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# Capital Stock Estimates for Manufacturing Industries:

## Methods and Data

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This paper documents the methods and source data used by Federal Reserve Board staff to estimate U.S. manufacturers' real net capital stocks of equipment and structures. The estimates, which begin in 1958 and cover 164 detailed manufacturing industries, are developed by the perpetual inventory method from investment data. Conceptually, the measures are designed to represent the available productive capability of the manufacturing capital stock; in developing the net stocks, each vintage of investment is adjusted for expected losses in efficiency owing to the joint effects of discards and economic decay.

The capital estimates described in this paper are part of the Federal Reserve's industrial production and capacity utilization system. Measures of capital, which are used to develop the manufacturing capacity indexes, had been provided by the Commerce Department's Office of Business Analysis (OBA) until the office ceased its capital measurement program in 1990.[1] For the February 1994 industrial production and capacity utilization revision, Board staff reconstructed the entire dataset and have regularly extended and refined the estimates as part of the annual revision program for industrial production and capacity utilization. Typically, the complete Federal Reserve manufacturing real net capital stocks dataset is available through the most recent year for which industry investment data are available from the Census Bureau, currently, 1993. [2]

The new Federal Reserve measures update the OBA estimates in several ways. They incorporate industry investment data adjusted to the 1987 Standard Industrial Classification, new information on service lives, investment, and price deflators by asset type from the Bureau of Economic Analysis (BEA)[3], and more recent information on the distribution of industry investment by asset type.[4] In the future, the system described in this paper will be updated

1. The original capital stock estimates were derived from a system developed by Jack Faucett Associates. [See Jack Faucett Associates, "Development of Capital Stock Series by Industry Sector", Report TR-81 (prepared for the Executive Office of the President, Office of Emergency Preparedness, March 1973).] Until 1980, the Bureau of Labor Statistics' Office of Economic Growth maintained and updated the Faucett system. [See "Capital Stock Estimates for Input-Output Industries: Methods and Data," Bulletin 2034 (Bureau of Labor Statistics, 1979).]

2. The dataset is available on written request to the Industrial Output Section, Mail Stop 82, Division of Research and Statistics, Board of Governors of the Federal Reserve System, Washington, DC 20551.

3. BEA's service lives, investment data and price deflators are discussed in *Fixed Reproducible Tangible Wealth in the United States, 1925–89* (Bureau of Economic Analysis, January 1993).

4. The OBA capital stock estimates were based on BEA's benchmark capital flows tables for 1963, 1967, and 1972. The new FRB estimates also incorporate BEA's 1977 table [ see "New Structures and Equipment by Using Industries, 1977," *Survey of Current Business* (November 1985), pp. 26–34.] as well as BEA's most recent, unpublished 1982 table.

with information assembled by BEA for its recent comprehensive revision of the National Income and Product Accounts.

**I. The FRB-PIMS Model**

The FRB capital stock estimates are based on a variant of the perpetual inventory model system (PIMS)[5] designed to provide capital measures for the analysis and measurement of capacity. In the broadest sense, the analysis and estimation of capacity requires a concept of productive capital stock, that is, a measure of capital that adjusts for losses of production capability due to economic decay rather than for losses of value due to economic depreciation.[6] Capacity estimation also requires capital measures that correspond at least to a three-digit level of industry detail in manufacturing and that represent the capital used, rather than owned, by each industry. By contrast, a national wealth concept is measured by the capital stocks compiled by BEA, and their manufacturing net wealth stocks, which are adjusted for expected losses in market value due to economic depreciation, provide detail at only the two-digit level.

The FRB-PIMS model constructs estimates of manufacturers' real net capital stocks from seven types of information:

- annual current-dollar new investment in equipment and structures by industry
- annual current-dollar new investment by asset type for the total economy
- annual capital flows tables that decompose industry investment into investment by asset type
- annual price deflators for each type of asset
- a mean service life for each type of asset
- a discard function to account for expected losses due to retirements from each type of asset as it ages

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5. BEA first published PIMS-based capital stock estimates in 1962; see "Expansion of Fixed Business Capital in the United States, Rapid Postwar Growth-Rise Slackens," *Survey of Current Business* (November 1962), pp. 9–18.

6. Economic decay represents the loss in asset efficiency (reflected in reduced output and/or increased input requirements) as the asset ages. Depreciation represents loss in asset value due to current and expected future declines in asset efficiency as the asset ages. Conceptual and statistical issues involved in capital measurement are discussed in J.E. Triplett, "Depreciation in Production Analysis and in Income and Wealth Accounts: Resolution of an Old Debate," *Economic Inquiry* (forthcoming) (manuscript prepared at Bureau of Economic Analysis, January 1995); C.R. Hulten, "The Measurement of Capital," in E.R. Berndt and J.E. Triplett, eds., *Fifty Years of Economic Measurement* (University of Chicago Press, 1990), pp. 119–52; D.W. Jorgenson, "Capital as a Factor of Production," in D.W. Jorgenson and R.W. Landau, eds., *Technology and Capital Formation* (MIT Press, 1989), pp. 1–36; and M.F. Mohr, "Capital Depreciation and Related Issues: Definitions, Theory, and Measurement," Discussion Paper 28 (Bureau of Economic Analysis, June 1988).

- an economic decay function to account for losses of economic efficiency experienced by each type of asset as it ages.

The sources of the information are summarized briefly in table 1. Capital stock estimates are constructed for each three-digit SIC manufacturing industry and twenty-four four-digit industries using separate PIMS models and industry investment data for structures and equipment. (The industries are listed in appendix table A.1). Information on a total of 29 asset types—four types of structures and 25 types of equipment—also is incorporated in the FRB stock estimates. (The asset types and their service lives are listed in appendix table A.2). The remainder of this paper describes more fully how these data were assembled and how the adjustments for discards and economic decay were generated.

1. Data Sources for Estimates of Manufacturers' Capital Stocks

| Type of information | 1890–1948 | 1949–71 | 1972–86 | 1987–93 |
|---|---|---|---|---|
| Investment by industry | JFA, OBA | JFA, OBA | COM/ASM | COM/ASM |
| Investment by asset type | BEA | BEA | BEA | BEA |
| Deflators by asset type | BEA | BEA | BEA | BEA |
| Service lives by asset type | BEA, JFA | BEA, JFA | BEA, JFA | BEA, JFA |
| Capital flows tables | FRB | BEA, FRB | BEA, FRB | FRB |

Note—More detailed documentation is provided in appendix table A.3 .
Abbreviations: BEA, Bureau of Economic Analysis; COM/ASM, Census of Manufactures and Annual Survey of Manufactures; FRB, Federal Reserve Board; JFA, Jack Faucett Associates; OBA, Office of Business Analysis, U.S. Department of Commerce.

## II. Creation of Industry and Asset-Type Investment Controls

The PIMS model requires the development of two different but complementary sets of investment controls: (1) a time series of annual investment controls by industry and (2) a time series of annual investment controls by asset type. The first set of controls represents the annual expenditures by each industry for new equipment and for new structures; the second set represents the annual, economy-wide private investment in asset types used in manufacturing industries.

The earliest value of the time series required for both sets of controls depends on the starting date for the capital estimates; the year in which investments in a particular asset type begins; and the upper bound of the distribution around the mean service life for the longest-lived asset type. Because the FRB capital stock estimates begin in 1958 and include a class of structures (other nonresidential non-building structures) that has a maximum possible service life of sixty years, the PIMS model for structures incorporates industry and asset-type controls that span the period 1890 to 1993. For similar reasons, the PIMS model for equipment incorporates controls that span the period 1921 to 1993.

The pre-1987 industry controls were developed from estimates of investment by three- and four-digit industries on a 1972 SIC basis provided by Jack Faucett Associates and OBA, as described in appendix table A.3. The estimates were converted to a 1987 SIC basis using a matrix of total investment conversion ratios constructed from data published in the 1987 Census of Manufactures (COM). The 1987–93 industry controls are the data published in the 1987 and 1992 COM and in the 1988-91 and 1993 Annual Survey of Manufactures (ASM). All asset-type controls were either supplied by or developed from data provided by BEA and represent new gross private fixed investment by asset type before any adjustment for the sale of used assets.[7]

Although the FRB and BEA estimates of manufacturing capital stock are derived from a PIMS methodology that exploits much the same source data, the respective methods differ in important details. Some of the differences are significant enough to cause the BEA estimates of annual investment by manufacturing industries to be substantially larger than the COM/ASM estimates used by FRB.

The FRB methodology either eliminates or implicitly forces out of the manufacturing sector and into the nonmanufacturing sector certain BEA adjustments that are appropriate for the owning-industry orientation of the NIPA but are either inappropriate or impractical for the detailed, establishment-based and using-industry orientation of the FRB estimates. The BEA adjustments include revisions to manufacturing industry column controls to reflect : (1) inclusion of large benchmarking revisions to manufacturers' investments in structures since 1971;[8] (2) inclusion of salesman-owned autos; (3) inclusion of computers owned by manufacturers but leased to others; (4) inclusion of assets owned by central and administrative offices and auxiliaries (CAOs) of manufacturers; (5) exclusion of used autos sold by car rental or leasing subsidiaries of manufacturers;[9] and (6) a reconciling item to assure the industry investment totals for equipment and for structures are equal to the total asset flows in the NIPAs.[10]

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7. In BEA's published estimates of private purchases of producers durable equipment, investment in autos is reported net of sales of used autos.

8. The reasons for BEA's revisions to its estimates of new investment in structures are presented in "The Comprehensive Revision of the U.S. National Income and Product Accounts: A review of Revisions and Major Statistical Changes," *Survey of Current Business* (December 1991), pp. 24–46.

9. BEA estimates that new autos purchased by car rental or leasing firms are sold to the household sector after two years.

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10. When estimating investment in detailed asset types by industry (for use in the RAS method described in section III below), the sum of all investment by industry must equal the sum of all investment by asset type. BEA achieves this constraint by selectively adjusting investment totals for all industries; the FRB only adjusts the investment by industries outside manufacturing.

## 2. Reconciliation of ASM and BEA Estimates of Investment by Manufacturers in 1991

Billions of dollars

| | Equipment | Structures |
|---|---|---|
| ASM investment | 84.0 | 14.9 |
| Plus adjustment for: | | |
| Investment by CAOs | 5.0[e] | 1.4[e] |
| Salesman-owned autos<br>Computers leased to others } | 4.1[e] | NA |
| Industrial buildings | NA | 16.6 |
| Office buildings<br>Nonbuilding structures } | NA | 8.0[e] |
| Less adjustment for: | | |
| Sales of used autos | 8.0 | NA |
| Equals: BEA investment [1] | 85.1 | 40.9 |

Addendum: BEA benchmark adjustments to 1991 investment in structures types used in manufacturing

| Structures Type | BEA investment [2] | Census VPIP investment [3] | BEA adjustment |
|---|---|---|---|
| Industrial | 38.9 | 22.3 | 16.6 |
| Office | 28.6 | 23.0 | 5.6 |
| Other commercial | 25.5 | 25.5 | 0.0 |
| Nonbuildings structures | 8.4 | 2.9 | 5.5 |
| Total | 101.4 | 73.7 | 27.7 |

1. CAO = central and administrative officers and auxiliaries.

2. Unpublished data and data from *Survey of Current Business* (July 1994), table 5.6.

3. From Current Construction Reports C30/94–5 “Value of New Construction Put in Place May 1994” (Bureau of the Census), table 1.

NA = not applicable.
e = estimated.

Table 2 indicates that the cumulative effect of the BEA adjustments to 1991 ASM equipment investment by manufacturers was quite small: BEA’s estimate exceeds the ASM estimate by only 1.3 percent. For the years 1950–91, the COM/ASM estimates for equipment ranged between plus and minus 6 percent of the BEA estimate.[11] In contrast, the typical annual discrepancy for investment in structures is much larger, and over the years the discrepancy has increased. For example, in 1970 the BEA estimates of structures investment by the manufacturing sector was 20 percent greater than the COM/ASM estimate; by 1991 it was 174 percent greater. As suggested by table 2, the growing discrepancy is largely explained

11. The adjustment for sales of used autos explains about half of the variation.

by the adjustments BEA made, during its 1991 benchmark revision, to the industrial structures component of the Census Bureau's estimates of the value of new construction put in place (VPIP).[12] Given the disparity between the BEA and COM/ASM estimates for structures and and the current joint effort by Census and BEA to develop a methodology for benchmarking structures investment by manufacturers, the BEA adjustments to structures investment were not incorporated into the FRB capital stock estimates.[13]

## III. Decomposition of Industry Investment Controls into Asset Type Bundles

In the second step of the FRB-PIMS model, the industry annual investment controls were decomposed into twenty-nine detailed asset-type bundles—twenty-five equipment types and four structures types (appendix table A.2). Although the decomposition is done separately for equipment and structures, both decompositions are achieved by using a biproportional matrix balancing technique known as the RAS method.[14] Under this method, an initial $m \times n$ investment distribution matrix, $A_1$, can be iterated between a set of row controls and a set of column controls until on the kth iteration a unique matrix, $A_k$, that satisfies both sets of controls is achieved. A necessary condition for using the RAS method is that the sum of the row controls must equal the sum of the column controls.

The RAS method was implemented by first constructing an initial $A_1$ matrix for the benchmark years—1963, 1967, 1972, 1977, and 1982—from the eighty-industry capital flows tables (CFTs) prepared by BEA from its benchmark input-output (I-O) tables. The CFT shows how new gross private fixed investment by each using I-O industry was distributed across detailed BEA asset types. To transform I-O level CFTs into $A_1$ matrices that are compatible with the FRB industry detail and control totals, the I-O industries in each benchmark CFT

12. In the VPIP estimates, the Census Bureau classifies as industrial any building constructed at the site of a manufacturing establishment, regardless of whether the building is a factory, a warehouse, an office building, or a research facility. Conversely, office buildings and research facilities not constructed at the site of a manufacturing establishment are not classified as industrial, even if they are owned by a manufacturer.

13. The Census Bureau is currently reviewing BEA's benchmarking procedures for structures investment; see, for example, discussion in "Value of New Construction Put in Place May 1994," Current Construction Reports C30/94-5 (Bureau of the Census), pp. 1–2.

14. Implementing the RAS technique requires the following steps:

(1) create a $m \times n$ matrix $A_{1t}$ whose cells represent initial estimates of how the new gross investment made by each of n industries in year t is distributed across m asset types; (2) assemble a $m \times 1$ vector of row controls and a $1 \times n$ vector of column controls for each year, subject to the adding-up restriction that the sum of the annual row controls equals the sum of the annual column controls; (3) premultiply the $A_{1t}$ matrix by a $m \times m$ diagonal matrix $R_{1t}$, where each element on the diagonal is the scalar adjustment factor required to force the sum of total investment in asset type i by all industries in $A_{1t}$ to equal the ith row control; (4) postmultiply the $R_{1t}\, A_{1t}$ matrix by a $n \times n$ diagonal matrix $S_{1t}$, where each element on the diagonal is the scalar adjustment factor required to force the sum of total investment by industry j in the RA matrix to equal the ith column control; and (5) set the matrix $R_{1t}\, A_{1t}\, S_{1t} = A_{2t}$ and iterate over steps 3 and 4 until on the kth iteration $R_{kt}$ and $S_{kt}$ converge to identity matrices. The final decomposition matrix for each year is thus defined as

$$A_{kt} = R_{k-1t}\, A_{k-1t}\, S_{k-1t}\ .$$

For a detailed discussion of the RAS method, see M.B. Bacharach, "Estimating Nonnegative Matrices from Marginal Data," *International Economic Review*, vol.6 (1965), pp. 294–310.

were first regrouped into manufacturing and nonmanufacturing blocks of industries, and the columns of the latter were collapsed to form a total nonmanufacturing sector. Next, the industry detail in the manufacturing block was expanded to the detail shown in table 2; this was done by assigning the parent I-O industry investment by asset type to all its component table 2 industries and then scaling each column of the expanded matrix to the corresponding industry column control for each benchmark year.

The adding-up restriction required by the RAS model was satisfied by using the difference between the sum of the BEA row controls by detailed asset type and the sum of the Census-based column controls by detailed manufacturing industry as the column control for the nonmanufacturing sector. After obtaining the benchmark-year $A_k$ matrices for equipment and structures, the corresponding matrices for years between and beyond the benchmark years were constructed as follows. First, for the years between benchmarks, $A_1$ matrices were constructed by linear interpolation between the benchmark $A_k$ matrices.[15] Second, for the years 1983–93, $A_1$ matrices were constructed by sequentially using the $A_k$ matrix from the previous year as the $A_1$ matrix for the subsequent year.[16] Third, the $A_1$ matrices for 1890 to 1963 were derived by sequentially using the $A_k$ matrix from the succeeding year as the $A_1$ matrix for the previous year. Finally, given the time series of $A_1$ matrices, the $A_k$ matrices were derived by applying the RAS method, using each year's row and column controls after they were reconciled to each other as described above.

## IV. Deflation of Investment by Asset Type

In the third step of the FRB-PIMS model, the cells of the annual $A_k$ matrices for equipment and structures were transformed from a current-dollar to a constant (1987)-dollar basis to obtain estimates of real investment by asset type that can be compared across time. Annual investment price deflators for 1929–93 for each of the twenty-seven broad asset types listed in appendix table A.2 were assembled or were constructed from matching asset-type deflators supplied by BEA. Estimates for earlier years were constructed, if necessary, by using the movements in corresponding implicit investment deflators from BEA's capital wealth model to extrapolate the revised deflators for 1929 back in time.

## V. Accounting for Discards

In the fourth step of the FRB-PIMS model, the deflated cells of the annual $A_k$ matrices are assigned mean service lives that are specific to each asset type and adjusted to account for

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15. $A_1$ matrices for structures were derived by interpolating only between the $A_k$ matrices for 1963, 1972, and 1977.

16. $A_1$ matrices for structures since 1977 were constructed in this manner.

expected retirements from each asset-type investment bundle around its mean life.[17] The mean service life estimates for the twenty-nine asset types used in manufacturing industries are shown in appendix table A.2; the estimates for all asset types except autos were supplied by BEA.

**A. Discard Density Function**

Although each asset-type in the annual $A_k$ matrices is identified with a specific mean service life, the actual service life is assumed to be a random variable; the events causing this randomness include fire, theft, accident, unanticipated obsolescence, and poor asset construction (the “lemon” problem). The effect of such events is embodied in a discard density function, which gives the probability that an asset will be retired at different ages in the domain of possible service lives for the asset class. Translated to an asset-type bundle, the density at each age can be interpreted as the expected proportion of the bundle that will be retired at that age. The FRB-PIMS model, like its predecessor Faucett model, uses a truncated normal distribution to define the discard density function for each asset-type bundle:[18]

(1)
$$P_D(S \mid \bar{S}, \sigma) = \begin{cases} \dfrac{e^{\frac{-(S-\bar{S})^2}{2\sigma^2}}}{\sigma \displaystyle\int_{-\frac{\delta}{\sigma}}^{\frac{\delta}{\sigma}} e^{-X^2} dX}, & .5\bar{S} \leq S \leq 1.5\bar{S} \\ 0, & \text{otherwise} \end{cases}$$

where the numerator on the right-hand side of equation 1 represents the standard normal density associated with service life S; the denominator represents the area under the truncated curve between the cutoff points— $\frac{\delta}{\sigma} = \pm\ 2$; and

$$X^2 = \left(\frac{S - \bar{S}}{\sigma}\right)^2$$
$$\sigma = \text{standard deviation}$$

17. Alternatively, deflation to constant dollars could occur after the cells of each $A_k$ matrix have been adjusted for discards. In this case, the aggregate of the discard-adjusted cells represents the gross stock of capital in historical dollars, which is conceptually analogous to the gross book value of capital reported in financial statements.

18. In contrast, BEA uses a bell-shaped curve, the Winfrey S-3 distribution, which is a discrete frequency distribution that is based on the service lives for 176 asset classes used primarily by farms, railroads, and utilities observed by Robley Winfrey. [See R. Winfrey, “Statistical Analysis of Industrial Property Retirement,” Bulletin 125 (Iowa Engineering Experiment Station, December 1935)]. The two types of distributions produce similar estimates of discards, and the truncated normal distribution can be interpreted as the asymptotic probability distribution that the Winfrey distribution would approach as the sample size on which it is based became increasingly larger.

$$= .25\,\overline{S}$$
$$\delta = \pm\,.5\,\overline{S}.$$

**B. Estimates of Real Gross Capital Stocks**

Given the mean service life for each asset type and the discard function defined by equation 1 that corresponds to each mean life, the cells of the vintage sequence of matrices, $A_{kt}$ (t = 0,1,...,T), are adjusted annually to remove the incremental retirements expected from each vintage-by-asset-type bundle as it ages over the domain of its possible service lives. Gross capital stock estimates for industry j in year T are then calculated by first summing over the adjusted cells in column j of $A_{kt}$ (t = 0,1,..,T) and then aggregating across the vintage sums. The FRB-PIMS model generates direct estimates, for 1958–93, of both the real gross equipment stocks and the real gross structures stocks held by each industry listed in appendix table A.1.

## VI. Accounting for Economic Decay

Although the estimates of gross capital stocks derived from the PIMS model correspond closely to the capital measures found in financial statements, they are generally considered inappropriate measures of the productive capital stock used as an input to an industry's production process. From this "production function" perspective, we desire a stock measure—real net capital stock—that is also adjusted for losses of the productive efficiency of capital assets that come with age. Thus, in the fifth and final step of the PIMS model, the cells of the vintage $A_{kt}$ matrices are adjusted for the joint effects of the decay and discard processes rather than just the latter. The methodology for these adjustments follows.

**A. Economic Decay Function**

The economic decay function defines the portion of an asset's original economic efficiency remaining at each age during its service life. In 1970, Faucett introduced into the literature a parametric expression for the decay function, known as the hyperbolic or beta-decay function:[19]

(2) $$\phi_a = \frac{S - a}{S - \beta a}\,, \qquad 0 \le a \le S$$

where

$\phi_a$ = proportion of asset's original productive efficiency remaining at age a

S = asset's service life

19. See Jack Faucett Associates, "Capital Stocks, Production Functions and Investment Functions for Selected Input-Output Sectors," Report 355 (U.S. Department of Labor, Bureau of Labor Statistics, 1970), p. 39.

a = asset's age

β = curvature parameter, which determines the rate at which the asset loses efficiency as it ages.

By setting different values for β ($-\infty < \beta < 1$), a whole family of decay functions can be generated. Examples for an asset with a mean life of fifteen years are illustrated in figure 1 below; the values on the y-axis represent the percentage of the asset's original efficiency that remains when the asset has reached age a. These examples span the patterns usually found in the literature: BEA's one-hoss-shay ($\beta = 1$) economic decay;[20] straightline ($\beta = 0$) economic decay; and the BLS-OBA assumptions ($\beta = 0.9$, 0.75, and 0.5).[21] The assumption of accelerated efficiency decay is illustrated by $\beta = -1.0$.

**B. Age-Efficiency Function**

The discard density and economic decay functions of the PIMS model interact to form the age-efficiency function, which describes the net product-producing capability that an asset-type bundle is expected to have at each age, a, in the domain of its stochastic service life, S. As such, it differs from the deterministic concept of efficiency represented by the decay function.

The age-efficiency function for each asset-type bundle is computed by integrating the product of the discard density function and the beta-decay function over the domain of the former function:

$$\Phi_a = E[\epsilon(a)] = \int_{.5\overline{S}}^{1.5\overline{S}} \phi(a \mid S,\ \beta) P_D(S \mid \overline{S},\ \sigma)\ dS \tag{3}$$

where

E = the expectations operator

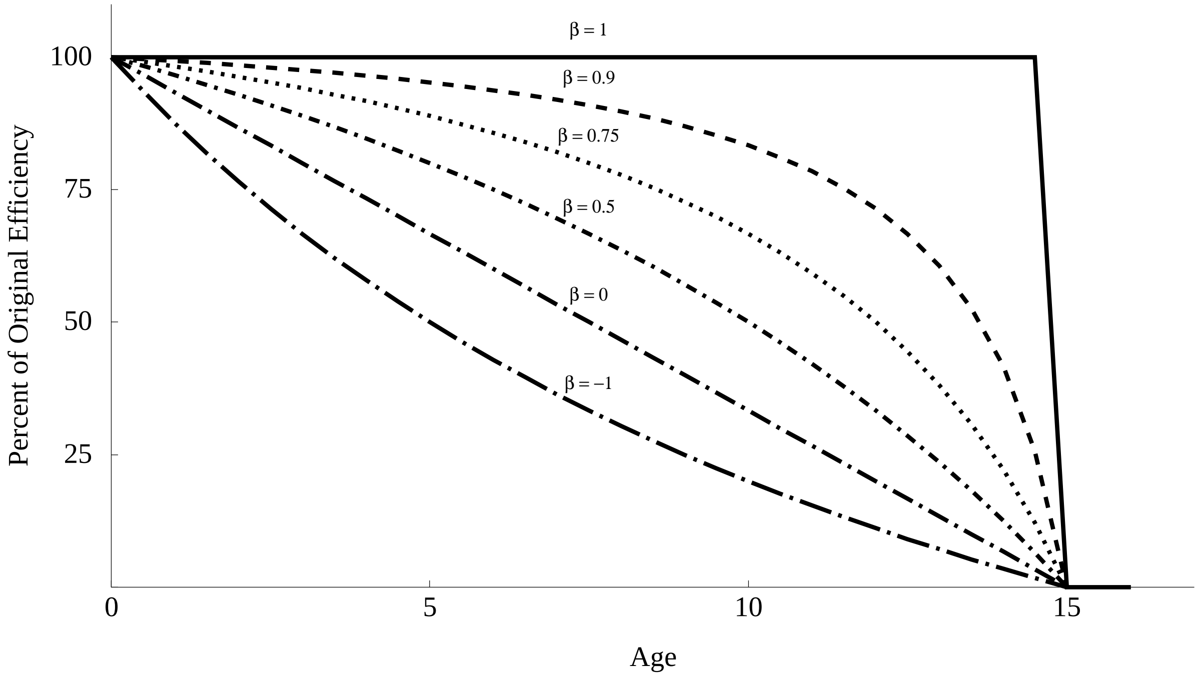
$\Phi_a = E[\varepsilon(a)]$ = expected asset efficiency at age a

$\phi(a \mid S, \beta)$ = value of beta-decay function at age a, given service life *S* and curvature parameter β

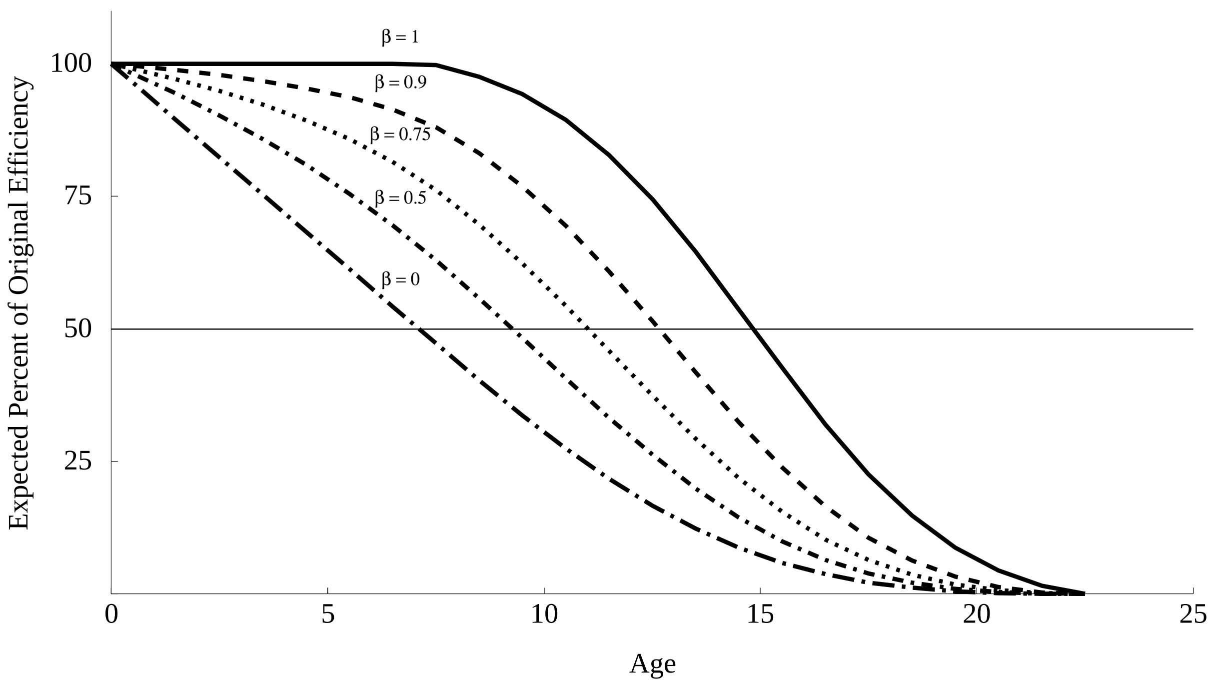
20. BEA's estimates of net wealth stocks are derived under the assumption that depreciation rather than economic decay occurs in a straightline pattern over an asset's mean service life. Implicit in this assumption, however, is the further assumption that the asset experiences no economic decay (one-hoss-shay) during its service life. For further discussion on this point, see A.H. Young and J.C. Musgrave, "Estimation of Capital Stock in the United States," in D. Usher, ed., *Measurement of Capital* (University of Chicago Press, 1980), pp. 23–81.

21. The BLS Office of Economic Growth and the Commerce Department's Office of Business Analysis used $\beta = 0.9$ for structures and $\beta = 0.75$ for equipment, while BLS's Office of Productivity and Technology uses $\beta = 0.75$ for structures and $\beta = 0.5$ for equipment.

## 1. Alternative Beta-Decay Functions for an Asset with a Service Life of Fifteen Years




## 2. Alternative Age-Efficiency Functions for an Asset with a Mean Service Life of Fifteen Years



$P_D(S \mid \bar{S}, \sigma) =$ discard density function derived from the truncated normal distribution.

The difference between the value of the age-efficiency function at age a at the start of year T and age a+1 at the end of year T, $[\Phi_a - \Phi_{a+1}]$, represents the expected percentage of the asset-type bundle's original efficiency units that will have to be replaced (by new investment) in order to maintain the effective stock of capital available during period T+1 at the level available during period T.

Figure 2 illustrates a family of possible age-efficiency functions for an asset bundle with a mean service life of fifteen years; pictured are five alternative profiles corresponding to $\beta$ = 1.0, 0.9, 0.75, 0.5, and 0.0. In the FRB-PIMS model, $\beta$ is set at 0.9 for structures and at 0.75 for equipment. The selection of these values was determined by a combination of technological, empirical, and judgment criteria. The first factor considered was the shape of the decay function implied by alternative $\beta$s. After a review of the arguments and empirical evidence on this point, the set of feasible $\beta$s was restricted to those producing concave decay functions ($0 < \beta < 1$), implying the common-sense rule that efficiency first declines slowly and then more rapidly as the asset ages. Faucett provided a technological justification for a concave decay function:[22]

> This viewpoint can by defended on purely technological grounds: that with reasonable care and maintenance this is what capital goods do and there is nothing that can be done about it. However, economic considerations strongly reinforce the technological arguments.

With respect to economic considerations, Faucett noted that a concave decay function is consistent with two different assumptions concerning maintenance and repair practices:

> [I]t represents either: the decline in efficiency [from losses of productive service units of capital—output decay] with any uniform level of maintenance and repair; or the increasing costs [input decay] of maintenance and repair required to maintain 100% of efficiency measured at age a ....

The limited empirical evidence also supports a concave decay function. For example, the evidence compiled by Barna and by Bitros strongly suggests that entrepreneurs attempt to preserve the productive efficiency of capital during its useful service life and that this behavior is modified only by negative demand prospects, by increases in the cost of capital, or, at the margin, by the effects of technological obsolescence operating to increase the opportunity cost

22. See Jack Faucett Associates, "Capital Stocks, Production Functions and Investment Functions for Selected Input-Output Sectors," Report 355 (U.S. Department of Labor, Bureau of Labor Statistics, 1970), p.35.

of additional maintenance expenditures.[23] A concave decay function is also supported by evidence obtained by BLS from the firms represented in its Business Research Advisory Council.[24]

The second factor considered in selecting the $\beta$ values for the FRB-PIMS model was the decay rates implied by empirical age-price profiles constructed by Hulten and Wykoff.[25] Using simulation analysis, BLS' Office of Productivity and Technology concluded that $\beta = 1.0$ worked best for retail and factory structures; $\beta = 0.75$ for office buildings; $\beta = 0.5$ for warehouses; and $0.25 < \beta < 0.75$ for trucks.[26]

The final factor considered in selecting the $\beta$ values was the "half-life" of the age-efficiency functions derived from alternative $\beta$s. Half-life is defined here as the age when half of an asset-type bundle's initial efficiency is expected to be exhausted. For example, the age-efficiency functions in figure 2 for assets with a mean life of fifteen years have the following half-lives: $\beta = 1.0$, fifteen years; $\beta = 0.9$, thirteen years; $\beta = 0.75$, eleven years; $\beta = 0.5$, nine years; and $\beta = 0.0$, seven years. On this criterion, the half-lives implied by $\beta$s greater than 0.75 were judged too long and those implied by $\beta$s less than 0.5 were judged too short to be realistic for short-lived equipment assets. From a similar analysis applied to assets with a mean-life of forty years, the half lives implied by $\beta$s greater than 0.9 and less than 0.75 were judged unrealistic for longer-lived structures assets. Corresponding to the $\beta$ values noted for the fifteen-year mean lives, the half-lives for forty-year assets were forty, thirty-four, thirty, twenty-five, and twenty-two years.

**C. Estimates of Net Capital Stocks**

Given the mean service life for each asset class and the age-efficiency function defined by equation 3 that corresponds to each mean life, the cells in the vintage sequence of $A_{kt}(t = 0,1,...,T)$ matrices are adjusted to remove the joint incremental effects of the discard and economic decay processes that are expected to occur over the domain of the asset-type bundle's possible service lives. Net capital stock estimates for industry j in year T are then calculated by first summing over the adjusted asset-type cells in column j of each vintage of $A_k$ and then aggregating across the vintage sums. The FRB-PIMS model generates direct estimates, for

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23. See T.S. Barna, "On Measuring Capital," in F.A. Lutz and D.C. Hague, eds., *The Theory of Capital* (Macmillian, 1962); and G.C. Bitros, "A Statistical Theory of Expenditures on Capital Maintenance and Repair," *Journal of Political Economy* (October 1976), pp. 917–36.

24. See "Trends in Multifactor Productivity, 1948–81," Bulletin 278 (U.S. Department of Labor, Bureau of Labor Statistics, 1983), p. 43.

25. See C.R. Hulten and F.C. Wykoff, "The Measurement of Economic Depreciation," in C.R. Hulten, ed., *Depreciation, Inflation, and the Taxation of Income from Capital* (Urban Institute Press, 1981), pp. 81–125.

26. See "Trends in Multifactor Productivity, 1948–81," Bulletin 2178 (U.S. Department of Labor, Bureau of Labor Statistics, 1983), p. 44.

1958–93, of both the real net equipment stocks and the real net structures stocks held by each industry listed in appendix table A.1.

**A.1 Industries Covered in FRB Estimates of Capital Stocks**

| SIC | Industry | SIC | Industry |
|---|---|---|---|
| 201 | Meat products | 273 | Books |
| 202 | Dairy products | 274 | Miscellaneous publishing |
| 203 | Preserved fruits and vegetables | 275 | Commercial printing |
| 204 | Grain mill products | 276 | Manifold business forms |
| 205 | Bakery products | 277 | Greeting cards |
| 206 | Sugar and confectionery products | 278 | Blankbooks and bookbinding |
| 207 | Fats and oils | 279 | Printing trade services |
| 208 | Beverages | 281 | Industrial inorganic chemicals |
| 209 | Misc. food and kindred products | 282 | Plastics materials and synthetics |
| 211 | Cigarettes | 2821 | Plastics materials and resins |
| 212 | Cigars | 2822 | Synthetic rubber |
| 213 | Chewing and smoking tobacco | 2823 | Cellulosic manmade fibers |
| 214 | Tobacco stemming and redrying | 2824 | Organic fibers, noncellulosic |
| 221 | Broadwoven fabric mills, cotton | 283 | Drugs |
| 222 | Broadwoven fabric mills, manmade | 284 | Soap, cleaners, and toilet goods |
| 223 | Broadwoven fabric mills, wool | 285 | Paints and allied products |
| 224 | Narrow fabric mills | 286 | Industrial organic chemicals |
| 225 | Knitting mills | 287 | Agricultural chemicals |
| 226 | Textile finishing, except wool | 2873 | Nitrogenous fertilizers |
| 227 | Carpets and rugs | 2874 | Phosphatic fertilizers |
| 228 | Yarn and thread mills | 2875 | Fertilizers, mixing only |
| 229 | Miscellaneous textile goods | 2879 | Agricultural chemicals, n.e.c. |
| 231 | Men’s and boys’ suits and coats | 289 | Miscellaneous chemical products |
| 232 | Men’s and boys’ furnishings | 291 | Petroleum refining |
| 233 | Women’s and misses’ outerwear | 295 | Asphalt paving and roofing materials |
| 234 | Women’s and children’s undergarments | 299 | Misc. petroleum and coal products |
| 235 | Hats, caps, and millinery | 301 | Tires and inner tubes |
| 236 | Girls’ and children’s outerwear | 302 | Rubber and plastics footwear |
| 237 | Fur goods | 305 | Hose & belting & gaskets & packing |
| 238 | Miscellaneous apparel and accessories | 306 | Fabricated rubber products, n.e.c. |
| 239 | Misc. fabricated textile products | 308 | Miscellaneous plastics products, n.e.c. |
| 241 | Logging | 311 | Leather tanning and finishing |
| 242 | Sawmills and planing mills | 313 | Footwear cut stock |
| 243 | Millwork, plywood & structural members | 314 | Footwear, except rubber |
| 244 | Wood containers | 315 | Leather gloves and mittens |
| 245 | Wood buildings and mobile homes | 316 | Luggage |
| 249 | Miscellaneous wood products | 317 | Handbags and personal leather goods |
| 251 | Household furniture | 319 | Leather goods, n.e.c. |
| 252 | Office furniture | 321 | Flat glass |
| 253 | Public building & related furniture | 322 | Glass and glassware, pressed or blown |
| 254 | Partitions and fixtures | 323 | Products of purchased glass |
| 259 | Miscellaneous furniture and fixtures | 324 | Cement, hydraulic |
| 261 | Pulp mills | 325 | Structural clay products |
| 262 | Paper mills | 326 | Pottery and related products |
| 263 | Paperboard mills | 327 | Concrete, gypsum, and plaster products |
| 265 | Paperboard containers and boxes | 328 | Cut stone and stone products |
| 267 | Misc. converted paper products | 329 | Misc. nonmetallic mineral products |
| 271 | Newspapers | 331 | Blast furnace and basic steel products |
| 272 | Periodicals | 332 | Iron and steel foundries |

**A.1 continued**

| SIC | Industry | SIC | Industry |
|---|---|---|---|
| 333 | Primary nonferrous metals | 3671 | Electron tubes |
| 3331 | Primary copper | 3672 | Printed circuit boards |
| 3334 | Primary aluminum | 3674 | Semiconductors and related devices |
| 3339 | Primary nonferrous metals, n.e.c. | 3675 | Electronic capacitors |
| 334 | Secondary nonferrous metals | 3676 | Electronic resistors |
| 335 | Nonferrous rolling and drawing | 3677 | Electronic coils and transformers |
| 336 | Nonferrous foundries (castings) | 3678 | Electronic connectors |
| 339 | Miscellaneous primary metal products | 3679 | Electronic components, n.e.c. |
| 341 | Metal cans and shipping containers | 369 | Misc. electrical equipment & supplies |
| 342 | Cutlery, handtools, and hardware | 371 | Motor vehicles and equipment |
| 343 | Plumbing and heating, except electric | 3711 | Motor vehicles and car bodies |
| 344 | Fabricated structural metal products | 3713 | Truck and bus bodies |
| 345 | Screw machine products, bolts, etc. | 3714 | Motor vehicle parts and accessories |
| 346 | Metal forgings and stampings | 3715 | Truck trailers |
| 347 | Metal services, n.e.c. | 3716 | Motor homes |
| 348 | Ordnance and accessories, n.e.c. | 372 | Aircraft and parts |
| 349 | Misc. fabricated metal products | 373 | Ship and boat building and repairing |
| 351 | Engines and turbines | 374 | Railroad equipment |
| 352 | Farm and garden machinery | 375 | Motorcycles, bicycles, and parts |
| 353 | Construction and related machinery | 376 | Guided missiles, space vehicles, parts |
| 354 | Metalworking machinery | 379 | Miscellaneous transportation equipment |
| 355 | Special industry machinery | 381 | Search and navigation equipment |
| 356 | General industrial machinery | 382 | Measuring and controlling devices |
| 357 | Computer and office equipment | 384 | Medical instruments and supplies |
| 358 | Refrigeration and service machinery | 385 | Ophthalmic goods |
| 359 | Industrial machinery, n.e.c. | 386 | Photographic equipment and supplies |
| 361 | Electric distribution equipment | 387 | Watches, clocks, watchcases & parts |
| 362 | Electrical industrial apparatus | 391 | Jewelry, silverware, and plated ware |
| 363 | Household appliances | 393 | Musical instruments |
| 364 | Electric lighting and wiring equipment | 394 | Toys and sporting goods |
| 365 | Household audio and video equipment | 395 | Pens, pencils, office, & art supplies |
| 366 | Communications equipment | 396 | Costume jewelry and notions |
| 367 | Electronic components and accessories | 399 | Miscellaneous manufactures |

**A.2 Mean Service Life of Assets**

Years

| Asset type | Mean service life |
|---|---|
| **Structures** | |
| Industrial buildings | 31 |
| Office buildings | 36 |
| Other buildings | 37 |
| Other nonresidential nonbuilding structures | 40 |
| **Equipment** | |
| Furniture and fixtures | 14 |
| Fabricated metal products | 18 |
| Steam engines and turbines | 32 |
| Internal combustion engines | 8 |
| Farm tractors | 9 |
| Construction tractors | 8 |
| Agricultural machinery, except tractors | 14 |
| Construction machinery, except tractors | 10 |
| Metalworking machinery used in: | |
| SIC 20 | 20 |
| SIC 21 | 21 |
| SIC 22 | 16 |
| SIC 23 | 15 |
| SIC 24 | 12 |
| SIC 25 | 14 |
| SIC 26 | 16 |
| SIC 27 | 15 |
| SIC 28 | 16 |
| SIC 29 | 22 |
| SIC 30 | 14 |
| SIC 31 | 15 |
| SIC 32 | 19 |
| SIC 33 | 27 |
| SIC 34 | 24 |
| SIC 35 | 25 |
| SIC 36 | 14 |
| SIC 371 | 14 |
| SIC 37 except 371 | 17 |
| SIC 38 | 14 |
| SIC 39 | 17 |
| Special industry machinery, n.e.c. used in: | |
| SIC 20 | 20 |
| SIC 21 | 21 |
| SIC 22 | 16 |
| SIC 23 | 15 |
| SIC 24 | 12 |
| SIC 25 | 14 |
| SIC 26 | 16 |
| SIC 27 | 15 |
| SIC 28 | 16 |
| SIC 29 | 22 |
| SIC 30 | 14 |

**A.2 continued**

Years

| Asset type | Mean service life |
|---|---|
| **Equipment continued** | |
| Special industry machinery, n.e.c. used in: | |
| SIC 31 | 15 |
| SIC 32 | 19 |
| SIC 33 | 27 |
| SIC 34 | 24 |
| SIC 35 | 25 |
| SIC 36 | 14 |
| SIC 371 | 14 |
| SIC 37 except 371 | 17 |
| SIC 38 | 14 |
| SIC 39 | 17 |
| General industrial, including materials handling, equipment used in: | |
| SIC 20 | 20 |
| SIC 21 | 21 |
| SIC 22 | 16 |
| SIC 23 | 15 |
| SIC 24 | 12 |
| SIC 25 | 14 |
| SIC 26 | 16 |
| SIC 27 | 15 |
| SIC 28 | 16 |
| SIC 29 | 22 |
| SIC 30 | 14 |
| SIC 31 | 15 |
| SIC 32 | 19 |
| SIC 33 | 27 |
| SIC 34 | 24 |
| SIC 35 | 25 |
| SIC 36 | 14 |
| SIC 371 | 14 |
| SIC 37 except 371 | 17 |
| SIC 38 | 14 |
| SIC 39 | 17 |
| Office, computing and accounting machinery | |
| (pre-1978) | 8 |
| (post-1977) | 7 |
| Service industry machinery | 10 |
| Electrical transmission, distribution, and industrial apparatus | 33 |
| Communication equipment | 15 |
| Household appliances | 10 |
| Other electrical equipment | 9 |
| Trucks, buses, and truck trailers | 9 |
| Autos | 7 |
| Aircraft | |
| (pre-1960) | 16 |
| (post-1959) | 20 |
| Ships and boats | 27 |
| Railroad equipment | 28 |
| Instruments other than copiers | 12 |
| Photocopy and related equipment | 9 |
| Other nonresidential equipment | 11 |

## A.3 Detailed Sources of Data for FRB Capital Stock Estimates

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Investment Controls by Industry

Controls on a 1987 standard industrial classification (SIC) were assembled for the 164 three- and four-digit industries listed in appendix table A.1. These controls were either taken or constructed from data on equipment investment and structures investment provided by the Bureau of the Census in Censuses and Annual Surveys of Manufactures (COM/ASM), Jack Faucett Associates (JFA), and the Office of Business Analysis (OBA), U.S. Department of Commerce. The data obtained from each source follows.

**1890−1948:** Three-digit 1972-SIC investment series constructed by OBA and corresponding four-digit series constructed by JFA. [1 2] Series used for the FRB capital estimates were converted to a 1987 SIC by FRB, using conversion ratios derived from the 1987 COM.

**1949−71:** Three-digit 1972-SIC investment series constructed from COM/ASM data by OBA and corresponding four-digit series constructed by JFA.[3] Series used for the FRB capital estimates were converted to a 1987 SIC by FRB, using conversion ratios derived from the 1987 COM.

**1972−86:** Three-digit 1972-SIC COM/ASM investment supplied by OBA and corresponding four-digit investment supplied by JFA. Series used for the FRB capital estimates were converted to a 1987 SIC by FRB, using conversion ratios derived from the 1987 COM.[4]

**1987−93:** Three- and four-digit investment series from COM/ASM.

Investment Controls by Asset Type

Control for the 29 asset types listed in appendix table A.2 were obtained or developed from data supplied by the Bureau of Economic Analysis (BEA).

**1890−1928:** Estimates developed by extrapolating BEA investment by asset type for 1929 back, using corresponding asset-type investment from BEA's capital wealth stocks model.[5]

**1929−92:** Published and unpublished data on investment by asset type from the BEA, National Income and Wealth Division.

Price Deflators By Asset Type

Deflators for the 29 asset types listed in appendix table A.2 were obtained or developed from data supplied by BEA.

**1890−1928:** Estimates derived by extrapolating BEA deflators by asset type for 1929 back, using corresponding asset-type deflators derived from BEA's capital wealth stock model.

**1929−92:** Published and unpublished deflators by asset type from the BEA, National Income and Wealth Division.

Mean Service Lives by Asset Type

Except for autos, the mean service lives by asset types shown in appendix table A.2 are from BEA's capital wealth stock model; the mean service life for autos is based on a study by JFA for the U.S. Department of Transportation .[6]

**A.3 continued**

Capital Flows Tables

Benchmark capital flows tables (CFTs) for 1963, 1967, 1972, 1977, and 1982 were derived from those developed by the BEA, Interindustry Division, in conjunction with its benchmark input–output tables; computerized versions of the 1963, 1967, and 1972 tables on a 1972-SIC were supplied by OBA.[7]

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1. The four-digit estimates supplied by JFA for 1890–1986 were constructed, as a by-product of contract work performed for OBA, to be generally consistent with the three-digit estimates supplied by OBA. The 1890–1986 controls for the four-digit industries displayed in appendix table A.1 were obtained by splitting 3-digit controls with four-digit proportions developed from the JFA estimates.

2. The methods used to create the pre–1949 OBA estimates follows that outlined in "Capital Stock Estimates for Input-Output Industries: Methods and Data," Bulletin 2034, pp. 7–13 (U.S. Department of Labor, Bureau of Labor Statistics, 1979).

3. During the period 1949–53, the ASM reported investment only at the 3–digit level and, during 1951–57, it did not split total investment by industry into equipment and structures. The procedures used by OBA to fill these gaps and make the requisite SIC conversions follow those discussed in "Capital Stock Estimates for Input-Output Industries: Methods and Data," Bulletin 2034, pp. 6–9 (U.S. Department of Labor, Bureau of Labor Statistics, 1979).

4. The 1890–1986 data provided by JFA and OBA were compared at the three-digit level. Where discrepancies occurred, the choice between the competing series join 1958–86 was settled by comparing them both to the latest available versions published in the COM or ASM, while the choice between competing series prior to 1958 was based on judgement by FRB staff. Occasionally, both series were replaced with published COM/ASM data. For SIC 371, however, COM/ASM data for equipment investment prior to 1972 were rejected in favor of OBA estimates which were driven by adjusting the pre-1972 COM/ASM data to include investment in short-lived special tools and dies for new vehicle models that is included in the post-1971 COM/ASM data.

5. See *Fixed Reproducible Tangible Wealth in the United States, 1925–89* (U.S. Department of Commerce, Bureau of Economic Analysis, January 1993)

6. See *Capital Stock Measures for Transportation: A Study in Five Volumes* (U.S. Department of Transportation, December 1974); BEA does not use a service life when estimating auto stocks held by businesses.

7. BEA's 1982 CFT is unpublished; its published CFTs are described in the following sources: The 1977 table, in "New Structures and Equipment by Using Industries, 1977," *Survey of Current Business* (November 1985); 26–35; the 1972 table, in U.S. Department of Commerce, Bureau of Economic Analysis, *New Structures and Equipment by Using Industries, 1972: Detailed Estimates and Methodology*, BEA staff paper no. 35 (Washington, D.C. U.S. Government Printing Office, 1980); and the 1963 and 1967 tables, in U.S. Department of Commerce, Bureau of Economic Analysis, *Interindustry Transactions in New Structures and Equipment 1963 and 1967*, 2 volumes (Springfield, VA; National Technical Information Service, 1975).