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Effect of the shape of the soil hydraulic functions near saturation on variably-saturated flow predictions

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Abstract

Relatively small changes in the shape of the soil water retention curve near saturation can significantly affect the results of numerical simulations of variably saturated flow, including the performance of the numerical scheme itself in terms of stability and rate of convergence. In this paper, we use a modified form of the van Genuchten–Mualem (VGM) soil hydraulic functions to account for a very small, but non-zero minimum capillary height, h_s, in the soil water retention curve. The modified VGM model is contrasted with the original formulation by comparing simulation results for infiltration in homogeneous soils assuming both constant pressure and constant flux boundary conditions. The two models gave significantly different results for infiltration in fine-textured soils, even for h_s-values as small as −1 cm. Incorporating a small minimum capillary height in the hydraulic properties leads to less non-linearity in the hydraulic conductivity function near saturation and, because of this, to more stable numerical solutions of the flow equation. This study indicates an urgent need for experimental studies that assess the precise shape of the hydraulic conductivity curve near saturation, especially for relatively fine-textured soils. For one example we found considerable improvement in the predicted conductivity function when a value of −2 cm for h_s was used in the modified VGM model.

Keywords: Soil hydraulic properties; Unsaturated flow; Numerical solutions

1. Introduction

Numerical models simulating the movement of water and chemicals in variably saturated soils are increasingly used in a variety of research and engineering projects. Unsaturated flow is usually described with the Richards' equation which, for one-dimensional vertical flow, is given by

$$\frac{\partial \theta(h)}{\partial t} = \frac{\partial}{\partial z}\left[K(h)\frac{\partial h}{\partial z} - K(h)\right], \qquad (1)$$

where θ is the volumetric water content, h the soil water pressure head, K the hydraulic conductivity, z the distance (positive downward), and t is the time. The use of Eq. (1) requires knowledge of two soil hydraulic functions: the soil water retention curve, $\theta(h)$, and the unsaturated hydraulic conductivity function, $K(h)$. Accurate measurement of the unsaturated conductivity is generally cumbersome, costly, and very time-consuming. Consequently, many attempts have been made to develop indirect methods which predict the conductivity function from the more easier measured water retention curve [2,5,10,16,]. Most of the predictive conductivity models are based on the assumption of having an ideal capillary medium characterized by a certain pore-size distribution function. Although not necessarily valid for all soils, this assumption is widely accepted as an effective working hypothesis [16].

A variety of empirical equations have been used to describe the soil water retention curve. One of the most popular equations is the power-law function initially proposed by Brooks and Corey [1]. This equation leads to an air-entry value, h_a, in the soil water retention curve above which the soil is assumed to be saturated. The Brooks and Corey equation was modified by van Genuchten [26] to enable a more accurate description of observed soil hydraulic data near saturation, especially

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for undisturbed and many fine-textured soils. As shown by van Genuchten and Nielsen [27], Vogel et al. [29] and Vogel and Cislerova [30], among others, the choice of the analytical model for $\theta(h)$ can significantly affect the predicted $K(h)$ function obtained with one of the statistical pore-size distribution models. One reason for this is that the predicted conductivity function is extremely sensitive to small changes in the shape of the retention curve near saturation. This sensitivity is a major cause of the sometimes significant differences between predicted $K(h)$ functions obtained with the Brooks and Corey and van Genuchten retention functions. The differences are specially important for fine-textured soils which can exhibit extreme non-linearity in $K(h)$ close to saturation when van Genuchten's equations are used [27,28]. The differences are generally much less severe for coarse-textured soils.

As will be shown further in this paper, the presence of highly non-linear $K(h)$ relationships near saturation can also substantially impact the performance of numerical solutions of Eq. (1) in terms of the accuracy, stability, and rate of convergence of the invoked numerical scheme. While numerical solutions for solving the variably saturated flow equation have been significantly improved in recent years [4,6,9,14], most improvements focused on numerical problems associated with the infiltration of water in very dry coarse-textured soils [6,7,25]. By comparison, less attention has been paid to numerical problems encountered when simulating infiltration in fine-textured soils.

In this paper, we focus on the non-linear description of the soil hydraulic properties near saturation. Specifically, we address two closely related issues: (1) how uncertainty in the shape of the hydraulic functions, notably the hydraulic conductivity, near saturation translates into uncertainty in predicting soil water contents during infiltration, and (2) how extremely small changes in the shape of the water retention curve near saturation can impact the stability and convergence of numerical solutions of Eq. (1). Results will be presented in terms of van Genuchten's [26] expressions for the soil hydraulic properties, as well as modifications thereof.

2. Description of the unsaturated soil hydraulic properties

The predictive model of Mualem [16] for the relative hydraulic conductivity function, $K_r(h)$, may be written in the form:

$$K_r(S_e) = S_e^l \left[\int_0^{S_e} \frac{dx}{h(x)} \right]^2 \bigg/ \left[\int_0^1 \frac{dx}{h(x)} \right]^2, \tag{2}$$

where $K_r = K/K_s$, K is the unsaturated hydraulic conductivity, K_s the saturated hydraulic conductivity, l the pore connectivity parameter usually assumed to be 0.5 following Mualem [16], and S_e is the effective saturation given by

$$S_e(h) = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r}, \tag{3}$$

where θ is the volumetric water content, and θ_r and θ_s are the residual and saturated water contents, respectively. Substituting van Genuchten's [26] expression for the soil water retention curve, i.e.,

$$\theta(h) = \begin{cases} \theta_r + \frac{\theta_s - \theta_r}{[1+(\alpha h)^n]^m}, & h < 0, \\ \theta_s, & h \geqslant 0. \end{cases} \tag{4}$$

into Eq. (2) leads to the following equation for the unsaturated hydraulic conductivity

$$K(h) = \begin{cases} K_s K_r(h), & h < 0, \\ K_s, & h \geqslant 0, \end{cases} \tag{5}$$

where

$$K_r(S_e) = S_e^l \left[1 - \left(1 - S_e^{1/m}\right)^m\right]^2 \tag{6}$$

in which n and α are empirical shape parameters (with $n > 1$), and $m = 1 - 1/n$. The restriction $m = 1 - 1/n$ is necessary to permit direct integration of Eq. (2) once the inverse of Eq. (4) is substituted into Eq. (2). Much more complicated expressions for $K_r(h)$ result when the parameters m and n are assumed to be mutually independent [28]. We will refer to Eqs. (4)–(6) as the original VGM model.

To increase the flexibility of the VGM model in describing retention and hydraulic conductivity data near saturation, the following modifications in Eqs. (4)–(6) were suggested by Vogel et al. [29] and Vogel and Cislerova [30]:

$$\theta(h) = \begin{cases} \theta_r + \frac{\theta_m - \theta_r}{[1+(-\alpha h^n)]^m}, & h < h_s, \\ \theta_s, & h \geqslant h_s, \end{cases} \tag{7}$$

$$K(h) = \begin{cases} K_s K_r(h), & h < h_s, \\ K_s, & h \geqslant h_s, \end{cases} \tag{8}$$

where

$$K_r(S_e) = S_e^l \left[\frac{1 - F(S_e)}{1 - F(1)} \right]^2, \tag{9}$$

$$F(S_e) = \left(1 - S_e^{*1/m}\right)^m, \tag{10}$$

$$S_e^* = \frac{\theta_s - \theta_r}{\theta_m - \theta_r} S_e \tag{11}$$

in which as before $m = 1 - 1/n$ and $n > 1$. The parameter h_s is denoted here as the minimum capillary height. Notice from Fig. 1 that the parameter θ_s in the modified retention function is replaced by a fictitious (extrapolated) parameter $\theta_m \geqslant \theta_s$. This approach maintains the physical meaning of θ_s as a measurable quantity, while the definition of the effective degree of saturation, S_e, is also not affected. The minimum capillary height,

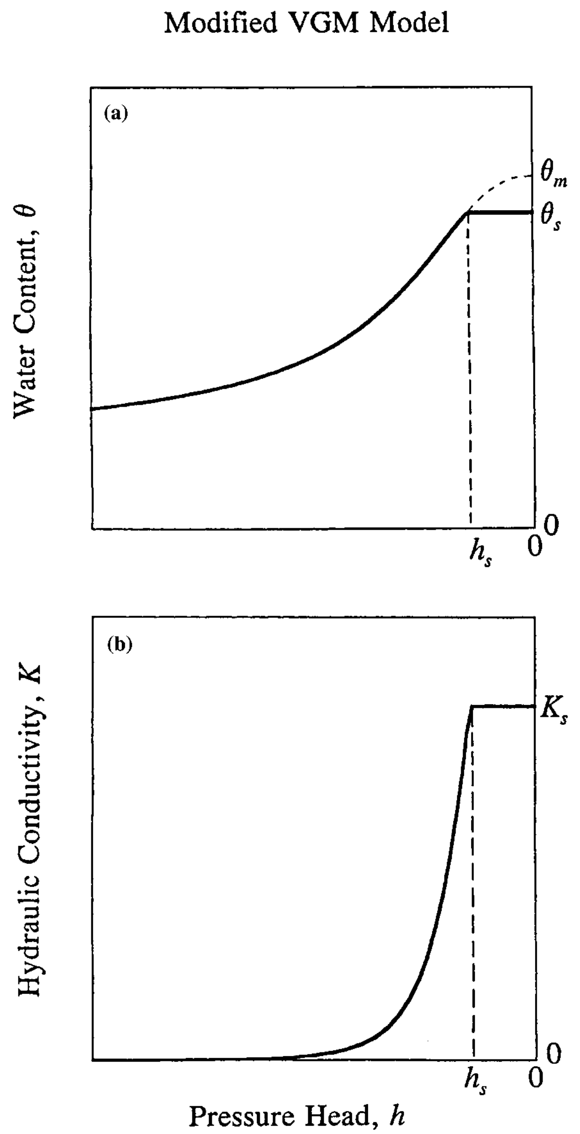

Fig. 1. Schematics of the soil water retention and hydraulic conductivity functions as described with the modified VGM model.

h_s, becomes zero when $\theta_m = \theta_s$, in which case Eqs. (7)–(11) reduce to the original VGM model.

Plots of the modified VGM model in Fig. 1 suggest that, at least phenomenologically, the parameter h_s serves the same purpose as the air-entry value, h_a, in the soil hydraulic equations of Brooks and Corey [1], that is to introduce a break in the functions at some negative pressure head. This is only partially true. Whereas Brooks and Corey's air entry value is usually somewhere between −10 and −1000 cm depending upon soil type (e.g., [24]), h_s in our study will be much smaller in absolute value (e.g., −1 to −2 cm) so that the retention function will retains its more-or-less S-shaped form typical of the VGM model (this will be further discussed later). The parameter h_s is a direct consequence of the soil system being a capillary medium which always should have some non-zero minimum capillary height ($h_s < 0$) associated with the largest pore of the soil. For example, the equation of capillary rise (e.g., [8, p. 23]) shows that water in a capillary of radius 1 mm should have a capillary elevation of approximately 15 mm. While it may appear here that we are trying to assign physical significance to h_s, this coefficient in reality is probably best treated pragmatically as an additional fitting parameter that enables a more precise description of the retention and hydraulic conductivity curves near saturation.

3. Shape of the hydraulic functions near saturation

We now address the question as to how well the original and modified VGM equations can describe the soil hydraulic functions close to saturation. The presence in Eq. (7) of a non-zero minimum capillary height has little or no effect on the retention function, but strongly affects the predicted shape of the unsaturated hydraulic conductivity function. This is demonstrated by a comparison of Figs. 2 and 3 which, for the original and modified VGM models, respectively, show variations in the soil hydraulic characteristics for a series of n-values, while keeping all other parameters constant ($\alpha = 0.05$ cm^{-1}, $\theta_s = 0.40$ and $\theta_r = 0.10$). Notice that the $K_r(h)$ function in Fig. 2 exhibits an abrupt drop at saturation when n becomes less than about 1.5. The abrupt near-instantaneous and highly non-linear decrease in $K_r(h)$ is caused by the $1/h$ term in Eq. (2) which goes to infinity at $S_e = 1$, and hence leads to a singularity in the original VGM conductivity equation. The magnitude of the decrease at $h = 0$ depends not only on the parameter n, but to a lesser extent also on α, which may be viewed as a scaling factor for h. The extreme non-linearity occurs only when $1 < n < 2$. Close inspection of Figs. 2(b) and (c) shows that the soil water capacity function, $C(h) = \mathrm{d}\theta/\mathrm{d}h$, and the relative conductivity function, $K_r(h)$, both change their shape near saturation when n changes from $n < 2$ to $n > 2$. The slopes of these two functions change from $-\infty$ for $C(h)$ and ∞ for $K_r(h)$ when $n < 2$ to some non-zero finite values when $n = 2$, and to zero when $n > 2$. This effect of the value of n on the shape of the retention curve near saturation was discussed previously by Luckner et al. [12] and Nielsen and Luckner [20]. They showed that the restriction $n > 1$ guarantees only first-order continuity in $\theta(h)$ at saturation, while $n > 2$ ensures also second-order continuity in $\theta(h)$, and hence first-order continuity in $C(h)$.

A comparison of Figs. 2 and 3 shows that the introduction of a very small but non-zero value of the minimum capillary height, h_s, in the VGM model leads to significant differences in the predicted $K(h)$ functions for n values close to 1 (i.e., for many fine-textured and/or

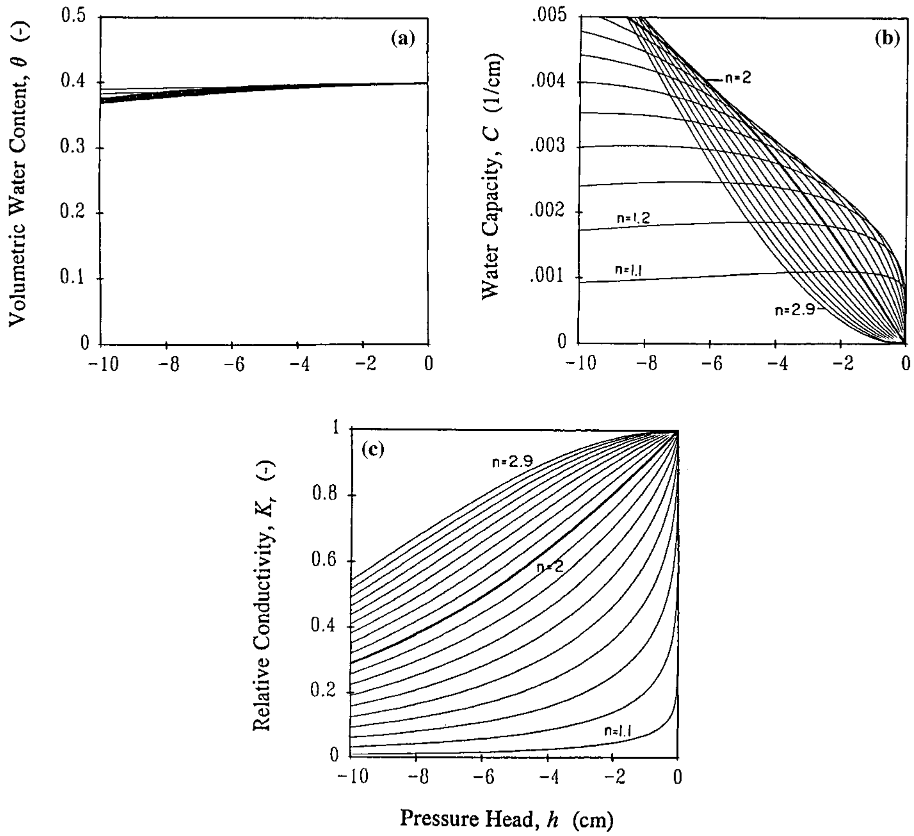

Fig. 2. Plots of the soil hydraulic properties near saturation as a function of the parameter n (original VGM model).

undisturbed soils characterized by relatively broad pore-size distributions), and to far less dramatic or even insignificant differences when n is large (i.e., for more coarse-textured, disturbed and/or compacted soils having relatively narrow pore-size distributions). By comparison, corresponding differences between the retention curves for the two models (Figs. 2(a) and 3(a)) are negligible, and definitely within the range of routine measurement errors. However, notice from Fig. 3(b) that the modified VGM soil water capacity curve is now discontinuous at $h = h_s$. Also, Fig. 3(c) shows that the modified hydraulic conductivity curve now no longer exhibits the abrupt decrease when the pressure head becomes equal to $h_s < 0$. This result is due to the fact that Mualem-type unsaturated conductivity functions are obtained by evaluating the integrals in Eq. (2) from 0 to S_e (or 1), which is equivalent to a pressure head range from $-\infty$ to h (or 0). When $h_s < 0$, the singularity in Eq. (2) at $h = 0$ is avoided and integration will always lead to a only modestly increasing $K(h)$ curve near saturation. In other words, the slope $K'(h)$ of the conductivity function remains finite when h reaches any $h_s < 0$, as opposed to becoming infinite when $h \to h_s = 0$ for $n < 2$ as shown in Fig. 2(c).

The effects of having a small non-zero h_s on the hydraulic properties is illustrated here with one example. Fig. 4 shows measured water retention and hydraulic conductivity data for Beit Netofa clay as obtained by Rawitz [23] and cited in Mualem [17]. The data are described in terms of both the original and modified VGM equations. The data set is well suited for our purposes here because several unsaturated conductivity data points were measured close to saturation. Hydraulic parameters for Eqs. (4) and (7) are given in Table 1. The parameters α and n were optimized using least-squares [28]. The parameter θ_m was adjusted so that the value of h_s in (7) would become equal to −2 cm. Note that, as expected, the original and modified VGM retention curves resulting are visually indistinguishable, whereas differences in the predicted hydraulic conductivity functions are significant and manifested over the entire range of pressure heads (i.e., not only near saturation). From Figs. 4(a) and (b), it is evident that the modified VGM method for this clay soil leads to a much better agreement between the predicted and measured conductivity curves.

4. Numerical simulations

The effects of changes in the shape of the soil hydraulic functions near saturation on simulated pressure

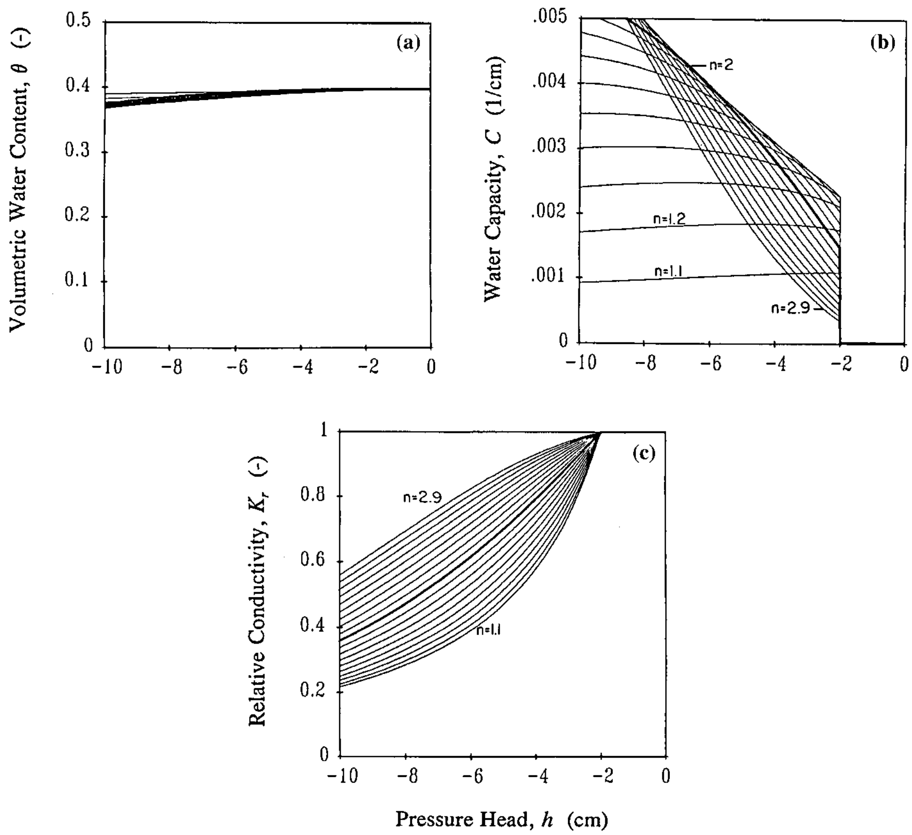

Fig. 3. Plots of the soil hydraulic properties near saturation as a function of the parameter n (modified VGM model with $h_s = -2$ cm).

head profiles were further analyzed by numerically simulating infiltration into an initially unsaturated homogeneous soil column. Results were obtained with the HYDRUS (version 5.0) one-dimensional Galerkin finite element computer program documented by Vogel et al. [31]. The solution invokes mass lumping of the time derivative in the flow equation, and incorporates the mass-conservative numerical scheme of Celia et al. [4]. We considered infiltration in a 100-cm long soil column that initially was assumed to be in equilibrium with an imposed pressure head of −1000 cm at the bottom of the column. Calculations were performed using nodal spacing of 0.5 and 1.0 cm, depending upon the non-linearity of the simulated problem, and an adaptive time stepping algorithm as described by Vogel et al. [31]. Three combinations of boundary conditions were tested: (1) a zero pressure head at the bottom of the column combined with a zero flux at the top of the column, leading to upward infiltration against gravity; (2) a zero pressure head at the top of the column combined with a zero flux at the bottom, leading to downward infiltration, and (3) constant flux infiltration at the top of the soil profile, combined with a zero flux at the bottom. Calculations were carried out for many soil types using n values between 1 and 1.5. Additional simulations were carried out for upward infiltration using the soil hydraulic data listed in Table 2, as well as by varying n while keeping the other hydraulic parameters constant. The data in Table 2 were obtained by Carsel and Parrish [3] by analyzing a large database of observed soil water retention data. We believe that the entries in Table 2 provide reasonable estimates of the VGM hydraulic parameters for different soil textural groups. As an illustrative example we present here detailed results only for the clay soil (Soil no. 12) whose n-value is 1.09. The hydraulic properties of this soil are displayed in Fig. 5 in terms of the original VGM formulation with $h_s = 0$, and the modified VGM model with $h_s = -2$ cm. Note again the insignificant differences between the two $\theta(h)$ curves, and the large differences between the $K_r(h)$ curves over the entire range of h-values.

The simulation results for upward and downward infiltration are presented in Figs. 6 and 7, respectively, while those for constant flux infiltration assuming a surface water flux, q_0 of $K_s/2$ are given in Fig. 8. The figures compare results obtained with the original VGM formulation with a zero minimum capillary height ($h_s = 0$), with calculations obtained with the modified VGM model assuming $h_s = -2$ cm. Figs. 6 and 7 show significant differences in the position of wetting front between the original and modified hydraulic models.

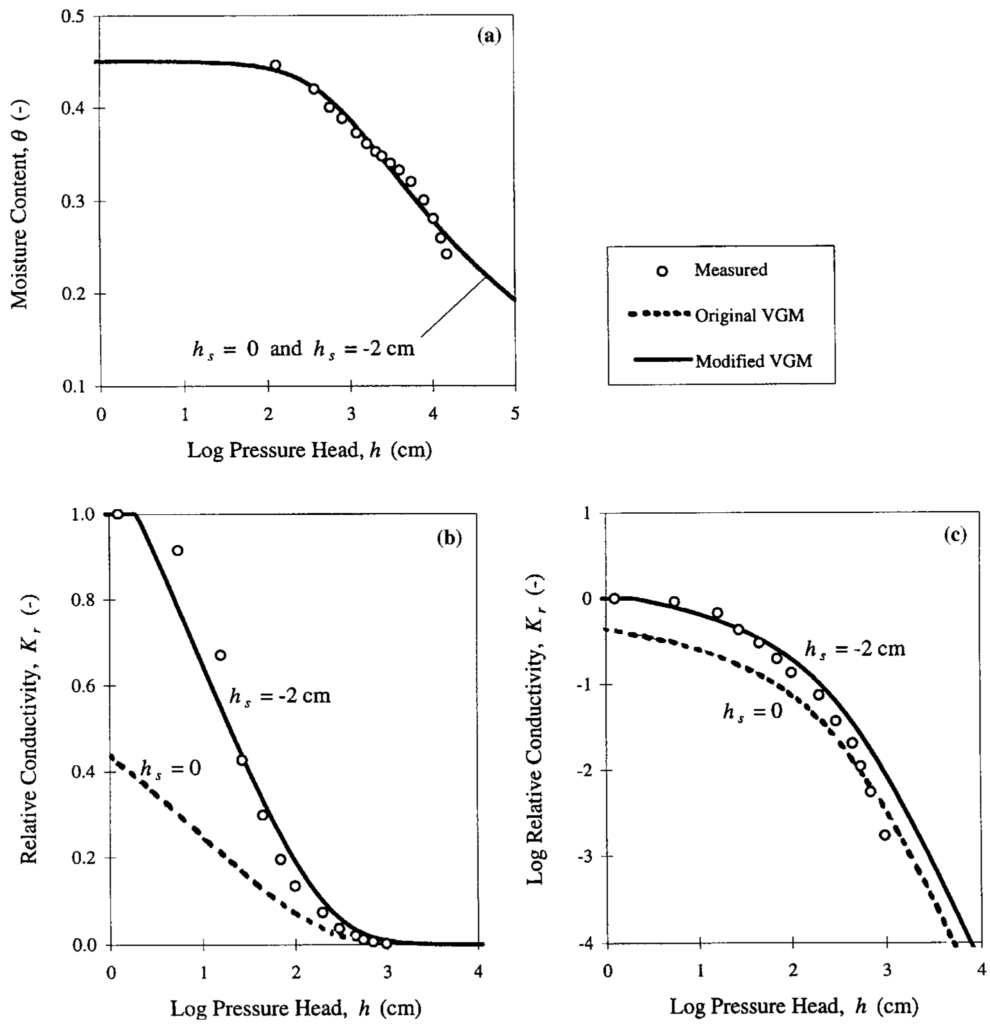

Fig. 4. Comparison of measured soil hydraulic data for Beit Netofa clay with calculated curves using: (i) the original VGM equations assuming a zero minimum capillary height, $h_s = 0$; (ii) the modified VGM with $h_s = -2$ cm.

Table 1
Estimated hydraulic parameters for Beit Netofa clay

θ_r	θ_s	θ_m	α	n	Method
0.010	0.450	–	0.00173	1.170	Original VGM
0.010	0.450	0.45008	0.00170	1.172	Modified VGM

The calculated infiltration rates are also quite different. These differences are consistent with the different shapes of the original and modified VGM hydraulic conductivity functions in Fig. 5.

Fig. 7(c) shows severe oscillations in the calculated infiltration rate for the $h_s = 0$ case when downward infiltration with a constant pressure head boundary condition was simulated. Some oscillations also developed in the calculated pressure heads for the constant flux boundary condition simulation (Fig. 8). Numerical instabilities associated with these oscillations are a direct consequence of the non-linear shape of the hydraulic conductivity function near saturation. For example, as the infiltration process shown in Fig. 7 for $h_s = 0$ proceeds and the top part of the soil column approaches saturation, the pressure head goes to zero and the surface flux slowly approaches the value of K_s. At this time the numerical scheme becomes affected by the extremely non-linear shape of the conductivity function near saturation (Fig. 5), and starts to produce oscillations. When $h_s = -2$ cm, on the other hand, the conductivity function decreases much more gradually and no oscillations occur. When downward infiltration with a constant flux boundary condition is simulated (Fig. 8), the

Table 2
Average values of VGM soil hydraulic parameters for 12 major soil textural groups according to Carsel and Parrish, 1988 [3]

Texture	θ_r (m^3/m^3)	θ_s (m^3/m^3)	α (1/cm)	n	K_s (cm/day)
(1) Sand	0.045	0.430	0.145	2.68	712.80
(2) Loamy sand	0.057	0.410	0.124	2.28	350.20
(3) Sandy loam	0.065	0.410	0.075	1.89	106.10
(4) Loam	0.078	0.430	0.036	1.56	24.96
(5) Silt	0.034	0.460	0.016	1.37	6.00
(6) Silt loam	0.067	0.450	0.020	1.41	10.80
(7) Sandy clay loam	0.100	0.390	0.059	1.48	31.44
(8) Clay loam	0.095	0.410	0.019	1.31	6.24
(9) Silty clay loam	0.089	0.430	0.010	1.23	1.68
(10) Sandy clay	0.100	0.380	0.027	1.23	2.88
(11) Silty clay	0.070	0.360	0.005	1.09	0.48
(12) Clay	0.068	0.380	0.008	1.09	4.80

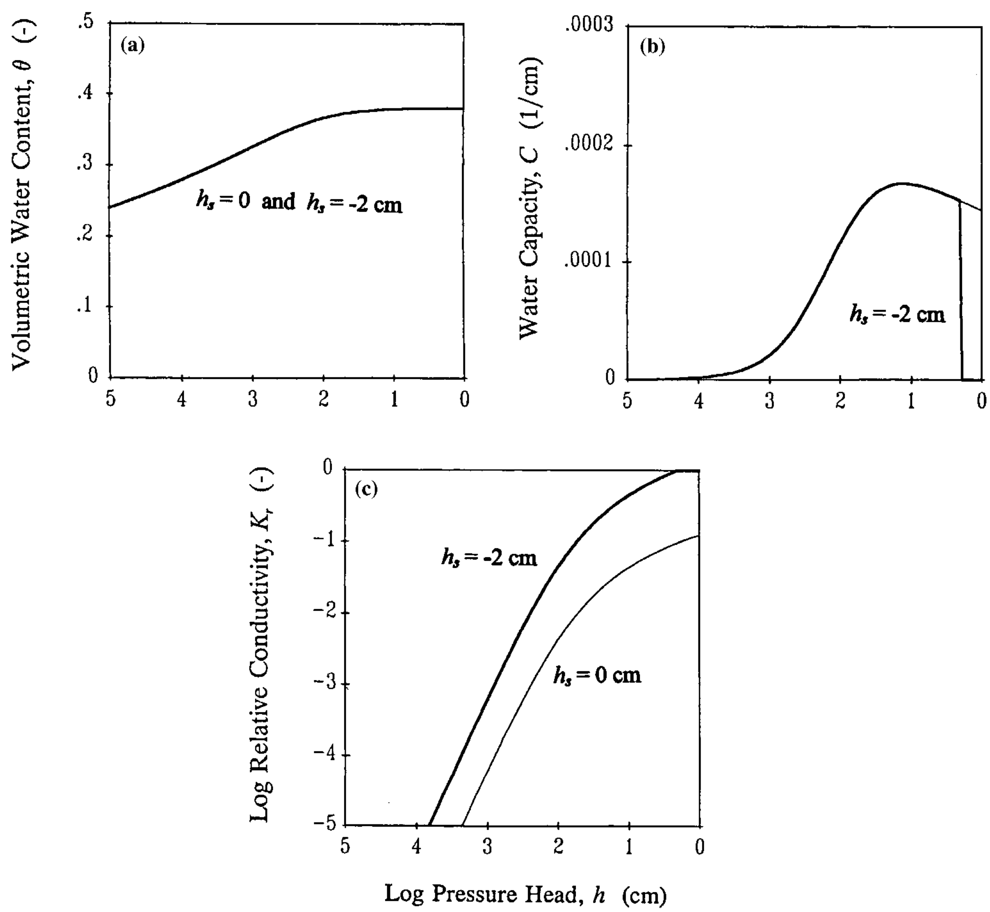

Fig. 5. Soil hydraulic properties of a clay soil (Soil no. 12 in Table 2) assuming $h_s = 0$ and $h_s = -2$ cm.

pressure head gradient again goes to zero as the top part of the column fills with water and the water flux becomes equal to the imposed surface flux, $q_0 = K_s/2$. The steep part of the conductivity function close to $K_s/2$ now again causes small pressure head oscillations (Fig. 8(c)). By contrast, no oscillations occur for upward infiltration (Fig. 6); this because the pressure head gradient in the more saturated bottom part of the column goes to +1 instead of zero. The upward water flux rate through the bottom of the column decreases quickly to values outside of the most non-linear portion of the $K_r(h)$ curve in Fig. 5, leading to numerically more stable simulations. The pressure head distributions in Fig. 6 eventually would reach equilibrium.

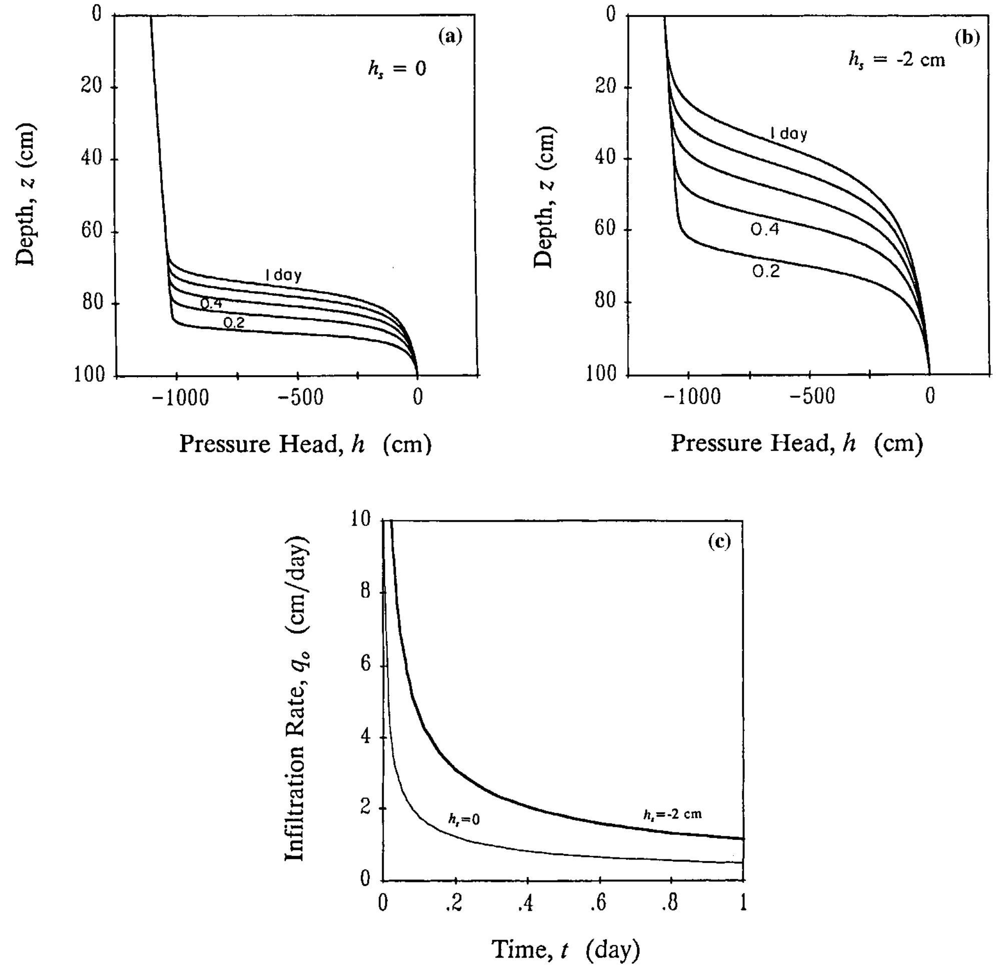

Fig. 6. Calculated upward infiltration into a clay soil column (Soil no. 12) subject to a zero pressure head at the lower boundary.

To assess the relative importance of the effects of h_s on water flow in different soils, we next carried out a series of calculations for upward infiltration assuming for all cases minimum capillary heights, h_s, of both 0 and −2 cm. The total volume of infiltrated water after one day of infiltration (denoted by I and I^* for the original and modified VGM models) were calculated for all soil types listed in Table 2. Fig. 9 gives a plot of the relative change, $(I^* - I)/I$, in the infiltrated amount of water as a function of the parameter n. Each pair of simulations resulted in an asterisk in Fig. 9 (one for each soil in Table 2). Relative changes in infiltration were also calculated by fixing the parameters $\alpha = 0.01$ cm, $\theta_s = 0.40$, $\theta_r = 0.10$, $K_s = 10\,\text{cm day}^{-1}$, but letting the parameter n vary between 1.0 and 3.0. The entire sequence was repeated once more with $\alpha = 0.1\,\text{cm}^{-1}$. The results in Fig. 9 indicate that a hyperbolic type function accurately bounds the most sensitive areas of the numerical algorithm. The figure shows that differences in the amount of simulated infiltration resulting from the original and modified VGM models can reach more than one hundred percent for soils having n values close to 1.

5. Discussion and conclusions

In this study, we showed that extremely small changes in the shape of the soil water retention curve near saturation can lead to significantly different curves for the unsaturated hydraulic conductivity function when incorporated into the predictive $K_r(h)$ model of Mualem [16]. The modified conductivity curves can lead to remarkably different simulation results for fine-textured soils. These findings were shown to hold for both upward and downward ponded infiltration, and downward constant-flux infiltration. The differences are a consequence of the extreme sensitivity of Mualem's model to small changes in $\theta(h)$ close to saturation. The sensitivity is relatively low for $n > 2$, but increases dramatically when n approaches 1.0, i.e., for most fine-textured and/or certain undisturbed soils. The sensitivity also depends to some extent on the value of scaling parameter α (Fig. 9).

We found that the non-linear nature of the original VGM conductivity function with $h_s = 0$ can cause serious numerical convergence problems for n values

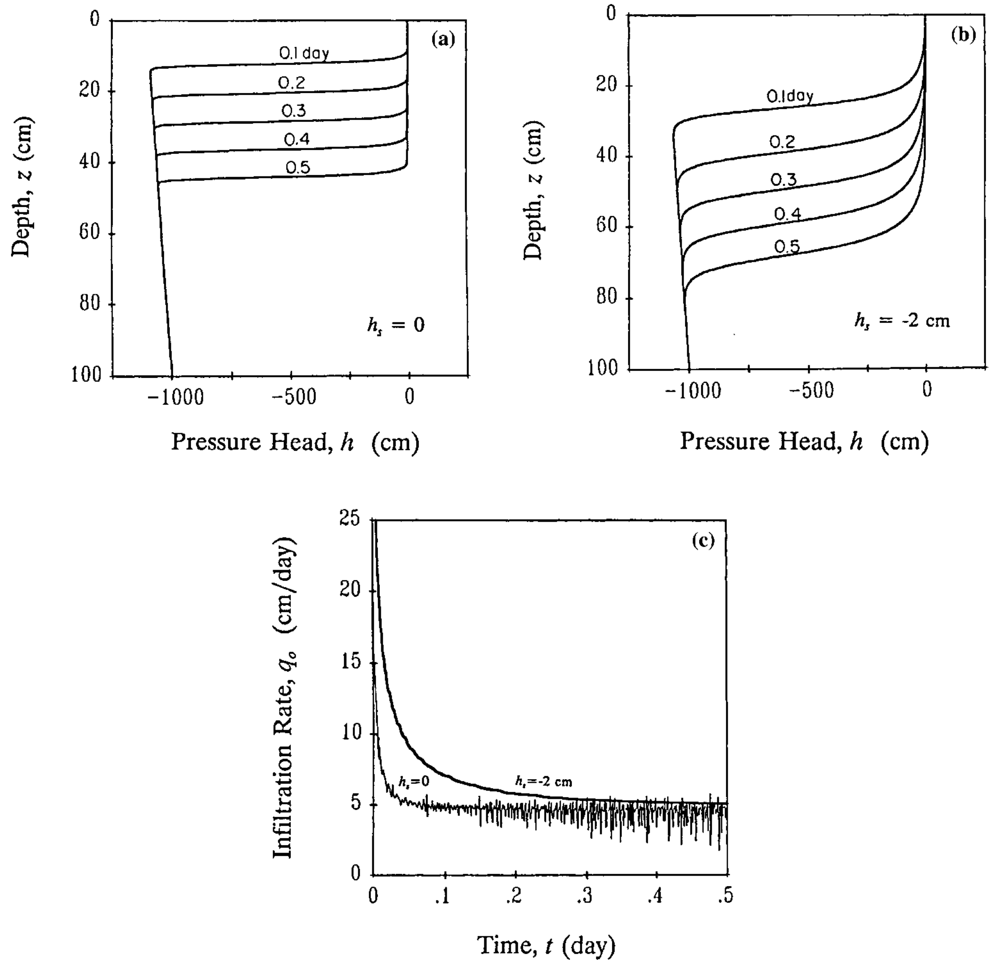

Fig. 7. Calculated downward infiltration into a clay soil (Soil no. 12) subject to ponded infiltration at the surface.

close to 1. Even if convergence is achieved, numerical oscillations may develop in the infiltration rate for a constant pressure head boundary condition, and in the calculated pressure head when a constant flux boundary condition is imposed. The convergence and oscillation problems were effectively removed by using the modified VGM model with a small non-zero minimum capillary height, h_s.

This study indicates an urgent need for very precise measurements of the unsaturated hydraulic conductivity function close to saturation for fine-textured soils (i.e., for n values less than about 1.3). Theoretical conductivity models, such as the Brooks–Corey and VGM models, are increasingly used in numerical models predicting subsurface water flow and chemical transport. While the equations have been validated against observed data for a large number of soils, most or all of these have been medium-or coarse-textured soils [11,16,18,19,28]. The soils in many cases were also repacked in the laboratory. In contrast, there is a lack of validation for fine-textured soils, mostly because of a conspicuous absence of reliable experimental data in the literature. This is unfortunate since, as shown in this paper, the greatest sensitivities of the predictive theories to changes in the retention curve occur for such fine-textured soils. Current predictions of the hydraulic conductivity function using the original VGM model may well be unreliable for n values close to 1.0. One example soil, Beit Netofa clay, was used in this paper to illustrate the type of differences that can be expected when the original VGM is modified to include a minimum capillary height, h_s. For this soil we used a value of −2 cm for h_s. Preliminary analyses of the UNSODA unsaturated soil hydraulic property database [11,19] involving some 235 combined retention and unsaturated hydraulic conductivity data sets showed that: (1) h_s values between 0 (including zero) and −5 cm produce essentially the same $\theta(h)$ function; (2) the use of any $h_s < 0$ (i.e., excluding zero) has a very significant effect on the calculated $K(h)$ function as compared to the original MVG model; (3) the effect of changing h_s on the $K(h)$ curve of a particular soil is about the same over the range $-10 < h_s < -1$ cm.

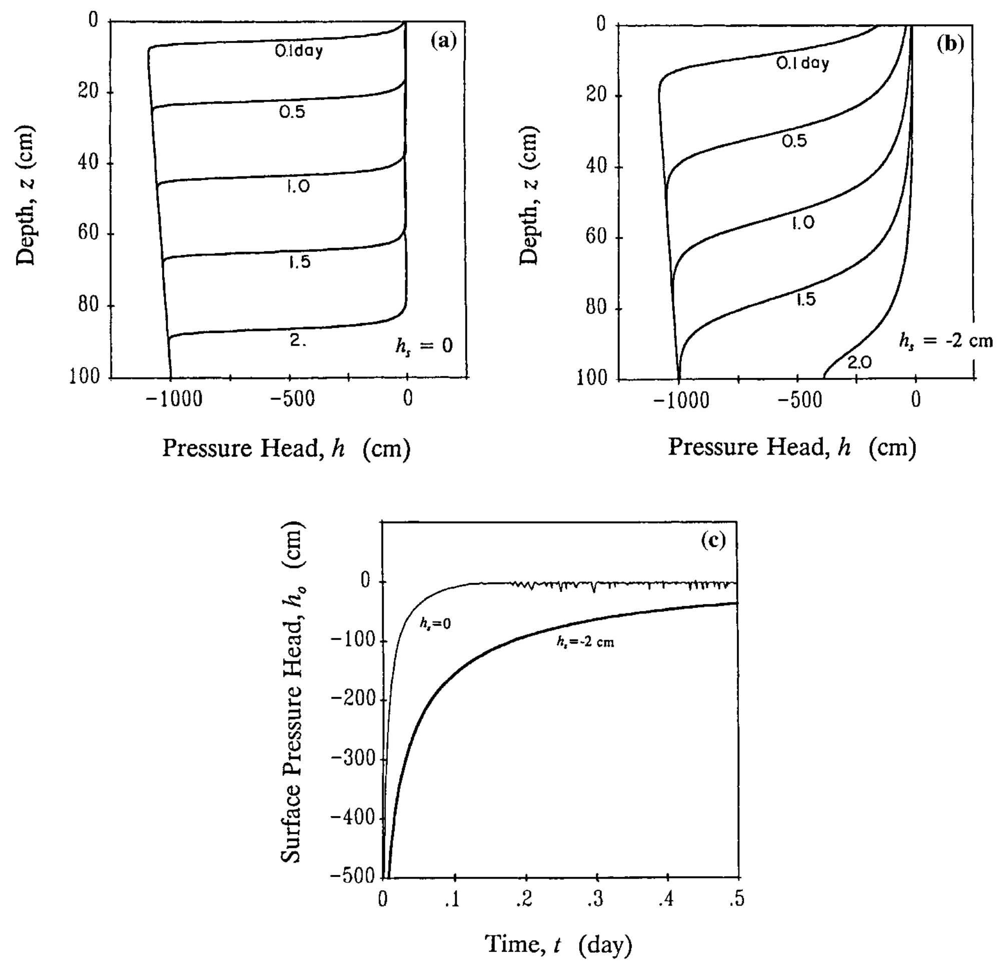

Fig. 8. Calculated downward infiltration into a clay soil (Soil no. 12) subject to flux-controlled infiltration at the soil surface (constant flux, $q = K_s/2$).

We emphasize here that our study focused on unstructured, mostly non-swelling soils. Additional complications arise for structured (macroporous or fractured) media associated with the presence of decayed root channels, earthworm channels, natural inter-aggregate pores, rock fractures, and/or drying cracks. Alternative formulations may be needed for such structured media, possibly using sums of several VGM type or other models [13,15,21,22,32]. Like the introduction of slightly negative h_s value, bimodal or multi-model functions lead to significant changes in $K(h)$ near saturation, but generally at slightly lower pressure heads (e.g., −3 to−10 cm, or lower, depending upon the type of macropores of fractures present). Conceptually, the reasons for such changes are of course completely different for structured as compared to fine-textured porous media.

Based on this study, as well as ongoing work, we recommend to always use the modified VGM model with a small value (e.g., −2 cm) for h_s when no precisely measured unsaturated hydraulic conductivity data are available and the parameter n (as estimated from retention data only) is less than about 1.3. When an optimization algorithm is used to determine the VGM-model parameters (either by fitting retention data or by solving the inverse problem by fitting the hydraulic parameters to measured time–space variations in pressure heads, water contents and/or fluxes), we recommend to use an additional optimized parameter θ_m (Eq. (7)) which during the optimization should remain slightly larger than θ_s (e.g. $\theta_m \approx \theta_s + 0.0001$). Alternatively, the following approximation may be used for θ_m:

$$\theta_m = \theta_r + (\theta_s - \theta_r)[1 + (2\alpha)^n]^m, \tag{12}$$

which follows from Eq. (7) with $h_s = -2$ cm. Our recommendation for θ_m is based in part on conceptual issues (including the Beit Netofa Clay example), and in part on more pragmatic considerations to remove the extreme non-linearity in the predicted $K(h)$ function, and hence to minimize numerical solution problems.

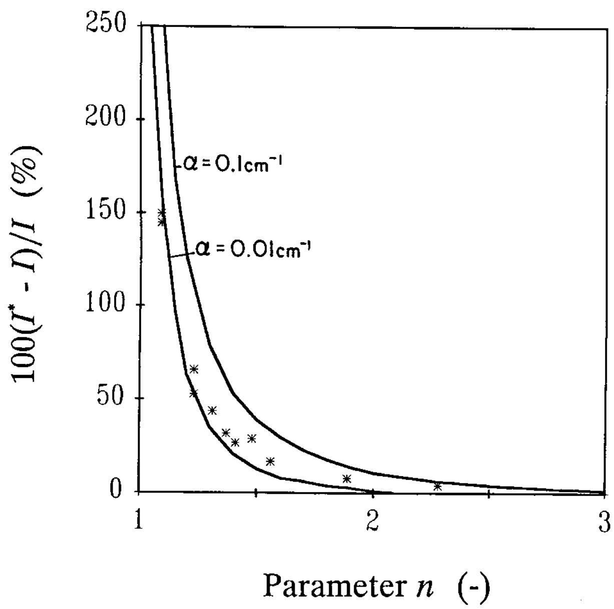

Fig. 9. Calculated relative increase in the amount of infiltrated water when using a non-zero capillary height, h_s, of −2 cm as compared to $h_s = 0$ (the asterisks represent results for the soils listed in Table 2; solid lines were obtained using two values of α in the VGM model and varying the value of n between 1 and 3).

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