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Integration of an Economy under Imperfect Competition with a Twelve-Cell Ecological Model

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Integration of an Economy under Imperfect Competition with a Twelve-Cell Ecological Model

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Notice

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Foreword

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Sally Gutierrez, Director
National Risk Management Research Laboratory

Abstract

This report documents the scientific research work done to date on developing a generalized mathematical model depicting a combined economic-ecological-social system with the goal of making it available to the scientific community. The model is preliminary and has not been tested or fully explored. The model system described here is intended to represent the first steps in combining (in simple fashion) the basic dynamic elements of an ecosystem functioning with a human society and an economy in a closed system with a non-limiting supply of energy (the model is based on flows of mass between system compartments while the total mass is conserved). In this preliminary model, optimizing economic agents (firms and households) interact in specific markets and with an ecological system consisting of resource pools and several domesticated and wild species. The result is an interdependent system that attempts to model macroeconomic variables (based on underlying property rights) and environmental stocks and flows. The report contains four chapters. Chapter 1 introduces the project's objective and approach; provides a background on sustainability as a complex system composed of several dimensions interacting through time; and provides a brief overview of the integrated economic/ecological model and its development. Chapter 2 gives a detailed description of the integrated model as it currently exists with equations and explanations. Chapter 3 provides the model solution and operational equations. Chapter 4 provides a summary of the report. The appendices contain a glossery of terms and the computer code used to implement the model in a form suitable for simulating different scenarions.

A portion of this report was submitted in partial fulfillment of Simplified Acquisition Order Number 4C-R101-NASA by H. W. Whitmore, under the sponsorship of the United States Environmental Protection Agency. This report covers a period from March 3, 2004 to August 15, 2005. The work was completed as of February 28, 2006.

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Chapter 1

Introduction

Objective and Approach

In many attempts to model economic and ecological systems, the coupling of the economic to the ecological in one model is limited, typically interacting through a single link or pathway. This method of modeling is inherently inappropriate and perpetuates the misconception that human activities can operate outside of the ecological systems on which the economic system and human societies depend (O'Neill and Kahn 2000, Rees 2002). In other cases, economic decision making is integrated into an ecosystem model, although the modeling of market mechanisms and the feedbacks between these and the ecosystem are limited (e.g., the lake eutrophication and fisheries models of Carpenter et al. 1999, Brock and Starrett 2003, Ludwig et al. 2003, and Carpenter and Brock 2004; Brock and Xepapadeas 2002; and the Patuxent River watershed models of Voinov et al. 1999 and Costanza et al. 2002). All of these models were developed to better understand and manage human impacts on specific ecological systems; however, they generally have very high data requirements for estimating model parameters.

Several more abstract models have been developed to understand the general behaviors of an integrated ecological/economic system, bypassing the need for large amounts of ecosystem-specific data. Van den Bergh (1996) offers a basic model that integrates the essential features of an ecological system with a fairly simplified production economy. His production functions account for materials balance, waste and recycling. In addition, he allows for renewable resources to regenerate and for the biosphere to assimilate a portion of pollution created during the production process. However, the economic portion of his model does not allow for labor services, a pricing mechanism or endogenous consumption. The extremely large GUMBO (Global Unified Metamodel of the Biosphere) includes an elaborate ecosystem together with production functions and an economic welfare function, but no explicit system of market prices (see, for instance, Baumans et al. 2002). A wide variety of Computable General Equilibrium (CGE) models exist which integrate certain features of the environment into an economic framework; they are lucidly reviewed by Conrad (2002). These models do account for the fact that emissions and the stock of pollution adversely affect the environment's ability to provide basic services. In addition, they permit environmental quality to influence economic efficiency or human welfare. These models contain well developed input-output matrices that include material flows across industries. Consumption, production and investment are derived endogenously using optimization techniques. Typically however the price of labor and the price of capital are exogenously determined. Product prices are determined either through perfect competition or by assuming that price is set equal to a constant average (and marginal) cost. In the latter case, product demand alone determines the level of production. CGE models do not incorporate the dynamics of the regeneration of renewable resources, endogenous population growth or the capacity of the biosphere to

assimilate a portion of the pollution generated.

The preliminary model described here differs in several respects. First, resource limits are addressed by explicitly modeling the system as closed to mass; economic decisions and biological interactions between species determine how mass is distributed throughout the system, and consequentially whether some resources are scarce or some species go extinct (i.e., species with zero mass). Second, a legal foundation is incorporated by identifying the mass in terms of its property type. Third, an explicit market system of decision making is implemented in the form of a price setting model. The model introduced here has evolved from much simpler, ecosystem-only models (see Cabezas et al. 2003, Fath et al. 2003) and from an ecosystem with economic-like behavior (Cabezas et al. 2005). This model also includes an industrial process subsystem in addition to the macroeconomic decision-making system and legal foundation.

The ultimate objective of this modeling effort is to gain general insight as to which legal and economic strategies might contribute to or degrade desirable system regimes and resilience in an integrated ecological-economic-legal system. It also hoped that some understanding is gained about which policies or laws (theoretically) could change economic and social behavior to increase the stability of this system. At this point, the model is a preliminary one and has not been tested or fully explored. Nor is the model calibrated or tied to any real-world systems; rather, it is meant to retain a degree of abstractness and generality that may lead to the drawing of general conclusions about the interaction between economic markets, ecosystems, and the law.

Background

Interest in sustainability has grown exponentially as it has become increasingly obvious that the supporting biological systems of the Earth can not indefinitely support current rates of human population growth and resource consumption (Millennium Ecosystem Assessment Synthesis Reports, 2005). Consider, for example, that according to the United States Census Bureau (2005) the human population of the earth grew from 2.5 billion in 1950 to about 6.4 billion in 2005. Human population growth continues. Also consider that from 1970 to 1995, consumption expenditures in 1995 U.S. dollars increased from $8.3 to $16.5 trillions in industrialized nations, and from $1.9 to $5.2 trillions in developing nations (United Nations Development Programme 1998). Lastly consider that the human population presently appropriates about 20% of the world net terrestrial primary production, leaving a vastly reduced resource pool for all other species (Imhoff et al., 2004, Haberl et al. 2004). Here, net primary production is defined as the net amount of solar energy used to convert mass to terrestrial plant organic matter by photosynthesis.

Sustainability is fundamentally an effort to create and maintain a regime in which the human population and its necessary energy and material consumption can be supported indefinitely by the biological system of the Earth. Hence, sustainability is not a goal but a path or corridor through time. Figure 1-1 illustrates conceptually the sustainable path

through time of a system through a corridor in a space where the coordinates are measurable ecological, industrial, economic and other variables. A sustainability corridor is, therefore, defined such that, for example, biodiversity and human population sizes are appropriate, the industrial processes perform at high efficiency with minimal environmental impacts, and the level of economic activity is adequate to provide employment and meet human needs. However, in an integrated system such as this one, deviations in any dimension have repercussions elsewhere. For example, inefficient and wasteful production causes pollution which damages ecosystems. Therefore, the construction of a sustainable corridor requires at least a basic understanding of the relationship between production processes, ecosystems, and economies.

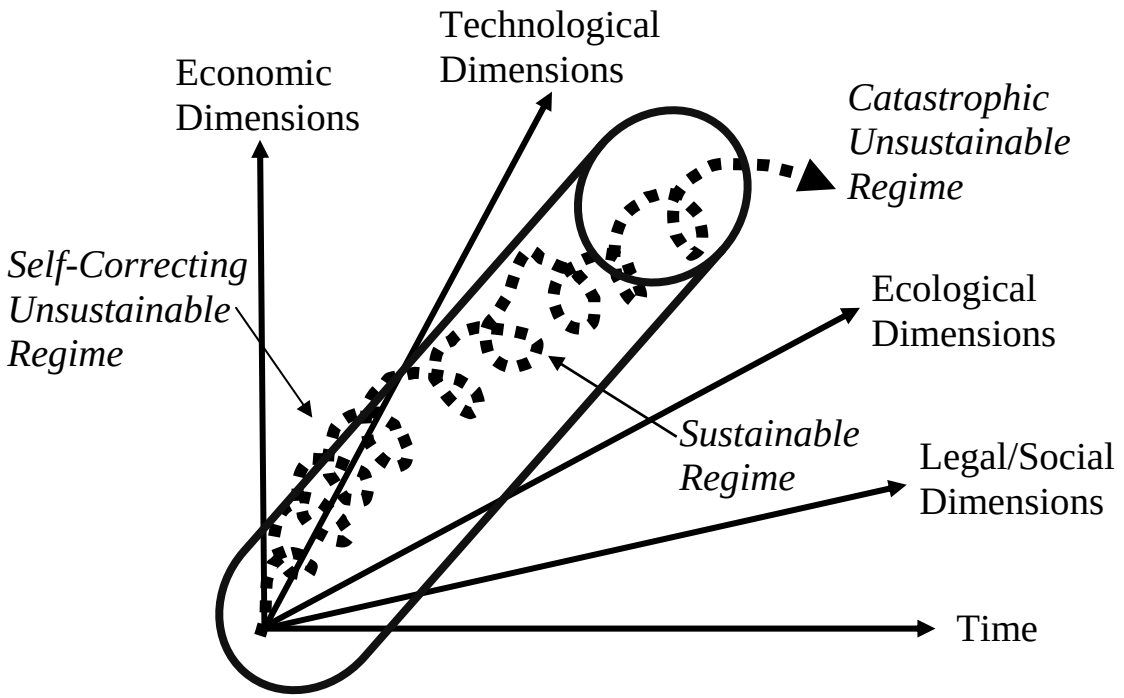

Figure 1-1 Conceptual path in time for a complex cyclic dynamic system having economic, technological, ecological, legal, and social components. Note the conceptual limits that define sustainable regimes shown here as a tunnel, and the three categories of regimes: sustainable, self-correcting unsustainable, and catastrophic unsustainable.

Hence, as already discussed the definition, assessment, and attainment of sustainability are by nature multidisciplinary (Goodland and Daly 1996, Dasgupta et al. 2000, Cabezas et al. 2003, McMichael et al. 2003, Cabezas et al. 2005; Figure 1). Popular definitions of

environmental sustainability abound, some of which can be contradictory (World Commission on Environment and Development 1987, Gatto 1995, Goodland and Daly 1996). The overarching concept of sustainability for all disciplines, when applied to humanity as a whole or a particular society, highlights the level of activity that can be sustained for a given length of time without diminishing the productivity of the system or its capacity to recover function following disturbances. While the time frame over which this concept is applied can differ markedly between disciplines, all disciplines approach sustainability with a reasonably consistent idea of which characteristics of the system are desirable, and which are not. From the many possible dimensions of sustainability, three which most relevant to the work in this report (ecological, economic, and social/legal) are described below.

Ecological

Ecological sustainability usually infers that an ecosystem can retain an ability to function through environmental changes and disturbances, and over the long term has the evolutionary capacity (through genetic and species diversity) to form adaptive ecosystem structures and functions. Ecosystems are comprised of collections of species that operate in fairly characteristic trophic (feeding) levels, and that engage in a wide variety of positive, neutral, and negative interactions. The stability and productivity of ecosystems is dictated by the biodiversity of a system, although the role of a specific species or population in an ecosystem is not known in all cases with certainty (Tilman 1999, Bond and Chase 2002, Hooper et al. 2005). However, due to habitat loss, overharvesting, invasion by non-native species, and other anthropogenically induced pressures, many species are currently at risk of extinction (Ehrlich 1995).

As species are lost from systems, the connectivity (and perhaps redundancy) of the system declines, and critical functions (such as nutrient recycling, waste treatment or pollination) may either no longer be provided (Kearns 1997, Hooper et al. 2005) or are provided at reduced capacity. Extinction rates have generally risen proportionately to the area of natural habitat impacted by human activity (MacArthur and Wilson 1967, Doak and Mills 1994, Pimm and Askins 1995). Extinction (either at the population or species level) reduces the evolutionary capacity of ecosystems, and with it their ability to adapt to changing environments. Loss of biodiversity in ecosystems would only be sustainable if immediate restoration of these systems were possible, but this is unlikely as restoration of ecosystems depends heavily on the order of species reintroduced into a system. Community assembly rules, resistance to invasion, and the characteristics of each species are all critical determinants of restoration efforts, but are at best vaguely understood for selected ecosystems (Levin et al. 2001, Sakai et al. 2001, Ferenc et al. 2002).

Ecosystems are complex, dynamic systems that often display characteristic regimes of behavior dictated by their internal dynamics and the disturbances that act on them (Scheffer et al. 2001, Mayer and Rietkerk 2004). A dynamic regime, or “alternative stable state,” is characterized formally by a particular multidimensional neighborhood or set of values over which the system state varies. About each set of such steady states, a

basin of attraction is formed such that absent any changes in external disturbances or random variations within the system, the system will remain in that basin of attraction and tend toward the steady state. In this analogy, a change in regime is a change to another basin of attraction. The size of disturbance that can be tolerated by an ecosystem before a change in regime occurs is a measure of its resilience (Holling 1973, 1996, Gunderson 2000). Disturbances can range in size, intensity, and frequency, and can originate from natural (e.g., lightning strike, fires, floods) or anthropogenic sources (e.g., agriculture, deforestation). Although ecosystems may naturally pass through many regimes, the functions and services that ecosystems can provide human societies do vary under different regimes (Wardle et al. 2000, Portela and Rademacher 2001). In this respect, some regimes may be more desirable to humans than other regimes (Carpenter et al. 2001).

Economic

All economic activity is dependent upon natural resources provided by the environment, both as inputs and as sinks for waste mass, e.g., pollutant treatment. Over human time scales, these resources can either be renewable (such as fish populations, timber stands, or the capacity of the environment to absorb some forms of pollution) or non-renewable (such as coal or copper), but renewable resources can be exhausted if harvest rates are too high (Slade 1982, Reed 1986, Pauly et al. 2002). Depletion rates of natural resource stocks are often dictated by the scarcity of the stock (which influences its price), or the economic discount rate used by the industry, instead of the biological limits of regeneration of the stock. This discrepancy can lead to extraction and harvest rates for renewable resources that are not sustainable.

Goods and services that ecological systems provide to human economies and societies are rarely directly valued in economic markets (Daily 1997). Historically, economists have not internalized pollution and other negative impacts to ecosystems in economic analyses, but rather have treated these impacts as externalities (Samuelson 1954, Freeman 1984, Bird 1987). However, feedback loops between ecological and economic systems can significantly alter projected resource availability and economic productivity, and are important in the determination of sustainable resource use (Settle et al. 2002). Several studies have attempted to make the costs of these services explicit, and estimate that the planetary value of all ecosystem services is around $16-54 trillion per year (Costanza et al. 1997, although see Heal 2000). Some of these estimates are based on the cost of designing and building a technological system that could provide the same services, such as reverse osmosis or desalinization technology to produce freshwater in the place of wetlands (Postel and Carpenter 1997). Other estimates are based on a variety of hedonic pricing, contingent valuation or other more indirect methods (Farber et al. 2002). The purpose for valuing ecosystem goods and services is not necessarily to include them in regular market transactions, but rather to provide an accounting system that is recognizable to economic analysts and can be used to monitor rapid depletion or unsustainable use of these goods and services.

In many analyses of economic sustainability, the discount rate chosen and the substitutability of environmental capital for other resources and for manmade capital can greatly influence the perceived sustainability of an activity. Higher discount rates cause firms to extract natural resources faster earlier, saving less for later, and thus leaving a reduced (or exhausted) resource base for future generations (Costanza et al. 1997, Barbier and Markandya 1998). On the other hand, resources for which there are many substitutable sources may be more likely to be used sustainably, as economic agents can more easily shift from one resource to another when prices rise (due to increasing scarcity). However, as some things have no substitution, such as breathable air or fresh water, and economists have long been at odds regarding the degree of substitutability between natural and manmade capital (Krutilla 1967).

Social/Legal

Since sustainability is ultimately about human well-being, the social aspect in general and its legal component in particular must also be considered. Human societies have always interacted with and depended upon ecosystems, but the nature of the relationship changed dramatically with the onset of agriculture about 10,000 years ago and with the evolving concept of property rights.

The domestication of animals and plants allowed for an exponential increase in human population size and density. Food surpluses resulted in job specialization, centralized conflict resolution and decision making, collection and redistribution of wealth, technology development, and amalgamation of smaller territories into larger ones (Diamond 1999). To reflect modern society and the current state of affairs, the plant and animal species can be divided into two categories: domesticated and non-domesticated. Transfers of mass associated with domesticated species are substantially affected by society's economic system. Transfers of mass associated with non-domesticated species are more often affected by the legal and political systems, and/or by biological rules alone. Humans have also become adept at appropriating and storing a large portion of the nutrient pool by making nutrients unavailable to themselves and the rest of the ecosystem for long periods of time (e.g. covering soil with asphalt, water consumed by industry or contaminated by pollutants is no longer available for drinking). This is in addition to the massive physical infrastructure and energy needed to transport, process, and distribute the agricultural products and other resources used to support other societal goals (i.e., production of non-food goods and services).

From a legal standpoint, the ecosystem is less about mass and more about the rights (or the lack of rights) associated with the mass. Property is commonly divided into three general types: private, state (also referred to as government or public) and commons (Heller 2000). Private property has come to be considered a "bundle of rights" that can be separated (or "unbundled") and owned and transferred separately. An example of this unbundling, one person (the landlord) may have the ownership rights to a parcel of land, while another (the renter) holds the right of possession, and a third (the landlord's heir) has a right to own the parcel at some time in the future. State or government property is

owned or controlled by the sovereign, e.g., a national park. Some legal scholars refer to "the commons," as meaning property which is available to everyone in the world and use the term, "common property" to denote a separate category, meaning property available to a specific group and excluding others outside that group (Yandle and Morriss 2001). Others refer to the former as "open-access commons" and the latter as "closed-access commons." In any case, the difference between the two types of commons is in who has the right to use the resource and who has the legal right to exclude others from using it.

Additional property types have also been identified. Yandle (1999) uses "regulatory property" to refer to property which is created and allocated by the government. Air pollution permits are an example of this type of property. Heller (1998) describes a type of property he calls "anticommons" to account for situations in which property rights are so divided among individuals that any one of them can exclude others from effective use and the result is underuse of the resource. This is the opposite of open-access commons, the well known subject of the Tragedy of the Commons (Hardin 1968), in which no one can be excluded and everyone maximizes his/her use of the resource resulting in its overuse.

In many ways, the property rights assigned to the mass in the ecosystem form the basis for the economic system because "the process of defining property rights defines wealth and its distribution in society" (Yandel and Morriss 2001). Using barbed wire as an example, the authors illustrate that it is the transaction costs of defining, defending, and devising the property that ultimately determines its fate. Before the invention of barbed wire, it was too expensive to enclose and defend large areas of rangeland and so it remained open-access. Once the wire was created, it became economically feasible to enclose the area (define it) and enforce the exclusion of others (defend it). At that point, the rancher had sufficient ownership control to have something worth selling to others (devise it). The other option would have been for the rangeland to stay in or revert to government ownership with grazing rights sold as "regulatory property." There are still transaction costs involved in this option, but they are borne by taxpayers as well as the permit purchaser.

Transaction costs for open-access commons resources are lower because no rights must be defined, defended, or devised, but mass in private hands is more subject to control and manipulation for maximum utility. Therefore, the hunter-gatherer has more in common with the other mobile species of the foodweb. Both value open-access commons or common property because enclosures and exclusive rights pose problems for moving across space. The food producer has opposite goals, because enclosures and exclusive rights increase wealth--the food producer benefits most by separating its domesticated species from others. The modern consumer is aligned with the food producer, since food production allows for a much more stable and convenient lifestyle.

Thus, human society with its legal system of property rights affects ecosystem sustainability in several ways. It manipulates foodwebs in favor of domesticated species, potentially affecting resources available to non-domesticated species. It appropriates portions of the resource pool for physical and social infrastructure. Finally, it raises the

bar on sustainability by a cyclical process of continually increasing its needs through population growth made possible by food surpluses brought about by manipulation of the food web.

Model Overview

The model (Figure 1-2) was developed in stages. The process began by establishing a basic ecological model with the total mass distributed among all compartments within a foodweb (Fath et al. 2003). The compartments were then differentiated in terms of human control over them by identifying species as being either domesticated or non-domesticated, assigning property rights to each compartment, and indicating intentional changes to the natural flows of mass, such as fences. Finally, a generic industrial process was added to account for resources diverted to human (non-food) consumption and use (Cabezas et al. 2005).

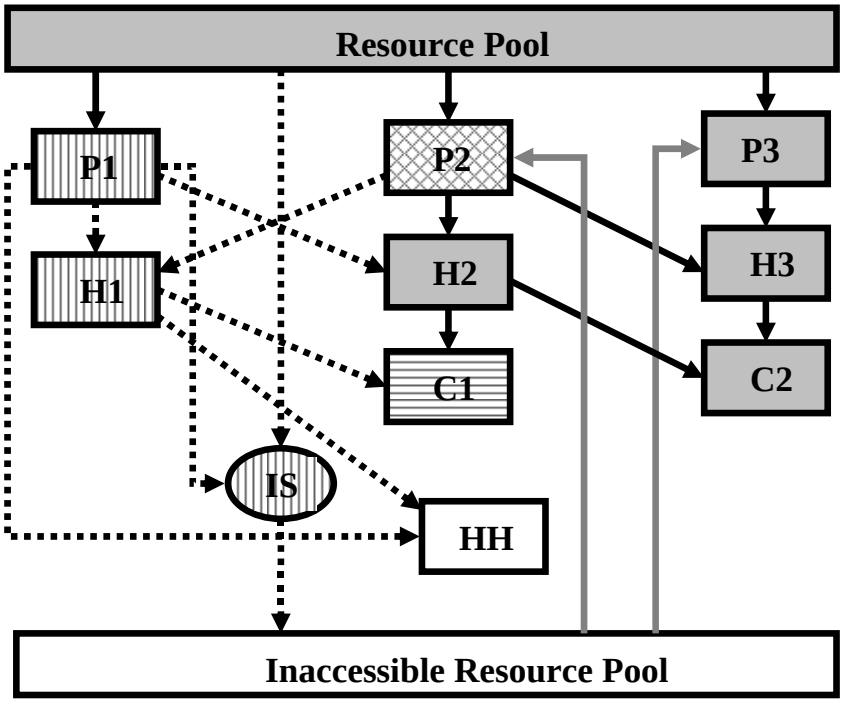

Figure 1-2 Integrated model showing flows of mass between compartments and property types. P1, H1, and IS are private property. P2 is state-owned property, but H1 has access to it through grazing leases issued by the government. C1 is a protected species, thus, H1's access to it is limited. P3, H2, H3, C2 are open-access commons to which no property rights attach. The resource pool in this model is open-access commons. Dotted lines indicate mass flows that occur under anthopogenic influence. Gray lines from the inaccessible resource pool to P2 and P3 indicate slow transfers of mass as a result of bacterial decay (this is the only natural outlet for mass to escape the inaccessible resource pool). Some mass is also transferred from each compartment to the resource pool representing the death of biological mass; however, to avoid confusing lines, these transfers to RP are not shown.

The resulting integrated model comprises twelve compartments, including two resource pools (RP and IRP), three primary producers (plants P1, P2, and P3), three herbivores (H1, H2, and H3), two carnivores (C1 and C2), an industrial sector (IS) and humans (HH). The system flows throughout are specified in terms of mass. The system is closed to mass (i.e., mass is conserved) and open to energy. Individual compartments, including those composed of private property, observe conservation of mass; that is, any difference between input and output in a compartment results in a corresponding change in mass in the compartment. Primary producers make available resources from an accessible resource pool (RP) to the rest of the food web. Although not shown on Figure 1-2, all biological compartments (i.e., all compartment except for IRP and IP) recycle mass back to the RP through death. There is a flow of mass from all biological compartments to RP proportional to the death rate of the species represented by the compartment. Mass from the IRP is recycled very slowly back to P2 and P3 through the action of bacteria; thus, it is "inaccessible"to the other compartments. The flow of mass through the biological part of the system is determined by a set of Lotka-Volterra type expressions, while the resource pools (RP and IRP) simply follow a simple mass balance, and the industrial sector (IP) follows a simple flow through with no mass accumulation.

As discussed, the model structure depicted in Figure 1-2 does not represent any particular real ecosystem. Rather it is meant to capture some of the typical features of combined ecological-economic-social systems such as: (1) an organization based on trophic levels, (2) decreasing number of species with higher tropic levels, (3) species specific preferences for food source (e.g., C1 consumes H2 but not H1), (4) the presence of humans with industrial production, agricultural production, an economy, and law including private property, and (5) the presence of mass that is biologically unavailable as result of industrial activity. While the specific structure is arbitrary, it is carefully constructed to try to capture as many of these typical features as possible. It is hoped that it has enough generality to produce some basic insights into the mechanics of combined ecological-economic-social systems. In essence, the model is the analog of a simple machine (e.g., a pendulum), which while simple and arbitrary, can still be used to study and illustrate basic laws of mechanics that are applicable to more complex machines (e.g., a gear box). It is expected that the model, perhaps with some modification, can be used to simulate the ecological consequences of different economic and regulatory strategies. In this sense this model would be a valuable tool for generically exploring sustainable environmental management strategies, but without the risk of experimenting with real ecosystems and people.

Chapter 2

Integrating the Economic Sector and Ecological Base

This chapter provides the equations and rationale for integrating the economic and ecological aspects of the model. It includes the optimal economic behavior of industry, government, and households, the natural growth of non-domesticated species and the human population, as well as the growth and depletion of the resource pools.

From an economic perspective the model contains human households (HH), an industrial sector (IS), and two private firms: one a producer of plants (P1) and one a producer of herbivores (H1). The households are the ultimate owners of the factors of production used to produce the goods that are traded in explicit markets. In addition to markets for the three goods (P1, H1, IS), there is a labor market. Households must decide between devoting time to working for one of the three industries or to leisure. Household income comprises labor income and profits generated by the three industries (P1, H1 and IS). In this model the stock of physical capital is held constant. A single period planning horizon is assumed, therefore savings and investment are ignored. Any dividend income is divided equally among the households.

The P1 firm uses resources (RP) and labor to produce plant inventory (P1). Households, the H1 firm and the industrial sector are economic consumers of this P1 inventory. H2, a wild herbivore, preys on P1. The P1 producer devotes labor to build and maintain fences to keep H2 out.. The H1 firm uses P1 and P2 to produce an inventory of herbivores, H2. The households are the only economic consumers of this inventory. A carnivore (C1) preys on the H1 inventory, but the H1 firm cannot kill or otherwise interfere with C1 because C1 is a protected species. The H1 firm's only recourse is to invest labor in fences. The H1 firm pays a fee to have grazing access to P2. It is assumed that this access is limited, and that the H1 firm takes the maximum that it is allowed. The industrial sector combines resources (RP) with plants (P1) and labor to produce goods consumed directly by the households. Unlike the P1 and H1 inventory, which transferred to their respective consumers, the mass associated with the consumption of IS inventory is not resident in the human compartment. Rather, it goes directly to an inaccessible resource pool.

The circular flow of income and spending is illustrated in Figure 2-1. Symbols used in this figure and throughout this report are in Appendix A.

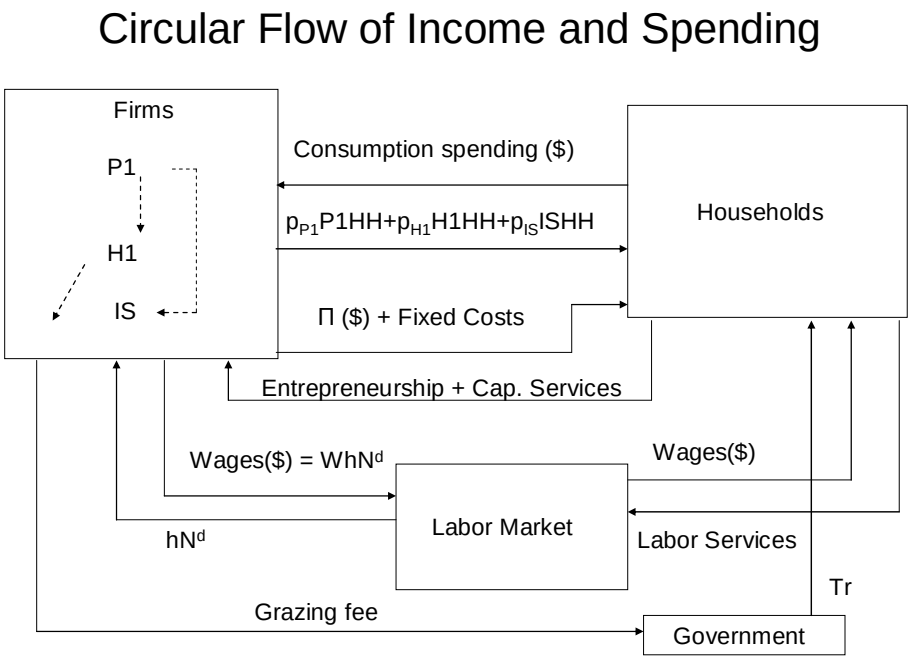

Figure 2-1. The economic model illustrating the flows of income and spending by Firms, Labor, Government, and Households. Note that in this model, taxes and "social costs" are not included.

Economic Decisions of Firms and Households

The households are the ultimate owners of the factors of production used to produce the goods that are traded in explicit markets. Factor incomes consist of labor incomes and profit incomes generated by the three industries, P1, H1 and IS. All production functions are specified in accordance with the law of conservation of mass: the mass emerging from a production process cannot exceed the total mass entering that process as inputs. In this initial model, the stock of physical capital is held constant. One possibility for introducing labor and capital into the production process is to view these inputs as simply reducing the amount of mass wasted in the production process. However, labor and capital provide a much greater service than merely reducing waste occurring in the production process. In particular, while labor and capital cannot create mass, they are nevertheless essential to the creation of value. Labor and capital provide necessary transformation services that alter the location of or the physical, chemical, biological, and/or aesthetic properties of the raw materials applied to the production function. Labor and capital can be substituted for each other in providing these transformation services, but neither labor nor capital nor a combination of the two factors can be substituted for raw materials in creating a given output mass. One possible exception is the extent that labor and capital can reduce waste, a possibility that is ignored here.

Therefore, for this purpose, the appropriate production function views transformation services and raw materials as being combined in fixed-proportions. However, the transformation services alone may be provided by a number of combinations of labor and

capital. It is further assumed that each industry and the households operate within a single-period planning horizon. Therefore, all saving and real investment in man-made capital are ignored. However, because they announce the price of their products before they know the actual demand for their goods, every industry may engage in unintended (dis)investment as they accumulate inventories (experience unfilled orders) over the current period. Each industry has a fixed number of equity shares (stock certificates) outstanding. Every household holds the same number of shares issued by a particular industry. Any dividend income the households receive is divided equally among the households. No market exists for these shares. Prices are set in terms of an abstract unit of account ($). Otherwise, the financial sectors are ignored. A stock of money presumably exists that facilitates transactions among the firms and the households, but any decision making as to the amount of money the agents desire to hold at any point in time is abstracted from. Also ignored are the social costs that the private industries producing plants and animals impose upon the ecosystem by growing their products, thereby removing nutrients from the resource pool. As a result, the market prices paid by the households and by the H1 industry for P1 reflect only the private out-of-pocket costs associated with the production of those plants.

Also, the market price that the H1 industry charges for its product will not reflect the full social cost of depleting the resource pool to produce food for the domesticated herbivores. In addition, the households are not required to reckon with the social cost they impose upon the ecosystem by adding to the inaccessible resource pool as a result of their consumption of the various plants and animals, either through an increase in consumption per capita or as a result of human population growth. The H1 industry does pay a grazing fee to the government for access to P2, but otherwise all economic agents in this model ignore the ecological benefits of P2, H2 and C2.

Households are free to decide how many hours they prefer to work based upon their preferences for P1, H1, IS, and leisure as well as upon the product prices, wage rate and the non-wage income they face. However, at the wage rate set by the IS firms, involuntary unemployment may result. The private industries use at least some of the revenue they receive from their current-period sales to pay for labor services. The P1 industry pays part of its revenue to labor. The H1 industry also uses some of its revenue to pay the P1 industry for the plants it purchases from that industry and to pay a grazing fee to the government for access to P2. The IS industry pays for labor and for the P1 it buys. All three private industries then distribute any remaining profits to the households as dividends. In the present model, the P1, H1 and IS industries set the price of their respective products before trade takes place during the current period. The prices they set conform to their forecasts of the demand functions they will face during the current period. Trade takes place in the domesticated sector even though markets may fail to clear. Industries may hold unanticipated inventories (unfilled orders) at the end of the period. It is assumed that the IS industry sets the money wage rate at the beginning of the current period, based upon its forecasts of the household sector's supply of labor and the demand for labor by the P1 and H1 industries. Involuntary unemployment (or unfilled job vacancies) is possible. Note that in the following equations, g_i is the growth parameter of species i and and m_i is the mortaility parameter of species i.

Optimal Economic Behavior for P1 Industry

The P1 industry applies a variable amount of labor and a fixed amount of capital to transform the mass of P1 that would otherwise grow naturally into a marketable product. The producer of P1 must also deal with the fact that H2 eats some P1 during the production process. The P1 industry hires labor to reduce the consumption of P1 by H2.

The production function for P1 is given by (2-1):

$$P1 = \min[g_{P1}\cdot P1_t \cdot RP_t - m_{P1}\cdot P1_t - c_{P1H2}(h^*_{P1}N_t)\cdot H2_t,\ P1(h_{P1}N_t, K^{P1}_t)] \qquad (2\text{-}1)$$

The production (growth) of P1 (g_{P1}) is viewed as positively related to: (a) the size of the resource pool at the beginning of the period, RP_t, (b) the initial stock of P1, $P1_t$, at the beginning of the period, and (c) the level of transformation services, $P1(\cdot)$, provided by (variable) labor hours, $h_{P1}N_t$, during the period and the (given amount of) physical capital, K^{P1}_t, held by the industry at the beginning of the period. From the point of view of the P1 industry, the amount of P1 lost during the current period (mortality, m_{P1}) due to consumption by H2, is assumed to be proportional to the initial stock of H2, $H2_t$, with the size of the "consumption coefficient," c_{P1H2}, and negatively related to the labor hours devoted to reducing the amount of P1 that $H2_t$ eats during the period. Assuming the P1 industry wastes neither the transformation services of labor and capital nor the net amount of P1 available for transformation (after deducting the loss due to the presence of H2), the amount of (transformed) P1 produced may be viewed as equal to $P1(h_{P1}N_t, K^{P1}_t)$:

$$P1 = P1(h_{P1}N_t, K^{P1}_t) \qquad (2\text{-}2)$$

Note that the characteristics of P1 may differ considerably from those that would occur if P1 were to grow naturally without the transformation services provided by labor and capital.

In addition, the amount of P1 transformed in the production process may be viewed as:

$$g_{P1}\cdot P1_t \cdot RP_t - m_{P1}\cdot P1_t - c_{P1H2}(h^*_{P1}N_t)\cdot H2_t = P1(h_{P1}N_t, K^{P1}_t) \qquad (2\text{-}3)$$

In principle (2-3) may be solved for $h^*_{P1}\cdot N_t$. To obtain the (linear approximation of) labor hours devoted to reducing H2's consumption of P1 (and holding g_{P1}, m_{P1} and K^{P1} constant), totally differentiate (2-3) and solve for $d(h^*_{P1}\cdot N_t)$:

$$d(h^*_{P1}\cdot N_t) = \big[g_{P1}\cdot P1_t \cdot d(RP_t) + [g_{P1}\cdot RP_t - m_{P1}]\cdot d(P1_t) - [\partial P1/\partial(h_{P1}N_t)]\cdot d(h_{P1}N_t) - c_{P1H2}\cdot(dH2_t)\big] / \{c'_{P1H2}\cdot H2_t\} \qquad (2\text{-}4)$$

where c'_{P1H2} denotes the derivative of the consumption coefficient with respect to $h^*_{P1}\cdot N_t$; this derivative is negative. Consequently, according to (2-5), the number of hours the P1 industry devotes to limiting the amount of P1 consumed by H2 is negatively related to

RP_t and $P1_t$, but positively related to both the number of labor hours it devotes to transforming P1 and the size of $H2_t$. Therefore, the following general function is given:

$$h^*_{P1} \cdot N_t = h^*_{P1} \cdot N_t(\underset{-}{P1_t}, \underset{-}{RP_t}, \underset{+}{H2_t}, \underset{+}{h_{P1}N_t}) \tag{2-5}$$

The economic stock of P1 at the end of the period, $P1_{t+1}$, is represented by (6):

$$P1_{t+1} = P1_t + P1(h_{P1}N_t, K^{P1}_t) - P1H1 - P1IS - P1HH \tag{2-6}$$

where P1H1 corresponds to the amount of P1 purchased by the H1 industry during the period, P1IS represents the amount of P1 purchased by the IS industry during the period and P1HH denotes the amount of P1 purchased by the households during the current period.

Taking the current wage rate and the P1 industry's forecast of the current period demand function for domesticated P1 plants as given, the P1 industry attempts to maximize the dividends they pay to its shareholders, subject to the restriction that the end-of-period stock of P1 is maintained at some predetermined level, $\overline{P}\ \overline{1}$. Then the firm's desired sales of P1 (in terms of mass) is given by:

$$(P1H1 + P1IS + P1HH)^{de} = P1(h_{P1}N_t, K^{P1}_t) - (\overline{P}\ \overline{1} - P1_t) \tag{2-7}$$

Dividends equal current net income minus net business saving. Net income is equal to the value of current production minus current expenses (wages and a fixed cost associated with the fixed stock of physical capital). Therefore, dividends are equal to the value of current production minus current expenses and minus net business saving. Net business saving is necessarily equal to the sum of the sector's net increase in assets minus the net increase in liabilities during the period. In the present model, the sector's planned accumulation of assets consists only of the market value of its planned accumulation of P1 during the current period; according to the assumptions, it does not plan to borrow during the current period. Therefore, current dividends are equal to the value of current production minus the current expenses and minus the value of the planned accumulation of the inventory of P1 during the period. From (2-7), the value of current production minus the value of the planned accumulation of inventory during the current period represents the value of desired current sales of P1 to other sectors during the period. Therefore, current dividends correspond to the current revenue from the sale of P1 to the producers of domesticated herbivores H1 and to the IS producers minus the current expenses of the P1 industry.

Maximize:

$$\begin{aligned}\Pi_{P1} &= p_{P1} \cdot \{P1(h_{P1}N_t, K^{P1}_t) - (\overline{P}\ \overline{1} - P1_t)\} \\ &\quad - W \cdot [h_{P1} \cdot N_t + h^*_{P1} \cdot N_t(P1_t, RP_t, H2_t, h_{P1}N_t)] - F_{P1}\end{aligned} \tag{2-8}$$

where F_{P1} denotes the current period's fixed cost. Since the P1 industry is viewed as

owning the P1 that it produces and harvests, the amount of P1 harvested does not enter directly into its cost considerations because the industry does not voluntarily consider the social cost of its production of P1.

At the beginning of the period, the P1 industry announces its product price, p_{P1}. Let the P1 industry's forecast of the current period demand for its product be represented by the following demand function:

$$P1^{de} = P1^{de}(\, p_{P1}, \omega_{H1}, \omega_{IS}, \omega_{P1HH}) = P1H1^{de}(p_{P1}, \omega_{H1}) + P1IS^{de}(p_{P1}, \omega_{IS}) + (P1HH/N)^{de}(p_{P1}, \omega_{P1HH})\cdot N_t \quad (2\text{-}9)$$

This demand function denotes the P1 industry's forecasted total (market) demand for P1 by the H1 and IS industries and by the households. It is assumed that the derivative of $P1^{de}$ with respect to p_{P1} is negative (the sign will be verified later when the economic behavior of the H1 and IS industries and the households is specified); ω_{H1} , ω_{IS} and ω_{P1HH} denote vectors of shift parameters. Solving the inverse function for p_{P1} yields:

$$p_{P1} = p_{P1}(P1^{de}, \omega_{H1}, \omega_{IS}, \omega_{P1HH}, N_t\ ) \quad (2\text{-}10)$$
$$\quad\ \ -\quad +\quad +\quad +\quad +$$

where:

$$\partial p_{P1}/\partial(P1^{de}) \equiv 1/\,[\partial(P1^{de})/\,\partial p_{P1}\,] < 0$$
$$\partial p_{P1}/\partial\omega_i \equiv -\,[\partial(P1^{de})/\,\partial\omega_i\,]/[\partial(P1^{de})/\,\partial p_{P1}\,] > 0$$
$$\text{and } \partial p_{P1}/\partial N_t \equiv -\,[(P1HH/N)^{de}]/[\partial(P1^{de})/\,\partial p_{P1}\,] > 0. \quad (2\text{-}11)$$

An increase in ω_i or N_t represents an outward shift in the P1 industry's forecast of the market demand for its product.

The price p_{P1} given by (2-10) represents the maximum uniform price the P1 sector expects it can value each alternative quantity of P1 it produces during the period. Based upon this function, the sector's forecasted current revenue function is given by:

$$R_{P1}^{\,e} = p_{P1}\cdot P1^{de} = p_{P1}(P1^{de}, \omega_{H1}, \omega_{IS}, \omega_{P1HH}, N_t\,)\cdot P1^{de}$$
$$= R_{P1}^{\,e}(P1^{de}, \omega_{H1}, \omega_{IS}, \omega_{P1HH}, N_t\,) \quad (2\text{-}12)$$

Substituting the right hand side of (2-7) for $P1^{de}$ in (2-12) yields:

$$R_{P1}^{\,e} = R_{P1}^{\,e}[P1(h_{P1}N_t, K^{P1}{}_t)\ -(\,\overline{P}\ \overline{1} - P1_t), \omega_{H1}, \omega_{IS}, \omega_{P1HH}, N_t] \quad (2\text{-}13)$$

Substituting the right hand side of (2-13) for the first term on the right hand side of (2-8) yields:

$$\Pi_{P1}\ = R_{P1}^{\,e}\{P1(h_{P1}N_t, K^{P1}{}_t) - (\,\overline{P}\ \overline{1} - P1_t), \omega_{H1}, \omega_{IS}, \omega_{P1HH}, N_t\,\}$$
$$-W\cdot[h_{P1}\cdot N_t\ + h^{*}{}_{P1}\cdot N_t(P1_t, RP_t, H2_t, h_{P1}N_t)] - F_{P1} \quad (2\text{-}14)$$

The only independent choice variable for the industry in (2-14) is the number of hours that its employees work transforming P1, $h_{P1}N_t$. Maximizing (2-14) with respect to $h_{P1}\cdot N_t$ yields the necessary condition:

$$[\partial R_{P1}^{e}/\partial(P1^{de})]\cdot[\partial(P1)/\partial(h_{P1}N_t)] = W\cdot\mathbf{[}\,1 + [\,\partial(h^{*}_{P1}\cdot N_t)/\partial(P1)]\cdot[\partial(P1)/\partial(h_{P1}\cdot N_t)]\mathbf{]}$$
$$= W\cdot\mathbf{[}\,1 - [\partial(P1)/\partial(h_{P1}\cdot N_t)]/\{c'_{P1H2}\cdot H2_t\,\}\mathbf{]} \qquad (2\text{-}15)$$

The left hand side of (2-15) represents the marginal revenue product of labor used in the P1 industry to transform P1 into a marketable product, and the condition requires the profit-maximizing firms in the industry to hire labor up to the point at which the marginal revenue product of labor equals the money wage times Equation 2-1 plus the extra labor required to reduce the amount that H2 eats as the sector adds to P1. Holding $P1_t$, RP_t, and $H2_t$ constant, if the firm hires more labor to transform P1 into a marketable product, it must also hire more labor to reduce the loss of P1 due to its consumption by H2, in order to obtain the extra P1 that will be used as the raw material input.

Assuming a diminishing marginal product of labor in transforming P1, a diminishing marginal product of labor in reducing loss, and that marginal revenue decreases as the number of units sold increases, then from (2-15), the profit maximizing level of labor to be used in the P1 industry becomes a decreasing function of the money wage. The P1 industry's demand functions for both types of labor are given by (2-16) and (2-17):

$$(h_{P1}\cdot N_t)^{d} = h_{P1}\cdot N_t^{d}(W, (\overline{P}\ \overline{1} - P1_t), \omega_{H1}, \omega_{IS}, \omega_{P1HH}, N_t, P1_t, RP_t, H2_t) \qquad (2\text{-}16)$$
$$\quad -\quad +\quad +\ +\ +\quad +\ +\ +\ -$$
$$(h^{*}_{P1}\cdot N_t)^{d} = h^{*}_{P1}\cdot N_t^{d}(W, (\overline{P}\ \overline{1} - P1_t), \omega_{H1}, \omega_{IS}, \omega_{P1HH}, N_t, P1_t, RP_t, H2_t) \qquad (2\text{-}17)$$
$$\quad -\quad +\quad +\ +\ +\quad +\ -\ -\ +$$

From (2-16) an increase in $(\overline{P}\ \overline{1} - P1_t)$ reduces, *ceteris paribus*, the amount of P1 the sector plans to sell, thereby raising the marginal revenue product of labor used to transform P1; an increase in ω_{H1}, ω_{IS}, ω_{P1HH}, or N_t shifts the market demand for P1 outward, thereby increasing the price that buyers are willing to pay for a given amount of P1, and raising the forecasted marginal revenue product for a given amount of labor used to transform P1. The effect upon the amount of labor used to conserve P1 responds in the same direction as the demand for labor used to transform P1. However, holding the amount of labor used to transform P1 unchanged, in accordance with (2-3), an increase in either $P1_t$ or RP_t reduces, *ceteris paribus*, the amount of labor required to conserve P1, causing the demand for that type of labor to diminish. As the quantity of $h^{*}_{P1}\cdot N_t$ decreases, its marginal product increases, thereby reducing the marginal cost of adding a unit of $h_{P1}\cdot N_t$; the industry's demand for $h_{P1}\cdot N_t$ increases with an increase in $P1_t$ or RP_t. The larger the stock of $H2_t$ at the beginning of the period, however, the greater will be the industry's demand for $h^{*}_{P1}\cdot N_t$ and the smaller will be its demand for $h_{P1}\cdot N_t$. Substituting the right hand side of (2-16) into the P1 industry's transformation function, (2-2), yields the P1 industry's current production of domesticated plants consistent with profit maximization in that industry:

$$P1^s = P1^s\,(h_{P1}\cdot N_t^d[W, (\overline{P}\ \overline{1} - P1_t), \omega_{H1}, \omega_{IS}, \omega_{P1HH,}\ N_t,\ P1_t, RP_t, H2_t], K^{P1}{}_t) \qquad (2\text{-}18)$$

The price that the P1 industry announces for the current period is then given by:

$$\begin{aligned} p_{P1} &= p_{P1}\{P1^{de}, \omega_{H1}, \omega_{IS}, \omega_{P1HH}, N_t\} \\ &= p_{P1}\{P1^s\,(h_{P1}\cdot N_t^d[W, (\overline{P}\ \overline{1} - P1_t), \omega_{H1}, \omega_{IS}, \omega_{P1HH}, N_t, P1_t, RP_t, H2_t], K^{P1}{}_t) \\ &\qquad -(\overline{P}\ \overline{1} - P1_t), \omega_{H1}, \omega_{IS}, \omega_{P1HH}, N_t\} \end{aligned} \qquad (2\text{-}19)$$

The actual inventory of P1 at the end of the period is given by:

$$P1_{t+1} = P1_t + P1^s - P1H1 - P1IS - P1HH. \qquad (2\text{-}20)$$

Optimal Economic Behavior for H1 Industry

H1 industry buys P1, labor and grazing rights to P2 and sells output to the households. The assumptions are that the government sector fixes the grazing fee, p_2, and that the H1 industry buys the maximum amount of P2, denoted by P2H1, permitted by the government. C1 also consumes H1; the H1 industry hires labor to limit the amount of H1 eaten by C1.

The production function for H1 is shown by (2-21):

$$H1 = \min[P1H1 + P2H1 - m_{H1}\cdot H1_t - c_{H1C1}(h^*_{H1}N_t)\cdot C1_t\ , H1(h_{H1}N_t, K^{H1}{}_t)] \qquad (2\text{-}21)$$

where $H1(\cdot)$ denotes the transformation services provided by labor and capital.

Assuming that the H1 industry wastes neither the transformation services provided by labor and capital nor the net amount of H1 available for transformation, the amount of H1 available to the market during the current period may be written as:

$$H1 = H1(h_{H1}N_t, K^{H1}{}_t) \qquad (2\text{-}22)$$

and also:

$$P1H1 + P2H1 - m_{H1}\cdot H1_t - c_{H1C1}(h^*_{H1}N_t)\cdot C1_t = H1(h_{H1}N_t, K^{H1}{}_t) \qquad (2\text{-}23)$$

The consumption coefficient, c_{H1C1}, is positive; the first derivative of this coefficient is negative and its second derivative is positive.

In principle, (2-23) may be solved for $h^*_{H1}N_t$ in terms of $h_{H1}N_t$, P1H1, $C1_t$, and P2H1. Totally differentiating (2-23) yields the following:

$$\begin{aligned} d(h^*_{H1}N_t) = [&d(P1H1) + d(P2H1) - m_{H1}\cdot d(H1_t) - [\partial H1/\partial(h_{H1}N_t)]\cdot d(h_{H1}N_t) \\ &- c_{H1C1}\cdot d(C1_t)] / \{c'_{H1C1}\cdot C1_t\} \end{aligned} \qquad (2\text{-}24)$$

Therefore, in general:

$$h^*_{H1}N_t = h^*_{H1}N_t(P1H1,\ P2H1,\ H1_t,\ h_{H1}N_t,\ C1_t) \qquad (2\text{-}25)$$
$$\qquad\qquad\qquad\qquad\quad -\quad -\quad +\quad +\quad +$$

The stock of H1 at the end of the current period is then given by:

$$H1_{t+1} = H1_t + H1(h_{H1}N_t,\ K^{H1}{}_t) \ - H1HH. \qquad (2\text{-}26)$$

H1HH denotes the amount of H1 sold to the households during the period.

Taking as given the current wage rate and its forecast of the current period demand function for domesticated herbivores, H1, the H1 firms attempt to maximize the dividends to their shareholders, subject to the restriction that they maintain the end-of-period stock of H1 at some predetermined level, $\overline{H}\ \overline{1}$. The firm's desired sales of H1 (in terms of mass) is given by:

$$(H1HH)^{de} = \ H1(h_{H1}N_t,\ K^{P2}{}_t) \ -(\overline{H}\ \overline{1} - H1_t\ ) \qquad (2\text{-}27)$$

Therefore, the objective of the H1 industry is to maximize:

$$\Pi_{H1} \ = p_{H1}\cdot\{H1(h_{H1}N_t,\ K^{H1}{}_t) \ - (\overline{H}\ \overline{1} - H1_t)\ \} - p_{P1}\cdot P1H1 - p_{P2}\cdot P2H1$$
$$-W\cdot[h_{H1}N_t + h^*_{H1}N_t(P1H1,\ P2H1,\ H1_t,\ h_{H1}N_t,\ C1_t)] \ - F_{H1} \qquad (2\text{-}28)$$

where F_{H1} represents the industry's fixed cost.

The product price, p_{H1}, is announced by the H1 industry at the beginning of the period. Let the H1 industry's forecast of the current period demand for its product be given by the following demand function:

$$H1HH^{de} = (H1HH/N_t)^{de}(p_{H1},\ \omega_{H1HH})\cdot N_t \qquad (2\text{-}29)$$

This demand function denotes the H1 industry's forecast of the households' demand for H1. The assumption is that the derivative of $H1^{de}$ with respect to p_{H1} is negative (the sign will be verified later when the economic behavior of the households is specified); ω_{H1HH} denotes a vector of shift parameters. Solving the inverse function for p_{H1} yields:

$$p_{H1} = p_{H1}(H1HH^{de},\ \omega_{H1HH},\ N_t) \qquad (2\text{-}30)$$

where

$$\partial p_{H1}/\partial(H1HH^{de}) \equiv 1/\ \{[\partial(H1HH/N)^{de}/\partial p_{H1}]\cdot N\} < 0 \qquad (2\text{-}31)$$
$$\partial p_{H1}/\partial\omega_{H1HH} \equiv -\ [\partial(H1HH/N)^{de}/\ \partial\omega_{H1HH}\ ]/\{[\partial(H1HH^{de})/\ \partial p_{H1}\ ]\cdot N\} > 0 \qquad (2\text{-}32)$$
$$\partial p_{H1}/\partial N_t \equiv -\ [(H1HH/N)^{de}]/\{[\partial(H1HH^{de})/\ \partial p_{H1}\ ]\cdot N\} > 0 \qquad (2\text{-}33)$$

and where a rise in ω_{H1HH} or N_t represents an outward shift in the demand for H1 function

facing the H1 industry.

The price p_{H1} given by (2-30) represents the maximum uniform price the H1 sector expects it can value each alternative quantity of H1 it produces during the period. Based upon this function, the sector's forecasted current revenue function is given by:

$$R_{H1}^{e} = p_{H1} \cdot H1HH^{de} = p_{H1}(H1HH^{de}, \omega_{H1HH}, N_t) \cdot H1HH^{de} = R_{H1}^{e}(H1HH^{de}, \omega_{H1HH}, N_t) \quad (2\text{-}34)$$

Substituting the right hand side of (2-27) for $H1HH^{de}$ in (2-34) yields:

$$R_{H1}^{e} = R_{H1}^{e}[H1(h_{H1}N_t, K^{P2}{}_t) - (\overline{H}\ \overline{1} - H1_t), \omega_{H1HH}, N_t] \quad (2\text{-}35)$$

Substituting the right hand side of (2-35) for the first term on the right-hand-side of (2-28) yields:

$$\begin{aligned}\Pi_{H1} = {} & R_{H1}^{e}\{H1(h_{H1}N_t, K^{P2}{}_t) - (\overline{H}\ \overline{1} - H1_t), \omega_{H1HH}, N_t\} - p_{P1} \cdot P1H1 - p_{P2} \cdot P2H1 \\ & - W \cdot [h_{H1}N_t + h^{*}{}_{H1}N_t(P1H1, P2H1, H1_t, h_{H1}N_t, C1_t)] - F_{H1}\end{aligned} \quad (2\text{-}36)$$

Given $\overline{H}\ \overline{1}$, two choice variables confront the H1 industry in this simplified model: the number of hours that employees work in the industry transforming P1 and P2 into H1; and the amount of the intermediate good, P1, the H1 industry buys from the P1 industry. The amount of labor the H1 industry employs to reduce C1's consumption of H1 then follows from (2-25). Maximizing (2-36) with respect to $h_{H1} \cdot N_t$ yields the necessary condition:

$$\begin{aligned}[\partial R_{H1}^{e}/\partial(H1HH^{de})] \cdot [\partial(H1)/\partial(h_{H1}N_t)] &= W \cdot [1 + [\partial(h^{*}{}_{H1}N_t)/\partial(H1)] \cdot [\partial(H1)/\partial(h_{H1}N_t)]] \\ &= W \cdot [1 - \{[\partial(H1)/\partial(h_{H1}N_t)]/(c'_{H1C1} \cdot C1_t)\}]\end{aligned} \quad (2\text{-}37)$$

The left hand side of (2-37) represents the marginal revenue product of labor used in the H1 industry to transform mass into marketable H1. The condition requires the profit-maximizing firms in the industry to hire this type of labor up to the point at which the marginal revenue product of labor equals the money wage, plus the marginal cost of adding as well the requisite amount of labor to further limit the consumption of H1 by C1 in order to obtain, *ceteris paribus*, the extra H1 mass to be transformed.

The first-order condition for P1H1 is given by:

$$-W \cdot [\partial(h^{*}{}_{H1}N_t)/\partial(P1H1)] = p_{P1} \quad (2\text{-}38)$$

or:

$$-W/\{c'_{H1C1} \cdot C1_t\} = p_{P1} \quad (2\text{-}39)$$

According to condition (2-38), the industry should continue to buy P1 up to the point at which the marginal revenue to the industry from an extra unit of P1, namely the wages it saves because it can reduce its use of labor to limit C1's consumption of H1, is equal to the price of p_1.

Assuming a diminishing marginal product of labor and assuming that marginal revenue decreases as the number of units sold increases, then taking total differentials of (2-37) and (2-39) yields a set of equations that can be solved for the responses in the choice variables to various parametric changes. In particular, the following demand functions are found for labor hours, $h_{H1}N_t$, and for P1H1:

$$(h_{H1}\cdot N_t)^d = h_{H1}\cdot N_t^d(W,\ p_{P1}\ ,\ (\overline{H}\ \overline{1} - H1_t\ ),\ \omega_{H1HH},\ N_t) \qquad (2\text{-}40)$$
$$\qquad\qquad\qquad\qquad\qquad\quad - \ -\qquad\quad +\qquad\quad +$$

$$(P1H1)^d = P1H1^d(W,\ p_{P1},\ (\overline{H}\ \overline{1} - H1_t\ ),\ \omega_{H1HH},\ N_t) \qquad (2\text{-}41)$$
$$\qquad\qquad\qquad\qquad\quad - \ -\qquad\quad +\qquad\quad +$$

Since, *ceteris paribus*, the inputs P1H1 and labor that is used to transform H1, $h_{H1}N_t$, are technical complements, an increase in the price of either input results in the H1 industry demanding less of both inputs. Substituting the right-hand sides of (2-40) and (2-41) into (2-25) yields the H1 industry's demand for labor for the purpose of limiting the amount of H1 consumed by C1:

$$(h^*_{H1}N_t)^d = (h^*_{H1}N_t)^d\,[(P1H1)^d(W,\ p_{P1},\ (\overline{H}\ \overline{1} - H1_t\ ),\ \omega_{H1HH},\ N_t),\ P2H1,\ H1_t,$$
$$(h_{H1}N_t)^d(W,\ p_{P1},\ (\overline{H}\ \overline{1} - H1_t\ ),\ \omega_{H1HH},\ N_t),\ C1_t] \qquad (2\text{-}42)$$

or:

$$(h^*_{H1}N_t)^d = (h^*_{H1}N_t)^d\,[W,\ p_{P1},\ (\overline{H}\ \overline{1} - H1_t\ ),\ \omega_{H1HH},\ N_t\ P2H1,\ H1_t,\ C1_t] \qquad (2\text{-}43)$$
$$\qquad\qquad\qquad\quad - \ +\qquad \pm\qquad\qquad \pm\qquad \pm\ -\qquad +\quad +$$

where it has been assumed that the own-price effects upon $(P1H1)^d$ and $(h_{H1}N_t)^d$ dominate the indirect, or cross-price effects.

Substituting the right hand side of (2-40) into (2-22) yields the amount of H1 the industry desires to supply to the market during the current period:

$$H1^s = H1^s[h_{H1}\cdot N_t^d(W,\ p_{P1}\ ,\ (\overline{H}\ \overline{1} - H1_t\ ),\ \omega_{H1HH},\ N_t)\ ,\ K^{H1}{}_t]$$
$$\qquad\qquad\qquad\qquad\quad - \ -\qquad\quad +\qquad\quad +\qquad +$$

$$= H1^s(W,\ p_{P1}\ ,\ (\overline{H}\ \overline{1} - H1_t\ ),\ \omega_{H1HH},\ N_t)\ ,\ K^{H1}{}_t) \qquad (2\text{-}44)$$
$$\qquad\quad - \ -\qquad\quad +\qquad\quad +$$

The price that the H1 industry announces for the current period is then given by:

$$p_{H1} = p_{H1}(H1HH^{de},\ \omega_{H1HH})$$
$$= p_{H1}(H1^s(W,\ p_{P1}\ ,\ (\overline{H}\ \overline{1} - H1_t\ ),\ \omega_{H1HH},\ N_t)\ ,\ K^{H1}{}_t) - (\overline{H}\ \overline{1} - H1_t\ ),\ \omega_{H1HH},\ N_t) \qquad (2\text{-}45)$$
$$\qquad\qquad\quad - \ -\qquad\quad +\qquad\quad +$$

The actual inventory of H1 at the end of the period is given by:

$$H1_{t+1} = H1_t + H1^s - H1HH. \qquad (2\text{-}46)$$

Optimal Economic Behavior for IS Industry

The IS industry buys P1 and combines P1 and RP in fixed proportions. Since RP is "free" the industry would use only RP if it could produce IS using variable proportions of P1 and RP. P1 and RP are combined with a variable amount of labor that is used to transform P1 and RP into IS. This arrangement is slightly more complicated than the H1 industry, since both P1 and RP vary. The IS industry buys P1, combines it with RP using variable labor and a fixed amount of capital to produce IS, which it sells to the households. Therefore the production function for IS may be written as:

$$IS = \min[P1IS/\theta, RP/\lambda, IS(h_{IS}N_t, K^{IS}_t)] \quad (2\text{-}47)$$

where θ denotes the amount of P1 necessary to produce a unit of IS and λ denotes the amount of RP necessary to produce a unit of IS. The function $IS(\cdot)$ denotes the transformation services provided by labor and capital in the process of producing a marketable unit of IS.

Assuming that the IS industry wastes neither the transformation services provided by labor and capital, nor P1 nor RP, the number of units of IS available to the market during the current period may be written as:

$$IS = IS(h_{IS}N_t, K^{IS}_t) \quad (2\text{-}48)$$

where,

$$P1IS = \theta \cdot IS(h_{IS}N_t, K^{IS}_t) \quad (2\text{-}49)$$
$$RPIS = \lambda \cdot IS(h_{IS}N_t, K^{IS}_t) \quad (2\text{-}50)$$

In terms of mass, the stock of IS at the end of the current period is given by:

$$(\theta+\lambda)IS_{t+1} = (\theta+\lambda)IS_t + (\theta+\lambda)\cdot IS(h_{IS}N_t, K^{IS}_t) - (\theta+\lambda)\cdot ISHH. \quad (2\text{-}51)$$

where ISHH denotes the number of units of IS purchased by the households and $(\theta+\lambda)\cdot ISHH$ represents the amount of IS purchased by the households during the period in terms of mass.

Assuming that the IS industry sets the current wage rate, the assumption is that the industry formulates an anticipated net supply of labor function, N^S, which presumably reflects the industry's estimate of (a) the amount of labor the household sector is willing to supply to the labor market during the current period at each money wage, W, less (b) the amount of labor that the P1 and H1 industries demand at each money wage. It is assumed that the estimated net supply function is given by:

$$(h_{IS}N)^{se} = (h_{IS}N)^{se}(W) \qquad (2\text{-}52)$$
$$+$$

Taking the industry's estimated labor supply function and its forecast of the household sector's current period demand function for IS as given, the IS firms attempt to maximize the dividends to their shareholders, subject to the restriction that they maintain the end-of-period stock of IS at some predetermined level, $\bar{I}\ \bar{S}$. The firm's desired sales of IS (in terms of units of IS) is given by:

$$(ISHH)^{de} = IS[(h_{IS}N)^{se}(W), K^{IS}_t] - (\bar{I}\ \bar{S} - IS_t) \qquad (2\text{-}53)$$

Therefore, the objective of the IS industry is to maximize:

$$\Pi_{IS} = p_{IS}\cdot\{IS[(h_{IS}N)^{se}(W), K^{IS}_t] - (\bar{I}\ \bar{S} - IS_t)\} - W\cdot(h_{IS}N)^{se}(W) - F_{IS} - p_{P1}\cdot P1IS \qquad (2\text{-}54)$$

or:

$$\Pi_{IS} = p_{IS}\cdot\{IS[(h_{IS}N)^{se}(W), K^{IS}_t] - (\bar{I}\ \bar{S} - IS_t)\} - W\cdot(h_{IS}N)^{se}(W) - F_{IS} - p_{P1}\cdot\theta\cdot IS[(h_{IS}N)^{se}(W), K^{IS}_t] \qquad (2\text{-}55)$$

where F_{IS} represents the industry's fixed cost.

The product price, p_{IS}, is announced by the IS industry at the beginning of the period. Let the IS industry's forecast of the current period demand for its product be given by the following demand function:

$$ISHH^{de} = (ISHH/N)^{de}(p_{IS}, \omega_{ISHH})\cdot N_t \qquad (2\text{-}56)$$
$$-\quad +$$

This demand function denotes the IS industry's forecast of the households' demand for IS. It is assumed (it will be verified later when the economic behavior of the households is specified) that the derivative of $(ISHH/N)^{de}$ with respect to p_{IS} is negative; ω_{ISHH} denotes a vector of shift parameters. Solving the inverse function for p_{IS} yields:

$$p_{IS} = p_{IS}(ISHH^{de}, \omega_{ISHH}, N_t) \qquad (2\text{-}57)$$
$$-\quad +\quad +$$

where:

$$\partial p_{IS}/\partial(ISHH^{de}) \equiv 1/\{[\partial(ISHH/N)^{de}/\partial p_{IS}]\cdot N_t\} < 0$$
$$\partial p_{IS}/\partial\omega_{ISHH} \equiv -[\partial(ISHH/N)^{de}/\partial\omega_{ISHH}]/\{[\partial(ISHH/N)^{de}/\partial p_{IS}]\cdot N_t\} > 0$$

and
$$\partial p_{IS}/\partial N_t \equiv -(ISHH/N)^{de}/\{[\partial(ISHH^{de})/\partial p_{IS}]\cdot N_t\} > 0$$

and where a rise in ω_{ISHH} or N_t represents an outward shift in the demand for IS function

facing the IS industry.

The price p_{IS} given by (2-57) represents the maximum uniform price the IS sector expects it can value each alternative amount of IS (in units) it produces during the period. Based upon this function, the sector's forecasted current revenue function is given by:

$$R_{IS}^{e} = p_{IS} \cdot ISHH^{de} = p_{IS}(ISHH^{de}, \omega_{ISHH}, N_t) \cdot ISHH^{de} = R_{IS}^{e}(ISHH^{de}, \omega_{ISHH}, N_t) \qquad (2\text{-}58)$$

Substituting the right hand side of (53) for $ISHH^{de}$ in (58) yields:

$$R_{IS}^{e} = R_{IS}^{e}[IS[(h_{IS}N)^{se}(W) , K^{IS}{}_t] \quad -(\bar{I}\ \bar{S} - IS_t), \omega_{ISHH}, N_t] \qquad (2\text{-}59)$$

Substituting the right-hand-side of (2-59) for the first term on the right hand side of (2-55) yields:

$$\begin{aligned}\Pi_{IS} \ &= R_{IS}^{e}[IS[(h_{IS}N)^{se}(W) , K^{IS}{}_t] \quad -(\bar{I}\ \bar{S} - IS_t), \omega_{ISHH}, N_t] \\ &\quad -W \cdot (h_{IS}N)^{se}(W) - F_{IS} \ - p_{P1} \cdot \theta \cdot IS[(h_{IS}N)^{se}(W) , K^{IS}{}_t]\end{aligned} \qquad (2\text{-}60)$$

Given $\bar{I}\ \bar{S}$, the only choice variable for the IS industry in this simplified model consists of the wage rate that it announces to attract the appropriate number of employees to transform P1 and RP into IS products; the amount of the intermediate good P1 the IS sector buys, P1IS, must then conform to the amount of mass necessary to produce that amount of IS. Maximizing (2-60) with respect to W yields the necessary condition:

$$\begin{aligned}&[\partial R_{IS}^{e}/\partial(ISHH^{de})] \cdot [\partial(IS)/\partial(h_{IS}N_t)] \cdot [\partial(h_{IS}N_t)/\partial W] \\ &\quad = (h_{IS}N)^{se}(W) + W \cdot [\partial(h_{IS}N_t)/\partial W] + p_{P1} \cdot \theta \cdot [\partial(IS)/\partial(h_{IS}N_t)] \cdot [\partial(h_{IS}N_t)/\partial W]\end{aligned} \qquad (2\text{-}61)$$

The left hand side of (2-61) represents the marginal revenue product of labor in the IS industry in terms of transforming P1 and RP into IS; the condition requires that the profit-maximizing firms in the industry continue to raise the wage rate, and therefore continue to hire labor up to the point at which the marginal revenue product of from the extra labor induced by the higher wage rate equals the marginal cost of raising the money wage, $(h_{IS}N)^{se}(W) + W \cdot [\partial(h_{IS}N_t)/\partial W]$, plus the marginal cost of adding as well the requisite amount of P1 to the production process to obtain the extra IS. The extra amount of RP that must accompany the extra P1 is considered to be a free good and is given by (2-50).

Assuming a diminishing marginal product of labor and that marginal revenue decreases as the number of units sold increases, then from (2-61), the profit maximizing wage to be announced by the IS industry is a decreasing function of the price of p_{P1} and an increasing function of the shift parameters, $(\bar{I}\ \bar{S} - IS_t)$ and ω_{ISHH}. The money wage function for the IS industry becomes:

$$\begin{aligned}W = W(\ & p_{P1} \cdot \theta, \ (\bar{I}\ \bar{S} - IS_t), \ \omega_{ISHH}, \ N_t \ ) \\ & - \qquad + \qquad + \qquad +\end{aligned} \qquad (2\text{-}62)$$

Substituting the right hand side of (2-62) into (2-53) yields the amount of IS the industry

desires to supply to the market during the current period:

$$(ISHH)^s = IS[(h_{IS}N)^{se}[W(p_{P1}\cdot\theta, (\bar{I}\ \bar{S} - IS_t), \omega_{ISHH})], K^{IS}{}_t] \quad -(\bar{I}\ \bar{S} - IS_t) \tag{2-63}$$

Substituting the right hand side of (2-63) into (2-57) yields the price p_{IS} announced by the IS industry:

$$p_{IS} = p_{IS}(IS[(h_{IS}N)^{se}[W(p_{P1}\cdot\theta, (\bar{I}\ \bar{S} - IS_t), \omega_{ISHH}, N_t)], K^{IS}{}_t], \omega_{ISHH}, N_t) \tag{2-64}$$

In terms of mass, the stock of IS at the end of the period is given by:

$$(\theta+\lambda)\cdot IS_{t+1} = (\theta+\lambda)\cdot IS_t + (\theta+\lambda)\cdot IS[(h_{IS}N)^{se}[W(p_{P1}\cdot\theta, (\bar{I}\ \bar{S} - IS_t), \omega_{ISHH}, N_t)], K^{IS}{}_t] - (\theta+\lambda)\cdot ISHH. \tag{2-65}$$

From (2-49) and (2-50), the amount of mass transferred from P1 and RP to IS is then given by:

$$P1IS = \theta\cdot IS((h_{IS}N_t)^d(W, K^{IS}{}_t, p_{P1}\cdot\theta, (\bar{I}\ \bar{S} - IS_t), \omega_{ISHH}, N_t), K^{IS}{}_t) \tag{2-66}$$

$$RPIS = \lambda\cdot IS((h_{IS}N_t)^d(W, K^{IS}{}_t, p_{P1}\cdot\theta, (\bar{I}\ \bar{S} - IS_t), \omega_{ISHH}, N_t), K^{IS}{}_t) \tag{2-67}$$

Government Sector

The only revenue the government sector receives is from the grazing rights it grants to the H1 industry equal to $p_{P2}\cdot P2H1$. For the time being, the assumption is that the government transfers this revenue to the households:

$$Tr = p_{P2}\cdot P2H1. \tag{2-68}$$

GDP

The level of nominal GDP is defined as the total market value of all units of final goods produced in the economy during the period, which also corresponds to the sum of all spending on final goods plus the values of the changes in inventories of all goods:

$$GDP = p_{P1}\cdot\{P1HH\} + p_{H1}\cdot\{H1HH\} + p_{IS}\cdot\{ISHH\} + p_{P1}(P1_{t+1} - P1_t) + p_{H1}(H1_{t+1} - H_t) + p_{IS}(IS_{t+1} - IS_t) \tag{2-69}$$

Optimal Economic Behavior of the Household Sector

In the present section the household sector's market demands for P1, H1, and IS as well as its market supply of labor is obtained. It is assumed that the objective of the household sector is to maximize current utility, which is assumed to be a positive increasing

function of consumption per capita of P1, H1, IS and leisure, *l*. It is assumed that marginal utility is positive, but decreasing. For simplicity, cross-partials are ignored.

Maximize:

$$U = U(P1HH/N_t\ ,\ H1HH/N_t\ ,\ ISHH/N_t,\ l\ ) \tag{2-70}$$

The households attempt to maximize utility subject to their budget constraint. This constraint stipulates that total household spending on all three products equals its current income, consisting of wage and non-wage income. It is assumed that the households take their current non-wage incomes as given, but attempt to choose their wage income. The households are not permitted to make adjustments based on how their current non-wage income may be affected by their work/leisure choice, or by the market basket of goods they decide to purchase. In terms of the abstract unit of account, the budget constraint facing the household sector is given by:

$$\Pi + Tr + W\cdot(h^{*} - l)\cdot N_t = p_{P1}\cdot(P1HH/N_t)\cdot N_t + p_{H1}\cdot(H1HH/N_t)\cdot N_t + p_{IS}\cdot(ISHH/N_t)\cdot N_t \tag{2-71}$$

where:

$$\Pi = \Pi_{PP1} + F_{PP1} + \Pi_{H1} + F_{H1} + \Pi_{IS} + F_{IS}. \tag{2-72}$$

Equation (2-70) is rewritten by dividing all terms by WN_t, thereby expressing the terms in the constraint in units of labor per capita:

$$(\Pi + Tr)/(W\cdot N_t) + (h^{*} - l) = (p_{P1}/W)\cdot(P1HH/N_t) + (p_{H1}/W)\cdot(H1HH/N_t) + (p_{IS}/W)\cdot(ISHH/N_t). \tag{2-73}$$

The following Lagrangean function is set up:

$$\begin{aligned}\text{Max } \mathbf{U} &= U(P1HH/N_t\ ,\ H1HH/N_t\ ,\ ISHH/N_t,\ l\ ) \\ &+\lambda\{(\Pi+Tr)/(W\cdot N_t) + (h^{*} - l) - (p_{P1}/W)\cdot(P1HH/N_t) - (p_{H1}/W)\cdot(H1HH/N_t) \\ &\qquad - (p_{IS}/W)\cdot(ISHH/N_t)\}\end{aligned} \tag{2-74}$$

The first-order necessary conditions for the maximization of (2-74) with respect to $P1HH/N_t$, $H1HH/N_t$, $ISHH/N_t$ and *l* are given by (2-75)–(2-78). From (2-71), the sum inside braces in (2-74) must equal zero; this restriction is added as condition (2-79):

$$\partial U/\partial(P1HH/N_t) - \lambda\,(p_{P1}/W) = 0. \tag{2-75}$$

$$\partial U/\partial(H1HH/N_t) - \lambda\,(p_{H1}/W) = 0. \tag{2-76}$$

$$\partial U/\partial(ISHH/N_t) - \lambda\,(p_{IS}/W) = 0. \tag{2-77}$$

$$\partial U/\partial l - \lambda = 0. \tag{2-78}$$

$$\begin{aligned}&(\Pi+Tr)/(W\cdot N_t) + (h^{*} - l) - (p_{P1}/W)\cdot(P1HH/N_t) \\ &\qquad - (p_{H1}/W)\cdot(H1HH/N_t) - (p_{IS}/W)\cdot(ISHH/N_t) = 0.\end{aligned} \tag{2-79}$$

According to (2-75)–(2-79), only non-wage income (in labor units, $(\Pi+Tr)/W$), and the relative prices, p_{P1}/W, p_{H1}/W, and p_{IS}/W (expressed here in labor units as well) matter in

terms of the optimal combinations of l, P1HH/N_t, H1HH/N_t and ISHH/N_t selected by the households. The budget constraint, expression (2-79), is homogeneous of degree zero in W, Π+Tr, p_{P1}, p_{H1} and p_{IS}. A doubling of all prices (including W) and non-wage income is consistent with the same combination of l, P1HH/N_t , H1HH/N_t and ISHH/N_t. Taking the total differentials of each of (2-75)–(2-79) yields the following system of equations, which are written in matrix form as (2-80):

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} & -(p_{P1}/W) \\ u_{21} & u_{22} & u_{23} & u_{24} & -(p_{H1}/W) \\ u_{31} & u_{32} & u_{33} & u_{34} & -(p_{IS}/W) \\ u_{41} & u_{42} & u_{43} & u_{44} & -1 \\ -(p_{P1}/W) & -(p_{H1}/W) & -(p_{IS}/W) & -1 & 0 \end{bmatrix} \begin{bmatrix} d(P1HH/N_t) \\ d(H1HH/N_t) \\ d(ISHH/N_t) \\ d(l) \\ d\lambda \end{bmatrix} \tag{2-80}$$

$$= \begin{bmatrix} \lambda\, d(p_{P1}/W) \\ \lambda\, d(p_{H1}/W) \\ \lambda\, d(p_{IS}/W) \\ 0 \\ -d[(\Pi+Tr)/(W\cdot N_t)] + (P1HH/N_t)\cdot d(p_{P1}/W) + (H1HH/N_t)\cdot d(p_{H1}/W) + (ISHH/N_t)\cdot d(p_{IS}/W) \end{bmatrix}$$

The coefficients u_{ij} denote the second partials of utility; the own-second partials, u_{ii}, are assumed to be negative. For simplicity, the cross-partials are treated as small relative to the own-partials as the signs of the effects of the parametric changes upon the choice variables are determined. Pre-multiplying the column vector on the right hand side of (2-79) by the inverse of the bordered Hessian matrix yields total differentials of the household sector's demands for P1HH/N_t, H1HH/N_t, ISHH/N_t and l. Assuming that the substitution effects dominate the income effects, these total differentials indicate the following responses in the household sector's demand functions for P1HH/N_t, H1HH/N_t, ISHH/N_t and l respectively:

$$(P1HH/N_t)^d = (P1HH/N)^d[(\Pi+Tr)/WN_t,\ p_{P1}/W,\ p_{H1}/W,\ p_{IS}/W) \tag{2-81}$$
$$\qquad\qquad\qquad\qquad +\quad -\quad +\quad +$$

$$(H1HH/N_t)^d = (H1HH/N)^d(\Pi+Tr/WN_t,\ p_{P1}/W,\ p_{H1}/W,\ p_{IS}/W) \tag{2-82}$$
$$\qquad\qquad\qquad\qquad +\quad +\quad -\quad +$$

$$(ISHH/N_t)^d = (ISHH/N)^d(\underset{+}{\Pi+Tr/WN_t},\ \underset{+}{p_{P1}/W},\ \underset{+}{p_{H1}/W},\ \underset{-}{p_{IS}/W}) \qquad (2\text{-}83)$$

$$l^d = l^d(\underset{+}{\Pi/WN_t},\ \underset{+}{p_{P1}/W},\ \underset{+}{p_{H1}/W},\ \underset{+}{p_{IS}/W}) \qquad (2\text{-}84)$$

Expression (2-79) imposes restrictions on the responses in (2-81)–(2-84). From (2-79), and the fact that the hours spent working, h, equals total hours in the period, h^*, minus leisure time, l, the household sector's per capita supply of labor, h^s, is given by (2-85):

$$h^s = h^s[\underset{-}{(\Pi+Tr)/WN_t},\ \underset{-}{p_{P1}/W},\ \underset{-}{p_{H1}/W},\ \underset{-}{p_{IS}/W}). \qquad (2\text{-}85)$$

Substituting (2-81)–(2-84) into constraint (2-79) yields:

$$\begin{aligned}&(\Pi+Tr)/(W\cdot N_t) + h^* - l^d((\Pi+Tr)/WN_t,\ p_{P1}/W,\ p_{H1}/W,\ p_{IS}/W)\\ &- (p_{P1}/W)\cdot(P1HH/N)^d((\Pi+Tr)/WN_t,\ p_{P1}/W,\ p_{H1}/W,\ p_{IS}/W)\\ &-(p_{H1}/W)\cdot(H1HH/N)^d((\Pi+Tr)/WN_t,\ p_{P1}/W,\ p_{H1}/W,\ p_{IS}/W)\\ &-(p_{IS}/W)\cdot(ISHH/N)^d((\Pi+Tr)/WN_t,\ p_{P1}/W,\ p_{H1}/W,\ p_{IS}/W) \quad = 0. \qquad (2\text{-}86)\end{aligned}$$

Since (2-85) must hold identically, the following restrictions apply:

$$\begin{aligned}1 \equiv\ & \partial l^d/\partial(\Pi/WN_t) + (p_{P1}/W)\cdot[\partial(P1HH/N)^d/\partial(\Pi/WN_t)]\\ &+(p_{H1}/W)\cdot[\partial(H1HH/N)^d/\partial(\Pi/WN_t)] + (p_{IS}/W)\cdot[\partial(ISHH/N)^d/\partial(\Pi/WN_t)] \qquad (2\text{-}87)\\ 0 \equiv\ & \partial l^d/\partial(p_{P1}/W) + (p_{P1}/W)\cdot[\partial(P1HH/N)^d/\partial(p_{P1}/W)] + (P1HH)^d\\ &+(p_{H1}/W)\cdot[\partial(H1HH/N)^d/\partial(p_{P1}/W) + (p_{IS}/W)\cdot[\partial(ISHH/N)^d/\partial(p_{P1}/W)] \qquad (2\text{-}88)\\ 0 \equiv\ & \partial l^d/\partial(p_{H1}/W) + (p_{P1}/W)\cdot[\partial(P1HH/N)^d/\partial(p_{H1}/W)]\\ &+(p_{H1}/W)\cdot[\partial(H1HH/N)^d/\partial(p_{H1}/W) + (H1HH)^d + (p_{IS}/W)\cdot[\partial(ISHH/N)^d/\partial(p_{H1}/W)] \qquad (2\text{-}89)\\ 0 \equiv\ & \partial l^d/\partial(p_{IS}/W) + (p_{P1}/W)\cdot[\partial(P1HH/N)^d/\partial(p_{IS}/W)]\\ &+(p_{H1}/W)\cdot[\partial(H1HH/N)^d/\partial(p_{IS}/W) + (p_{IS}/W)\cdot[\partial(ISHH/N)^d/\partial(p_{IS}/W)] + (ISHH)^d \qquad (2\text{-}90)\end{aligned}$$

Gross Domestic Product

As shown in expression (2-69), repeated here for convenience, nominal gross domestic product, GDP, is defined as the total market value of all units of final goods and services produced during a period of time. In the present context, the market value of the goods the households purchase from the private firms plus the changes in inventories of all goods represent nominal GDP:

$$\begin{aligned}GDP = &\ p_{P1}\cdot\{P1HH\} + p_{H1}\cdot\{H1HH\} + p_{IS}\cdot\{ISHH\}\\ &+ p_{P1}(P1_{t+1} - P1_t) + p_{H1}(H1_{t+1} - H_t) + p_{IS}(IS_{t+1} - IS_t) \qquad (2\text{-}69)\end{aligned}$$

Profits are defined as value added in production minus current expenses. In this model, current expenses consist of wages, rental income to the owners of physical capital ("fixed cost"), and fees paid to the government for grazing rights. Therefore, profits plus wages, plus rental income to the owners of physical capital, and fees paid to the government for

grazing rights are equal to the value added in production (value of the goods produced in a stage of production minus the purchase of intermediate goods entering that stage). In this model, the fees paid by the H1 industry for grazing rights are returned to the households as transfer payments. Therefore, the sum of profits plus wages plus rental incomes plus transfers is equal to the value of production minus the purchase of intermediate goods. Since the value of production coincides with the value of sales plus the change in inventories, the following three relationships hold for the P1, H1, and IS industries respectively:

$$\Pi^*_{P1} + F_{P1} + W\cdot[h_{P1}\cdot N_t + h^*_{P1}\cdot N_t] = p_{P1}\cdot\{P1HH + P1H1 + P1IS + (P1_{t+1} - P1_t)\} \quad (2\text{-}91)$$

$$\Pi^*_{H1} + F_{H1} + W\cdot[h_{H1}N_t + h^*_{H1}N_t] + Tr = p_{H1}\cdot\{H1HH + (H1_{t+1} - H_t)\} - p_{P1}\cdot P1H1 \quad (2\text{-}92)$$

$$\Pi^*_{IS} + F_{IS} + W\cdot(h_{IS}N) = p_{IS}\cdot\{ISHH + (IS_{t+1} - IS_t)\} - p_{P1}\cdot P1IS \quad (2\text{-}93)$$

Summing across (91)–(93) yields:

$$\Pi^{**} + Tr + WhN_t = p_{P1}\cdot\{P1HH + (P1_{t+1} - P1_t)\} + p_{H1}\cdot\{H1HH + (H1_{t+1} - H_t)\} + p_{IS}\cdot\{ISHH + (IS_{t+1} - IS_t)\} \quad (2\text{-}94)$$

Since the right hand side of (2-94) corresponds to the right hand side of (2-69), nominal GDP is also equal to total non-wage income, Π^{**}, plus transfers plus total wages:

$$GDP = \Pi^{**} + Tr + W\cdot h\cdot N. \quad (2\text{-}95)$$

Note that non-wage income corresponds to profits plus the rental income to owners:

$$\Pi^{**} = \Pi^*_{P1} + \Pi^*_{H1} + \Pi^*_{IS} + F_{P1} + F_{H1} + F_{IS} \quad (2\text{-}96)$$

Profits of the individual industries correspond to the dividends they pay to the households plus net business saving (additions to retained earnings). In this model net business saving corresponds to the change in inventories. Therefore,

$$\Pi^*_{P1} = \Pi_{P1} + p_{P1}\cdot(P1_{t+1} - P1_t) \quad (2\text{-}97)$$

$$\Pi^*_{H1} = \Pi_{H1} + p_{H1}\cdot(H1_{t+1} - H1_t) \quad (2\text{-}98)$$

$$\Pi^*_{IS} = \Pi_{IS} + p_{IS}\cdot(IS_{t+1} - IS_t) \quad (2\text{-}99)$$

Real GDP, or RGDP, written in terms of units of goods, is given by:

$$RGDP = P1HH + H1HH + ISHH + (P1_{t+1} - P1_t) + (H1_{t+1} - H_t) + (IS_{t+1} - IS_t) \quad (2\text{-}100)$$

Naturally Occurring Changes in Biological and Physical Resources

Growth of Non-domesticated Plants, P2

The production (natural growth) of non-domesticated plant P2 during the current period

is viewed as proportional to both the volume of P2 at the beginning of the current period and the size of the resource pool at the beginning of the current period. In addition, since plant P2 is able to draw mass from the inaccessible resource pool, plant P2's growth is also proportional to the size of the inaccessible resource pool as well. But the growth of P2 is diminished by the amount of the plant eaten by the domesticated, H1, and non-domesticated herbivores, H2 and H3, during the period. Net growth of P2 is shown by (2-101).

$$P2 = P2_t \cdot (g_{P2} \cdot RP_t + g_{IRPP2} \cdot IRP_t) - m_{P2} \cdot P2_t - P2H1 - H2_t \cdot g_{H2} \cdot P2_t - H3_t \cdot g_{P2H2} \cdot P2_t. \quad (2\text{-}101)$$

The volume of P2 at the end of the current period is equal to the volume at the beginning of the current period, plus the net natural growth of P2, minus the amount of the plant eaten by the domesticated and non-domesticated herbivores, H2 and H3, during the period:

$$P2_{t+1} = P2_t + P2_t \cdot (g_{P2} \cdot RP_t + g_{IRPP2} \cdot IRP_t) - m_{P2} \cdot P2_t - P2H1 - H2_t \cdot g_{H2} \cdot P2_t - H3_t \cdot g_{P2H2} \cdot P2_t. \quad (2\text{-}102)$$

Growth of Non-domesticated Herbivores, H2

The production (natural growth) of H2 during the current period is viewed as proportional to both the amount of H2 at the beginning of the current period and the amount of plant P2 at the beginning of the current period. In addition the growth of H2 is increased by the amount of P1 that H2 eats during the current period, but diminished by the amount of H2 eaten by C1 and C2. Net growth is given by (2-103):

$$H2 = P1H2 + H2_t \cdot g_{H2} \cdot P2_t - m_{H2} \cdot H2_t - H2C1 - C2_t \cdot g_{H3C2} \cdot H3_t \quad (2\text{-}103)$$

and the end-of-period stock of H2 is given by:

$$H2_{t+1} = H2_t + P1H2 + H2_t \cdot g_{H2} \cdot P2_t - m_{H2} \cdot H2_t - H2C1 - C2_t \cdot g_{H3C2} \cdot H3_t. \quad (2\text{-}104)$$

Growth of Non-domesticated Carnivores, C1

The mass of C1 increases during the period by the amount of mass that C1 eats of H1, $c_{H1C1} \cdot C1_t$, and H2, H2C1; its mass decreases by its natural mortality, $m_{C1} \cdot C1_t$. Net growth of C1 during the period is represented by (2-105) and the end-of-period stock of C1 is given by (2-106):

$$C1 = c_{H1C1} \cdot C1_t + H2C1 - m_{C1} \cdot C1_t \quad (2\text{-}105)$$

and the end-of-period stock of C1 is given by:

$$C1_{t+1} = C1_t + c_{H1C1} \cdot C1_t + H2C1 - m_{C1} \cdot C1_t \quad (2\text{-}106)$$

Growth of Non-domesticated Plants, P3

The production (natural growth) of non-domesticated plant P3 during the current period is viewed as proportional to both the volume of P3 at the beginning of the current period and the size of the resource pool at the beginning of the current period. In addition, since plant P3 is able to draw mass from the inaccessible resource pool, plant P3's growth is also proportional to the size of the inaccessible resource pool as well. The growth in P3 is diminished by the amount of the plant eaten by the non-domesticated herbivores H3 during the period. Net growth of P3 is shown by (2-107):

$$P3 = P3_t \cdot (g_{P3} \cdot RP_t + g_{IRPP3} \cdot IRP_t) - m_{P3} \cdot P3_t - H3_t \cdot g_{P3H3} \cdot P3_t. \quad (2\text{-}107)$$

The volume of P3 at the end of the current period is equal to the volume at the beginning of the current period plus the net natural growth of P3, minus the amount of the plant eaten by the non-domesticated herbivores H3 during the period:

$$P3_{t+1} = P3_t + P3_t \cdot (g_{P3} \cdot RP_t + g_{IRPP3} \cdot IRP_t) - m_{P3} \cdot P3_t - H3_t \cdot g_{P3H3} \cdot P3_t. \quad (2\text{-}108)$$

Growth of Non-domesticated Herbivores, H3

The production (natural growth) of H3 during the current period is viewed as proportional to both the amount of H3 at the beginning of the current period and the amounts of plants P2 and P3 at the beginning of the current period. In addition, the growth of H3 is diminished by the amount of H3 eaten by C2. Net growth is given by (109):

$$H3 = H3_t \cdot (g_{P2H3} \cdot P2_t + g_{P3H3} \cdot P3_t) - m_{H3} \cdot H3_t - C2_t \cdot g_{H3C2} \cdot H3_t \quad (2\text{-}109)$$

and the end-of-period stock of H3 is given by:

$$H3_{t+1} = H3_t + H3_t \cdot (g_{P2H3} \cdot P2_t + g_{P3H3} \cdot P3_t) - m_{H3} \cdot H3_t - C2_t \cdot g_{H3C2} \cdot H3_t \quad (2\text{-}110)$$

Growth of Non-domesticated Carnivores, C2

The mass of C2 increases during the period by the amount of mass that C2 eats of H2 and H3; its mass decreases by its natural mortality, $m_{C2} \cdot C2_t$. Net growth of C2 during the period is represented by (2-111) and the end-of-period stock of C1 is given by (2-112):

$$C2 = C2_t \cdot (g_{H2C2} \cdot H2_t + g_{H3C2} \cdot H3_t) - m_{C2} \cdot C2_t \quad (2\text{-}111)$$

and the end-of-period stock of C2 is given by:

$$C2_{t+1} = C2_t + C2_t \cdot (g_{H2C2} \cdot H2_t + g_{H3C2} \cdot H3_t) - m_{C2} \cdot C2_t \quad (2\text{-}112)$$

Growth in Human Population and Growth in Human Mass

Human population at the end of the current period is equal to the human population at the beginning of the current period, plus the number of people born during the current period, minus human mortality during the period. The human birthrate, η, during the period is assumed to be a negative function of the real wage prevailing during the current period. The rationale for this assumption is that the real wage represents the opportunity cost of opting to remain outside the labor force (at least part-time) for the purpose of rearing children.

$$N_{t+1} = N_t + [\eta(W/P^*) - m_N] \cdot N_t + P1HH + H1HH \qquad (2\text{-}113)$$

where P^* denotes a weighted average of the prices the households pay for P1, H1, and IS:

$$P^* = [p_{P1} \cdot P1HH + p_{H1} \cdot H1HH + p_{IS} \cdot ISHH] / [P1HH + H1HH + ISHH] \qquad (2\text{-}114)$$

The net addition to human mass is equal to human consumption of plants P1 and herbivores H1 during the current period, minus human mortality in terms of mass during the current period. The net growth in human mass, $N^{\wedge}$, is given by (2-115):

$$N^{\wedge}_{t+1} = N^{\wedge}_t + P1HH + H1HH - m_N \cdot N^{\wedge}_t \qquad (2\text{-}115)$$

Mass per human at time t, μ_t , is given by:

$$\mu_t = N^{\wedge}_t / N_t \qquad (2\text{-}116)$$

Resource Pool

The size of the resource pool at the end of the current period is equal to its size at the beginning of the current period, plus the mass flowing to the resource pool generated by plant and animal mortality, minus the amount by which the resource pool declines during the current period in direct response to the growth in the domesticated and non-domesticated plants during the current period. In addition the resource pool increases over time because of mass transfer from the inaccessible resource pool. This is assumed to be in proportion to the size of the inaccessible resource pool. Finally, mass is transferred from the accessible resource pool to the inaccessible resource pool in direct proportion to the level of production by the IS sector.

$$RP_{t+1} = RP_t + m_{P1} \cdot P1_t + m_{P2} \cdot P2_t + m_{H1} \cdot H1_t + m_{H2} \cdot H2_t + m_{C1} \cdot C1_t + m_N \cdot N_t - P1_t \cdot g_{P1} \cdot RP_t - P2_t \cdot g_{P2} \cdot RP_t + r_{IRPRP} \cdot IRP_t - \lambda \cdot IS. \qquad (2\text{-}117)$$

Inaccessible Resource Pool

The size of the inaccessible resource pool at the end of the current period is equal to its size at the beginning of the current period, plus the volume of waste contributed to the pool due to the scale of operations of the IS industry, minus the amount that the non-domesticated plants P2 are able to recycle from the inaccessible resource pool during the current period and minus the amount of natural decay that moves mass directly from the inaccessible resource pool to the resource pool.

$$IRP_{t+1} = IRP_t + (\theta+\lambda)\cdot ISHH - P2_t \cdot g_{IRPP2} \cdot IRP_t - r_{IRPRP} \cdot IRP_t. \quad (2\text{-}118)$$

Chapter 3

Model Solution and Operational Equations

This chapter provides a model solution based on the assumptions that firms attempt to maximize the difference between the sale of their products less the costs (of materials and labor), while households maximize their utility (incoming flow of goods balanced against leisure). The details and operational equations for the solution are described here.

Model Solution

Each firm (P1, H1, IS) seeks to maximize its profit (income minus wages paid and purchases of supplies). These industries set their prices in accordance with their current forecasts of demand for their products before the market clears. A price setting mechanism is used so that trades occur even if markets fail to clear (this is in contrast to a general equilibrium setting). Excess inventory is carried to the next time step, and shortfalls made up with current inventory. It is assumed that the industrial sector sets the money wage rate at the beginning of the current period. In this solution, one of the industries, say the IS industry, becomes the dominant industry in the labor market. Before any other price is announced, it sets the wage rate, W, at the rate at which it anticipates that the amount of labor supplied, net of the sum of the amounts of labor demanded by the P1 and H1 industries, will be sufficient to provide the level of labor services the IS industry demands. There are several approaches of constructing a solution to the model. For example, one could use P1 or H1 as the dominant industries for setting wages. However, IS was chosen as the dominant industry because in most modern societies, wages tend to be set by the industrial or service sectors rather than the agricultural sectors which PI and H1 represent.

Once the IS industry has announced the wage rate, all industries and the households take that wage rate as given. Given the wage rate, the P1, H1 and IS industries decide the amount of labor that they want to hire, and the households decide the amount of labor they wish to supply to the labor market. It is assumed that the supply of labor exceeds the market demand for labor. Thus the amount of labor hired by each industry is the amount that each industry demands, given the wage rate announced by the IS industry. The three industries simultaneously decide how much labor to hire, decide how much output to produce and announce the prices of their respective products, in light of the money wage set by the IS industry. At the same time, the IS and H1 industries decide their demands for P1. The wage incomes the households receive from the three industries, plus their non-wage incomes from the three industries, plus the transfers the households receive from the government, plus the revenue from the industries' fixed costs, equals the nominal GDP of the aggregate economy. From (2-95), in real terms, GDP and real disposable income, also amounts to the sum: P1HH + H1HH +ISHH. Given their real disposable incomes, and the prices the industries set for their products, the households then decide their demands for the three products P1, H1, and IS. In each

case, the end of period stocks of P1, H1 and IS correspond to the amount of the product produced during the period minus the amount of the item purchased by the various sectors.

Operational Equations

Linear forms of demand and supply functions are used to simplify calculation. The following are consistent with the optimization goal of the firms and the households. Firms attempt to maximize the difference between the sale of their products less the costs (of materials and labor), while households maximize their utility (incoming flow of goods balanced against leisure). The planning horizon in the current model is one time step.

The model has 19 state variables and 33 outputs. The state variables are given in Table 3-1. These are in terms of mass, with the exception of the human population. The deficits refer to the accumulated difference between the demand for a good and the actual quantity of that good delivered in a given time step. Deficits are restricted to being non-positive. Any surplus of a good increases the inventory of that good. The compartment masses are restricted to being non-negative. In addition to the system state variables, 14 other internal system flows are output. These additional outputs are given in Table 3-2. All system flows are restricted to being non-negative.

Table 3-1 State variables, on a mass basis unless otherwise indicated.

1	P1	8	C2	15	P1 deficit for HH ($P1HH^{def}$)
2	P2	9	Humans (HH)	16	H1 deficit for HH ($H1HH^{def}$)
3	P3	10	IS	17	IS deficit for HH ($ISHH^{def}$)
4	H1	11	RP	18	Number of Humans (N_{HH})
5	H2	12	IRP	19	Human per cap mass (μ)
6	H3	13	P1 deficit for H1 ($P1H1^{def}$)		
7	C1	14	P1 deficit for IS ($P1IS^{def}$)		

In what follows, the sum of total deficits of either P1, H1 or IS and their current inventories represents what the industry must make up in production for past over-demand (or under-supply), in addition to what must be produced to meet the current demand. These total deficits to be made up are designated with a star superscript in the following ($P1_t^*$, $H1_t^*$, IS_t^*):

$$P1_t^* = P1_t + P1H1_t^{def} + P1IS_t^{def} + P1HH_t^{def}$$
$$H1_t^* = H1_t + H1HH_t^{def}$$
$$IS_t^* = IS_t + ISHH_t^{def}$$

Table 3-2. Additional model outputs

20	RPP1	25	RPIS	30	pP1
21	P1H2	26	ISHH	31	pH1
22	P1IS	27	P2H1	32	pIS
23	P1H1	28	H1C1	33	W
24	P1HH	29	H1HH		

At a given time step *t* the IS industry first sets the wage (W_t):

$$W_t = a_w - c_w(IS_t^* - \overline{IS})/(\lambda + \theta) - d_w N_{HH\,t} \quad (3\text{-}1)$$

In the above, θ and λ convert IS mass to unit quantities. In what follows, conversion factors for the other industries are assumed to be unity. The term $\overline{(\cdot)}$ is the end-of-period inventory target. Deficits are made up in subsequent time steps, although customers already have paid for the goods.

Based on the wage, the P1, H1 and IS firms set their prices, $p_t^{(\cdot)}$, and production targets, $(\cdot)_t^p$, (how much they should produce) in order to maximize profit, given their forecasts of demands. All other symbols in the equations below are constant parameters.

$$p_t^{P1} = a_{P1} + b_{P1}W_t - c_{P1}(P1_t^* - \overline{P1}) \quad (3\text{-}2)$$
$$P1_t^p = 3d - 3eW_t - c_{P1p}(P1_t^* - \overline{P1}) \quad (3\text{-}3)$$

$$p_t^{H1} = a_{H1} + b_{H1}W_t - c_{H1}(H1_t^* - \overline{H1}) \quad (3\text{-}4)$$
$$H1_t^p = d - eW_t - c_{H1p}(H1_t^* - \overline{H1}) \quad (3\text{-}5)$$

$$p_t^{IS} = a_{IS} + b_{IS}W_t - c_{IS}(IS_t^* - \overline{IS})/(\lambda + \theta) \quad (3\text{-}6)$$

$$IS_t^p = d - eW_t - c_{ISp}(IS_t^* - \overline{IS})/(\lambda + \theta) \quad (3\text{-}7)$$

Next, the industry and household demands for input goods are calculated. These are expressed as flows between compartments. $P1H1_t$ is the flow of P1 product to the H1 industry at time *t* in units of units, for example:

$$P1H1_t^{de} = d - eW_t - fp_t^{P1} - g_{P1H1}(H1_t^* - \overline{H1}) \quad (3\text{-}8)$$

$$P2H1_t^{de} = \hat{k} \quad (3\text{-}9)$$

$$P1HH_t^{de} = d - kp_t^{P1} + \tfrac{1}{2} mp_t^{H1} + \tfrac{1}{2} np_t^{IS} + z(P1HH_t^{de} + H1HH_t^{de} + ISHH_t^{de}) \quad (3\text{-}10)$$

$$H1HH_t^{de} = d + \tfrac{1}{2} kp_t^{P1} - mp_t^{H1} + \tfrac{1}{2} np_t^{IS} + z(P1HH_t^{de} + H1HH_t^{de} + ISHH_t^{de}) \quad (3\text{-}11)$$

$$ISHH_t^{de} = d + \tfrac{1}{2} kp_t^{P1} + \tfrac{1}{2} mp_t^{H1} - np_t^{IS} + z(P1HH_t^{de} + H1HH_t^{de} + ISHH_t^{de}) \quad (3\text{-}12)$$

The last three equations are in terms of units per capita and must be solved simultaneously. $ISHH^{de}$ represents the demand for IS by the humans. The flow of goods *ISHH* passes through the human compartment (i.e., these goods are used by humans), but does not contribute to the mass of humans. Instead, the mass flows into the inaccessible resource pool (IRP).

Next, the flows that involve labor are calculated. Labor effort is expended to meet production targets $P1_t^p$ and $H1_t^p$ as discussed above. The labor goes into fences to keep the wild herbivores (H2) and carnivores (C1) from eating stocks:

$$P1H2_t = g_{RPP1} P1_t RP_t - P1_t^p - m_{P1} P1_t \quad (3\text{-}13)$$

$$H1C1_t = P1H1_t + P2H1_t - H1_t^p - m_{H1} H1_t \quad (3\text{-}14)$$

This is completed by calculating IS industry demands flows. IS demand and supply is in individual units and thus requires a conversion factor to mass for each input. This is true of all goods, but conversions for P1 and H1 are assumed to be unity (one unit requires 1 unit of mass).

$$P1IS_t^{de} = IS_t^p \theta \quad (3\text{-}15)$$

$$RPIS_t^{de} = IS_t^p \lambda \quad (3\text{-}16)$$

For the next time step, in the following, g_i are growth rates and m_i are mortality rates for the individual species i. All state variables are in terms of mass, except for $N_{HH\,t}$. Checks are done so that all flows are positive, and that compartments maintain a non-negative mass (except the deficit state variables). This latter requires that flows sometimes do not meet demands. Such deficits are accumulated in the deficit state variables and contribute to the production functions as outlined above. These deficit state variables are given by:

$$P1H1_{t+1}^{def} = P1H1_t^{def} + P1H1_t - P1H1_t^{de} \quad (3\text{-}17)$$

$$P1IS_{t+1}^{def} = P1IS_t^{def} + P1IS_t - P1IS_t^{de} \quad (3\text{-}18)$$

$$P1HH_{t+1}^{def} = P1HH_t^{def} + P1HH_t - P1HH_t^{de} N_{HH\,t} \quad (3\text{-}19)$$

$$H1HH_{t+1}^{def} = H1HH_t^{def} + H1HH_t - H1HH_t^{de} N_{HH\,t} \tag{3-20}$$

$$ISHH_{t+1}^{def} = ISHH_t^{def} + ISHH_t - (\theta + \lambda) ISHH_t^{de} * N_{HH\,t} \tag{3-21}$$

The above simply track the difference between what was actually delivered versus demanded. For example, $P1IS$ refers to the flow from P1 to IS, which may be less or greater than $P1IS_t^{de}$, depending on the situation at that time step.

Excess surplus is carried over to the next time step, or is used to make up deficits flows from previous time steps. The remaining state equations are given as follows.

$$P1_{t+1} = P1_t + P1_t(g_{RPP1} RP_t - m_{P1}) - P1H1_t - P1H2_t - P1HH_t - P1IS_t \tag{3-22}$$

$$P2_{t+1} = P2_t + P2_t(g_{RPP2} RP_t + r_{IRPP2} \frac{100}{100 + IRP^2} IRP_t - m_{P2} - g_{P2H2} H2_t - g_{P2H3} H3_t) - P2H1_t \tag{3-23}$$

$$P3_{t+1} = P3_t + P3_t(g_{RPP3} RP_t + r_{IRPP3} \frac{100}{100 + IRP^2} IRP_t - m_{P3} - g_{P3H3} H3_t) \tag{3-24}$$

$$H1_{t+1} = H1_t + P1H1_t + P2H1_t - m_{H1} H1_t - H1C1_t - H1HH_t \tag{3-25}$$

$$H2_{t+1} = H2_t + P1H2_t + H2_t(g_{P2H2} P2_t - m_{H2} - g_{H2C1} C1_t - g_{H2C2} C2_t) \tag{3-26}$$

$$H3_{t+1} = H3_t + H3_t(g_{P2H3} P2_t + g_{P3H3} P3_t - m_{H3} - g_{H3C2} C2_t) \tag{3-27}$$

$$C1_{t+1} = C1_t + H1C1_t + C1_t(g_{H2C1} H2_t - m_{C1}) \tag{3-28}$$

$$C2_{t+1} = C2_t + C2_t(g_{H2C2} H2_t + g_{H3C2} H3_t - m_{C2}) \tag{3-29}$$

$$HH_{t+1} = HH_t + P1HH_t + H1HH_t - m_{HH} N_{HH\,t} \mu_t \tag{3-30}$$

$$N_{HH\,t+1} = N_{HH\,t} + (\eta(W_t / P_t^*) - m_{HH} - \varphi(\mu_t - \mu_I)^2) N_{HH\,t} \tag{3-31}$$

$$\mu_{t+1} = HH_{t+1} / N_{HH\,t+1} \tag{3-32}$$

$$IS_{t+1} = IS_t + P1IS_t + RPIS_t - ISHH_t \tag{3-33}$$

$$RP_{t+1} = RP_t + m_{P1}P1_t + m_{P2}P2_t + m_{P3}P3_t + m_{H1}H1_t + m_{H2}H2_t + m_{H3}H3_t + m_{C1}C1_t + m_{C2}C2_t + m_{HH}HH_t + m_{IRPRP}IRP_t - RP_t(g_{RPIRP}IRP_t + g_{RPP1}P1_t + g_{RPP2}P2_t + g_{RPP3}P3_t) - RPIS_t \quad (3\text{-}34)$$

$$IRP_{t+1} = IRP_t + ISHH_t - IRP_t(r_{IRPP2}P2_t + r_{IRPP3}P3_t + m_{IRPRP}) \quad (3\text{-}35)$$

Where

$$P_t^* = (p_t^{P1}P1HH_t + p_t^{H1}H1HH_t + p_t^{IS}ISHH_t)/(P1HH_t + H1HH_t + ISHH_t) \quad (3\text{-}36)$$

$$\eta(W_t/P_t^*) = a_\eta - b_\eta(W_t/P_t^*)^{1/2} \quad (3\text{-}37)$$

All parameters and their nominal values are given in Table 3-3 along with the corresponding state variable initial conditions in Table 3-4.

Table 3-3. Parameters and their nominal values*

Ecological Parameters	Economic Parameters	
1. gRPP2=2.861325e-2	1. Aw=4.832249e-1	32. mP1HH=1.997642e-4
2. gP2H2=2.934352e-2	2. Cw=1.357181e-1	33. nP1HH=7.776176e-5
3. gP2H3=0.042	3. aP1=1.0	34. dH1HH=1.9108e-004
4. gRPP3=5.732733e-3	4. bP1=1.0	35. zH1HH=1.4745e-001
5. gP3H3=3.131235e-1	5. cP1=7.737088e-1	36. kH1HH=2.1204e-004
6. gH2C1=9.174907e-1	6. aP1p=5.732311e-1	37. mH1HH=3.9953e-004
7. gH2C2=1.312728e-1	7. bP1p=1.497375e-1	38. nH1HH=7.7762e-005
8. gH3C2=2.938371e-1	8. cP1p=3.380538e-2	39. dISHH=1.9108e-004
9. rIRPP2=1.081656e-2	9. aH1=7.524099e-1	40. zISHH=1.4745e-001
10. rIRPP3=0.9	10. bH1=0.001	41. kISHH=2.1204e-004
11. mP2=4.932829e-1	11. cH1=2.527165e-1	42. mISHH=1.9976e-004
12. mP3=4.658138e-1	12. aH1p=1.910770e-1	43. nISHH=1.5552e-004
13. mH2=0.001	13. bH1p=4.991250e-2	44. K^=3.0000e-001
14. mH3=4.903092e-1	14. cH1p=9.926235e-1	45. θ (Theta)=1.0199e-001
15. mC1=2.302639e-1	15. aIS=6.081040e-1	46. λ (Lambda)=6.7668e-001
16. mC2=4.286472e-1	16. bIS=2.972103e-1	47. gRPP1=0.09
20. gP1H2=0.1	17. cIS=0.001	48. mP1=1.018295e-3
21. gH1C1=0.2	18. aISp=1.910770e-1	49. mH1=9.838862e-3
	19. bISp=4.991250e-2	50. mHH=0.22
	20. cISp=5.646818e-1	51. $\bar{P}\ \bar{1}$ (P1bar)mass=0
	21. dP1H1=1.910770e-4	52. $\bar{H}\ \bar{1}$ (H1bar)mass=0.4
	22. eP1H1=4.991250e-2	53. $\bar{I}\ \bar{S}$ (ISbar)mass=0
	23. fP1H1=4.240783e-1	54. Dw=4.507354e-6
	24. gP1H1=1.9	55. η (Eta)a=1.5000e+000
	29. dP1HH=1.910770e-4	56. η (Eta)b=8.3333e-001
	30. zP1HH=1.474467e-1	57. η (Eta)c=1.6667e-002
	31. kP1HH=4.240783e-4	58. φ (Phi) =1.0000e+001
		59. Idealpercapmass=4.0556e-003

* Note, these are grouped by their context in the software, not necessarily by number.

Table 3-4. Corresponding state variable initial conditions.

1	P1	1.55031758212386	8	C2	1.32270575776743	15	$P1HH^{def}$	0
2	P2	10.00000000000000	9	HH	0.45073543305286	16	$H1HH^{def}$	0
3	P3	1.46577689115230	10	IS	0.10089700803626	17	$ISHH^{def}$	0
4	H1	0.14002360565832	11	RP	19.14708496882624	18	N_{HH}	10
5	H2	0.25097142084356	12	IRP	0.86413620441690	19	μ	0.004055586 60633
6	H3	1.34666957783805	13	$P1H1^{def}$	0			
7	C1	0.12948354945840	14	$P1IS^{def}$	0			

Chapter 4

Summary

Sustainability is fundamentally an effort to create and maintain a regime in which the human population and its necessary energy and material consumption can be supported indefinitely by the biological system of the Earth. The model system presented in this report consists of a foodweb with an integrated industrial sector, agricultural production, a very simple economy, and the rudiments of a legal system. As such, it represents a promising first step in understanding the relationship between ecology, economics, technology, and society. The purpose of this report is to document the model so as to make it accessible to the larger user community in the hopes of stimulating further research work.

The preliminary model described here was developed in stages. First, a basic ecological model was created with the total mass distributed among all compartments within a foodweb. The compartments were then differentiated in terms of human control over them (by identifying species as being either domesticated or non-domesticated, assigning property rights to the mass in certain compartments, and indicating intentional changes to the natural flows of mass, such as fences). Finally, a generic industrial process (with private property rights) was added that diverts resources to human (non-food) consumption and use.

From an ecological perspective, the model contains two resource pools (one biologically accessible and one inaccessible), three primary producers (P1, P2, and P3); three herbivores (H1, H2, and H3), two carnivores (C1 and C2), and humans (HH). The system flows are specified in terms of mass. The system is closed to mass (i.e., mass is conserved) and open to a non-limiting source of energy.

From an economic perspective the model contains human households, an industrial sector (IS), and two private agricultural firms: one a producer of plants (P1) and one a producer of herbivores (H1). The households are the ultimate owners of the factors of production used to produce the goods that are traded in explicit markets. In addition to markets for the three goods (P1, H1, IS), there is a labor market.

The optimal economic behavior of industry, government, and households is as follows. The P1 industry applies a variable amount of labor and a fixed amount of capital to transform the mass of P1 that would otherwise grow naturally into a marketable product. The P1 industry hires labor to reduce the consumption of P1 by H2. The H1 industry buys P1, labor and grazing rights to P2 and sells output to the households. The H1 industry hires labor to limit the amount of H1 eaten by C1. The IS industry buys P1 and combines it with mass from the resource pool (RP) using variable labor and a fixed amount of capital to produce IS, which it sells to the households. The government sector receives revenue from the grazing rights it grants to the H1 industry and transfers this revenue to the households. The objective of the household sector is to maximize current

utility, which is assumed to be a positive increasing function of consumption per capita of P1, H1, IS and leisure. This economic activity is reflected in the nominal gross national product (GDP), which is the total market value of all units of final goods produced in the economy during the period, which also corresponds to the sum of all spending on final goods plus the values of the changes in inventories of all goods.

The production (natural growth) of non-domesticated species during the period is governed by biological rules, but is influenced by human economic behavior (i.e., fences to protect domesticated species).

The human population at the end of the current period is equal to the human population at the beginning of the current period, plus the number of people born during the current period, minus human mortality during the period. The human birthrate during the period is assumed to be a negative function of the real wage prevailing during the current period. The net addition to human mass is equal to human consumption of plants P1 and herbivores H1 during the current period, minus human mortality in terms of mass during the current period.

The size of the resource pool at the end of the current period is equal to its size at the beginning of the current period, plus the mass flowing to the resource pool generated by plant and animal mortality, minus the amount by which the resource pool declines during the current period in direct response to the growth in the domesticated and non-domesticated plants during the current period. In addition the resource pool increases over time because of mass transfer from the inaccessible resource pool. Mass is transferred from the resource pool to the inaccessible resource pool in direct proportion to the level of production by the IS sector.

The size of the inaccessible resource pool at the end of the current period is equal to its size at the beginning of the current period, plus the volume of waste contributed to the pool due to the scale of operations of the IS industry, minus the amount that the non-domesticated plants P1 and P2 are able to recycle from the inaccessible resource pool during the current period and minus the amount of natural decay that moves mass directly from the inaccessible resource pool to the resource pool.

This study is a preliminary one and did not include all possible parameters or model behaviors. For example, it is known that changes in the recycling parameter for P2 and P3 can cause the system to crash entirely. The solution used in this report consisted of the following iterative steps.

- The industrial sector sets the wage rate. The assumption is that this sector dominates the labor market.
- Based on the wage, all industries set their prices and their production targets according to their own utility functions and internal models of demand of their products.
- Humans determine their demands for goods.
- Industries determine their demands for goods and labor.

- Checks are done for internal consistencies of flows (to be sure they meet positivity constraints on flows and compartment masses).
- The next step is taken (flows are transferred) for both the economic and ecological parts of the model.

Some preliminary simulations have been conducted using this model (Cabezas et al. 2006). However, the modle behavior has not been explored in detail. Future uture work entails further refinement of the model and methods of analysis. Several options are being considered for expanding the model and attempting to use it to suggest or evaluate possible policy changes that would allow all species to continue while maintaining a viable economic environment for the human population. Some of these options include adding economic and regulatory tools such a fee on waste generation, futher legal protections of wild species, and inclusion of environmental quality valuation. The overall general goal is to use the model as rough generic screening tool for sustainable environmental management strategies. This might include the use of economic incentives and regulation.

Appendix A

Symbols Used in the Report

$C1_t$ = the stock of C1 at the beginning of the period

c_{P1H2} = consumption of P1 per unit of H2 (depends on the restrictions imposed by P1 industry)

g_i = growth parameter for species i

g_{P1} = growth parameter of P1

$H1^s$ = H1 industry's supply of H1 during the current period

$H1_t$ = stock of H1 at the beginning of the period

$\overline{H}\ \overline{1}$ = the H1 industry's desired end of period inventory of H1

H1HH = the amount of H1 consumed by the households

$(H1HH)^d$ = household sector's demand for H1

$H2_t$ = stock of H2 at the beginning of the period

h_{H1} = average number of hours employees work in the H1 industry producing H1

h^*_{H1} = average number of hours employees are used in the H1 industry to reduce the amount of H1 eaten by C1

h_{IS} = average number of hours employees work in the IS industry producing IS

hN^d = labor hours demanded

h_{P1} = average number of hours employees work in the P1 industry producing P1

h^*_{P1} = average number of hours employees are used in the P1 industry to reduce the amount of P1 eaten by H2

h^s = average number of hours the households would like to work during the period (for all employers combined)

IS_t = the initial stock of IS (in units of goods) held by the IS industry

$(\theta+\lambda)\cdot IS_t$ = the initial stock of IS (in units of mass) held by the IS industry

$\bar{I}\ \bar{S}$ = the ending inventory of IS (in units of goods) that the IS industry plans for the end of the period

$(ISHH)^s$ = the amount of IS the IS industry desires to supply to the households

$(ISHH)^d$ = the household sector's demand for IS

K^{H1}_t = stock of physical capital held by the H1 industry at the beginning of the period

K^{IS}_t = stock of physical capital held by the IS industry at the beginning of the period

K^{P1}_t = stock of physical capital held by the P1 industry at the beginning of the period

K^ = P2H1 (the amount of P2 the government permits the H1 industry to take)

m_i = mortality parameter for species i

m_{P1} = mortality parameter of P1

m_{H1} = mortality parameter of H1

N^S = net supply of labor function

N_t = human population at time t

$N^{\wedge}_t$ = total human mass at time t

P^* = (weighted) average price of consumer goods

P1H1 = the amount of P1 consumed by the H1 industry

$(P1H1)^d$ = the H1 industry's demand for P1

P1HH = the amount of P1 consumed by the households

$(P1HH)^d$ = the household sector's total demand for P1

$(P1IS)^d$ = the IS industry's demand for P1

$P1^s$ = P1 industry's supply of P1 during the period

$P1_t$ = stock of P1 at the beginning of the period

$\bar{P}\ \bar{1}$ = desired inventory of P1 for the end of the period

P2H1 = amount of P2 the government permits the H1 industry to take

p_{H1} = price of H1 announced by the H1 industry

p_{P1} = price of P1 announced by the P1 industry

RP_t = mass of the resource pool at the beginning of the period

Tr = the transfer payments received by the households from the government sector

W = money wage rate

WhN^d = total wage bill (wage rate times number of labor hours)

$\eta(\bullet)$ = human birthrate

μ_t = average human mass per capita at time t = $\hat{N}_t / N_t$

Π = total dividend income received by the households from all industries

Π^*_{P1} = profits of the P1 industry
= dividends plus additions to retained earnings for P1 industry
= dividends plus unintended real investment in inventories in the P1 industry

Π^{**} = total non-wage income earned by all industries
= profits plus rental income to fixed factors earned by all industries

φ = weighting factor for the difference between the current per capita weight of a human and the ideal weight

λ = the amount of RP necessary to produce a unit of IS

θ = the amount of P1 necessary to produce a unit of IS

$\theta \cdot IS$ = the amount of P1 used by the IS industry

$(\theta+\lambda) \cdot IS$ = the amount of mass appearing in the IS produced during the period

$(\theta+\lambda) \cdot ISHH$ = the amount of mass the households buy during the current period in the form of IS

ω_{H1} = a vector of parameters other than p_{P1} that the P1 industry believes affects the H1 industry's demand for P1

ω_{IS} = a vector of parameters other than p_{P1} that the P1 industry believes affects the IS industry's demand for P1

ω_{ISHH} = a vector of parameters other than p_{IS} that the IS industry believes affects the households' demand for IS

ω_{HH} = a vector of parameters other than p_{P1} that the P1 industry believes affects the households' demand for P1

ωH1HH = a vector of parameters other than pH1 that the H1 industry believes affects the households' demand for H1

Appendix B

SIMULINK Graphical Model

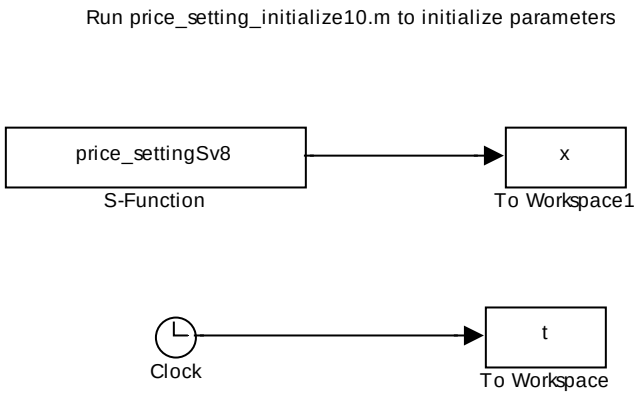

Figure B-1. Root level SIMULINK model (file price_setting2.mdl as opened in SIMULINK)

The S-Function contains a pointer to the file price_settingSv8.m
It is a masked system, with the mask defining the parameter names (values loaded via initialization file price_setting_initialization10.m).

To Workspace1 defines a MATLAB workspace variable x which holds all the model output.

To Workspace defines a MATLAB workspace variable t which holds the time variable.

Appendix C

SIMULINK Code

Text contained in and defining the graphical model of Appendix 1. Filename: price_setting2.mdl

```
Model {
  Name                    "price_setting2"
  Version           6.1
  MdlSubVersion           0
  GraphicalInterface {
    NumRootInports          0
    NumRootOutports         0
    ParameterArgumentNames  ""
    ComputedModelVersion    "1.151"
    NumModelReferences      0
    NumTestPointedSignals   0
  }
  SavedCharacterEncoding  "ibm-5348_P100-1997"
  SaveDefaultBlockParams  on
  SampleTimeColors        off
  LibraryLinkDisplay      "none"
  WideLines         off
  ShowLineDimensions      off
  ShowPortDataTypes       off
  ShowLoopsOnError        on
  IgnoreBidirectionalLines off
  ShowStorageClass        off
  ShowTestPointIcons      on
  ShowViewerIcons   on
  SortedOrder             off
  ExecutionContextIcon    off
  ShowLinearizationAnnotations on
  RecordCoverage    off
  CovPath           "/"
  CovSaveName             "covdata"
  CovMetricSettings       "dw"
  CovNameIncrementing     off
  CovHtmlReporting        on
  covSaveCumulativeToWorkspaceVar on
  CovSaveSingleToWorkspaceVar on
  CovCumulativeVarName    "covCumulativeData"
  CovCumulativeReport     off
  CovReportOnPause        on
  ScopeRefreshTime        0.035000
  OverrideScopeRefreshTime on
  DisableAllScopes        off
  DataTypeOverride        "UseLocalSettings"
  MinMaxOverflowLogging   "UseLocalSettings"
  MinMaxOverflowArchiveMode "Overwrite"
  BlockNameDataTip        off
  BlockParametersDataTip  off
```

```
BlockDescriptionStringDataTip     off
ToolBar           on
StatusBar         on
BrowserShowLibraryLinks off
BrowserLookUnderMasks   off
Created           "Tue Aug 12 09:54:50 2003"
UpdateHistory           "UpdateHistoryNever"
ModifiedByFormat        "%<Auto>"
LastModifiedBy    "CPAWLOWS"
ModifiedDateFormat      "%<Auto>"
LastModifiedDate        "Mon Mar 28 14:46:08 2005"
ModelVersionFormat      "1.%<AutoIncrement:151>"
ConfigurationManager    "None"
LinearizationMsg        "none"
Profile           off
ParamWorkspaceSource    "MATLABWorkspace"
AccelSystemTargetFile   "accel.tlc"
AccelTemplateMakefile   "accel_default_tmf"
AccelMakeCommand        "make_rtw"
TryForcingSFcnDF        off
ExtModeBatchMode        off
ExtModeEnableFloating   on
ExtModeTrigType   "manual"
ExtModeTrigMode   "normal"
ExtModeTrigPort   "1"
ExtModeTrigElement      "any"
ExtModeTrigDuration     1000
ExtModeTrigDurationFloating "auto"
ExtModeTrigHoldOff      0
ExtModeTrigDelay        0
ExtModeTrigDirection    "rising"
ExtModeTrigLevel        0
ExtModeArchiveMode      "off"
ExtModeAutoIncOneShot   off
ExtModeIncDirWhenArm    off
ExtModeAddSuffixToVar   off
ExtModeWriteAllDataToWs off
ExtModeArmWhenConnect   on
ExtModeSkipDownloadWhenConnect off
ExtModeLogAll           on
ExtModeAutoUpdateStatusClock on
BufferReuse             on
StrictBusMsg            "None"
ProdHWDeviceType        "32-bit Generic"
ShowModelReferenceBlockVersion off
ShowModelReferenceBlockIO off
Array {
  Type              "Handle"
  Dimension                1
  Simulink.ConfigSet {
    $ObjectID               1
    Version                 "1.0.4"
    Array {
    Type              "Handle"
    Dimension         7
    Simulink.SolverCC {
      $ObjectID         2
```

```
  Version           "1.0.4"
  StartTime         "0.0"
  StopTime          "200"
  AbsTol            "auto"
  FixedStep         "auto"
  InitialStep             "auto"
  MaxNumMinSteps    "-1"
  MaxOrder          5
  ExtrapolationOrder      4
  NumberNewtonIterations  1
  MaxStep           "auto"
  MinStep           "auto"
  RelTol            "1e-3"
  SolverMode              "Auto"
  Solver            "FixedStepDiscrete"
  SolverName              "FixedStepDiscrete"
  ZeroCrossControl        "UseLocalSettings"
  PositivePriorityOrder   off
  AutoInsertRateTranBlk   off
  SampleTimeConstraint    "Unconstrained"
  RateTranMode            "Deterministic"
}
Simulink.DataIOCC {
  $ObjectID         3
  Version           "1.0.4"
  Decimation              "1"
  ExternalInput           "[t, u]"
  FinalStateName    "xFinal"
  InitialState            "xInitial"
  LimitDataPoints   on
  MaxDataPoints           "1000"
  LoadExternalInput       off
  LoadInitialState        off
  SaveFinalState    off
  SaveFormat              "Array"
  SaveOutput              on
  SaveState         off
  SignalLogging           on
  SaveTime          on
  StateSaveName           "xout"
  TimeSaveName            "tout"
  OutputSaveName    "yout"
  SignalLoggingName       "sigsOut"
  OutputOption            "RefineOutputTimes"
  OutputTimes             "[]"
  Refine            "1"
}
Simulink.OptimizationCC {
  $ObjectID         4
  Version           "1.0.4"
  BlockReduction    on
  BooleanDataType   on
  ConditionallyExecuteInputs on
  ConditionalExecOptimization "on_for_testing"
  InlineParams            off
  InlineInvariantSignals  on
  OptimizeBlockIOStorage  on
```

```
  BufferReuse             on
  EnforceIntegerDowncast  on
  ExpressionFolding       on
  FoldNonRolledExpr       on
  LocalBlockOutputs       on
  ParameterPooling        on
  RollThreshold           5
  SystemCodeInlineAuto    off
  StateBitsets            off
  DataBitsets             off
  UseTempVars             off
  ZeroExternalMemoryAtStartup on
  ZeroInternalMemoryAtStartup on
  InitFltsAndDblsToZero   on
  NoFixptDivByZeroProtection off
  OptimizeModelRefInitCode off
  LifeSpan          "inf"
}
Simulink.DebuggingCC {
  $ObjectID         5
  Version           "1.0.4"
  RTPrefix          "error"
  ConsistencyChecking     "none"
  ArrayBoundsChecking     "none"
  AlgebraicLoopMsg        "warning"
  ArtificialAlgebraicLoopMsg "warning"
  CheckSSInitialOutputMsg on
  CheckExecutionContextPreStartOutputMsg off
  CheckExecutionContextRuntimeOutputMsg   off
  SignalResolutionControl "TryResolveAllWithWarning"
  BlockPriorityViolationMsg "warning"
  MinStepSizeMsg    "warning"
  SolverPrmCheckMsg       "none"
  InheritedTsInSrcMsg     "warning"
  DiscreteInheritContinuousMsg "warning"
  MultiTaskDSMMsg   "warning"
  MultiTaskRateTransMsg   "error"
  SingleTaskRateTransMsg  "none"
  TasksWithSamePriorityMsg "warning"
  CheckMatrixSingularityMsg "none"
  IntegerOverflowMsg      "warning"
  Int32ToFloatConvMsg     "warning"
  ParameterDowncastMsg    "error"
  ParameterOverflowMsg    "error"
  ParameterPrecisionLossMsg "warning"
  UnderSpecifiedDataTypeMsg "none"
  UnnecessaryDatatypeConvMsg "none"
  VectorMatrixConversionMsg "none"
  InvalidFcnCallConnMsg   "error"
  FcnCallInpInsideContextMsg "Use local settings"
  SignalLabelMismatchMsg  "none"
  UnconnectedInputMsg     "warning"
  UnconnectedOutputMsg    "warning"
  UnconnectedLineMsg      "warning"
  SFcnCompatibilityMsg    "none"
  UniqueDataStoreMsg      "none"
  RootOutportRequireBusObject "warning"
```

```
  AssertControl           "UseLocalSettings"
  EnableOverflowDetection off
  ModelReferenceIOMsg     "none"
  ModelReferenceVersionMismatchMessage "none"
  ModelReferenceIOMismatchMessage "none"
  ModelReferenceCSMismatchMessage "none"
  ModelReferenceSimTargetVerbose off
  UnknownTsInhSupMsg      "warning"
  ModelReferenceDataLoggingMessage "warning"
  ModelReferenceSymbolNameMessage "warning"
}
Simulink.HardwareCC {
  $ObjectID         6
  Version           "1.0.4"
  ProdBitPerChar    8
  ProdBitPerShort   16
  ProdBitPerInt           32
  ProdBitPerLong    32
  ProdIntDivRoundTo       "Undefined"
  ProdEndianess           "Unspecified"
  ProdWordSize            32
  ProdShiftRightIntArith  on
  ProdHWDeviceType        "32-bit Generic"
  TargetBitPerChar        8
  TargetBitPerShort       16
  TargetBitPerInt   32
  TargetBitPerLong        32
  TargetShiftRightIntArith on
  TargetIntDivRoundTo     "Undefined"
  TargetEndianess   "Unspecified"
  TargetWordSize    32
  TargetTypeEmulationWarnSuppressLevel 0
  TargetPreprocMaxBitsSint 32
  TargetPreprocMaxBitsUint 32
  TargetHWDeviceType      "Specified"
  TargetUnknown           on
  ProdEqTarget            on
}
Simulink.ModelReferenceCC {
  $ObjectID         7
  Version           "1.0.4"
  UpdateModelReferenceTargets "IfOutOfDateOrStructuralChange"
  CheckModelReferenceTargetMessage "error"
  ModelReferenceNumInstancesAllowed "Multi"
  ModelReferencePassRootInputsByReference on
  ModelReferenceMinAlgLoopOccurrences off
}
Simulink.RTWCC {
  $BackupClass            "Simulink.RTWCC"
  $ObjectID         8
  Version           "1.0.4"
  SystemTargetFile        "grt.tlc"
  GenCodeOnly             off
  MakeCommand             "make_rtw"
  TemplateMakefile        "grt_default_tmf"
  GenerateReport    off
  SaveLog           off
```

```
RTWVerbose              on
RetainRTWFile           off
ProfileTLC              off
TLCDebug          off
TLCCoverage             off
TLCAssert         off
ProcessScriptMode       "Default"
ConfigurationMode       "Optimized"
ConfigAtBuild           off
IncludeHyperlinkInReport off
LaunchReport            off
Array {
  Type              "Handle"
  Dimension               2
  Simulink.CodeAppCC {
    $ObjectID               9
    Version                 "1.0.4"
    ForceParamTrailComments off
    GenerateComments        on
    IgnoreCustomStorageClasses on
    IncHierarchyInIds       off
    MaxIdLength       31
    PreserveName            off
    PreserveNameWithParent  off
    ShowEliminatedStatement off
    IncAutoGenComments      off
    SimulinkDataObjDesc     off
    SFDataObjDesc           off
    IncDataTypeInIds        off
    PrefixModelToSubsysFcnNames on
    CustomSymbolStr         "$R$N$M"
    MangleLength            1
    DefineNamingRule        "None"
    ParamNamingRule         "None"
    SignalNamingRule        "None"
    InsertBlockDesc         off
    SimulinkBlockComments   on
    EnableCustomComments    off
    InlinedPrmAccess        "Literals"
    ReqsInCode        off
  }
  Simulink.GRTTargetCC {
    $BackupClass            "Simulink.TargetCC"
    $ObjectID               10
    Version                 "1.0.4"
    TargetFcnLib            "ansi_tfl_tmw.mat"
    GenFloatMathFcnCalls    "ANSI_C"
    UtilityFuncGeneration   "Auto"
    GenerateFullHeader      on
    GenerateSampleERTMain   off
    IsPILTarget       off
    ModelReferenceCompliant off
    IncludeMdlTerminateFcn  on
    CombineOutputUpdateFcns off
    SuppressErrorStatus     off
    IncludeFileDelimiter    "Auto"
    ERTCustomFileBanners    off
```

```
          SupportAbsoluteTime     on
          LogVarNameModifier      "rt_"
          MatFileLogging          off
          MultiInstanceERTCode    off
          SupportNonFinite        on
          SupportComplex          on
          PurelyIntegerCode       off
          SupportContinuousTime   on
          SupportNonInlinedSFcns  on
          ExtMode                 off
          ExtModeStaticAlloc      off
          ExtModeTesting          off
          ExtModeStaticAllocSize  1000000
          ExtModeTransport        0
          ExtModeMexFile          "ext_comm"
          RTWCAPISignals          off
          RTWCAPIParams           off
          RTWCAPIStates           off
          GenerateASAP2           off
        }
        PropName                "Components"
      }
    }
    PropName          "Components"
    }
    Name              "Configuration"
    SimulationMode          "normal"
    CurrentDlgPage          "Solver"
  }
  PropName                "ConfigurationSets"
}
Simulink.ConfigSet {
  $PropName               "ActiveConfigurationSet"
  $ObjectID               1
}
BlockDefaults {
  Orientation             "right"
  ForegroundColor         "black"
  BackgroundColor         "white"
  DropShadow              off
  NamePlacement     "normal"
  FontName                "Helvetica"
  FontSize                10
  FontWeight              "normal"
  FontAngle               "normal"
  ShowName                on
}
BlockParameterDefaults {
  Block {
    BlockType               Clock
    DisplayTime       off
  }
  Block {
    BlockType               "S-Function"
    FunctionName            "system"
    SFunctionModules        "''"
    PortCounts        "[]"
```

```
    }
    Block {
      BlockType               ToWorkspace
      VariableName            "simulink_output"
      MaxDataPoints           "1000"
      Decimation        "1"
      SampleTime        "0"
      FixptAsFi               off
    }
  }
  AnnotationDefaults {
    HorizontalAlignment     "center"
    VerticalAlignment       "middle"
    ForegroundColor         "black"
    BackgroundColor         "white"
    DropShadow              off
    FontName                "Helvetica"
    FontSize                10
    FontWeight              "normal"
    FontAngle               "normal"
  }
  LineDefaults {
    FontName                "Helvetica"
    FontSize                9
    FontWeight              "normal"
    FontAngle               "normal"
  }
  System {
    Name              "price_setting2"
    Location                [368, 122, 948, 423]
    Open              on
    ModelBrowserVisibility  off
    ModelBrowserWidth       200
    ScreenColor             "white"
    PaperOrientation        "landscape"
    PaperPositionMode       "auto"
    PaperType               "usletter"
    PaperUnits              "inches"
    ZoomFactor              "100"
    ReportName              "simulink-default.rpt"
    Block {
      BlockType               Clock
      Name              "Clock"
      Position                [160, 175, 180, 195]
      Decimation        "10"
    }
    Block {
      BlockType               "S-Function"
      Name              "S-Function"
      Ports             [0, 1]
      Position                [85, 80, 235, 110]
      FunctionName            "price_settingSv8"
      Parameters
"ecolparams,econparams,ic,MH_updown,Lamb_updown,"
"Thet_updown, tlo, thi"
      MaskPromptString        "Human Mortality|Lambda, RP input for IS
industr"
```

```
"y|Theta, P1 input for IS industry|tlo, time to start change|thi, time
to end "
"change"
      MaskStyleString         "popup(unchanging|increasing to not more
than 2 "
"times|decreasing to not less than 1/10th),popup(unchanged|increasing
w/o boun"
"d|decreasing to not less than 1/10th),popup(unchanging|increasing w/o
bound|d"
"ecreasing to not less than  1/10th),edit,edit"
      MaskTunableValueString  "off,off,off,off,off"
      MaskCallbackString      "||||"
      MaskEnableString        "on,on,on,on,on"
      MaskVisibilityString    "on,on,on,on,on"
      MaskToolTipString       "on,on,on,on,on"
      MaskVarAliasString      ",,,,"
      MaskVariables
"MH_updown=@1;Lamb_updown=@2;Thet_updown=@3;tlo="
"@4;thi=@5;"
      MaskIconFrame           on
      MaskIconOpaque          on
      MaskIconRotate          "none"
      MaskIconUnits           "autoscale"
      MaskValueString         "unchanging|unchanged|unchanging|10|10"
      MaskTabNameString       ",,,,"
    }
    Block {
      BlockType               ToWorkspace
      Name              "To Workspace"
      Position                [325, 170, 385, 200]
      VariableName            "t"
      MaxDataPoints           "inf"
      SampleTime        "1"
      SaveFormat        "Array"
    }
    Block {
      BlockType               ToWorkspace
      Name              "To Workspace1"
      Position                [335, 80, 395, 110]
      VariableName            "x"
      MaxDataPoints           "inf"
      SampleTime        "1"
      SaveFormat        "Array"
    }
    Line {
      SrcBlock                "Clock"
      SrcPort                 1
      DstBlock                "To Workspace"
      DstPort                 1
    }
    Line {
      SrcBlock                "S-Function"
      SrcPort                 1
      DstBlock                "To Workspace1"
      DstPort                 1
    }
    Annotation {
```

```
      Name              "Run price_setting_initialize10.m to initialize
"
"parameters"
      Position                [258, 26]
    }
  }
}
```

Appendix D

MATLAB Code

The first file is an initialization file, price_setting_initialize10.m, which initializes the parameters and initial state conditions. This is a convoluted way of loading the parameters listed above.

The next file is the core of the model, written as an S function. The file name is price_settingSv8.m

Initialization file (price_setting_initialize10.m). Run before running SIMULINK model

```
% price_setting_initialize10.m
%initialization file for price_settingSv8.m
%


 ecolparams=[2.861325308717139e-002 %found 30 Aug 2004
    1.734351784507601e-002
    3.265224224356716e-002
    5.732733145333690e-003
    3.131235080180339e-001
    9.174906638699569e-001
    3.327275637705977e-001
    2.350696927025789e-001
    5.081656269272850e-002
    8.688268022462951e-001
    4.932828648894071e-001
    4.658138102587053e-001
    1.000000000000000e-003
    5.030915711147533e-001
    2.302639355221487e-001
    4.286472149788781e-001
    0.0%6.602439763447394e-006 %background feedback of IRP to RP set to
% zero
    1.233437625619434e+000
    1.000000000000000e+001
    .1                          %gP1H2 %when humans die off
     .2];                        %gH1C1

ecolparams(10)=.9;
ecolparams(7)=1.3127275637705977e-001;

ecolparams(14)=4.9030915711147533e-001;
ecolparams(3)=3.7e-2;
ecolparams(9)=1.081656269272850e-002;
ecolparams(2)=2.934351784507601e-002;
ecolparams(3)=4.2e-2; %6 Oct 2004
```

```
% ecological params =
% [gRPP2;gP2H2;gP2H3;gRPP3;gP3H3;gH2C1;gH2C2;gH3C2;
% rIRPP2;rIRPP3;mP2;mP3;mH2;mH3;mC1;mC2;mIRPRP;RPIRP;P2;gP1H2;gH1C1];
% Here, P2 is a
% choice variable, controlling the total system mass
%
% gRPP2=ecolparams(1);
%
% gP2H2=ecolparams(2);gP2H3=ecolparams(3);gRPP3=ecolparams(4);
% gP3H3=ecolparams(5);
% gH2C1=ecolparams(6);gH2C2=ecolparams(7);gH3C2=ecolparams(8);
% rIRPP2=ecolparams(9);rIRPP3=ecolparams(10);
% mP2=ecolparams(11);mP3=ecolparams(12);mH2=ecolparams(13);
% mH3=ecolparams(14);mC1=ecolparams(15);mC2=ecolparams(16);
% mIRPRP=ecolparams(17);RPIRP=ecolparams(18); %not used in full econ-
% ecol model
% P2=ecolparams(19);
% gP1H2=ecolparams(20);gH1C1=ecolparams(21); %13 Aug 2004 natural
% predation of P1 and H1



%
% economic parameters, found using ga_find_econV1.m:
%
% aw=econparams(1);cw=econparams(2);
% aP1=econparams(3);bP1=econparams(4);cP1=econparams(5);
% aP1p=econparams(6);bP1p=econparams(7);cP1p=econparams(8);
% aH1=econparams(9);bH1=econparams(10);cH1=econparams(11);
% aH1p=econparams(12);bH1p=econparams(13);cH1p=econparams(14);
% aIS=econparams(15);bIS=econparams(16);cIS=econparams(17);
% aISp=econparams(18);bISp=econparams(19);cISp=econparams(20);
%
% dP1H1=econparams(21);eP1H1=econparams(22);fP1H1=econparams(23);
% gP1H1=econparams(24);
%
% *****dP1IS=econparams(25);eP1IS=econparams(26);fP1IS=econparams(27);
% gP1IS=econparams(28) no longer used****;
% dP1HH=econparams(29);zP1HH=econparams(30);kP1HH=econparams(31);
% mP1HH=econparams(32);nP1HH=econparams(33);
% dH1HH=econparams(34);zH1HH=econparams(35);kH1HH=econparams(36);
% mH1HH=econparams(37);nH1HH=econparams(38);
% dISHH=econparams(39);zISHH=econparams(40);kISHH=econparams(41);
% mISHH=econparams(42);nISHH=econparams(43);
% khat=econparams(44);theta=econparams(45);lambda=econparams(46);
% gRPP1=econparams(47);mP1=econparams(48);mH1=econparams(49);
% mHH=econparams(50);
% P1barmass=econparams(51);H1barmass=econparams(52);
% ISbarmass=econparams(53);dw=econparams(54);
% etaa=econparams(55);etab=econparams(56);etac=econparams(57);
% phi=econparams(58);
% idealpercapmass=econparams(59);

sol=[4.832248668616482e-001
    1.357181037387526e-001
    1.000000000000000e+000
```

```
    1.000000000000000e+000
    7.737088487049635e-001
    1.910770291618713e-001
    4.991249741897157e-002
    3.380538111199238e-002
    7.524098562805767e-001
    1.000000000000000e-003
    2.527165128459011e-001
    9.926234817539428e-001
    6.081040091094971e-001
    2.972103070158917e-001
    1.000000000000000e-003
    5.646817673172643e-001
    4.240782789383344e-001
    19.000000000000000e-001
    1.286507253686182e-001
    1.474467159083802e-002
    3.995283985198757e-001
    1.555235114499388e-001
    1.019919611353713e-001
    6.766772330024032e-001
    1.018295456166137e-003
    9.838862467656756e-003
    9.000000000000011e-001];

% sol(6)=1.25e-001;

%
% assign parameters
%
econparams(1:5)=sol(1:5);
econparams(6)=3*sol(6);econparams(7)=3*sol(7);
econparams(8:11)=sol(8:11);
econparams(12)=sol(6);
econparams(13)=sol(7);
econparams(14:17)=sol(12:15);
econparams(18)=sol(6);
econparams(19)=sol(7);
econparams(20)=sol(16);
dP1H1=sol(6);econparams(21:4:29)=[dP1H1/1000;dP1H1/1000;dP1H1/1000];eco
nparams(34:5:39)=[dP1H1/1000;dP1H1/1000];
eP1H1=sol(7);econparams(22:4:26)=[eP1H1;eP1H1];
fP1H1=sol(17);econparams(23:4:27)=[fP1H1/1;fP1H1/1];econparams(31:5:41)
=[fP1H1/1000;1/2*fP1H1/1000;1/2*fP1H1/1000];
econparams(24)=sol(18);econparams(28)=sol(19);
econparams(30:5:40)=[sol(20)/1000;sol(20)/1000;sol(20)/1000]*10000;
mH1HH=sol(21);econparams(32:5:42)=[1/2*mH1HH/1000;mH1HH/1000;1/2*mH1HH/
1000];
nISHH=sol(22);econparams(33:5:43)=[1/2*nISHH/1000;1/2*nISHH/1000;nISHH/
1000];
econparams(45)=sol(23);econparams(46)=sol(24);
econparams(48)=sol(25);econparams(49)=sol(26);econparams(50)=.22;

econparams(44)=0.3; %khat
econparams(47)=.09;%1.05e-2; %growth of P1
```

```
econparams(51)=0;%1; %P1barmass
econparams(52)=.4;%0.125; %H1barmass
econparams(53)=0; %ISbarmass
econparams(54)=0.01*4.507354330528614e-004; %dw, second number in
% product is ic_econ(4)/numHH

econparams(55)=9/6; %etaa
econparams(56)=5/6; %etab
econparams(57)=.1/6;%etac
econparams(58)=1e1; %phi

%
% initialize economic state [P1;H1;IS;HH]
%
ic_econ=[1.550317582123858e+000
    1.400236056583227e-001
    1.008970080362582e-001
    4.507354330528614e-001];

numHH=10; %initial number of humans

econparams(59)=4.055586606327234e-003;%ic_econ(4)/numHH; %ideal per
% capita human mass

%
% calculate ecological equilibrium, without any of the domesticated or
% industrial agents
%
ecolparams(8)=ecolparams(8)+.25*ecolparams(8);
ic_ecol=eq21apr2004V2(ecolparams);


ecolparams(18)=0; %set RPIRP to zero, without changing original
% equilibrium.
                  %comment this out to run ecological model itself


%   econparams=zeros(50,1);       %uncomment these two lines to run the
%   econparams(45:46,1)=[1;1];    %ecological model with no economics


% P1, P2, P3, H1, H2, H3, C1, C2, HH, IS, RP, IRP, P1H1massdeficit,
% P1ISmassdeficit,...
% P1HHmassdeficit, H1massdeficit, ISmassdeficit, numHH, percapmass
ic=[ic_econ(1);ic_ecol(1);ic_ecol(2);ic_econ(2);ic_ecol(3);ic_ecol(4);i
c_ecol(5);ic_ecol(6);ic_econ(4);...
        ic_econ(3);ic_ecol(7);ic_ecol(8);0;0;0;0;0; numHH;
4.055586606327234e-003];%ic_econ(4)/numHH];
```

The ecological part of the model has its equilibrium calculated by the function eq21apr2004V2.m, given below.

```
function out = eq21apr2004V2(params)
%
% eq21apr2004V2.m gives the equilibrium of the simple foodweb in
% equations3.nb, in C:\chrisp\EPA\projects\agent
% model\mnsimple_21_apr_2004
%
%
% First written 28 Jul 2004 CWP

%
% assign parameters
%
%
% params =
% [gRPP2;gP2H2;gP2H3;gRPP3;gP3H3;gH2C1;gH2C2;gH3C2;
% rIRPP2;rIRPP3;mP2;mP3;mH2;mH3;mC1;mC2;mIRPRP;gRPIRP;P2]; Here, P2 is
% a choice variable, controlling the total system mass
%
gRPP2=params(1);
gP2H2=params(2);gP2H3=params(3);gRPP3=params(4);gP3H3=params(5);
gH2C1=params(6);gH2C2=params(7);gH3C2=params(8);
rIRPP2=params(9);rIRPP3=params(10);
mP2=params(11);mP3=params(12);mH2=params(13);
mH3=params(14);mC1=params(15);mC2=params(16);
mIRPRP=params(17);RPIRP=params(18);P2=params(19); %gRPIRP not used here

%
% calculate equilibria as determined analytically in the Mathematica
% notebook
%
C1=(1/gH2C1)*(-mH2+gP2H2*P2-(1/gH3C2)*...
(gH2C2*(-mH3+gP2H3*P2+...
(gP3H3*((-gH2C2)*gP3H3*gRPP2*mC1*mIRPRP-...
gH3C2*gP2H2*gRPP3*mC1*mIRPRP+gH2C2*gP2H3*...
gRPP3*mC1*mIRPRP+gH2C1*gP3H3*gRPP2*mC2*...
mIRPRP-gH2C1*gP2H3*gRPP3*mC2*mIRPRP-...
gH2C1*gH3C2*gRPP3*mIRPRP*mP2+gH2C1*gH3C2*...
gRPP2*mIRPRP*mP3-gH2C2*gP3H3*gRPP2*mC1*P2*...
rIRPP2-gH3C2*gP2H2*gRPP3*mC1*P2*rIRPP2+...
gH2C2*gP2H3*gRPP3*mC1*P2*rIRPP2+...
gH2C1*gP3H3*gRPP2*mC2*P2*rIRPP2-...
gH2C1*gP2H3*gRPP3*mC2*P2*rIRPP2-...
gH2C1*gH3C2*gRPP3*mP2*P2*rIRPP2+...
gH2C1*gH3C2*gRPP2*mP3*P2*rIRPP2+...
gH2C1*gH3C2*gRPP3*rIRPP2*RPIRP-...
gH2C1*gH3C2*gRPP2*rIRPP3*RPIRP))/...
((gH2C2*gP3H3*gRPP2*mC1+gH3C2*gP2H2*gRPP3*mC1-...
gH2C2*gP2H3*gRPP3*mC1-gH2C1*gP3H3*gRPP2*mC2+...
gH2C1*gP2H3*gRPP3*mC2+gH2C1*gH3C2*gRPP3*mP2-...
gH2C1*gH3C2*gRPP2*mP3)*rIRPP3))));

C2=(1/gH3C2)*(-mH3+gP2H3*P2+...
(gP3H3*((-gH2C2)*gP3H3*gRPP2*mC1*mIRPRP-...
gH3C2*gP2H2*gRPP3*mC1*mIRPRP+gH2C2*gP2H3*gRPP3*...
```

```
mC1*mIRPRP+gH2C1*gP3H3*gRPP2*mC2*mIRPRP-...
gH2C1*gP2H3*gRPP3*mC2*mIRPRP-gH2C1*gH3C2*gRPP3*...
mIRPRP*mP2+gH2C1*gH3C2*gRPP2*mIRPRP*mP3-...
gH2C2*gP3H3*gRPP2*mC1*P2*rIRPP2-...
gH3C2*gP2H2*gRPP3*mC1*P2*rIRPP2+...
gH2C2*gP2H3*gRPP3*mC1*P2*rIRPP2+...
gH2C1*gP3H3*gRPP2*mC2*P2*rIRPP2-...
gH2C1*gP2H3*gRPP3*mC2*P2*rIRPP2-...
gH2C1*gH3C2*gRPP3*mP2*P2*rIRPP2+...
gH2C1*gH3C2*gRPP2*mP3*P2*rIRPP2+...
gH2C1*gH3C2*gRPP3*rIRPP2*RPIRP-...
gH2C1*gH3C2*gRPP2*rIRPP3*RPIRP))/...
((gH2C2*gP3H3*gRPP2*mC1+gH3C2*gP2H2*gRPP3*mC1-...
gH2C2*gP2H3*gRPP3*mC1-gH2C1*gP3H3*gRPP2*mC2+...
gH2C1*gP2H3*gRPP3*mC2+gH2C1*gH3C2*gRPP3*mP2-...
gH2C1*gH3C2*gRPP2*mP3)*rIRPP3));

H2=mC1/gH2C1;

H3=((-gH2C2)*mC1 + gH2C1*mC2)/(gH2C1*gH3C2);

P3=((-gH2C2)*gP3H3*gRPP2*mC1*mIRPRP-gH3C2*gP2H2*gRPP3*mC1*...
mIRPRP+gH2C2*gP2H3*gRPP3*mC1*mIRPRP+...
gH2C1*gP3H3*gRPP2*mC2*mIRPRP-gH2C1*gP2H3*gRPP3*mC2*...
mIRPRP-gH2C1*gH3C2*gRPP3*mIRPRP*mP2+...
gH2C1*gH3C2*gRPP2*mIRPRP*mP3-gH2C2*gP3H3*gRPP2*mC1*...
P2*rIRPP2-gH3C2*gP2H2*gRPP3*mC1*P2*rIRPP2+...
gH2C2*gP2H3*gRPP3*mC1*P2*rIRPP2+...
gH2C1*gP3H3*gRPP2*mC2*P2*rIRPP2-...
gH2C1*gP2H3*gRPP3*mC2*P2*rIRPP2-...
gH2C1*gH3C2*gRPP3*mP2*P2*rIRPP2+...
gH2C1*gH3C2*gRPP2*mP3*P2*rIRPP2+...
gH2C1*gH3C2*gRPP3*rIRPP2*RPIRP-...
gH2C1*gH3C2*gRPP2*rIRPP3*RPIRP)/...
((gH2C2*gP3H3*gRPP2*mC1+gH3C2*gP2H2*gRPP3*mC1-...
gH2C2*gP2H3*gRPP3*mC1-gH2C1*gP3H3*gRPP2*mC2+...
gH2C1*gP2H3*gRPP3*mC2+gH2C1*gH3C2*gRPP3*mP2-...
gH2C1*gH3C2*gRPP2*mP3)*rIRPP3);

RP=((-gH2C2)*gP3H3*mC1*rIRPP2+gH2C1*gP3H3*mC2*rIRPP2+...
gH2C1*gH3C2*mP3*rIRPP2-gH3C2*gP2H2*mC1*rIRPP3+...
gH2C2*gP2H3*mC1*rIRPP3-gH2C1*gP2H3*mC2*rIRPP3-...
gH2C1*gH3C2*mP2*rIRPP3)/(gH2C1*gH3C2*...
(gRPP3*rIRPP2-gRPP2*rIRPP3));

IRP=((-gH2C2)*gP3H3*gRPP2*mC1-gH3C2*gP2H2*gRPP3*mC1+...
gH2C2*gP2H3*gRPP3*mC1+gH2C1*gP3H3*gRPP2*mC2-...
gH2C1*gP2H3*gRPP3*mC2-gH2C1*gH3C2*gRPP3*mP2+...
gH2C1*gH3C2*gRPP2*mP3)/(gH2C1*gH3C2*...
((-gRPP3)*rIRPP2+gRPP2*rIRPP3));

out=[P2;P3;H2;H3;C1;C2;RP;IRP];
```

S- Function, core of model (file: price_settingSv8.m)

```
function [sys,y0,str,ts] =
price_settingSv8(t,y,u,flag,ecolparams,econparams,...
    iconditions,MH_updown,Lamb_updown,Thet_updown, tlo, thi)
% price_settingSv8.m is based on price_settingSv7.m.  corrects v7
% See summary summtoaustriav5.doc
%
%
%
%
%
% 18 Jan 2005 CWP
%

%
% The following outlines the general structure of an S-function.
%
switch flag,

  %%%%%%%%%%%%%%%%%%
  % Initialization %
  %%%%%%%%%%%%%%%%%%
  case 0,

[sys,y0,str,ts]=mdlInitializeSizes(iconditions,ecolparams,econparams);

  %%%%%%%%%%%%%%%
  % Derivatives %
  %%%%%%%%%%%%%%%
  case 1,
    sys=mdlDerivatives(t,y,u);

  %%%%%%%%%%
  % Update %
  %%%%%%%%%%
  case 2,

sys=mdlUpdate(t,y,u,ecolparams,econparams,MH_updown,Lamb_updown,Thet_up
down, tlo, thi);

  %%%%%%%%%%
  % Outputs %
  %%%%%%%%%%
  case 3,
    sys=mdlOutputs(t,y,u);

  %%%%%%%%%%%%%%%%%%%%%%%
  % GetTimeOfNextVarHit %
  %%%%%%%%%%%%%%%%%%%%%%%
  case 4,
    sys=mdlGetTimeOfNextVarHit(t,y,u);

  %%%%%%%%%%%%
  % Terminate %
  %%%%%%%%%%%%
```

```
  case 9,
    sys=mdlTerminate(t,y,u);

  %%%%%%%%%%%%%%%%%%%%
  % Unexpected flags %
  %%%%%%%%%%%%%%%%%%%%
  otherwise
    error(['Unhandled flag = ',num2str(flag)]);

end

% end sfuntmpl

%
%=============================================================================
% mdlInitializeSizes
% Return the sizes, initial conditions, and sample times for the S-
% function.
%=============================================================================
%
function
[sys,y0,str,ts]=mdlInitializeSizes(iconditions,ecolparams,econparams)

global RPP1 P1H2 P1IS P1H1 P1HH RPIS ISIRP P2H1 H1C1 H1HH pP1 pH1 pIS W
ISHHflow

%
% call simsizes for a sizes structure, fill it in and convert it to a
% sizes array.
%
sizes = simsizes;

sizes.NumContStates  = 0;
sizes.NumDiscStates  = 19;
sizes.NumOutputs     = 33;
sizes.NumInputs      = 0;
sizes.DirFeedthrough = 0;
sizes.NumSampleTimes = 1;   % at least one sample time is needed

sys = simsizes(sizes);

%
% assign initial conditions
%
y0=iconditions;

%
% str is always an empty matrix
%
str = [];

%
% initialize the array of sample times
%
ts  = [1 0];
```

```
%
% Do the following to find initial transfer flows for the global
% variables
%
belownoreproduction=1e-4; %level below which the natural ecosystem
% elements do not reproduce

% assign parameters
%
%
% ecolparams =
% [gRPP2;gP2H2;gP2H3;gRPP3;gP3H3;gH2C1;gH2C2;gH3C2;
% rIRPP2;rIRPP3;mP2;mP3;mH2;mH3;mC1;mC2;mIRPRP;RPIRP;P2]; Here, P2 is a
% choice variable, controlling the total system mass
%
gRPP2=ecolparams(1);
gP2H2=ecolparams(2);gP2H3=ecolparams(3);gRPP3=ecolparams(4);gP3H3=ecolp
arams(5);
gH2C1=ecolparams(6);gH2C2=ecolparams(7);gH3C2=ecolparams(8);
rIRPP2=ecolparams(9);rIRPP3=ecolparams(10);
mP2=ecolparams(11);mP3=ecolparams(12);mH2=ecolparams(13);
mH3=ecolparams(14);mC1=ecolparams(15);mC2=ecolparams(16);
mIRPRP=ecolparams(17);RPIRP=ecolparams(18); %this last constant changed
% 2 Aug 2004
gP1H2=ecolparams(20);gH1C1=ecolparams(21); %13 Aug 2004 natural
% predation of P1 and H1


%
% econparams are economic parameters:
%
aw=econparams(1);cw=econparams(2);
aP1=econparams(3);bP1=econparams(4);cP1=econparams(5);
aP1p=econparams(6);bP1p=econparams(7);cP1p=econparams(8);
aH1=econparams(9);bH1=econparams(10);cH1=econparams(11);
aH1p=econparams(12);bH1p=econparams(13);cH1p=econparams(14);
aIS=econparams(15);bIS=econparams(16);cIS=econparams(17);
aISp=econparams(18);bISp=econparams(19);cISp=econparams(20);
dP1H1=econparams(21);eP1H1=econparams(22);fP1H1=econparams(23);gP1H1=ec
onparams(24);
% dP1IS=econparams(25);eP1IS=econparams(26);fP1IS=econparams(27);
% gP1IS=econparams(28);
dP1HH=econparams(29);zP1HH=econparams(30);kP1HH=econparams(31);mP1HH=ec
onparams(32);nP1HH=econparams(33);
dH1HH=econparams(34);zH1HH=econparams(35);kH1HH=econparams(36);mH1HH=ec
onparams(37);nH1HH=econparams(38);
dISHH=econparams(39);zISHH=econparams(40);kISHH=econparams(41);mISHH=ec
onparams(42);nISHH=econparams(43);
khat=econparams(44);theta=econparams(45);lambda=econparams(46);
gRPP1=econparams(47);mP1=econparams(48);mH1=econparams(49);mHH=econpara
ms(50);
P1bar=econparams(51);H1bar=econparams(52);ISbar=econparams(53);dw=econp
arams(54);
etaa=econparams(55);etab=econparams(56);etac=econparams(57);phi=econpar
ams(58);
```

```
idealpercapmass=econparams(59);

%
% assign state
%
P1=y0(1);P2=y0(2);P3=y0(3);
H1=y0(4);H2=y0(5);H3=y0(6);
C1=y0(7);C2=y0(8);
HH=y0(9);ISmass=y0(10); %ISmassdeficit keeps track of deficits
RP=y0(11);
IRP=y0(12);
P1H1massdeficit=y0(13);P1ISmassdeficit=y0(14);P1HHmassdeficit=y0(15);
H1massdeficit=y0(16);ISmassdeficit=y0(17);
numHH=y0(18);percapmass=y0(19);
P1massdeficit=P1H1massdeficit+P1ISmassdeficit+P1HHmassdeficit;

rIRPP2=rIRPP2*(10^2/(10^2+IRP^2));
rIRPP3=rIRPP3*(10^2/(10^2+IRP^2));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                             %
%       Economics                                                             %
%                                                                             %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%
% Industrial sector sets the wage rate
%
W=max(aw+cw*(ISbar-(ISmassdeficit+ISmass))/(theta+lambda)-dw*numHH,0);

%
% Based on the Wage, industries set prices and their production (how
% much they would like to produce to maximize their profits based on
% their assumption as to what the demand for their products will be).
% Here a linear functional form for the supply is assumed.
%
if P1==0
    pP1=0;
    P1production=0;
else
    pP1=max(aP1+bP1*W-cP1*((P1massdeficit+P1)-P1bar),0);
    P1production=max(aP1p-bP1p*W-cP1p*((P1massdeficit+P1)-P1bar),0);
end

if H1==0
    pH1=0;
    H1production=0;
else
    pH1=max(aH1+bH1*W-cH1*((H1massdeficit+H1)-H1bar),0);
    H1production=max(aH1p-bH1p*W-cH1p*((H1massdeficit+H1)-H1bar),0);
end

pIS=max(aIS+bIS*W+cIS*(ISbar-(ISmassdeficit+ISmass))/(theta+lambda),0);
ISproduction=max(aISp-bISp*W+cISp*(ISbar-
(ISmassdeficit+ISmass))/(theta+lambda),0);
```

```
if HH==0|numHH<2
    pIS=0;
    ISproduction=0;
end

%
% Next, a calculation is made of how much each industry is going to
% demand of its suppliers. The following are in units of mass, unless
% otherwise noted
%
if H1==0|HH==0|numHH<2
    P1H1demand=0;
    P2H1=0;
else
    P1H1demand=max(dP1H1-eP1H1*W-fP1H1*pP1-gP1H1*((H1massdeficit+H1)-
H1bar),0);
    P2H1=khat;
end



%
% expressions for P1HH, H1HH and ISHH reflect constraint on human
% spending.
% These expressions were determined in MATHEMATICA under the file
% C:\chrisp\EPA\projects\agent model\mnsimple_21_apr_2004
% solve_humans.nb
% These are per capita, so must be multiplied by population later
%
P1HHdemand=max((1/(-1+zP1HH+zH1HH+zISHH))*(-dP1HH-mP1HH*pH1-
nP1HH*pIS+...
kP1HH*pP1-dH1HH*zP1HH-dISHH*zP1HH+mH1HH*pH1*zP1HH-...
mISHH*pH1*zP1HH-nH1HH*pIS*zP1HH+nISHH*pIS*zP1HH-...
kH1HH*pP1*zP1HH-kISHH*pP1*zP1HH+dP1HH*zH1HH+...
mP1HH*pH1*zH1HH+nP1HH*pIS*zH1HH-kP1HH*pP1*zH1HH+...
dP1HH*zISHH+mP1HH*pH1*zISHH+nP1HH*pIS*zISHH-kP1HH*pP1*zISHH),0);

H1HHdemand=max((1/(-1+zP1HH+zH1HH+zISHH))*(-dH1HH+mH1HH*pH1-nH1HH*pIS-
...
kH1HH*pP1+dH1HH*zP1HH-mH1HH*pH1*zP1HH+nH1HH*pIS*zP1HH+...
kH1HH*pP1*zP1HH-dISHH*zH1HH-dP1HH*zH1HH-mISHH*pH1*zH1HH-...
mP1HH*pH1*zH1HH+nISHH*pIS*zH1HH-nP1HH*pIS*zH1HH-...
kISHH*pP1*zH1HH+kP1HH*pP1*zH1HH+dH1HH*zISHH-...
mH1HH*pH1*zISHH+nH1HH*pIS*zISHH+kH1HH*pP1*zISHH),0);

ISHHdemand=max(-((dISHH+mISHH*pH1-nISHH*pIS+kISHH*pP1-dISHH*zP1HH-
mISHH*pH1*zP1HH+...
nISHH*pIS*zP1HH-kISHH*pP1*zP1HH-dISHH*zH1HH-
mISHH*pH1*zH1HH+nISHH*pIS*zH1HH-...
kISHH*pP1*zH1HH+dH1HH*zISHH+dP1HH*zISHH-
mH1HH*pH1*zISHH+mP1HH*pH1*zISHH+...
nH1HH*pIS*zISHH+nP1HH*pIS*zISHH+kH1HH*pP1*zISHH-kP1HH*pP1*zISHH)/...
(-1+zP1HH+zH1HH+zISHH)),0); %in units of units
                            % corrected 9 Aug 2004
if HH==0|numHH<2
    P1HHdemand=0;
```

```
    H1HHdemand=0;
    ISHHdemand=0;
end

%
% The flows that involve labor to keep the wild
% from taking domestics, namely P1H2 and H1C2 must then be calculated.
%
if P1==0|H2==0
    P1H2=0;
else
    P1H2=max((gRPP1*P1*RP-mP1*P1-P1production),0);
end

if HH==0|numHH<2
    P1H2=gP1H2*P1*H2;
end

if H1==0|C1==0
    H1C1=0;
else
    H1C1=max((P1H1demand+P2H1-mH1*H1-H1production),0);
end

if HH==0|numHH<2
    H1C1=gH1C1*H1*C1;
end

P1ISdemand=theta*ISproduction;
RPISdemand=lambda*ISproduction;



%
% calculate all but next state, according to system
% equations (pricesettingequations3.doc, and Whitmore's paper
% C:\chrisp\EPA\projects\agent model\mnsimple_21_apr_2004\whitdocs\12
% cell imp comp EPA 5-04-04.doc)
%
%
% P1
%
P1RP=max(mP1*P1,0);RPP1=max(gRPP1*P1*RP,0);
P1H1=P1H1demand;
P1IS=P1ISdemand;
P1HH=P1HHdemand*numHH;
if P1+RPP1-P1RP-P1H2-P1H1-P1HH-P1IS<0 %if statement to deal with going
% negative
    if P1+RPP1-P1RP<0
        P1RP=P1+RPP1;
        P1H2=0;P1H1=0;P1HH=0;P1IS=0;
    else
        totP1demand=P1H2+P1H1+P1HH+P1IS;
        P1avail=P1+RPP1-P1RP;
        P1H2=P1avail*P1H2/totP1demand;
```

```
        P1H1=P1avail*P1H1/totP1demand;
        P1HH=P1avail*P1HH/totP1demand;
        P1IS=P1avail-(P1H2+P1H1+P1HH);
    end
else
    if P1massdeficit<0 %if there is an accumulated deficit between
% demand for P1 try to
                  %make this up if there is extra stock
    P1surplus=min(P1+RPP1-P1RP-P1H2-P1H1-P1HH-P1IS,-P1massdeficit);
%only what you need to make up deficit
    P1H1=P1H1+P1surplus*P1H1massdeficit/P1massdeficit;
    P1IS=P1IS+P1surplus*P1ISmassdeficit/P1massdeficit;
    P1HH=P1HH+P1surplus*P1HHmassdeficit/P1massdeficit;
    end
end

P2H2=gP2H2*P2*H2;P2H3=gP2H3*P2*H3;P2RP=max(mP2*P2,0);RPP2=max(gRPP2*RP*
P2,0);
IRPP2=max(rIRPP2*P2*IRP,0);
P3RP=max(mP3*P3,0);P3H3=gP3H3*P3*H3;RPP3=max(gRPP3*RP*P3,0);
IRPP3=max(rIRPP3*P3*IRP,0);
if IRP<=0
    IRPP2=0;
    IRPP3=0;
elseif IRP-IRPP2-IRPP3-max(IRP*mIRPRP,0)+RPIRP<0
    if P2~=0
        IRPP2=rIRPP2*(IRP-max(IRP*mIRPRP,0)+RPIRP)/(rIRPP2+rIRPP3);
    end
    if P3~=0
        IRPP3=rIRPP3*(IRP-max(IRP*mIRPRP,0)+RPIRP)/(rIRPP2+rIRPP3);
    end
end
if P2+IRPP2+RPP2-P2RP-P2H2-P2H3-P2H1<belownoreproduction
    if P2+IRPP2+RPP2-P2RP<belownoreproduction
        P2RP=P2+IRPP2+RPP2;
        P2H2=0;P2H3=0;P2H1=0;
    else
    totP2demand=P2H2+P2H3+P2H1;
    P2avail=P2+IRPP2+RPP2-P2RP;
    P2H2=P2H2*P2avail/totP2demand;
    P2H3=P2H3*P2avail/totP2demand;
    P2H1=P2avail-(P2H2+P2H3);
    end
end

if P3+IRPP3+RPP3-P3RP-P3H3<belownoreproduction
    if P3+IRPP3+RPP3-P3RP<belownoreproduction
        P3RP=P3+IRPP3+RPP3;
        P3H3=0;
    else
        totP3demand=P3H3;
        P3avail=P3+IRPP3+RPP3-P3RP;
        P3H3=P3H3*P3avail/totP3demand;
    end
end
```

```
H1RP=max(mH1*H1,0);
H1HH=H1HHdemand*numHH;
if H1+P1H1+P2H1-H1RP-H1C1-H1HH<0
    if H1+P1H1+P2H1-H1RP<0
        H1RP=H1+P1H1+P2H1;
        H1C1=0;H1HH=0;
    else
        totH1demand=H1C1+H1HH;
        H1avail=H1+P1H1+P2H1-H1RP;
        H1C1=H1avail*H1C1/totH1demand;
        H1HH=H1avail-H1C1;
    end
else
    if H1massdeficit<0
        H1HH=H1HH+min(H1+P1H1+P2H1-H1RP-H1C1-H1HH,-H1massdeficit);
%only what you need to make up deficit
    end
end



H2C1=gH2C1*C1*H2;H2C2=gH2C2*H2*C2;H2RP=max(mH2*H2,0);
if H2+P1H2+P2H2-H2RP-H2C1-H2C2<belownoreproduction
    if H2+P1H2+P2H2-H2RP<belownoreproduction
        H2RP=H2+P1H2+P2H2;
        H2C1=0;H2C2=0;
    else
        totH2demand=H2C1+H2C2;
        H2avail=H2+P1H2+P2H2-H2RP;
        H2C1=H2C1*H2avail/totH2demand;
        H2C2=H2avail-H2C1;
    end
end

H3RP=max(mH3*H3,0);H3C2=gH3C2*H3*C2;
if H3+P2H3+P3H3-H3RP-H3C2<belownoreproduction
    if H3+P2H3+P3H3-H3RP<belownoreproduction
        H3RP=H3+P2H3+P3H3;
        H3C2=0;
    else
        totH3demand=H3C2;
        H3avail=H3+P2H3+P3H3-H3RP;
        H3C2=H3C2*H3avail/totH3demand;
    end
end

C1RP=max(mC1*C1,0);
if C1+H1C1+H2C1-C1RP<belownoreproduction
    C1RP=C1+H1C1+H2C1;
end

C2RP=max(mC2*C2,0);
if C2+H2C2+H3C2-C2RP<belownoreproduction
    C2RP=C2+H2C2+H3C2;
end

HHRP=mHH*numHH*percapmass;
```

```
IRPRP=max(IRP*mIRPRP,0);
RPIS=min(lambda*P1IS/theta,RPISdemand);
stockRP=RP+P1RP+P2RP+P3RP+H1RP+H2RP+H3RP+C1RP+C2RP+HHRP+IRPRP;
if stockRP<0
    stockRP=0;
end
if stockRP-(RPP1+RPP2+RPP3)-RPIRP-RPIS<=0&RPIRP==0 %this line changed
% 26 Aug 2004
    RPdemand=RPP1+RPP2+RPP3+RPIS;
        RPP1=RPP1*stockRP/RPdemand;
            RPP2=RPP2*stockRP/RPdemand;
            RPP3=RPP3*stockRP/RPdemand;
        if RPIS~=0
            RPIS=stockRP-(RPP1+RPP2+RPP3);
        else
            RPIS=0;
        end
end
P1IS=min(theta*RPIS/lambda,P1IS);
% t
% stockRP
% nextRP = stockRP-(RPP1+RPP2+RPP3)-RPIRP-RPIS


ISHHflow=max(0,(theta+lambda)*ISHHdemand*numHH);
ISIRP=ISHHflow;
if ISmass+P1IS+RPIS-ISIRP<=0
    ISIRP=ISmass+P1IS+RPIS;
else
    if ISmassdeficit<0&numHH>=2 %if there is an accumulated deficit
% between demand for IS by the HH and IS
        %supplied, try to make this up if there is extra stock
        ISIRP=ISIRP+min(ISmass+P1IS+RPIS-ISIRP,-ISmassdeficit); %only
% what you need to make up deficit
    end
end

if (P1HHdemand+H1HHdemand+ISHHflow)==0
    weightedprice=0;
    percapbirths=0;
elseif (pP1*P1HHdemand+pH1*H1HHdemand+pIS*ISHHflow)==0
    weightedprice=0;
    percapbirths=0;
else

%weightedprice=(pP1*P1HHdemand+pH1*H1HHdemand+pIS*ISHHflow)/(P1HHdemand
% +H1HHdemand+ISHHflow);
    weightedprice=(pP1*P1HH+pH1*H1HH+pIS*ISIRP)/(P1HH+H1HH+ISIRP);
        %percapbirths=max(etaa-
% etab*W/weightedprice+etac*(W/weightedprice)^2,0);
    percapbirths=max(etaa-etab*sqrt(W/weightedprice),0);
%     if etab<=2*etac*(W/weightedprice)|etaa<=etac*(W/weightedprice)^2
%         error('pricesettingmodel:growtherror','Percapbirths function
% parameters give bad function form')
%     end
```

```
end

% end mdlInitializeSizes

%
%=============================================================================
% mdlDerivatives
% Return the derivatives for the continuous states.
%=============================================================================
%
function sys=mdlDerivatives(t,y,u)

sys=[];



% end mdlDerivatives




%
%=============================================================================
% mdlUpdate
% Handle discrete state updates, sample time hits, and major time step
% requirements.
%=============================================================================
%
function
sys=mdlUpdate(t,y,u,ecolparams,econparams,MH_updown,Lamb_updown,Thet_up
down, tlo, thi)

global RPP1 P1H2 P1IS P1H1 P1HH RPIS ISIRP P2H1 H1C1 H1HH pP1 pH1 pIS W
ISHHflow

belownoreproduction=1e-4; %level below which the natural ecosystem
% elements do not reproduce

% assign parameters
%
%
% ecolparams =
% [gRPP2;gP2H2;gP2H3;gRPP3;gP3H3;gH2C1;gH2C2;gH3C2;
% rIRPP2;rIRPP3;mP2;mP3;mH2;mH3;mC1;mC2;mIRPRP;RPIRP;P2]; Here, P2 is a
% choice variable, controlling the total system mass %gRPIRP changed 2
% Aug 2004 to RPIRP
%
gRPP2=ecolparams(1);
gP2H2=ecolparams(2);gP2H3=ecolparams(3);gRPP3=ecolparams(4);gP3H3=ecolp
arams(5);
gH2C1=ecolparams(6);gH2C2=ecolparams(7);gH3C2=ecolparams(8);
rIRPP2=ecolparams(9);rIRPP3=ecolparams(10);
mP2=ecolparams(11);mP3=ecolparams(12);mH2=ecolparams(13);
```

```
mH3=ecolparams(14);mC1=ecolparams(15);mC2=ecolparams(16);
mIRPRP=ecolparams(17);RPIRP=ecolparams(18);
gP1H2=ecolparams(20);gH1C1=ecolparams(21); %13 Aug 2004 natural
predation of P1 and H1


%
% econparams are economic parameters:
%
aw=econparams(1);cw=econparams(2);
aP1=econparams(3);bP1=econparams(4);cP1=econparams(5);
aP1p=econparams(6);bP1p=econparams(7);cP1p=econparams(8);
aH1=econparams(9);bH1=econparams(10);cH1=econparams(11);
aH1p=econparams(12);bH1p=econparams(13);cH1p=econparams(14);
aIS=econparams(15);bIS=econparams(16);cIS=econparams(17);
aISp=econparams(18);bISp=econparams(19);cISp=econparams(20);
dP1H1=econparams(21);eP1H1=econparams(22);fP1H1=econparams(23);gP1H1=ec
onparams(24);
%dP1IS=econparams(25);eP1IS=econparams(26);fP1IS=econparams(27);gP1IS=e
% conparams(28);
dP1HH=econparams(29);zP1HH=econparams(30);kP1HH=econparams(31);mP1HH=ec
onparams(32);nP1HH=econparams(33);
dH1HH=econparams(34);zH1HH=econparams(35);kH1HH=econparams(36);mH1HH=ec
onparams(37);nH1HH=econparams(38);
dISHH=econparams(39);zISHH=econparams(40);kISHH=econparams(41);mISHH=ec
onparams(42);nISHH=econparams(43);
khat=econparams(44);theta=econparams(45);lambda=econparams(46);
gRPP1=econparams(47);mP1=econparams(48);mH1=econparams(49);mHH=econpara
ms(50);
P1bar=econparams(51);H1bar=econparams(52);ISbar=econparams(53);dw=econp
arams(54);
etaa=econparams(55);etab=econparams(56);etac=econparams(57);phi=econpar
ams(58);
idealpercapmass=econparams(59);

%  gRPP1=gRPP1*(1+1/100*sin(2*pi*t/12));
%  gRPP2=gRPP2*(1+1/100*sin(2*pi*t/12));
%  gRPP3=gRPP3*(1+1/100*sin(2*pi*t/12));

lambdaz=lambda;thetaz=theta;mHHz=mHH;


if Lamb_updown~=1
    if Lamb_updown==2
        if t>=tlo&t<=thi
            lambda=lambdaz+.001*(t-tlo);
        elseif t>thi
            lambda=lambdaz+.001*(thi-tlo);
        end
    else
        if t>=tlo
            lambda=max(lambdaz-.001*(t-tlo),lambda/10);
        end
    end
end
```

```
if Thet_updown~=1
    if Thet_updown==2
        if t>=tlo
            theta=thetaz+.001*(t-tlo);
        end
    else
        if t>=tlo
            theta=max(thetaz-.001*(t-tlo),theta/10);
        end
    end
end


if MH_updown~=1
    if MH_updown==2
        if t>=tlo&t<=thi
            mHH=min(mHH*2,mHHz+.001*(t-tlo));
        elseif t>thi
            mHH=min(mHH*2,mHHz+.001*(thi-tlo));
        end
    else
        if t>=tlo
            mHH=max(mHH/10,mHHz-.001*(t-tlo));
        end
    end
end


%
% assign state
%
P1=y(1);P2=y(2);P3=y(3);
H1=y(4);H2=y(5);H3=y(6);
C1=y(7);C2=y(8);
HH=y(9);ISmass=y(10);
RP=y(11);
IRP=y(12);
P1H1massdeficit=y(13);P1ISmassdeficit=y(14);P1HHmassdeficit=y(15);
H1massdeficit=y(16);ISmassdeficit=y(17);
numHH=y(18);percapmass=y(19);
P1massdeficit=P1H1massdeficit+P1ISmassdeficit+P1HHmassdeficit;


rIRPP2=rIRPP2*(10^2/(10^2+IRP^2));
rIRPP3=rIRPP3*(10^2/(10^2+IRP^2));


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                             %
%       Economics                                                             %
%                                                                             %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%
```

```
% Industrial sector sets the wage rate
%
W=max(aw+cw*(ISbar-(ISmassdeficit+ISmass))/(theta+lambda)-dw*numHH,0);

%
% Based on the Wage, industries set prices and their production (how
% much they would like to produce to maximize their profits based on
% their assumption as to what the demand for their products will be).
% Here, a linear functional form is assumed for the supply.
if P1==0
    pP1=0;
    P1production=0;
else
    pP1=max(aP1+bP1*W-cP1*((P1massdeficit+P1)-P1bar),0);
    P1production=max(aP1p-bP1p*W-cP1p*((P1massdeficit+P1)-P1bar),0);
end

if H1==0
    pH1=0;
    H1production=0;
else
    pH1=max(aH1+bH1*W-cH1*((H1massdeficit+H1)-H1bar),0);
    H1production=max(aH1p-bH1p*W-cH1p*((H1massdeficit+H1)-H1bar),0);
end

pIS=max(aIS+bIS*W+cIS*(ISbar-(ISmassdeficit+ISmass))/(theta+lambda),0);
ISproduction=max(aISp-bISp*W+cISp*(ISbar-
(ISmassdeficit+ISmass))/(theta+lambda),0);
if HH==0|numHH<2
    pIS=0;
    ISproduction=0;
end

%
% Next, how much each industry is going to demand of its
% suppliers is calculated.  The following are in units of mass, unless
% otherwise noted
%
if H1==0|HH==0|numHH<2
    P1H1demand=0;
    P2H1=0;
else
    P1H1demand=max(dP1H1-eP1H1*W-fP1H1*pP1-gP1H1*((H1massdeficit+H1)-
H1bar),0);
    P2H1=khat;
end



%
% expressions for P1HH, H1HH and ISHH reflect constraint on human
% spending.
% These expressions were determined in MATHEMATICA under the file
% C:\chrisp\EPA\projects\price_setting\mnsimple_21_apr_2004
% solve_humans.nb
% These are per capita, so must be multiplied by population later
```

```
%
P1HHdemand=max((1/(-1+zP1HH+zH1HH+zISHH))*(-dP1HH-mP1HH*pH1-
nP1HH*pIS+...
kP1HH*pP1-dH1HH*zP1HH-dISHH*zP1HH+mH1HH*pH1*zP1HH-...
mISHH*pH1*zP1HH-nH1HH*pIS*zP1HH+nISHH*pIS*zP1HH-...
kH1HH*pP1*zP1HH-kISHH*pP1*zP1HH+dP1HH*zH1HH+...
mP1HH*pH1*zH1HH+nP1HH*pIS*zH1HH-kP1HH*pP1*zH1HH+...
dP1HH*zISHH+mP1HH*pH1*zISHH+nP1HH*pIS*zISHH-kP1HH*pP1*zISHH),0);

H1HHdemand=max((1/(-1+zP1HH+zH1HH+zISHH))*(-dH1HH+mH1HH*pH1-nH1HH*pIS-
...
kH1HH*pP1+dH1HH*zP1HH-mH1HH*pH1*zP1HH+nH1HH*pIS*zP1HH+...
kH1HH*pP1*zP1HH-dISHH*zH1HH-dP1HH*zH1HH-mISHH*pH1*zH1HH-...
mP1HH*pH1*zH1HH+nISHH*pIS*zH1HH-nP1HH*pIS*zH1HH-...
kISHH*pP1*zH1HH+kP1HH*pP1*zH1HH+dH1HH*zISHH-...
mH1HH*pH1*zISHH+nH1HH*pIS*zISHH+kH1HH*pP1*zISHH),0);

ISHHdemand=max(-((dISHH+mISHH*pH1-nISHH*pIS+kISHH*pP1-dISHH*zP1HH-
mISHH*pH1*zP1HH+...
nISHH*pIS*zP1HH-kISHH*pP1*zP1HH-dISHH*zH1HH-
mISHH*pH1*zH1HH+nISHH*pIS*zH1HH-...
kISHH*pP1*zH1HH+dH1HH*zISHH+dP1HH*zISHH-
mH1HH*pH1*zISHH+mP1HH*pH1*zISHH+...
nH1HH*pIS*zISHH+nP1HH*pIS*zISHH+kH1HH*pP1*zISHH-kP1HH*pP1*zISHH)/...
(-1+zP1HH+zH1HH+zISHH)),0); %in units of units
                            % corrected 9 Aug 2004

if HH==0|numHH<2
    ISHHdemand=0;
    P1HHdemand=0;
    H1HHdemand=0;
end
%    ISHHdemand+P1HHdemand+H1HHdemand

% P1HHdemand
%
% The flows that involve labor to keep the wild
% from taking domestics, namely P1H2 and H1C2 must then be calculated.
%
if P1==0|H2==0
    P1H2=0;
else
    P1H2=max((gRPP1*P1*RP-mP1*P1-P1production),0);
end

if HH==0|numHH<2
    P1H2=gP1H2*P1*H2;
end

if H1==0|C1==0
    H1C1=0;
else
    H1C1=max((P1H1demand+P2H1-mH1*H1-H1production),0);
end
```

```
if HH==0|numHH<2
    H1C1=gH1C1*H1*C1;
end

%
% the ISproduction is checked again below as well, after checks for
% realistic mass transfers
%
if HH==0|numHH<2
    ISproduction=0.0;
end
P1ISdemand=theta*ISproduction;
RPISdemand=lambda*ISproduction;

%
% calculate next state, according to system
% equations (pricesettingequations3.doc, and Whitmore's paper
% C:\chrisp\EPA\projects\agent model\mnsimple_21_apr_2004\whitdocs\12
% cell imp comp EPA 5-04-04.doc)
% Here, check to see that these transfers won't violate conservation of
% mass

%
% P1
%
P1RP=max(mP1*P1,0);RPP1=max(gRPP1*P1*RP,0);
P1H1=P1H1demand;
P1IS=P1ISdemand;
P1HH=P1HHdemand*numHH;
if P1+RPP1-P1RP-P1H2-P1H1-P1HH-P1IS<0 %if statement to deal with going
% negative
    if P1+RPP1-P1RP<0
        P1RP=P1+RPP1;
        P1H2=0;P1H1=0;P1HH=0;P1IS=0;
    else
        totP1demand=P1H2+P1H1+P1HH+P1IS;
        P1avail=P1+RPP1-P1RP;
        P1H2=P1avail*P1H2/totP1demand;
        P1H1=P1avail*P1H1/totP1demand;
        P1HH=P1avail*P1HH/totP1demand;
        P1IS=P1avail-(P1H2+P1H1+P1HH);
    end
else
    if P1massdeficit<0 %if there is an accumulated deficit between
% demand for P1 try to
                  %make this up if there is extra stock
    P1surplus=min(P1+RPP1-P1RP-P1H2-P1H1-P1HH-P1IS,-P1massdeficit);
%only what you need to make up deficit
    P1H1=P1H1+P1surplus*P1H1massdeficit/P1massdeficit;
    P1IS=P1IS+P1surplus*P1ISmassdeficit/P1massdeficit;
    P1HH=P1HH+P1surplus*P1HHmassdeficit/P1massdeficit;
    end
end

%
% P2
```

```
%
P2H2=gP2H2*P2*H2;P2H3=gP2H3*P2*H3;P2RP=max(mP2*P2,0);RPP2=max(gRPP2*RP*
P2,0);
IRPP2=max(rIRPP2*P2*IRP,0);
P3RP=max(mP3*P3,0);P3H3=gP3H3*P3*H3;RPP3=max(gRPP3*RP*P3,0);
IRPP3=max(rIRPP3*P3*IRP,0);
if IRP<=0
    IRPP2=0;
    IRPP3=0;
elseif IRP-IRPP2-IRPP3-max(IRP*mIRPRP,0)+RPIRP<0
    if P2~=0
        IRPP2=rIRPP2*(IRP-max(IRP*mIRPRP,0)+RPIRP)/(rIRPP2+rIRPP3);
    end
    if P3~=0
        IRPP3=rIRPP3*(IRP-max(IRP*mIRPRP,0)+RPIRP)/(rIRPP2+rIRPP3);
    end
end
if P2+IRPP2+RPP2-P2RP-P2H2-P2H3-P2H1<belownoreproduction
    if P2+IRPP2+RPP2-P2RP<belownoreproduction
        P2RP=P2+IRPP2+RPP2;
        P2H2=0;P2H3=0;P2H1=0;
    else
    totP2demand=P2H2+P2H3+P2H1;
    P2avail=P2+IRPP2+RPP2-P2RP;
    P2H2=P2H2*P2avail/totP2demand;
    P2H3=P2H3*P2avail/totP2demand;
    P2H1=P2avail-(P2H2+P2H3);
    end
end

%
% P3
%
if P3+IRPP3+RPP3-P3RP-P3H3<belownoreproduction
    if P3+IRPP3+RPP3-P3RP<belownoreproduction
        P3RP=P3+IRPP3+RPP3;
        P3H3=0;
    else
        totP3demand=P3H3;
        P3avail=P3+IRPP3+RPP3-P3RP;
        P3H3=P3H3*P3avail/totP3demand;
    end
end

%
% H1
%
H1RP=max(mH1*H1,0);
H1HH=H1HHdemand*numHH;
if H1+P1H1+P2H1-H1RP-H1C1-H1HH<0
    if H1+P1H1+P2H1-H1RP<0
        H1RP=H1+P1H1+P2H1;
        H1C1=0;H1HH=0;
    else
        totH1demand=H1C1+H1HH;
        H1avail=H1+P1H1+P2H1-H1RP;
```

```
        H1C1=H1avail*H1C1/totH1demand;
        H1HH=H1avail-H1C1;
    end
else
    if H1massdeficit<0
        H1HH=H1HH+min(H1+P1H1+P2H1-H1RP-H1C1-H1HH,-H1massdeficit);
%only what you need to make up deficit
    end
end

%
% H2
%
H2C1=gH2C1*C1*H2;H2C2=gH2C2*H2*C2;H2RP=max(mH2*H2,0);
if H2+P1H2+P2H2-H2RP-H2C1-H2C2<belownoreproduction
    if H2+P1H2+P2H2-H2RP<belownoreproduction
        H2RP=H2+P1H2+P2H2;
        H2C1=0;H2C2=0;
    else
        totH2demand=H2C1+H2C2;
        H2avail=H2+P1H2+P2H2-H2RP;
        H2C1=H2C1*H2avail/totH2demand;
        H2C2=H2avail-H2C1;
    end
end

%
% H3
%
H3RP=max(mH3*H3,0);H3C2=gH3C2*H3*C2;
if H3+P2H3+P3H3-H3RP-H3C2<belownoreproduction
    if H3+P2H3+P3H3-H3RP<belownoreproduction
        H3RP=H3+P2H3+P3H3;
        H3C2=0;
    else
        totH3demand=H3C2;
        H3avail=H3+P2H3+P3H3-H3RP;
        H3C2=H3C2*H3avail/totH3demand;
    end
end

%
% C1
%
C1RP=max(mC1*C1,0);
if C1+H1C1+H2C1-C1RP<belownoreproduction
    C1RP=C1+H1C1+H2C1;
end

%
% C2
%
C2RP=max(mC2*C2,0);
if C2+H2C2+H3C2-C2RP<belownoreproduction
    C2RP=C2+H2C2+H3C2;
end
```

```
%
% HH
%
HHRP=ceil(mHH*numHH)*percapmass;


%
% RP
%
IRPRP=max(IRP*mIRPRP,0);
RPIS=min(lambda*P1IS/theta,RPISdemand);
stockRP=RP+P1RP+P2RP+P3RP+H1RP+H2RP+H3RP+C1RP+C2RP+HHRP+IRPRP;
if stockRP<0
    stockRP=0;
end
if stockRP-(RPP1+RPP2+RPP3)-RPIRP-RPIS<=0&RPIRP==0 %this line changed
% 26 Aug 2004
    RPdemand=RPP1+RPP2+RPP3+RPISdemand;
            RPP1=RPP1*stockRP/RPdemand;
            RPP2=RPP2*stockRP/RPdemand;
            RPP3=RPP3*stockRP/RPdemand;
            if RPIS~=0
                RPIS=stockRP-(RPP1+RPP2+RPP3);
            else
                RPIS=0;
            end
end
P1IS=min(theta*RPIS/lambda,P1IS);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% make checks again, to balance flows
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%
% P1
%
if P1+RPP1-P1RP-P1H2-P1H1-P1HH-P1IS<0 %if statement to deal with going
% negative
    if P1+RPP1-P1RP<0
        P1RP=P1+RPP1;
        P1H2=0;P1H1=0;P1HH=0;P1IS=0;
    else
        totP1demand=P1H2+P1H1+P1HH+P1IS;
        P1avail=P1+RPP1-P1RP;
        P1H2=P1avail*P1H2/totP1demand;
        P1H1=P1avail*P1H1/totP1demand;
        P1HH=P1avail*P1HH/totP1demand;
        P1IS=P1avail-(P1H2+P1H1+P1HH);
    end
else
    if P1massdeficit<0 %if there is an accumulated deficit between
% demand for P1 try to
                   %make this up if there is extra stock
    P1surplus=min(P1+RPP1-P1RP-P1H2-P1H1-P1HH-P1IS,-P1massdeficit);
%only what you need to make up deficit
```

```
    P1H1=P1H1+P1surplus*P1H1massdeficit/P1massdeficit;
    P1IS=P1IS+P1surplus*P1ISmassdeficit/P1massdeficit;
    P1HH=P1HH+P1surplus*P1HHmassdeficit/P1massdeficit;
    end
end

%
% P2
%
if IRP<=0
    IRPP2=0;
    IRPP3=0;
elseif IRP-IRPP2-IRPP3-max(IRP*mIRPRP,0)+RPIRP<0
    if P2~=0
        IRPP2=rIRPP2*(IRP-max(IRP*mIRPRP,0)+RPIRP)/(rIRPP2+rIRPP3);
    end
    if P3~=0
        IRPP3=rIRPP3*(IRP-max(IRP*mIRPRP,0)+RPIRP)/(rIRPP2+rIRPP3);
    end
end
if P2+IRPP2+RPP2-P2RP-P2H2-P2H3-P2H1<belownoreproduction
    if P2+IRPP2+RPP2-P2RP<belownoreproduction
        P2RP=P2+IRPP2+RPP2;
        P2H2=0;P2H3=0;P2H1=0;
    else
    totP2demand=P2H2+P2H3+P2H1;
    P2avail=P2+IRPP2+RPP2-P2RP;
    P2H2=P2H2*P2avail/totP2demand;
    P2H3=P2H3*P2avail/totP2demand;
    P2H1=P2avail-(P2H2+P2H3);
    end
end

%
% P3
%
if P3+IRPP3+RPP3-P3RP-P3H3<belownoreproduction
    if P3+IRPP3+RPP3-P3RP<belownoreproduction
        P3RP=P3+IRPP3+RPP3;
        P3H3=0;
    else
        totP3demand=P3H3;
        P3avail=P3+IRPP3+RPP3-P3RP;
        P3H3=P3H3*P3avail/totP3demand;
    end
end

%
% H1
%
if H1+P1H1+P2H1-H1RP-H1C1-H1HH<0
    if H1+P1H1+P2H1-H1RP<0
        H1RP=H1+P1H1+P2H1;
        H1C1=0;H1HH=0;
    else
        totH1demand=H1C1+H1HH;
```

```
        H1avail=H1+P1H1+P2H1-H1RP;
        H1C1=H1avail*H1C1/totH1demand;
        H1HH=H1avail-H1C1;
    end
else
    if H1massdeficit<0
        H1HH=H1HH+min(H1+P1H1+P2H1-H1RP-H1C1-H1HH,-H1massdeficit);
%only what you need to make up deficit
    end
end

%
% H2
%
if H2+P1H2+P2H2-H2RP-H2C1-H2C2<belownoreproduction
    if H2+P1H2+P2H2-H2RP<belownoreproduction
        H2RP=H2+P1H2+P2H2;
        H2C1=0;H2C2=0;
    else
        totH2demand=H2C1+H2C2;
        H2avail=H2+P1H2+P2H2-H2RP;
        H2C1=H2C1*H2avail/totH2demand;
        H2C2=H2avail-H2C1;
    end
end

%
% H3
%
if H3+P2H3+P3H3-H3RP-H3C2<belownoreproduction
    if H3+P2H3+P3H3-H3RP<belownoreproduction
        H3RP=H3+P2H3+P3H3;
        H3C2=0;
    else
        totH3demand=H3C2;
        H3avail=H3+P2H3+P3H3-H3RP;
        H3C2=H3C2*H3avail/totH3demand;
    end
end

%
% C1
%
if C1+H1C1+H2C1-C1RP<belownoreproduction
    C1RP=C1+H1C1+H2C1;
end

%
% C2
%
if C2+H2C2+H3C2-C2RP<belownoreproduction
    C2RP=C2+H2C2+H3C2;
end
```

```
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

ISHHflow=max(0,(theta+lambda)*ISHHdemand*numHH);
ISIRP=ISHHflow;
if ISmass+P1IS+RPIS-ISIRP<=0
    ISIRP=ISmass+P1IS+RPIS;
else
    if ISmassdeficit<0&numHH>=2 %if there is an accumulated deficit
% between demand for IS by the HH and IS
        %supplied, try to make this up if there is extra stock
        ISIRP=ISIRP+min(ISmass+P1IS+RPIS-ISIRP,-ISmassdeficit);%only
% what you need to make up deficit
    end
end

if (P1HH+H1HH+ISIRP)==0
    weightedprice=0;
    percapbirths=0;
elseif (pP1*P1HH+pH1*H1HH+pIS*ISIRP)==0
    weightedprice=0;
    percapbirths=0;
else

%weightedprice=(pP1*P1HHdemand+pH1*H1HHdemand+pIS*ISHHflow)/(P1HHdemand
% +H1HHdemand+ISHHflow);
    weightedprice=(pP1*P1HH+pH1*H1HH+pIS*ISIRP)/(P1HH+H1HH+ISIRP);
    %percapbirths=max(etaa-
% etab*W/weightedprice+etac*(W/weightedprice)^2,0);
    percapbirths=max(etaa-etab*sqrt(W/weightedprice),0);
%     if etab<=2*etac*(W/weightedprice)|etaa<=etac*(W/weightedprice)^2
%         error('pricesettingmodel:growtherror','Percapbirths function
% parameters give bad function form')
%     end
end




%
%%%%% changed 26 AUgust 2004
%
nextP1 = P1+RPP1-P1RP-P1H2-P1H1-P1HH-P1IS;

if P1==0
    P1H1demand=0;P1ISdemand=0;P1HHdemand=0;
end
nextP1H1massdeficit=P1H1massdeficit+P1H1-P1H1demand;

nextP1ISmassdeficit=P1ISmassdeficit+P1IS-P1ISdemand;

nextP1HHmassdeficit=P1HHmassdeficit+P1HH-P1HHdemand*numHH;
```

```
nextP2 = P2+IRPP2+RPP2-P2RP-P2H2-P2H3-P2H1;

nextP3 = P3+IRPP3+RPP3-P3RP-P3H3;

nextH1 = H1+P1H1+P2H1-H1RP-H1C1-H1HH;

if H1==0
    H1HHdemand=0;
end
nextH1massdeficit=H1massdeficit+H1HH-H1HHdemand*numHH;

nextH2 = H2+P1H2+P2H2-H2RP-H2C1-H2C2;

nextH3 = H3+P2H3+P3H3-H3RP-H3C2;

nextC1=C1+H1C1+H2C1-C1RP;

nextC2=C2+H2C2+H3C2-C2RP;

nextHH = HH+P1HH+H1HH-HHRP;

nextISmass=ISmass+P1IS+RPIS-ISIRP; %keep track of actual mass in IS

nextISmassdeficit = ISmassdeficit+ISIRP-ISHHflow;
%keep track of deficit in IS, what is supplied minus the demand

nextIRP = IRP-IRPP2-IRPP3+RPIRP+ISIRP-IRPRP; %this line changed 2 Aug
% 2004

nextRP = stockRP-(RPP1+RPP2+RPP3)-RPIRP-RPIS;

nextnumHH=max(numHH+ceil(percapbirths*numHH)-ceil(mHH*numHH)-
ceil(numHH*phi*(percapmass-idealpercapmass)^2),1);

nextpercapmass=nextHH/nextnumHH;

sys =
[nextP1;nextP2;nextP3;nextH1;nextH2;nextH3;nextC1;nextC2;nextHH;nextISm
ass;nextRP;nextIRP;...

nextP1H1massdeficit;nextP1ISmassdeficit;nextP1HHmassdeficit;nextH1massd
eficit;nextISmassdeficit;...
    nextnumHH;nextpercapmass];


% end mdlUpdate

%
%=============================================================================
% mdlOutputs
% Return the block outputs.
%=============================================================================
%
```

```
function sys=mdlOutputs(t,y,u)

global RPP1 P1H2 P1IS P1H1 P1HH RPIS ISIRP P2H1 H1C1 H1HH pP1 pH1 pIS W
ISHHflow

sys =
[y;RPP1;P1H2;P1IS;P1H1;P1HH;RPIS;ISIRP;P2H1;H1C1;H1HH;pP1;pH1;pIS;W];


% end mdlOutputs

%
%=============================================================================
% mdlGetTimeOfNextVarHit
% Return the time of the next hit for this block. Note that the result
% is absolute time. Note that this function is only used when you
% specify a variable discrete-time sample time [-2 0] in the sample
% time array in
% mdlInitializeSizes.
%=============================================================================
%
function sys=mdlGetTimeOfNextVarHit(t,y,u)

sampleTime = 1;    %  Example, set the next hit to be one second later.
sys = t + sampleTime;

% end mdlGetTimeOfNextVarHit

%
%=============================================================================
% mdlTerminate
% Perform any end of simulation tasks.
%=============================================================================
%
function sys=mdlTerminate(t,y,u)

sys = [];

% end mdlTerminate
```

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