What Mathematics Knowledge is Needed for Teaching Mathematics?

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$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline \end{array}$$

What is Necessary Mathematics Knowledge for Teaching?

1. <u>Examine teaching</u>, and identify the mathematical work that teaching entails

2. Analyze <u>what</u> mathematical knowledge –– topics and skills –– is needed to do that work

3. Analyze also the qualities of that knowledge –– <u>how</u> it must be understood and known to be serviceable for the work

Evaluating and interpreting common incorrect student answers

$$
\begin{array}{r}
35 \\
\times\ 25 \\
\hline
175 \\
70 \\
\hline
245
\end{array}
$$

What is the misunderstanding here?

Explaining mathematical concepts and procedures

$$
\begin{array}{r}
35 \\
\times\ 25 \\
\hline
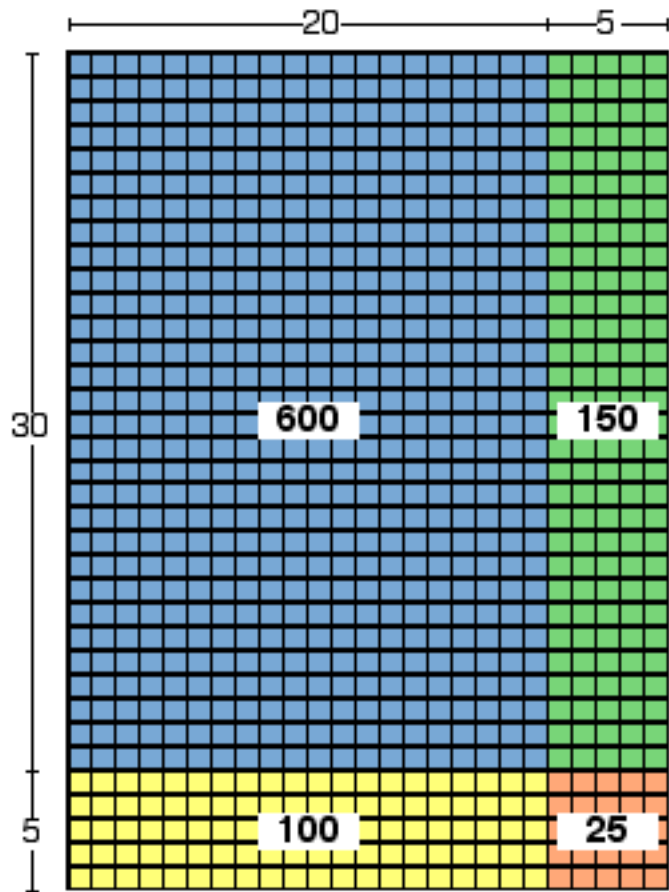175 \\
70 \quad\longleftarrow \\
\hline
875
\end{array}
$$

Why do we "move over" before writing 70?

Evaluating and appraising different methods

Student A	Student B	Student C
35	35	35
×25	×25	×25
1 25	175	25
+75	+700	150
875	875	100
		+600
		875

Which of these students is using a method that could be used to multiply any two whole numbers?

# Using representations to model and reason about procedures

Qualities of Mathematics Knowledge for Effective Instruction

- Respectful of the integrity of the discipline
- Able to be extended and opened up for learners — "unpacked"
- Justified, reasoned
- Connected within and across domains, building on earlier ideas and anticipating more advanced topics
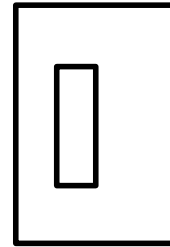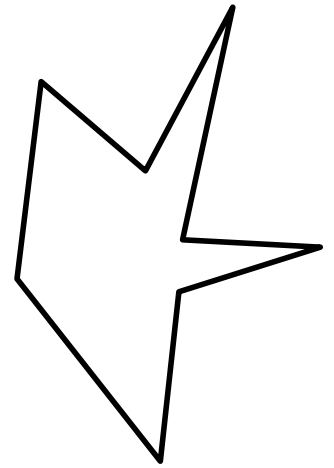- Organized psychologically as well as logically
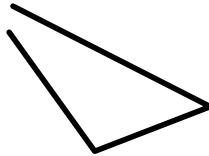
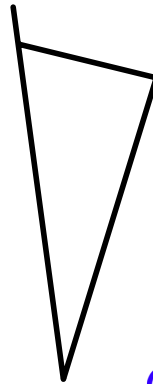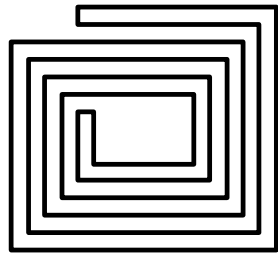# Using definitions
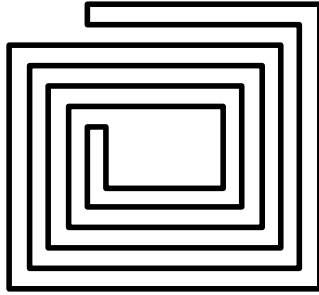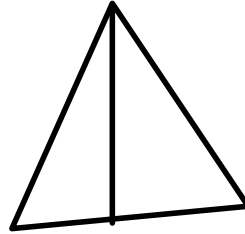
(a)   (b)   (c)   (d)

(e)   (f)   (g)

Which shapes are polygons?
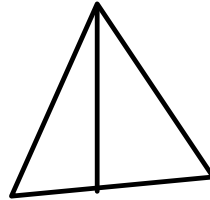
Textbook definition: A closed flat two-dimensional shape whose sides are formed by line segments.

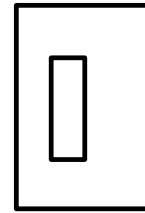# Definition: A simple closed plane curve formed by straight line segments.

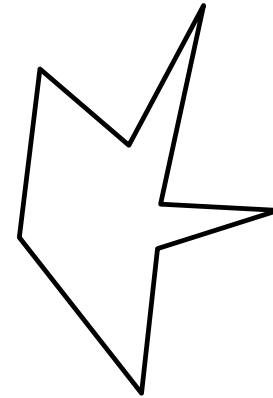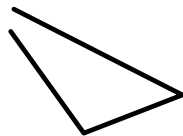But — what's a mathematically acceptable definition of "polygon" that is usable by fifth graders?
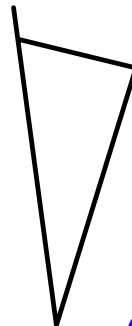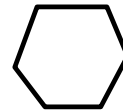

(a)   (b)   (c)   (d)

(e)   (f)   (g)

Looking Back: What Mathematics Do Teachers Have to <u>Do</u>?

Looking Back: What Mathematics Do Teachers Have to **Do**?

- Design mathematically accurate explanations that are comprehensible and useful for students

- Use mathematically appropriate and comprehensible definitions

- Represent ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process

- Interpret and make mathematical and pedagogical judgments about students' questions, solutions, problems,and insights (both predictable and unusual)

- Be able to respond productively to students' mathematical questions and curiosities

- Make judgments about the mathematical quality of instructional materials and modify as necessary

- Be able to pose good mathematical questions and problems that are productive for students' learning

- Assess students' mathematics learning and take next steps

Looking Back: What Mathematics Do Teachers Need to **Know** to Do Those Things?

Looking Back: What Mathematics Do Teachers Need to **Know** to Do Those Things?

In general:

- Topics and ideas that are fundamental to the school curriculum–– and beyond

- Tools and skills for reasoning about mathematical claims, ideas, representations, and solutions; and sensibility about what constitutes adequate proof

- Fluency and care with mathematical language and notation

- Familiarity with applications of mathematics

Looking Back: What Mathematics Do (Elementary) Teachers Need to Know?

What provides mathematical leverage?

- Concepts of number and place value notation
- Operations
- Number theory and number systems
- Common algorithms and how and why they work
- Concepts and tools of algebra
- Geometric concepts and reasoning
- Concepts and tools of statistics and probability

- Representing and connecting representations (e.g., symbols, graphs, geometric models)
- Mathematical language and definitions
- Mathematical reasoning and justification
- Good sense about mathematical precision
- Mathematical curiosity and interest

Conclusion:
Knowing Mathematics for Teaching — Three Core Principles

1. Teachers need to know the same things and be able to do the same things as any adult, and <u>much more</u>.

2. Knowledge needed for teaching is <u>different</u> from what is needed for other occupations or professions where mathematics is used (e.g., physics, mathematics, carpentry, tailoring, business).

3. Knowledge needed for teaching must be <u>usable</u> for the mathematical work that <u>teachers</u> have to do.