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Z^0 photoproduction at high energy*

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We study the photoproduction of the so-called Z^0, conjectured in certain theories of the weak interactions, in the limit of incident photon energy much larger than the Z^0 mass and the momentum transfer to the target. The incident photon creates a lepton-antilepton pair which exchanges photons with the target before combining to form the Z^0. The Z^0 generally has both charge conjugation +1 and −1, so we study both one- and two-photon exchange. The latter is essentially a generalization of Delbrück scattering. The amplitudes are complex due to the leptonic decay of the Z^0. If the unknown couplings are not much bigger than the electric charge, then our quantitative results will be extremely small due to the large Z^0 mass, even in the limit of infinite energy. We also sum over all exchanged photons, obtaining a well-known form for the amplitude, which may merit further study in the context of photoproduction.

I. INTRODUCTION

Much work has been concentrated on the asymptotic properties of quantum electrodynamics in attempts both to test the theory[1] and to find qualitative ideas which might apply to strong interactions.[2] It is interesting to consider such questions in the context of renormalizable Weinberg-Salam models which unify weak and electromagnetic interactions in spontaneously broken gauge symmetries.[3] The electron, muon, and photon become members of gauge multiplets involving a large spectrum of charge, mass, and spin states. These may cause deviations in the tests of "isolated" quantum electrodynamics and may shed some light on its qualitative features as well.

If charged vector bosons $W^{\pm}$ are exchanged in the known weak processes, then a neutral lepton L^0 or a neutral vector boson Z^0 must be exchanged in $e^+e^- \to W^+W^-$ if the Froissart bound is to be satisfied.[4] Existence of the Z^0 would change the electromagnetic interaction Lagrangian to

$$\mathcal{L}_{\rm em} = -\bar{L}\,[e\gamma_\mu A^\mu + (g_V - g_A\gamma_5)\gamma_\mu Z^\mu]\,L\ , \qquad (1.1)$$

where L is a lepton field of charge e, A and Z are the photon and Z^0 fields, and g_V and g_A are model-dependent constants.

In order to really know if the Z^0 exists, one would like to produce it and detect its decay products. It can be electromagnetically produced in two ways. In colliding electron beams one has[5] $e^+e^- \to Z^0$, and in photon beams on nuclear targets one has $\gamma N \to Z^0 N$. Here we study the high-energy behavior of the photoproduction process, neglecting the nuclear structure. This generalizes previous results for γe elastic and Delbrück scattering[6] by giving the final photon a mass[7] and also an axial coupling.

It should always be kept in mind that the Z^0 mass will be at least on the order of a few GeV.[8] This will have significant effects on (a) the required incident energy, (b) the range of momentum transfers involved, (c) the role of the nucleus, (d) the size of the cross sections, and (e) the decay.

This paper is organized as follows. Section II summarizes the relevant kinematics and the infinite-momentum technique. Sections III and IV give the derivations of the one- and two-photon amplitudes. Section V is a discussion of results and various extensions. Finally, evaluation of some traces and discussion of numerical details are given in Appendixes A and B.

II. KINEMATICS

The general Z^0 photoproduction process is shown in Fig. 1. A photon of four-momentum q is incident upon some target of four-momentum p and mass m, producing a Z^0 of four-momentum q' and mass M plus missing mass m'. In the laboratory the photon energy and the energy transferred to the target are given by

$$\omega = \frac{s - m^2}{2m}\ , \qquad (2.1)$$

$$\nu = \frac{m'^2 - m^2 - t}{2m}\ , \qquad (2.2)$$

respectively, where $s = (p+q)^2$ is the total center-of-mass energy squared and $t = (q'-q)^2$ is the center-of-mass momentum-transfer squared. The primary question here is whether the Z^0 can be produced even under ideal conditions. So we consider the limit $\omega \to \infty$, with the other variables held fixed. Corrections of $O(M/\omega)$ will tend to decrease the result.

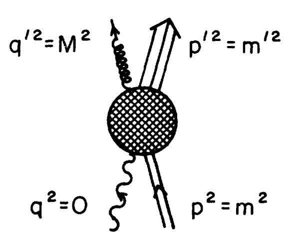

FIG. 1. The general Z^0 photoproduction process.

A. Scales for the momentum transfer

In the forward direction t is given by

$$t_{\min}=-M^2\left(\frac{M^2}{4\omega^2}+\frac{\nu}{\omega}\right) . \tag{2.3}$$

In terms of the natural variable t/M^2, we have elastic scattering when $t/M^2 \sim O(M^2/\omega^2)$, pion production when $t/M^2 \sim O(m_\pi/\omega)$, and a third region in which $t/M^2 \sim O(1)$. Which region is of interest depends on the backgrounds to the possible Z^0 decays, and all possibilities should be considered. There will be a peak in the forward direction as in Delbrück scattering, growing like $\ln^2(\omega)$ due to virtual production of lepton pairs in the photon. In this study we concentrate only on such photon-exchange contributions to Fig. 1. To lowest order in the fine-structure constant α and to all orders in $Z\alpha$, where Z is the number of unit charges in the target, the process involves a single lepton loop as shown in Fig. 2.

Photon-exchange processes are also of interest outside the $t_{\min}$ region. Larger t values may be helpful in eliminating some background, and may provide qualitative ideas for more general Compton-type processes. Here we study the region $t/M^2 \sim O(1)$. As $t/M^2 \to 0$ the result must join smoothly with the forward behavior, so that

$$\frac{d\sigma}{dt} \to \ln^2(-t/M^2) \to \ln^2(-t_{\min}/M^2) . \tag{2.4}$$

For a given value of t we imagine integrating over all accessible values of m'^2 or ν. The electron-proton inelastic experiments[9] indicate that the result may be on the same order as the pointlike process. Information on the region intermediate between the forward and scaling regions will depend on details of the target. These may well be important, but we make no attempt to include them at this stage. As a first step, then, we study the pointlike process $\gamma e \to Z^0 e$.

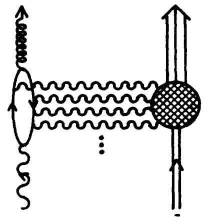

FIG. 2. Multiphoton exchange contributions to Fig. 1.

B. Infinite-momentum technique

We choose the three-axis in the direction of the incident beam and, instead of writing four-vectors as $p=(p_0, p_1, p_2, p_3)$, use the plus-minus components[10]

$$p=(p_0+p_3, p_1, p_2, p_0-p_3)=(p_+, \vec{p}, p_-) . \tag{2.5}$$

The notation $\vec{p}$ will always indicate a two-vector transverse to the three-axis. In terms of these components the Dirac matrices obey

$$\begin{aligned}&\{\gamma_\pm, \vec{\gamma}\}=0 ,\\ &\gamma_\pm\gamma_\mp\gamma_\pm = 4\gamma_\pm ,\\ &\gamma_\pm{}^2=0 ,\end{aligned} \tag{2.6}$$

and the scalar product becomes

$$p_1 \cdot p_2 = \tfrac{1}{2}(p_{1+}p_{2-}+p_{1-}p_{2+}) - \vec{p}_1\cdot\vec{p}_2 . \tag{2.7}$$

The scalar product is clearly invariant under the scale transformation

$$p \to (p_+/\eta, \vec{p}, p_-\eta) , \tag{2.8}$$

which is equivalent to a boost along the three-axis with rapidity $\xi=\ln(\eta)$.

The central advantage of the $p_\pm$ variables in $2\to 2$ processes at high energy is the fact that in the center-of-mass frame, one particle carries only a large p_+ and the other carries only a large p_-. Since the momentum transfer tends to be small and purely transverse, this separation between p_+ and p_- tends to be unaffected by the interaction.

The center-of-mass frame is shown in Fig. 3, with the beam directed along the three-axis with momentum

$$q_3=\frac{s-m^2}{2\sqrt{s}}=\omega\left(\frac{m^2}{s}\right)^{1/2} \equiv \bar{\omega} . \tag{2.9}$$

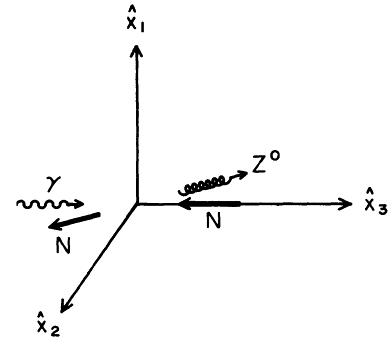

FIG. 3. The center-of-mass frame.

In the notation of Eq. (2.5) we have

$$q = (2\tilde{\omega}, \vec{0}, 0)\,,$$
$$p = \left(\frac{m\tilde{\omega}}{\omega}, \vec{0}, \frac{m}{\tilde{\omega}}\,\omega\right), \tag{2.10}$$

so the beam has only large q_+ and the target has only large p_-. (The apparent dependence on m eventually cancels for pointlike targets.) If the four-momentum transfer is written

$$k = p' - p\,, \quad k^2 = t\,, \tag{2.11}$$

then the mass-shell conditions for q' and p' imply

$$k_- = \frac{M^2 - k^2}{q_+}\,,$$
$$k_+ = \frac{k^2 - p_+ k_-}{p_-}\,.$$

If we require

$$\frac{M^2}{\omega} \ll |\vec{k}| \ll \omega\,, \tag{2.12}$$

then $k_\pm$ can be neglected and we have

$$t = -\vec{k}^2\,. \tag{2.13}$$

Condition (2.12) will be assumed true here.

When t is finite and transverse, the s dependence of photon-exchange amplitudes can be scaled out with Lorentz transformations as in (2.8). The incoming momentum q (or p) is finite in a frame moving with the photon (or target). For example, the matrix element for one-photon exchange has the form

$$\mathfrak{M}_1 = A_\mu(q, k)\,\frac{-i}{k^2 + i\epsilon}\,B^\mu(k, p)\,. \tag{2.14}$$

Here A may be evaluated by boosting with rapidity[11] $\xi' = \ln(q_+)$ so that $A = (A'_+ q_+, A, A'_-/q_+)$, where $A'_\pm$ is finite. Similarly B may be evaluated by boosting with rapidity $\xi'' = \ln(1/p_-)$ so that $B = (B''_+/p_-, B, B''_- p_-)$, where $B''_\pm$ is finite. Then we have

$$A_\mu B^\mu = \tfrac{1}{2}\,q_+ p_- A'_+ B''_- + \frac{1}{2q_+p_-}\,A'_- B''_+ - \vec{A}\cdot\vec{B}$$
$$= m\,\omega\,[A'_+ B''_- + O(1/m\,\omega)]\,. \tag{2.15}$$

In the frame defined by ξ' the arguments of A are

$$\text{standard frame for the photon}\begin{cases} q = (1, \vec{0}, 0)\,, \\ k = (0, \vec{k}, \vec{k}^2 + M^2), \end{cases} \tag{2.16}$$

where $k_+ \sim O(1/2m\omega)$ and k_- is fixed by $q'^2 = M^2$. In the frame defined by ξ'' the arguments of B are

$$\text{standard frame for the target}\begin{cases} k = (\vec{k}^2, \vec{k}, 0)\,, \\ p = (m^2, \vec{0}, 1)\,, \end{cases} \tag{2.17}$$

where $k_- \sim O(1/2m\omega)$ and k_+ is fixed by $p'^2 = m^2$. When n photons are exchanged, Eq. (2.15) applies to each pair of indices, so that

$$A_{\mu_1 \cdots \mu_n} B^{\mu_1 \cdots \mu_n} = (m\,\omega)^n\,[A'_{+ \cdots +} B''_{- \cdots -} + O(1/m\,\omega)]\,, \tag{2.18}$$

and there are $n-1$ integrations:

$$d^4k_i = \frac{dk'_{i-}\,dk''_{i+}\,d^2k_i}{2m\,\omega}\,, \tag{2.19}$$

so the result is still linear in ω.

This simple factorization of the energy dependence breaks down near $t_{\min}$ when $k_\pm$ become important, but as long as (2.12) holds, all calculations may be done in terms of the energy-independent momenta in the standard frames.

Two additional standard-frame results are needed. Firstly, if u_λ is a Dirac spinor of helicity λ, then in the target frame one has

$$\bar{u}''_{\lambda'}\,\gamma_-\,u''_\lambda = \delta_{\lambda\lambda'}/m\,, \tag{2.20}$$

so that the target helicity is always conserved. Secondly, if $\epsilon_\mu(r)$ is a spin-one polarization vector with four-momentum r, then in the photon frame one finds

$$\epsilon_{\pm 1}(r) = (0, \vec{\epsilon}_{\pm 1}, 2\vec{r}\cdot\vec{\epsilon}_{\pm 1})\,,$$
$$\epsilon_0(r) = -2M(0, \vec{0}, 1) + r/M\,, \tag{2.21}$$

where the transverse helicity vector is

$$\vec{\epsilon}_{\pm 1} = \mp\frac{1}{\sqrt{2}}\,(1, \pm i)\,, \tag{2.22}$$

and M is the mass.

III. ONE-PHOTON EXCHANGE

The one-photon exchange term of Z^0 photoproduction from a point target is shown in Fig. 4. By charge-conjugation invariance only the axial-vector current contributes at the Z^0 vertex, and the second diagram just gives a factor of 2. By power counting the loop appears to be linearly divergent, but conservation of the two vector currents renders it finite. However, as emphasized by Adler,[12] a shift of integration may introduce a surface term.

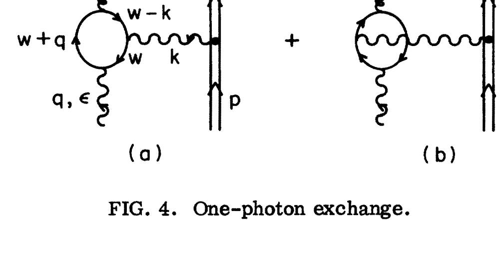

FIG. 4. One-photon exchange.

The ambiguity is resolved by routing the loop momentum so as to give the photons Bose symmetry. A second apparent divergence comes when $t=-\vec{k}^2 \to 0$, because of the infinite range of the interaction. When the Z^0 mass is nonzero this is also eliminated by current conservation, because in the photon frame one has

$$k^{\mu'} A'_{\mu} = \tfrac{1}{2}(\vec{k}^2 + M^2) A'_{+} - \vec{k}\cdot\vec{A} = 0 \ , \tag{3.1}$$

so that A'_+ must vanish as $\vec{k}^2 \to 0$.

A. Derivation of the amplitude

Combining Eqs. (2.14), (2.15), and (2.20) we find that the infinite-energy amplitude has the form

$$\mathfrak{M}_1 = \frac{\omega}{\vec{k}^2} A'_+ e \delta_{\lambda\lambda'} \ . \tag{3.2}$$

Note that the dependence on the target mass cancels, indicating that our results are equivalent to those for a fixed potential.

A'_+ can be written in terms of the VVA vertex function[12] $R_{\sigma\rho\mu}$ as

$$A'_+ = \frac{-ie^2}{(2\pi)^4}(+ig_A)\left[-\epsilon^i R_{i+\mu}\epsilon^{*\mu}\right], \tag{3.3}$$

where e is the electric charge, g_A is the axial coupling constant appearing in Eq. (1.1), ϵ is the photon polarization vector (purely transverse since $\vec{q}=0$), and ϵ^* is the Z^0 polarization vector. In writing down $R_{i+\mu}$ we let

$$w_+ = -\beta' \, , \, 1 - w_+ = \beta \ . \tag{3.4}$$

Factoring β and β' out of the denominators of the lepton propagators, we can write

$$R_{i+\mu} = \int d\beta\, d\beta'\, \delta(1-\beta-\beta') \int d^2 w \int dw_- \frac{T_{i+\mu}}{\beta\beta'^2}$$
$$\times \frac{1}{\left(w_- - \dfrac{\vec{w}^2 + m_l^2}{\beta} + i\dfrac{\epsilon}{\beta}\right)\left(w_- - \dfrac{\vec{w}^2+m_l^2}{-\beta'} + i\dfrac{\epsilon}{-\beta'}\right)\left[w_- - (\vec{k}^2 + M^2) - \dfrac{(\vec{w}-\vec{k})^2 + m_l^2}{-\beta'} + i\dfrac{\epsilon}{-\beta'}\right]}, \tag{3.5}$$

where $T_{i+\mu}$ is the trace of the numerator factors and m_l is the lepton mass. Consider the contour integral of w_-. Since $\gamma_+{}^2 = 0$, the γ_+ coupling of the exchanged photon eliminates two factors of $w_-\gamma_+$ in the numerator, so only the poles contribute. The poles lie in the same half plane unless $\beta, \beta' > 0$. Closing the contour in the lower half plane picks out

$$w_- = \frac{\vec{w}^2 + m_l^2}{\beta} \ . \tag{3.6}$$

Combining the two remaining denominator factors with a Feynman parameter x, we obtain

$$R_{i+\mu} = -2\pi i \int d\beta\, d\beta'\, \delta(1-\beta-\beta') \int_0^1 dx \int d^2 w \frac{\beta T_{i+\mu}}{[(\vec{w} - x\beta\vec{k})^2 + x(1-x)\beta^2\vec{k}^2 + m_l^2 - x\beta\beta' M^2 - i\epsilon]^2} \ . \tag{3.7}$$

Note that $x\beta\beta' > \frac{1}{4}$, so when $M^2 > (2m_l)^2$ the denominator can vanish. Therefore the vertex function has an imaginary part due to the decay $Z^0 \to l^+ l^-$.

Inserting the Z^0 polarization vectors into Eq. (3.3), the factor in square brackets has the form

$$\begin{aligned} f_T &= -\epsilon^i (R_{i++}\vec{k}\cdot\vec{\epsilon}^* - R_{i+j}\epsilon^{*j}) \ , \\ f_L &= \epsilon^i (R_{i++}M - R_{i+\mu}q'^{\mu}/M) \ , \end{aligned} \tag{3.8}$$

associated with transverse (T) and longitudinal (L) Z^0, respectively. Only the longitudinal factor, which involves R_{i+-} in the last term, has the ambiguous linear divergence mentioned above. This can be handled with Adler's "anomalous Ward identity,"[13] expressed in the photon frame as

$$R_{i+\mu} q'^{\mu}/M = R_{i+}(2m_l/M) + 8\pi^2 \epsilon_{ij} k^j / M \ , \tag{3.9}$$

where the first term is the "naive" divergence obtained by replacing γ_μ by $2m_l$, and the second term is the "anomaly."

The numerators of the remaining terms in Eq. (3.8), T_{i++} and T_{i+j}, are evaluated in Appendix A. No linear divergence appears, and letting

$$\vec{w} = \vec{w}' + x\beta\vec{k} \ , \tag{3.10}$$

we obtain

$$f_T=-2\pi i(4i)\int d\beta\, d\beta'\,\delta(1-\beta-\beta')\int_0^1 dx\int d^2w'\,\beta$$
$$\times\frac{-2x\beta\beta'[(\vec\epsilon\times\vec k)\,\vec\epsilon^*\cdot\vec k-\vec\epsilon\cdot\vec k\,(\vec\epsilon^*\times\vec k)]\cdot\hat x_3+[((\beta'-\beta)/\beta)(\vec w'^2+m_l^2)+((\beta'-\beta)x+1)\,x\beta\vec k^2](\vec\epsilon\times\vec\epsilon^*)\cdot\hat x_3}{[\vec w'^2+x(1-x)\beta^2\vec k^2+m_l^2-x\beta\beta' M^2-i\epsilon]^2}, \tag{3.11}$$

$$f_L=-2\pi i(4i)\int d\beta\, d\beta'\,\delta(1-\beta-\beta')\int_0^1 dx\int d^2w'\frac{\beta(-2x\beta\beta' M^2)(\vec\epsilon\times\vec k/M)\cdot\hat x_3}{[\vec w'^2+x(1-x)\beta^2\vec k^2+m_l^2-x\beta\beta' M^2-i\epsilon]^2}$$
$$-\epsilon^i R_{i+}\frac{2m_l}{M}-8\pi^2(\vec\epsilon\times\vec k/M)\cdot\hat x_3\,. \tag{3.12}$$

The transverse integral in f_T appears logarithmically divergent at the upper limit, but since the coefficient is antisymmetric in β and β', it integrates to zero. The logarithmic term from the lower limit can be integrated by parts over x as follows:

$$-\int_0^1 dx\ln(x(1-x)\beta^2\vec k^2+m_l^2-x\beta\beta' m^2-i\epsilon)=-\ln(m_l^2-\beta\beta' M^2-i\epsilon)+\int_0^1 dx\,\frac{x(1-2x)\beta^2\vec k^2-x\beta\beta' M^2}{x(1-x)\beta^2\vec k^2+m_l^2-x\beta\beta' M^2-i\epsilon}, \tag{3.13}$$

and again the first term does not contribute by symmetry. After some algebra we find

$$f_T=8\pi^2\int d\beta\, d\beta'\,\delta(1-\beta-\beta')\int_0^1 dx\beta\,\frac{-2x\beta\beta'[(\vec\epsilon\times\vec k)\,\vec\epsilon^*\cdot\vec k-\vec\epsilon\cdot\vec k(\vec\epsilon^*\times\vec k)]\cdot\hat x_3+x\beta\vec k^2(\vec\epsilon\times\vec\epsilon^*)\cdot\hat x_3}{x(1-x)\beta^2\vec k^2+m_l^2-x\beta\beta' M^2-i\epsilon} \tag{3.14}$$

and

$$f_L=8\pi^2\int d\beta\, d\beta'\,\delta(1-\beta-\beta')\int_0^1 dx\beta\,\frac{-2x\beta\beta' M^2(\vec\epsilon\times\vec k/M)\cdot\hat x_3}{x(1-x)\beta^2\vec k^2+m_l^2-x\beta\beta' M^2-i\epsilon}-\epsilon^i R_{i+}\frac{2m_l}{M}-8\pi^2(\vec\epsilon\times\vec k/M)\cdot\hat x_3. \tag{3.15}$$

These equations have a number of interesting properties. They vanish when $\vec k\to 0$, as we have seen from current conservation. Since $\vec\epsilon^*_\pm=-\vec\epsilon_\mp$, we find that $\Delta\mu\neq 2$ in Eq. (3.14) and the polarization vectors can be replaced by $-i\delta_{\mu\mu'}$, where μ and μ' are the photon and Z^0 helicities. This is essentially the Primakoff effect[14] and does not hold for two-photon exchange. Finally, if we neglect the lepton mass in Eq. (3.15) then as $\vec k^2\to 0$ the first term is canceled by the Adler anomaly, thus verifying angular momentum conservation. Since m_l/M is small for the known leptons, we examine that limit in more detail.

B. Zero-lepton-mass helicity amplitudes

Taking the limit $m_l\to 0$ in the last two equations and canceling a factor x in numerator and denominator, we can carry out the x integral trivially. The result is

$$f_T=8\pi^2 i\delta_{\mu\mu'}\int d\beta\, d\beta'\,\delta(1-\beta-\beta')(\beta'-\beta)$$
$$\times\ln(\beta^2(\vec k^2/M^2)-\beta\beta'-i\epsilon)\,, \tag{3.16}$$

$$f_L=-8\pi^2\left(\vec\epsilon\times\frac{\vec k}{M}\right)\left[\int d\beta\, d\beta'\,\delta(1-\beta-\beta')\,2\beta'\frac{M^2}{\vec k^2}\right.$$
$$\left.\times\ln\left(\frac{\beta^2(\vec k^2/M^2)-\beta\beta'-i\epsilon}{\beta\beta'}\right)+1\right], \tag{3.17}$$

where $\ln(x-i\epsilon)=\ln|x|+i\pi\theta(x)$. If we define

$$if_{T,L}=8\pi^2\frac{\vec k^2}{M^2}f_{\Delta\mu}\left(\frac{\vec k^2}{M^2}\right), \tag{3.18}$$

then Eq. (3.2) becomes

$$\mathfrak{M}_1=-i\omega\delta_{\lambda\lambda'}\frac{8\hbar c\alpha^2}{M^2}\frac{g_A'}{e}f_{\Delta\mu}\left(\frac{\vec k^2}{M^2}\right), \tag{3.19}$$

where we have inserted the units and also set $e^2/4\pi=\alpha$. The remaining integral in Eqs. (3.16) and (3.17) gives the final results in the form

$$f_0\left(\frac{\vec k^2}{M^2}\right)=\frac{1+(\vec k^2/M^2)+\ln(\vec k^2/M^2)+i\pi}{[1+(\vec k^2/M^2)]^2}, \tag{3.20}$$

$$f_1\left(\frac{\vec k^2}{M^2}\right)=\left(i\vec\epsilon\times\frac{\vec k}{M}\right)\cdot\hat x_3 f_0\left(\frac{\vec k^2}{M^2}\right). \tag{3.21}$$

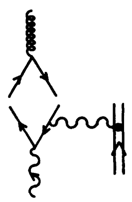

FIG. 5. On-shell lepton pair in one-photon exchange.

The most striking result here is the rapid damping of the amplitude with increasing M^2. This is simply a result of current conservation and dimensional analysis, and therefore it is true for multiphoton exchange as well. It is unfortunate that even at infinite energy and forward angles we cannot overcome this rapid damping.

As $\vec{k}^2/M^2 \to 0$ we see from Eq. (3.20) that

$$f_0 \to \ln\frac{\vec{k}^2}{M^2} \to \ln\left(\frac{2m_l}{M}\right)^2 , \qquad (3.22)$$

where we assume that the behavior is smooth as $|\vec{k}| \to k_{\min} = M^2/2\omega$. The amplitude is purely imaginary in this limit and associated with the production of the lepton pair as shown in Fig. 5. The total cross section for pair production is given by the imaginary part of the forward Delbrück amplitude[15] and can be written as

$$\sigma(l^+l^-) = \tfrac{14}{9}\frac{\alpha^3}{m_l^2}\left[\ln\left(\frac{2\omega}{m_l}\right)^2 - \tfrac{109}{21}\right]. \qquad (3.23)$$

The characteristic logarithmic behavior which is reflected in (3.22) is not affected by the outgoing particle or, as we will see, by the number of photons exchanged. In Sec. IV we study the two-photon case, generalizing the Delbrück scattering by giving the final photon a mass.

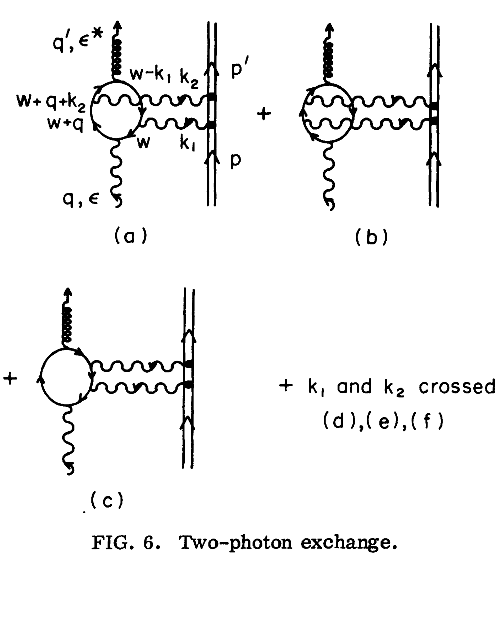

FIG. 6. Two-photon exchange.

IV. TWO-PHOTON EXCHANGE

In this section the high-energy limit of the amplitude represented by Fig. 6 is derived. The algebra closely parallels the one-photon case. The limit $m_l/M \to 0$ is then taken and the results given in a form like Eqs. (3.19)–(3.21), with additional integrals over the relative transverse momenta of the exchanged photons and a Feynman parameter. A graph of the momentum transfer behavior of f_0', obtained by numerical integration, is shown in order to exhibit the characteristic logarithmic behavior.

A. Derivation of the amplitude

The two-photon amplitude has the form

$$\mathfrak{M}_2 = \frac{1}{8}\frac{m\omega}{(2\pi)^4}\int dk_{1-}'\, dk_{1+}''\, d^2k_1 \frac{-i}{k_1^2+i\epsilon}\,\frac{-i}{k_2^2+i\epsilon} \times A_{++}' B_{--}'' , \qquad (4.1)$$

according to the discussion of Sec. II, and where $k_1 + k_2 = k$. Since $k_{1\pm}$ must be finite in the center-of-mass system, the behavior in the standard frames is

$$k_{1+}', k_{1-}'' \sim O(1/\omega) ,$$
$$k_{1-}', k_{1+}'' \sim O(\omega) . \qquad (4.2)$$

B_{--}'' is the sum of two terms in which the target emits k_1 before k_2, and then k_2 before k_1 (labeled "k_1 and k_2 crossed" in Fig. 6). According to Eqs. (2.6), (2.20), and (4.2), these two terms have the form

$$\frac{\bar{u}_{\lambda'}''\gamma_-(\not{p}-\not{k}_1+m)\gamma_- u_\lambda''}{(p-k_1)^2-m^2+i\epsilon} = \frac{2\delta_{\lambda\lambda'}/m}{-k_{1+}''+i\epsilon} ,$$

$$\frac{\bar{u}_{\lambda'}''\gamma_-(\not{p}-\not{k}_2+m)\gamma_- u_\lambda''}{(p-k_2)^2-m^2+i\epsilon} = \frac{2\delta_{\lambda\lambda'}/m}{k_{1+}''+i\epsilon} ,$$

so that the sum is purely imaginary, giving

$$B_{--}'' = -4\pi i(-ie)^2\,\delta(k_{1+}'')\,\delta_{\lambda\lambda'}/m . \qquad (4.3)$$

The k_{1+}'' integral is therefore trivial and Eq. (4.1) becomes

$$\mathfrak{M}_2 = -i\omega\frac{\pi e^2}{2(2\pi)^4}\delta_{\lambda\lambda'}\int d^2k_1 \frac{1}{\vec{k}_1^2}\frac{1}{\vec{k}_2^2}\int dk_{1-}' A_{++}' . \qquad (4.4)$$

As in the one-photon exchange, current conservation requires that A_{++}' vanish as either $\vec{k}_1^2$ or $\vec{k}_2^2$ vanishes. So when $\vec{k}=0$, Eq. (4.4) is logarithmically divergent as $\vec{k}_1^2 \to 0$, which implies that $\mathfrak{M}_2 \sim \ln(\vec{k}^2)$ as expected. Current conservation also implies that only the first diagram in Fig. 6 need be evaluated. The second two are easily seen to be only functions of k, so if we call the contribution from the first $A(\vec{k}_1, \vec{k}_2)$ (symmetric in $\vec{k}_1$ and $\vec{k}_2$) then the total must have the form

$$\int dk'_{1-} A'_{++} = A(\vec{k}_1, \vec{k}_2) - A(\vec{0}, \vec{k}) \tag{4.5}$$

in order to satisfy current conservation.

The function $A(\vec{k}_1, \vec{k}_2)$ can be written similarly to Eq. (3.3) as

$$A(\vec{k}_1, \vec{k}_2) = \frac{-ie^3}{(2\pi)^4}(-ig_V)\left[-\epsilon^i \int dk'_{1-} R_{+i+\mu}\,\epsilon^{*\mu}\right], \tag{4.6}$$

and with the notation of Eq. (3.4) we have

$$\int dk'_{1-} R_{+i+\mu} = \int d\beta\, d\beta' \int d^2w \int dw_- \int dk_{1-} \frac{T_{+i+\mu}}{\beta^2\beta'^2} \frac{1}{\left(w_- - \frac{\vec{w}^2 + m_l^2}{\beta} + \frac{i\epsilon}{\beta}\right)\left(w_- - \frac{\vec{w}^2 + m_l^2}{-\beta'} + \frac{i\epsilon}{-\beta'}\right)}$$
$$\times \frac{1}{\left[w_- - k_{1-} - \frac{(\vec{w} - \vec{k}_1)^2 m_l^2}{-\beta'} + \frac{i\epsilon}{-\beta'}\right]\left[w_- - k_{1-} - (\vec{k}^2 + M^2) - \frac{(\vec{w} + \vec{k}_2)^2 + m_l^2}{\beta} + \frac{i\epsilon}{\beta}\right]}. \tag{4.7}$$

The numerator here is independent of the minus components because $\gamma_+{}^2 = 0$. The variables w_- and $y_- = w_- - k_{1-}$ decouple and their poles lie in the same half plane unless $\beta, \beta' > 0$. Closing each contour in the lower half plane picks out

$$w_- = (\vec{w}^2 + m_l^2)/\beta\ ,$$
$$y_- = \vec{k}^2 + M^2 + [(\vec{w} + \vec{k}_2)^2 + m_l^2]/\beta\ . \tag{4.8}$$

Using a Feynman parameter x to combine the two remaining denominators, we find

$$\int dk'_{1-} R_{+i+\mu} = (-2\pi i)^2 \int d\beta\, d\beta'\, \delta(1-\beta-\beta') \int_0^1 dx \int d^2w \frac{T_{+i+\mu}}{[(\vec{w} + x\vec{R})^2 + x(1-x)\vec{R}^2 + m_l^2 - x\beta\beta' M^2 - i\epsilon]^2}\ , \tag{4.9}$$

where

$$\vec{R} = \beta'\vec{k}_2 - \beta\vec{k}_1\ . \tag{4.10}$$

This is a simple generalization of the one-photon expression, Eq. (3.7), and again it has an imaginary part due to Z^0 decay when $M^2 > (2m_l)^2$.

As before, we define the term in square brackets in Eq. (4.6) as f'_T or f'_L for transverse or longitudinal Z^0. Since the vector current is conserved we have

$$R_{+i+\mu}\, q'^\mu = 0, \tag{4.11}$$

so the second term of $\epsilon_0(q')$ does not contribute to f_L. The traces T_{+i++} and T_{+i+j} are evaluated in Appendix A, and letting

$$\vec{w} = \vec{w}' - x\vec{R}\ , \tag{4.12}$$

we obtain

$$f'_T(\vec{k}_1, \vec{k}_2) = 8(2\pi)^2 \int d\beta\, d\beta'\, \delta(1-\beta-\beta')$$
$$\times \int_0^1 dx \int d^2w' \frac{4\beta\beta' x(1-x)\vec{R}\cdot\vec{\epsilon}\,\vec{R}\cdot\vec{\epsilon}^* + [(1-2\beta\beta')\vec{w}'^2 + m_l^2 - x(1-x)\vec{R}^2]\,\vec{\epsilon}\cdot\vec{\epsilon}^*}{[\vec{w}'^2 + x(1-x)\vec{R}^2 - x\beta\beta' M^2 - i\epsilon]^2}\ , \tag{4.13}$$

$$f'_L(\vec{k}_1, \vec{k}_2) = 8(2\pi)^2 \int d\beta\, d\beta'\, \delta(1-\beta-\beta') \int_0^1 dx \int d^2w' \frac{-2M\beta\beta'(\beta-\beta')\,x\vec{R}\cdot\vec{\epsilon}}{[\vec{w}'^2 + x(1-x)\vec{R}^2 - x\beta\beta' M^2 - i\epsilon]^2}\ . \tag{4.14}$$

The logarithmic divergence in f'_T will clearly cancel when the subtraction indicated by Eq. (4.5) is made. The logarithm from the lower limit can be written as in Eq. (3.13) and the first term will not contribute to Eq. (4.5). If we redefine Eqs. (4.13) and (4.14) through

$$(g_V/e)\,(\alpha/\pi)\, f'_{T,L} = 4(2\pi)^3 I_{T,L}\ , \tag{4.15}$$

then the two-photon amplitude takes the form

$$\mathfrak{M}_2 = i\,\omega\delta_{\lambda\lambda'}\,e^4\int[d^2k_1/(2\pi)^2]\;(1/\vec{k}_1^{\,2})(1/\vec{k}_2^{\,2})\;[I(\vec{k}_1,\vec{k}_2)-I(\vec{0},\vec{k})]\;, \tag{4.16}$$

and the result of the w' integral is

$$I_T(\vec{k}_1,\vec{k}_2)=\frac{g_V}{e}\frac{\alpha}{\pi}\int d\beta\,d\beta'\,\delta(1-\beta-\beta')$$
$$\times\int_0^1 dx\,\frac{4\beta\beta' x(1-x)\vec{R}\cdot\vec{\epsilon}\,\vec{R}\cdot\vec{\epsilon}^{\,*}-\frac{1}{2}[\vec{R}^2(1-8\beta\beta'(x-\frac{1}{2})^2)-\beta\beta' M^2(1-2\beta\beta'+4\beta\beta' x)]\,\vec{\epsilon}\cdot\vec{\epsilon}^{\,*}}{x(1-x)\vec{R}^2+m_l^{\,2}-x\beta\beta' M^2-i\epsilon}\,, \tag{4.17}$$

$$I_L(\vec{k}_1,\vec{k}_2)=-\frac{g_V}{e}\frac{\alpha}{\pi}\int d\beta\,d\beta'\,\delta(1-\beta-\beta')\int_0^1 dx\,\frac{2M\beta\beta'(\beta-\beta')\,x\vec{R}\cdot\vec{\epsilon}}{x(1-x)\vec{R}^2+m_l^{\,2}-x\beta\beta' M^2-i\epsilon}\,. \tag{4.18}$$

These equations can now be directly compared with the known result for γe elastic and Delbrück scattering.[16] In the limit $M\to 0$ and $g_V/e=1$ the longitudinal term vanishes and the transverse term reproduces the known result. Then the $i\epsilon$ can be dropped, so that I is purely real and the amplitude is purely imaginary. In the present case we are interested in the opposite limit $M\to\infty$, or equivalently $m_l\to 0$.

B. Zero-lepton-mass helicity amplitudes

The limit $m_l\to 0$ is straightforward except for the two coefficients of $\vec{\epsilon}\cdot\vec{\epsilon}^{\,*}$ in Eq. (4.17), which are badly behaved as $x\to 0$. They involve

$$\int_0^1 dx\,\frac{\vec{R}^2-\beta\beta' M^2}{x(1-x)\vec{R}^2+m_l^{\,2}-x\beta\beta' M^2-i\epsilon}\,, \tag{4.19}$$

which diverges logarithmically when the lepton mass vanishes. However, the divergence will be canceled in Eq. (4.5). We extract the finite part as follows. In the numerator of (4.19) we subtract $1/x$ times the denominator. Such a term does not contribute to Eq. (4.5). After the subtraction (4.19) becomes

$$\int_0^1 dx\,\frac{x\vec{R}^2-(1/x)m_l^{\,2}}{x(1-x)\vec{R}^2+m_l^{\,2}-x\beta\beta' M^2-i\epsilon}$$
$$=\int_0^1 dx\,\frac{x\vec{R}^2}{x(1-x)\vec{R}^2+m_l^{\,2}-x\beta\beta' M^2-i\epsilon}$$
$$-\int_0^1\frac{dx}{x}\,\frac{m_l^{\,2}}{x(1-x)\vec{R}^2+m_l^{\,2}-x\beta\beta' M^2-i\epsilon}\,. \tag{4.20}$$

In the first term of Eq. (4.20) the $m_l^{\,2}$ in the denominator can safely be neglected. After canceling out a factor of x in the numerator and denominator, we can carry out the x integral trivially. In the second term of Eq. (4.20) it can be shown that, to $O(m_l^{\,2})$, the $x^2\vec{R}^2$ in the denominator can be dropped and the upper limit of the integral can be extended to infinity. We then encounter an integral of the form $\int_0^\infty (dx/x)f(ax)$, where $f(ax)=m_l^{\,2}/(ax+m_l^{\,2})$ and $a=\vec{R}^2-\beta\beta' M^2$. Substituting this second term into Eq. (4.5) and combining it with a similar term from $A(\vec{0},\vec{k})$, we can evaluate the resulting form via the identity

$$\int_0^\infty\frac{dx}{x}[f(ax)-f(bx)]=[f(0)-f(\infty)]\ln\left(\frac{b}{a}\right), \tag{4.21}$$

which is valid whenever df/dx is continuous and $\int_1^\infty (dx/x)[f(x)-f(\infty)]$ converges.

By the method outlined above, the x integration in Eqs. (4.17) and (4.18) can be done in the limit $m_l\to 0$ to give

$$I_T(\vec{k}_1,\vec{k}_2)=\frac{g_V}{e}\frac{\alpha}{\pi}\int d\beta\,d\beta'\,\delta(1-\beta-\beta')$$
$$\times\left\{2\beta\beta'\left(\frac{2\vec{R}\cdot\vec{\epsilon}\,\vec{R}\cdot\vec{\epsilon}^{\,*}}{\vec{R}^2}-\vec{\epsilon}\cdot\vec{\epsilon}^{\,*}\right)\left[1+\frac{\beta\beta' M^2}{\vec{R}^2}\ln\left(\frac{\vec{R}^2-\beta\beta' M^2-i\epsilon}{-\beta\beta' M^2}\right)\right]\right.$$
$$\left.-(1-2\beta\beta')\,\vec{\epsilon}\cdot\vec{\epsilon}^{\,*}\ln\left(\frac{\vec{R}^2-\beta\beta' M^2-i\epsilon}{-\beta\beta' M^2}\right)\right\}\,, \tag{4.22}$$

$$I_L(\vec{k}_1,\vec{k}_2)=\frac{g_V}{e}\frac{\alpha}{\pi}\int d\beta\,d\beta'\,\delta(1-\beta-\beta')\,2M\beta\beta'(\beta-\beta')\frac{\vec{R}\cdot\vec{\epsilon}}{\vec{R}^2}\ln\left(\frac{\vec{R}^2-\beta\beta' M^2-i\epsilon}{-\beta\beta' M^2}\right), \tag{4.23}$$

where again $\ln(x - i\epsilon) = \ln|x| + i\pi\theta(x)$. The second term in Eq. (4.22) comes from the above identity and turns out to be just the helicity-conserved term.

If we now scale all the momenta by M, defining

$$\vec{\kappa}_{1,2} = \frac{\vec{k}_{1,2}}{M}, \quad \vec{\kappa} = \vec{\kappa}_1 + \vec{\kappa}_2 \tag{4.24}$$

and

$$I_{T,L}(\vec{k}_1, \vec{k}_2) = \frac{g_Y}{e} \frac{\alpha}{\pi} I_{\Delta\mu}(\vec{\kappa}_1, \vec{\kappa}_2), \tag{4.25}$$

then the two-photon amplitude of Eq. (4.16) takes the form

$$\mathfrak{M}_2 = i\,\omega\delta_{\lambda\lambda'} \frac{4\hbar c\alpha^3}{M^2} \frac{g_Y}{e} \frac{1}{\pi} f'_{\Delta\mu}(\vec{\kappa}^2), \tag{4.26}$$

where

$$f'_{\Delta\mu}(\vec{\kappa}^2) = \int d^2\kappa_1 \frac{1}{\vec{\kappa}_1^{\,2}} \frac{1}{\vec{\kappa}_2^{\,2}} [I_{\Delta\mu}(\vec{\kappa}_1, \vec{\kappa}_2) - I_{\Delta\mu}(\vec{0}, \vec{\kappa})]. \tag{4.27}$$

Making the change of variables

$$\beta = \tfrac{1}{2}(1 - \alpha), \tag{4.28}$$

we obtain

$$I_0(\vec{\kappa}_1, \vec{\kappa}_2) = \tfrac{1}{4} \int_{-1}^{1} d\alpha (1 + \alpha^2) \ln\left(\frac{4\vec{R}^2 - (1 - \alpha^2) - i\epsilon}{1 - \alpha^2}\right), \tag{4.29}$$

$$I_1(\vec{\kappa}_1, \vec{\kappa}_2) = \tfrac{1}{4} \int_{-1}^{1} d\alpha\, \alpha (1 - \alpha^2) \frac{\vec{R}\cdot\vec{\epsilon}}{\vec{R}^2} \times \ln\left(\frac{4\vec{R}^2 - (1 - \alpha^2) - i\epsilon}{1 - \alpha^2}\right), \tag{4.30}$$

$$I_2(\vec{\kappa}_1, \vec{\kappa}_2) = \tfrac{1}{4} \int_{-1}^{1} d\alpha (1 - \alpha^2) \frac{2\vec{R}\cdot\vec{\epsilon}_+ \vec{R}\cdot\vec{\epsilon}^*_-}{\vec{R}^2} \times \left[1 + \frac{1 - \alpha^2}{4R^2} \ln\left(\frac{4\vec{R}^2 - (1 - \alpha^2) - i\epsilon}{1 - \alpha^2}\right)\right], \tag{4.31}$$

where now

$$2\vec{R} \equiv (1 + \alpha)\vec{\kappa}_2 - (1 - \alpha)\vec{\kappa}_1. \tag{4.32}$$

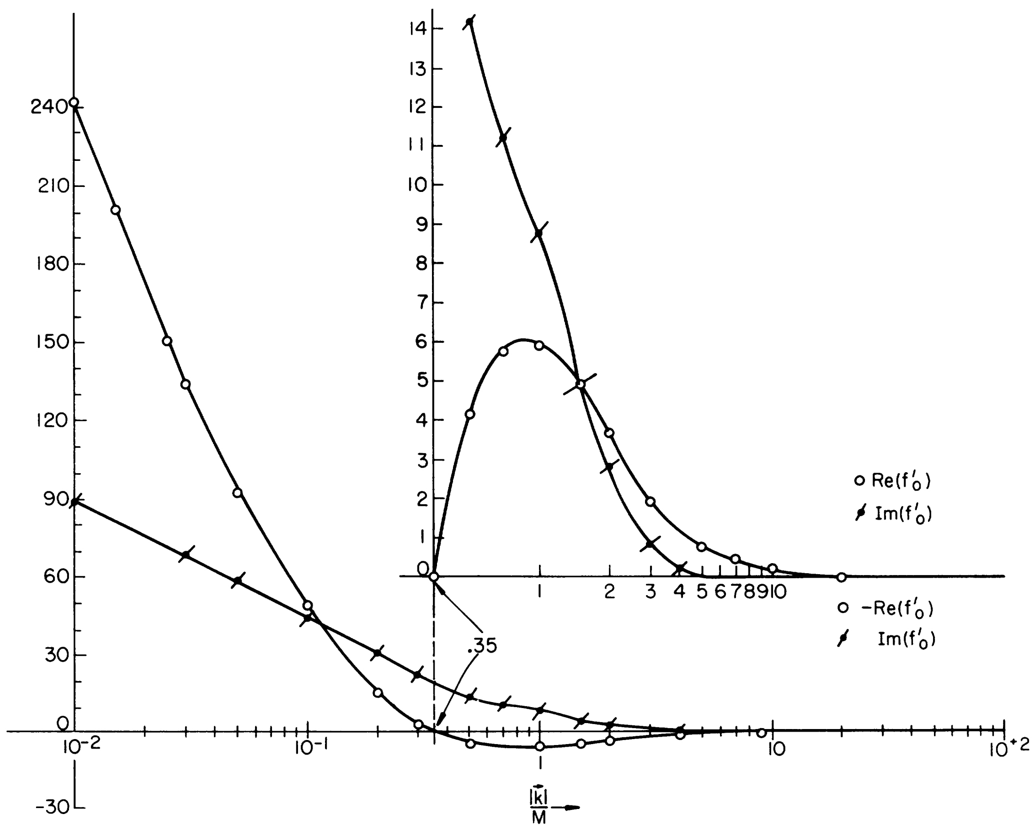

FIG. 7. Momentum transfer dependence of helicity-conserved two-photon exchange. The imaginary part of the amplitude comes from the real part of f_0'.

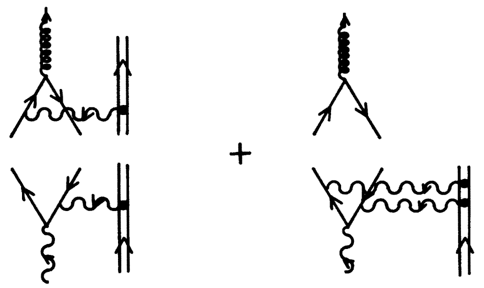

FIG. 8. On-shell lepton pairs in two-photon exchange.

The three-dimensional integrals appearing in these equations have been studied numerically, as discussed in Appendix B. As $\vec{k} \to 0$ only f_0' has the logarithmic behavior and it is shown in Fig. 7. The limiting form is

$$\frac{1}{\pi} f_0' \to (16 - 3i) \ln (\vec{k}^2) \to - 16 \ln \left(\frac{2\omega}{M}\right)^2 - 3i \ln \left(\frac{2m_l}{M}\right)^2 . \quad (4.33)$$

The real and imaginary parts are associated with the one- and two-photon Bethe-Heitler pair production, respectively, shown in Fig. 8. Of course in the Delbrück case the second process cannot occur and the first process leads to the cross section of Eq. (3.23) by the optical theorem.

V. CONCLUSIONS AND EXTENSIONS

We have obtained the one- and two-photon exchange contributions to Z^0 photoproduction at high energy. They have some simple properties which should be emphasized. First of all, the amplitudes factorize in the form

$$\mathfrak{M} = i\omega f(\vec{k}^2, M^2) , \quad (5.1)$$

where f is generally complex due to Z^0 decay, and the lepton mass may be neglected. Second, although f contains the singular Coulomb potential, current conservation implies that the behavior is softened to $\ln(\vec{k}^2)$. As $|\vec{k}| \to k_{\min}$ this becomes $\ln(\omega)$ and is associated with production of the lepton pair. Then the dimensions of f require that it vary inversely with M^2. This makes the production extremely rare, as we can see by considering the pointlike cross sections.

A. Pointlike cross sections

The differential cross sections at high energy have the form

$$\frac{d\sigma}{d\vec{k}^2} = \frac{1}{16\pi\omega^2} |\mathfrak{M}|^2 . \quad (5.2)$$

Inserting the one- and two-photon exchange contributions from Eqs. (3.19) and (4.26) we have

$$\frac{d\sigma^{(1)}_{\Delta\mu}}{d\vec{k}^2} = \frac{1}{\pi} \left| \frac{2\hbar c \alpha^2}{M^2} \frac{g_A}{e} f_{\Delta\mu} \left(\frac{\vec{k}^2}{M^2}\right)\right|^2 , \quad (5.3)$$

$$\frac{d\sigma^{(2)}_{\Delta\mu}}{d\vec{k}^2} = \frac{1}{\pi} \left| \frac{\hbar c \alpha^3}{M^2} \frac{g_V}{e} \frac{1}{\pi} f'_{\Delta\mu} \left(\frac{\vec{k}^2}{M^2}\right)\right|^2 . \quad (5.4)$$

When the momentum transfer is small enough to neglect form factors, the helicity is conserved and the momentum-transfer dependence is simply logarithmic and given to within a constant by (3.22) and (4.33). The controlling behavior is therefore the over-all $1/M^4$. By contrast, in Delbrück scattering it is the *lepton* mass which provides the scale for the cross section. Then when $|\vec{k}| \ll m_e$, where m_e is the electron mass, 5×10^{-4} GeV, we have

$$\frac{d\sigma}{d\vec{k}^2} (\text{Delbrück}) \sim \left(\frac{\hbar c \alpha^3}{m_e^2}\right)^2 \sim 1 \frac{\text{mb}}{\text{GeV}^2} . \quad (5.5)$$

If $g_{V,A}/e \sim O(1)$ then the Z^0 photoproduction is suppressed relative to the Delbrück scattering by a factor $[m_e^2/M^2]^2 \sim 10^{-12}$ for vector coupling and $10^{-12}/\alpha^2 \sim 10^{-8}$ for axial-vector coupling.

In the present case, however, the Z^0 mass also provides the scale of momentum transfer. We are therefore led to consider values of $\vec{k}^2$ large on the hadronic scale in the hope that summing over final states of the nucleus may not greatly decrease the cross sections. The integrated cross sections are

$$\sigma_{\Delta\mu} = \int_0^\infty d\vec{k}^2 \frac{d\sigma_{\Delta\mu}}{d\vec{k}^2} = M^2 \int_0^\infty d\left(\frac{\vec{k}^2}{M^2}\right) \frac{d\sigma_{\Delta\mu}}{d\vec{k}^2} , \quad (5.6)$$

which fall off like $1/M^2$. In the helicity-conserved case we find[17]

$$\sigma_0^{(1)} = \frac{1+\pi^2}{3\pi} \left(2\hbar c \alpha^2 \frac{g_A/e}{M}\right)^2 = 5.09 \left(\frac{g_A/e}{M}\right)^2 \times 10^{-6} \ \mu\text{b} , \quad (5.7)$$

$$\sigma_0^{(2)} = 1.17 \left(\frac{g_V/e}{M}\right)^2 \times 10^{-9} \ \mu\text{b} , \quad (5.8)$$

where M is in GeV.[18] Of course in the absence of form factors these results should not be taken too seriously.

B. Z^0 decay

For any decay of the Z^0 into much smaller masses we expect a width on the order of

$$\Gamma \sim \alpha M \sim 10\text{–}1000 \text{ MeV}. \quad (5.9)$$

The lifetime is therefore extremely short, and all possible backgrounds to a particular decay mode must be considered.

In terms of the purely electromagnetic production considered here, the obvious decay mode is

$$Z^0 \to \mu^+\mu^- . \tag{5.10}$$

The Bethe-Heitler pair production will occur simultaneously, so we must consider the square of terms like those of Fig. 9. The lowest-order term produces a charge-symmetric pair. It interferes with the second Born contribution to produce an asymmetry at the level of α^4. This has been studied in the case of wide-angle electron pairs and momentum transfers to the nucleus small enough to neglect incoherent effects.[19] If $g_A/e \sim O(1)$ (and it may be larger) term (c) of Fig. 9 will produce an asymmetry at the same level of α. Its dependence on momentum transfer is presently being studied.

C. Multiphoton and Z^0 exchange

The results of this work can easily be generalized to include the multiphoton exchange of Fig. 2 with a pointlike target. The analysis of Chang and Ma[6] applies unchanged. Defining the two-dimensional Fourier-Bessel transformation

$$F(\vec{b}) = \int \frac{d^2k}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} F(\vec{k}), \tag{5.11}$$

one obtains the impact-parameter representation

$$\mathfrak{M}(\vec{b}) = i\omega\delta_{\lambda\lambda'} \int d^2b_1\, d^2b_2 \left(e^{i\chi(\vec{b}-\vec{b}_1)-i\chi(\vec{b}-\vec{b}_2)}-1\right) \times [I(\vec{b}_1,\vec{b}_2) - I_5(\vec{b}_1,\vec{b}_2)]. \tag{5.12}$$

Here $\chi(b)$ is the transform of the "potential"

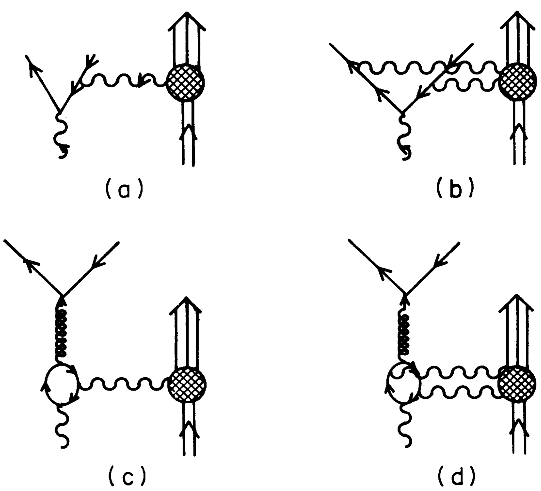

FIG. 9. Contributions to pair production.

$$V(\vec{k}) = -\frac{e^2}{\vec{k}^2} \tag{5.13}$$

and $I(\vec{b}_1,\vec{b}_2)$ and $I_5(\vec{b}_1,\vec{b}_2)$ are the transforms of the lepton loop factors of Fig. 6(a), with vector and axial-vector coupling, respectively.

Applying charge conjugation to Fig. 6 (a) shows that the scalar loop is symmetric in k_1 and k_2, while the pseudoscalar loop is antisymmetric. Therefore

$$\begin{aligned} I(\vec{b}_1,\vec{b}_2) &= I(\vec{b}_2,\vec{b}_1), \\ I_5(\vec{b}_1,\vec{b}_2) &= -I_5(\vec{b}_2,\vec{b}_1), \end{aligned} \tag{5.14}$$

so that in Eq. (5.12) the coefficient of I must be even in $\chi(\vec{b}-\vec{b}_1)-\chi(\vec{b}-\vec{b}_2)$ and the coefficient of I_5 must be odd. This is just Furry's theorem, which implies that an even number of photons are exchanged to produce a vector Z^0, and an odd number to produce an axial-vector Z^0.

Expanding the exponential in Eq. (5.12), the first term is given by

$$\mathfrak{M}_1 = i\omega\delta_{\lambda\lambda'} \int \frac{d^2k}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} \times \left(\frac{-e^2}{\vec{k}^2}\right)[I_5(\vec{0},\vec{k}) - I_5(\vec{k},\vec{0})]; \tag{5.15}$$

current conservation for k_1 implies that $I_5(\vec{0},\vec{k})$ can be evaluated from the loop in Fig. 6(b) [see discussion above Eq. (4.5)], and similarly current conservation for k_2 implies that $I_5(\vec{k},\vec{0})$ can be evaluated from Fig. 6(c). The factors of γ_+ can then be combined and (using $\beta+\beta'=1$) the contributions of Fig. 4 are reproduced. In other words, we have

$$-ie[I_5(\vec{0},\vec{k}) - I_5(\vec{k},\vec{0})] = A'_+, \tag{5.16}$$

so that Eq. (5.15) reproduces Eq. (3.2).

More generally, we may assume that the Z^0 interacts with the target with both vector and axial-vector couplings g'_V and g'_A. In the axial case we have

$$\bar{\mu}'_{\lambda'}\gamma_-\gamma_5\mu'_\lambda = -\lambda\delta_{\lambda\lambda'}/m, \tag{5.17}$$

so that the target helicity remains conserved. The eikonal picture is preserved under inclusion of multi-Z^0 exchange. The only modification is in the potential, which becomes

$$V(\vec{k}) = -\frac{e^2}{\vec{k}^2} - \frac{(g_V - g_A\gamma_5)^1(g'_V - g'_A\gamma_5)^2}{\vec{k}^2+M^2}. \tag{5.18}$$

Here the superscripts 1 and 2 denote the γ matrices for the photon and target, respectively. Note that outside of the scaling region the Z^0-exchange effect is negligible due to the additional $1/M^2$. The effect of Z^0 exchange on Delbrück scattering is just given by setting $M^2=0$ in $I(\vec{b}_1, \vec{b}_2)$.

The Z^0 may well be expected to have significant interactions with hadrons, just as its massless partner does. Rather than photon exchange then, we should consider more general Compton-type mechanisms. The eikonal picture may be useful here, and may be applied to other products of high-energy photoproduction.

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APPENDIX A

The traces used in Secs. III and IV of the text are evaluated here. They have the form

$$T_{i+\mu} = \mathrm{Tr}\,[(\not{w}+\not{q} + m_i)\gamma_i\,(\not{w}+m_i)\gamma_+(\not{w}-\not{k}+m_i)\gamma_5\gamma_\mu], \tag{A1}$$

$$T_{+i+\mu} = \mathrm{Tr}\,[\gamma_+(\not{w}+\not{q}+m_i)\gamma_i(\not{w}+m_i) \times \gamma_+(\not{w}-\not{k}_1+m_i)\gamma_\mu(\not{w}+\not{q}+\not{k}_2+m_i)], \tag{A2}$$

where $k=k_1+k_2$, $\mu=+,j$, and $i,j=1,2$. From the discussion in the text we also have $k_+, k_{1+}, \vec{q}=0$, and we employ the notation

$$\begin{aligned} &w_+ = -\beta', \\ &q_+ + w_+ = 1-\beta' = \beta . \end{aligned} \tag{A3}$$

Repeated use of Eqs. (2.6) will be made.

Equation (A1) can be simplified by writing

$$\begin{aligned} &\gamma_5 = \tfrac{1}{4}(\gamma_-\gamma_+ - \gamma_+\gamma_-)\gamma_{12}, \\ &\gamma_{12} = i\gamma_1\gamma_2 , \end{aligned} \tag{A4}$$

and it is easily shown that

$$\mathrm{Tr}(\gamma_i\gamma_{12}\gamma_j) = 4i\epsilon_{ij} . \tag{A5}$$

In this notation we find

$$T_{i+\mu} = \mathrm{Tr}\,[(\not{w}+\not{q}+m_i)\gamma_i(-\tfrac{1}{2}\beta'\gamma_- - \vec{w}\cdot\vec{\gamma}+m_i) \times\gamma_+(\tfrac{1}{2}\beta'\gamma_- - (\vec{w}-\vec{k})\cdot\vec{\gamma}+m_i)\gamma_{12}\gamma_\mu] . \tag{A6}$$

When $\mu=+$ this becomes

$$\begin{aligned} T_{i++} &= 2\beta'\,\mathrm{Tr}[(\not{w}+\not{q}+m_i)\gamma_i(-\tfrac{1}{2}\beta'\gamma_- - \vec{w}\cdot\vec{\gamma}+m_i)\gamma_+\gamma_{12}] \\ &= 2\beta'\{\tfrac{1}{2}\beta\,\mathrm{Tr}\,[\gamma_-\gamma_i(-\vec{w}\cdot\vec{\gamma}+m_i)\gamma_+\gamma_{12}] \\ &\quad -\tfrac{1}{2}\beta'\,\mathrm{Tr}\,[(-\vec{w}\cdot\vec{\gamma}+m_i)\gamma_i\gamma_-\gamma_+\gamma_{12}]\} . \end{aligned}$$

Now γ_+ must dot into γ_-, giving

$$T_{i++} = 8i\beta'\epsilon_{ia}w^a . \tag{A7}$$

Putting $\mu=j$ into Eq. (A6) and expanding the first factor gives

$$\begin{aligned} T_{i+j} = {} & \frac{\vec{w}^2+m_i^2}{2\beta}\,\mathrm{Tr}\,[\gamma_+\gamma_i(-\beta'^2\gamma_-)\gamma_{12}\gamma_j] + \tfrac{1}{2}\beta\,\mathrm{Tr}\,[\gamma_-\gamma_i(-\vec{w}\cdot\vec{\gamma}+m_i)\gamma_+(-(\vec{w}-\vec{k})\cdot\vec{\gamma}+m_i)\gamma_{12}\gamma_j] \\ & + \mathrm{Tr}\{(-\vec{w}\cdot\vec{\gamma}+m_i)\gamma_i[-\tfrac{1}{2}\beta'\gamma_-\gamma_+(-(\vec{w}-\vec{k})\cdot\vec{\gamma}+m_i) + (-\vec{w}\cdot\vec{\gamma}+m_i)\gamma_+\tfrac{1}{2}\beta'\gamma_-]\gamma_{12}\gamma_j\} . \end{aligned}$$

Again γ_+ must dot into γ_-, so that

$$\begin{aligned} T_{i+j} &= (\vec{w}^2+m_i^2)\frac{\beta'^2}{\beta}\,\mathrm{Tr}\,[\gamma_i\gamma_{12}\gamma_j] + \beta\,\mathrm{Tr}\,[\gamma_i(-\vec{w}\cdot\vec{\gamma}-m_i)(-(\vec{w}-\vec{k})\cdot\vec{\gamma}+m_i)\gamma_{12}\gamma_j] + \beta'\,\mathrm{Tr}\,[(-\vec{w}\cdot\vec{\gamma}+m_i)\gamma_i(-\vec{k}\cdot\vec{\gamma})\gamma_{12}\gamma_j] \\ &= (\vec{w}^2+m_i^2)((\beta'^2/\beta)-\beta)\,\mathrm{Tr}\,[\gamma_i\gamma_{12}\gamma_j] + \mathrm{Tr}\,[(-\beta\gamma_i\,\vec{w}\cdot\vec{\gamma}+\beta'\,\vec{w}\cdot\vec{\gamma}\gamma_i)\vec{k}\cdot\vec{\gamma}\gamma_{12}\gamma_j] . \end{aligned}$$

Commuting the γ_i to the left in the last term and using $\beta+\beta'=1$, we obtain

$$T_{i+j} = 4i\left[\frac{\beta'-\beta}{\beta}(\vec{w}^2+m_i^2)\epsilon_{ij} - 2\beta' w_i\epsilon_{aj}k^a\right] - \mathrm{Tr}\,[\gamma_i\,\vec{w}\cdot\vec{\gamma}\,\vec{k}\cdot\vec{\gamma}\gamma_{12}\gamma_j] . \tag{A8}$$

Finally, putting

$$\vec{w} = \vec{w}' + x\beta\vec{k} \tag{A9}$$

into Eqs. (A7) and (A8) and dropping terms linear in w', we obtain

$$T_{i++}=4i(2x\beta\beta'\epsilon_{ia}k^a)\,, \tag{A10}$$

$$T_{i+j}=4i\left[\frac{\beta'-\beta}{\beta}(\vec{w}'^2+m_t^2)\epsilon_{ij} - 2x\beta\beta' k_i\epsilon_{aj}k^a\right] + 4i\epsilon_{ij}[(\beta'-\beta)x+1]x\beta\vec{k}^2. \tag{A11}$$

Now we evaluate Eq. (A2). When $\mu=+$ we obtain

$$\begin{aligned}T_{+i++}&=-4\beta\beta'\,\mathrm{Tr}\,[(\not w+\not q+m_t)\gamma_i(\not w+m_t)\gamma_+]\\ &=16\beta\beta'(\beta'-\beta)w_i\,.\end{aligned} \tag{A12}$$

When $\mu=j$ in Eq. (A2) we write it as

$$\begin{aligned}T_{+i+j}&=\mathrm{Tr}[\gamma_+\not L_i\gamma_+\not R_j]\\ &=\tfrac{1}{2}\mathrm{Tr}\,[(\gamma_+\not L_i-\not L_i\gamma_+)(\gamma_+\not R_j-\not R_j\gamma_+)]\,,\end{aligned} \tag{A13}$$

where the last equality follows from $\gamma_+{}^2=0$. In the present case Eq. (A13) involves combinations of the form

$$\gamma_+\not A\gamma_i\not B-\not A\gamma_i\not B\gamma_+=2A_+\gamma_i\not B+2B_+\not A\gamma_i\,. \tag{A14}$$

The coefficient of γ_- cancels, and γ_+ can also be dropped since it has nothing to dot into. We therefore have

$$\begin{aligned}T_{+i+j}&=2\,\mathrm{Tr}\{(\beta\gamma_i(-\vec{w}\cdot\vec{\gamma}+m_t)-\beta'(-\vec{w}\cdot\vec{\gamma}-m_t)\gamma_i)\\ &\qquad\times[-\beta'\gamma_j(-(\vec{w}+\vec{k}_2)\cdot\vec{\gamma}+m_t)\\ &\qquad\quad+\beta(-(\vec{w}-\vec{k}_1)\cdot\vec{\gamma}-m_t)\gamma_j]\}\\ &=2\,\mathrm{Tr}\{(\gamma_i(-\vec{w}\cdot\vec{\gamma}+m_t)+2\beta' w_i)\\ &\qquad\times[2\beta'(w+R+\beta k)_j+(-(\vec{w}+\vec{R})\cdot\vec{\gamma}+m_t)\gamma_j]\}\,,\end{aligned} \tag{A15}$$

where

$$\vec{R}=\beta'\vec{k}_2-\beta\vec{k}_1\,. \tag{A16}$$

Finally, putting

$$\vec{w}=\vec{w}'-x\vec{R} \tag{A17}$$

into Eqs. (A12) and (A15) and dropping terms linear in w', we obtain

$$T_{+i++}=-16\beta\beta'(\beta'-\beta)xR_i\,, \tag{A18}$$

$$T_{+i+j}=8\{[(1-2\beta\beta')\vec{w}'^2+m_t^2-x(1-x)\vec{R}^2]\,\delta_{ij} + 4\beta\beta' x(1-x)R_iR_j-2\beta\beta'(\beta'-\beta)xR_ik_j\}\,. \tag{A19}$$

APPENDIX B

This appendix gives a brief description of the numerical evaluation of the $f'_{\Delta\mu}(\vec{\kappa}^2)$ given in Eqs. (4.27)–(4.31). The integrand diverges when $\vec{\kappa}_1$, $\vec{\kappa}_2$, or the argument of the logarithm vanishes. The logarithm will be discussed first since it is the easiest to handle.

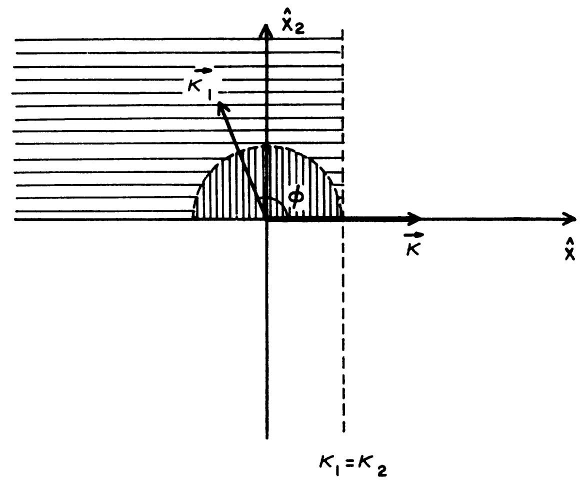

FIG. 10. The transverse-momentum plane.

The logarithmic divergence at $\alpha=\pm1$ is canceled in I_1 and I_2 by the $1-\alpha^2$, and in the case of I_0 we show below that the integral can be done analytically because of the simple $\vec{R}$ dependence. With $\vec{\kappa}$ along the 1 axis and calling ϕ the angle between $\vec{\kappa}$ and $\vec{\kappa}_1$, as in Fig. 10, we can factor the argument of the logarithm as follows:

$$4\vec{R}^2-(1-\alpha^2)=(1+\kappa^2)(\alpha+\alpha_+)(\alpha+\alpha_-)\,, \tag{B1}$$

where

$$\alpha_\pm=\delta\pm\gamma\,, \tag{B2}$$

$$\delta=\frac{\kappa(\kappa-2\kappa_1\cos\phi)}{1+\kappa^2}\,, \tag{B3}$$

$$\begin{aligned}\gamma&=|1-\delta|\left[1-\frac{4\vec{\kappa}_1{}^2}{(1+\kappa^2)(1-\delta)^2}\right]^{1/2}\\ &=|1+\delta|\left[1-\frac{4(\vec{\kappa}-\vec{\kappa}_1)^2}{(1+\kappa^2)(1+\delta)^2}\right]^{1/2},\end{aligned} \tag{B4}$$

and we use the notation $r=|\vec{r}|$ for the transverse vectors. The divergence problems at $\alpha=-\alpha_\pm$ can be simply eliminated by writing the logarithm as a sum and subtracting, from the coefficient of each log, the coefficient evaluated at the respective divergent point. At the same time the two terms can be added on with the logarithm integrated analytically.

The κ_1 integral is simplified by two properties of the integrand: It is even in ϕ and symmetric in $\vec{\kappa}_1$ and $\vec{\kappa}_2$. Therefore, in the transverse-momentum plane shown in Fig. 10 we need only integrate above the 1 axis and to the left of the line $\kappa_1=\kappa_2$, in the shaded region of Fig. 10, and at the same time dropping the $\frac{1}{4}$ in Eqs. (4.29)–(4.31). The point $\vec{\kappa}_2=0$ is thereby avoided and we need only worry about the divergence at $\vec{\kappa}_1=0$. [The variable $\vec{\kappa}'=\vec{\kappa}_1+\frac{1}{2}\vec{\kappa}$ may seem more convenient, since the integrand is even in κ', but the point $\vec{\kappa}_1=0$ is numerically easier to handle if the origin is chosen there.]

As $\kappa\to 0$ we have

$$f'_{\Delta\mu}\to\int\frac{d\kappa_1}{\kappa_1}\int d\phi\frac{I_{\Delta\mu}(\vec{\kappa}_1,\vec{\kappa}_1)-I_{\Delta\mu}(\vec{0},\vec{0})}{\vec{\kappa}_1^{\,2}}\ ,$$

which is logarithmically divergent at the lower limit due to the assumption $\kappa \gg M/\omega$, as discussed in the text. Examining the integrand in more detail, we can see from Eq. (4.32) that when κ is small $\vec{R}\sim-\kappa_1$, independent of α. Then the integrand of I_1 is odd in α and therefore integrates to zero. The integrand of I_2 has the factor $\vec{R}\cdot\vec{\epsilon}\vec{R}\cdot\vec{\epsilon}^*\sim(\cos\phi+i\sin\phi)^2$, so f_2 vanishes by the angular integration when $\kappa\to 0$. So we verify angular momentum conservation and $f'_0\sim\ln(\kappa^2)$. Evaluating f'_0 for small values of $\vec{\kappa}$ is much simplified by the fact that the α integration can be performed analytically.

Factoring the logarithm as in Eq. (B1), we have

$$I_0(\vec{\kappa}_1,\vec{\kappa}_2)-I_0(\vec{0},\vec{\kappa})=\int_{-1}^{1}d\alpha(1+\alpha^2)\{[\ln|\alpha+\alpha_+|+\ln|\alpha+\alpha_-|+i\pi\theta((\alpha+\delta)^2-\gamma^2)]-(\kappa_1=0)\},\tag{B5}$$

where we have dropped the $\frac{1}{4}$ as mentioned above. The integral of the logarithms can eventually be written as

$$\begin{aligned}\mathrm{Re}[I_0(\vec{\kappa}_1,\vec{\kappa}_2)-I_0(\vec{0},\vec{\kappa})]=&-\tfrac{16}{3}\frac{\kappa_1}{(1+\kappa^2)^2}[(1-\kappa^2)(\kappa\cos\phi-\kappa_1)-2\kappa_1\kappa^2\sin^2\phi]+\tfrac{4}{3}\ln(\vec{\kappa}_1^{\,2}\vec{\kappa}_2^{\,2})\\&+\mathrm{Re}(\alpha_+)[1+\tfrac{1}{3}(\mathrm{Re}(\alpha_+))^2]\ln\left|\frac{1+\alpha_+}{1-\alpha_+}\right|+\mathrm{Re}(\alpha_-)[1+\tfrac{1}{3}(\mathrm{Re}(\alpha_-))^2]\ln\left|\frac{1+\alpha_-}{1-\alpha_-}\right|\\&-\tfrac{4}{3}[2(1+\kappa^2)^2+1-\kappa^2]\frac{\kappa^2\ln\kappa^2}{(1+\kappa^2)^3}\\&+2\,\mathrm{Im}(\alpha_+)[1+(\mathrm{Re}(\alpha_+))^2-\tfrac{1}{3}(\mathrm{Im}(\alpha_+))^2]\left[\tan^{-1}\left(\frac{1+\mathrm{Re}(\alpha_+)}{\mathrm{Im}(\alpha_+)}\right)+\tan^{-1}\left(\frac{1-\mathrm{Re}(\alpha_+)}{\mathrm{Im}(\alpha_+)}\right)\right]\\&-2\mathrm{Re}(\alpha_+)[\mathrm{Im}(\alpha_+)]^2\ln\left|\frac{1+\alpha_+}{1-\alpha_+}\right|.\end{aligned}\tag{B6}$$

In evaluating the imaginary part it is convenient to write $\theta(x)=1-\theta(-x)$, and letting $\alpha'=\alpha+\delta$, we have

$$\mathrm{Im}[I_0(\vec{\kappa}_1,\vec{\kappa}_2)-I_0(\vec{0},\vec{\kappa})]=-\pi\int_{-(1-\delta)}^{1+\delta}d\alpha'[1+(\alpha'-\delta)^2]\,\theta(\gamma^2-\alpha'^2)-(\kappa_1=0)\tag{B7}$$

In the second term ($\kappa_1=0$) we have $\gamma=1-\delta=1/1+\kappa^2<1+\delta$. So we just replace the upper limit by $1-\delta$ and drop the θ function. The first term vanishes unless $\gamma^2>0$ and since $\delta>0$ in the shaded region of Fig. 10 we have in this case $-(1-\delta)<-\gamma<\gamma<1+\delta$ so that Eq. (B7) becomes

$$\begin{aligned}\mathrm{Im}[I_0(\vec{\kappa}_1,\vec{\kappa}_2)-I_0(\vec{0},\vec{\kappa})]&=-\pi\left\{\theta(\gamma^2)\int_{-\gamma}^{\gamma}d\alpha'[1+(\alpha'-\delta)^2]-\int_{-1/(1+\kappa^2)}^{1/(1+\kappa^2)}d\alpha'\left[1+\left(\alpha'-\frac{\kappa^2}{1+\kappa^2}\right)^2\right]\right.\\&=-2\pi\left\{\theta(\gamma^2)\gamma[1+\delta^2+\tfrac{1}{3}\gamma^2]-\frac{1}{1+\kappa^2}\left[1+\frac{\kappa^4+\frac{1}{3}}{(1+\kappa^2)^2}\right]\right\}.\end{aligned}\tag{B8}$$

Now consider the $\vec{\kappa}_1$ integral for f'_0. It is clear from Eq. (4.29) that as $\kappa_1\to 0$,

$$I_0(\vec{\kappa}_1,\vec{\kappa}_2)-I_0(\vec{0},\vec{\kappa})\sim 2\kappa\kappa_1\cos\phi\,C(\kappa^2)+O(\kappa_1^{\,2}),\tag{B9}$$

so the angular integral should vanish near the origin of Fig. 10. However, the numerical error in the ϕ integral is multiplied by $1/\kappa^2$. In order to make the integrand vanish explicitly near the origin, we can divide the integral into $\kappa_1 < \kappa/2$ and $\kappa_1 > \kappa/2$, and write

$$f_0(\kappa) = \int_0^{\kappa/2} \frac{d\kappa_1}{\kappa_1} \int_0^{\pi} d\phi \frac{1}{\vec{\kappa}_2^{\,2}}\, 2\kappa\kappa_1 \cos\phi\, C(\kappa^2) + \int_0^{\kappa/2} \frac{d\kappa_1}{\kappa_1} \int_0^{\pi} d\phi \frac{1}{\vec{\kappa}_2^{\,2}} [I_0(\vec{\kappa}_1, \vec{\kappa}_2) - I_0(\vec{0}, \vec{\kappa}) - 2\kappa\kappa_1 \cos\phi\, C(\kappa^2)]$$
$$+ \int_{\kappa/2}^{\infty} d\phi \frac{d\kappa_1}{\kappa_1} \int_{\phi_{\min}}^{\pi} d\phi \frac{1}{\vec{\kappa}_2^{\,2}} [I_0(\kappa_1, \kappa_2) - I_0(\vec{0}, \vec{\kappa})]\,. \quad \text{(B10)}$$

The first term here integrates to

$$-\pi\, C(\kappa^2) \ln(\tfrac{3}{4}) \quad \text{(B11)}$$

and taking $\kappa_1 \to 0$ in Eqs. (B6) and (B8) eventually gives

$$C(\kappa^2) = \frac{-1}{(1+\kappa^2)^2} \left[1 - \kappa^2 + \frac{1+\kappa^4}{1+\kappa^2} (\ln\kappa^2 - i\pi) \right]. \quad \text{(B12)}$$

The integrand of the second term in Eq. (B10) vanishes as $\kappa_1 \to 0$. In the last term ($\kappa_1 > \kappa/2$) the angular integral starts at

$$\phi_{\min} = \cos^{-1}\left(\frac{\kappa/2}{\kappa_1} \right), \quad \text{(B13)}$$

and we let

$$\kappa_1 = \tfrac{1}{2}\kappa + CM \frac{x}{1-x}\ . \quad \text{(B14)}$$

We integrate x from 0 to 1 by Simpson's rule with equally spaced points, choosing the variable CM for each value of κ to equalize the κ_1 integral on either side of $x = \frac{1}{2}$. For $\kappa_1 < \kappa/2$ we have

$$\kappa_1 = (\kappa/2)x\,. \quad \text{(B15)}$$

Double precision was employed in order to effect the necessary cancellations for small κ, with well-determined results well into the logarithmic region as shown in Fig. 7.

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[1]S. Brodsky and S. Drell, Annu. Rev. Nucl. Sci. 20, 147 (1970).

[2]For a recent example see S.-J. Chang and P. M. Fishbane, Phys. Rev. D 2, 1104 (1970), and references therein.

[3]For a review see B. W. Lee, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 249.

[4]M. Gell-Mann, M. L. Goldberger, N. Kroll, and F. E. Low, Phys. Rev. 179, 1518 (1969). For a discussion of this point, a study of the heavy leptons, a summary of various models, and many excellent references, see J. D. Bjorken and C. H. Llewellyn Smith, Phys. Rev. D 7, 887 (1973).

[5]For the Z^0 effect in $e^+e^- \to \mu^+\mu^-$ see R. Brown, V. Cung, K. Mikaelian, E. A. Paschos, Phys. Lett. B43, 403 (1973), and references therein.

[6]The lowest-order result was first given in H. Cheng and T. T. Wu, Phys. Rev. 182, 1873 (1969). The multi-photon eikonal form is obtained in S.-J. Chang and S. K. Ma, *ibid.* 188, 2385 (1969). We follow the method of the second paper quite closely.

[7]Delbrück scattering of spacelike photons has been studied in the context of electroproduction by H. Cheng and T. T. Wu, Phys. Rev. 183, 1324 (1969); J. D. Bjorken, J. B. Kogut, and D. E. Soper, Phys. Rev. D 3, 1382 (1971); and J. B. Kogut, *ibid.* 5, 1152 (1972).

[8]From elastic e^+e^- scattering at high energy and momentum transfer, a heavy photon must have $M > 2.5$ GeV. (See Ref. 1.)

[9]See H. Kendall, in *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies,* edited by N. B. Mistry (Laboratory of Nuclear Studies, Cornell University, Ithaca, N. Y., 1972), p. 148 ff.

[10]L. Susskind, Phys. Rev. 165, 1535 (1968); K. Bardakci and M. K. Halpern, *ibid.* 176, 1686 (1968); and S.-J. Chang and S. K. Ma, *ibid.* 180, 1506 (1969).

[11]Since this is just a scale transformation the units are irrelevant.

[12]S. L. Adler, in *Lectures on Elementary Particles and Quantum Field Theory,* edited by S. Deser *et al.* (MIT Press, Cambridge, Mass., 1970), Vol. 1.

[13]Reference 12, Eq. (63). Our definition of $R_{\sigma\rho\mu}$ differs by a sign since we write $\gamma_5\gamma_\mu$.

[14]A. Halprin, C. M. Anderson, and H. Primakoff, Phys. Rev. 152, 1295 (1966).

[15]H. Cheng and T. T. Wu, Phys. Rev. D 2, 2444 (1969).

[16]Cf. Eqs. (4.33) and (4.44) of Chang and Ma, Ref. 6.

[17]Note that for large $|\vec{k}|/M$ one has $f_1 \sim |\vec{k}|/M$ due to the anomaly. The $\vec{k}^2$ integral therefore diverges in this case. The value of $\sigma_0^{(1)}$ is given by derivatives of a beta function. The value of $\sigma_0^{(2)}$ was found numerically from the results shown in Fig. 7 with the change of variables

$|\vec{k}|/M=(1-z)^2/z$.

[18]In the original Weinberg model, for example, if we call the $Z^0\gamma$ mixing angle θ, then one has $g_A/e = \frac{1}{4}/\sin\theta\cos\theta$, $g_V/e=(\sin^2\theta-\frac{1}{4})/\sin\theta\cos\theta$, and for the Z^0 mass $M=37$ GeV$/\sin\theta\cos\theta$. The factor $(g_A/e)/M$ appearing in Eq. (5.7) is then independent of θ. The parameters of the other models described in Ref. 4 are not significantly different. The value of M need not be so large, however. See, for example, M. Bég and A. Zee, Phys. Rev. Lett. 30, 675 (1973).

[19]S. Brodsky and J. R. Gillespie, Phys. Rev. 173, 1011 (1968).

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$\bar{N}N$ binding energies*

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A reported bound state of $\bar{p}n$ at −83 MeV is compared with calculations on the boundary-condition model (BCM) for NN scattering. The most probable orbital assignment for the bound state appears to be $^3S_1$ or $^1P_1$, but it is not possible to distinguish between these two assignments by binding energies alone. At least half a dozen other bound states are predicted. The imaginary part of the core boundary condition is quite small in the present case. Comparison with published calculations using the Bryan-Phillips static (BPS) potential shows that the latter has a somewhat richer spectrum and includes $^3D_1$ as equally probable orbital for the observed state. The BCM and BPS potentials differ substantially in their spin-isospin dependence, but distinguishing data do not now exist.

A recent report[1] of a bound $\bar{p}n$ state at $B=-E=83$ MeV with $\Gamma\lesssim 8$ MeV encourages the hope that other such lightly bound states may be found. Their study has a twofold interest: For $B\lesssim 300$ MeV, they should[2] provide further insight into meson-exchange NN potentials, which have given fits to scattering data over a comparable energy range and can be related to $\bar{N}N$ potentials mainly by reversing the signs of appropriate terms. Second, the comparison of calculation and experiment may ultimately indicate the extent to which bosons can be described as $\bar{N}N$ bound states—at least in the region around $m_B=m_N+m_{\bar{N}}\simeq 1877$ MeV.

I. BCM POTENTIAL

The relatively small values of B and Γ quoted above suggest that it should be possible to interpret this state mainly in terms of an $\bar{N}N$ potential at large range (i.e., $r\gtrsim 0.5$ F). We report here some calculations using an $\bar{N}N$ potential taken from the boundary-condition model[3] (BCM) for NN interactions. General parameters are as in Table III of Ref. 3, with the following modifications:

(i) sign changes of $g^2=g_\pi^2$ and $N'^2=\frac{4}{9}g_\omega^2$;
(ii) no tensor coupling of states with different L, although diagonal tensor terms are retained;
(iii) variation of g_ω^2 over the range 0 to 30 and free variation of the boundary parameter f.

The sign changes are those associated with $N\to\bar{N}$: $(-1)^G$, where G is the G parity of the exchanged meson combination. Tensor coupling is omitted because we seek only first approximations to states with binding energies of order 100 MeV, so that additional binding from tensor mixing can generally be neglected. The situation is the opposite of the deuteron, where the tensor force makes the difference between a bound and an unbound state. The diagonal tensor terms were retained to give some qualitative J splitting for fixed L, since Ref. 3 does not include a spin-orbit potential. The BCM calculations use $g_\omega^2=3$, which is smaller than that given by most meson potential fits to the NN data or by direct ω production measurements.[4] A range $0\leq g_\omega^2\lesssim 30$ was therefore searched. There is no *a priori* reason to expect boundary condition values f to be identical for $\bar{N}N$ and NN states with the same $LSJT$; in any case, half the states present in the $\bar{N}N$ system are excluded from NN by the Pauli principle. As a first approximation, the f values are taken entirely real, since for a narrow resonance the correction to the binding energy will be of order $(\Gamma/2E)^2$, which is $<10^{-2}$ in the present case.[1]

The two most immediate assignments for the observed state with this potential are $^3S_1$ and $^1P_1$.