Development and Comparison of Cloud Particle Size Distribution Fitting and Analysis Techniques

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1. Overview

Knowledge of particle size distributions (PSDs) is needed for determining cloud radiative properties and sedimentation rates, and for the development and evaluation of remote sensing retrieval schemes and model parameterizations. PSDs are commonly represented as analytical functions. Methods are needed to accurately characterize observed PSDs as exponential or gamma distributions.

Objectives:

1. Develop a technique for determining fit parameters of a gamma distribution that match the moments of observed PSDs.

2. Compare the accuracy of the new fitting technique with that of previously developed techniques.

Data:

60 second averaged PSDs observed on the 27 Jan, 29 Jan and 2 Feb 2006 TWP-ICE cirrus flights are used. Data from the Cloud and Aerosol Spectrometer (CAS) were used to characterize particles with maximum dimensions D < 50 μm and from the Cloud Imaging Probe (CIP) to characterize particles with D > 50 μm.

2. Incomplete Gamma Fitting (IGF) Method

The PSDs are represented by a gamma function

$$N(D) = N_0 D^{\mu} e^{-\lambda D} \qquad (1)$$

$N(D)$ - number distribution
N_0 - intercept; λ - slope
μ - shape parameter

The IGF method calculates N_0, λ and μ by minimizing the differences between the moments calculated by integrating the PSD and those calculated using an incomplete gamma distribution whose use accounts for the fact that observed PSDs do not cover the complete range of particle sizes.

2.a Formulation:

The xth moment M_x of a PSD is calculated using

$$M_x = \int_{D_{min}}^{D_{max}} N(D) D^x dD \qquad (2)$$

D_{min} - minimum particle diameter
D_{max} - maximum particle diameter

Substituting Eq. (1) into Eq. (2) gives

$$M_x = \frac{N_0}{\lambda^{x+\mu+1}} \left[\gamma(x+\mu+1, \lambda D_{max}) - \gamma(x+\mu+1, \lambda D_{min})\right] \qquad (3)$$

where $\gamma(u,v) = \int_0^v x^{u-1} e^{-x} dx$ is the incomplete gamma function.

Using Eq. (3), the moments for a given N_0, λ and μ can be calculated ($M_{x_{calc}}$) and used in the procedure to give the closest match to moments computed from the observed PSDs ($M_{x_{obs}}$).

2.b Minimization

N_0, λ and μ are determined by minimizing

$$\chi^2 = \frac{M_{x_{obs}} - M_{x_{calc}}}{\sqrt{M_{x_{obs}} M_{x_{calc}}}} \qquad (4)$$

using an iterative procedure.

3. Application

- IGF set to minimize difference between observed and calculated 1st, 2nd and 6th moments (IGF-126).
- Gamma LSQ is the standard gamma fitting method where the least squares difference between observed and parameterized N(D) is minimized
- In general, Gamma LSQ gives PSDs and calculated moments (Table 1) further from those observed compared to IGF.

Table 1: Moments of Observed PSD, Gamma LSQ and IGF-126 shown in Figure 1.

	M1	M2	M3	M4	M5	M6
Observed	1.39E+04	2.12E+05	5.00E+06	2.12E+08	1.67E+10	2.20E+12
Gamma LSQ	1.27E+04	1.87E+05	3.48E+06	7.89E+07	2.12E+09	6.45E+10
IGF-12	1.39E+04	2.12E+05	6.40E+06	3.23E+08	2.34E+10	2.20E+12

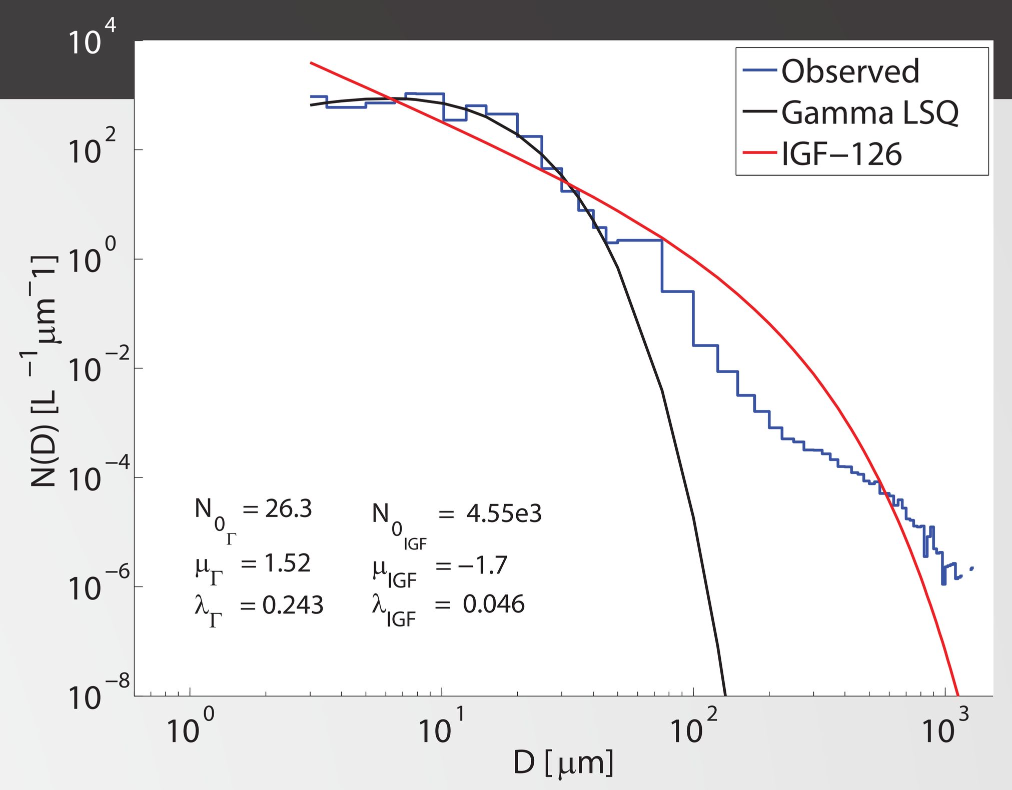

Figure 1: IGF method and standard least squares fit applied to a 60 s PSD measured 0805-0806 UTC on 29 Jan 2006 during TWP-ICE.

4. Comparison of Fitting Techniques

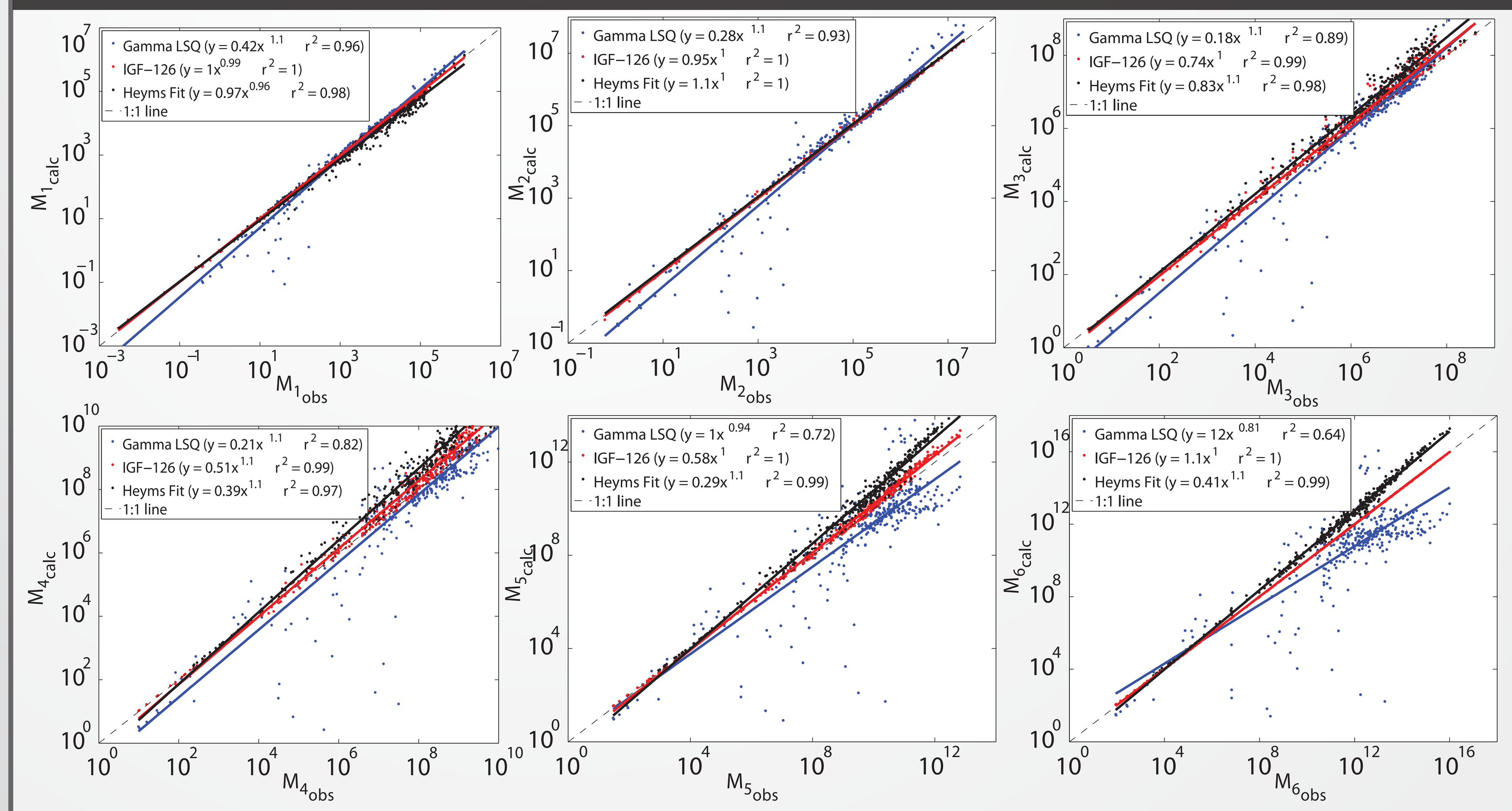

Figure 2: Comparisons of moments calculated from observed PSDs, $M_{x_{obs}}$, against moments calculated from fit distributions, $M_{x_{calc}}$, for moments 1 through 6.

4.a Dependence on Fitting Algorithm

- Fits to observed PSDs on 27 Jan., 29 Jan. and 2 Feb. were performed using IGF-126, the standard LSQ fit, and a technique developed by Heymsfield et al. (2002).
- Results show that the IGF consistently provides the best match to observed moments.

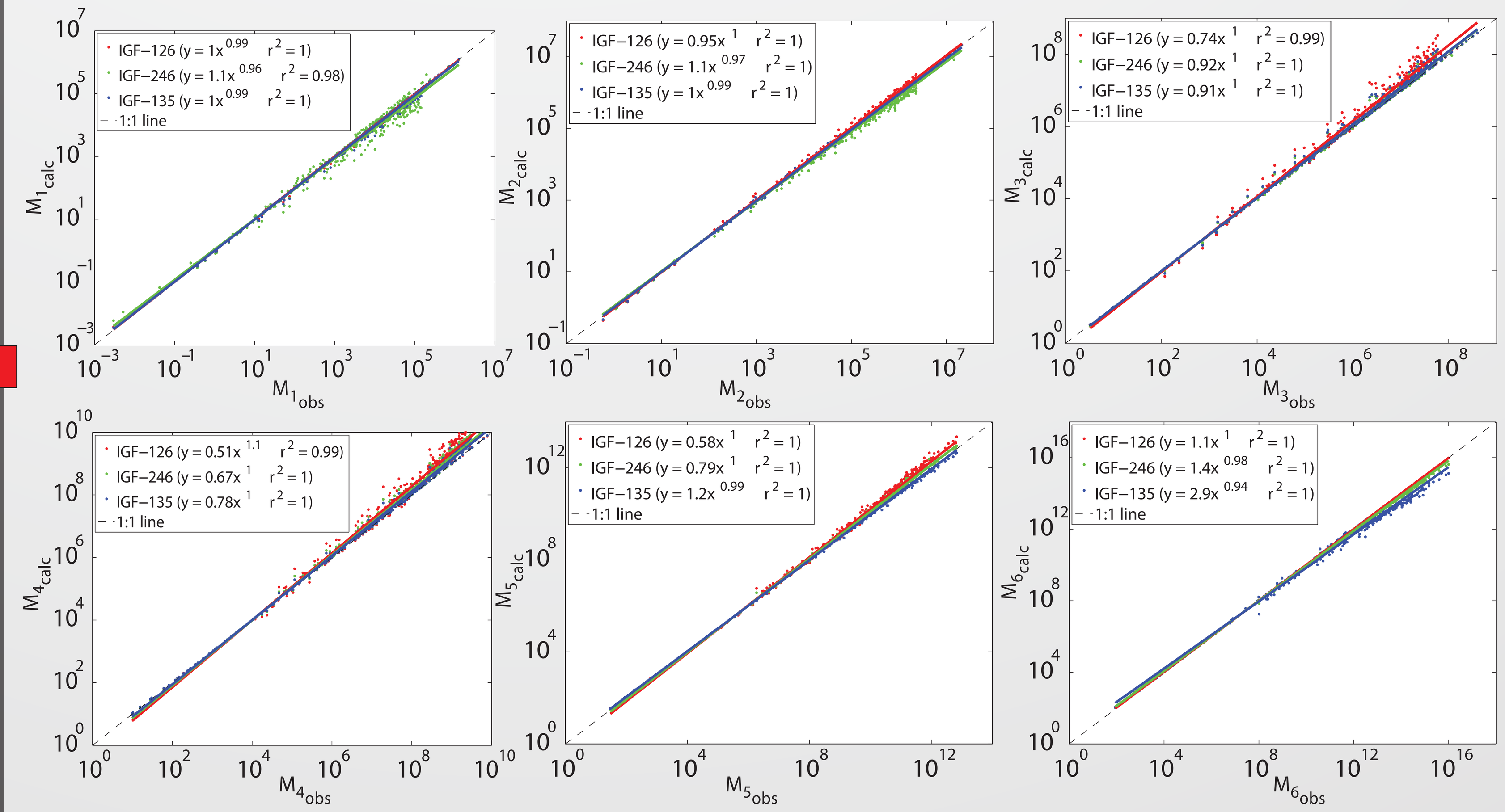

Figure 3: As in Fig. 2, except different colors represent IGF method performed using moments 1, 2 and 6 (IGF-126), moments 2, 4, and 6 (IGF-246) and moments 1, 3, and 5 (IGF-135).

4.b. Dependence on Moments used in IGF method

- The IGF method can be configured to minimize difference in any 3 moments since there are 3 free parameters in the gamma distribuiton.
- The three moments chosen for the IGF fit best match the observed moments
- In general, there is good agreement between observed and calculated moments for all moments irrespective of chosen moments.

5. Summary

The IGF fitting technique provides more accurate estimate of the moments of observed size distributions than standard least squares fitting techniques and outperformed the Heymsfield et al (2002) fitting technique in all situations.

Varying the IGF method moments had minimal impacts on the overall agreement of calculated and observed moments.

Application of IGF technique to observed PSDs will ensure any resulting parameterizations will be more consistent with observations.

References

Heymsfield, A.J., et al., 2002: Observations and Parameterizations of Particle Size Distributions in Deep Tropical Cirrus and Stratiform Precipitating Clouds: Results from In Situ Observations in TRMM Field Campaigns. *J. Atmos. Sci.*, **59** 3457-3491.

McFarquhar, G.M., and M. Freer, 2008: Development of a new fitting technique for cloud particle size distributions using incomplete gamma functions. *J. Atmos. Sci.*, In preparation.

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