ANALYTIC EXPRESSIONS FOR THE LIGHT-SCATTERING CROSS SECTION AND ÅNGSTRÖM EXPONENT OF AN AEROSOL

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BACKGROUND

For an aerosol consisting of spherical particles with size distribution of number concentration $dN(r)/dr$ and real index of refraction m (thus no absorption), the light-scattering coefficient σ_{sp} for wavelength λ is defined by

$$\sigma_{sp} = \int \pi r^2 Q_{sp}(r/\lambda, m) \frac{dN(r)}{dr} dr,$$

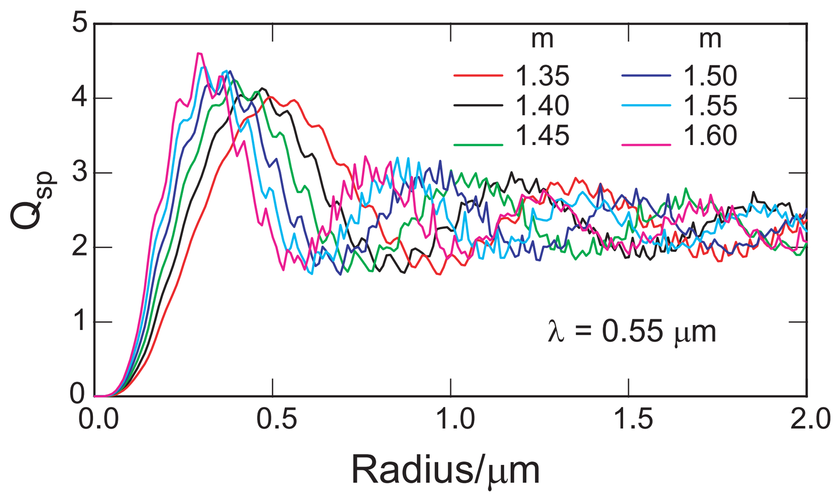
where Q_{sp}, the scattering efficiency of a particle, depends on the ratio of the radius r to λ and on m.

- Q_{sp} varies as $(r/\lambda)^4$ when $r << \lambda$
- Q_{sp} approaches the value 2 when $r >> \lambda$

The wavelength dependence of σ_{sp} is characterized by the Ångström exponent $å$, defined by

$$å \equiv \frac{-\partial \ln \sigma_{sp}}{\partial \ln \lambda} = \frac{-\lambda}{\sigma_{sp}} \int \pi r^2 \frac{\partial Q_{sp}(r/\lambda, m)}{\partial \lambda} \frac{dN(r)}{dr} dr.$$

- $å = 4$ when scattering is from particles with $r << \lambda$
- $å = 0$ when scattering is from particles with $r >> \lambda$



Q_{sp} is determined by Mie theory. It is a complicated function of r/λ and m, with many "wiggles" on many scales, and its computation, though straightforward, is time intensive.

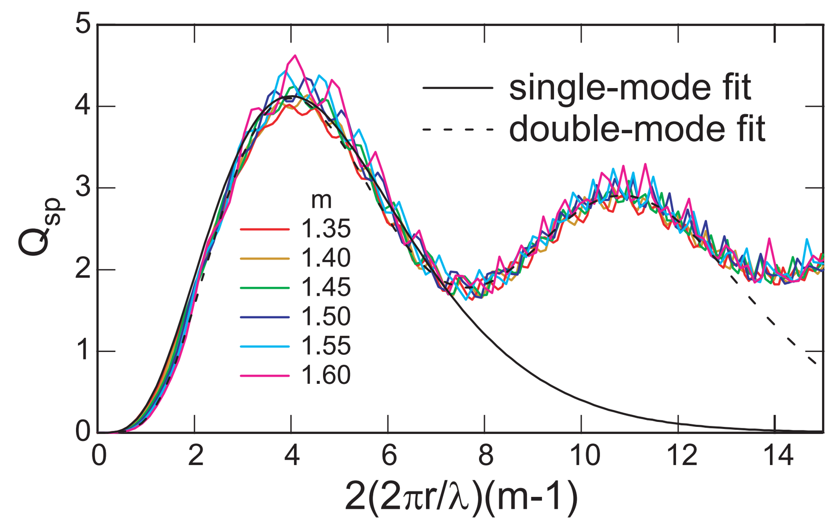
A simple expression for Q_{sp} would simplify calculation of many aerosol optical properties and would provide insight into the dependence of these properties on wavelength, index of refraction, RH, and the characteristic size of the aerosol.

APPROXIMATION of Q_{sp}

Much of the variability in Q_{sp} can be eliminated by plotting it against the quantity

$$z = 2(2\pi r/\lambda)(m - 1).$$

- For $\lambda = 0.50$ μm and $m = 1.40$, $z \approx 10(r/\mu m)$.
- Wiggles are unimportant, as they are smoothed out by the spread in the size distribution.



In many situations the majority of the scattering is from the range $0.1 < r/\mu m < 0.7$ ($1 < z < 7$), over which Q_{sp} can be accurately approximated by a gamma function:

$$Q_\Gamma = 0.85\, z^4 \exp(-z).$$

For a size distribution which is a gamma function with effective radius r_{eff} and effective variance v_{eff}, this approximation yields an analytic expression for the scattering cross section

$$\sigma_{sp} = 0.85\left(\pi r_{eff}^2 N_{tot}\right) v_{eff}^2 \frac{\Gamma\left(\frac{1}{v_{eff}}+4\right)}{\Gamma\left(\frac{1}{v_{eff}}-2\right)} \frac{\left[v_{eff}\frac{2(2\pi r_{eff})(m-1)}{\lambda}\right]^4}{\left[1+v_{eff}\frac{2(2\pi r_{eff})(m-1)}{\lambda}\right]^{1/v_{eff}+4}}$$

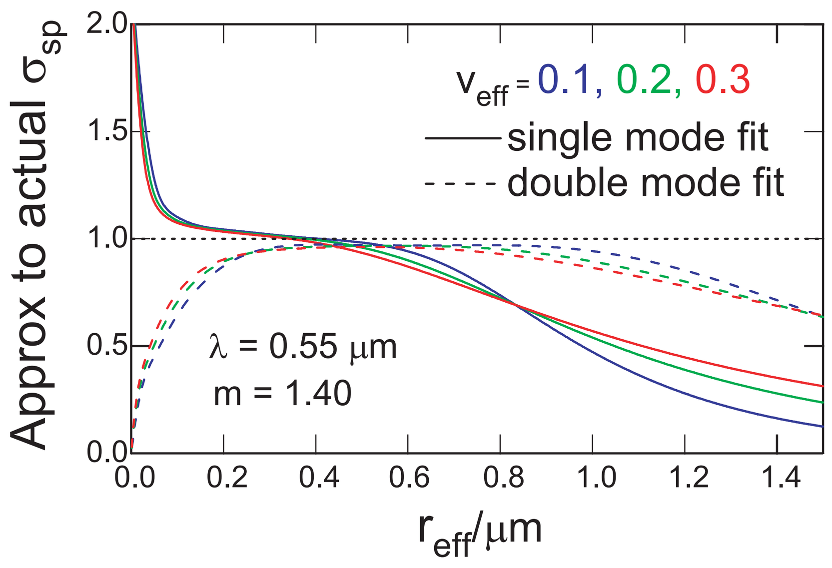
and for the Ångström exponent

$$å = \frac{4-\left[\frac{2(2\pi r_{eff})(m-1)}{\lambda}\right]}{1+v_{eff}\left[\frac{2(2\pi r_{eff})(m-1)}{\lambda}\right]},$$

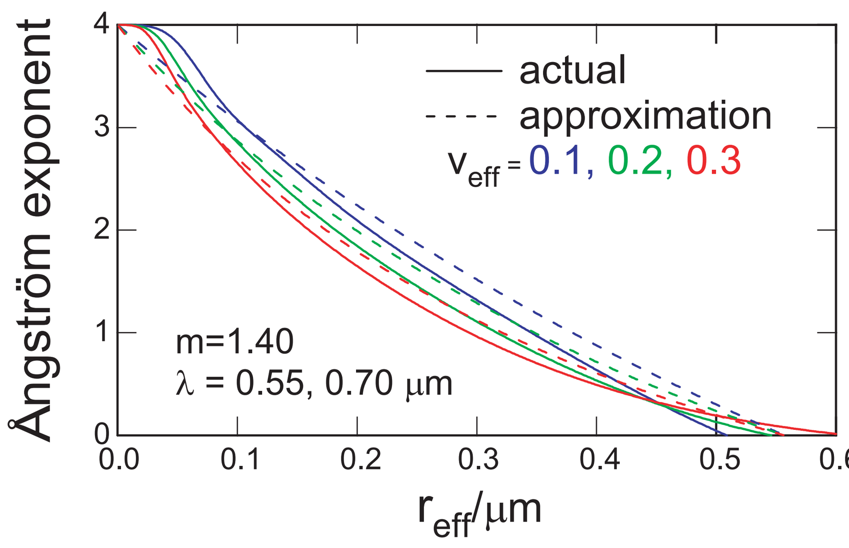
explicitly showing the dependences of these quantities on properties of the size distribution (i.e., r_{eff} and v_{eff}), wavelength, refractive index, and RH (through r_{eff}, m).

- $å$ increases with increasing λ.
- The dependence of $å$ on RH is not simple, as r_{eff} increases but m decreases with increasing RH.

COMPARISON of σ_{sp}, $å$



The single mode fit provides a good approximation (within 10-20%) for ~0.05 μm < r_{eff} < ~0.7 μm, and the double mode fit for ~0.1 μm < r_{eff} < ~1.2 μm.



Most measurements of $å$ yield values between 0 and 2. The approximation is quite accurate.

CONCLUSIONS

- Q_{sp} depends mostly on $z \equiv 4\pi(r/\lambda)\cdot(m-1)$.
- Q_{sp} can be accurately approximated by a sum of gamma functions over the range of interest.
- Analytic results for σ_{sp} and $å$ can be obtained for size distributions which are gamma functions or sums of gamma funcions.
- This method can also be applied to lognormals.