

Upscaling Schemes and Relationships for the Gardner and van Genuchten Hydraulic Functions for Heterogeneous Soils

Jianting Zhu,* Michael H. Young, and Martinus Th. van Genuchten

ABSTRACT

Upscaled soil hydraulic properties are needed for many large-scale hydrologic applications such as regional and global climate studies and investigations of land–atmosphere interactions. Many larger scale subsurface flow and contaminant transport studies also require upscaled hydraulic property estimates. The objectives of this study were to develop a methodology for upscaling hydraulic property functions using a p -norm approach, to examine how p -norm values differ for two commonly used soil hydraulic property models (the Gardner and van Genuchten functions), and to investigate the relative sensitivities of p -norms and the effective hydraulic parameters to the degree of soil heterogeneity (expressed in terms of variances and auto-correlation lengths of the hydraulic parameters) and other environmental conditions. The p -norm approach expresses upscaling schemes such that it reduces their sensitivity to uncertainties (heterogeneities) in site conditions. The upscaling schemes are obtained as the result of two new criteria proposed to upscale soil hydraulic properties in this study—one preserving the ensemble vertical moisture flux across the land–atmosphere boundary, and a second preserving the ensemble soil surface moisture content. The effective soil hydraulic parameters of a heterogeneous soil formation are then derived by conceptualizing the formation as an equivalent homogeneous medium that satisfies the upscaling criteria. Upscaling relationships between the Gardner and van Genuchten models can then also be established for steady-state vertical flow using the statistical structures of the hydraulic parameters of these two models as estimated from field measurements. The upscaling scheme is demonstrated using hydraulic property data collected at 84 locations across a site in the Mojave Desert. Our results show that the p -norms generally vary less in magnitude than the effective parameters when the variances of the hydraulic parameters increase. We also show that, in general, p -norm values are better defined for the van Genuchten model than the Gardner model. Hydraulic parameter auto correlations, as defined by correlation lengths, were found to have little impact in relating the upscaling schemes (p -norm values) for the two hydraulic property models, but correlation between the hydraulic parameters within the hydraulic property models can significantly affect p -norm relationships.

UNSATURATED flow in the vadose zone is often simulated using closed-form expressions for the soil hydraulic properties involving the soil water retention and unsaturated hydraulic conductivity functions. Some of the more commonly used hydraulic functions include the Gardner–Russo (Gardner, 1958; Russo, 1988), Brooks–Corey (Brooks and Corey, 1964), and van Genuchten (van Genuchten, 1980) models. Soil hydraulic property

functions are generally measured at relatively small (local) scales. Major questions remain about how to best aggregate the spatially variable hydraulic properties within a heterogeneous soil volume to obtain effective hydraulic properties (or hydraulic parameters) at the large (field or watershed) scale. Effective parameters are essential for upscaling point-level measurements and processes to larger scales, and for reducing the complexity of hydrologic systems.

During the last several decades, much work has been done to develop and examine different averaging (and hence upscaling) schemes. Sharma and Luxmoore (1979) pointed out that the soil–plant–atmospheric interactions are very complex in terms of evaluating the influence of soil heterogeneity on water budget. They found that results very much depended on the coefficient of variation and the frequency distribution function of the Miller–Miller scaling factor, as well as on the specific soil–plant–atmospheric conditions involved. Kim and Stricker (1996) used Monte Carlo simulations to investigate both the separate and simultaneous effects of horizontal heterogeneity in the soil hydraulic properties and rainfall intensity on the statistical properties for various components of the one-dimensional water balance. Their results showed that heterogeneity has a stronger effect on the annual water balance for loam than for sand. Kim et al. (1997) further investigated the impact of heterogeneity on the spatially averaged water budget of the unsaturated zone using an analytical framework (Kim et al., 1996). Zhu et al. (2004) established optimal averaging schemes and probability distribution functions for parameters of several hydraulic functions, using correspondence among those hydraulic models by preserving the macroscopic capillary length and predicting the same vertical moisture flux. In more recent studies, Zhu and Mohanty (2006) investigated the effective hydraulic parameters for transient infiltration in terms of the optimal averaging schemes for the input hydraulic and environmental parameter fields. Specifically, they explored the effects of microtopography (as reflected in surface ponding depth) and hydraulic parameter correlation on the ensemble-mean behavior, as well as on the optimal effective schemes. Zhu et al. (2006) additionally examined the impact of the skewness (third-order moment) of hydraulic parameter distributions on “effective” soil hydraulic parameter averaging schemes for heterogeneous soils in a flat landscape. Numerical or field experimental results showed that the distribution skewness is also important for determining the upscaled effective parameters, in addition to the mean and variance.

Several studies have investigated conditions for which different hydraulic property functions produce the same or similar hydrologic responses. For example, Warrick (1995) discussed some of the features that need to be

J. Zhu and M.H. Young, Desert Research Institute, Division of Hydrologic Sciences, 755 East Flamingo Rd., Las Vegas, NV 89119; M.Th. van Genuchten, U.S. Salinity Lab., 450 West Big Springs Rd., Riverside, CA 92507-4617. Received 14 Mar. 2006. *Corresponding author (Jianting.Zhu@dri.edu).

Published in Vadose Zone Journal 6:186–195 (2007).

Original Research

doi:10.2136/vzj2006.0041

© Soil Science Society of America

677 S. Segoe Rd., Madison, WI 53711 USA

preserved in order for different hydraulic property functions to give similar or identical results for a given flow scenario. Lenhard et al. (1989) related the van Genuchten (1980) and Brooks and Corey (1964) parameters by assuming similar shapes of the soil water retention curves. Lenhard et al. (1989) obtained equivalence by equating the specific moisture capacity halfway between the saturated (θ_s) and residual (θ_r) water contents, and minimizing the difference between the water retention curves. Morel-Seytoux et al. (1996) converted Brooks–Corey parameters to van Genuchten parameters by preserving the macroscopic capillary length and keeping the same asymptotic behavior of the soil water retention curve at low water contents. Zhu et al. (2004) similarly established parameter equivalence among the hydraulic conductivity functions by preserving the macroscopic capillary length and predicting the same vertical moisture flux across the soil surface. Their results showed that the hydraulic parameters correspond very well when predicting evaporation rates from heterogeneous soils having a relatively large soil water pressure head at the soil surface (e.g., a dry surface or a shallow groundwater table).

A main objective of this study was to express effective hydraulic parameters in terms of certain types of averages (using the averaging or upscaling schemes) of individual parameter values obtained for a heterogeneous soil formation. We examined, for this purpose, the upscaling schemes for both the Gardner and van Genuchten functions using a data set of field-measured hydraulic properties for steady-state vertically dominated flow. We propose two new criteria to upscale the Gardner and van Genuchten models, and also establish the upscaling relationships between the two models. Our criteria are based on two important hydrologic processes: the moisture flux across the land–atmosphere boundary and the soil surface moisture content. The criteria require that the upscaled hydraulic properties for both models predict the ensemble vertical water flux across the land–atmosphere boundary, and produce the ensemble surface reduced soil moisture content. These two hydrologic quantities are important in that they are key inputs for land–atmosphere feedback schemes in soil–vegetation–atmosphere transfer models in terms of partitioning upward and downward water fluxes at the land surface. Water table depth (finite) and soil water pressure head at the soil surface are assumed homogenous for typical remote-sensing pixel or large-scale hydro-climate model grid scales as considered in this study, since the scale of variations for these two quantities are usually larger than that of hydraulic properties.

In this study, we interpret hydraulic property upscaling as deriving effective homogeneous soil hydraulic parameters to account for uncertainties in the spatially variable hydraulic parameters (e.g., Zhu and Mohanty, 2002, 2003). Effective or average hydraulic properties are those that can be used for an entire field (e.g., typical remote-sensing pixel or a larger scale model grid) based on measurements at point scales, or using estimates obtained in some other way such as from soil texture. Effective soil hydraulic parameters of a heterogeneous soil formation are derived by conceptualizing the het-

erogeneous soil formation as an equivalent homogeneous medium such that the equivalent homogeneous soil will produce the same ensemble-mean flux across the land–atmosphere boundary and the same ensemble-mean surface moisture content as the heterogeneous medium. Mathematically, we express the effective parameters using the p -norm, or the p -order power average of the local-scale hydraulic parameter sets. The p -norm is a generalization of commonly used averaging schemes, which includes the arithmetic, geometric, and harmonic averages as particular cases. We hypothesized that the p -norm should be less sensitive to the variation in random hydraulic parameter fields than the effective parameters since the larger random parameter variance will probably offset the variability of the p -norm. Therefore, the p -norm should be better defined than the effective parameter values when describing hydraulic parameter averaging schemes. Quantitatively, the upscaling schemes are considered “better defined” if they (expressed in terms of the p -norm values) are less sensitive to the variability of the hydraulic parameters and other related soil-water conditions such as water table depth and the pressure head at the soil surface. In other words, if the upscaling schemes of the hydraulic parameters for large-scale applications are less sensitive to the soil heterogeneity and other conditions, conceptualization of the heterogeneous medium as an equivalent homogeneous medium is more feasible and the upscaling schemes are easier to implement.

One complication with upscaling schemes for larger scale hydrologic applications is that they differ when different soil hydraulic property functions are used. While significant efforts have been made to establish the equivalence among many hydraulic property models at the local scale (e.g., Warrick, 1995; Lenhard et al., 1989; Morel-Seytoux et al., 1996), this study deals with equivalence in an upscaling context, which is a significant and still largely unresolved issue. Therefore, our first specific objective was to develop a methodology for calculating the p -norm values (and therefore the upscaling schemes) based on the two new upscaling criteria. A second objective was to determine whether the optimal averaging schemes differ for different forms of the hydraulic properties, and the relationships of the averaging schemes if they indeed differ. Finally, a third objective was to examine whether the p -norms are less sensitive to variations in random hydraulic parameter fields compared with the effective parameters.

MATERIALS AND METHODS

Soil Hydraulic Property Models and Steady-State Local-Scale Processes

The soil hydraulic functions consist of the soil water retention function, which defines the water content as a function of the soil water pressure head, and the hydraulic conductivity function, which relates the hydraulic conductivity with the water content or soil water pressure head. A brief discussion of the soil hydraulic models used in this study is presented below. We refer to Leij et al. (1997) and Warrick (2003) for a more comprehensive overview of alternative functional expres-

sions for the soil hydraulic properties, including those used in this study.

Gardner Model

The hydraulic functions used by Gardner (1958) and Russo (1988) are given by

$$S_e = [\exp(-0.5\alpha_G h)(1 + 0.5\alpha_G h)]^{0.8} \quad [1]$$

$$K = K_{sG} \exp(-\alpha_G h) \quad [2]$$

Van Genuchten Model

Van Genuchten (1980) combined his S-shaped soil water retention function with the statistical pore-size distribution model of Mualem (1976) to obtain the following functions:

$$S_e = \frac{1}{[1 + (\alpha_{vG} h)^n]^m} \quad [3]$$

$$K = \frac{K_{svG} \{1 - (\alpha_{vG} h)^{mm} [1 + (\alpha_{vG} h)^n]^{-m}\}^2}{[1 + (\alpha_{vG} h)^n]^{0.5m}} \quad [4]$$

In Eq. [1] to [4], $S_e = (\theta - \theta_r)/(\theta_s - \theta_r)$ is the effective degree of saturation, θ is the volumetric water content, θ_r is the residual volumetric water content, θ_s is the saturated volumetric water content, h is the soil water pressure head (in this study assumed to be positive for unsaturated conditions), K is the hydraulic conductivity, K_s is the saturated hydraulic conductivity; α , m , and n are empirical hydraulic shape parameters, and $m = 1 - 1/n$, while the subscripts G and vG refer to Gardner and van Genuchten model parameters, respectively. Being the hydraulically active portion of the soil moisture regime, S_e is an important parameter in many water balance studies and land-atmosphere parameterizations. For this reason, we use S_e here to more effectively express soil moisture conditions in this study.

When the soil water pressure head at the land surface, h_0 , is given, application of Darcy's law leads to (e.g., Zaslavsky, 1964; Warrick and Yeh, 1990; Zhu and Mohanty, 2002)

$$z_0 = \frac{1}{\alpha} F(q/K_s) \quad [5]$$

where z_0 is the distance from the soil surface to the water table, q is the steady-state flux rate for either evaporation (positive value) or infiltration (negative value), and the function F is defined as

$$F(q/K_s) = \int_0^{\alpha h_0} \frac{K_r(s) ds}{K_r(s) + q/K_s} \quad [6]$$

in which s is a dummy integration variable denoting the dimensionless quantity αh in the hydraulic conductivity functions. For the Gardner model, $K_r(s) = \exp(-s)$, in which case Eq. [6] can be integrated analytically to give

$$F_G(q/K_s) = \ln \left[\frac{1 + q/K_s}{\exp(-\alpha h_0) + q/K_s} \right] \quad [7]$$

which, with Eq. [5], leads to

$$q = K_s \frac{1 - \exp[\alpha(z_0 - h_0)]}{\exp(\alpha z_0) - 1} \quad [8]$$

For the van Genuchten model, $F(q/K_s)$ in Eq. [6] requires numerical integration, which is achieved in this study using Romberg integration (e.g., Scheid, 1968; Stoer and Bulirsch,

1980). The steady-state vertical flux for either evaporation or infiltration for the van Genuchten function is calculated iteratively using Eq. [5] and [6]. The dimensionless flux q/K_s for a given soil water pressure head, h_0 , and water table depth, z_0 , is obtained by assuming some initial value for the flux and then iteratively adjusting its value until Eq. [5] is satisfied within a prescribed tolerance.

Field Measurements

In this study, we used measured hydraulic properties from the Corn Creek Fan Complex located in the Desert National Wildlife Refuge, approximately 50 km north of Las Vegas, NV (see Fig. 1). The soil hydraulic properties of the surface soil at each field site were determined using tension infiltrometry (Ankeny et al., 1988; Reynolds and Elrick, 1991). Data were collected using differential pressure transducers (Casey and Derby, 2002) at 15-s intervals. Manual readings were taken periodically to verify operation and to better identify when the intake rate was at or near steady state. At that time, the tension infiltrometer was reset to a lower tension level. Four to five tension steps were used for each test, typically at 12, 9, 6, 3, and ~0 (saturation) cm.

Two methods of analysis were used to obtain the soil hydraulic properties from the tension infiltrometer data. This included the semiempirical approach of Wooding (1968) and numerical inversion of the data using parameter estimation. Assuming applicability of Gardner's exponential $K(h)$ function (Eq. [2]), three-dimensional, steady-state infiltration from a circular source is given by (Wooding, 1968)

$$i(h) = K_{sG} \exp(-\alpha_G h) \left(1 + \frac{4}{\pi r \alpha_G} \right) \quad [9]$$

where $i(h)$ is the steady-state infiltration rate at supply tension h , and r is the radius of the tension infiltrometer. The first term of Eq. [9] within the second set of parentheses accounts for vertical gravitational flow, and the second term for lateral flow due to capillarity. Equation [9] contains two unknowns: K_{sG} and α_G . Using the infiltrometer for a range of tensions, paired values of the flux $[i(h)]$ and the tension step (h) can be obtained. These paired values are subsequently used in an iterative nonlinear least-squares regression approach to estimate K_{sG} and α_G in Eq. [9] (e.g., Logsdon and Jaynes, 1993).

In addition to Wooding's solution to obtain K_{sG} and α_G , a complete set of van Genuchten soil hydraulic parameters (i.e., K_{svG} , α_{vG} , n , and θ_s) were also identified through numerical inversion of the cumulative infiltration data, given known changes in the boundary conditions (Simunek et al., 1998; Young et al., 2004). We used, for this purpose, the axisymmetric option of the HYDRUS-2D software package of Simunek et al. (1999). Additional details of the field test methodology and procedures are given by Young et al. (2005).

Results of the data analysis produced a series of fitting parameters and soil hydraulic properties that were used for developing the upscaling rules and relationships. A total of 84 hydraulic parameter sets for both the Gardner and van Genuchten models were derived from the field measurements. Basic statistical data of the 84 field-measured hydraulic parameters are listed in Table 1, including mean values, variances, standard deviations, and coefficients of variation. We additionally calculated coefficients of correlation (CC) between the saturated hydraulic conductivity, K_s , and the α parameter for both models, obtaining $CC(K_{sG}, \alpha_G) = 0.24$, and $CC(K_{svG}, \alpha_{vG}) = 0.78$. The parameter sets in Table 1 involve the Gardner and van Genuchten saturated hydraulic conductivities (K_{sG} and K_{svG}), the α parameter for the two models (α_G and α_{vG}), and

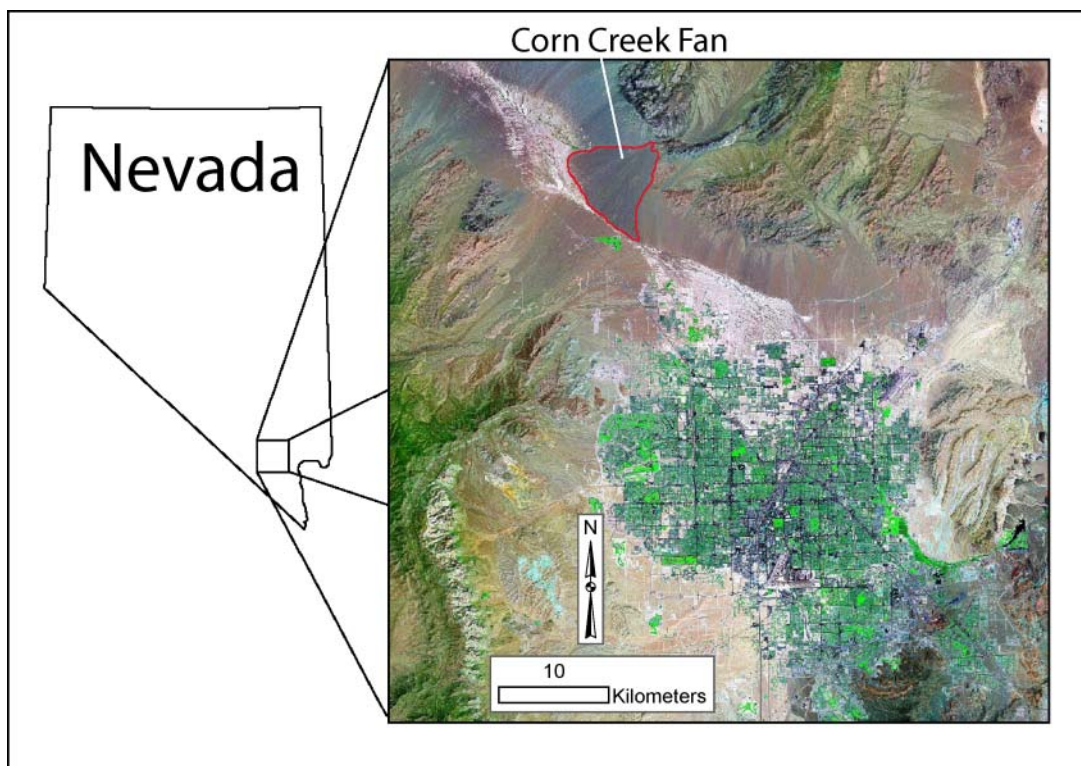


Fig. 1. Location map of the Corn Creek Fan where the field hydraulic properties were measured.

the van Genuchten *n* parameter. Since we were interested in the effective degree of saturation at the soil surface, the θ_s and θ_r parameters of the hydraulic property function are not relevant in this study.

Effective Hydraulic Parameters

Because the van Genuchten model contains one more parameter than the Gardner model and our proposed new criteria will invoke only two equations, we assumed that the van Genuchten *n* parameter is a deterministic parameter for which we used the average value from our field-measured data. In practice, *n* can be determined with greater certainty than the other van Genuchten parameters (e.g., Schaap and Leij, 1998). In a study of van Genuchten’s hydraulic model, Hills et al. (1992) also demonstrated that random variability in the water retention characteristics could be adequately modeled using either a variable parameter α_{vG} together with a constant parameter *n*, or a variable *n* with a constant α_{vG} , but achieved better results using the first option. Following these findings, we treated *n* as a deterministic parameter in this study.

To examine the appropriateness of using only the field-measured hydraulic parameters for establishing the upscaling relationships, we also used regenerated fields of 10 000 ran-

dom values for the hydraulic parameters that may be cross-correlated. The cross-correlated random fields of K_s and α were generated using the spectral method proposed by Robin et al. (1993). Random fields were obtained with the power spectral density function using exponentially decaying covariance functions. As an indicator of correlation between the two random parameter fields, we used the coherency spectrum given by

$$R(\mathbf{f}) = \frac{\phi_{12}(\mathbf{f})}{[\phi_{11}(\mathbf{f})\phi_{22}(\mathbf{f})]^{1/2}} \quad [10]$$

where $\phi_{11}(\mathbf{f})$ and $\phi_{22}(\mathbf{f})$ are the power spectra of random Fields 1 and 2, respectively, and $\phi_{12}(\mathbf{f})$ is the cross spectrum between Fields 1 and 2. Having $|R|^2 = 1$ indicates perfect linear correlation between the random fields. In this study, 10 000 realizations of hydraulic parameter values were regenerated to match the field-measured parameter target statistics to various degrees. The parameters K_s and α can be satisfactorily fitted using lognormal distributions (Smith and Diekkruger, 1996; Nielsen et al., 1973). In this study, both K_s and α were assumed to obey lognormal distributions.

A total of six hydraulic parameter sets of 10 000 data points was generated for both the Gardner and van Genuchten models. The first four sets all assumed nearly full correlation between K_s and α , with correlation coefficients being in the range between 0.95 and 0.99. The differences among the four parameter sets are the normalized autocorrelation lengths, CL (i.e., the autocorrelation length divided by the grid length), which were set at 2, 5, 10, and 50 for the four sets. The fifth set had a CL of 5 and the same values of correlation coefficients between K_s and α as the field-measured data (about 0.24 and 0.74 for the Gardner and van Genuchten models, respectively). The sixth set had the same correlation characteristics as the first four data sets, but variances that were four times the actual field-measured variance values.

Table 1. Basic statistics of 84 field-measured soil hydraulic parameter† values.

	K_{sG}	α_G	K_{svG}	α_{vG}	<i>n</i>	θ_s
	cm d ⁻¹	cm ⁻¹	cm d ⁻¹	cm ⁻¹		cm ³ cm ⁻³
Mean	212.433	0.227	176.731	0.092	2.565	0.212
Variance	25579.630	0.010	21883.190	6.96E-4	0.387	2.33E-3
SD	159.936	0.098	147.930	0.026	0.622	0.048
CV	0.753	0.433	.837	0.287	0.242	0.228

† G, Gardner model; vG, van Genuchten model; K_s , saturated hydraulic conductivity; α and *n*, shape parameters; θ_s , saturated volumetric water content.

where β denotes any quantity of interest (i.e., any of $p_{K_s G}$, $p_{\alpha G}$, $p_{K_s vG}$, $p_{\alpha vG}$, $K_{s \text{Geff}}$, α_{Geff} , $K_{s \text{vGeff}}$, and α_{vGeff}). The subscript X in Eq. [20] indicates that the quantity was obtained using variances of the field data, while the subscript 4X indicates input parameter variances that were four times those of the field measurements (i.e., the coefficient of variation was two times larger). A fourfold difference in the variances is relatively large and hence should provide a good indication of whether p -norms are indeed less sensitive to parameter variances.

RESULTS AND DISCUSSION

Figure 2 shows a scatter plot of relative differences in effective parameter values vs. corresponding p -norms for the two assumed variance levels (i.e., one and four times those of the field data). Results are plotted for the Gardner and van Genuchten models assuming different dimensionless water table depths, z_0^* (10, 5, and 2.5). We did run simulations for a range of h_0^* values (i.e., pressure heads at the land surface) for each combination of model (Gardner or van Genuchten) and parameter type (α or K_s). Depending on the value of h_0^* , flow can either represent evaporation (when $h_0^* > z_0^*$) or infiltration (when $h_0^* < z_0^*$). In general, relative differences in the p -norms between the two variance levels are smaller than relative differences in the effective hydraulic parameter values, especially for the saturated hydraulic conductivity. The figure shows that p -norms vary far less than the effective parameter values.

Figure 2 indicates that results are scattered mostly below the 1:1 line, which means that p -norm relative differences are generally smaller than those for the effective parameters. In view of the large difference in the input parameter variances (a fourfold difference), a much smaller variation in the p -norms than in the effective parameters is encouraging when trying to estimate the effective hydraulic parameters of relatively large land areas. The results imply that upscaling schemes in terms of p -norms will absorb some hydraulic parameter vari-

ability, which should make them less sensitive to the parameter variances, and hence to soil heterogeneity.

The lesser variability of p -norm values relative to the variabilities (heterogeneities) of the input hydraulic parameters is important when trying to define effective media. A motivation for using effective parameters is that the larger scale process of interest in heterogeneous soils can be captured by a process that assumes only one set of soil parameters. Thus, the heterogeneous system is replaced by an equivalent homogeneous system with one constant set of hydraulic parameters. The largest relative difference in effective parameters for the two scenarios was about 0.5 (for the Gardner K_s when $z_0^* = 10.0$), while the maximum relative difference for the p -norm values was about 0.25 (for the Gardner α when $z_0^* = 10$). Figure 2 shows that the p -norms for the α parameter are generally more variable than the effective parameters. In fact, the p -norm variability in α is larger than the effective parameter variability mainly only for relatively high pressure heads at the soil surface (i.e., when the surface soil is relatively dry). Figure 2 also shows that the p -norm of the Gardner α has more variability relative to the van Genuchten α . We believe that this is related to the rapid decrease in the Gardner water retention curve when soils are relatively dry (i.e., at large pressure heads).

Two additional points from Fig. 2 are worth mentioning. First, p -norms (or the optimal upscaling schemes) are less well defined for the Gardner model than for the van Genuchten model, and may in fact be more difficult to use than the van Genuchten model in the upscaling context, especially for dry scenarios such as for many arid and semiarid conditions. Also, dry conditions are generally more difficult to deal with both in the upscaling process and when trying to establish upscaling relationship between the two soil hydraulic models.

Figure 3 shows p -norm values of K_s and α for both the Gardner and van Genuchten models as functions of the

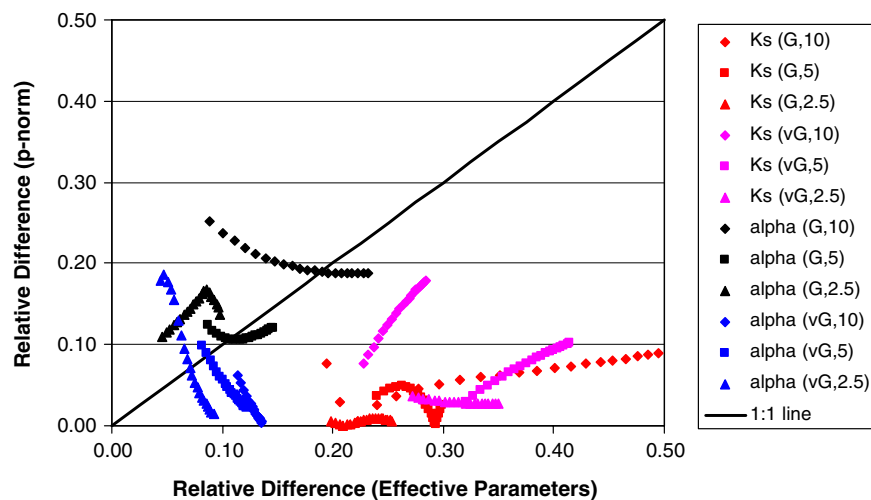


Fig. 2. Scatter plot of relative differences in effective values of the saturated hydraulic conductivity, K_s , and shape parameter α vs. corresponding p -norms for two variance levels (one and four times those observed for the field data). G: Gardner model, vG: van Genuchten model. The numbers in parentheses indicate the dimensionless water table depth, $z_0^* = \bar{\alpha}_{vG} z_0$, where $\bar{\alpha}_{vG}$ is the mean of the field-measured van Genuchten α_{vG} values and z_0 is the water table depth.

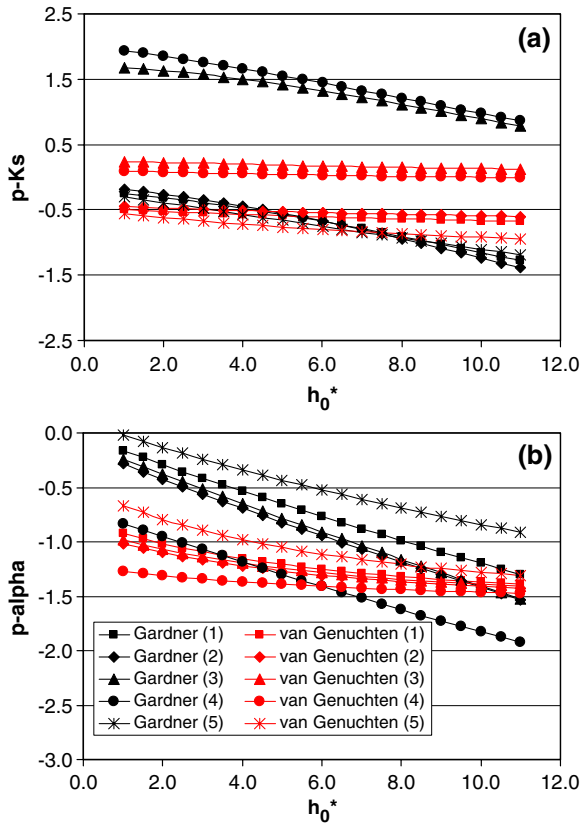


Fig. 3. Plots showing p -norms of (a) the saturated hydraulic conductivity, K_s , and (b) the α shape parameter for the Gardner (G) and van Genuchten (vG) models when the dimensionless water table depth $z_0^* = 5$: (1) 10 000 regenerated points, correlation length divided by grid length (CL) = 5, correlation coefficient between α and K_s (CC)(G) > 0.9, CC(vG) > 0.9; (2) 10 000 regenerated points, CL = 50, CC(G) > 0.9, CC(vG) > 0.9; (3) 84 measured points, CC(G) = 0.24, CC(vG) = 0.74; (4) 10 000 regenerated points, CL = 5, CC(G) = 0.24, CC(vG) = 0.75; (5) 10 000 regenerated points, CL = 5, CC(G) > 0.9, CC(vG) > 0.9, variance four times measured value.

dimensionless pressure head at the soil surface h_0^* , for conditions when the dimensionless water table depth $z_0^* = 5$. The figure legends (1) through (5) indicate various statistical structures of the heterogeneous local-scale hydraulic parameters. Since $z_0^* = 5$, the left half of the figures holds mostly for infiltration ($h_0^* < z_0^*$) and the right half for evaporation ($h_0^* > z_0^*$). As seen by the lower slopes of the red (van Genuchten) traces vs. the black (Gardner) traces, the results indicate that upscaling schemes for the van Genuchten model are less affected by pressure head conditions at the soil surface than the Gardner model. Considering different statistical structures of hydraulic parameters, the p -norms for the Gardner model are also more variable than those for the van Genuchten model, especially for the saturated hydraulic conductivity, K_s . In general, the Gardner model p -norms vary more than the van Genuchten model p -norms when the statistical features of the hydraulic parameters change (i.e., the black traces vary much more in both plots). These results suggest that the upscaling schemes for the van Genuchten model are better defined and less variable when both the environmental condi-

tions (e.g., the pressure head values at the soil surface) and the hydraulic parameter variabilities change.

Figures 4, 5, and 6 show p -norm relationships between the Gardner and van Genuchten parameters for three progressively smaller dimensionless depths of the water table (z_0^*): 10, 5, and 2.5, respectively. In each figure, we plotted a total of seven curves. Of these, six curves resulted from the 10 000 randomly regenerated values of the input hydraulic parameters, while the seventh curve shows results based on the measured field data. Note that all of the curves in Fig. 4 to 6 have a mean input parameter value equal to the field-measured data. Among the six curves calculated from the 10 000 regenerated values, those with solid symbols depict situations where α and K_s are correlated, with the coefficients (CC) in the range between 0.95 and 0.99.

Because the measured field data are spatially too sparse for a meaningful characterization of parameter spatial correlation structures, we used a wide range of parameter autocorrelation lengths to investigate the importance of autocorrelation length. For Fig. 4, 5, and 6, we used four parameter autocorrelation lengths with a relatively large range (from 2 to 50 times the grid length values)

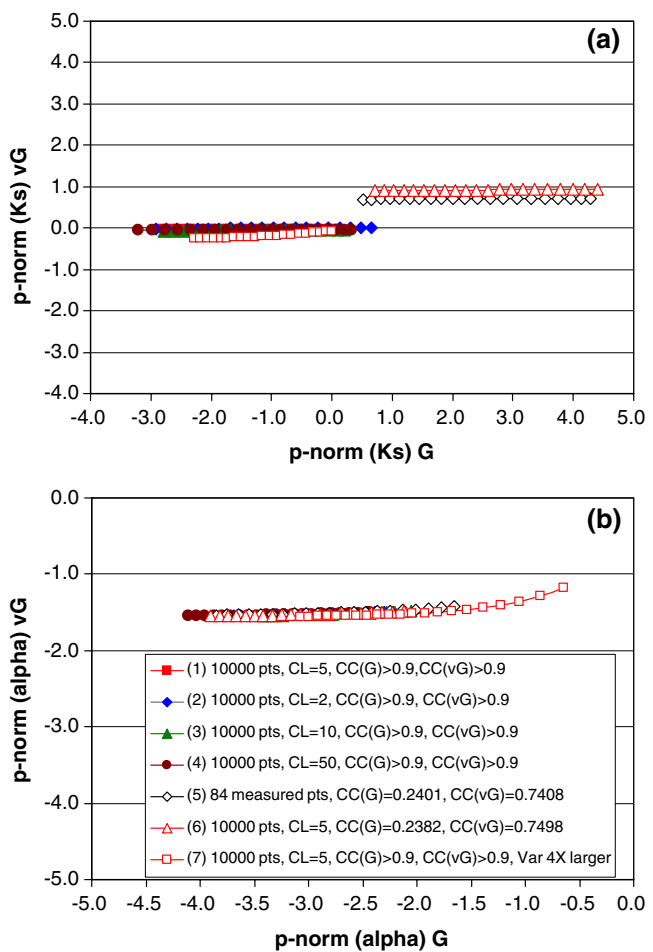


Fig. 4. The p -norm relationships between the Gardner (G) and van Genuchten (vG) models for (a) saturated hydraulic conductivity, K_s , and (b) the α shape parameter when the dimensionless water table depth $z_0^* = 10$. CL is correlation length divided by grid length and CC is the correlation coefficient between α and K_s .

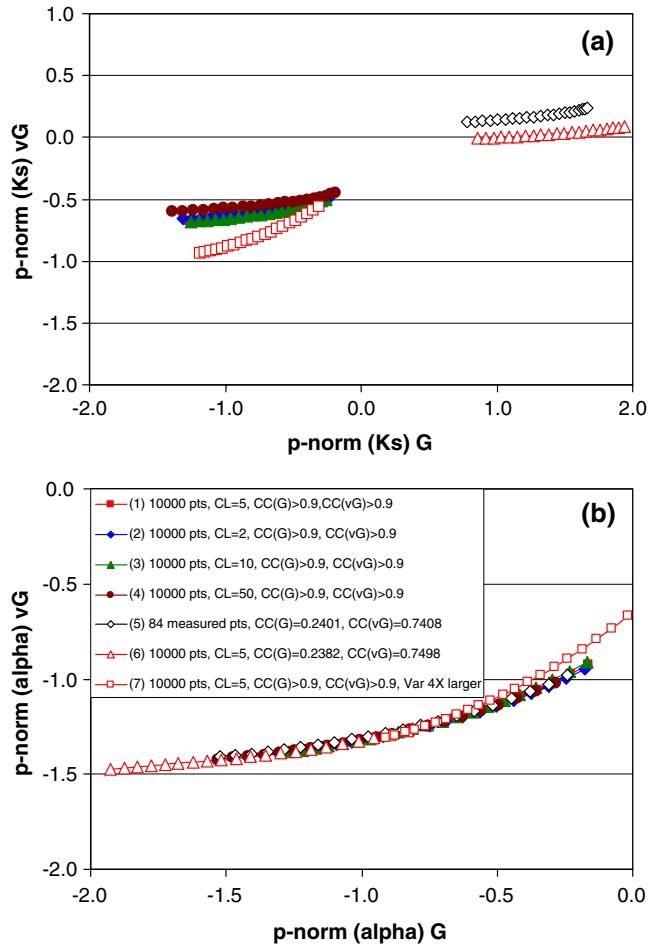


Fig. 5. The p -norm relationships between the Gardner (G) and van Genuchten (vG) models for (a) saturated hydraulic conductivity, K_s , and (b) the α shape parameter when the dimensionless water table depth $z_0^* = 5$. CL is correlation length divided by grid length and CC is the correlation coefficient between α and K_s .

so as to capture possible practical field correlation conditions. The results are shown as Curves (1) through (4). Curve (5) in all figures shows results based on only 84 measured parameter points. Curve (6) represents the situation where 10000 points were regenerated using a correlation coefficient that was the same as for the field-measured parameters. Curve (7) shows results when the variance was four times the field-measured variance.

Figure 4 shows p -norm relationships between the Gardner and van Genuchten models for K_s and α when $z_0^* = 10$ (note that $z_0^* = 10$ is approximately equivalent to z_0 [dimensional] = 10 m). Our calculations showed that the p -norm relationships are very similar in cases of deeper water tables (i.e., $z_0^* > 10$, results not shown) compared with cases for which $z_0^* = 10$ (shown in Fig. 4). However, the p -norms span a wider range when $z_0^* > 10$ than when $z_0^* = 10$, especially for the Gardner model. When the water table was relatively deep ($z_0^* \geq 10$), p -norms for the van Genuchten model were well defined and relatively constant for a wide range of h_0^* values, covering both evaporation and infiltration conditions. In comparison, p -norms for the Gardner model were not well defined in that they tended to vary quite significantly

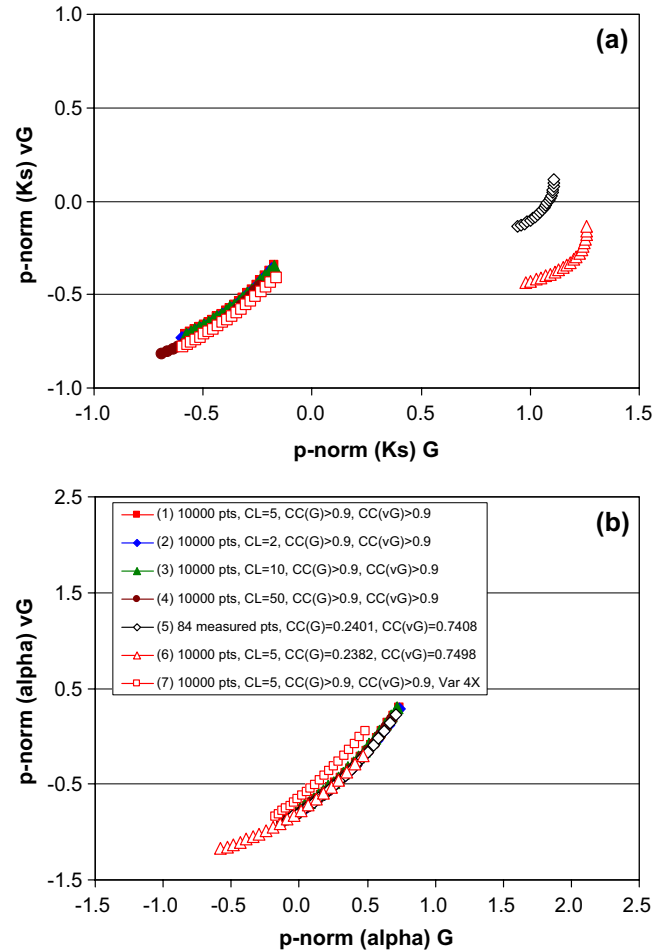


Fig. 6. The p -norm relationships between the Gardner (G) and van Genuchten (vG) models for (a) saturated hydraulic conductivity, K_s , and (b) the α shape parameter when the dimensionless water table depth $z_0^* = 2.5$. CL is correlation length divided by grid length and CC is the correlation coefficient between α and K_s .

as scenarios changed from evaporation to infiltration. These results suggest that the van Genuchten functions generally work better than the Gardner functions for the hydrologic processes considered in this study, especially for relatively high pressure heads at the soil surface (dry surfaces) such as those typical in arid and semiarid environments. Again, we believe that the steeper decline in the exponential hydraulic conductivity function in the dry range is responsible for the poorer performance of the Gardner relative to the van Genuchten function.

Figure 5 shows p -norm relationships between the Gardner and van Genuchten models for both α and K_s when $z_0^* = 5$ (i.e., a shallower water table in comparison with Fig. 4). A noticeable difference compared with Fig. 4 is that the p -norm for the van Genuchten parameters varied across a slightly wider range of values when the flow scenarios ranged from evaporation to infiltration, especially for situations with larger variances in the input hydraulic parameters. Conversely, p -norms for the Gardner model spanned a much smaller range of values than the larger z_0^* (i.e., deeper water table) case.

Figure 6 presents p -norm relationships between the Gardner and van Genuchten models for both α and K_s

when $z_0^* = 2.5$. As z_0^* becomes smaller, the p -norm patterns continue to shift to a narrower range for the Gardner p -norm and a broader range for the van Genuchten p -norm. Thus, for shallower water table conditions, and when the pressure head at the surface soil is lower, p -norms for either the Gardner or van Genuchten models would be comparable. We believe that the behavior of the Gardner model p -norm for large z_0^* can be also partly attributed to the fact that the Gardner α is typically about twice as large as the van Genuchten α . The Gardner α has first-order effects on the hydraulic conductivity functions, which makes the flux very sensitive to the value of the Gardner α . Therefore, direct use of α in the Gardner hydraulic conductivity model makes p -norm schemes very sensitive to changes in z_0^* .

The results from Fig. 4 to 6 show that the p -norms, and their relationships, for the two hydraulic property functions are similar when comparing Curves (5) and (6). Recall that those two curves have same the means, variances, and correlation coefficients (CC), the only difference being that Curve (5) is based on only 84 measured parameter values while Curve (6) is based on 10000 randomly regenerated parameter realizations. In other words, the two vastly different data sets (in terms of data quantity) produce quite comparable results when upscaling the flux across the land-atmospheric boundary and the surface effective degree of saturation. Another finding is that correlation between the hydraulic parameters within the same hydraulic property model significantly impacts the p -norm relationships as well as the p -norm values. For all scenarios considered in this study, smaller correlation levels that are strictly based on field-measured data cause the p -norm relationships to shift significantly compared with hydraulic parameters that have correlation coefficients between 0.95 and 0.99. Moreover, correlation between the hydraulic parameters within the same hydraulic property model has a more significant impact on the p -norm values (especially for the Gardner model) and p -norm relationships than do the variances of the hydraulic parameters.

From Fig. 4 through 6, hence, an important pattern emerges. Results indicate that even when variances in the input hydraulic parameter are four times the original measured data, the p -norm relationships are still clustered in the same neighborhood as long as the correlation coefficients remain unchanged. This suggests that hydraulic parameter variances have little impact on the p -norms, as well as on the corresponding p -norm relationships between the Gardner and van Genuchten models. On the other hand, when the parameter correlations between α and K_s change, the corresponding upscaling p -norm relationships shift significantly, thus illustrating the importance of parameter correlation in terms of dictating scaling relationships for different hydraulic property models. The results also show that correlation length (CL) of hydraulic parameters has a relatively minor impact in determining the p -norm relationships of the Gardner and van Genuchten functions. This fact can be seen from the four solid curves in Fig. 4, 5, and 6. When CL varies from 2 to 50 (a difference of 25 times!), all p -norm relationships studied

still cluster together and can be described by very similar relationships.

CONCLUSIONS

In this study we investigated hydraulic property upscaling schemes and possible relationships between different hydraulic functions, based on a data set of field-measured hydraulic properties, by applying two new upscaling criteria. While the established framework is quite generic, we used the field-measured data for two purposes: (i) to demonstrate how limited field data can be used in a relatively large-scale upscaling context, and (ii) to determine if limited field characterization is sufficient for upscaling considerations in a wide range of hydrologic processes ranging from infiltration to evaporation. Some of the main findings of this study are summarized as follows:

1. For the steady-state flow problem considered in this study, we showed that p -norms and their relationships were similar using 84 field-measured hydraulic parameter values and 10000 randomly regenerated hydraulic parameter realizations when upscaling the flux across the land-atmospheric boundary and the surface effective degree of saturation. For more heterogeneous sites, however, more field measurements may be required to characterize the hydraulic parameter fields in the upscaling context.
2. The upscaling schemes were in general better defined, and had less variability, in terms of p -norms than when effective parameter values were used (see Fig. 2).
3. For the α parameters, the p -norm relationships between the Gardner and van Genuchten models were typically similar for a broad range of scenarios considered in this study. The p -norms may diverge, however, for different levels of variability in the input hydraulic parameters and other hydrologic conditions.
4. In general, p -norms for the Gardner model were less well defined than for the van Genuchten model, and may in fact be more difficult to use than the van Genuchten model in the upscaling context when the water table is relatively deep (such as under many arid and semiarid conditions). The exponential decline of Gardner's hydraulic conductivity with the soil water pressure head is a property that could not be handled well in the upscaling schemes for deep water table conditions (such as often occurs in arid and semiarid regions).
5. For deep water tables (at least equivalent to 10 m), p -norms for van Genuchten parameters were relatively constant, while p -norms for Gardner parameters varied significantly, especially as flow scenarios shifted from evaporation to infiltration. As the water table became shallower, p -norms for the van Genuchten model became also less well defined and more sensitive to changes in the pressure head at the soil surface.

- Correlations between the hydraulic parameters within a given hydraulic property model was important for determining p -norm relationships between the Gardner and van Genuchten models.
- Hydraulic parameter correlation lengths appeared to have a relatively minor impact in determining the p -norm relationships between the Gardner and van Genuchten models.

ACKNOWLEDGMENTS

Funding support from the Water Resources Research Act, Section 104, research grant program of the U.S. Geological Survey, DRI's Applied Research Initiative, start-up funds to the first author, and funds from the Urban Flood Demonstration Project, U.S. Army Corp of Engineers, contract no. DACW 42-03-2-0003, are greatly appreciated. The support from NSF EPSCoR Grant no. EPS-0447416 is also acknowledged.

REFERENCES

- Ankeny, M.D., T.C. Kaspar, and R. Horton. 1988. Design for an automated tension infiltrometer. *Soil Sci. Soc. Am. J.* 52:893–896.
- Brooks, R.H., and A.T. Corey. 1964. Hydraulic properties of porous media. *Hydrol. Pap. 3*. Colorado State Univ., Fort Collins.
- Casey, F.X.M., and N.E. Derby. 2002. Improved design for automated tension infiltrometer. *Soil Sci. Soc. Am. J.* 66:64–67.
- Gardner, W.R. 1958. Some steady state solutions of unsaturated moisture flow equations with applications to evaporation from a water table. *Soil Sci.* 85:228–232.
- Green, T.R., J.E. Contantz, and D.L. Freyberg. 1996. Upscaled soil-water retention using van Genuchten's function. *J. Hydrol. Eng.* 1(3):123–130.
- Hills, R.G., D.B. Hudson, and P.J. Wierenga. 1992. Spatial variability at the Las Cruces trench site. p. 529–538. *In* M.Th. van Genuchten et al. (ed.) *Indirect methods for estimating the hydraulic properties of unsaturated soils*. Univ. of California, Riverside.
- Kim, C.P., and J.N.M. Stricker. 1996. Influence of spatially variable soil hydraulic properties and rainfall intensity on the water budget. *Water Resour. Res.* 32:1699–1712.
- Kim, C.P., J.N.M. Stricker, and R.A. Feddes. 1997. Impact of soil heterogeneity on the water budget of the unsaturated zone. *Water Resour. Res.* 33:991–999.
- Kim, C.P., J.N.M. Stricker, and P.J.J.F. Torfs. 1996. An analytical framework for the water budget of the unsaturated zone. *Water Resour. Res.* 32:3475–3484.
- Korvin, G. 1982. Axiomatic characterization of the general mixture rule. *Geoexploration* 19:267–276.
- Leij, F.J., W.B. Russell, and S.M. Lesch. 1997. Closed-form expressions for water retention and conductivity data. *Ground Water* 35: 848–858.
- Lenhard, R.J., J.C. Parker, and S. Mishra. 1989. On the correspondence between Brooks-Corey and van Genuchten models. *J. Irrig. Drain. Eng.* 115:744–751.
- Logsdon, S.D., and D.B. Jaynes. 1993. Methodology for determining hydraulic conductivity with tension infiltrometers. *Soil Sci. Soc. Am. J.* 57:1426–1431.
- Morel-Seytoux, H.J., P.D. Meyer, M. Nachabe, J. Touma, M.Th. van Genuchten, and R.J. Lenhard. 1996. Parameter equivalence for the Brooks-Corey and van Genuchten soil characteristics: Preserving the effective capillary drive. *Water Resour. Res.* 32:1251–1258.
- Mualem, Y. 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. *Water Resour. Res.* 12:513–522.
- Nielsen, D.R., J.W. Biggar, and K.T. Erh. 1973. Spatial variability of field-measured soil-water properties. *Hilgardia* 42(7):215–260.
- Reynolds, W.D., and D.E. Elrick. 1991. Determination of hydraulic conductivity using a tension infiltrometer. *Soil Sci. Soc. Am. J.* 55: 633–639.
- Robin, M.J.L., A.L. Gutjahr, E.A. Sudicky, and J.L. Wilson. 1993. Cross-correlated random field generation with the direct Fourier transform method. *Water Resour. Res.* 29:2385–2397.
- Russo, D. 1988. Determining soil hydraulic properties by parameter estimation: On the selection of a model for the hydraulic properties. *Water Resour. Res.* 24:453–459.
- Schaap, M.G., and F.J. Leij. 1998. Database-related accuracy and uncertainty of pedotransfer functions. *Soil Sci.* 163:765–779.
- Scheid, F. 1968. *Theory and problems of numerical analysis*. Schaum's Outline Ser. McGraw Hill Book Co., New York.
- Sharma, M.L., and R.J. Luxmoore. 1979. Soil spatial variability and its consequences on simulated water balance. *Water Resour. Res.* 15: 1567–1573.
- Simunek, J., K. Huang, M. Sejna, and M.Th. van Genuchten. 1999. The HYDRUS-2D software package for simulating two-dimensional movement of water, heat and multiple solutes in variably-saturated media. Version 2.0. IGWMC-TPS-53. Int. Ground Water Model. Ctr., Colo. School of Mines, Golden.
- Simunek, J., D. Wang, P.J. Shouse, and M.Th. van Genuchten. 1998. Analysis of field tension disc infiltrometer data by parameter estimation. *Int. Agrophys.* 12:167–180.
- Smith, R.E., and B. Diekkruger. 1996. Effective soil water characteristics and ensemble soil water profiles in heterogeneous soils. *Water Resour. Res.* 32:1993–2002.
- Stoer, J., and R. Bulirsch. 1980. *Introduction to numerical analysis*. Springer-Verlag, New York.
- van Genuchten, M.Th. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Sci. Soc. Am. J.* 44:892–898.
- Warrick, A.W. 1995. Correspondence of hydraulic functions for unsaturated soils. *Soil Sci. Soc. Am. J.* 59:292–299.
- Warrick, A.W. 2003. *Soil water dynamics*. Oxford Univ. Press, New York.
- Warrick, A.W., and T.-C.J. Yeh. 1990. One-dimensional, steady vertical flow in a layered soil profile. *Adv. Water Resour.* 13:207–210.
- Wooding, R.A. 1968. Steady infiltration from a large shallow circular pond. *Water Resour. Res.* 4:1259–1273.
- Young, M.H., K. Adams, S. Mizell, L. Chen, T. Bullard, and S. Grant. 2005. Assessment and improvement of hydraulic characterization of alluvial fans to support estimates of recharge and initial abstraction. DRI Tech. Rep. DHS Publ. 41218. Desert Res. Inst., Las Vegas, NV.
- Young, M.H., E.V. McDonald, T.G. Caldwell, S.G. Benner, and D.G. Meadows. 2004. Hydraulic properties of a desert soil chronosequence in the Mojave Desert, USA. *Vadose Zone J.* 3:956–963.
- Zaslavsky, D. 1964. Theory of unsaturated flow into a non-uniform soil profile. *Soil Sci.* 97:400–410.
- Zhu, J., and B.P. Mohanty. 2002. Spatial averaging of van Genuchten hydraulic parameters for steady state flow in heterogeneous soils: A numerical study. *Vadose Zone J.* 1:261–272.
- Zhu, J., and B.P. Mohanty. 2003. Effective hydraulic parameters for steady state flows in heterogeneous soils. *Water Resour. Res.* 39(8): 1178, doi:10.1029/2002WR001831.
- Zhu, J., and B.P. Mohanty. 2006. Effective scaling factor for transient infiltration in heterogeneous soils. *J. Hydrol.* 319:96–108.
- Zhu, J., B.P. Mohanty, and N.N. Das. 2006. On the effective averaging schemes of hydraulic properties at the landscape scale. *Vadose Zone J.* 5:308–316.
- Zhu, J., B.P. Mohanty, A.W. Warrick, and M.Th. van Genuchten. 2004. Correspondence and upscaling of hydraulic functions for heterogeneous soils. *Vadose Zone J.* 3:527–533.