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Adverse Selection and Pay Equity

Jay Stewart

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Jay Stewart*
Bureau of Labor Statistics
Office of Employment Research
and Program Development
Room 4945 - Postal Square Building
2 Massachusetts Ave., NE
Washington, D.C. 20212-0001
ph. (202) 606-7376

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Abstract

Previous studies have shown that adverse incentives can lead firms to weaken the link between pay and performance, which leads to more equal pay across workers. In these models, high-powered incentives encourage workers to neglect some aspects of their job, or to sabotage their co-workers' efforts. This paper offers another explanation for the weak link between pay and performance. When labor contracts are contests, the Nash equilibrium often pools workers. This implies that incentives are weaker than would be the case if firms could observe workers types before contracting and offer each type their respective optimal contests.

I. Introduction

A fundamental result of the principal-agent literature is that pay will be linked to performance when it is difficult for the principal to monitor the agent's actions. But in the real world, we often find this link to be weak. Several authors (Lazear [1989], Holmstrom and Milgrom [1991], and Garvey and Swan [1992]) have shown that weaker incentives may be optimal when performance pay leads to undesired behavior on the part of workers.

In Lazear's [1989] model workers compete in a tournament and can sabotage the efforts of their opponents. Stronger incentives (larger winning prizes) encourage more sabotage, which reduces the firm's total output. The firm could eliminate sabotage completely by paying workers a salary that does not depend on performance. But this would eliminate legitimate production activities as well. Hence, the firm reduces the winning prize relative to the prize in a world without sabotage.

Holmstrom and Milgrom [1991] show that straight salaries are optimal when the worker's job includes some activities that cannot be easily monitored. For example, piece rates are not optimal if quality is hard to monitor, or if equipment maintenance is important; workers have an incentive to reduce quality and neglect equipment in order to increase output.[1]

In Garvey and Swan [1992], there are three sources of moral hazard: two from workers and one from management. Workers control the amount of effort they exert on the job, and the degree to which they help their co-workers (cooperation is simply negative sabotage). Obviously, firm owners like to see greater effort and more

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[1] In an earlier paper, Barron and Loewenstein [1986] show that when performance measures are noisy and workers are risk averse, optimal contracts weaken the link between pay and performance. Here, the weak link between pay and performance arises because the imperfect performance evaluations impose added risk, not because of any adverse incentives. However, they do note that high ability workers sort out of jobs where the link between pay and performance is weak.

cooperation. The third source of moral hazard arises because the firm's owner hires a manager to administer the workers' contract. The manager values a harmonious working environment, which gives him an incentive to "give away the firm." As in Lazear's model, a pure tournament gives workers an incentive to exert effort, but not to cooperate. Garvey and Swan propose a hybrid compensation scheme that is part tournament, part managerial discretion. Since managers benefit from a more harmonious work environment, they tend to weaken the link between pay and performance by increasing the base pay. This also encourages more cooperation from workers. What keeps the manager from "giving away the firm" is that he incurs a large loss if the firm goes bankrupt. The firm owner manipulates the probability that the firm goes bankrupt by varying the level of debt incurred by the firm. The greater the level of debt, the greater is the threat of bankruptcy, which curbs the manager's tendency to overpay workers. Garvey and Swan show that it is optimal for the firm's owner *not* to pay the manager based on the firm's profits, but rather vary the level of debt so that the manager administers the workers' contract optimally.

The common thread in these papers is that principals weaken the link between pay and performance to mitigate adverse incentives. However, these papers ignore another possible source of what Garvey and Swan call pay compression: worker heterogeneity. In what follows, I show that worker heterogeneity can lead principals to weaken the link between pay and performance, even if there are no adverse incentives.

I consider a labor market where workers are heterogeneous with respect to ability (measured as cost of effort: more able workers have a lower cost of effort), and jobs have a single dimension. In this market a pooling equilibrium is common, and the equilibrium contest caters to low ability workers. This implies that the difference between the winning and losing prize is smaller in the equilibrium pooling contest than would be the case if firms could observe workers' abilities before contracting. Pay compression results naturally in equilibrium even though all parties involved (firms, low ability agents, and high ability agents) would opt for greater incentives in the absence of adverse selection.

II. The Model

Firms produce output according to a production function that is additively separable across workers and linear in effort. In particular, worker *i* produces output (q_i) according to $q_i = a_i + \theta + e_i$, where a_i is an action (effort) taken by the worker, θ is a random shock that is common to all workers in the firm, and e_i is an individual specific random shock. The θ term may be thought of as factors that affect the profitability of the firm, while the e_i term represents factors that affect the individual worker's performance. The random variable θ is distributed according to $H(\cdot)$ over $[\theta_0, \theta_1]$, and the e_i are i.i.d. according to $F(\cdot)$ on the interval $[e_0,e_1]$. Also $E(e_i) = E(\theta) = 0$.[2] A firm's total output is given by $Q = \sum_i q_i$. Firms maximize $E(\Pi) = vQ - \sum_i r_i$, where v is the price of output. In addition, firms can freely enter and exit the market so that $E(\Pi) = 0$ in equilibrium.

The labor market is composed of two types of workers, high (H) ability and low (L) ability, who differ in their cost of effort. All workers are risk averse and maximize utility of wealth net of the cost of exerting effort. If worker *i* is a type J worker (J = H,L), this is given by: $U_J = V(r_i) - C_J(a_i)$, where $a \in [a_0,a_1]$, and r_i is the income received by worker i. Utility of wealth, $V(\cdot)$, is increasing, concave, and twice differentiable, while the cost of effort, $C_J(\cdot)$, is increasing, convex, and twice differentiable. High ability workers

[2] The assumption that $E(e_i) = E(\theta) = 0$ is harmless, since contracts can be adjusted to account for any values of $E(e)$ and $E(\theta)$.

have a lower cost of effort. Specifically, $C_L(a_0) = C_H(a_0) = 0$ and $C'_L(a) > C'_H(a)$ for all $a \in [a_0,a_1]$. Firms cannot distinguish between high and low ability workers before hiring.

Nalebuff and Stiglitz [1983] and Green and Stokey [1983] have shown that when the variance of θ is sufficiently large, contests dominate individualistic contracts because they impose less risk on workers. Under individualistic contracts such as piece rates, a high variance of θ results in high variance in compensation. But in a contest θ does not affect relative standings, and hence, does not affect compensation. Throughout the rest of the paper, I assume that the variance of θ is large enough so that contests dominate individualistic contracts.[3]

In a contest, two workers, *i* and *j*, compete for a prize that is determined by the firm. The worker who produces the most wins so that worker *i* wins if $q_i > q_j$. This implies $a_i + \theta + e_i > a_j + \theta + e_j$, which reduces to $e_j - e_i < a_i - a_j$. If $(e_j - e_i)$ is distributed according to $G(\cdot)$, then Prob(worker *i* wins) = $G(a_i-a_j)$. And since e_i and e_j are i.i.d., the density $G'(\cdot) = g(\cdot)$ is symmetric around $(e_j-e_i) = 0$. I also assume that the density $g(\cdot)$ has a single mode, and that it reaches its strict global maximum at $(e_j-e_i) = 0$.[4]

Letting $(Y+x)$ and $(Y-x)$ denote payments to the winner and loser, a contest τ is defined by the pair (x,Y). Note that total prize money in the contest is $2Y$, and $x > 0$ is the amount paid to the winner over and above Y. Hereafter, I refer to x as the prize.[5]

Now consider a situation in which both types compete in the same contest. Let $\pi \in [0,1]$ be the proportion of H workers in the labor market, and a_H and a_L denote the

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3 Alternatively, one may assume that firms can only determine which worker produced the most output, but not how much was actually produced. This assumption also rules out individualistic contracts.

4 This implies that when worker *i* exerts more effort than worker *j* $(a_i>a_j)$, worker *i*'s probability of winning increases with effort, but at a decreasing rate. Thus *i*'s marginal gain to increasing effort is greatest when levels of effort are the same, and decreases monotonically as he increases effort. Since $G(a_i-a_j) = 1-G(a_j-a_i)$, a similar argument applies for worker *j*.

5 I follow the terminology of Nalebuff and Stiglitz [1983] in referring to x as the prize. Lazear and Rosen [1981] refer to (Y+x) and Y-x) as the winning and losing prizes.

optimal level of effort of H and L workers. (For the remainder of the paper, I will index workers by their types.) Then the expected utility of a type J worker in a contest becomes

$$U_J(\tau,\pi) = \text{Max}\,\pi\{V(Y+x)G(a_J - a_H) + V(Y+x)[1 - G(a_J - a_H)]\} + \\ (1-\pi)\{V(Y+x)G(a_J - a_L) + V(Y+x)[1 - G(a_J - a_L)]\} - C_J(a_J),$$

which reduces to

$$(1) \quad U_J(\tau,\pi) = \text{Max}\,\{\Delta(x,Y)[\pi G(a_J - a_H) + (1-\pi)G(a_J - a_L)] + V(Y-x) - C_J(a_J)\},$$

where $\Delta(x,Y) = V(Y+x) - V(Y-x)$, and the term in square brackets is the probability of winning. Note that if τ earns zero expected profit then $Y = v[\pi a_H + (1-\pi)a_L]$.

The reaction function for a type J worker is

$$(2) \quad \Delta(x,Y)[\pi g(a_J - a_H) + (1-\pi)g(a_J - a_L)] = C_J'(a_J)$$

Assuming all workers of the same type choose the same action, (2) becomes

$$(3H) \quad \Delta(x,Y)[\pi g(0) + (1-\pi)g(a_H - a_L)] = C_J'(a_J)$$

$$(3L) \quad \Delta(x,Y)[\pi g(a_L - a_H) + (1-\pi)g(0)] = C_J'(a_J)$$

The optimal levels of effort are denoted as $a_H(\tau;\pi)$ and $a_H(\tau;\pi)$.

There are two special cases worth noting. If $\pi = 1$, then all workers are high ability workers. Similarly, if $\pi = 0$, then all workers are low ability. In these cases, Equation (3) reduces to

$$(4) \quad \Delta(x, Y)g(0) = C_J'(a_J), \qquad \text{for } J = H, L.$$

There is no adverse selection, so that firms maximize the following program:

$$(5) \quad \text{Max } U_J(\tau, J) \quad \text{s.t. (i)} \quad \Delta(x, Y)g(0) = C_J'(a_J)$$
$$\text{(ii)} \quad va_J - Y = 0$$

where J as the second argument indicates the opponent's type.[6] The solution to (5) is the "pure moral hazard" contest for a type J worker, and is denoted as $\bar{\tau}^J$ (J=H,L).

If firms observed workers' abilities before contracting, they could offer the "pure moral hazard" contest for each type. Instead, they compete for workers in a screening market similar to the one described by Rothschild and Stiglitz [1976].[7]

Workers and firms play the game in three stages. First, each firm offers a contest and agrees to hire any worker who accepts. I will refer to the set of all contests offered as the market tournament, T. In equilibrium, the market tournament either separates workers, $T^S = \{\tau^H, \tau^L\}$, or pools workers, $T^P = \{\tau^P\}$. Second, each worker determines the utility maximizing level of effort in each contest, and enters the contest that provides the highest level of utility. Finally, workers compete against each other in their respective contests, and firms award prizes to the winners and losers.

Since a worker's expected payoff in a contest depends on his opponent's ability, his choice of a contest depends on his expectations about the decisions of other workers. I

[6] This notation is convenient when discussing out of equilibrium contests later in the paper.

[7] Papers by Wilson [1977] and Riley [1979] examine similar markets, but use different equilibrium concepts. In contrast to Rothschild and Stiglitz, who use a Nash equilibrium concept, Wilson uses an "anticipatory equilibrium" concept, while Riley uses "reactive equilibrium." Both anticipatory and reactive equilibrium concepts attempt to account for the reactions of firms to defections from equilibrium.

Other, more recent, articles by Baron and Besanko [1987], Laffont and Tirole [1986], McAfee and McMillan [1987], Picard [1987], Riordan and Sappington [1987], and Stewart [1994] examine screening markets when firms offer incentive contracts.

assume that workers' expectations regarding their prospective opponent in a given contest coincide with those of the offering firm. For example, an L worker expects that he will compete against L workers in a contest designed for L workers and against H workers in a contest designed for H workers. In effect, workers do not account for possible defections by other workers. This assumption is important when determining the response of workers to the offering of non-equilibrium contests, because whether a non-equilibrium contest upsets the equilibrium set of contests depends on workers' actions, which in turn, depend on their expectations.

Note that this dependent utility feature means that the "single crossing property" of screening models does not apply to the full mapping of preferences. As a result, it is possible for a pooling Nash equilibrium to exist.[8] Since the focus of this paper is on pay compression, I do not give a full description of conditions under which each type of equilibrium exists. Instead, I describe the pooling equilibrium, show that it exists under a wide range of conditions, and show that a pooling equilibrium implies pay compression.

[8] Recall from Rothschild and Stiglitz [1976] that the single crossing property implies that a pooling Nash equilibrium can never exist because it is always possible to "skim the cream" (attact the better workers) from any pooling contract by offering a contact in a small neighborhood around the pooling contract. (See Cooper [1984] for a description of the single crossing property and its implications.

III. Equilibrium Pooling Contests and Pay Compression

Lazear and Rosen [1981] have shown that firms cannot choose x and Y so that a pooling contest is efficient even when workers are risk neutral. Consequently, the equilibrium pooling contest trades off the welfare of L workers against the welfare of H workers. If pay were based on individual performance, the equilibrium contract would maximize the utility of high ability workers.[9] But when the contract is a contest the equilibrium contract maximizes the utility of low ability workers. Consider the following maximization program:

$$(6) \quad \underset{\tau^P}{\text{Max}}\ U_L(a_L;\tau,\pi) \quad \text{s.t.}$$

$$\text{(i)} \quad \Delta(x,Y)\left[\pi g(0)+(1-\pi)g(a_H-a_L)\right]=C_H'(a_H)$$

$$\text{(ii)} \quad \Delta(x,Y)\left[\pi g(a_L-a_H)+(1-\pi)g(0)\right]=C_L'(a_L)$$

$$\text{(iii)} \quad v\left[\pi a_H+(1-\pi)a_L\right]-Y\geq 0.$$

The solution to equation (6), which is denoted as $\hat{\tau}^P$, maximizes the utility of low ability workers, subject to incentive compatibility constraints (the reaction functions of H and L workers), and a profitability condition.

Proposition 1: Let T* = {τ*} be an equilibrium pooling tournament. Then $\tau^* = \hat{\tau}^P$.

A sketch of the proof provides the intuition behind Proposition 1 (see Appendix 1 for a formal proof). Suppose $\tau^* \neq \hat{\tau}^P$. Then there exists a contest τ^0, in the constraint set of equation (6), that gives L workers a higher level of utility, and H workers a lower

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[9] See Wilson [1977].

level of utility. If τ^0 is offered, all L workers enter. The desertion of L workers from τ^* reduces the utility of H workers in τ^* by a fixed amount. However, the reduction in the utility of H workers who move from τ^* to τ^0 can be made arbitrarily small. Thus, if τ^0 is properly constructed, H workers prefer to compete in τ^0 against both types, rather than in τ^* against only H workers. This implies that τ^* cannot be an equilibrium contest unless it maximizes the utility of L workers.[10]

In standard screening models low ability agents impose a negative externality, because they are subsidized by high ability agents. Firms minimize this externality by designing the contract to maximize the utility of high ability agents. Once the contract is offered, an agent's payoff does not depend on the actions of other agents. This is not true in a contest. Since the presence of low ability workers increases the probability that a high ability worker wins, the externality is positive *once the contest is offered*. Hence, firms maximize the utility of low ability workers to assure their participation.

Figure 1 illustrates the pooling equilibrium. The curves labeled $\Pi(\tau;H)=0$ and $\Pi(\tau;P)=0$ indicate the set of contests that earn zero expected profit when accepted by H workers and by a pool of both types. The $U_J(\tau;P)=\hat{U}_J^P$ indifference curve passes through $\hat{\tau}^P$ and indicates the set of pooling contests that generate the same utility as $\hat{\tau}^P$ for type J (=L,H) workers. The $U_H(\tau;H)=\hat{U}_H^P$ indifference curve indicates the set of *non-pooling* contests (competing against other H workers) that generate the same level of utility as $\hat{\tau}^P$ for type H workers. As one would expect, utility increases as Y increases and as x decreases. And since L workers have a higher cost of effort, they have steeper indifference curves. Note that effort is not constant along the indifference curves and the zero profit curves; at each point on both curves workers exert the optimal level of effort given (x,Y).

[10] This also implies that, even if an efficient pooling contest exists, it is not an equilibrium contest unless it also solves equation (8).

A pooling Nash equilibrium exists if two conditions are satisfied. First, L workers must weakly prefer $\hat{\tau}^{P}$ to $\overline{\tau}^{L}$. If this condition does not hold, it is possible for some firm to upset $\hat{\tau}^{P}$ by offering a profitable non-pooling contest that attracts L workers from the pooling contest.[11]

The second condition requires that no defecting firm be able to skim the cream from $\hat{\tau}^{P}$ by offering a profitable non-pooling contest that attracts H workers. To illustrate, suppose that some firm offers a contest, τ^{B}, in region **B** of Figure 1.[12] Such a contest attracts H workers because $U_H(\tau^B;H) > U_H(\hat{\tau}^P;P)$. It also earns a positive profit as long as no L workers enter. To determine whether L workers enter τ^{B}, one must compare the utility from competing against H workers in τ^{B} to the utility from competing against L workers in $\hat{\tau}^{P}$. If $U_L(\tau^B;H) > U_L(\hat{\tau}^P;L)$ for every τ^{B} in **B**, then no firm can skim the cream and a pooling equilibrium exists (as long as the first condition outlined above also holds).

Unfortunately, it is impossible to determine analytically when a pooling equilibrium exists. However, it is possible to specify functional forms and solve the model numerically. Appendix 2 presents evidence from these numerical examples showing that a pooling equilibrium exists under a wide range of conditions, which supports the following conjecture:

Conjecture 1: A pooling Nash equilibrium exists when the proportion of high ability workers is small and the cost of effort for high ability workers is low.

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[11] To avoid clutter, I have omitted this condition from Figure 1. Although it is important, the second condition is more interesting.

[12] Region B is bounded by the $U_H(\tau;H) = \hat{U}_H^P$ indifference curve and the $\Pi(\tau;H)$ isoprofit curve.

Pay Compression

Given that the equilibrium pooling contest maximizes the utility of L workers, it is straightforward to establish that a pooling equilibrium implies wage compression. That is, if firms could observe workers' types and offer the pure moral hazard contest (described in equation (5)) for high and low ability workers, the prize in both contests would be larger than in the equilibrium pooling contest. Figure 2 illustrates this result using the simulations described in Appendix 2.

Figure 2 shows the prize in the equilibrium pooling contest as a function the proportion of H workers in the labor market (π) for different sets of parameters of the cost functions.[13] When there are no H workers in the market ($\pi = 0$), the contest is simply the pure moral hazard contest (for L workers) described in equation (5).

There are two effects that determine whether $\hat{x}^P$ increases or decreases as π increases. As the proportion of H workers increases, the probability that an L worker wins the contest decreases. This tends to decrease $\hat{x}^P$, the prize in the equilibrium pooling contest. But as π increases, expected output (and hence, Y) increases, which increases total payments to workers. When π becomes sufficiently large, the optimal prize increases because the increase in Y more than offsets the increase in x. As can be seen in Figure 2, for values where a pooling equilibrium exists the prize decreases. For some values of the cost parameter on L workers ($\beta = 4,5,6$), $\hat{x}^P$ begins to increase, but never to the point where it exceeds the level at $\pi = 0$. Hence, the prize in the equilibrium pooling contest is always less than the prize in the pure moral hazard contest for L workers. Since the optimal prize in the pure moral hazard contest increases with ability, $\hat{x}^P < \overline{x}^L$ implies that $\hat{x}^P < \overline{x}^H$. Hence pay in the pooling equilibrium contest is compressed relative to the pure moral hazard contests of both types. This supports Conjecture 2.

[13] In all examples in Figure 2, α (the cost parameter for high ability workers) equals 2 and β (the cost parameter for low ability workers) ranges between 3 and 7.

Conjecture 2: $\hat{x}^P < \overline{x}^H, \overline{x}^L$. The prize in the equilibrium pooling contest is smaller than the prizes in the pure moral hazard contests for high and low ability workers.

The simulations indicate that a pooling equilibrium results in pay compression relative to the pure moral hazard contests characterized in equation (5). But if adverse selection is a problem, one might be interested in comparing the pooling equilibrium to the separating equilibrium. It is easily shown that if the prize in the pooling contest is smaller than those in the pure moral hazard contests, then it is also smaller than the prizes in the separating contests.

Figure 3 illustrates the equilibrium separating tournament, where $\hat{\tau}^L = (\hat{x}^L, \hat{Y}^L)$ and $\hat{\tau}^H = (\hat{x}^H, \hat{Y}^H)$ denote the contests designed for L and H workers. The contest offered to L workers is the moral hazard contest described in Equation (5) (i.e.,$\hat{\tau}^L = \overline{\tau}^L$). The contest designed for H workers ($\hat{\tau}^H$) is determined by the intersection of the $U_L(\tau;H) = \overline{U}_L$ indifference curve and the Π(τ;H)=0 isoprofit curve.

Since $\hat{\tau}^L = \overline{\tau}^L$, the previous discussion implies that $\hat{x}^P < \hat{x}^L$. It remains to be shown that $\hat{x}^P < \hat{x}^H$. Since $\hat{x}^P = \overline{x}^H$, it is sufficient to show that $\overline{x}^H \leq \hat{x}^H$. Now suppose, to the contrary, that $\overline{x}^H > \hat{x}^H$ (i.e., that $\overline{\tau}^H$ lies to the right of the intersection of the U (τ;H)=U indifference curve and the Π(τ;H)=0 isoprofit curve in Figure 3). Then if L workers are indifferent between $\hat{\tau}^L$ (= $\overline{\tau}^L$) and $\hat{\tau}^H$ they strictly prefer $\hat{\tau}^L$ to $\overline{\tau}^H$, which implies that $\{\overline{\tau}^H, \overline{\tau}^L\}$ also separates workers. And since H workers prefer $\overline{\tau}^H$ to $\hat{\tau}^H$, $\{\overline{\tau}^H, \overline{\tau}^L\}$ dominates $\{\hat{\tau}^H, \overline{\tau}^L\}$ and $\{\hat{\tau}^H, \overline{\tau}^L\}$ cannot be an equilibrium.[14] Hence $\hat{x}^P < \overline{x}^H \leq \hat{x}^H$, and the pooling equilibrium results in pay compression relative to the separating equilibrium.

[14] This outcome is possible in screening models where the contracts are incentive contracts (see Stewart [1994]).

IV. Summary and Conclusions

If contracts are contests, the Nash equilibrium pools workers in many cases. Although low ability workers impose a negative externality *ex ante*, firms must insure their participation because they impose a positive externality once the contest is offered. That is, high ability workers will not enter the pooling contest unless low ability workers also enter. As a result, the equilibrium pooling contest maximizes the utility of low ability workers. Pay compression results because low ability workers, being less likely to win, prefer a smaller spread between the winning and losing prizes.

This provides an alternative explanation for the weak link frequently observed between pay and performance. Previous work has emphasized adverse incentives created by jobs that have more than one task (Holmstrom and Milgrom [1991]) or require cooperation among workers (Lazear [1989] and Garvey and Swan [1992]). I have shown that even when jobs have a single dimension, pay compression can arise naturally when the market is characterized by adverse selection.

Appendix 1: Proof of Proposition 1

Proposition 1: Let $T^* = \{\tau^*\}$ be an equilibrium pooling tournament. Then $\tau^* = \hat{\tau}^P$.

Proof: I show that if $\tau^* \neq \hat{\tau}^P$, there exists a contest τ^0, in the constraint set of Equation (6), that yields L workers a higher level of utility, and H workers a lower level of utility. If τ^0 is offered, all L workers enter. The desertion of L workers from τ^* reduces the utility of H workers in τ^* by a fixed amount. However, the reduction in the utility of H workers from τ^* to τ^0 can be made arbitrarily small. Thus, if τ^0 is properly constructed, H workers prefer to compete in τ^0against both types, rather τ^* against only H workers.

Note that for any contest τ^P, $U_H(\tau^P, P) > U_H(\tau^P, H)$ and $U_L(\tau^P, L) > U_L(\tau^P, P)$. The first inequality states that H workers would rather compete in a pool of both types than against only other H workers, while the second states that L workers prefer playing against other L workers to playing in a pool.

Suppose that $\tau^* \neq \hat{\tau}^P$. Then there exists a profitable contest, τ^0, such that $U_L(\tau^0, P) > U_L(\tau^*, P)$. For H workers, either $U_H(\tau^0, P) < U_H(\tau^*, P)$ or $U_H(\tau^0, P) \geq U_H(\tau^*, P)$.

Suppose $U_H(\tau^0, P) \geq U_H(\tau^*, P)$. Then $\hat{\tau}^P$ cannot be an equilibrium contest since τ^0 offers L workers a higher level of utility and H workers are no worse off. Hence $U_H(\tau^0, P) < U_H(\tau^*, P)$. Since $U_H(\tau, P)$ and $U_L(\tau, P)$ are continuous in x and Y, I may choose x and Y so that $U_L(\tau^0, P) > U_L(\tau^*, P)$ and $U_H(\tau^0, P) = U_H(\tau^*, P) - \varepsilon$ for any $\varepsilon > 0$.[15] If I choose x and Y so that $\varepsilon < U_H(\tau^*, P) - U_H(\tau^*, H)$, H workers prefer to play in τ^0 against a pool rather than in τ^* against only H workers. Since τ^0 is profitable, this contradicts the assumption that τ^* is an equilibrium contest. Hence $\tau^* = \hat{\tau}^P$. **QED**

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[15] This condition is stronger than necessary. An H worker enters τ^0 as long as $U_H(\tau^0; L) > U_H(\hat{\tau}^P; H)$ (*i.e.*, each H worker considers only the effect of her own action).

Appendix 2: Existence of a Pooling Equilibrium

Each entry in Table A1 gives the largest value of π (multiplied by 100) for which a pooling equilibrium exists. An "N" indicates a pooling equilibrium does not exist for any values of π, while entries followed by a "+" sign indicate that a pooling equilibrium exists when π is greater than or equal to the entry.

Inspection of table A1 reveals that for 106 out of the 144 sets of cost parameters, a pooling equilibrium exists for some values of π. However, the most striking feature of table 1 is that, except for three cases (indicated by the "+" sign), a pooling equilibrium exists when π is sufficiently *small*.

For values of π for which a pooling equilibrium does not exist, either a separating equilibrium exists or no equilibrium exists. A separating equilibrium exists when no pooling contest dominates the set of separating contests. Such a contest may exist even though $\hat{\tau}^P$ is dominated by the separating tournament described above.[16] Consider a pooling contest, τ', that maximizes the utility of H workers subject to the constraint that L workers receive at least as much utility as in the L contest ($\hat{\tau}^L$). When the proportion of H workers (π) is sufficiently large, H workers prefer τ' to $\hat{\tau}^H$. And since L workers weakly prefer τ' to $\hat{\tau}^L$, τ' upsets separating equilibrium. Hence, no equilibrium exists for large values of π (except as noted in Table A1), and a separating equilibrium exists for intermediate values of π.

[16] Recall that $\hat{\tau}^P$ maximizes the utility of *low* ability workers.

The following functional forms were used for the simulations:

$$G(u) = \begin{cases} \frac{1}{2} + \gamma u + \frac{1}{2}\gamma^2 u^2 & \text{for } u \in [-\frac{1}{\gamma}, 0] \\ \frac{1}{2} - \gamma u + \frac{1}{2}\gamma^2 u^2 & \text{for } u \in [0, -\frac{1}{\gamma}] \end{cases} \qquad \gamma = \tfrac{1}{2}$$

$$V(W) = b - be^{-AW} \qquad A = \tfrac{1}{2}, 1, 2, 3$$

$$C_H(a) = \alpha a^2$$

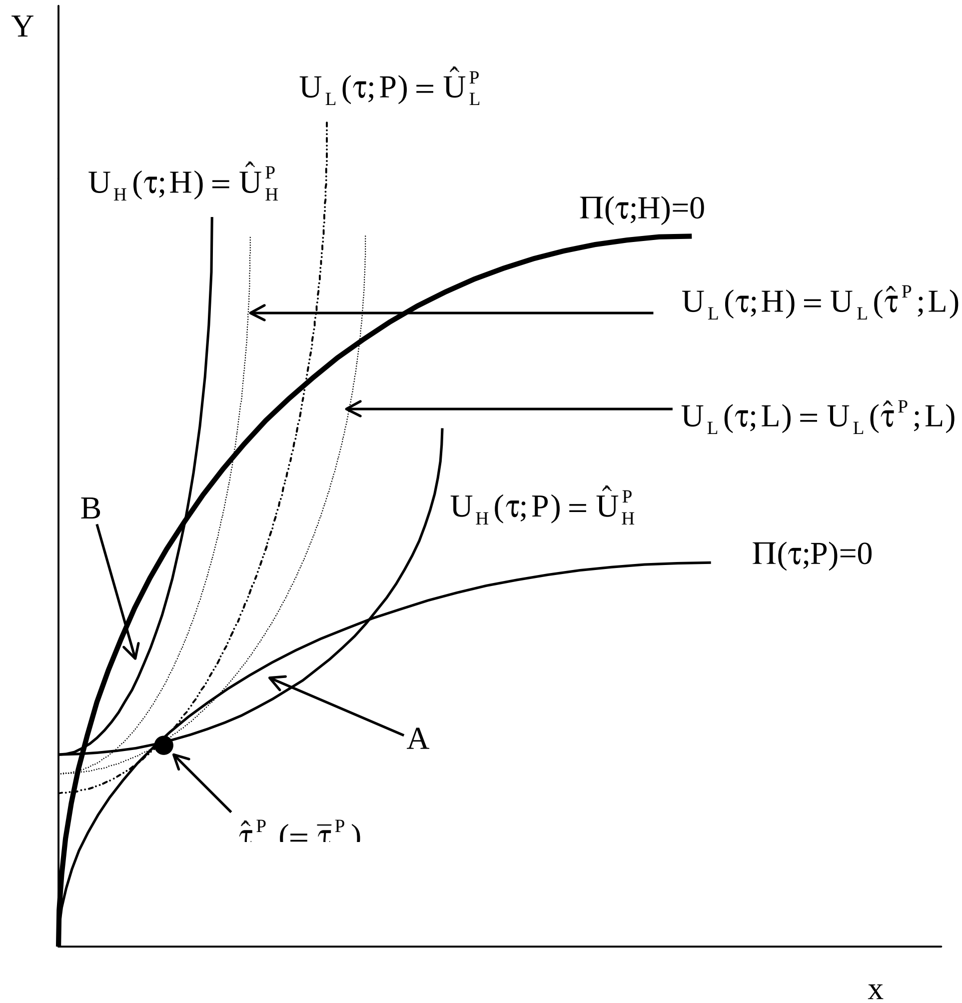
$$\alpha < \beta$$

$$C_L(a) = \beta a^2$$

Table A1

	β = 3		β = 4		β = 5		β = 6		β = 7		β = 8		β = 9		β = 10	
α = 2	70	62	63	59	59	57	56	55	54	53	53	52	51	51	50	50
	37	23	44	31	45	35	45	36	45	37	45	38	45	38	44	39
α = 3			49	42	48	47	47	47	46	47	44	46	43	46	42	45
			17	7	28	18	33	24	35	27	37	29	37	30	38	31
α = 4					16	15	27	29	30	34	31	36	31	36	30	37
					N	2	14	9	21	14	24	18	27	20	28	22
α = 5							N	2	N	12	6	18	10	21	12	24
							N	N	5	N	10	8	15	10	18	14
α = 6									N	N	N	N	N	2	N	8
									N	N	N	N	4	N	6	5
α = 7											N	N	N	N	N	N
											N	N	N	N	N	N
α = 8	A	=	.5	1									N	N	N	N
			2	3									*95*	N	N	N
α = 9															N	N
															93	*96*

Figure 1



Y
$U_L(\tau;P) = \hat{U}_L^P$
$U_H(\tau;H) = \hat{U}_H^P$
$\Pi(\tau;H)=0$
$U_L(\tau;H) = U_L(\hat{\tau}^P;L)$
$U_L(\tau;L) = U_L(\hat{\tau}^P;L)$
B
$U_H(\tau;P) = \hat{U}_H^P$
$\Pi(\tau;P)=0$
A
$\hat{\tau}^P\ (= \bar{\tau}^P)$
x

Figure 2

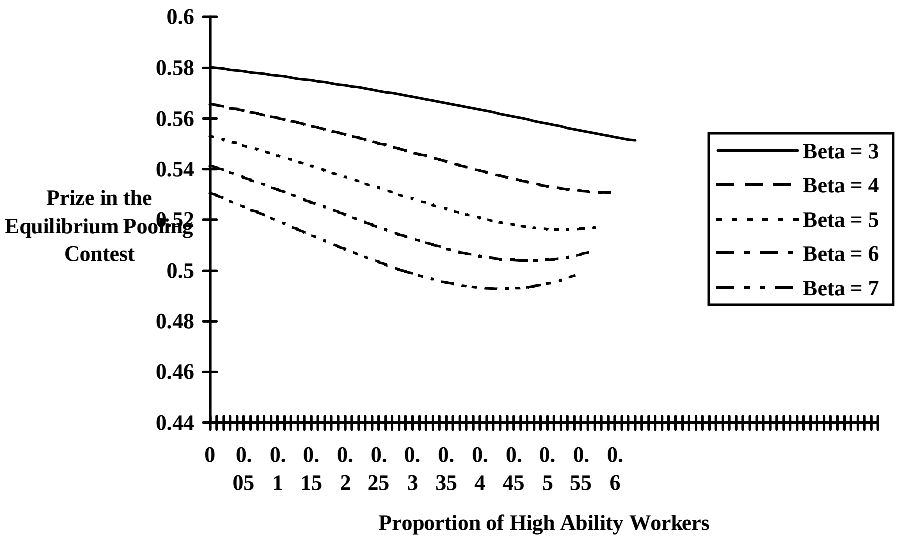
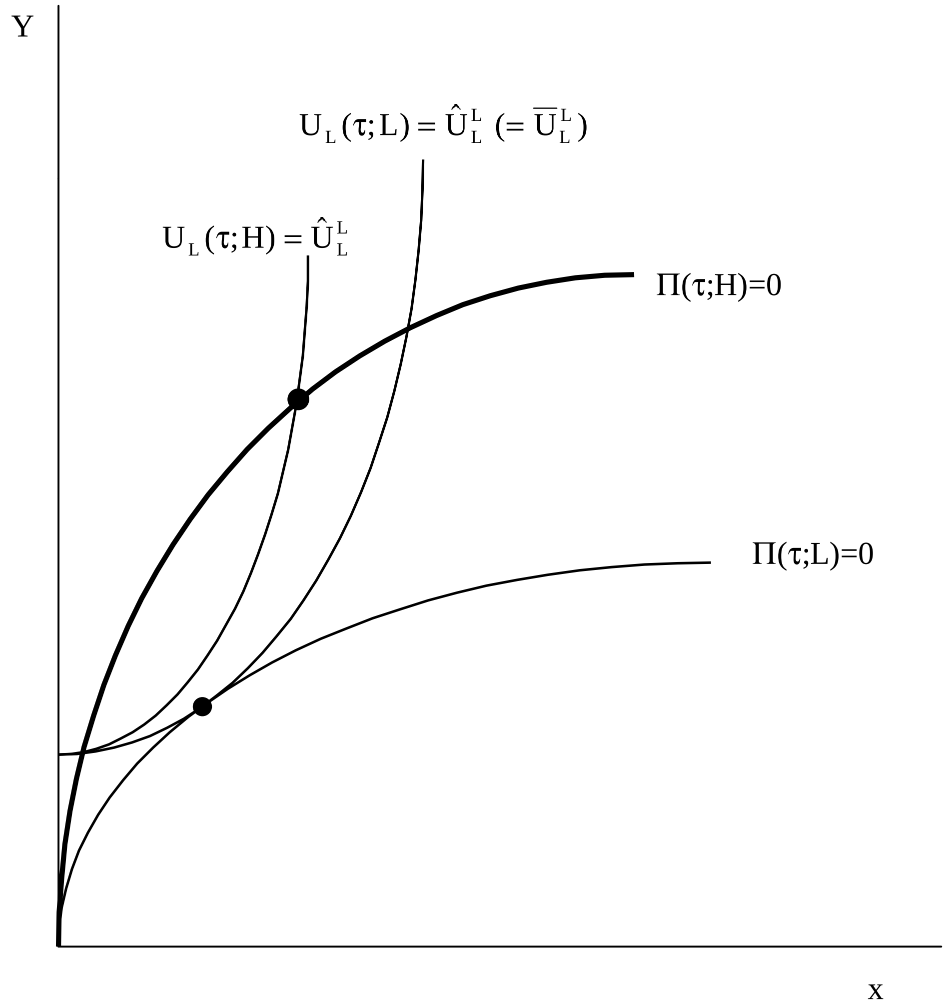

0.6
0.58
0.56
0.54
0.52
0.5
0.48
0.46
0.44
Prize in the Equilibrium Pooling Contest
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
Proportion of High Ability Workers
Beta = 3
Beta = 4
Beta = 5
Beta = 6
Beta = 7

Figure 3



Y
$U_L(\tau;L) = \hat{U}_L^L \ (= \overline{U}_L^L)$
$U_L(\tau;H) = \hat{U}_L^L$
$\Pi(\tau;H)=0$
$\Pi(\tau;L)=0$
x

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