Z^0 effect in lepton-pair photoproduction*

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We study the modifications of lepton-pair photoproduction due to the spin-one Z^0 appearing in gauge theories of the weak interactions. It gives rise to a resonance in the invariant mass of the pair. To obtain exact results we consider neutral currents involving only the known leptons, and still we find an effect significantly above the electromagnetic Bethe-Heitler background. Aside from the resonance, a unique parity-violating signal occurs in the form of an azimuthal asymmetry. For a typical Z^0 mass of 10 GeV and beam energy of 400 GeV, it is on the order of 5%. Our results are easily adapted to any model of leptons and would probably be much increased by the hadronic part of the neutral current. The leptonic mechanism involves the *VVA* triangle, for which a previous result is extended to any energy, momentum transfer, and lepton mass.

I. INTRODUCTION

The ongoing neutrino experiments at CERN[1] and FNAL[2] indicate the existence of weak neutral currents. The precise nature and magnitude of such effects is very much open, but the most natural existing framework for them is a renormalizable gauge field theory.[3] Here a massive gauge boson Z^0 couples with roughly electromagnetic strength to the neutral current J^0, which is generally a linear combination of vector and axial-vector terms containing any of the known leptons and additional hypothetical particles. Models may be constructed with almost any given structure for J^0. In order to pin it down we need more data involving leptons at high energy and momentum transfer, and more theoretical work on possible Z^0 effects.

The present paper is a study of the Z^0 effect in lepton-pair photoproduction. The leptons are mainly produced by the purely electromagnetic Bethe-Heitler process shown in Fig. 1.[4] (Permutations of vertices on charged lines are understood for every Feynman diagram in this paper.) Here the pair transforms under charge conjugation like two photons, so that $C=+1$. The target form factors are completely determined by electron scattering. Interference with the next-order term (Fig. 2) produces a charge asymmetry,[5] and the form factors are not explicitly known in this case. There are also resonances in the invariant mass of the pair due to leptonic decays of diffractively produced hadrons (Fig. 3).

Weak interactions may enter pair production in various ways. A possible signal for production of charged vector bosons or heavy leptons of mass m_λ (Fig. 4) would be a pair close together on the same side of the beam with a transverse-momentum spectrum increasing up to a sharp cutoff at $\frac{1}{2}m_\lambda$.[6] On the other hand, a neutral vector boson, the Z^0, may well be diffractively produced like the hadrons in Fig. 3. One may be tempted to assume that the high-energy limit of Z^0 photoproduction would be equal to that of Compton scattering. However, this may be tested in a quantum electrodynamical model[7] by computing the ratio of the amplitude of Fig. 5 to the Delbrück amplitude, and one finds that cancellations due to the gauge invariance of photon exchange give a result of $O(m^2/M^2)$, where m is the "constituent" mass and M is the Z^0 mass. Perhaps this just means that we have chosen a bad model for the Pomeron, but it is certainly a warning against overly optimistic predictions. Perhaps a safer assumption is that the coupling of a highly spacelike Z^0 to hadrons will depend mainly on its vector nature, so that the structure functions will resemble those of the photon. Then Z^0 exchange (Fig. 6) will have sizable effects on wide-angle leptons.[8]

In order to obtain less model-dependent results, we study the case in which the neutral current contains only the known leptons in the vector and axial-vector parts. The results are quite small but the calculation is clean and presumably provides a lower bound to the case in which the hadronic neutral current is included. The lowest-order corrections are then those of Fig. 7. We will consider only lepton pairs with large invariant mass, such as those produced in the decay of the heavy Z^0. Diagrams (a), (b), and (c) of Fig. 7 are $O(G_F)$ smaller than the electromagnetic background, while diagram (d), on which we concentrate, has the possibility of a resonance enhancement. We do not attempt to calculate the last diagram, which can also produce a Z^0 very near its mass shell,

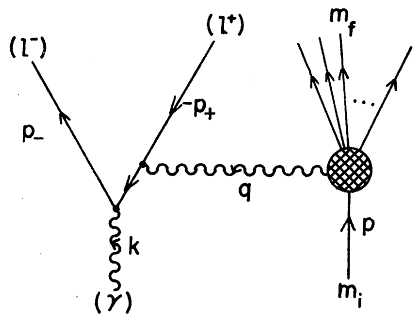

FIG. 1. The Bethe-Heitler mechanism for photoproduction of lepton pairs $\gamma N \to l^+ l^- X$. Here m_i and m_f are the masses of N and X, respectively, and the lepton pair is in a $C = +1$ state.

for two reasons. First, the massive timelike photon propagator in this diagram tends to damp out its contribution. Second, one cannot reliably calculate the amplitude for virtual Compton scattering off hadrons.

The outline of the paper is as follows. We treat the photoproduction separately in Sec. II, giving the exact lowest-order amplitude for $\gamma N \to Z^0 X$. In Sec. III we include the decay for a general off-shell Z^0. The lepton polarization and charge asymmetry are determined. Since the latter also has contributions from Figs. 2 and 3, the parity-violating part is isolated in the form of an azimuthal asymmetry. Section IV concludes with some numerical results and discussions.

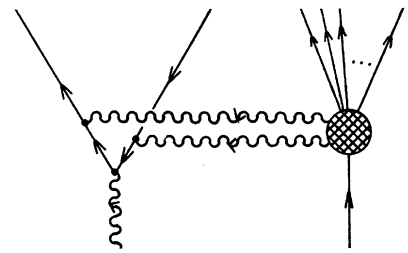

FIG. 2. Photoproduction of lepton pairs with two photons exchanged. The lepton pair is in a $C = -1$ state.

II. TRIANGLE CONTRIBUTION TO Z^0 PHOTOPRODUCTION

Here we compute the exact amplitude corresponding to Fig. 8. The loop may represent an electron or muon, and only the axial-vector part of the neutral current contributes due to C invariance. One has

$$\mathfrak{M} = \frac{e^3 g_A}{(2\pi)^4} \epsilon_\sigma(k) J_\rho(q) \epsilon^*_\mu(k') R^{\sigma\rho\mu}(k, q, m)/q^2 , \tag{2.1}$$

where e, ϵ_σ and g_A, ϵ_μ are the leptonic couplings and polarization vectors of the photon and Z^0, respectively, J_ρ is the electromagnetic current of the target over e, and $R^{\sigma\rho\mu}$ is the vertex function of the VVA triangle which has been extensively studied.[9] Lorentz invariance, gauge invariance, and Bose symmetry determine the general triangle vertex in terms of two functions of k^2, q^2, k'^2, and the loop mass m. Considerable simplification occurs in the present case due to the conditions

$$k^2 = k^\sigma \epsilon_\sigma = q^\rho J_\rho = k'^\mu \epsilon^*_\mu = 0 . \tag{2.2}$$

One then finds

$$\epsilon_\sigma J_\rho \epsilon^*_\mu R^{\sigma\rho\mu} = [k_\alpha \epsilon_\sigma J_\rho \epsilon^*_\mu \epsilon^{\alpha\sigma\rho\mu}] \frac{8\pi^2 q^2}{k'^2} \Delta(q^2, k'^2, m^2), \tag{2.3}$$

where

$$\Delta(q^2, k'^2, m^2) = k'^2 \int_0^1 dx \int_0^{1-x} dy \frac{2x(1-x-y)}{-x(1-x-y)q^2 + m^2 - xyk'^2 - i\epsilon} . \tag{2.4}$$

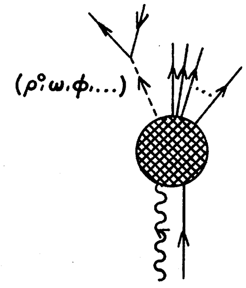

FIG. 3. Production of $C = +1$ lepton pairs by diffractively produced hadrons.

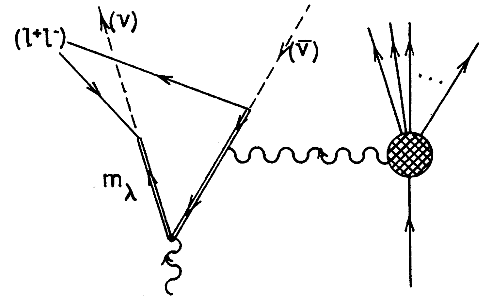

FIG. 4. Photoproduction and decay of heavy leptons.

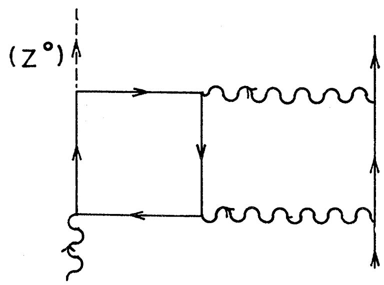

FIG. 5. Model for diffractive production of Z^0.

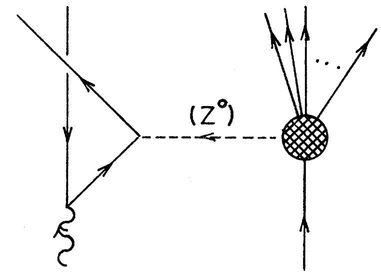

FIG. 6. Z^0 exchange in photoproduction of lepton pairs.

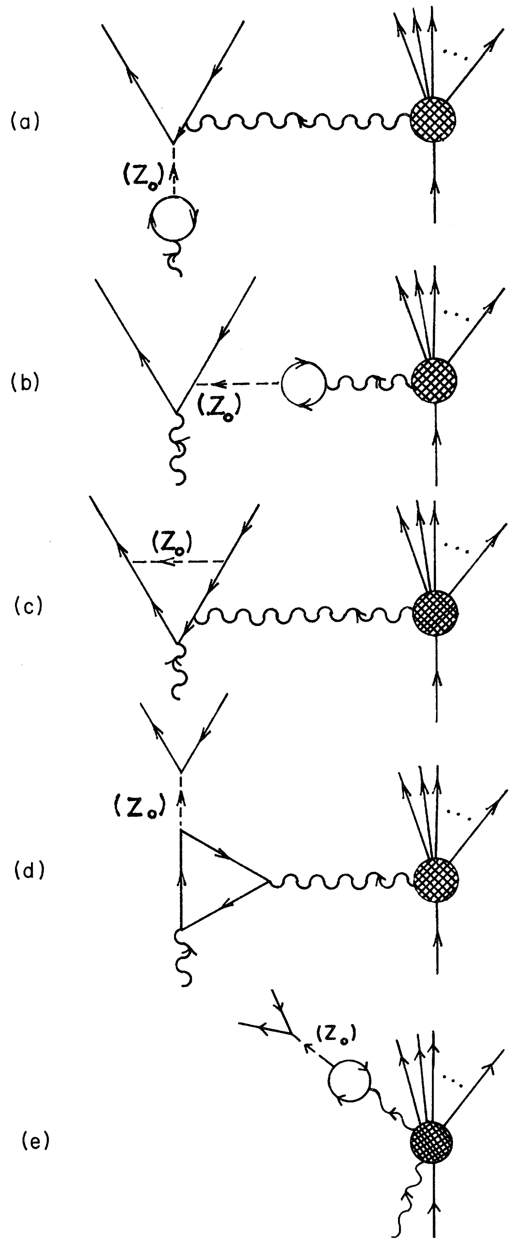

FIG. 7. Purely leptonic Z^0 effects in photoproduction of lepton pairs.

The explicit factor of q^2 in Eq. (2.3) comes from the gauge invariance of photon exchange and also guarantees that the Z^0 cannot decay into two photons.[10] This is the same mechanism which damps out Fig. 5, and here it cancels the one-photon exchange peak. Δ is constructed to be dimensionless. The denominator has a lower limit of $m^2 - \frac{1}{4}k'^2$, so when $k'^2 > (2m)^2$, as we assume here, then Δ has an imaginary part due to the decay $Z^0 \to l^+ l^-$.

The result of the y integral may be written

$$\Delta(q^2, k'^2, m^2) = \frac{k'^2}{k'^2 - q^2}\left(1 + 2\,\frac{k'^2 I_1 - m^2 I_2}{k'^2 - q^2}\right), \quad (2.5)$$

where

$$I_1 = \int_0^1 dx(1-x)\ln\left(\frac{1 - x(1-x)q^2/m^2 - i\epsilon}{1 - x(1-x)k'^2/m^2 - i\epsilon}\right), \quad (2.6)$$

$$I_2 = \int_0^1 \frac{dx}{x}\ln\left(\frac{1 - x(1-x)q^2/m^2 - i\epsilon}{1 - x(1-x)k'^2/m^2 - i\epsilon}\right). \quad (2.7)$$

The imaginary parts are easily evaluated by using $\ln(z - i\epsilon) = \ln|z| + i\pi\theta(-z)$. The real part of I_1 is also straightforward. In I_2 we may write

$$\frac{-q^2}{4m^2} = \sinh^2(\theta) > 0, \quad (2.8)$$

and then we have[11]

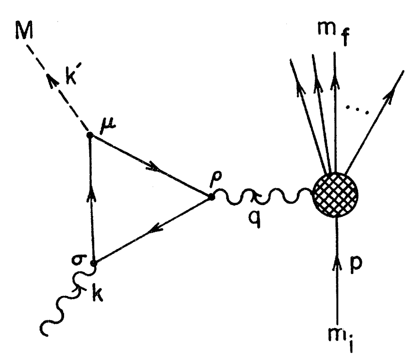

FIG. 8. Triangle contribution to photoproduction of a Z^0 of mass M. Permutations of vectices on the lepton loop are understood, and m_i and m_f are initial and final target masses.

$$\int_0^1 \frac{dx}{x}\ln\left(1-x(1-x)\frac{q^2}{m^2}\right)$$
$$=\int_0^1 \frac{dx}{x}\ln\{[1+x(e^{2\theta}-1)][1+x(e^{-2\theta}-1)]\}$$
$$=\int_1^{e^{2\theta}} \frac{dt}{t}\ln(t)$$
$$=\tfrac{1}{2}(2\theta)^2 . \tag{2.9}$$

Similarly we may set

$$\frac{k'^2}{4m^2}=\cosh^2(\theta')>1, \tag{2.10}$$

leading to the same form as Eq. (2.9) with $e^{2\theta}$ replaced by $-e^{2\theta'}$. Taking the real part gives

$$\int_0^1 \frac{dx}{x}\ln\left|1-x(1-x)\frac{k'^2}{m^2}\right|=\tfrac{1}{2}[(2\theta')^2-\pi^2]. \tag{2.11}$$

The final result is then

$$\Delta(q^2,k'^2,m^2)=\left(1-\frac{q^2}{k'^2}\right)^{-2}\left\{1-\frac{q^2}{k'^2}+2\left(1-\frac{4m^2}{q^2}\right)^{1/2}\ln\left(\frac{1+(1-4m^2/q^2)^{1/2}}{(4m^2/-q^2)^{1/2}}\right)\right.$$
$$-2\left(1-\frac{4m^2}{k'^2}\right)^{1/2}\left[\ln\left(\frac{1+(1-4m^2/k'^2)^{1/2}}{(4m^2/k'^2)^{1/2}}\right)-\tfrac{1}{2}i\pi\right]$$
$$+\frac{4m^2}{k'^2}\left[\{\cosh^{-1}[(k'^2/4m^2)^{1/2}]\}^2-\{\sinh^{-1}[(-q^2/4m^2)^{1/2}]\}^2\right.$$
$$\left.\left.-\tfrac{1}{4}\pi^2-\pi i\ln\left(\frac{1+(1-4m^2/k'^2)^{1/2}}{(4m^2/k'^2)^{1/2}}\right)\right]\right\}. \tag{2.12}$$

The main limit of interest for Δ is that of small lepton mass. When $m^2\ll k'^2$, the second logarithm becomes just $\ln(m^2/k'^2)$ and the terms within the last set of large square brackets are negligible. If in addition $m^2\ll -q^2$, then the first logarithm becomes $\ln(-q^2/m^2)$, combining with the second to give

$$\Delta(q^2,k'^2,0)=\frac{1-q^2/k'^2+\ln(-q^2/k'^2)+i\pi}{(1-q^2/k'^2)^2}, \tag{2.13}$$

which was given previously by one of the authors.[12]

In terms of Δ and $\alpha=e^2/4\pi$ the amplitude may be written

$$\mathfrak{M}=\frac{8\alpha^2}{k'^2}\frac{g_A}{e}\Delta(q^2,k'^2,m^2)\epsilon^{\alpha\sigma\rho\mu}q_\alpha\epsilon_\sigma J_\rho\epsilon^*_\mu . \tag{2.14}$$

It is now straightforward to obtain the differential cross section for Z^0 photoproduction. Averaging and summing over photon and Z^0 helicities and setting $k'^2=M^2$, we find

$$\frac{d\sigma^Z}{d^3k'/k'_0}=\frac{4\alpha^4}{\pi^2}\frac{m_i}{s-m_i{}^2}\left|\frac{g_A/e}{M^2}\Delta(q^2,k'^2,m^2)\right|^2$$
$$\times\left\{\tfrac{1}{2}(M^2-q^2)^2\left(\frac{1}{M^2}+\frac{1}{q^2}\right)W_1(q^2,\nu)\right.$$
$$\left.+\left[\left(2-\frac{q^2}{M^2}\right)\left(\frac{s-m_i{}^2}{2m_i}+\frac{\nu}{2q^2}(M^2-q^2)\right)^2-\left(1-\frac{\nu^2}{q^2}\right)\left(\frac{M^2-q^2}{2M}\right)^2\right]W_2(q^2,\nu)\right\}, \tag{2.15}$$

where s is the center-of-mass energy squared,

$$\omega=\frac{s-m_i{}^2}{2m_i}$$
$$=\text{laboratory energy of the photon}, \tag{2.16}$$

$$\nu=\frac{m_f{}^2-m_i{}^2-q^2}{2m_i}$$
$$=\text{laboratory energy transferred}, \tag{2.17}$$

and $W_{1,2}$ are the target structure functions with dimensions of m^{-1}.

III. LEPTON PAIR EFFECTS

Here we include the decay of a general off-shell Z^0 arising from the mechanism of the preceding section. Since the resonance effect may be either above the available invariant mass or poorly resolved, we isolate the Z^0 effect in polarization dependence and charge asymmetry. In the present case these effects are not important very far below the resonance, but hadronic neutral currents may alter that.

The amplitude $\mathfrak{M}^Z$ corresponding to Fig. 7(d) can be obtained from the amplitude for Z^0 photoproduction by the substitution

$$\epsilon_\mu^*(k') \to t_\mu = -\frac{g_{\mu\nu} - k'_\mu k'_\nu /M^2}{k'^2 - M^2 + i\Gamma M} \times \bar{u}_{\lambda_-}(p_-)\gamma_\nu(g_V - g_A\gamma_5)v_{\lambda_+}(p_+), \quad (3.1)$$

where Γ is the total Z^0 decay width, $\bar{u}$ and v are Dirac spinors for the lepton and antilepton of helicity and 4-momentum λ_-, p_- and λ_+, p_+, respectively, and $g_{V,A}$ are the leptonic couplings of the Z^0. Note that instead of $k'^\mu \epsilon_\mu^* = 0$, we now have

$$k'^\mu t_\mu = -\frac{2m}{M^2} g_A \bar{u}_{\lambda_-}(p_-)\gamma_5 v_{\lambda_+}(p_+). \quad (3.2)$$

We therefore neglect the final lepton mass (but not necessarily the loop mass), so that Eq. (2.14) gives directly

$$\mathfrak{M}^Z = -\frac{8\alpha^2}{k'^2}\frac{g_A}{e}\frac{\Delta(q^2, k'^2, m^2)}{k'^2 - M^2 + i\Gamma M}\epsilon^{\alpha\sigma\rho\mu}k_\alpha\epsilon_\sigma J_\rho \times \bar{u}_{\lambda_-}(p_-)\gamma_\mu(g_V - g_A\gamma_5)v_{\lambda_+}(p_+). \quad (3.3)$$

Now $\mathfrak{M}^Z$ must be added to the Bethe-Heitler amplitude, given by

$$\mathfrak{M}^{\text{B.H.}} = ie^3\epsilon_\sigma(k)J_\rho(q)\bar{u}_{\lambda_-}(p_-)\left[\frac{\gamma^\sigma(\not{p}_- - \not{k})\gamma^\rho}{2k\cdot p_-} - \frac{\gamma^\rho(\not{p}_+ - \not{k})\gamma^\sigma}{2k\cdot p_+}\right]v_{\lambda_+}(p_+)/q^2\,, \quad (3.4)$$

which leads to a differential cross section of the form

$$d\sigma_{\lambda_+,\lambda_-}(p_+,p_-) = d\sigma^{\text{B.H.}}_{\lambda_+,\lambda_-}(p_+,p_-) + d\sigma^{\text{int.}}_{\lambda_+,\lambda_-}(p_+,p_-) + d\sigma^Z_{\lambda_+,\lambda_-}(p_+,p_-), \quad (3.5)$$

the terms representing pure Bethe-Heitler, interference, and pure Z^0, respectively. (Contributions from Figs. 2 and 3 will not enter the final results.) The helicity projection operators for zero lepton mass satisfy $P(\lambda)\gamma_\mu = \gamma_\mu P(-\lambda)$ and can therefore be brought together to give

$$P(\lambda_-)P(-\lambda_+) = \delta_{-\lambda_+,\lambda_-}P(\lambda_-). \quad (3.6)$$

Thus the leptons must have opposite helicities. Terms which are even under C are either independent of λ_- and symmetric in p_+ and p_- or linear in λ_- and antisymmetric in p_+ and p_-, and vice versa for terms which are odd under C.

In particular,

$$d\sigma^{\text{B.H.}}_{\lambda_+,\lambda_-}(p_+,p_-) = \tfrac{1}{2}\delta_{-\lambda_+,\lambda_-}d\sigma^{\text{B.H.}}(p_+,p_-), \quad (3.7)$$

where $d\sigma^{\text{B.H.}}$ is the unpolarized result.

The interference term has the form

$$\frac{d\sigma^{\text{int.}}_{\lambda_+,\lambda_-}(p_+,p_-)}{(d^3p_+/2E_+)(d^3p_-/2E_-)} = \frac{4\alpha^4}{\pi^3}\frac{g_A}{e}\delta_{-\lambda_+,\lambda_-}\frac{1}{k'^2q^2}\frac{2m_i}{s - m_i{}^2}\operatorname{Im}\left(\frac{\Delta\cdot Y}{k'^2 - M^2 + i\Gamma M}\right), \quad (3.8)$$

where

$$Y = \frac{g_V}{e}(A + i\lambda_- S) - \frac{g_A}{e}(\lambda_- A + iS)\,, \quad (3.9)$$

with the definitions

$$A = \epsilon_{\mu\nu\alpha\beta}k^\mu p^\nu p_+^\alpha p_-^\beta\left(\frac{l\cdot p_+}{k\cdot p_+} + \frac{l\cdot p_-}{k\cdot p_-}\right)W_2(q^2,\nu)/m_i{}^2, \quad (3.10)$$

$$\begin{aligned} S = {} & \left\{4p_+\cdot p_- - 3k\cdot(p_+ + p_-) - \frac{1}{q^2}\left[(k\cdot q)^2 + (q\cdot p_+)^2\left(1 + \frac{k\cdot p_-}{k\cdot p_+}\right) + (q\cdot p_-)^2\left(1 + \frac{k\cdot p_+}{k\cdot p_-}\right)\right.\right. \\ & \left.\left. - p_+\cdot p_- k\cdot q\left(\frac{q\cdot p_+}{k\cdot p_+} + \frac{q\cdot p_-}{k\cdot p_-}\right)\right]\right\}W_1(q^2,\nu) \\ & - \left\{[2p_+\cdot p_- - k\cdot(p_+ + p_-)]\,l^2 + (l\cdot k)^2 + (l\cdot p_+)^2\left(1 + \frac{k\cdot p_-}{k\cdot p_+}\right)\right. \\ & \left. + (l\cdot p_-)^2\left(1 + \frac{k\cdot p_+}{k\cdot p_-}\right) - p_+\cdot p_- l\cdot k\left(\frac{l\cdot p_+}{k\cdot p_+} + \frac{l\cdot p_-}{k\cdot p_-}\right)\right\}W_2(q^2,\nu)/m_i{}^2, \end{aligned} \quad (3.11)$$

$$l = p - \frac{p \cdot q}{q^2} q . \qquad (3.12)$$

Note that A is antisymmetric in p_+ and p_-, while S is symmetric, so that the g_V part of Y is odd under C and the g_A part is even, as expected. Finally, the pure Z^0 term is given by

$$\frac{d\sigma^Z_{\lambda_+,\lambda_-}(p_+,p_-)}{(d^3p_+/2E_+)(d^3p_-/2E_-)} = \frac{8\alpha^5}{\pi^4}\left(\frac{g_A}{e}\right)^2 \delta_{\lambda_-,-\lambda_+}\frac{1}{k'^4}\frac{2m_i}{s-m_i{}^2}\left[\left(\frac{g_V}{e}\right)^2+\left(\frac{g_A}{e}\right)^2-2\lambda_-\frac{g_A}{e}\frac{g_V}{e}\right]\left|\frac{\Delta}{k'^2-M^2+i\Gamma M}\right|^2$$
$$\times\left\{\left[k\cdot p_+ k\cdot p_- + \frac{k\cdot q}{q^2}(k\cdot p_- q\cdot p_+ + k\cdot p_+ q\cdot p_-)\right]W_1(q^2,\nu)\right.$$
$$\left.+\left[l\cdot k(k\cdot p_- l\cdot p_+ + k\cdot p_+ l\cdot p_-) - l^2 k\cdot p_+ k\cdot p_-\right]W_2(q^2,\nu)/m_i{}^2\right\} . \qquad (3.13)$$

One way to isolate the Z^0 effect is through its dependence on lepton helicity. The polarization is given by

$$\langle\lambda_-\rangle = \frac{d\sigma_{1,-1}-d\sigma_{-1,1}}{d\sigma_{1,-1}+d\sigma_{-1,1}}$$
$$= \frac{\frac{8\alpha^4}{\pi^3}\frac{g_A}{e}\frac{1}{k'^2q^2}\frac{2m_i}{s-m_i{}^2}\,\mathrm{Im}\left(\frac{\Delta\cdot[-(g_V/e)iS+(g_A/e)A]}{k'^2-M^2+i\Gamma M}\right)+O(\alpha^5)}{d\sigma^{\mathrm{B.H.}}+O(\alpha^4)+O(\alpha^5)} . \qquad (3.14)$$

On the other hand, if λ_- is summed on, the only surviving term which may be isolated from the Bethe-Heitler term is the part of the interference term proportional to A, which is clearly parity-violating. It contributes in $O(\alpha^4)$ even off-resonance since Δ has an imaginary part. To isolate it, consider first the charge asymmetry given by

$$\text{charge asymmetry} = \frac{d\sigma(p_+,p_-)-d\sigma(p_-,p_+)}{d\sigma(p_+,p_-)+d\sigma(p_-,p_+)}$$
$$= \frac{\frac{8\alpha^4}{\pi^3}\frac{g_A}{e}\frac{1}{k'^2q^2}\frac{2m_i}{s-m_i{}^2}\,\mathrm{Im}\left(\frac{\Delta\cdot(g_V/e)A}{k'^2-M^2+i\Gamma M}\right)+\mathrm{QED}+\mathrm{res.}}{d\sigma^{\mathrm{B.H.}}+O(\alpha^4)+O(\alpha^5)} . \qquad (3.15)$$

Here "QED + res." refer to contributions to the charge asymmetry from Fig. 2 and Fig. 3, respectively. These are even under parity. Parity changes the "handedness" of a coordinate system, so in terms of the laboratory variables defined in Fig. 9, $\phi \to -\phi$ under parity. In particular at the "symmetry point" where

$$E_+ = E_- = E ,$$
$$\theta_+ = \theta_- = \theta , \qquad (3.16)$$

even-parity contributions to the charge asymmetry must vanish. More generally, the Z^0 effect may be isolated in the "azimuthal asymmetry" given by

$$A(\phi) = \frac{d\sigma(\phi)-d\sigma(-\phi)}{d\sigma(\phi)+d\sigma(-\phi)} , \qquad (3.17)$$

which equals the first term in Eq. (3.15). Thus all $E_\pm$ and $\theta_\pm$ may be included, as long as ϕ is known precisely. In the laboratory $A(\phi)$ is proportional to

$$-m_i \vec{k}\cdot\vec{p}_+\times\vec{p}_- = (m_i\,\omega E_+E_-\sin\theta_+\sin\theta_-)\sin\phi . \qquad (3.18)$$

For the numerical estimates given in the next section it is convenient to reduce the number of variables by choosing the symmetry point. Then

$$k'^2 \to 4E^2\sin^2\theta\sin^2(\phi/2) , \qquad (3.19)$$

$$q^2 \to -8\omega E\sin^2(\theta/2)[1-2E/\omega\cos^2(\theta/2)\sin^2(\phi/2)] , \qquad (3.20)$$

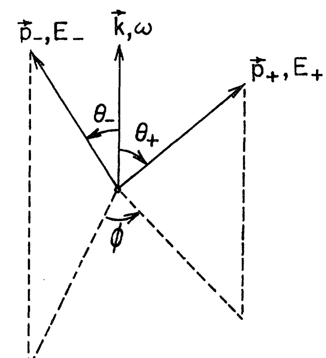

FIG. 9. Laboratory coordinates for leptons of energies $E_\pm$ with respect to a photon beam of energy ω.

$$\frac{d\sigma^{\text{B.H.}}}{(d^3p_+/2E_+)(d^3p_-/2E_-)} \to \frac{4\alpha^3}{\pi^2 q^4 \omega^3}(2[\omega - 4E\cos^2(\theta/2)\sin^2(\phi/2)]^2 W_1(q^2,\nu) + \cot^2(\theta/2)\{[\omega - 4E\cos^2(\theta/2)\sin^2(\phi/2)][\omega + 4E\sin^2(\theta/2)\sin^2(\phi/2)] - 4E^2\sin^2(\phi/2)\} W_2(q^2,\nu)), \tag{3.21}$$

$$A \to \frac{2\omega^2 E^2}{q^2}\sin^2\theta\left\{1 + \frac{2E}{\omega}[1 - 2\cos^2(\theta/2)\sin^2(\phi/2)]\right\}\sin\phi\, W_2(q^2,\nu). \tag{3.22}$$

IV. DISCUSSION

Here we give some numerical estimates of the results of the last two sections and discuss the model independence and possible extensions. For simplicity we consider only a hydrogen target. Hadronic neutral currents have been omitted so that only the well-known electromagnetic structure enters. Photon exchange tends to suppress the region of $-q^2 \ll k'^2$, so the main contribution is from inelastic processes. Neglecting longitudinal photon exchange gives

$$\frac{\nu W_2}{2m_i W_1} = \frac{\nu}{2m_i}\left(1 - \frac{\nu^2}{q^2}\right)^{-1} = x\left(1 + x\frac{2m_i}{\nu}\right)^{-1}, \tag{4.1}$$

$$x = \frac{-q^2}{2m_i\nu}, \tag{4.2}$$

which is quite accurate in the scaling region, where the data have the simple form[13]

$$\nu W_2 \sim (1-x)^3[1.274 + 0.5989(1-x) - 1.675(1-x)^2]. \tag{4.3}$$

Table I shows the results of integrating Eq. (2.15) to obtain the total Z^0 photoproduction cross section as a function of beam energy for two values of the Z^0 mass. For comparison we also list the result for transverse Z^0, pointlike target, and infinite beam energy, given by[14]

$$\sigma(\infty) = 20\left(\frac{g_A/e}{M\ (\text{GeV})}\right)^2 \times 10^{-6}\ \mu\text{b}. \tag{4.4}$$

Roughly speaking, the effect of including longitudinal Z^0 and reasonable form factors and beam energies is a 10% reduction in cross section at $M \sim 10$ GeV and a much faster falloff in M.

To obtain some typical results for lepton pair photoproduction, we choose

$$\omega = 400\ \text{GeV}, \quad E_+ = E_- = E, \quad \theta_+ = \theta_- = 10°, \quad \phi = 90°. \tag{4.5}$$

Then Eqs. (3.19)–(3.22) apply. We may expect that $\Gamma \sim \alpha M$, and we consider the range

$$M = 10\text{–}15\ \text{GeV}, \quad \Gamma = 100\text{–}200\ \text{MeV}. \tag{4.6}$$

The distance from resonance may be measured by

$$\delta = \frac{E - E_0}{\Gamma}, \tag{4.7}$$

where E_0 is the value of E for which $k'^2 = M^2$.

Far below resonance the effects are extremely small due to the high power of coupling involved. For example,

$$A(\phi) \xrightarrow[k'^2 \ll M^2]{} \frac{g_A g_V}{\pi M^2} 2E^2 \sin^2(\theta/2)\cos^4(\theta/2)\sin^2(\phi/2)\sin\phi[1 + y - 2y\cos^2(\theta/2)\sin^2(\phi/2)] \times\left[[1 - 2y\cos^2(\theta/2)\sin^2(\phi/2)]^2\left(\sin^2(\theta/2) + \frac{(1-y)^2}{4y[1 - y\cos^2(\theta/2)\sin^2(\phi/2)]}\right) + \frac{[1 + y - 2y\cos^2(\theta/2)\sin^2(\phi/2)]^2}{4y[1 - y\cos^2(\theta/2)\sin^2(\phi/2)]}\right]^{-1}, \tag{4.8}$$

$$y = 2E/\omega. \tag{4.9}$$

We must warn the reader, however, that as one goes off the resonance region, the neglected diagrams in Fig. 7 become more and more important and their contributions must be added to Eq. (4.8). The order of magnitude of course remains the same.

The situation is quite different near the resonance. Figure 10 shows the relative sizes and

TABLE I. Total cross section $\sigma(M)$ for photoproduction of a Z^0 of mass M by the mechanism of Fig. 8 [Eq. (2.15)] for various values of beam energy ω. The target is a proton and the units are $(g_A/e)^2 \times 10^{-8}$ μb, where g_A/e is the ratio of axial-vector to electromagnetic coupling of electron and muon. Also shown is the value of p if $\sigma(M) \sim M^{-p}$.

M (GeV) \ ω (GeV)	100	200	300	400	500	∞
10	0.01	0.46	1.26	2.07	2.78	20.0
15		0.002	0.04	0.14	0.28	8.9
$p \simeq \dfrac{-\ln[\sigma(10)/\sigma(15)]}{\ln(10/15)}$		20.5	8.5	6.66	5.64	2.0

shapes of the unpolarized Bethe-Heitler, symmetric and antisymmetric interference, and pure Z^0 terms as functions of δ. This is for $M = 10$ GeV ($E_0 = 40.7$ GeV) and the two cases $\Gamma = 200$ MeV when the Z^0 couplings equal e and $\Gamma = 100$ MeV when $g_A/e = -1/2$, $g_V/e = 1/10$. (The latter case resembles the Weinberg model with $\sin^2\theta_W = 0.3$, although then $M = 80$ GeV.) On the resonance the $O(\alpha^5)$ pure Z^0 term is about the same size as the lower orders.

Near resonance then, the higher-order terms indicated in Eqs. (3.14) and (3.15) must be included. In the notation of Fig. 10 the lepton polarization and azimuthal asymmetry are

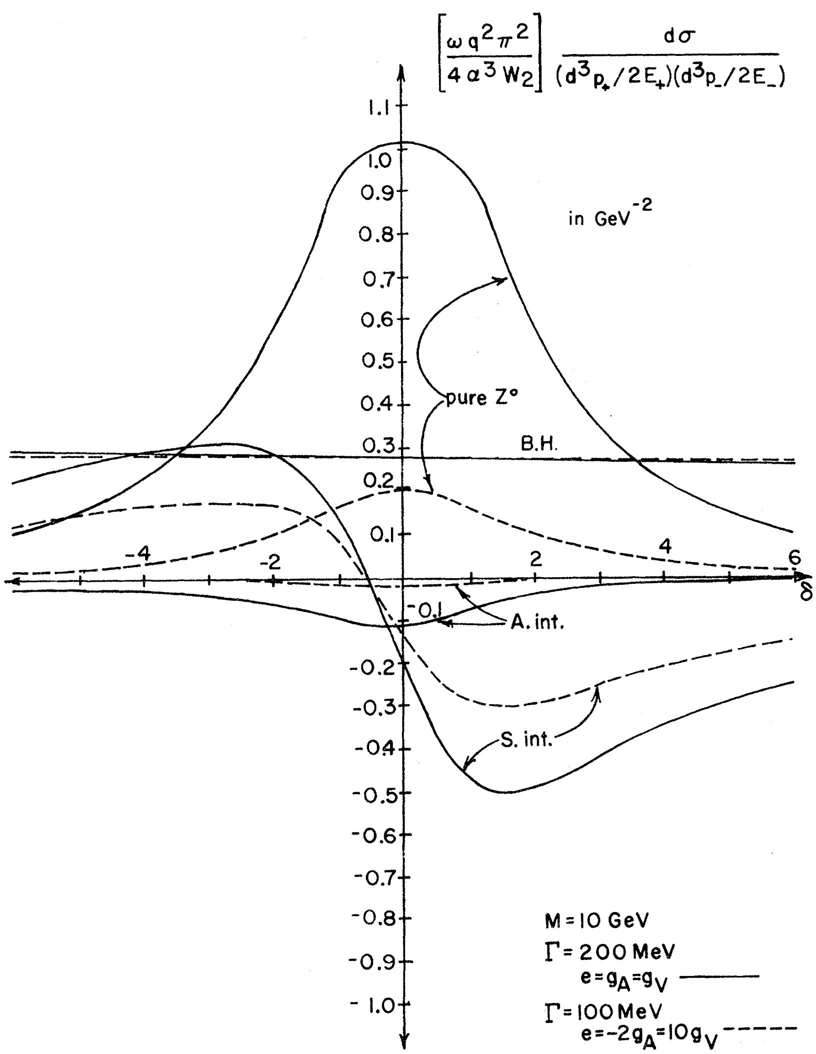

FIG. 10. Relative contributions from Fig. 1 and Fig. 7(d) as the invariant mass of the lepton pair goes through the Z^0 resonance. The relevant quantities are defined in the text. (The dashed curves require an overall minus sign.)

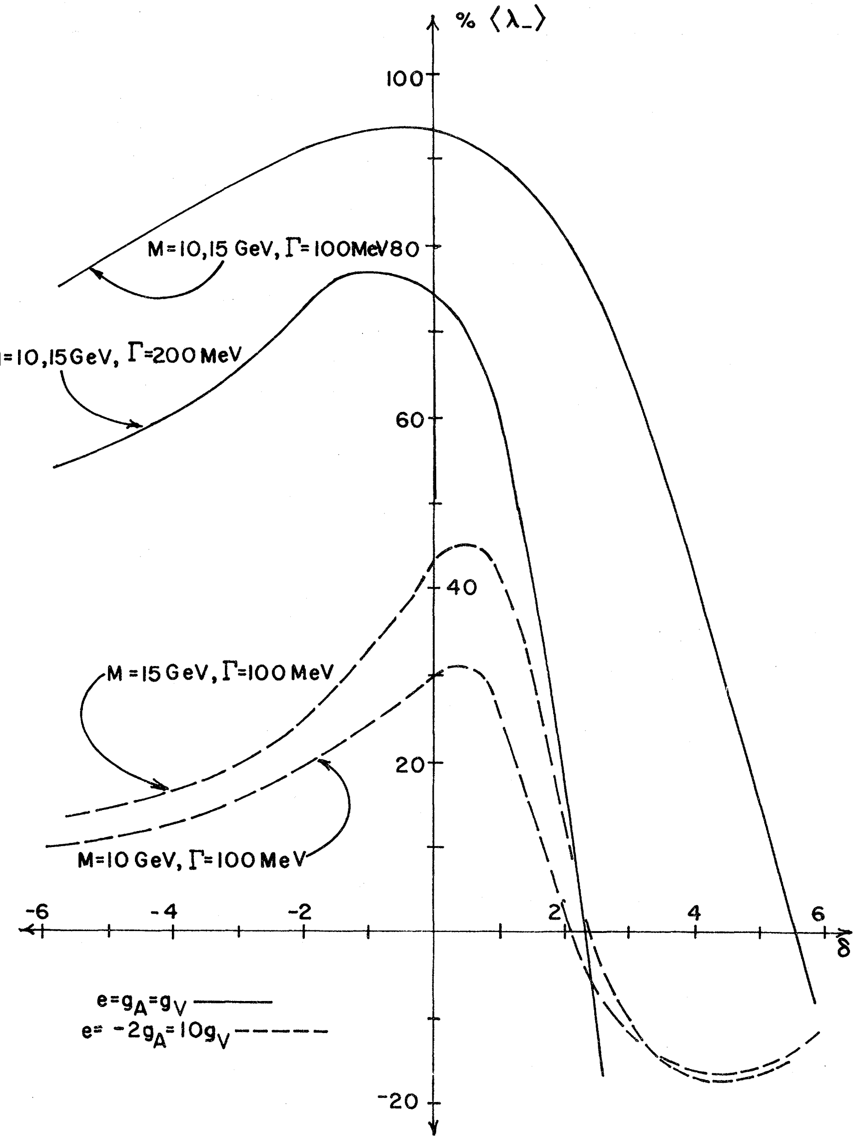

FIG. 11. Lepton polarization for the situation defined in the text. (The dashed curves require an overall minus sign.)

$$\langle \lambda_- \rangle = \frac{(\text{A. int.})g_A/g_V + (\text{S. int.})g_V/g_A + (\text{pure } Z^0)2g_Ag_V/(g_A{}^2+g_V{}^2)}{\text{B.H.} + \text{A. int.} + \text{S. int.} + \text{pure } Z^0}, \tag{4.10}$$

$$A(\phi) = \frac{-\text{A. int.}}{\text{B.H.} + \text{A. int.} + \text{S. int.} + \text{pure } Z^0}. \tag{4.11}$$

These are shown as a function of δ in Figs. 11 and 12, respectively. For the case of $e=g_A=g_V$ we also show the effect of reducing Γ by a factor of 2. The results are very weakly dependent upon the precise choice of Γ or the choice of symmetric detection. They primarily depend upon the Z^0 mass and couplings.

If there exist other leptons besides the electron

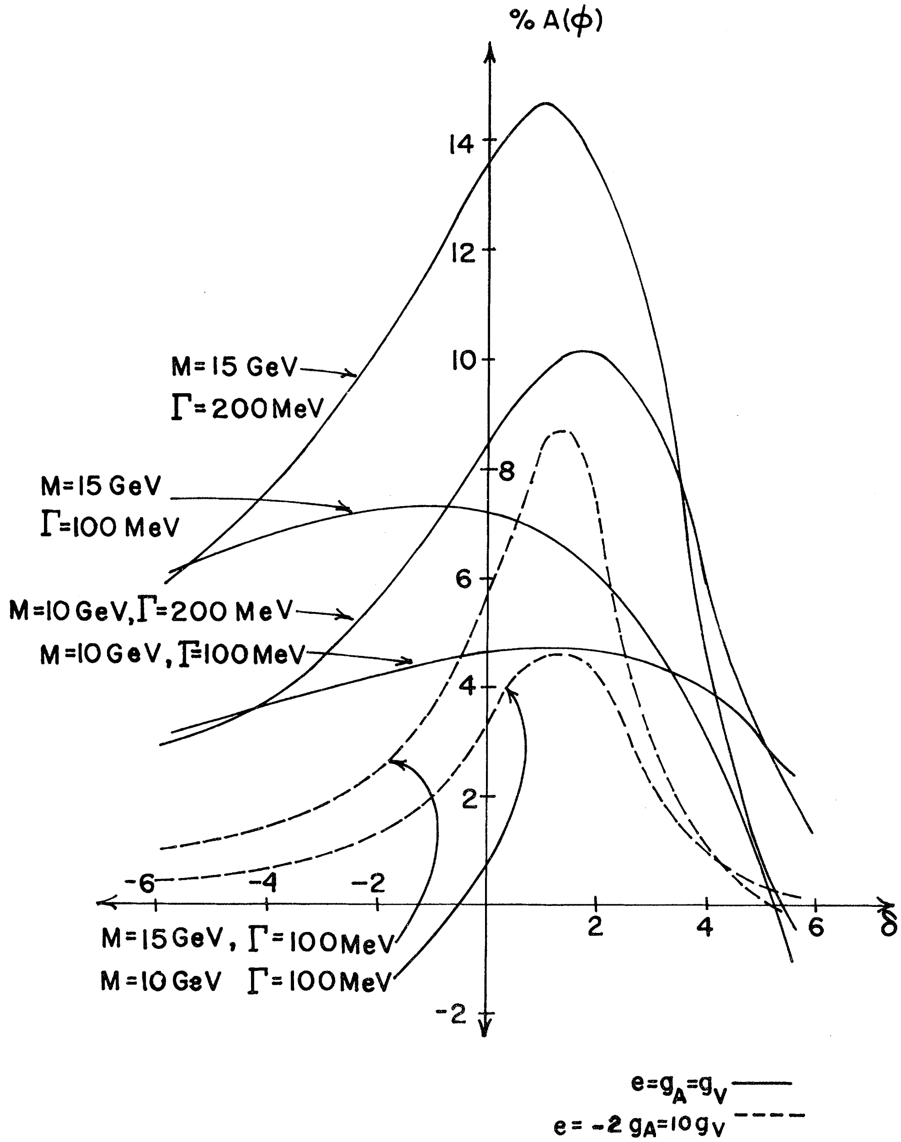

FIG. 12. Azimuthal asymmetry for the situation defined in the text. (The dashed curves require an overall minus sign.)

and muon, then the vertex function Δ becomes a linear combination of terms each with the appropriate value of m and g_A. Various models exhibit three circumstances in which our results will be significantly lowered: (i) The lower bound on the Z^0 mass may be prohibitively large. (ii) The Z^0 may have negligible axial-vector (and/or vector) coupling to charged leptons. (iii) Cancellation between loops could give results of $O(m^2/M^2)$.

On the other hand, if the Z^0 couples to hadrons the effects may be significantly larger. [A related situation is that of wide-angle leptons from colliding proton beams.[15] The triangle contribution in Fig. 13(a) is relatively model-independent, but if one assumes a quark-parton model for the hadronic neutral current then the annihilation diagram in Fig. 13(b) is much larger.[16]] For Z^0 photoproduction the simplest diagram would be a Compton-type Born term,[17] but this falls with photon energy and is soon superseded by a triangle- or Delbrück-type contribution with vector gluons or something more general in the t channel. If the Z^0 is not

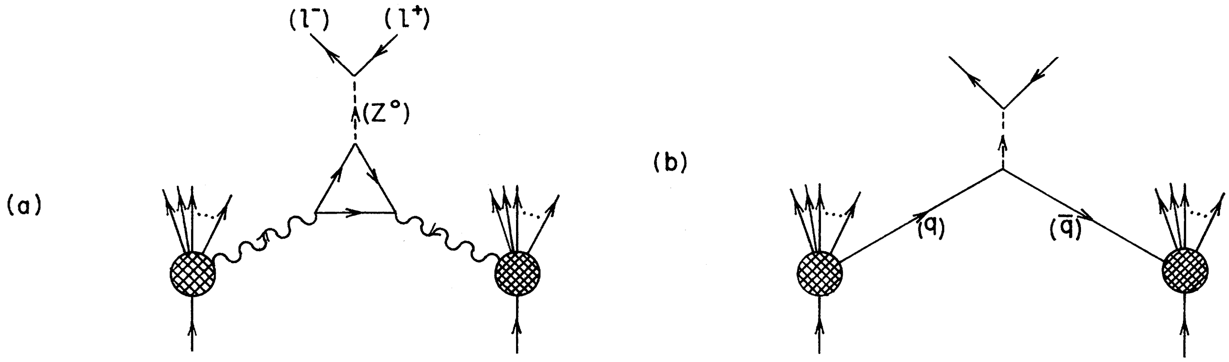

FIG. 13. Contributions to production of lepton pairs in colliding proton beams.

produced in the photon beam then we must rely on Z^0 exchange (Fig. 6). This gives polarization and charge asymmetry but cannot contribute to $A(\phi)$. More specific results on possible hadronic neutral current effects in photoproduction will appear in a separate paper.[18]

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[11] Each factor in the logarithm gives a dilogarithm integral.

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