

# Stochastic Approach to Study Cirrus Structure Using ARM



## Millimeter Wavelength Radar Observations



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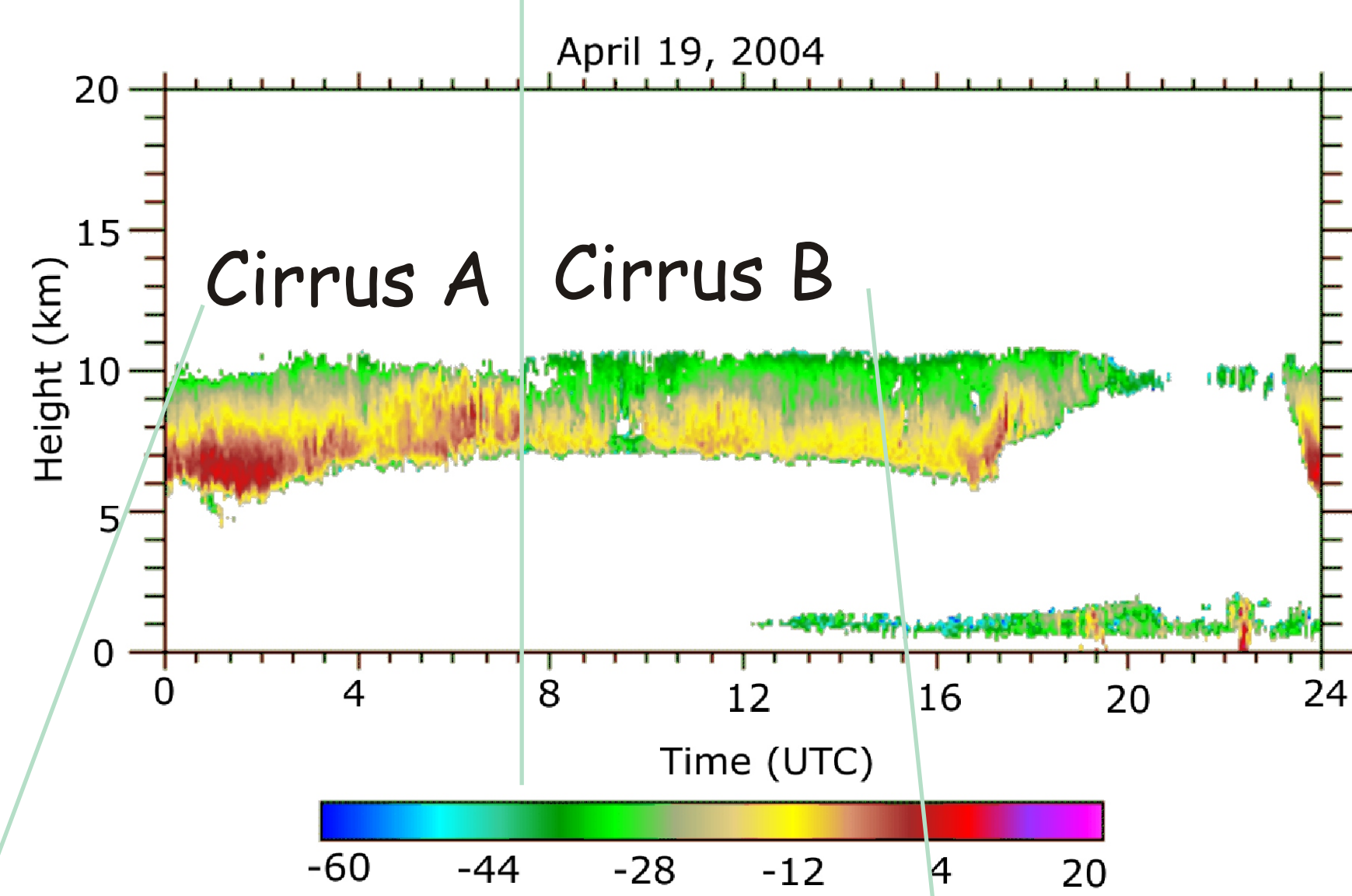
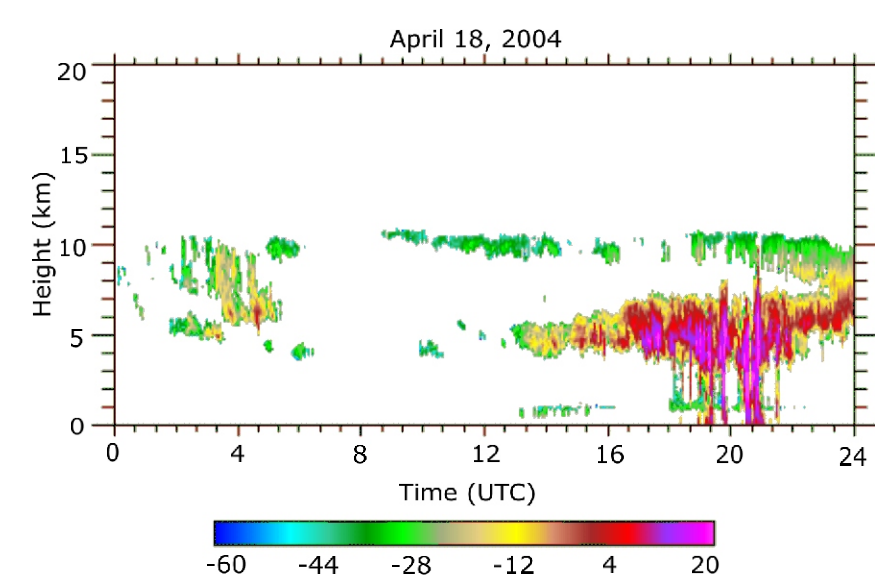
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### Motivation

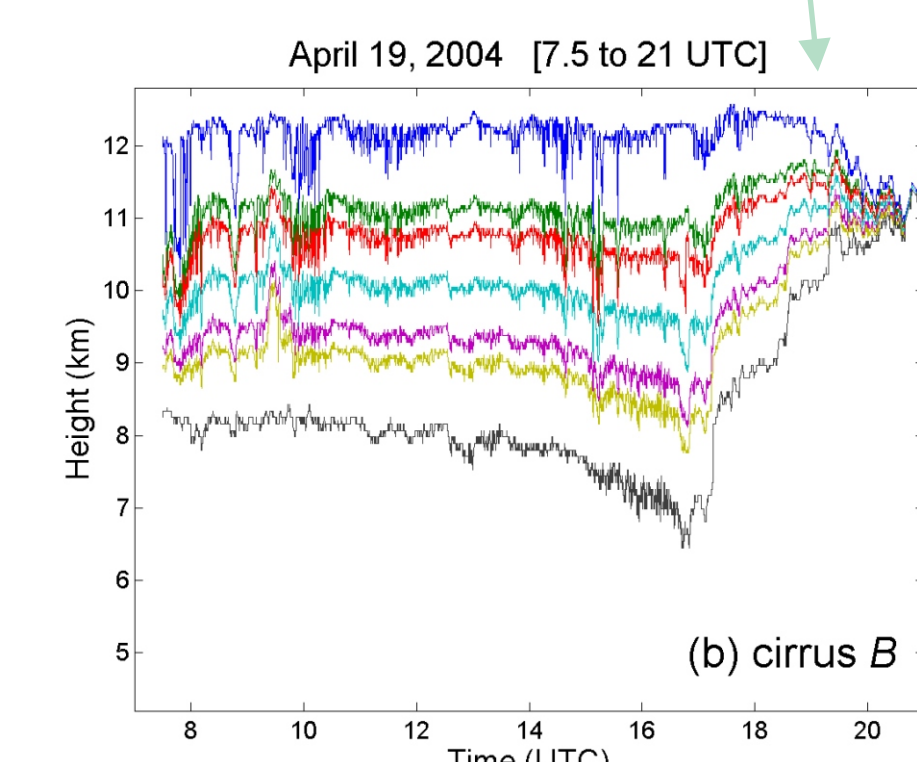
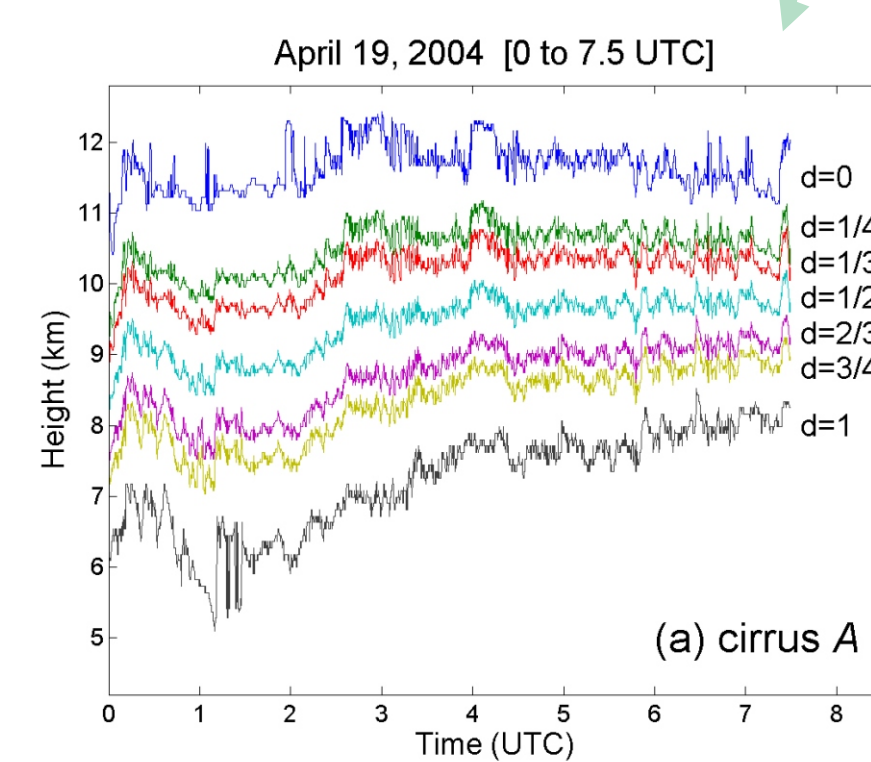
Knowledge of the local dynamical properties within cirrus and the radiative response due to these properties is still lacking. The interactions between the synoptic conditions in which cirrus form and maintain, and the small-scale of individual generating cells are still not quantitatively understood.

### Observations:

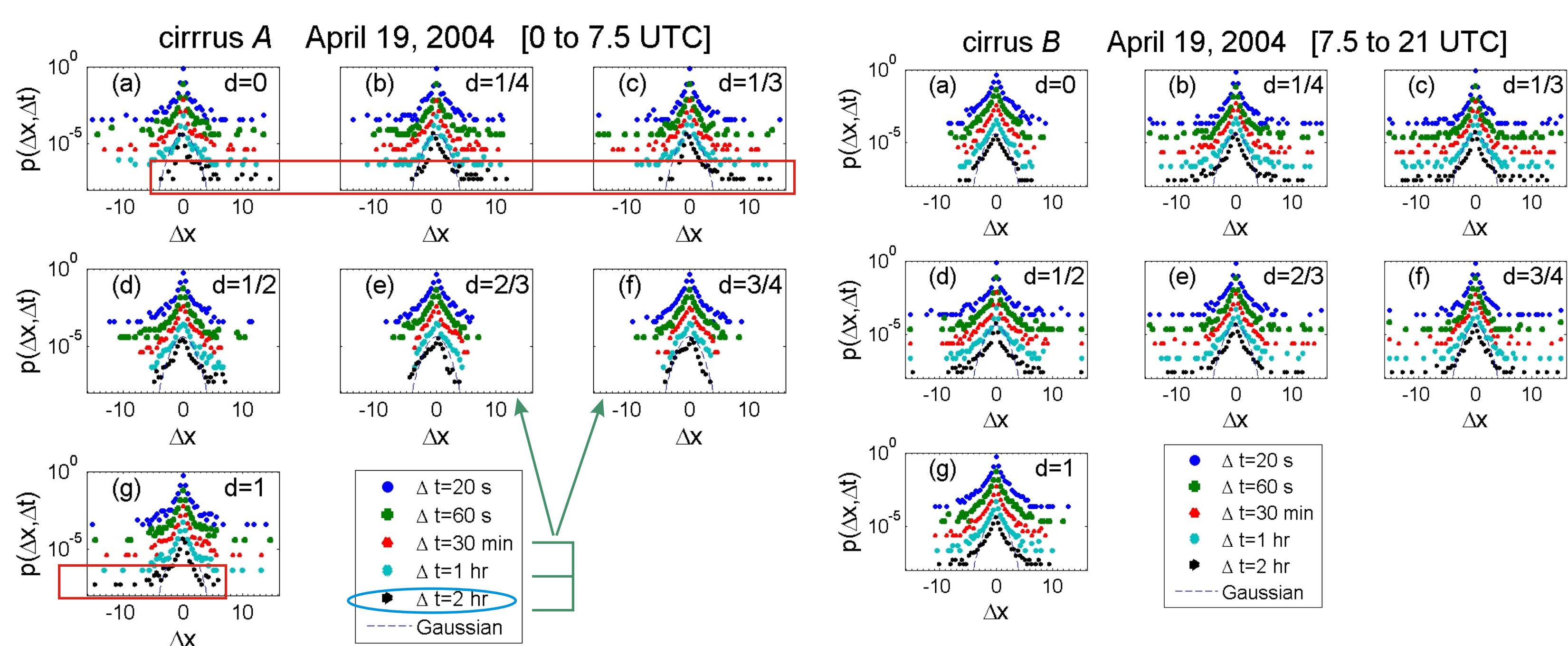
MMCR measurements at ARM SGP site



### Consider time series of radar reflectivity at depth d into cloud relative to cloud top



### Probability distribution functions (pdfs) of reflectivities for various delay times Δt



### Method of Analysis

It is known that two equations govern the dynamics of a system. The Fokker-Planck equation for the probability distribution functions

$$\frac{\partial p(x,t)}{\partial t} = \left[ -\frac{\partial}{\partial x} D_1(x,t) + \frac{\partial^2}{\partial x^2} D_2(x,t) \right] p(x,t)$$

And the Langevin equation for the state of the system

$$\frac{dx}{dt} = h(x,t) + g(x,t)W(t)$$

All slow, large scale processes Stochastic approximation to the fast nonlinear processes

Because we infer a stochastic model from discrete empirical data, the Ito calculus is more relevant to describe the dynamics of the system. The Kramers-Moyal expansion of the drift  $D_1$  and diffusion  $D_2$  coefficients is:

$$D_1(x,t) = h(x,t) + \frac{\partial g(x,t)}{\partial x} g(x,t)$$

$$D_2(x,t) = g^2(x,t)$$

The functional dependence of the drift  $D_1$  and diffusion  $D_2$  coefficients can be estimated directly from the moments of conditional probability distribution:

$$D_n(x,t) = \frac{1}{n!} \lim_{\Delta t \rightarrow 0} M_n(x,t,\Delta t),$$

Connects the probabilities at larger scales with all smaller, embedded scales

$$M_n(x,t,\Delta t) = \frac{1}{\Delta t} \int_{-\infty}^{+\infty} (x' - x)^n p(x',t - \Delta t|x,t) dx'$$

Therefore, the Fokker-Planck equation provides a method for obtaining the drift  $D_1$  and diffusion  $D_2$  coefficients directly from observations.

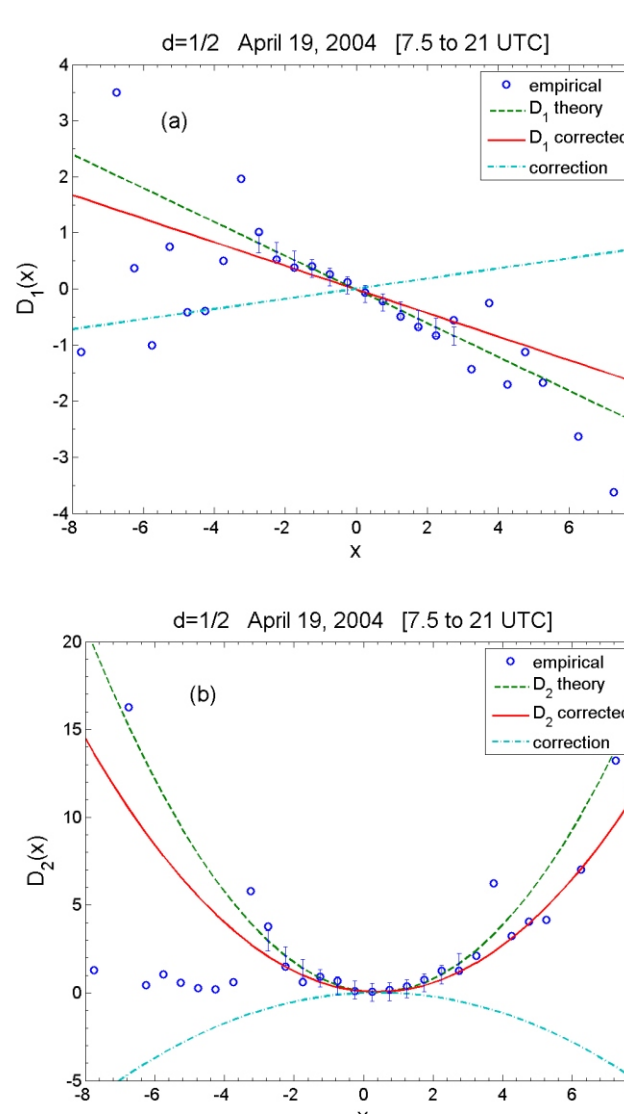
### Deterministic drift Effective drift

$$D_1^{eff}(x,t) = D_1^{d.d.}(x,t) + D_1^{n.i.}(x,t)$$

$$D_1^{d.d.}(x,t) = h(x,t)$$

$$D_1^{n.i.}(x,t) = \frac{\partial g(x,t)}{\partial x} g(x,t)$$

### Effective, deterministic, and noise-induced drift



$$\bar{D}_1^{eff}(x) = a_1x + a_0$$

$$\bar{D}_1^{n.i.}(x) = b_2x + b_1/2$$

$$\bar{D}_1^{d.d.}(x) = \bar{D}_1^{eff}(x) - \bar{D}_1^{n.i.}(x)$$

$$\bar{D}_1^{d.d.}(x) = h_1x + h_0$$

$$\bar{D}_2(x) = b_2x^2 + b_1x + b_0$$

### We demonstrate that

The time-dependent, non Gaussian pdfs can be produced by a linear system (linear drift  $D_1$ ) with multiplicative noise (quadratic  $D_2$ ).

The former is tentatively identified with larger-scale forcing and the latter with in-cloud circulation and turbulence.

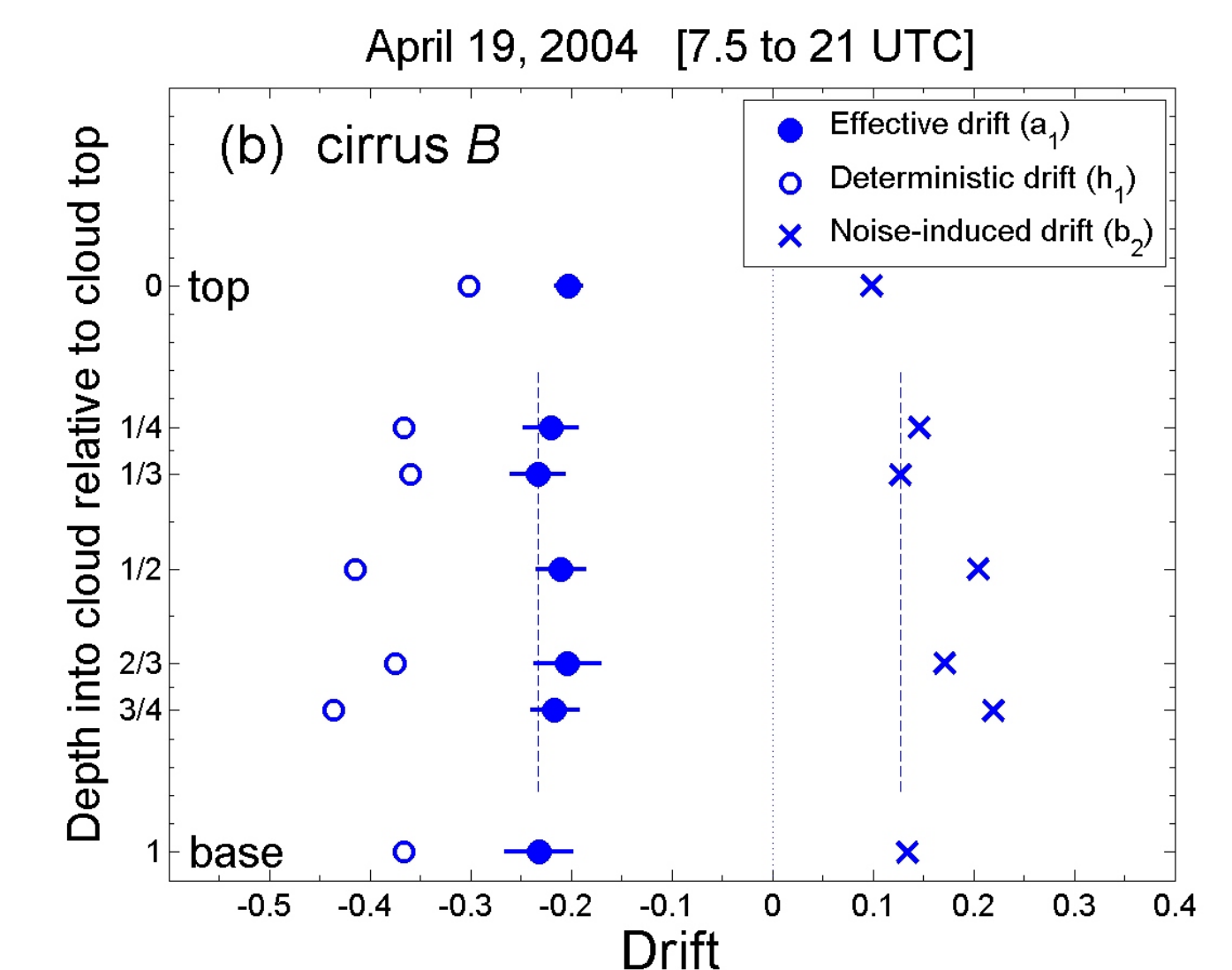
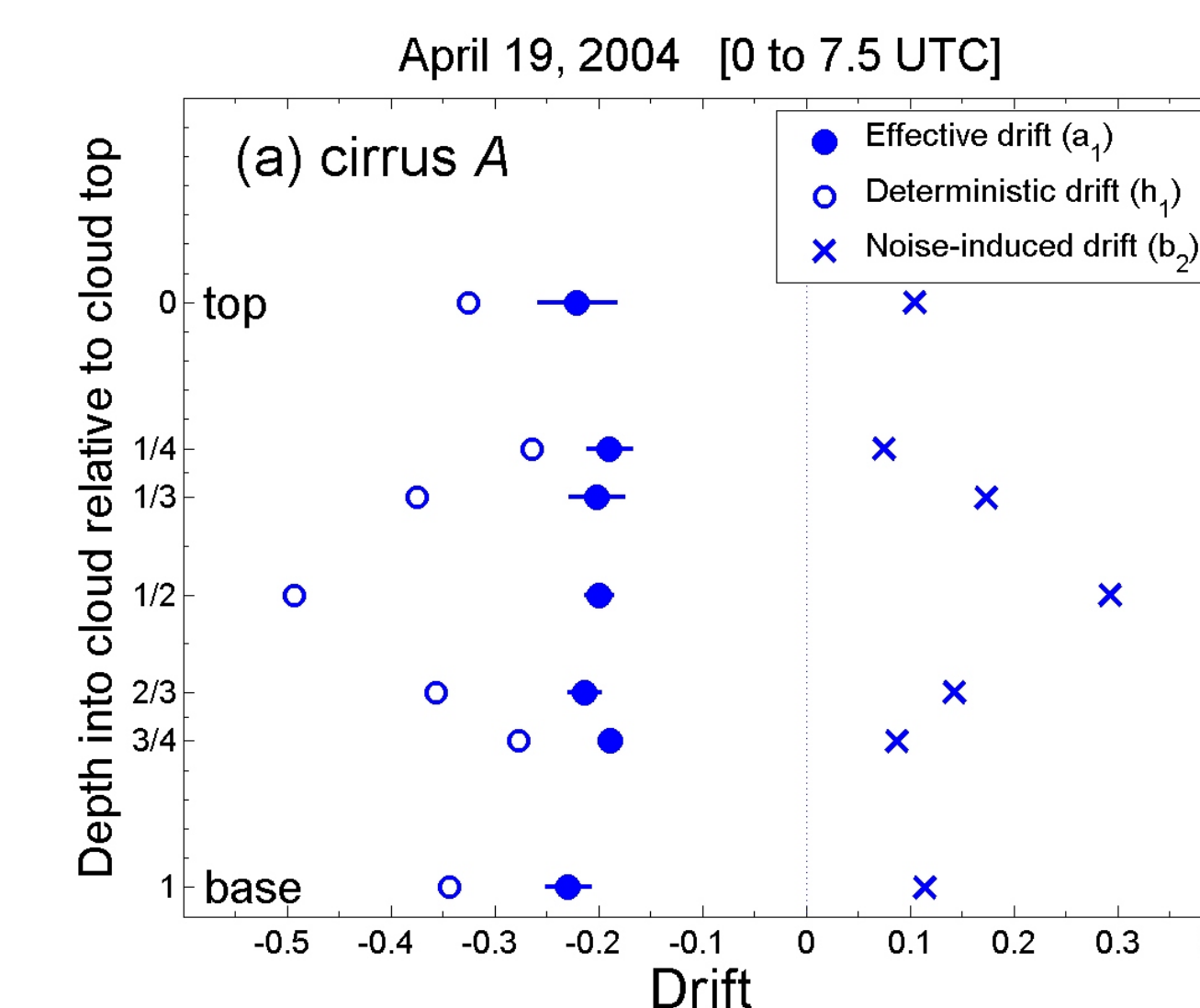
Obtained quadratic dependence of  $D_2$  leads to the existence of noise-induced drift term.

The noise-induced drift presents the stochasticity in the dynamics of the system, the influence of the small-scale noise on the slow, large-scale deterministic processes.

### We find that at the scale of the generating cells

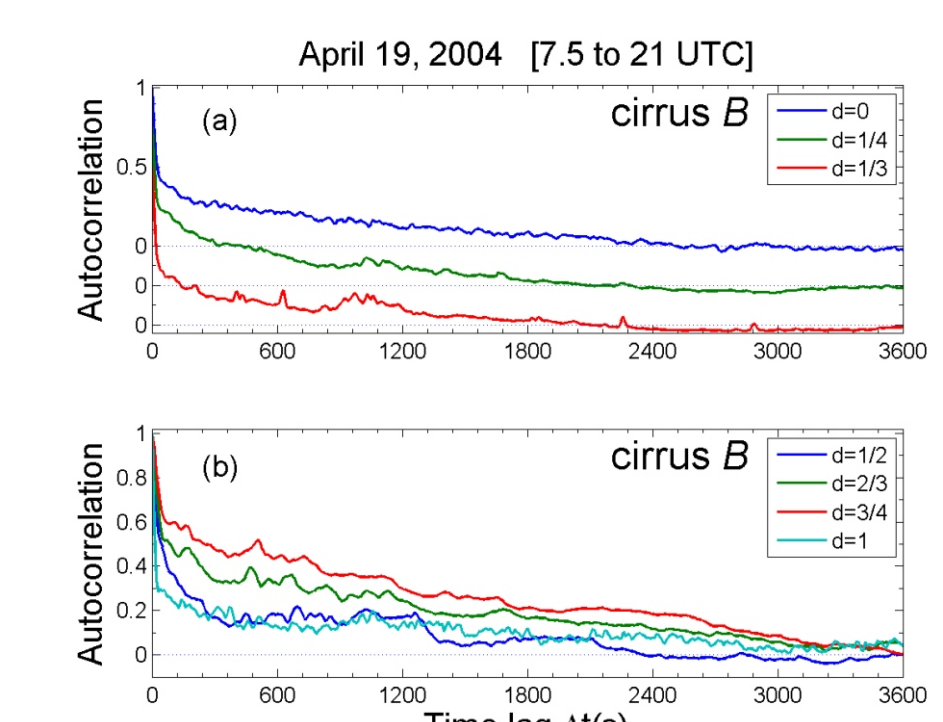
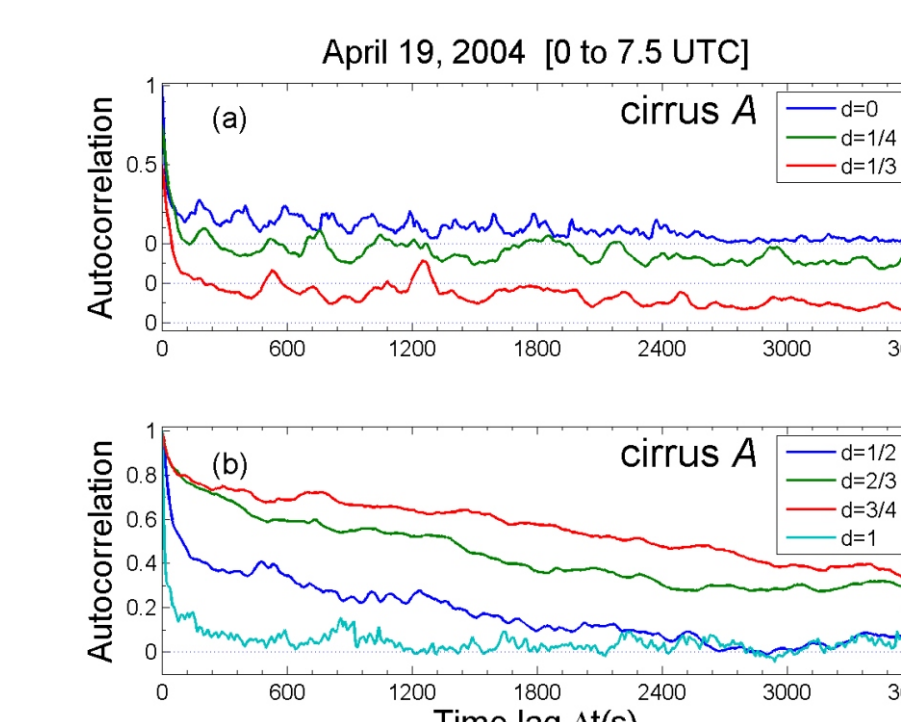
The effective drift has larger values at the top and the base because it is in a closer contact with the large-scale conditions.

The larger values of the noise-induced drift obtained for the middle 50% of both cirrus A and B are in accordance with what is expected from the ice crystal growth and deposition region.



We find correlations with period  $\omega_1=200s$  at the top and with period  $\omega_2=600s$  at the upper 33% of cirrus A. These scales are suggestive of phenomena that occur on larger scales simultaneously with the processes that are taking place in the generating cells.

We find no periodicity in cirrus B.



### Acknowledgments

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### We find that

2hr scale (~100 km) cirrus A

The pdfs are time-dependent and non-Gaussian for both cirrus A&B. The pdfs for delay times  $\Delta t=2hr$  for cirrus A exhibit behavior that is consistent with structure of cirrus based on aircraft in situ measurements [Heymsfield and Miloshevich, 1995], and with results from ground-based Raman lidar studies [Comstock et al., 2004].

The upper 33% of cirrus A shows behavior that is in accordance with what is expected from a region of nucleation with long deposition times as found in model studies [Khvorostyanov and Sassen, 1998].

The pdfs ( $\Delta t=2hr$ ) at the base ( $d=1$ ) are suggestive of sublimation.