Correspondence and Upscaling of Hydraulic Functions for Steady-State Flow in Heterogeneous Soils

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ABSTRACT

Soil hydraulic parameters at relatively large scales (e.g., remote sensing footprints) are important for land–atmosphere interaction and general circulation models as well as other applications. Within this context, we investigated two major issues involving soil hydraulic properties: (i) hydraulic parameter correspondence among some of the more commonly used soil hydraulic conductivity functions (i.e., the Gardner [G], Brooks–Corey [BC], and van Genuchten [VG] equations) and (ii) their application to upscaling of hydraulic properties for steady-state flow in heterogeneous soils. We first establish parameter equivalence among the conductivity functions based on preserving macroscopic capillary lengths and predicting the same vertical water flux. Next we investigate the significance of parameter equivalence on averaging schemes for the hydraulic parameters to allow predictions of the ensemble characteristics for steady-state flow. Results show that the hydraulic parameters correspond very well and that the same rules can be used for averaging the parameters of different hydraulic conductivity functions when predicting ensemble evaporation rates from heterogeneous soils having a relatively large suction at the soil surface (e.g., a dry surface condition and/or a shallow groundwater table). On the other hand, when the surface suction is finite (especially when the suction is relatively small and/or the groundwater table is deep), it is more difficult to obtain correspondence between the parameters of the different conductivity models. The hydraulic functions correspond especially poorly when infiltration is considered. Parameter equivalence between the hydraulic functions is always satisfied for the case of evaporation from a shallow water table, as long as the macroscopic capillary length is preserved.

Simulations of unsaturated flow in the vadose zone typically use closed-form functional relationships to represent the water-retention and unsaturated hydraulic conductivity functions. The Gardner exponential model (Gardner, 1958), the Brooks and Corey piecewise continuous model (Brooks and Corey, 1964), and the van Genuchten model (van Genuchten, 1980) are some of the more widely used hydraulic conductivity functions. Conditions for which alternative forms of the hydraulic functions give the same or similar responses are important in many applications. Warrick (1995) investigated the correspondence of hydraulic functions and discussed some of the features that need to be preserved in order for different types of hydraulic functions to correspond (i.e., to give similar or identical results for a given flow scenario). Lenhard et al. (1989) developed equivalent van Genuchten (1980) and Brooks and Corey (1964) parameters based on the shapes of retention curves. Morel-Seytoux et al. (1996) provided a way to convert Brooks–Corey parameters to van Genuchten parameters based on preserving the macroscopic capillary length and the asymptotic behavior of the soil water retention curve.

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Received 4 Feb. 2004. Special Section: Uncertainty in Vadose Zone Flow and Transport Properties.
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Published in Vadose Zone Journal 3:527–533 (2004).

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A related issue of concern for heterogeneous field soils is the upscaling of hydraulic parameters. Soil hydraulic functions are generally valid only at the point or local scale. When they are used in larger (plot, field, watershed or regional)–scale models, major questions remain about how best to average the spatially variable hydraulic properties over a heterogeneous soil volume.

The main objective of this study is to investigate how the hydraulic parameters of commonly used hydraulic functions correspond and what the correspondence implies in terms of averaging schemes of the hydraulic properties for steady-state vertical flow in heterogeneous soils at the larger scale. More specifically, we aim to establish relationships of probability distribution functions and averaging schemes (in terms of p norms, as will be discussed below) among the parameters of those hydraulic functions. Our study focuses especially on correspondence between the Brooks–Corey and van Genuchten models.

SOIL HYDRAULIC PROPERTY MODELS AND MACROSCOPIC CAPILLARY LENGTH

Soil hydraulic behavior is characterized by the soil water retention curve, which defines the water content as a function of suction, and the hydraulic conductivity function, which establishes the relationship between the hydraulic conductivity and the water content or suction. A brief review of hydraulic conductivity models used in this study is given below. Interested readers are referred to Leij et al. (1997) and Warrick (2003) for more comprehensive reviews and discussions of various closed-form expressions for the soil hydraulic properties, including the hydraulic conductivity models used here. The unsaturated hydraulic conductivity, K, is typically expressed by an equation of the form

$$K = K_s K_r(\alpha h, \eta) \quad [1]$$

where K_s is the saturated hydraulic conductivity, K_r is the relative hydraulic conductivity, α is known as the pore-size distribution parameter, h is the suction, and η is an empirical parameter characterizing the shape

Abbreviations: BC, Brooks–Corey; G, Gardner; pdf, probability distribution function; VG, van Genuchten.

(nonlinearity) of the hydraulic conductivity function. For the three models considered here, α is denoted as α_G for the Gardner model, α_{BC} for the Brooks–Corey model, and α_{vG} for the van Genuchten model. By analogy, η is nonexistent for the Gardner model, and denoted λ for the BC model and m for the VG model using the conventional nomenclature. Please note that αh is a dimensionless quantity.

Gardner Model

The relative hydraulic conductivity as used by Gardner (1958) is given by

$$K_r(\alpha_G h, \eta) = \exp(-\alpha_G h) \quad [2]$$

Brooks–Corey Model

Brooks and Corey (1964) used the following empirical relationship between K and h:

$$K_r(\alpha_{BC} h, \lambda) = (\alpha_{BC} h)^{-\beta} \quad \text{when} \quad \alpha_{BC} h > 1 \quad [3a]$$

$$K_r(\alpha_{BC} h, \lambda) = 1 \quad \text{when} \quad \alpha_{BC} h \leq 1 \quad [3b]$$

where $\beta = \lambda(\ell_B + 1) + 2$, and ℓ_B is the pore-connectivity parameter that accounts for the presence of a tortuous flow path, generally assumed to be 2.0 (Burdine, 1953). This model has been successfully used to describe conductivity data for relatively homogeneous, mostly coarse-textured soils. The model may not describe data well at or near the air-entry (or bubbling) suction ($h = 1/\alpha_{BC}$) where the curve is only zero-order continuous (i.e., the slope is discontinuous).

van Genuchten Model

van Genuchten (1980) combined his proposed S-shaped soil water retention function with the theoretical pore-size distribution model of Mualem (1976) to obtain the following hydraulic conductivity function:

$$K_r(\alpha_{vG} h, m) = \frac{\{1 - (\alpha_{vG} h)^{mn}[1 + (\alpha_{vG} h)^n]^{-m}\}^2}{[1 + (\alpha_{vG} h)^n]^{m\ell_M}} \quad [4]$$

where ℓ_M is a tortuosity or pore-connectivity parameter estimated by Mualem (1976) to be about 0.5 as an average for many soils, n is a shape factor, and $m = 1 - 1/n$.

Macroscopic Capillary Length

The macroscopic capillary length (H_c) as frequently used in analyses of unsaturated flow from a source at suction h_{wet} into a soil at suction h_{dry} is given by (e.g., Philip, 1985)

$$H_c = (K_{wet} - K_{dry})^{-1} \int_{h_{wet}}^{h_{dry}} K \mathrm{d}h \quad [5]$$

where $K_{wet} = K(h_{wet})$ and $K_{dry} = K(h_{wet})$. For systems that span across saturated ($h_{wet} = 0$) and very dry conditions ($h_{dry} \rightarrow \infty$), Eq. [5] reduces to the effective capillary drive as used by Morel-Seytoux and Khanji (1974, 1975) and Morel-Seytoux et al. (1996), among others:

$$H_c = K_s^{-1} \int_0^{\infty} K dh \quad [6]$$

Considering the case of evaporation from a very dry soil surface ($h_{dry} \rightarrow \infty$) of a soil profile that contains a water table ($h_{wet} = 0$) and substituting Eq. [2] and [3] into Eq. [5] then leads to the following expressions for the macroscopic capillary length for the G and BC models:

$$H_{cG} = 1/\alpha_G \quad [7]$$

and

$$H_{cBC} = \left(\frac{1}{\alpha_{BC}}\right)\frac{\lambda(\ell_B + 1) + 2}{\lambda(\ell_B +) + 1} \quad [8]$$

For the VG model, the macroscopic capillary length can be approximated fairly accurately using the expression (Morel-Seytoux et al., 1996)

$$H_{cvG} = \left(\frac{1}{\alpha_{vG}}\right)\left(\frac{0.046m + 2.07m^2 + 19.5m^3}{1 + 4.7m + 16m^2}\right) \quad [9]$$

HYDRAULIC PARAMETER CORRESPONDENCE

Synthesis of Earlier Results

Morel-Seytoux et al. (1996) proposed two criteria for equivalence of the BC and VG parameters. Their primary criterion was to preserve the effective capillary drive given by Eq. [6]. Their secondary criterion was to preserve the asymptotic behavior of the retention curve at low water contents. These two criteria lead to

$$\frac{\alpha_{vG}}{\alpha_{BC}} = \left(\frac{0.046m + 2.07m^2 + 19.5m^3}{1 + 4.7m + 16m^2}\right)\left(\frac{3\lambda + 1}{3\lambda + 2}\right) \quad [10a]$$

and

$$\lambda = n - 1 \quad [10b]$$

Other approximate relationships between the BC and VG parameters were suggested by van Genuchten (1980):

$$\alpha_{BC} = \alpha_{vG} \quad [11]$$

$$\lambda = n - 1 \quad [12]$$

Lenhard et al. (1989) obtained yet other relationships by equating the specific moisture capacity halfway between the saturated (θ_s) and residual (θ_r) water contents and minimizing the difference between the two BC and VG water retention curves:

$$\frac{\alpha_{vG}}{\alpha_{BC}} = [0.72 - 0.35\exp(-n^4)]^{1/\lambda} \left\{[0.72 - 0.35\exp(-n^4)]^{n/(1-n)} - 1\right\}^{1/n} \quad [13]$$

$$\lambda = (n - 1)(1 - 0.5^{n/(n-1)}) \quad [14]$$

While these relationships are potentially useful for establishing parameter correspondence among the different hydraulic property models, all are in essence mathematical manipulations based on matching of hydraulic property curves, and as such may lack physical meaning in hydrologic applications. In this study we propose an alternative primary equivalence criterion based on hydraulic behavior equivalence. Our approach forces the predicted flux across the soil surface to be the same for the different hydraulic conductivity functions, rather than matching the hydraulic property functions themselves. Warrick (1995) pointed out that hydraulic behavior equivalence depends on the hydrologic scenario being considered. We selected the flux across the soil surface for this purpose since this is an important quantity in upscaling of hydrologic process from local scale to footprint scale.

Our second equivalence criterion remains the same as that used by Morel-Seytoux et al. (1996), that is, preserving the macroscopic capillary length. This criterion is important since, as we will show later, for the asymptotic case when the flux is large (such as for evaporation from a soil with a shallow water table), flux equivalence of various hydraulic functions will always be satisfied as long as the macroscopic capillary length is preserved.

Correspondence with the Gardner Model

Since the Gardner model contains only one shape (nonlinearity) parameter, while the other two models contain two parameters, we first determine how Gardner's α_G should correspond with the α's of other two models using the primary criterion of flux equivalence. We then discuss conditions for which both criteria will be satisfied.

When the suction at the soil surface is relatively large ($h_0 \rightarrow \infty$), application of Darcy's Law leads to (Zhu and Mohanty, 2002)

$$z_0 = \frac{1}{\alpha} F(q/K_s,\eta) \quad [15]$$

where z_0 is the depth from the soil surface to the water table, q is the steady-state flux rate, and function F is defined by

$$F(q/K_s,\eta) = \int_0^\infty \frac{K_r(s,\eta)\mathrm{d}s}{K_r(s,\eta) + q/K_s} \quad [16]$$

where s is a dummy integration variable. For the G model, $K_r(s, \eta) = \exp(-s)$, in which case Eq. [16] can be integrated analytically to give

$$F_G(q/K_s,\eta) = \ln[1 + (q/K_s)^{-1}] \quad [17]$$

in which case Eq. [15] reduces to

$$z_0 = \frac{\ln[1 + (q/K_s)^{-1}]}{\alpha_G} \quad [18]$$

By equating Eq. [15] and [18], any arbitrary hydraulic conductivity function will produce equivalence to the Gardner function in terms of predicting the same dimensionless flux q/K_s if

$$\frac{\alpha}{\alpha_G} = \frac{F(q/K_s,\eta)}{\ln[1 + (q/K_s)^{-1}]} \quad [19]$$

Equation [19] was applied to both the BC and VG hydraulic conductivity equations. $F(q/K_s, \eta)$ in Eq. [19] required numerical integration for this purpose, which was achieved using Romberg integration (e.g., Scheid, 1968; Stoer and Bulirsch, 1980).

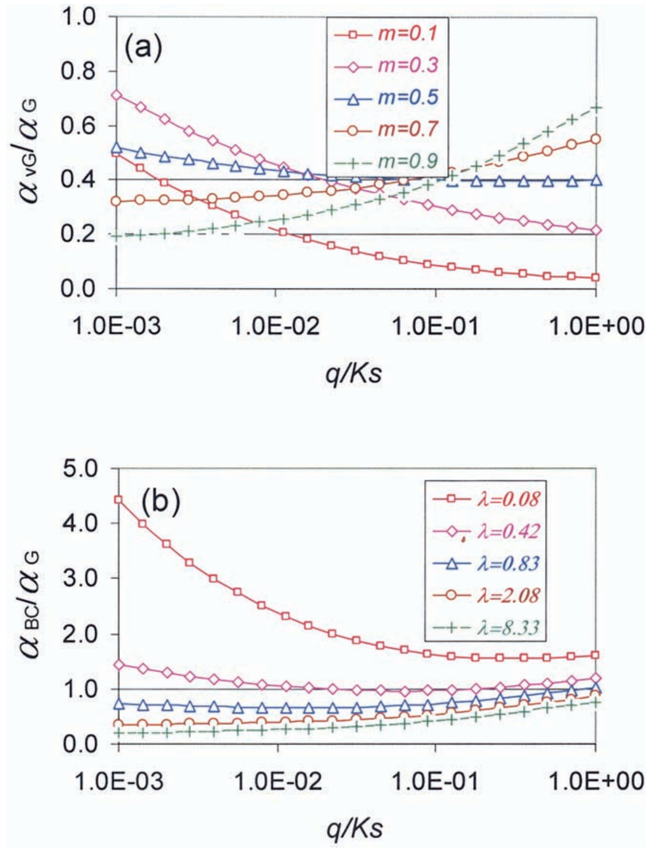

Fig. 1. Plots showing α parameter equivalence when $h_0 \rightarrow \infty$ between (a) the Gardner and van Genuchten models and (b) the Gardner and Brooks–Corey models.

Figure 1 shows α parameter equivalence based on Eq. [19] for large surface suctions ($h_0 \rightarrow \infty$) between the Gardner and van Genuchten models (Fig. 1a), and the Gardner and Brooks–Corey models (Fig. 1b). Note that parameter equivalence does not depend on the depth to the water table, z_0. The figure shows that in order for the Gardner and van Genuchten models to correspond, α_{vG} must be smaller than α_G, while for Brooks–Corey model α_{BC} can be either larger or smaller than Gardner α_G. Also, the ratio α_{BC}/α_G decreases as λ increases. Other major findings are that, for the VG model, when $m = 0.5$ (i.e., $n = 2$) the ratio α_{vG}/α_G varies over a relatively small range, from about 0.4 to 0.5, when q/K_s varies three orders of magnitude (Fig. 1a), and for the BC model, a λ value between 0.42 and 0.83 leads to only a small range in α_{BC}/α_G values as shown in Fig. 1b.

Equating Eq. [7] and [9] to preserve the macroscopic capillary length for the G and VG models leads to

$$\frac{\alpha_{vG}}{\alpha_G} = \frac{0.046m + 2.07m^2 + 19.5m^3}{1 + 4.7m + 16m^2} \quad [20]$$

Equation [20] shows that $\alpha_{vG}/\alpha_G = 0.4052$ for $m = 0.5$. In other words, when the m is about 0.5 for the VG model, a α_{vG}/α_G value of about 0.4 will produce the same evaporative flux and also preserve the macroscopic length when using the G and VG models.

Equating Eq. [7] and [8] similarly preserves the macroscopic capillary length for the G and BC models:

$$\frac{\alpha_{BC}}{\alpha_G} = \frac{\lambda(\ell_B + 1) + 2}{\lambda(\ell_B + 1) + 1} = \frac{3\lambda + 2}{3\lambda + 1} \quad [21]$$

This equation shows that $\alpha_{BC}/\alpha_G = 1.444$ for $\lambda = 0.42$, and 1.286 for $\lambda = 0.83$. These values are close to those obtained from Fig. 1b for $\lambda = 0.42$ and 0.83, respectively. This means that for λ values between approximately 0.42 and 0.83, the α parameter ratio required to preserve the macroscopic capillary length will also predict approximately the same flux.

For a special scenario where q/K_s is large (such as for evaporation from soil having a shallow water table), Eq. [16] can be approximated (with K_r neglected in the denominator) to

$$F(q/K_s,\eta) = \int_0^\infty \frac{K_r(s,\eta)\mathrm{d}s}{K_r(s,\eta) + q/K_s} \approx (q/K_s)^{-1} \int_0^\infty K_r(s,\eta)\mathrm{d}s = (q/K_s)^{-1}\eta H_c \quad [22]$$

Substituting Eq. [22] back into Eq. [15] leads to

$$\frac{q}{K_s} = \frac{H_c}{z_0} \quad [23]$$

An important conclusion of Eq. [23] is that flux equivalence of various hydraulic functions will always be satisfied as long as the macroscopic capillary length is preserved.

Correspondence between the Brooks–Corey and van Genuchten Models

If the suction at the soil surface is finite (h_0), application of Darcy's Law in a form similar to Eq. [15], and using the BC and VG functions leads to

$$z_0 = \frac{1}{\alpha_{BC}} F_{1BC}(q/K_s,\lambda) \quad [24]$$

and

$$z_0 = \frac{1}{\alpha_{vG}} F_{1vG}(q/K_s,m) \quad [25]$$

respectively, where

$$F_{1BC}(q/K_s,\lambda) = \int_0^{\alpha_{BC}h_0} \frac{K_r(s,\lambda)\mathrm{d}s}{K_r(s,\lambda) + q/K_s} \quad [26]$$

and

$$F_{1vG}(q/K_s,m) = \int_0^{\alpha_{vG}h_0} \frac{K_r(s,m)\mathrm{d}s}{K_r(s,m) + q/K_s} \quad [27]$$

By equating Eq. [24] and [25], predicting the same q/K_s requires

$$F_{1vG}(q/K_s,m)/\alpha_{vG} = F_{1BC}(q/K_s,\lambda)/\alpha_{BC} \quad [28]$$

and preserving the macroscopic capillary length implies that

$$\frac{\beta - (\alpha_{BC}h_0)^{1-\beta}}{\alpha_{BC}(\beta - 1)[1 - (\alpha_{BC}h_0)^{-\beta}]}$$

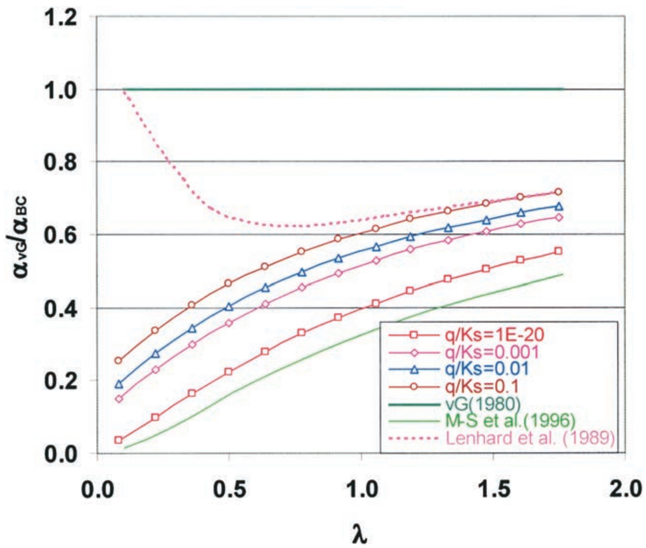

Fig. 2. Plots showing α parameter equivalence between the Brooks–Corey and van Genuchten models when $h_0 \rightarrow \infty$.

$$= \frac{1}{\alpha_{vG}[1 - K_r(\alpha_{vG}h_0,m)]} \int_0^{\alpha_{vG}h_0} K_r(s,m)\mathrm{d}s \quad [29]$$

The left-hand side of Eq. [29] for the BC model was obtained by substituting Eq. [3] into Eq. [5] and noting that $h_{dry} = h_0$ and $h_{wet} = 0$. The right-hand side of Eq. [29] was similarly obtained for the VG model by substituting Eq. [4] into Eq. [5]. Given q/K_s and the BC λ and α_{BC}, the equivalent VG m and α_{vG} must be solved simultaneously from Eq. [28] and [29]. The roots for m and α_{vG} were found by a combination of successive approximations and the van Wijngaarden–Dekker–Brent method (Brent, 1973; Press et al., 1992).

Please note that when $\alpha_{BC}h_0 \leq 1$, it is impossible to preserve the macroscopic capillary length between the BC and VG models since the BC model uses a function that is only piecewise continuous. To circumvent this problem, we used an alternate correspondence when $\alpha_{BC}h_0$ is less than some threshold αh_{crit} (the value of αh_{crit} is usually somewhat larger than 1). We simply assumed that α_{vG}/α_{BC} remains constant when $\alpha_{BC}h_0 < \alpha h_{crit}$. The correspondence between the VG n and BC λ is then established based on the requirement of having the same vertical flux.

Figure 2 shows α parameter equivalence (α_{vG}/α_{BC}) between the BC and VG models as a function of λ when $h_0 \rightarrow \infty$ for different q/K_s values. Figure 3 similarly demonstrates parameter equivalence when $h_0 \rightarrow \infty$ between the BC λ and VG n parameters. For comparison purposes, these two figures also show the three parametric relationships that were established previously (i.e., Eq. [10]–[14]). Notice that the VG n and BC λ parameters are linearly related. As the flux increases, both n and α_{vG}/α_{BC} must also increase to predict the same flux with the two models. When the flux is very small, such as for evaporation from a deep water table, the parametric relationships of Morel-Seytoux et al. (1996) are closest to our results (see Fig. 2). Also, please notice the rela-

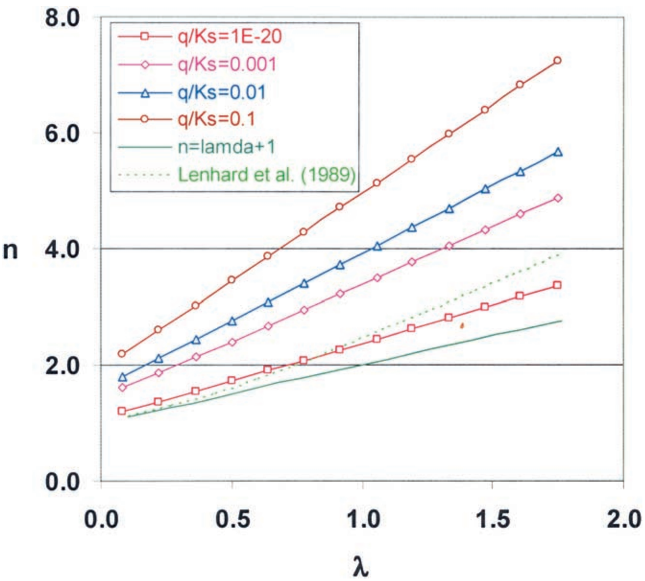

Fig. 3. Parameter equivalence when $h_0 \rightarrow \infty$ between the Brooks–Corey λ and van Genuchten *n*.

tively large deviations with our results in Fig. 2 when α_{vG} and α_{BC} are simply equated (Eq. [12]).

Figure 4 shows parameter equivalence between the Brooks–Corey and van Genuchten models when h_0 is finite and $\lambda = 0.83$. Results show that when $\alpha_{BC}h_0$ is small, the equivalent VG *n* needed to predict the same flux varies considerably as a function of $\alpha_{BC}h_0$. This is partly due to the fact that the BC and VG models correspond poorly for relatively small values of $\alpha_{BC}h_0$. In most practical applications, little or no evaporation probably will take place for small $\alpha_{BC}h_0$ values. When $\alpha_{BC}h_0$ is small enough, flow will shift from evaporation to infiltration. In other words, this shows that parameter correspondence between the BC and VG models is more difficult to achieve for infiltration than for evaporation.

IMPLICATIONS FOR HYDRAULIC PARAMETER UPSCALING

In this study we interpret hydraulic property upscaling as deriving "equivalent" homogeneous soil hydraulic properties to account for uncertainties in the spatially variable hydraulic parameters (e.g., Zhu and Mohanty,

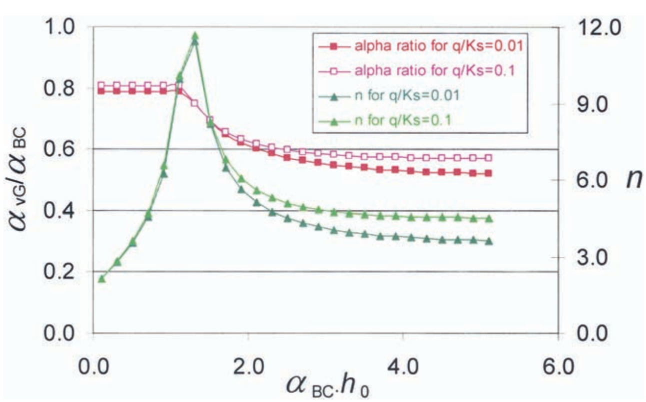

Fig. 4. Parameter equivalence between the Brooks–Corey and van Genuchten models when h_0 is finite and $\lambda = 0.83$.

2002; Zhu and Mohanty, 2003). The equivalent homogeneous soil hydraulic properties should produce the same ensemble flux as that corresponding to random fields of the soil hydraulic parameters. In practical applications, two special types of heterogeneity need to be distinguished: (i) vertical layering (heterogeneity) where variations in soil properties are in the vertical direction and (ii) vertically homogeneous soil columns with variations in soil properties in the horizontal plane. Our study focuses on the latter case, where the variability is in the horizontal plane. For meso- or regional-scale Soil–Vegetation–Atmosphere Transfer (SVAT) schemes in hydroclimatic models, pixel dimensions may range from several hundred square meters to several square kilometers, while the vertical scale of relevant subsurface flow processes near the land–atmosphere boundary (top few meters) will be relatively small. For such a large horizontal scale, areal heterogeneity in soil hydraulic properties is likely much more important than vertical heterogeneity. Our assumption of having predominant horizontal heterogeneity will not apply to scenarios where the vadose zone is deep and significantly layered, nor to regions where the topography varies considerably such that mutual lateral interactions occur between the parallel soil columns.

In previous studies (Zhu and Mohanty, 2002, 2003) we tried to answer the question: For a typical soil textural combination in a real field condition, what are the effective (or average) homogeneous hydraulic properties for the entire field (pixel) if the soil hydraulic properties can be estimated accurately for each individual texture? In this study the questions we try to address are:

1. If the optimal *p*-norm (best averaging scheme, as described and explained below) value is known for the parameters of one hydraulic function that will produce a certain ensemble flux into or from a heterogeneous field, what should be the corresponding *p*-norm of other hydraulic functions in order to produce the same ensemble flux?
2. If the probability distribution function (pdf) of hydraulic parameters from one function is known, what are the pdfs of the parameters of the other hydraulic functions in order to produce the same statistics of the flux field?

Specifically, we try to establish the probability distribution function and the *p*-norm relationships between the BC and VG hydraulic parameters based on the correspondence between these two models developed above.

The *p*-norm or *p*-order power average $\hat{\alpha}(p)$ for a set of *N* random parameter values α_i is given by (Korvin 1982; Green et al., 1996)

$$\hat{\alpha}(p) = \left[(1/N)\sum_{i=1}^{N}\alpha_i^p\right]^{1/p} \qquad [30]$$

The power average is a generalization of the arithmetic, geometric and harmonic averages. The arithmetic ($p = 1$), geometric ($p \rightarrow 0$), and harmonic ($p = -1$) means are all particular cases of the power average.

For our example calculations we adopt typical values of $\overline{\alpha_{BC}} = 0.0478$ (1/cm) and $CV(\alpha_{BC}) = 0.85$, where over-

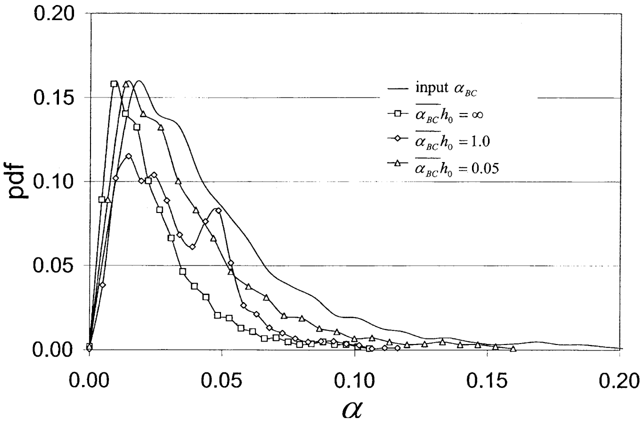

Fig. 5. Comparison of probability density distribution of the Brooks–Corey α_{BC} parameter and the van Genuchten α_{vG} parameter when $q/K_s = 0.01$ and $\lambda = 0.83$.

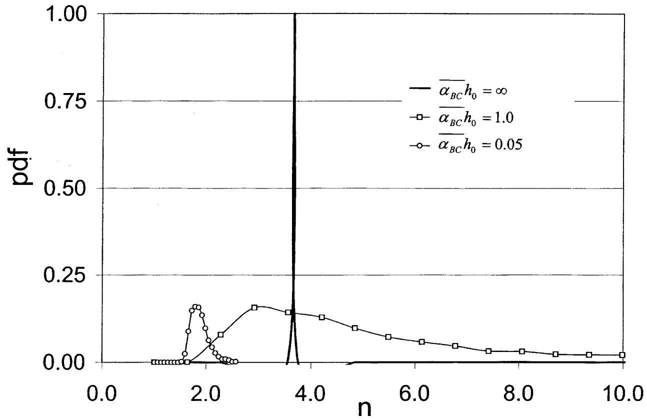

Fig. 6. Probability density distribution of the van Genuchten *n* parameter when $q/K_s = 0.01$ and $\lambda = 0.83$.

bars denote average values. Based on the input statistics, a random field of 10 000 values for α_{BC} was generated using the spectral method proposed by Robin et al. (1993). After generating the random field, the corresponding fields of the VG parameters that will produce the same flux can be calculated using the previously established relationships for parameter equivalence. After this the pdfs of the VG parameters can be calculated. The *p*-norm relationship between the BC and VG models that will produce the same ensemble flux can also be established.

Figure 5 is a comparison of pdfs of the α_{BC} and the α_{vG} parameters when $q/K_s = 0.01$ and $\lambda = 0.83$. Results show that for the two extremes of $\overline{\alpha_{BC}}h_0$ (very large and very small values; see Fig. 5) the corresponding α_{vG} distribution is the same as the input α_{BC}. The only difference is in their average values, with $\overline{\alpha_{vG}}$ being smaller than $\overline{\alpha_{BC}}$. The entire VG α pdf scales back to smaller values, which reflects the fact that for the two extremes of large and small surface suctions, the corresponding α_{vG} and α_{BC} parameters are proportional, while the ratio of α_{vG} over α_{BC} is always <1 (Fig. 4). In the vicinity of $h_0 = 1/\overline{\alpha_{BC}}$, the corresponding VG α changes from its highest limit at $h_0 \to 0$ to its lowest limit at $h_0 \to \infty$. This transition in the VG α explains the double-hump pdf shape for the van Genuchten α when $\overline{\alpha_{BC}}h_0 = 1.0$ (see curve denoted by diamonds in Fig. 5).

Figure 6 shows probability density distributions of the VG *n* when $q/K_s = 0.01$ and $\lambda = 0.83$. The structure of the VG *n* pdf can be explained by the local-scale correspondence of the models shown in Fig. 4. When $h_0 \to \infty$, the VG *n* will approach an asymptotic value of about 3.7, which is not related to α_{BC}. This fact is reflected in the delta function pdf distribution for the VG *n* seen in Fig. 6. When h_0 is small or when $h_0 = 1/\overline{\alpha_{BC}}$, the VG *n* pdf distribution is similar to the input lognormal distribution of α_{BC}. However, at and near the transition point (bubbling suction) of the Brooks–Corey model ($h_0 = 1/\overline{\alpha_{BC}}$), the value of *n* is large and its distribution more stretched, leading to a high mean value and large standard deviation for the corresponding VG *n*.

Figure 7 plots *p*-norm values for the α_{vG} parameter in comparison with those for the α_{BC} parameter when $q/K_s = 0.01$ and $\lambda = 0.83$. The comparison is not meant to suggest optimal *p*-norm values (or the best averaging scheme) that would produce the ensemble flux for heterogeneous soils. Rather the optimal *p*-norm values for the α_{BC} and α_{vG} parameters that would produce the same ensemble flux are compared for different surface suction conditions. Zhu and Mohanty (2002) established general guidelines for spatial averaging of VG parameters that will produce the same ensemble flux for steady-state evaporation and infiltration. It is interesting to observe that at both ends of the surface suction values, the optimal *p*-norm*s* for the VG model and those for the BC model are the same. In other words, the same averaging rules can be used for both models. However, in the middle range of the surface suction values, the *p*-norm*s* for the α_{vG} are generally higher than those for the α_{vG} to predict the same ensemble flux.

CONCLUSION

For a large surface suction (such as for a very dry surface), when *m* is approximately 0.5 ($n \approx 2$), the Gardner and van Genuchten models gave similar fluxes. When λ was between 0.42 and 0.83, the Gardner and Brooks–Corey models were found to correspond the best. In general, the α_{BC} and α_{vG} parameters corresponded very well and were found to be proportional. Furthermore, the VG *n* and BC λ parameters had a linear relationship. This means that the *p*-norm and the corresponding probability density functions of the BC and VG parameters should be the same. Since they have a linear relationship, we can use the same rules for upscaling of the van Genuchten and Brooks–Corey hydraulic parameters when predicting maximum potential evaporation rates (i.e., evaporation from soils having very large soil surface suctions).

For the more general case where the surface suction is finite, it is more difficult for the hydraulic parameters of different conductivity models to correspond in terms of generating the same fluxes. The correspondence depends on the value of h_0. The smaller h_0, (i.e., scenarios closer to infiltration), the more difficult it is to achieve correspondence. For infiltration, the most difficult as-

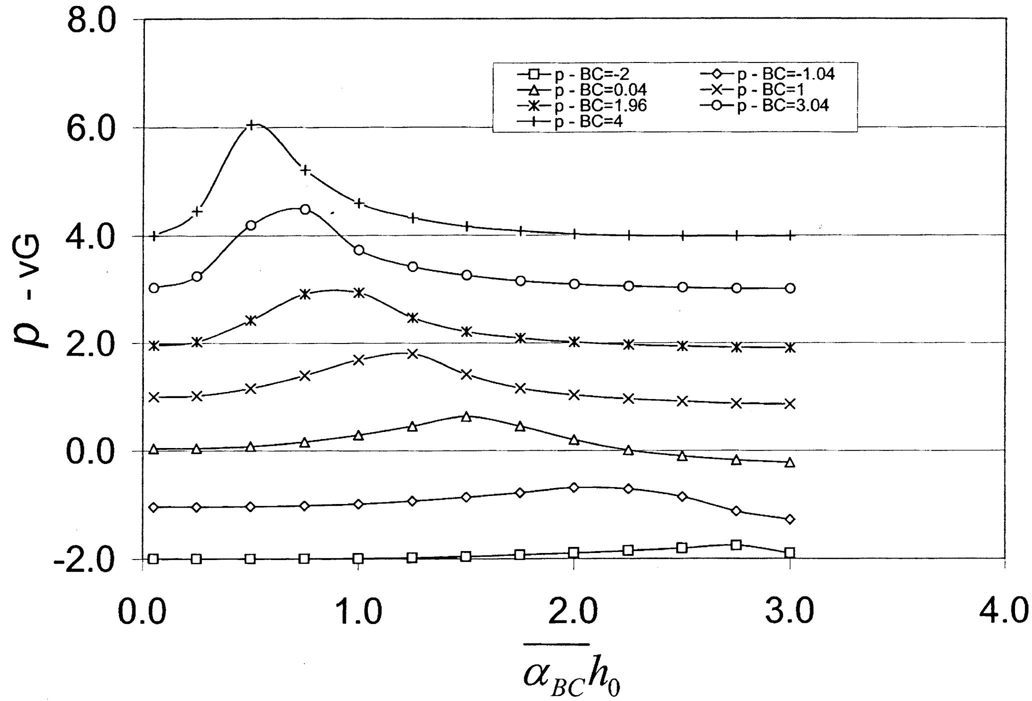

Fig. 7. Plots comparing values of the *p*-norm for the van Genuchten α parameter with those for the Brooks–Corey α parameter when q/K_s = 0.01 and λ = 0.83.

pect is the first-order discontinuous shape of the Brooks–Corey model. This feature makes its correspondence to the van Genuchten model difficult when the suction in the domain drops close to or below the threshold point ($1/\alpha_{BC}$) of the function, which is more relevant to infiltration scenario. In case of evaporation from a shallow water table, parameter equivalence of the different hydraulic functions will always be satisfied as long as the macroscopic capillary length is preserved.

ACKNOWLEDGMENTS

This project is supported by NASA (Grants NAG5-8682 and NAG5-11702) and also in part by SAHRA (Sustainability of Semi-Arid Hydrology and Riparian Areas) under the STC Program of the National Science Foundation, Agreement no. EAR-9876800.

REFERENCES

Brent, R. 1973. The use of successive interpolation for finding simple zeros of a function and its derivatives. p. 19–46. *In* Algorithms for minimization without derivatives. Prentice Hall, Englewood Cliffs, NJ.

Brooks, R.H., and A.T. Corey. 1964. Hydraulic properties of porous media. Hydrol. Paper 3. Colorado State Univ., Fort Collins.

Burdine, N.T. 1953. Relative permeability calculations from pore-size distribution data. Trans. AIME 198:71–77.

Gardner, W.R. 1958. Some steady state solutions of unsaturated moisture flow equations with applications to evaporation from a water table. Soil Sci. 85:228–232.

Green, T.R., J.E. Contantz, and D.L. Freyberg. 1996. Upscaled soil-water retention using van Genuchten's function. J. Hydrol. Eng. ASCE 1:123–130.

Korvin, G. 1982. Axiomatic characterization of the general mixture rule. Geoexploration 19:267–276.

Leij, F.J., W.B. Russell, and S.M. Lesch. 1997. Closed-form expressions for water retention and conductivity data. Ground Water 35: 848–858.

Lenhard, R.J., J.C. Parker, and S. Mishra. 1989. On the correspondence between Brooks-Corey and van Genuchten models. J. Irrig. Drain. Eng. ASCE 115:744–751.

Morel-Seytoux, H.J., and J. Khanji. 1974. Derivation of an equation of infiltration. Water Resour. Res. 10:795–800.

Morel-Seytoux, H.J., and J. Khanji. 1975. Prediction of imbibition in a horizontal column. Soil Sci. Soc. Am. Proc. 39:613–617.

Morel-Seytoux, H.J., P.D. Meyer, M. Nachabe, J. Touma, M.Th. van Genuchten, and R.J. Lenhard. 1996. Parameter equivalence for the Brooks-Corey and van Genuchten soil characteristics: Preserving the effective capillary drive. Water Resour. Res. 32:1251–1258.

Mualem, Y. 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour. Res. 12:513–522.

Philip, J.R. 1985. Reply to "Comments on 'Steady infiltration from spherical cavities.'". Soil Sci. Soc. Am. J. 49:788–789.

Press, W.H., S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery. 1992. Numerical recipes in Fortran: The art of scientific computing. 2nd ed. Cambridge University Press, Cambridge, UK.

Robin, M.J.L., A.L. Gutjahr, E.A. Sudicky, and J.L. Wilson. 1993. Cross-correlated random field generation with the direct Fourier transform method. Water Resour. Res. 29:2385–2397.

Scheid, F. 1968. Theory and problems of numerical analysis. Schaum's Outline Series. McGrawHill Book Co., New York.

Stoer, J., and R. Bulirsch. 1980. Introduction to numerical analysis. Springer-Verlag, New York.

van Genuchten, M.Th. 1980. A closed-form equation fro predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J. 44:892–898.

Warrick, A.W. 1995. Correspondence of hydraulic functions for unsaturated soils. Soil Sci. Soc. Am. J. 59:292–299.

Warrick, A.W. 2003. Soil water dynamics. Oxford Univ. Press, New York.

Zhu, J., and B.P. Mohanty. 2002. Spatial averaging of van Genuchten hydraulic parameters for steady state flow in heterogeneous soils: A numerical study. Available at www.vadosezonejournal.org. Vadose Zone J. 1:261–272.

Zhu, J., and B.P. Mohanty. 2003. Effective hydraulic parameters for steady state flows in heterogeneous soils. Water Resour. Res. 39(8). 1178. doi:10.1029/2002WR001831.