mathbf{\Omega}')$ = {two successive interactions between photons and phytoelements occuring in the neighborhoods of $\vec{r}'$ and $\vec{r} = \vec{r}' +$

$\xi'\mathbf{\Omega}'$, $\xi' > 0\}$ has occurred. This allows us to represent the extinction coefficient at $(\vec{r} + \xi\mathbf{\Omega},\mathbf{\Omega})$ for those photons with previous state $(\vec{r}',\mathbf{\Omega}')$ as a product of the extinction coefficient in the phytomedium $\tilde{\sigma}(\vec{r} + \xi\mathbf{\Omega},\mathbf{\Omega})$ (which does not depend on their previous state) and the probability $[1 - q(\vec{r} + \xi\mathbf{\Omega}, \mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')]$ of encountering a phytomedium (which depends on their previous state). The realization of the event $A(\vec{r}',\xi',\mathbf{\Omega}')$ means that there are no phytoelements between $\vec{r}'$ and $\vec{r} = \vec{r}' + \xi'\mathbf{\Omega}'$. In which case, the probability $q(\vec{r} - \xi\mathbf{\Omega}', -\mathbf{\Omega}'|\vec{r}',\mathbf{\Omega}') = 1$, if $0 \leqslant \xi \leqslant \xi'$, that is, a photon from $(\vec{r}, -\mathbf{\Omega}')$ can unimpededly reach the previous site of interaction $(\vec{r}',\mathbf{\Omega}')$. These considerations are the foundation of our development of the extinction and scattering coefficients.

The plan of this article is as follows. In the next section the extinction coefficient for a turbid phytomedium is described. The dependence on previous state for photon interactions is rationalized in the third section. In the fourth section, the architecture of a leaf canopy model is discussed. The fifth and sixth sections are devoted to the derivation and analysis of the probability $q(\vec{r} + \xi\mathbf{\Omega},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')$. In the seventh section, these results are applied to the scattering coefficient. A final section considers some numerical examples. The Appendix deals with the cross-shading effect that occurs naturally in the definition of the extinction coefficient in plate turbid medium models.

THE INTERACTION BETWEEN PHOTONS AND TURBID PHYTOMEDIUM

We suppose that the elementary volume $V(\vec{r},\mathbf{\Omega})$ belongs to the turbid plate phytomedium. The theory of radiative transfer is well developed for treating problems in astrophysics (Chandrasekhar, 1960) and atmospheric optics (Kondratyev, 1969). Ross (1981) generalized this theory to the plate turbid medium consisting of nondimensional but oriented scatterers. Attempts to include the leaf size in the framework of radiative transfer theory were made by Marshak (1989) and Myneni et al. (1991). In this section we shall closely follow the ideas presented in the latter.

Let $\tilde{\sigma}$ denote the extinction coefficient in the turbid plate medium. It consists of two parts—the first characterizes the scattering event and the second, the absorption event, for a photon traveling an elementary distance. In case of oriented plates, both events depend on the direction of photon travel, in addition to space variables—a point first emphasized by Ross (1981).

We begin with a quantitative description of the architecture of plate turbid medium. Let $[(2\pi)^{-1}h_L(\vec{r},a_L,\mathbf{\Omega}_L)]$ be the probability density that a leaf of area a_L at a point $\vec{r}$ has a normal $\mathbf{\Omega}_L \sim (\theta_L,\phi_L)$, directed away from its upper surface into a unit solid angle about $\mathbf{\Omega}_L$ in the upper hemisphere. Thus

$$\frac{1}{2\pi}\int_0^\infty\int_{2\pi^+} h_L(\vec{r}, a_L, \mathbf{\Omega}_L)\ da_L\ d\mathbf{\Omega}_L = 1,$$

where $2\pi^+$ is the upper hemisphere. We assume that the random variables a_L and $\mathbf{\Omega}_L$ are independently distributed; thus,

$$\frac{1}{2\pi}h_L(\vec{r}, a_L, \mathbf{\Omega}_L) \equiv \rho_L(\vec{r}, a_L)\frac{1}{2\pi}g_L(\vec{r}, \Omega_L).$$

Here, ρ_L is the probability density of leaf size distribution, and $g_L/2\pi$ is the probability density of leaf normal orientation distribution. Models for the latter are available in literature (Bunnik, 1978; Goel, 1988; Nilson and Kuusk, 1989).

Let $n_L(\vec{r},a_L)$ be a function that relates the number of leaves in the elementary volume to leaf size a_L. Then, $[n_L(\vec{r},a_L)\rho_L(\vec{r},a_L)g_L(\vec{r},\mathbf{\Omega}_L)a_L|\mathbf{\Omega}\cdot\mathbf{\Omega}_L|/2\pi]$ is the area projected on a plane perpendicular to $\mathbf{\Omega}$ by the leaves in elementary volume around $\vec{r}$ with size a_L and a normal directed into a unit solid angle about $\mathbf{\Omega}_L$. Integrating the above over all leaf areas a_L and over all orientations $\mathbf{\Omega}_L$ gives the extinction coefficient $\tilde{\sigma}$,

$$\tilde{\sigma}(\vec{r}, \mathbf{\Omega}) = G(\vec{r}, \mathbf{\Omega})\int_0^\infty a_L n_L(\vec{r}, a_L)\rho_L(\vec{r}, a_L)\ da_L, \quad (1)$$

where the function,

$$G(\vec{r}, \mathbf{\Omega}) = \frac{1}{2\pi}\int_{2\pi^+} g_L(\vec{r}, \mathbf{\Omega}_L)|\mathbf{\Omega}\cdot\mathbf{\Omega}_L|\ d\mathbf{\Omega}_L, \quad (2)$$

is the total leaf area projected on a plane perpendicular to the direction $\mathbf{\Omega}$, by leaves of all orientations (Ross, 1981). The leaf area density function $u_L(\vec{r})$ introduced by Ross (1981) is equivalent to

$$u_L(\vec{r}) = \int_0^\infty a_L n_L(\vec{r}, a_L)\rho_L(\vec{r}, a_L)\ da_L. \quad (3)$$

If all the leaves in $V(\vec{r},\mathbf{\Omega})$ are of the same size a_0, then $\rho_L(\vec{r},a_L) = \delta(a_L - a_0)$, and

$$\tilde{\sigma}(\vec{r}, \mathbf{\Omega}) = a_0 n_L(\vec{r}, a_0)G(\vec{r}, \mathbf{\Omega}) = u_L(\vec{r})G(\vec{r}, \mathbf{\Omega}), \quad (4)$$

where

$$u_L(\vec{r}) = a_0 n_L(\vec{r}, a_0) \tag{5}$$

for any a_0.

It should be noted that it is not easy to obtain the leaf area density u_L at a point $\vec{r}$ using formulae (3) or (5). To measure u_L, inclined point quadrats method and stratified clip method, or other methods can be employed (see Ross, 1981; Myneni et al., 1989). A recent development in models for leaf area density is fractal-based theory (Myneni et al., 1990). One important point, however, is that two finite volumes with the same leaf area density and orientation distribution might have different degrees of mutual shading. The use of Eq. (4) leads to the same extinction coefficient $\bar{\sigma}(\vec{r},\mathbf{\Omega})$. In order to account for overlapment or mutual shading between leaves in a finite volume, a dimensionless function χ was introduced by Myneni et al. (1991). Those ideas are expanded using a simple model for χ in the Appendix.

DEPENDENCE ON THE PREVIOUS STATE

In this section we rationalize the dependence of photon fate on its previous phase-space state. Since photons traveling in parallel or nearly parallel directions, separated by an infinitesimally small spatial extent, are likely to be intercepted by the same leaf or pass without hindrance through the same gap, Myneni et al. (1991) introduced photon interaction coefficients that depend on the previous point of interaction. This imbues the coefficients with an important property, namely, if after a scattering act, a photon were to trace its previous trajectory, it is likely to experience its recent history with unit probability. Since these arguments are intuitive, we shall give them a rigorous basis here.

We denote a photon by its phase-space location $(\vec{r},\mathbf{\Omega})$ and an elementary volume about $(\vec{r},\mathbf{\Omega})$ by $V(\vec{r},\mathbf{\Omega})$. Here, $\vec{r} \sim (x,y,z) \in D$ is a point of the three-dimensional space, D is a region in the leaf canopy, and $\mathbf{\Omega} = (\Omega_x,\Omega_y,\Omega_z)$ $(\Omega_x^2+\Omega_y^2+\Omega_z^2=1)$ is a unit vector along the direction of photon travel. The following definitions are necessary for further development.

Definition 1. We say that a photon $(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})$, $\xi > 0$, has a previous state $(\vec{r}',\mathbf{\Omega}')$, if two successive interactions between the photon and scattering

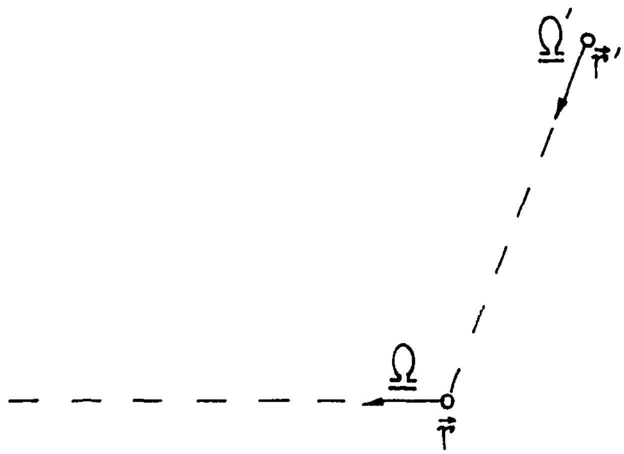

Figure 1. The elementary volumes $V(\vec{r},\mathbf{\Omega})$ and $V(\vec{r}',\mathbf{\Omega}')$, where interactions occuring are denoted as points. The point $(\vec{r}',\mathbf{\Omega}')$ preceeds point $(\vec{r},\mathbf{\Omega})$. The dotted line illustrates photon free path, that is, there are no interactions along it. All photons $(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})$, $\xi \geqslant 0$, have the same previous state $(\vec{r}',\mathbf{\Omega}')$.

elements occurred in the neighborhoods $V(\vec{r}',\mathbf{\Omega}')$ and $V(\vec{r},\mathbf{\Omega}')$, where $V(\vec{r},\mathbf{\Omega}') \cap V(\vec{r}',\mathbf{\Omega}') = \varnothing$ and $\vec{r} = \vec{r}' + |\vec{r}-\vec{r}'|\mathbf{\Omega}'$ (Fig. 1).

Definition 2. The function $\sigma(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')$ is the fraction of photons from $V(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})$, with previous state $(\vec{r}',\mathbf{\Omega}')$, attenuated in the medium while traversing an elementary distance $[\vec{r}+\xi\mathbf{\Omega},\vec{r}+(\xi+d\xi)\mathbf{\Omega}]$. This defines the extinction coefficient.

We emphasize that the extinction coefficient σ describes the attenuation not of all photons but only those with previous state $(\vec{r}',\mathbf{\Omega}')$. In other words, from all photons in the elementary volume $V(\vec{r},\mathbf{\Omega})$, we consider only those with previous state $(\vec{r}',\mathbf{\Omega}')$ and describe their attenuation along the line $\{\vec{r}+\xi\mathbf{\Omega},\ \xi \geqslant 0\}$. This distinction is necessary to derive the transport equation for photons with dependence on their previous state (Knyazikhin, 1990; Myneni et al., 1991). The value $\sigma(\vec{r},\mathbf{\Omega}|\vec{r},\mathbf{\Omega})$ can be defined as the limit

$$\sigma(\vec{r},\mathbf{\Omega}|\vec{r},\mathbf{\Omega}) = \lim_{\xi \to 0+;\, V(\vec{r},\mathbf{\Omega}) \cap V(\vec{r}-\xi\mathbf{\Omega},\mathbf{\Omega}) = \varnothing} \sigma(\vec{r},\mathbf{\Omega}|\vec{r}-\xi\mathbf{\Omega},\mathbf{\Omega}). \tag{6}$$

This definition of $\sigma(\vec{r},\mathbf{\Omega}|\vec{r},\mathbf{\Omega})$ has a physical interpretation. A photon can interact with the medium as a consequence of movement only. This fact is reflected in the definition of each coefficient by using the phrase "in traveling an elementary distance." Therefore, the coefficient $\sigma(\vec{r},\mathbf{\Omega}|\vec{r},\mathbf{\Omega})$ describes the attenuation of photons arriving from $V(\vec{r}-d\xi\mathbf{\Omega},\mathbf{\Omega})$ $(d\xi>0)$ to $V(\vec{r},\mathbf{\Omega})$. It is most important to note that $V(\vec{r},\mathbf{\Omega})\cap V(\vec{r}-\xi\mathbf{\Omega},\mathbf{\Omega})=\varnothing$. Say that this condition is violated, that is, $V(\vec{r},\mathbf{\Omega})\cap V(\vec{r}-\xi\mathbf{\Omega},\mathbf{\Omega})\neq\varnothing$, if $\xi\to 0+$. It is understood that "an elementary volume about the point $(\vec{r},\mathbf{\Omega})$" means the process of "squeezing" the neighborhood of this point to the $(\vec{r},\mathbf{\Omega})$ [a strict mathematical description of these concepts can be found in Smelov (1978)]. Then, both the condition $V(\vec{r},\mathbf{\Omega})\cap V(\vec{r}-\xi\mathbf{\Omega},\mathbf{\Omega})\neq\varnothing$, if $\xi\to 0+$, and degeneration of the neighborhood of the point $(\vec{r},\mathbf{\Omega})$ to itself does not exclude the photon $(\vec{r},\mathbf{\Omega})$. Thus, there may be a photon that can potentially interact with the medium without movement; but such photons must be excluded from our consideration. One can also note that equality (6) is an important topological characteristic of the physical process under study, and any change in its precise definition can lead to qualitive changes of the ensuing processes.

The (differential) scattering coefficient can be defined in a similar manner. Let $\sigma_s(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega}\to\mathbf{\Omega}^*|\vec{r}',\mathbf{\Omega}')\,d\mathbf{\Omega}$ be the fraction of photons in an elementary volume $V(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})$, with previous state $(\vec{r}',\mathbf{\Omega}')$, that is scattered into a unit solid angle about $\mathbf{\Omega}^*$ as a consequence of interaction with phytoelements (also consult the section on the scattering coefficient). It should be emphasized that scattering changes not only the direction of photon travel but also its previous state (Myneni et al., 1991).

Let the number of photons in the elementary volume $V(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})$ with previous state $(\vec{r}',\mathbf{\Omega}')$ be $dN(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')$. Then:

Definition 3. The function

$$\psi(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')=\frac{dN(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')}{d\vec{r}\,d\mathbf{\Omega}\,d\vec{r}'\,d\mathbf{\Omega}'} \tag{7}$$

is the conditional probability density of photons $(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})$ with previous state $(\vec{r}',\mathbf{\Omega}')$ and

$$\psi(\vec{r},\mathbf{\Omega}|\vec{r},\mathbf{\Omega})=\lim_{\xi\to 0+;\,V(\vec{r},\mathbf{\Omega})\cap V(\vec{r}-\xi\mathbf{\Omega},\mathbf{\Omega})=\varnothing}\psi(\vec{r},\mathbf{\Omega}|\vec{r}-\xi\mathbf{\Omega},\mathbf{\Omega}).$$

The above limit has the same interpretation as (6). A strict definition of (7) can be found in Smelov (1978). The function ψ (aside from a trivial factor) was interpreted by Myneni et al. (1991) as the partial intensity of photons with the same previous phase-space state.

It is not difficult to see that the density $\varphi(\vec{r},\mathbf{\Omega})$ and the conditional probability density $\psi(\vec{r},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')$ are connected by the following relationship:

$$\varphi(\vec{r},\mathbf{\Omega})=\int_{4\pi}d\mathbf{\Omega}'\int_0^{\xi(\mathbf{\Omega}')}\psi(\vec{r},\mathbf{\Omega}|\vec{r}-\xi'\mathbf{\Omega}',\mathbf{\Omega}')\,d\xi', \tag{8}$$

where $\xi(\mathbf{\Omega}')$ is the distance between $\vec{r}$ and the boundary of D along the direction $-\mathbf{\Omega}'$.

THE STRUCTURE OF A LEAF CANOPY

The definitions introduced in the previous section are necessary to highlight the fact that photons in an elementary volume are heterogeneous; that is, photons with different previous states have different probabilities of encountering voids (Kuusk, 1985; Myneni et al., 1991). In this section, the structure of such a binary medium is investigated.

The length η of the interval $\{\vec{r}'+\xi'\mathbf{\Omega}',\ 0\leqslant\xi'\leqslant|\vec{r}-\vec{r}'|\}$ between two successive interactions in elementary volumes $V(\vec{r}',\mathbf{\Omega}')$ and $V(\vec{r},\mathbf{\Omega}')$ is the sum of two random values—length of free space and length of photon mean free path in the phytomedium. Let $p(\vec{r}',\mathbf{\Omega}',\xi')$, $\xi'>0$, be the probability density of distribution of the random value η. The probability density $p(\vec{r}',\mathbf{\Omega}',\xi')$ and the conditional probability density $\psi(\vec{r},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')$ are related as

$$\psi(\vec{r},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')=\varphi(\vec{r}',\mathbf{\Omega}')p(\vec{r}',\mathbf{\Omega}',|\vec{r}-\vec{r}'|)\sigma_s(\vec{r}'+|\vec{r}-\vec{r}'|\mathbf{\Omega}',\mathbf{\Omega}'\to\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}'), \tag{9}$$

where the density $\varphi(\vec{r},\mathbf{\Omega})$ is defined by Eq. (8). The scattering coefficient σ_s will be defined later. Then, to find $p(\vec{r},\mathbf{\Omega},\xi)$ from Eq. (9), the conditional probability density $\psi(\vec{r},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')$ must be evaluated by solving the corresponding transport equation. We notice that if the region D consists of a phytomedium only (absence of voids), then the function p is the probability density of the length of photon free path in a turbid medium,

$$p(\vec{r}',\mathbf{\Omega}',\xi')=\tilde{\sigma}(\vec{r}'+\xi'\mathbf{\Omega}',\mathbf{\Omega}')\exp\left[-\int_0^{\xi'}\tilde{\sigma}(\vec{r}'+t\mathbf{\Omega}',\mathbf{\Omega}')\,dt\right].$$

Integrating both sides of Eq. (9) by analogue of Eq. (8) gives the "ordinary" integral transport equation (Smelov, 1978).

In order to separate photons with the same previous state from all photons in the elementary volume $V(\vec{r},\mathbf{\Omega})$, we represent the region D as a combination of all possible open oriented spheres,

$$\bigcup_\alpha S(\vec{r}_\alpha,\xi_\alpha,\mathbf{\Omega}_\alpha)\cap D,\ \vec{r}_\alpha\in D.$$

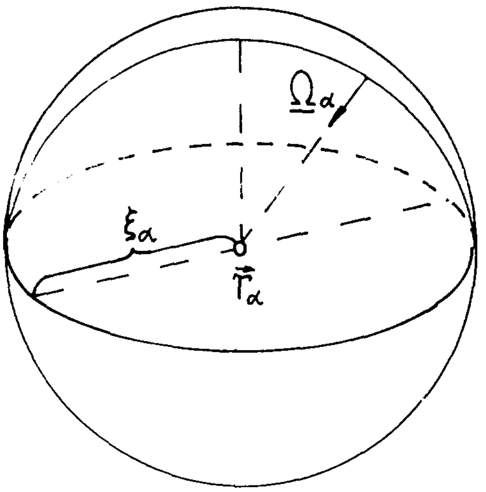

Figure 2. A sphere $S(\vec{r}_\alpha,\xi_\alpha,\mathbf{\Omega}_\alpha)$ with its center at $\vec{r}_\alpha$ of radius ξ_α and orientation $\mathbf{\Omega}_\alpha$. The center of the sphere belongs to the line $\xi\mathbf{\Omega}_\alpha$, $\xi > 0$.

Here, $S(\vec{r}_\alpha,\xi_\alpha,\mathbf{\Omega}_\alpha)$ denotes a sphere of radius ξ_α with its center at $\vec{r}_\alpha$ and of orientation $\mathbf{\Omega}_\alpha$. By orientation we mean the unit vector $\mathbf{\Omega}_\alpha = (\Omega_{\alpha x},\Omega_{\alpha y},\Omega_{\alpha z})$ $(\Omega^2_{\alpha x}+\Omega^2_{\alpha y}+\Omega^2_{\alpha z}=1)$, drawn from the surface and directed to the center of the sphere (Fig. 2). A photon $(\vec{r},\mathbf{\Omega})$ with previous state $(\vec{r}',\mathbf{\Omega}')$ belongs to the sphere $S(\vec{r},|\vec{r}-\vec{r}'|,\mathbf{\Omega}')$. Further, if a photon $(\vec{r},\mathbf{\Omega})$ changes position to $(\vec{r}^*,\mathbf{\Omega}^*)$ as a result of collision, then according to our abstraction it belongs to $S(\vec{r}^*,|\vec{r}-\vec{r}^*|,\mathbf{\Omega})$. The probability of photon transfer from one sphere to another is described by the probability density p.

Considering the nature of interactions between photons and phytoelements in a leaf canopy, we shall proceed with the assumption that the event $A(\vec{r}',|\vec{r}-\vec{r}'|,\mathbf{\Omega}') = \{$two successive points of the interaction between a photon and phytoelements occuring in $V(\vec{r}',\mathbf{\Omega}')$ and $V(\vec{r},\mathbf{\Omega}')$ respectively$\}$ is valid. It means that the interval between the points $\vec{r}$ and $\vec{r}'$ does not contain any phytoelements. The probability density of the event $A(\vec{r}',\xi',\mathbf{\Omega}')$ is simply the function $p(\vec{r}',\mathbf{\Omega}',\xi')$, $\xi' = |\vec{r}-\vec{r}'|$.

Let $q(\vec{r}+\xi\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')$ be the conditional probability of encountering a void in the elementary volume $V(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})$, provided that the event $A(\vec{r}',|\vec{r}-\vec{r}'|,\mathbf{\Omega}')$ was realized. Then, the extinction coefficient at $(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})$ for photons with previous state $(\vec{r}',\mathbf{\Omega}')$ is

$$\sigma(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}') = \bar{\sigma}(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})[1-q(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')],$$

where $\bar{\sigma}(\vec{r},\mathbf{\Omega})$ is the coefficient of extinction in the turbid phytomedium (see the second section). Since $A(\vec{r}',|\vec{r}-\vec{r}'|,\mathbf{\Omega}')$ is realized, $q(\vec{r}-\xi\mathbf{\Omega}',-\mathbf{\Omega}'|\vec{r}',\mathbf{\Omega}')=1$ and $\sigma(\vec{r}-\xi\mathbf{\Omega}',\mathbf{\Omega}',|\vec{r}',\mathbf{\Omega}')=0$, for $0\leqslant\xi\leqslant|\vec{r}-\vec{r}'|$. Proceed from the finite scatterers the gap between $\vec{r}$ and $\vec{r}'$ is construed to extend from the interval $\{(\vec{r}-\xi\mathbf{\Omega}',\mathbf{\Omega}');\ 0\leqslant\xi\leqslant|\vec{r}-\vec{r}'|\}$ to a sufficiently small spherical cone,

$$K(\vec{r},\vec{r}',-\mathbf{\Omega}',\zeta) = \{(\vec{r}-\xi\mathbf{\Omega},\mathbf{\Omega}'): 0\leqslant\xi \leqslant|\vec{r}-\vec{r}'|, (-\mathbf{\Omega}\cdot\mathbf{\Omega}')\geqslant\zeta\}. \quad (10)$$

The value ζ is the cosine of the angle between the height and base of the cone. It depends on the dimensions of the phytoelements. Thus, for example, if leaves are infinitely small, then $\zeta=1$, and the cone K degenerates to the interval $\{(\vec{r}-\xi\mathbf{\Omega}',\mathbf{\Omega}');\ 0\leqslant\xi\leqslant|\vec{r}-\vec{r}'|\}$. In this case, $\sigma=\bar{\sigma}$ almost everywhere. This corresponds to the transfer of photons in a turbid medium (standard transport theory).

With the above in mind, we postulate the following:

1. A photon $(\vec{r},\mathbf{\Omega})$ with previous state $(\vec{r}',\mathbf{\Omega}')$ can encounter a void only inside the sphere $S(\vec{r},|\vec{r}-\vec{r}'|,\mathbf{\Omega}')$.
2. The closer the direction $\mathbf{\Omega}$, of a photon $(\vec{r},\mathbf{\Omega})$ with previous state $(\vec{r}',\mathbf{\Omega}')$, to $\mathbf{\Omega}'$, the greater the probability of encountering a void when traveling along $\mathbf{\Omega}$.

Thus, the memory of a photon can be understood as follows. A photon at $\vec{r}$ "has" some information about the realization of the random value characterizing the spatial distribution of phytoelements. This information is meaningful only in the space encompassed by an oriented sphere with center at $\vec{r}$ and radius $|\vec{r}-\vec{r}'|$; here $\vec{r}$ and $\vec{r}'$ are two successive points of interaction. A photon "knows," for example, that the path back from $\vec{r}$ to $\vec{r}'$ is free of phytoelements. The information that the photon possesses depends on the previous phase-space point of its interaction. Consequently, two photons at $\vec{r}$ with different history will have different information regarding the distribution of phytoelements and/or voids around it. So, the length of their mean free path scattering at $\vec{r}$ can be different as well. After each interaction the information is updated (Myneni et al., 1991).

PROBABILITY OF ENCOUNTERING VOIDS

In this section we derive the probability that a photon traveling along $\mathbf{\Omega}$ will encounter a void inside the sphere $S(\vec{r},|\vec{r}-\vec{r}'|,\mathbf{\Omega}')$. Let $q(\xi)$ be the desired probability at $\vec{r}_\xi=\vec{r}+\xi\mathbf{\Omega}$, $0\leqslant\xi\leqslant\xi'=|\vec{r}-\vec{r}'|$. Suppose that the event $A(\vec{r}',|\vec{r}-\vec{r}'|,\mathbf{\Omega}')$ (after interaction at $\vec{r}'$, a photon traveling along $\mathbf{\Omega}'$ hits a phytoelement at $\vec{r}$; Fig. 1) is realized. This is only the information that is required to derive the distribution of voids.

Let $\kappa=\kappa[(\mathbf{\Omega}\cdot\mathbf{\Omega}')]$ be the rate of decrease of the probability q of encountering voids. Clearly, κ depends on the geometrical parameters of the canopy (next section). Also, we introduce the continuous weight function $\nu(x)$, $x=(\mathbf{\Omega}\cdot\mathbf{\Omega}')$, $-1\leqslant x\leqslant 1$, such that

$$\left\{\nu(-1)=1,\nu(1)=0,\int_{-1}^{1}\nu(x)\ dx=1\right\}. \quad (11)$$

Next, we propose that the voids are uniformly distributed with respect to azimuth $\varphi-\varphi'$; here φ is the azimuth of direction $\mathbf{\Omega}$ and, similarly, φ' of $\mathbf{\Omega}'$. It is seen that the weight $\nu[(\mathbf{\Omega}\cdot\mathbf{\Omega}')]$ continuously increases from the direction $\mathbf{\Omega}=\mathbf{\Omega}'$ (extention of the ray $\mathbf{\Omega}'$) to the direction $\mathbf{\Omega}=-\mathbf{\Omega}'$ (retrodirection). Now, a source function for voids can be defined as follows:

$$f(\xi)=\nu\ \exp(-\kappa\xi)\delta(\xi'-\xi),\ 0\leqslant\xi\leqslant\xi', \quad (12)$$

where δ is the Dirac delta function.

An expression for the probability q can be derived with the above in mind. It is not difficult to see that with increase in ξ from the center of the sphere to its surface, the probability of encountering a void decreases and

$$q(\xi+\Delta\xi)=q(\xi)+\Delta\xi\kappa q(\xi)+\Delta\xi f(\xi).$$

Dividing both sides of the above by $\Delta\xi$ and considering the process $\Delta\xi\rightarrow 0$, we obtain

$$q'(\xi)=\kappa q(\xi)+f(\xi). \quad (13)$$

As a boundary condition one can write

$$q(0)=\nu[(\mathbf{\Omega}\cdot\mathbf{\Omega}')], \quad (14)$$

since the probability of encountering a void at the center of the sphere along $\mathbf{\Omega}=-\mathbf{\Omega}'$ is equal to unity. This condition ensures that the probability of encountering a void at the center of the sphere along $\mathbf{\Omega}=\mathbf{\Omega}'$ is zero [cf. Eq. (11)]. Solving the initial-value problem [Eqs. (13) and (14)], we obtain

$$q(\xi)=\int_0^\xi f(\xi')\ \exp[-\kappa(\xi-\xi')]\ d\xi',$$

and with the source function (12) we obtain explicitly

$$q(\xi)=\nu\ \exp(-\kappa\xi)H(\xi'-\xi),\ 0\leqslant\xi\leqslant\xi', \quad (15)$$

where $H(x)$ is the Heaviside function

$$H(x)=\begin{cases}1, & \text{if}\, x\geqslant 0,\\ 0, & \text{if}\, x<0.\end{cases}$$

The Heaviside function assures consideration of voids inside the sphere $S(\vec{r},\xi',\mathbf{\Omega}')$ only (see the previous section).

Finally, an example of the weight function ν. In the simplest case, one can think of a linear function of the form

$$\nu(x)=(1-x)/2, x=(\mathbf{\Omega}\cdot\mathbf{\Omega}'). \quad (16)$$

Clearly, (16) satisfies the conditions implied by (11).

RATE OF CHANGE OF THE PROBABILITY OF ENCOUNTERING VOIDS

Consider a sphere $S(\vec{r},|\vec{r}-\vec{r}'|,\mathbf{\Omega}')$ containing phytoelements of finite dimensions. The leaf area density and leaf normal orientation are averaged over the sphere S and treated as constants, $\bar{u}_L$ and $\bar{g}_L$. For instance,

$$\bar{u}_L(\vec{r},|\vec{r}-\vec{r}'|,\mathbf{\Omega}') = \frac{3}{4\pi|\vec{r}-\vec{r}'|}\int_{S(\vec{r},|\vec{r}-\vec{r}'|,\mathbf{\Omega}')}u_L(\vec{r}'')\ d\vec{r}''.$$

Without loss of detail, the phytoelements are assumed circular in shape and of constant diameter d_L.

In the previous section we derived the probability of encountering a zone free of phytoelements for photons $(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})$ with previous state $(\vec{r}',\mathbf{\Omega}')$. Combining Eqs. (15) and (16) gives

$$q(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')=0.5[1-(\mathbf{\Omega}\cdot\mathbf{\Omega}')] \times\exp(-\kappa\xi)H(\xi'-\xi),\ 0\leqslant\xi\leqslant|\vec{r}-\vec{r}'|. \quad (17)$$

The complete argument list of the probability q are written to emphasize dependence on the previous state $(\vec{r}',\mathbf{\Omega}')$. The extinction coefficient (following its definition as the product of two probabilities) can be written as

$$\sigma(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')\ d\xi=\tilde{\sigma}(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega})\ d\xi \times[1-q(\vec{r}+\xi\mathbf{\Omega},\mathbf{\Omega}|\vec{r}',\mathbf{\Omega}')]. \quad (18)$$

The first term, $\tilde{\sigma}\, d\xi$ [defined by Eqs. (1) or (4)], is the probability that a photon will be captured while traveling a distance $d\xi$ in the phytomedium, which does not depend on the previous state. The second term, $1-q$, denotes the probability of encountering a phytomedium, and this depends on the previous direction of photon travel $\mathbf{\Omega}'$ and on the previous point of interaction $\vec{r}'$. One can conclude that the probability of encountering a void q is equal to the probability of dependence of the extinction coefficient on previous photon state. But this is the same as the probability of the dependence of photon mean free path on its previous state. Thus, we understand κ to be the rate of decrease of the probability of dependence.

There are four factors that influence κ—1) $\bar{u}_L$, leaf area density over the sphere S; 2) d_L, diameter of circular leaves; 3) $\bar{g}_L$, leaf normal distribution averaged over the sphere; 4) $\alpha = cos^{-1}(\mathbf{\Omega}\cdot\mathbf{\Omega}')$, the angle between $\mathbf{\Omega}$ and $\mathbf{\Omega}'$.

Nilson and Kuusk (1989) considered the correlation between two random indicator functions along $\mathbf{\Omega}$ and $\mathbf{\Omega}'$, and found a relationship between the radius of correlation, l_L, diameter of the leaf, d_L, and leaf orientation $\bar{g}_L$. For the case of spherical orientation they found

$$l_L = (\pi^2/16)\, d_L, \tag{19a}$$

and for horizontal leaves

$$l_L = (\pi/4)\, d_L. \tag{19b}$$

Thus, the number of parameters that influence κ can be reduced to three—$\bar{u}_L$, l_L, and α, where l_L can be considered as the length of mean chord. Let us consider these dependencies in some detail.

1. *Leaf area density* $\bar{u}_L$: It is obvious that the greater the density $\bar{u}_L$, the larger the cone K [Eq. (10)], and weaker the dependence of photon free path on $(\vec{r}',\mathbf{\Omega}')$. In the limit $\bar{u}_L = 0$ (absence of leaves), the rate $\kappa = \infty$, and $q(\vec{r}_\xi,\mathbf{\Omega}) = 0$, $\xi > 0$, $\mathbf{\Omega} \neq -\mathbf{\Omega}'$ (the cone K degenerates to the linear interval). On the other hand, if leaf area density is very large, the dependence is large as well and, in the limit, $\bar{u}_L = \infty$, the probability $q(\vec{r}_\xi,\mathbf{\Omega}) = 1$ for any direction $\mathbf{\Omega}$. So, it is reasonable to assume that $\kappa \sim \bar{u}_L^{-1}$.
2. *Size of leaves* l_L: In the case of infinitesimally small leaves ($l_L = 0$), dependence on the previous state is absent, and $q(\vec{r}_\xi,\mathbf{\Omega}) = 0$, which corresponds to an infinite rate of change of κ. Also, in this case the cone K degenerates to the linear interval. However, the presence of even a single but a large leaf that divides the sphere S into two hemispheres necessitates considerations of dependence for the entire sphere S. The K in this situation spans the entire hemisphere. So, the rate of change of the dependence of photon free path is inverse proportional to the average leaf size l_L, that is, $\kappa \sim l_L^{-1}$.
3. *Angle between successive directions of photon travel* α: We shall quantify angular spread between the vectors $\mathbf{\Omega}$ and $\mathbf{\Omega}'$ that pass through the center of the sphere S by the sine of the angle between them. In other words, the angle between the height and base of the cone is our parameter of choice. The smaller the value of $|\sin\,\alpha|$, the closer $\mathbf{\Omega}$ is to $\mathbf{\Omega}'$ and the smaller is the cone K. Consequently, the rate of weakening of their dependence is also smaller. It is clear that a maximum value is reached when $(\mathbf{\Omega}\cdot\mathbf{\Omega}') = 0$ and vice versa. Thus, $\kappa \sim |\sin\,\alpha|$.

From the above analysis one can write

$$\kappa \sim |\sin\,\alpha|\bar{u}_L^{-1} l_L^{-1}. \tag{20}$$

The coefficient of proportionality in Eq. (20) can be given a geometrical meaning. For example, $|\sin\,\alpha|/2\pi$ can be considered as a fraction of the triangle between $\mathbf{\Omega}$ and $\mathbf{\Omega}'$ (cross section of the cone) from the correspondent circle (cross section of the sphere). Thus, for instance,

$$\kappa = (1/2\pi)\,|\sin\,\alpha|\bar{u}_L^{-1} l_L^{-1}. \tag{21}$$

One can also consider the square of a sector instead of the triangle. In this case, one obtains $\alpha/2\pi$, $0<\alpha<\pi/2$, instead of $|\sin\,\alpha|/2\pi$. However, it is numerically less convenient. In fact, there are several ways of defining κ. The problem of a proper definition of κ is far from trivial and is best answered with experimental data.

SCATTERING COEFFICIENT

In this section we apply the foregoing analysis to the scattering of photons, and derive an expression for the scattering coefficient for a phytomedium. As with photon capture, the scattering interaction is described by a scattering coefficient $\tilde{\sigma}_{s'}$. The ratio $\tilde{\sigma}_{s'} / \tilde{\sigma}$ is the albedo of single scattering, ω, denoting the probability of scattering given that a collision has occurred. However, since the scattering event serves to change the direction of photon travel, it is convenient to introduce the differential scattering coefficient $\tilde{\sigma}_s$. It describes the probability of scattering from $\mathbf{\Omega}'$ to a unit solid angle about $\mathbf{\Omega}$ at $\vec{r}$. The coefficient $\tilde{\sigma}_s$ is related to $\tilde{\sigma}_{s'}$ as

$$\tilde{\sigma}_{s'}(\vec{r}, \mathbf{\Omega}') = \int_{4\pi} \tilde{\sigma}_s(\vec{r}; \mathbf{\Omega}' \rightarrow \mathbf{\Omega})\ d\mathbf{\Omega}. \tag{22}$$

For a leaf assembly, the differential scattering coefficient can be expressed as

$$\tilde{\sigma}_s(\vec{r}; \mathbf{\Omega}' \rightarrow \mathbf{\Omega}) = u_L\,(1/\pi)\,\Gamma(\vec{r}; \mathbf{\Omega}' \rightarrow \mathbf{\Omega}), \tag{23}$$

where u_L is the leaf area density [Eq. (3)]. The function Γ / π is the area scattering phase function (Ross, 1981)

$$(1/\pi)\,\Gamma(\vec{r}; \mathbf{\Omega}' \rightarrow \mathbf{\Omega}) = (1/2\pi) \int_{2\pi+} g_L(\vec{r}, \mathbf{\Omega}_L) |\mathbf{\Omega}' \cdot \mathbf{\Omega}_L| \times \gamma_L(\vec{r}, \mathbf{\Omega}_L; \mathbf{\Omega}' \rightarrow \mathbf{\Omega})\ d\mathbf{\Omega}_L, \tag{24}$$

which, in general, is not rotationally invariant because of the distribution function g_L. Here, γ_L is the single-leaf scattering phase function. For a leaf at $\vec{r}$ with outward normal $\mathbf{\Omega}_L$, this phase function is the fraction of the intercepted energy (from photons initially traveling in direction $\mathbf{\Omega}'$) that is scattered into a unit solid angle about $\mathbf{\Omega}$ (Shultis and Myneni, 1988). The leaf albedo ω_L can be defined as

$$\omega_L(\vec{r}, \mathbf{\Omega}'; \mathbf{\Omega}_L) = \int_{4\pi} \gamma_L(\vec{r}, \mathbf{\Omega}_L; \mathbf{\Omega}' \rightarrow \mathbf{\Omega})\ d\mathbf{\Omega}.$$

Let ρ_L and τ_L be the leaf hemispherical reflectance and transmittance, respectively. Then for the bi-Lambertian reflectance model (Ross, 1981),

$$\gamma_L(\vec{r}, \mathbf{\Omega}_L; \mathbf{\Omega}' \rightarrow \mathbf{\Omega}) = \begin{cases} \rho_L |\mathbf{\Omega} \cdot \mathbf{\Omega}_L| / \pi, & (\mathbf{\Omega} \cdot \mathbf{\Omega}_L)(\mathbf{\Omega}' \cdot \mathbf{\Omega}_L) < 0, \\ \tau_L |\mathbf{\Omega} \cdot \mathbf{\Omega}_L| / \pi, & (\mathbf{\Omega} \cdot \mathbf{\Omega}_L)(\mathbf{\Omega}' \cdot \mathbf{\Omega}_L) > 0, \end{cases}$$

we have

$$\omega_L(\vec{r}, \mathbf{\Omega}'; \mathbf{\Omega}_L) \equiv \omega_L(\vec{r}) = \omega(\vec{r}).$$

With this approximation the scattering coefficient for a phytomedium can be written as

$$\tilde{\sigma}_{s'}(\vec{r}, \mathbf{\Omega}') = \omega_L(\vec{r}) \tilde{\sigma}(\vec{r}, \mathbf{\Omega}'),$$

and, thus, the leaf-albedo ω_L is equivalent to the single scattering albedo ω admitted by the transport equation. It was proved by Marshak (1989) that the condition $\omega \leqslant 1$ guarantees the existence and uniqueness of the solution to the transport equation in a turbid plate medium. If mutual shading between leaves in an elementary volume is accounted for, the area scattering phase function [Eq. (24)] will change (details in the Appendix).

Now we apply results of the three previous sections to the scattering process. In the discussion leading to Eq. (18), we proposed that the extinction coefficient in a leaf medium consists of two probabilities—the probability of being captured in a phytomedium and the probability of encountering a phytomedium. In view of the fact that a collision preceeds a scattering interaction, the differential scattering coefficient σ_s can be written as

$$\sigma_s(\vec{r'} + \xi' \mathbf{\Omega}'; \mathbf{\Omega}' \rightarrow \mathbf{\Omega} | \vec{r''}, \mathbf{\Omega}'') = \tilde{\sigma}_s(\vec{r'} + \xi' \mathbf{\Omega}'; \mathbf{\Omega}' \rightarrow \mathbf{\Omega}) \times [1 - q(\vec{r'} + \xi' \mathbf{\Omega}', \mathbf{\Omega}' | \vec{r''}, \mathbf{\Omega}'')]. \tag{25}$$

Here, $\xi' = |\vec{r} - \vec{r'}|$ and $(\vec{r''}, \mathbf{\Omega}'')$ is the previous state of a photon $(\vec{r'}, \mathbf{\Omega}')$. The differential scattering coefficient $\tilde{\sigma}_s$ is defined by Eq. (23) and the probability of encountering a void q is defined by Eq. (17), with κ defined by Eq. (21). By analogy,

$$\sigma_{s'}(\vec{r'} + \xi' \mathbf{\Omega}', \mathbf{\Omega}' | \vec{r''}, \mathbf{\Omega}'') = \tilde{\sigma}_{s'}(\vec{r'} + \xi' \mathbf{\Omega}', \mathbf{\Omega}') \times [1 - q(\vec{r'} + \xi' \mathbf{\Omega}', \mathbf{\Omega}' | \vec{r''}, \mathbf{\Omega}'')] \tag{26}$$

is the new scattering coefficient. It is seen that (25) and (26) are connected by an expression analogous to Eq. (22).

NUMERICAL EXAMPLES

To illustrate dependence of the extinction coefficient on the parameters in its definition, we present results of numerical experiments here. Consider a plane parallel leaf canopy of depth $T = 100$ cm filled with circular leaves of diameter $d_L = 6$ cm. The leaf area index of the canopy is equal to 3. Assume that leaf area density is constant—$u_L(\vec{r}) = u_L = \bar{u}_L = \mathrm{LAI}/T$. Consider a sphere of radius $|\vec{r} - \vec{r'}| = 30$ cm located in the canopy, with an orientation given by $\mathbf{\Omega}' \sim (\theta', \varphi') = (139°, 105°)$. Without loss of detail, we assume that mutual shading between leaves in an elementary volume is absent. Then from Eqs. (4) and (21) we have

$$\sigma(\vec{r} + \xi \mathbf{\Omega}, \mathbf{\Omega} | \vec{r'}, \mathbf{\Omega}') = u_L G(\vec{r} + \xi \mathbf{\Omega}, \mathbf{\Omega})[1 - q(\vec{r} + \xi \mathbf{\Omega}, \mathbf{\Omega} | \vec{r'}, \mathbf{\Omega}')],$$

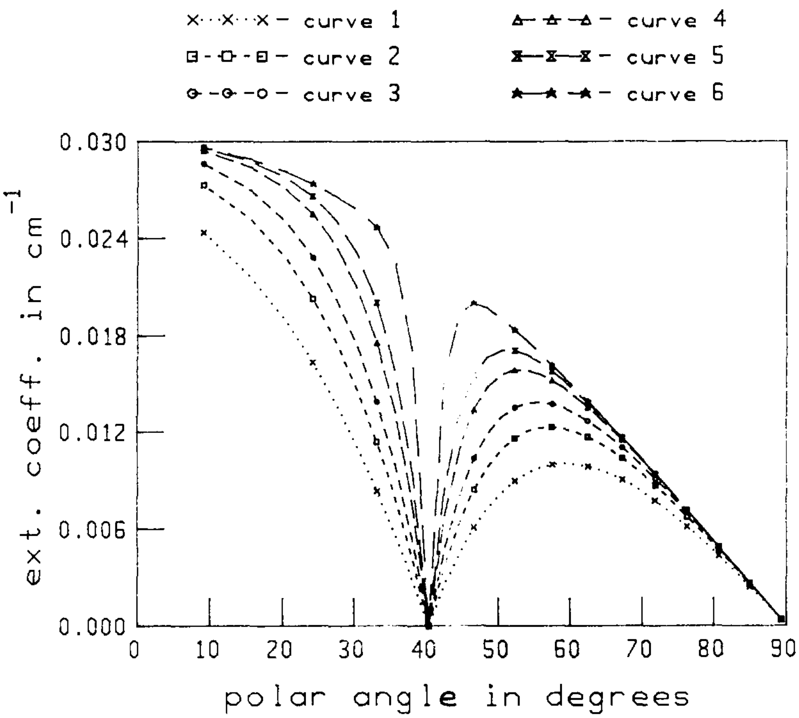

Figure 3. Dependence of the extinction coefficient σ on distance ξ from the point of interaction $\vec{r}$: 1) $\xi = 2.8$ cm, $C = 9.5\%$; 2) $\xi = 4.3$ cm, $C = 4.7\%$; 3) $\xi = 5.6$ cm, $C = 2.8\%$; 4) $\xi = 8.5$ cm, $C = 1.4\%$; 5) $\xi = 11.3$ cm, $C = 0.9\%$; 6) $\xi = 17.0$ cm, $C = 0.6\%$; 7) $\xi = 28.3$ cm, $C = 0.4\%$. The problem parameters are $T = 100$ cm, LAI = 3, $d_L = 6$ cm, $\theta' = 139°$, $\varphi' = 105°$, $\varphi = \varphi' + 180°$. Leaves are horizontal.

where G is defined by (2) and q is defined by (17) and (21).

Of special interest is the following question: How important is the consideration of the probability of encountering voids for photon transport in a canopy of finite dimensional scatterers. Consider the integral C of the probability q over the lower hemisphere (since $\mathbf{\Omega}$ is directed downwards),

$$C = \frac{100\%}{2\pi} \int_{2\pi^-} q(\vec{r} + \xi\mathbf{\Omega}, \mathbf{\Omega} | \vec{r}', \mathbf{\Omega}')\ d\mathbf{\Omega}'$$

$$= 100\% \left[1 - \frac{1}{2\pi} \int_{2\pi^-} \frac{\sigma(\vec{r} + \xi\mathbf{\Omega}, \mathbf{\Omega} | \vec{r}', \mathbf{\Omega}')}{\tilde{\sigma}(\vec{r} + \xi\mathbf{\Omega}, \mathbf{\Omega})}\ d\mathbf{\Omega} \right].$$

The value of C can be taken as an index of the influence of leaf size on the extinction coefficient; a smaller C indicates a smaller error if leaf size is ignored.

The dependence of extinction coefficient σ on polar angle θ is shown in Figure 3, for azimuth $\varphi = \varphi' + 180°$. The leaves were assumed to be horizontal. The curves correspond to different distances to the center of the sphere along $\mathbf{\Omega}$, that is, to different values of $\xi = |\vec{r}_\xi - \vec{r}|$. It is clear that as ξ increases, the probability q of encountering a void decreases, and σ tends to $\tilde{\sigma}$ for any $\mathbf{\Omega}$ except for the region around the retrodirection, $\mathbf{\Omega} = -\mathbf{\Omega}'$.

The values of C for different ξ and constant leaf dimensions are given in the caption in Figure 3. One can correlate these numbers and curves with constant ξ and varying d_L. However, it is sufficient to note the term ξ / d_L in the exponent [cf. Eqs. (17), (19), and (21)]. Thus, an increase of ξ by a factor is equivalent to a decrease in leaf diameter d_L by the same factor.

Figure 4 illustrates the influence of azimuth φ on σ, for $\xi = 2.8$ cm. Curve 1 in Figure 3 corresponds to curve 6 in Figure 4 ($\varphi = \varphi' + 180°$). The index $C = 9.5\%$ in this case. The dependence of the ratio $\sigma / \tilde{\sigma}$ or the difference $1 - q$ on the azimuth φ is shown in Figure 5. The minimum value corresponds to the polar angle of the retroincidence direction ($\theta = 41°$).

CONCLUDING REMARKS

In this article, we have developed a formalism for photon interactions in media with finite-dimensional scattering centers. In these media the assumption that each scattering center is in the far field of radiation scattered from all other scatterers is not valid. Although the details are those relating to a leaf canopy, it should be emphasized

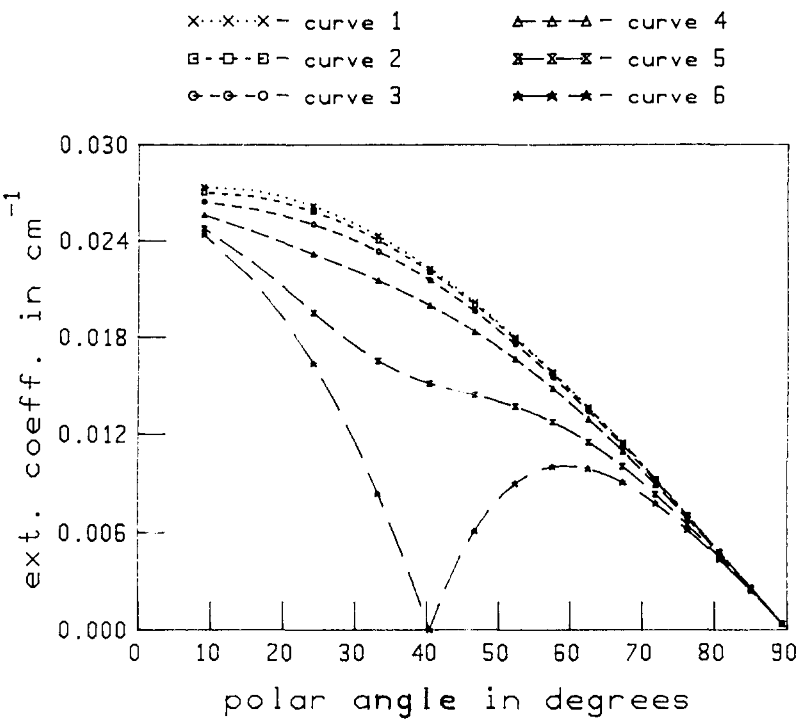

Figure 4. Dependence of the extinction coefficient σ on azimuth angle φ. 1) $\varphi = 135°$, 2) $\varphi = 165°$, 3) $\varphi = 195°$, 4) $\varphi = 225°$, 5) $\varphi = 255°$, 6) $\varphi = 285°$. Here $\xi = 2.8$ cm; the other parameters are the same as in Figure 3.

that the principles developed here are equally applicable in studies on light scattering from rough surfaces that show opposition brightening.

The transport of energy by radiation can be visualized as consisting of two events—the mean length of photon free path (along this length a photon streams without a change in its direction of flight) and the scattering event (where the direction of photon travel is altered). These two events are characterized by the extinction coefficient σ and the differential scattering coefficient σ_s.

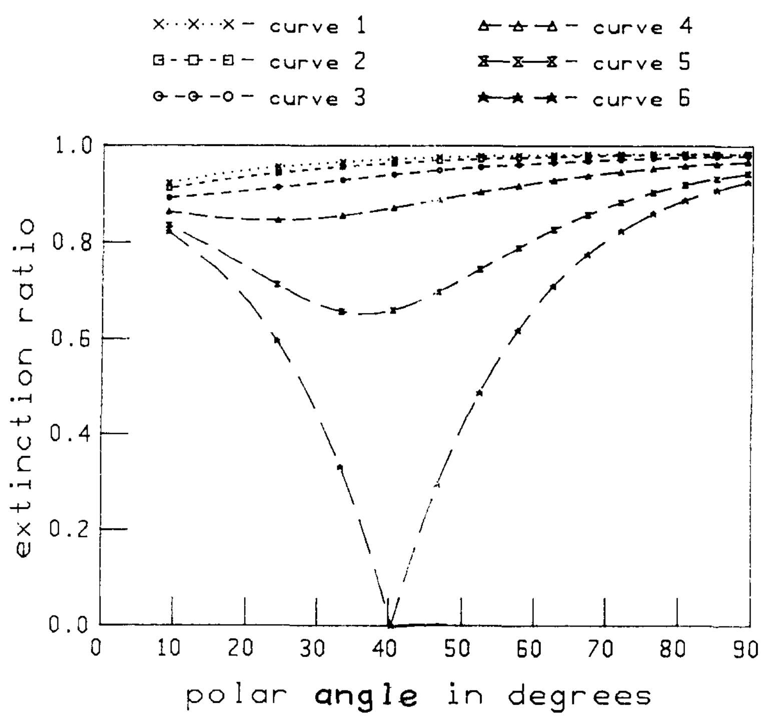

Figure 5. Relationship between the extinction coefficients σ and $\tilde{\sigma}$. The problem parameters are the same as in Figure 4.

As an abstraction, we propose that a leaf canopy consists of two nonoverlapping regions. The first is a turbid medium filled with phytoelements. The theory for a turbid plate medium developed by Ross (1981) is applicable with some minor modification. The second region is the randomly distributed zones devoid of phytoelements: the so-called empty zones or voids. Hence, photon free path in a leaf canopy is the sum of two random values. The first characterizes the phytomedium and the second the voids. Thus, the probability of photon capture is a product of two probabilities—the probability of being absorbed or scattered in a phytomedium and the probability of encountering a phytomedium. Since the first is known, we need to define the probability of a photon encountering a void. This probability at a phase-space point $(\vec{r},\boldsymbol{\Omega})$ depends on the previous state of a photon $(\vec{r}',\boldsymbol{\Omega}')$. It is postulated that the interval between the points $\vec{r}$ and $\vec{r}'$ is free of phytoelements. Considering the finite scatterers, this interval is assumed to extend to a small cone, depending on the dimensions of the phytoelements. It means that the probability for a photon to be captured while traveling from $\vec{r}$ to the neighborhood of $\vec{r}'$ along $-\boldsymbol{\Omega}'$ is equal to zero. Contrast this with the fact that in a turbid medium the probability of capture for a photon traveling an elementary distance is always greater than zero. This is the main difference between the theories in media with finite-dimensional and nondimensional scatterers.

In order to derive the probability of encountering a void, the leaf canopy was abstracted as a combination of oriented spheres. Two successive points of interactions are the center of a sphere and a point on its surface, respectively. For uniqueness, we propose that voids are contained inside the sphere only. This permits the derivation of an initial-value problem for the probability of encountering voids.

The numerical results presented indicate the boundaries of the applicability of turbid medium theory for media with finite-dimensional scatterers. In other words, they indicate how small the leaves should be to ignore their dimensions. The theory also helps us to describe the hot spot effect in an abstract way for rough surfaces that show opposition brightening.

The next step should be the derivation of the transport equation in a leaf canopy with finite-dimensional scattering centers and to deal with the interaction between a leaf canopy and the adjacent atmosphere. To solve the leaf canopy transport problem, we need to specify as initial data the conditional intensity incident from the atmosphere. This is discussed in detail by Myneni et al. (1991).

APPENDIX: CONSIDERATION OF MUTUAL SHADING EFFECTS

In the text we assumed that a leaf canopy has finite-dimensional scattering centers. However, two small finite volumes with the same leaf area density $u_L(\vec{r})$ and geometry factors $G(\vec{r},\boldsymbol{\Omega})$ might have different properties, depending on the overlapment or shading between leaves when viewed along $\boldsymbol{\Omega}$. The function χ was introduced by Myneni et al. (1991) to account for mutual shading effects. In this appendix, we present a model for χ and discuss the consequences.

In general, $\chi \equiv \chi(\vec{r},a_L,\boldsymbol{\Omega}_L,\boldsymbol{\Omega})$. The problem of specifying a strict definition for χ is not trivial, and we shall leave this topic for a detailed analysis at a later time. Here, we propose to approximate it by an exponential function, namely,

$$\chi(\vec{r}, a_L, \boldsymbol{\Omega}_L, \boldsymbol{\Omega}) = \exp[-A_{\mathrm{sh}}(\vec{r}, a_L, |(\boldsymbol{\Omega}_L \cdot \boldsymbol{\Omega})|], \tag{A.1}$$

where $0 \leqslant A_{\mathrm{sh}} < 1$ is a fraction of the shadow area of leaves with size a_L and normal $\boldsymbol{\Omega}_L$ in an finite volume around $\vec{r}$ when illuminated along $\boldsymbol{\Omega}$. If the scattering centers are nondimensional, then $A_{\mathrm{sh}} \equiv 0$, that is, no cross-shading between leaves and $\chi = 1$. The same is true when a photon grazes along the surface of a leaf.

With the above in mind, we introduce the function $G(\vec{r},\boldsymbol{\Omega},a_L)$ instead of (2). It characterizes the nonoverlapped area that is projected on a plane perpendicular to the direction $\boldsymbol{\Omega}$, namely (Myneni et al., 1991),

$$G(\vec{r}, \boldsymbol{\Omega}, a_L) = \frac{1}{2\pi}\int_{2\pi} g_L(\vec{r}, \boldsymbol{\Omega}_L)|\boldsymbol{\Omega} \cdot \boldsymbol{\Omega}_L|\chi(\vec{r}, a_L, \boldsymbol{\Omega}_L, \boldsymbol{\Omega})\, d\boldsymbol{\Omega}_L. \tag{A.2}$$

In case of nondimensional leaves, $\chi \equiv 1$, and

$$\lim_{a_L \to 0} G(\vec{r}, \boldsymbol{\Omega}, a_L) = G(\vec{r}, \boldsymbol{\Omega}).$$

It is not difficult to calculate the generalized geometry factor G for different leaf orientations.

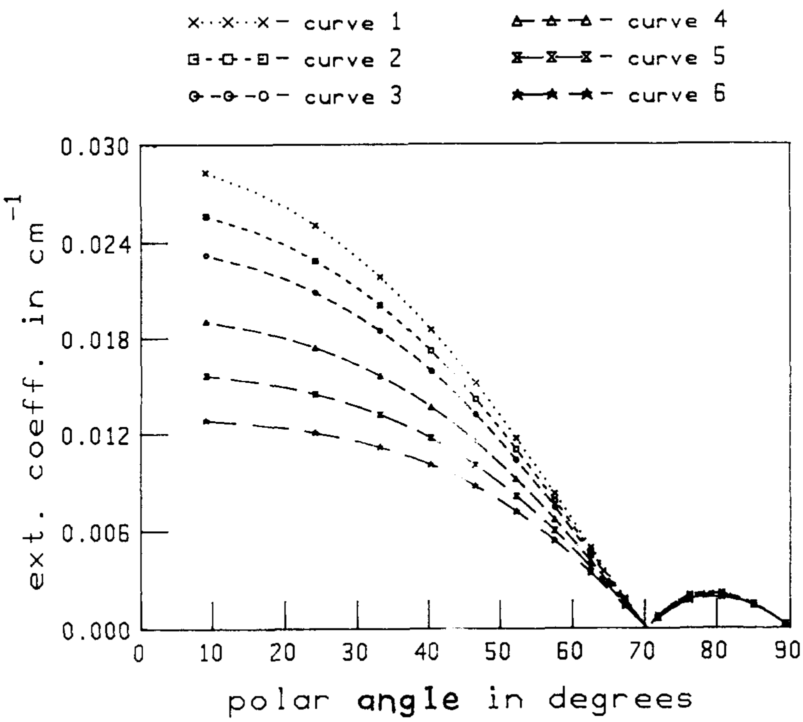

Figure 6. Consideration of the mutual shading function χ: 1) $A_{sh}=0.0$; 2) $A_{sh}=0.1$; 3) $A_{sh}=0.2$; 4) $A_{sh}=0.4$; 5) $A_{sh}=0.6$; 6) $A_{sh}=0.8$. Here $\theta'=110°$. Other problem parameters are the same as in Figure 4.

For instance, for horizontal leaves

$$G(\vec{r},\mathbf{\Omega},a_L)=\mu\ \exp(-A_{sh}\mu),$$

where $\cos^{-1}\mu$ is the direction of photon travel. If leaves have a spherical orientation, then $g(\vec{r},\mathbf{\Omega}_L)\equiv 1$ and

$$G(\vec{r},\mathbf{\Omega};a_L)=A_{sh}^{-2}\,[1-\exp(-A_{sh})-A_{sh}\ \exp(-A_{sh})].$$

Also, if $A_{sh}\to 0$, then $G\to 0.5$.

Following Myneni et al. (1991), the extinction coefficient $\tilde{\sigma}$ can be written as

$$\tilde{\sigma}(\vec{r},\mathbf{\Omega})=\int_0^\infty a_L n_L(\vec{r},a_L)\rho_L(\vec{r},a_L)G(\vec{r},\mathbf{\Omega},a_L)\ da_L.$$

Similarly, the differential scattering coefficient becomes

$$\tilde{\sigma}_s(\vec{r},\mathbf{\Omega}'\to\mathbf{\Omega})=\frac{1}{\pi}\int_0^\infty a_L n_L(\vec{r},a_L)\rho_L(\vec{r},a_L)\Gamma(\vec{r},\mathbf{\Omega}'\to\mathbf{\Omega},a_L)\ da_L,$$

where Γ/π is the finite-leaf analog of the area scattering phase function (Myneni et al. 1991):

$$(1/\pi)\Gamma(\vec{r};\mathbf{\Omega}'\to\mathbf{\Omega},a_L)=(1/2\pi)\int_{2\pi^+}g_L(\vec{r},\mathbf{\Omega}_L)|\mathbf{\Omega}'\cdot\mathbf{\Omega}_L|\times\gamma_L(\vec{r},\mathbf{\Omega}_L;\mathbf{\Omega}'\to\mathbf{\Omega})\chi(\vec{r},a_L,\mathbf{\Omega}_L,\mathbf{\Omega}')\ d\mathbf{\Omega}_L.$$

Now we consider a numerical experiment for estimating the contribution of χ to the extinction coefficient σ. For evaluation of the G function, we shall use (A.2) instead of (2), and approximate χ by (A.1). Six curves with different degree of mutual shading (0–80%) are shown in Figure 6. It is seen that when $\theta > 70°$, the role of mutual shading in aggregations of horizontal leaves is not significant. However, around the nadir, mutual shading effects are large for this particular case of constant leaf area density u_L. This leads to a greater probability of escape for photons traveling this volume, and therefore, a smaller extinction coefficient (curve 6).

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REFERENCES

Antyufeev, V. S., and Marshak, A. L. (1990), Monte Carlo method and transport equation in plant canopies, *Remote Sens. Environ.* 31:183–191.

Avaste, O. A., and Vainikko, G. M. (1973), Calculation of the mean values of intensities and fluxes in broken clouds, In *Noctilucent Clouds. Optical Properties*. Inst. of Phys. and Astronomy, Estonian Acad. Sci., Tallinn, pp. 98–117.

Bunnik, N. J. J. (1978), *The Multispectral Reflectance of*

Shortwave Radiation by Agricultural Crops in Relation with Their Morphological and Optical Properties, Mededelingen Landbouwhogeschool, Wageningen, The Netherlands, 78–175.

Chandrasekhar, S. (1960), *Radiative Transfer*, Dover, New York, 393 pp.

Goel, N. S. (1988), Models of vegetation canopy reflectance and their use in estimation of biophysical parameters from reflectance data, *Remote Sens. Rev.* 4:1–212.

Knyazikhin, Yu. V. (1990), Transfer of particles depended on previous states, in *Proc. of the All-Union Seminar on the Numerical Methods in Solving the Transport Equation*, Estonian Acad. Sci., Tartu, pp. 87–90. (Russian).

Kondratyev, K. Ya. (1969), *Radiation in the Atmosphere*, Academic, New York, 912 pp.

Kuusk, A. (1985), The hot-spot effect of a uniform vegetative cover, *Sov. J. Remote Sens.* 3:645–658.

Marshak, A. L. (1989), Effect of the hot spot on the transport equation in plant canopies, *J. Quant. Spectrosc. Radiat. Transfer* 42:615–630.

Myneni, R. B., Ross, J., and Asrar, G. (1989), A review of photon transport in leaf canopies, *Agric. For. Meteorol.* 45:1–153.

Myneni, R. B., Asrar G., and Gerstl, S. A. W. (1990), Radiative transfer in three dimensional leaf canopies, *Transport Theory Stat. Phys.* 19:205–250.

Myneni, R. B., Marshak, A. L., and Knyazikhin, Y. V. (1991), Transport theory for a leaf canopy of finite dimensional scattering centers, *J. Quant. Spectrosc. Radiat. Transfer* 46:259–280.

Nilson, T., and Kuusk, A. (1989), A reflectance model for the homogeneous plant canopy and its inversion, *Remote Sens. Environ.* 27:157–167.

Ross, J. (1981), *The Radiation Regime and Architecture of Plant Stands*, Dr. W. Junk Publ., The Hague, 391 pp.

Ross, J. K., and Marshak, A. L. (1989), The influence of leaf orientation and the specular component of leaf reflectance on the canopy bidirectional reflectance, *Remote Sens. Envir.* 27:251–260.

Shifrin, K. S. (1953), Concerning the theory of albedo, *Trans. Main Geophys. Obs.* 39(101):244–257 (Russian).

Shultis, J. K. and Myeni, R. B. (1988), Radiative transfer in vegetation canopies with an isotropic scattering, *J. Quant. Spectrosc. Radiat. Transfer* 39:115–129.

Smelov, V. V. (1978), *Lectures on the Theory of Neutron Transport*, Atomizdat, Novosibirsk, 216 pp. (Russian).

Vainikko, G. M. (1973), Calculation for mean value of intensity in broken clouds, *Meteorol. Res.* 21:28–37 (Russian).