Scaling Parameter to Predict the Soil Water Characteristic from Particle-Size Distribution Data

Lalit M. Arya,* Feike J. Leij, Martinus Th. van Genuchten, and Peter J. Shouse

ABSTRACT

The Arya-Paris model is an indirect method to estimate the soil water characteristic from particle-size data. The scaling parameter, α, in the original model was assumed constant for all soil textures. In this study, α is defined as $\alpha_i = (\log N_i/\log n_i)$, where n_i is the number of spherical particles in the ith particle-size fraction (determined by the fraction solid mass, w_i, and mean particle radius, R_i) and N_i is the number of spherical particles of radius R_i required to trace the pore length generated by the same solid mass in a natural structure soil matrix. An estimate for $\log N_i$ was obtained by either relating $\log N_i$ to $\log n_i$ using a logistic growth equation or by relating $\log N_i$ linearly to $\log (w_i/R_i^3)$ based on the similarity principle. For any given texture, both approaches showed that α was not constant but decreased with increasing particle size, especially for the coarse fractions. In addition, α was also calculated as a single-value average for a given textural class. The three formulations of α were evaluated on 23 soils that represented a range in particle-size distribution, bulk density, and organic matter content. The average α consistently predicted higher pressure heads in the wet range and lower pressure heads in the dry range. The formulation based on the similarity principle resulted in bias similar to that of the constant α approach, whereas no bias was observed for the logistic growth equation. The logistic growth equation implicitly accounted for bias in experimental procedures, because it was fitted to $\log N_i$ values computed from experimental soil water characteristic data. The formulation based on the similarity principle is independent of bias that might be inherent in experimental data.

THERE IS AMPLE JUSTIFICATION for indirect methods of estimating soil hydraulic properties from routinely available taxonomic data (e.g., Bouma and van Lanen, 1987; van Genuchten and Leij, 1992). The effects of texture, bulk density, and organic matter on soil water retention and hydraulic conductivity have long been recognized. However, an explicit formulation of the relationship between texture and hydraulic properties of the soil remains a challenge because of the very complex pore-particle geometry. Hence, empirical approaches to predicting hydraulic properties at specific points of the water content–pressure–hydraulic conductivity curves from texture, bulk density, mineralogy, and organic matter content by using multiple regression techniques or neural network analyses remain popular (e.g., Rajkai and Varallyay, 1992; Tietje and Hennings, 1995). Mathematical representations of the water content–pressure–hydraulic conductivity curves as a continuous function (Brooks and Corey, 1964; Mualem, 1976; and van Genuchten, 1980) require one or more fitting parameters, which are normally evaluated from the basic soil properties through regression (Kool et al., 1987; Rajkai et al., 1996) or neural network techniques (Schaap and Bouten, 1996). Because the soil water retention curve is essentially a pore-size distribution curve, it is required as the primary input in models of the hydraulic conductivity based on pore-size distribution (e.g., Marshall, 1958; Millington and Quirk, 1961; Mualem, 1976). Thus, accurate water retention curves are of great importance.

The first attempt to directly translate the particle-size distribution into a soil water characteristic was made by Arya and Paris (1981). The basis for their model is a close similarity between the shapes of the particle-size distribution and the water retention curve. In the model, the pore size that is associated with a pore volume is determined by scaling the pore length. Since particle size is normally expressed in terms of equivalent spheres, Arya and Paris (1981) estimated pore lengths for the various fractions of the particle-size distribution curve by summing the diameters of spherical particles in the fraction. Pore lengths based on spherical particles were scaled to natural pore lengths using a scaling parameter, α, with an average value of 1.38.

A similar model was later proposed by Haverkamp and Parlange (1986) and tested on a coarse-textured sand. Tyler and Wheatcraft (1989) interpreted α as being the fractal dimension of a tortuous pore. Since then, there has been a growing interest in the use of fractals to predict hydraulic properties from particle-size distributions (e.g., Rieu and Sposito, 1991; Tyler and Wheatcraft, 1992; Shepard, 1993). However, it should be noted that fractal scaling, inasmuch as it is concerned with the nature of fragmentation, accounts only for the effects of tortuosity of pore lengths, but not for other factors that influence water retention, such as packing density, chemical characteristics of solid surfaces, organic matter content, fluid properties, and air entrapment.

Later investigations by Arya et al. (1982) showed that the average α varied among textural classes and ranged in value from 1.1 for finer textures to 2.5 for coarse-textured materials. A similar range of values was reported by Tyler and Wheatcraft (1989) for the fractal dimension. Yoshida et al. (1985) also reported higher values of α for coarse-textured materials. Several researchers (e.g., Schuh et al., 1988; Mishra et al., 1989; Gupta and Ewing 1992; Jonasson, 1992; Basile and D'Urso, 1997; Nimmo, 1997) have suggested that predictions of water retention curves would improve if α were formulated such that it varies over the range of particle sizes.

The single values of α in the original study of Arya and Paris (1981) were obtained by minimizing the sum of squares of deviations between the measured and calculated pressures. The objective of the present study is to investigate of relationships between α and the particle-size distribution.

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THEORY

The Arya-Paris Model

In the following we will review pertinent aspects of the model. The particle-size distribution curve is divided into n size fractions, and the solid mass in each fraction is assembled to form a hypothetical, cubic close-packed structure consisting of uniform size spherical particles. The pore volume in each assemblage is calculated from the bulk density and particle density measured on the natural structure soil. Arya and Paris (1981) found $n = 20$ as a reasonable number of fractions, with fraction boundaries at particle diameters of 1, 2, 3, 5, 10, 20, 30, 40, 50, 70, 100, 150, 200, 300, 400, 600, 800, 1000, 1500, and 2000 μm. Starting with the first fraction, calculated pore volumes are progressively summed and considered filled with water. Summations of filled pore volumes are divided by the bulk volume to obtain volumetric water content at the upper bounds of successive mass fractions. An equivalent pore radius is calculated for each fraction and converted to soil water pressure head using the capillary equation. Calculated pressure heads are sequentially paired with calculated water contents to obtain a soil water characteristic curve. The matrix of the medium formed in this manner is termed the *ideal matrix* to indicate that the geometry of this matrix is well defined. Properties of the ideal matrix can be used as a basis to characterize deviations in the desorption behavior of more complex structures.

Basic Relationships

The pore volume, V_{pi} (cm^3 g^{-1}), associated with the solid mass in the ith particle-size fraction, is represented as a single cylindrical capillary tube, and is given by

$$V_{pi} = \left(\frac{w_i}{\rho_s}\right) e = \pi r_i^2 l_i \qquad [1]$$

where w_i is the fraction solid mass (g g^{-1}), ρ_s is the particle density (g cm^{-3}), e is the void ratio, r_i is the pore radius (cm), and l_i is the pore length (cm g^{-1}). The void ratio, e, is given by

$$e = (\rho_s - \rho_b)/\rho_b \qquad [2]$$

where ρ_b is the bulk density of the natural soil (g cm^{-3}). The water content, θ_i (cm^3 cm^{-3}), is obtained from successive summations of water-filled pore volumes according to

$$\theta_i = (\phi\ S_w)\sum_{j=1}^{j=i} w_j; \qquad i = 1,2...n \qquad [3]$$

where ϕ is the total porosity (cm^3 cm^{-3}), and S_w is the ratio of measured saturated water content to theoretical porosity. The number of spherical particles, n_i (g^{-1}), for each fraction of the particle-size distribution is calculated from

$$n_i = 3w_i/(4\pi\ \rho_s\ R_i^3) \qquad [4]$$

where R_i is the mean particle radius (cm) for the ith particle-size fraction. For an ideal soil consisting of uniform size spherical particles in a cubic close-packed assemblage, l_i can be estimated by $l_i = 2n_i\ R_i$, and the pore radius is related to the particle radius by

$$r_i = 0.816\ R_i\sqrt{e} \qquad [5]$$

For a natural soil, made up of the same solid mass but with nonspherical particles that are arranged randomly, l_i can be estimated by $l_i = 2n_i^{\alpha}R_i$, and the corresponding pore radius is related to the particle radius by

$$r_i = 0.816\ R_i\sqrt{e n_i^{(1-\alpha i)}} \qquad [6]$$

where α is the scaling parameter. Note that $n_i^{(1-\alpha)}$ is dimensionless. Calculated pore radii, r_i, are converted to equivalent pressure heads, $|h_i|$ (cm water), using the capillary equation:

$$h_i = \frac{2\gamma \cos \Theta}{\rho_w\ g\ r_i} \qquad [7]$$

where γ is the surface tension at the air–water interface (g s^{-2}), Θ is the contact angle, ρ_w is the density of water (g cm^{-3}), and g is the acceleration due to gravity (cm s^{-2}). The model assumes perfect wettability and, hence, $\Theta = 0°$. A complete list of variables used is given in Table 1.

Scaling of the Soil Water Characteristic

Scaling of the soil water characteristic from the ideal to the natural soil is schematically shown in Fig. 1. The curve for the ideal soil was calculated from the particle-size distribution data using Eq. [1] through [5] and Eq. [7]. The curve for the natural soil was drawn through the experimental data. Data used in Fig. 1 are from the loam soil 4101 in the UNSODA database (Leij et al., 1996). Scaling involves moving the soil water characteristic curve for the ideal soil along either the θ or h axis such that it closely matches the experimental curve for the natural soil. It requires mention that this scaling is not to be confused with the concept of geometric similitude of Miller and Miller (1956).

In this study, we scaled the pressure head, h. For any water content, θ_i, the pressure heads for the ideal and natural soils are different. They are denoted as h_i (ideal) and h_i (natural) in Fig. 1. In order to match h_i (ideal) to h_i (natural), the pore radii calculated from the capillary equation (Eq. [7]) must be scaled to equivalent pressure heads for the natural soil. Since volumetric water contents are obtained by progressive summation of fractional pore volumes, scaling can be accomplished by pairing each calculated fractional pore volume with the equivalent pore radius on the experimental soil water characteristic curve. In this study, the calculated effective pore volume was matched to experimental saturated water content, and the experimental saturated water content was assumed to correspond to $|h| = 10^{-2}$ cm water.

Table 1. Variables used in the equations.

Variable	Description	Dimension†
e	void ratio, defined as $(\rho_s - \rho_b)/\rho_b$	$L_p^3\ L_s^{-3}$
g	acceleration due to gravity	$L\ T^{-2}$
h_i	pressure head, ith fraction	L_w
l_i	pore length, ith fraction	$L_p\ M_s^{-1}$
n_i	number of spherical particles, ith fraction, obtained from the particle-size distribution data	M_s^{-1}
N_i	scaled number of spherical particles, ith fraction, to trace the pore length in the corresponding natural soil	M_s^{-1}
r_i	pore radius, ith fraction	L_p
R_i	particle radius, ith fraction	L_s
S_w	ratio of experimental saturated water content to porosity	
V_{pi}	pore volume, ith fraction	$L_p^3\ M_s^{-1}$
w_i	solid mass, ith fraction	$M_s\ M_s^{-1}$
α	scaling parameter	
γ	surface tension (air-water)	$M\ T^{-2}$
θ_i	volumetric water content, ith fraction	$L_w^3\ L_b^{-3}$
Θ	contact angle, degrees	
ρ_s	particle density	$M_s\ L_s^{-3}$
ρ_b	bulk density	$M_s\ L_b^{-3}$
ρ_w	density of water	$M_w\ L_w^{-3}$
ϕ	porosity	$L_p^3\ L_b^{-3}$

† For dimensional analysis, L = length, M = mass, and T = time, with subscripts b for bulk, p for pore, s for solid, and w for water.

The assumption that a pore volume can be represented by a single cylindrical capillary tube makes the scaling convenient. One needs to know the pore volume and pore length to calculate the pore radius. Pore volume can be easily calculated from the particle-size distribution data using Eq. [1]. However, pore lengths can only be approximated. Arya and Paris (1981) used the diameter of the spherical particle as a measuring unit of length. Equations [5] and [6] result from the argument that a pore length can be estimated by the number of particles that lie along the pore path times the length contributed by each particle; i.e., the particle diameter. Thus, for an ideal soil, the pore length is estimated by $2n_i R_i$. However, for the corresponding natural structure soil, we assume that n_i^{α} spherical particles of radius R_i are needed to trace the entire pore length; i.e., the pore length is approximated by $2n_i^{\alpha}R_i$.

The parameter α needs to be estimated in order to scale pore length in an ideal soil to that of a corresponding natural soil. In an ideal soil, the pore length is equal to the sum of physical lengths that are contributed by particle diameters arranged in straight columns. In contrast, particles in a natural soil may contribute to pore lengths in more than one dimension. In addition, the pressure head depends not only on the pore size but is also affected by organic matter, solutes, and electrochemical properties of the solid surfaces. Therefore, α needs to be estimated from experimental soil water characteristic data; it should be considered more of a fitting parameter than a geometrical constant.

MATERIALS AND METHODS

Experimental soil water characteristic, particle-size distribution, bulk density, and particle density data were obtained from the UNSODA hydraulic property database (Leij et al., 1996). Forty-seven data sets, representing a range of textures that include sand, sandy loam, loam, silt loam, and clay, were selected for this study. For each textural class, four to five soils were used to develop a formulation for α and another four to five soils were used for independent testing of the formulations. All soils are identified in Table 2. The particle-size distribution curves for the selected soils were divided into n fractions according to the method used by Arya and Paris (1981), and all parameters required in Eq. [1] through Eq. [6] were calculated.

Table 2. Textural classes and UNSODA codes for soils used for formulating and testing the scaling parameter α.

Textural class	UNSODA codes	Use
Sand	1050, 1460, 2100, 3132, 3340, 4650	Formulating α
	1464, 4000, 4521, 4661	Testing
Sandy loam	1130, 1131, 1381, 3310, 4160	Formulating α
	0000†, 1112, 1121, 1380, 1450, 4162	Testing
Loam	1370, 2531, 4610	Formulating α
	2530, 3303, 4101, 4600	Testing
Silt loam	1341, 4081, 4510, 4531, 4670	Formulating α
	2000, 3260, 4510, 4531, 4673	Testing
Clay	1400, 2361, 3282, 4121, 4681	Formulating α
	2360, 3281, 4120, 4680	Testing

† This soil is not in UNSODA.

Estimating Pore Length in Natural Structure Soil

The relationship between the number of spherical particles in the ideal soil, n_i (g^{-1}), and the number of spherical particles required to trace the pore length in the corresponding natural structure soil, N_i (g^{-1}), is given by

$$n_i^{\alpha i} = N_i \quad \text{or} \quad \alpha_i = \log N_i / \log n_i \qquad [8]$$

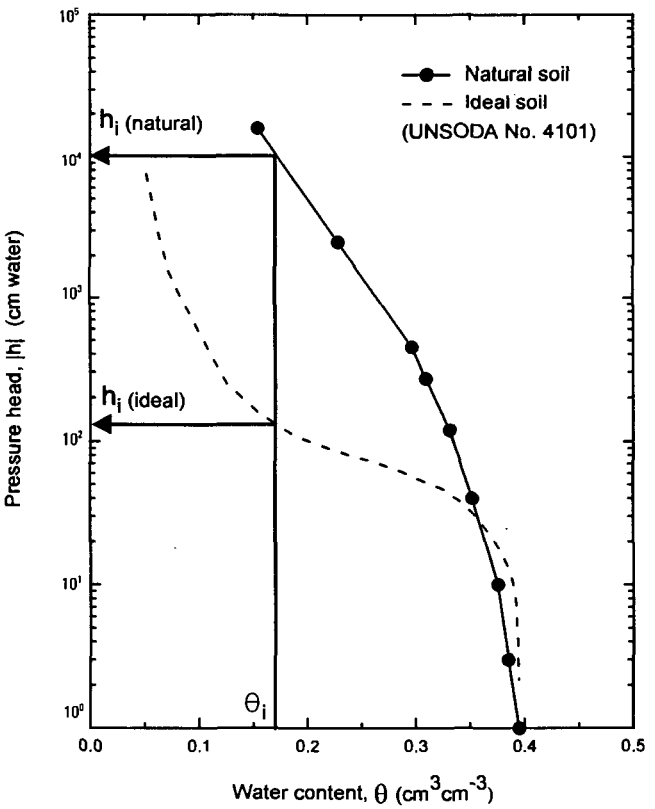

Fig. 1. Schematic representation of the relationship between water characteristics of an ideal soil and the corresponding natural structure soil.

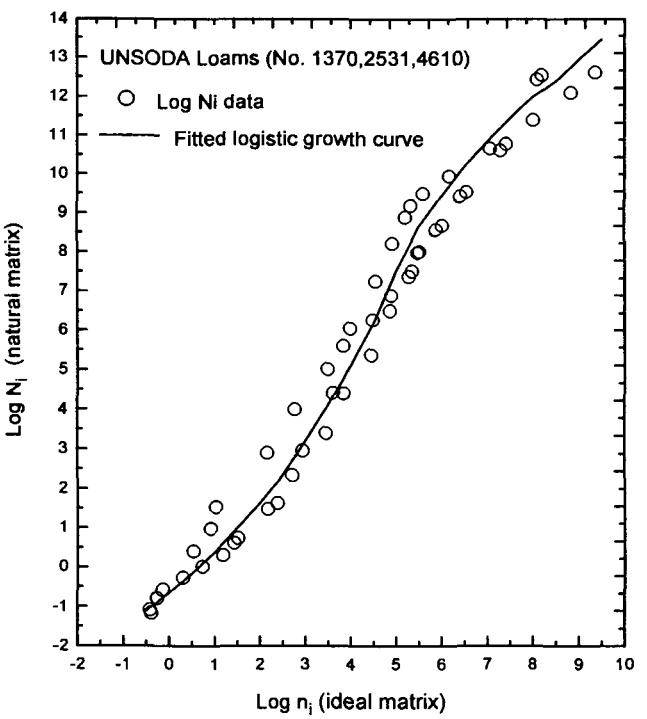

Fig. 2. Relationship between the number of spherical particles required to trace the pore length in a natural structure soil (log N_i) and the number of spherical particles forming the corresponding ideal soil (log n_i) for loam soils (from Table 2).

Table 3. Fitted values for (log N_i)$_i$, (log N_i)$_f$, μ, Δ log N_i, and Δ log n_i and goodness of fit, r^2, for five textural classes.

Textural class	(log N_i)$_i$	(log N_i)$_f$	μ	Δ log N_i	Δ log n_i	r^2
Sand	0.996	16.602	0.609	1.734	0.00032	0.880
Sandy loam	0.559	16.983	0.553	2.492	1.849	0.966
Loam	0.628	16.614	0.510	2.242	1.977	0.980
Silt loam	0.719	19.686	0.457	1.902	0.684	0.940
Clay	1.993	21.685	0.289	4.766	2.648	0.960

Values of n_i were calculated from the particle-size distribution data using Eq. [4]. To evaluate N_i, calculated water contents for the ideal soil were paired with pressure heads on the experimental soil water characteristic curve (see Fig. 1). These pressure heads were converted to pore radii using Eq. [7], and the corresponding pore lengths were calculated using Eq. [1]. The calculated pore lengths were divided by $2R_i$ to obtain N_i. These steps can be captured in the following expression for N_i.

$$N_i = 7.371\ \mathrm{w}_i\ e\ h_{\mathrm{m}i}^2/\rho_s\ R_i \qquad [9]$$

where $h_{\mathrm{m}i}$ is the measured pressure head. The number 7.371 represents a composite of the constants in Eq. [1] and Eq. [7] and has the unit of cm^{-4}.

Normally no retention data are available, and α or N_i must be estimated from the particle-size distribution data. We plotted log N_i vs. log n_i for different textural classes to investigate the behavior of α according to Eq. [8]. Figure 2 shows a somewhat nonlinear relationship between the two variables for loam soils (see Table 2 for UNSODA codes). Similar relationships were obtained for other textures that were examined. In the following we investigate the relationship of log N_i to parameters of the particle-size distribution.

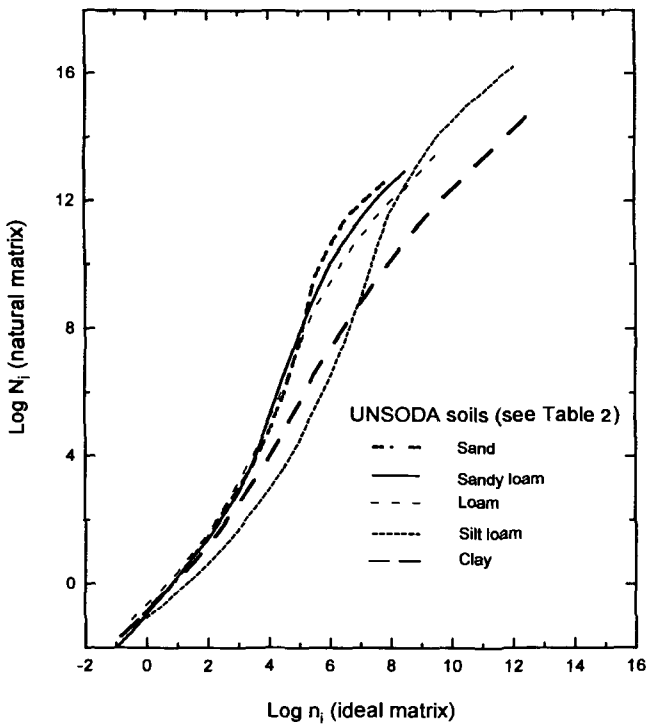

Fig. 3. Smooth curves (fitted logistic growth equation) showing the relationship between the number of spherical particles required to measure the pore length in a natural structure soil (log N_i) and the number of spherical particles forming the corresponding ideal soil (log n_i) for sand, sandy loam, loam, silt loam, and clay soils.

Method 1: Logistic Growth Curve

The relationship between log N_i and log n_i, as depicted in Fig. 2, closely follows the logistic growth equation (Thornley, 1990)

$$Y = \frac{Y_f Y_i}{Y_i + (Y_f - Y_i)\exp(-\mu x)} \qquad [10]$$

where Y represents the dependent variable log N_i, μ is the rate coefficient, and x is the independent variable log n_i. The subscripts i and f signify the initial and final values of log N_i, which correspond to particle-size fractions with the smallest and largest particle numbers, respectively. Equation [10] was fitted to data depicted in Fig. 2, using a least squares method (Marquardt, 1963). For a large number of soils, the initial values of log N_i and log n_i were negative and, therefore, we had to shift the X and Y axes, transforming Eq. [10] to

$$(Y + \Delta Y) = \frac{Y_f Y_i}{Y_i + (Y_f - Y_i)\exp[-\mu(x + \Delta x)]} \qquad [11]$$

The least squares procedure provided the fitted initial and final values of log N_i and μ, as well as those of Δ (log N_i) and Δ (log n_i). These values are summarized in Table 3 along with the coefficient of determination, r^2, between log N_i and log n_i, determined according to Eq. [9] and Eq. [4]. Smooth curves that show the relationship between log N_i and log n_i for all textural classes are presented in Fig. 3. Variables x and Y in Eq. [11] were replaced by log n_i and log N_i, respectively. An explicit expression for α_i was obtained by substituting α_i log n_i for Y (see Eq. [8]) on the left hand side of Eq. [11].

Method 2: Linear Fit between Log N_i and Log (w_i/R_i^3)

According to Eq. [4] log n_i is linearly dependent on log (w_i/R_i^3). Since N_i represents only a scaled-up number for each n_i, log N_i must exhibit a similar linear relationship with log (w_i/R_i^3). An example of the plot of log N_i against log (w_i/R_i^3) for clay soils is presented in Fig. 4. Similar relationships were found for sand, sandy loam, and silt loam textures. A linear relationship between log N_i and log (w_i/R_i^3) implies the existence of two particle-size distributions of similar shape, one represented by n_i and the other by N_i. Shape similarities between particle-size distributions and soil water characteristics support this contention. We, therefore, explored formulations for α based on a linear relationship between log N_i and log (w_i/R_i^3). The generalized form of this relationship, as illustrated in Fig. 4, is

$$\log N_i = a + b \log (\mathrm{w}_i/\mathrm{R}_i^3) \qquad [12]$$

Combining Eq. [12] with Eq. [4] and [8], α can be expressed as

$$\alpha_i = \left[\frac{a + b \log (\mathrm{w}_i/R_i^3)}{\log n_i}\right] \qquad [13]$$

Equation [13] represents an explicit formulation for α in terms of particle-size distribution parameters. Parameters of regression for Eq. [12] for five soil textures are summarized in Table 4.

Method 3: Constant α

For each soil texture, an average value of α was estimated by fitting a linear regression with zero intercept to the plots of log N_i vs. log n_i data. The slope of the line should represent an average α. This approach is essentially the same as that used by Arya and Paris (1981). Average α values estimated

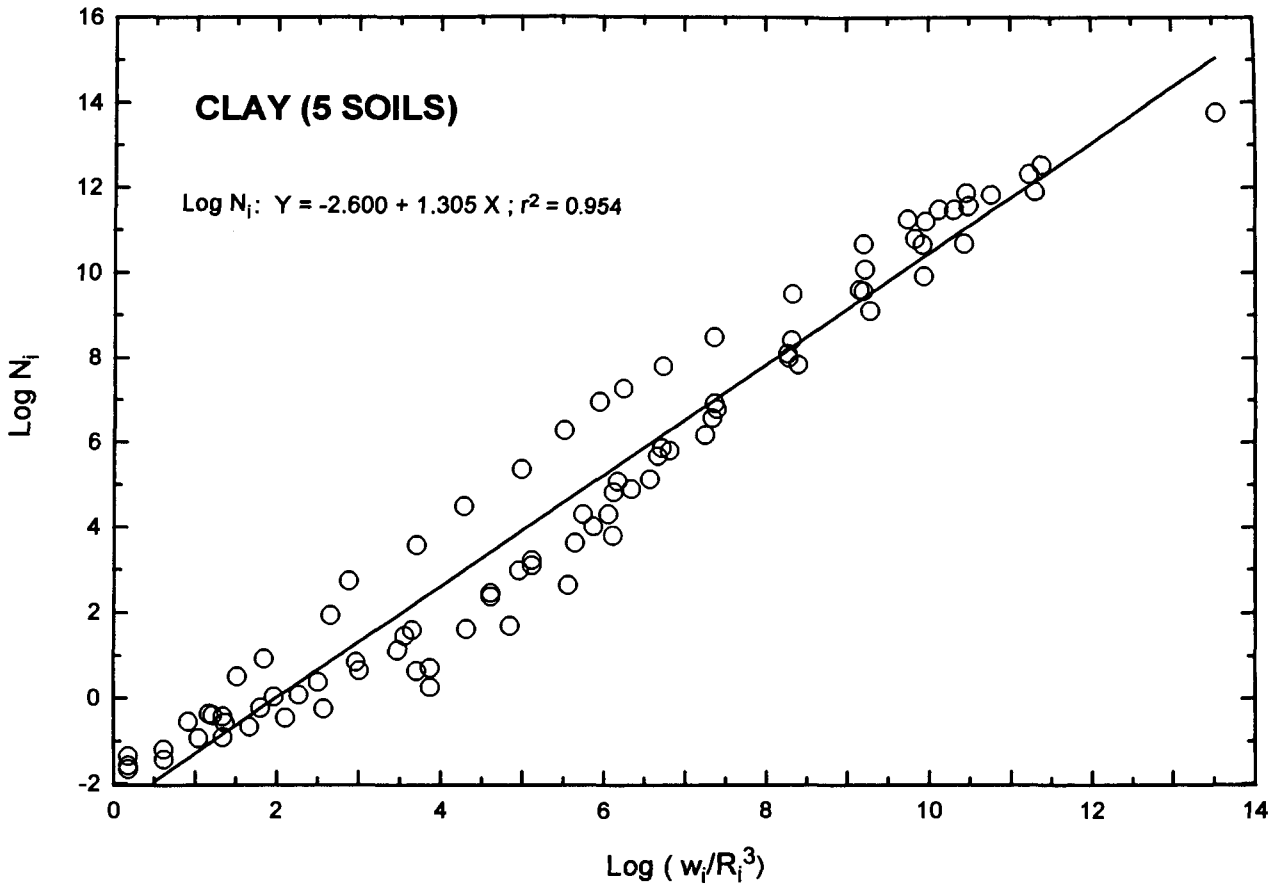

Fig. 4. Relationship between the number of spherical particles accounting for pore lengths in natural structure soils (log N_i) and the fraction solid mass (w_i) and particle size (R_i) for clay soils.

in this manner were 1.285, 1.459, 1.375, 1.150, and 1.160 for the sand, sandy loam, loam, silt loam, and clay textures, respectively.

Model Tests

The three formulations for α, described above, were tested independently on four to five soils for each textural class. These soils are also identified in Table 2. Calculated values of α were substituted in Eq. [6] to calculate pore radius, r_i, for each particle-size fraction. Combining Eq. [6] with Eq. [7] yields

$$h_i = 0.18/R_i\sqrt{en_i^{(1-\alpha_i)}} \qquad [14]$$

Equation [14] expresses h_i explicitly in terms of α_i, n_i, R_i, and e. The number 0.18 represents a composite of constants in Eq. [6] and Eq. [7], and has the unit of cm^2.

The three formulations of α were used to independently predict pressure heads for 23 soils, using particle-size distribution, bulk density, and particle density data. The calculated pressure heads were paired with corresponding water contents obtained from Eq. [3] to yield soil water characteristics. Predicted pressure heads were compared with the corresponding experimental pressure heads for each α formulation.

Table 4. Parameters of Eq. [12] that relate log N_i to log (w_i/R_i^3) and the goodness of fit, r^2, for five soil textures.

Textural class	No. of soils	Data pairs	a	b	r^2
Sand	6	62	−2.478	1.490	0.882
Sandy loam	6	75	−3.398	1.773	0.952
Loam	4	50	−1.681	1.395	0.936
Silt loam	5	77	−2.480	1.353	0.965
Clay	5	88	−2.600	1.305	0.954

RESULTS AND DISCUSSION

Nature of the Scaling Parameter α

Values of α, based on Methods 1 and 2 (Eq. [11] and Eq. [13]), were calculated as a function of log n_i for several soil textures. UNSODA soils used for this purpose are identified in Table 2 as those used for formulating α. Experimental values of α for the same soils were also calculated using Eq. [8], in which log N_i values were obtained directly from measured soil water characteristic curves. Examples of variations in α with log n_i are shown in Fig. 5a and 5b for sandy loam and clay textures, respectively. Note that large values of log n_i coincide with small particle sizes (Eq. [4]). Thus, one should expect to see a relationship between α and R_i, as well. We prefer the particle number n_i because it includes the effects of both the solid mass and the particle size. Similar results were obtained for the sand, loam, and silt loam textures. The trends in α values calculated from Methods 1 and 2 and those from experimental soil water characteristics are essentially identical. The experimental data show some scatter, probably because experimental α values were computed from different

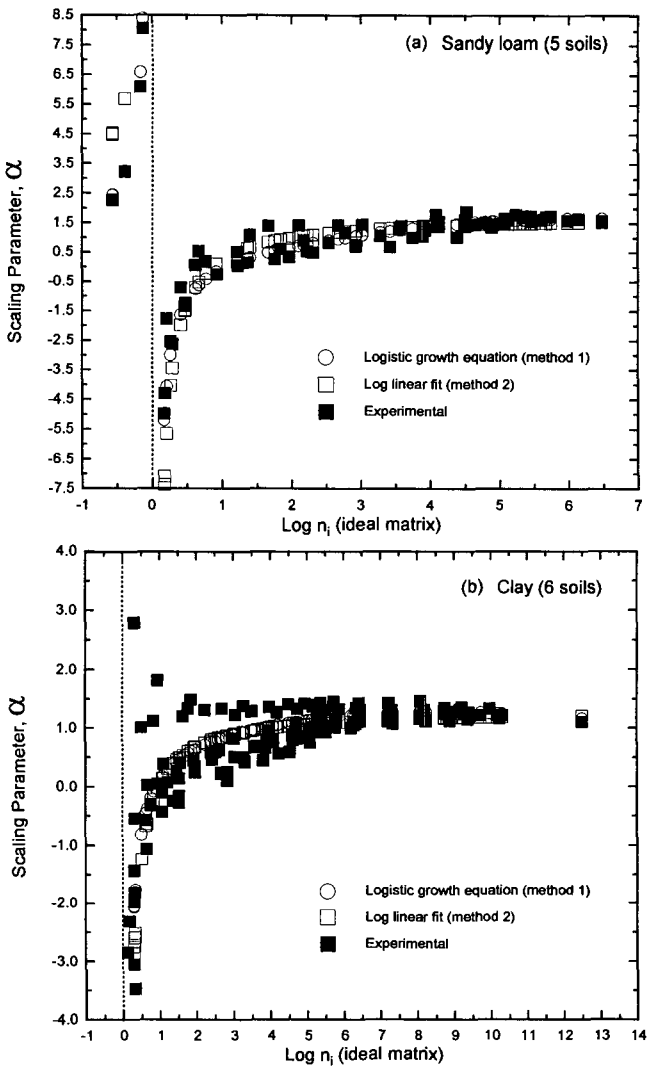

Fig. 5. Scaling parameter α as a function of the number of spherical particles in the ideal soil (log n_i): (a) sandy loam soils; and (b) clay soils.

soil water characteristics within each textural class. It is noteworthy that α values based on Methods 1 and 2 differ from each other only slightly, and both represent an average for the scatter of the experimental α values. Linear regression between Methods 1 and 2 gave $r^2 > 0.96$ for all textures. Those between Methods 1 or 2 and experimental data yielded r^2 from 0.78 to 0.99.

It is clear from Fig. 5 that α is not a constant; rather, α decreases as the number of particles decreases. The change in α with particle number is relatively small for most of the range of particle-size distribution. A sharp decrease in α, however, occurs at low particle numbers when log n_i is in the range of 0.5 to 2.0. The transition point is smallest for the sand texture and largest for the clay texture. Large negative and large positive values of α occur in an erratic manner below log $n_i < 0$. We attribute this behavior to errors in the estimation of very small numbers of particles. It is not unusual to compute particle numbers of <1 in the diameter range of 1500 to 2000 μm for many particle-size distributions. However, we believe the error is large. The diameter range of 1500 to 2000 μm is also the range of the particle-size distribution that coincides with the soil water characteristic near saturation. Because of experimental uncertainties in soil water characteristic data, estimating N_i accurately near saturation is relatively difficult. We, therefore, suggest that no significance be attached to values of α when log $n_i < 0$.

The degree of variation in α from clay to sand size particles is relatively small, and an average value of α may well represent the entire particle-size distribution, except near saturation. The constant value of 1.38 suggested by Arya and Paris (1981) worked well enough for many but not all particle-size distributions. The results in Fig. 5a and 5b and those for sand, loam, and silt loam textures, however, indicate that overall α values vary from one textural group to another, and that a single value may not be appropriate for a particular textural class. Arya et al. (1982) analyzed 181 New Jersey soils and found average α values that varied from 1.26 for silt loams to 2.10 for sandy clay loams. Schuh (1992) evaluated variation in α for a large number of soils in North Dakota. He concluded that α could vary from as low as 0.95 for silt loams to 1.3 for loams. In addition, he found that α was not constant over the full range of the soil water characteristic curve for several textural classes.

Effect of α on Predictions of Soil Water Characteristic

In the following we discuss the effects on predictions of the soil water characteristic of the use of a single value of α vs. considering α as a continuous function. Soil water characteristics were predicted for four to five test soils within each textural group using α values that were based on Methods 1, 2, and 3. Typical examples of predicted and experimental soil water characteristics for clay, loam, sandy loam, and sand textures are presented in Fig. 6. Results show that all three α functions adequately predicted the shape of the experimental soil water characteristic. However, the differences in predicted and experimental pressures varied in magnitude from soil to soil. Differences between predicted and experimental pressures occurred more frequently at or near the point where soil water characteristic curves exhibit a sharp change. However, no particular saturation could be identified where a given formulation for α produced consistently poor or good predictions, particularly for Methods 1 and 2. A constant α, on the other hand, yielded in a majority of cases lower absolute pressures in the dry range and higher absolute pressures in the wet range.

The overall predictive ability of α for the three formulations was evaluated by comparing experimental with calculated pressure heads on a 1:1 plot. Pairs of predicted and experimental pressures for all soils and all textural groups were pooled, and the logarithm of experimental pressure head was plotted as a function of the logarithm of calculated pressure head. The results are presented in Fig. 7. The large scatter in the data is a result of the fact that, while a wide range of soil water

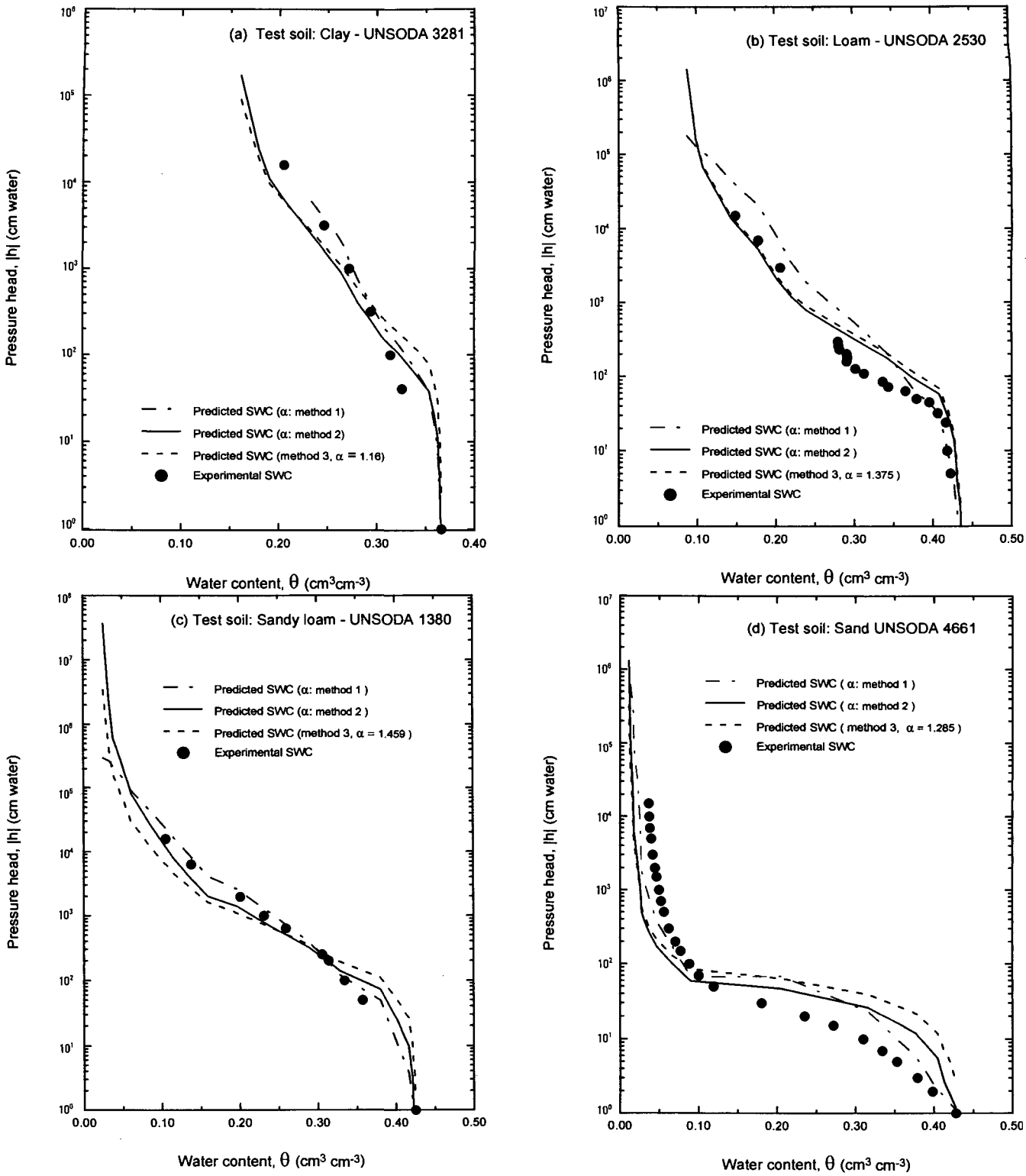

Fig. 6. Comparison of measured and predicted soil water characteristics (SWC): (a) clay soil, (b) loam soil, (c) sandy loam soil, and (d) sand.

characteristics and particle-size distributions occur within the same textural class, only textural class average values of α were used in calculating pressure heads. Despite the scatter, Method 1 (Fig. 7a) showed an overall good agreement between experimental and predicted pressures. The linear regression had an r^2 of 0.925 and the regression line differed only slightly from the 1:1 line. In contrast, Method 2 (Fig. 7b) appears to produce bias. Although the regression appears to be good with an r^2 of 0.892, the method over-predicted absolute pressure heads in the wet range and under-predicted in the dry range. The regression line deviated from the 1:1 line in the dry range. Predictions with constant α (Fig. 7c) produced results similar to those in Method 2. Although the value of r^2 in this case was 0.912, the deviation of the regression line from the 1:1 line was much more pronounced than for Method 2, as is reflected by the intercept of −0.795 and slope of 1.32. Additionally,

over- and under-predictions of pressures with constant α were also more pronounced than with Method 2. Thus, based on these results one would conclude that Method 1 is the most appropriate formulation for α and Method 3 the least, even when a different average α is used for different textural classes.

While comparing Methods 1 and 2, it must be noted that Method 1 is based on values of log N_i derived from the experimental soil water characteristic. Use of five parameters (Table 3) also assured a better fit of the logistic growth equation to experimental log N_i data. Naturally, then, α values based on this fitted relationship should produce the best overall agreement with the experimental data. Method 2, on the other hand, is not based on fitting an equation to a trend in the data, but on similarity in particle-size distributions between the ideal soil and its counterpart, natural soil. We postulated for this purpose a linear relationship between log N_i and $\log(w_i/R_i^3)$ and did not consider inherent nonlinearity. Therefore, results from Method 2 will conform to the experimental data only if the similarity principle is valid and experimental data have been obtained without bias. The behavior of the constant α function is partially clarified by the results in Fig. 5 and 6, which show that constant α is likely to introduce large errors in the wet range and some errors in the dry range.

At this point we may not be able to critically differentiate between Methods 1 and 2, while yet other methods may even be better suited to determine α. Although Method 1 appears more attractive, this formulation is not independent of the nature of experimental soil water characteristic data. Our data sets were heterogeneous, and they were contributed by researchers in the United States, Netherlands, United Kingdom, Germany, Belgium, Denmark, Russia, Italy, and Australia, who each used different experimental procedures. In addition, systematic and random errors are likely in experimental determination of the soil water characteristic. For measurements obtained with a pressure plate apparatus (Klute, 1986), loss of contact between the sample and the ceramic plate as the sample is progressively desaturated can result in serious overestimation of water content in the dry range. In the wet range, structural disturbances (e.g., puddling and sealing, incomplete saturation) may lead to under-estimation of equilibrium water contents. If these errors are present to some extent in all measured soil water characteristic data, then the bias apparent with Method 2 is understandable (Fig. 7b). Of course, other sources of variability exist that may mask any systematic trends. For example, grouping soils together solely on the basis of textural nomenclature may introduce variations $\leq$25 percentage points in the mass fraction for some particle-size ranges (Arya, unpublished data, 1983 based on USDA-SCS [1974]). Significant variations in bulk density and organic matter content also exist within a textural class. Thus, widely differing soil water characteristic curves may be obtained for soils of the same textural class.

SUMMARY AND CONCLUSIONS

This study proposes two new formulations for the scaling parameter α in the Arya and Paris (1981) model

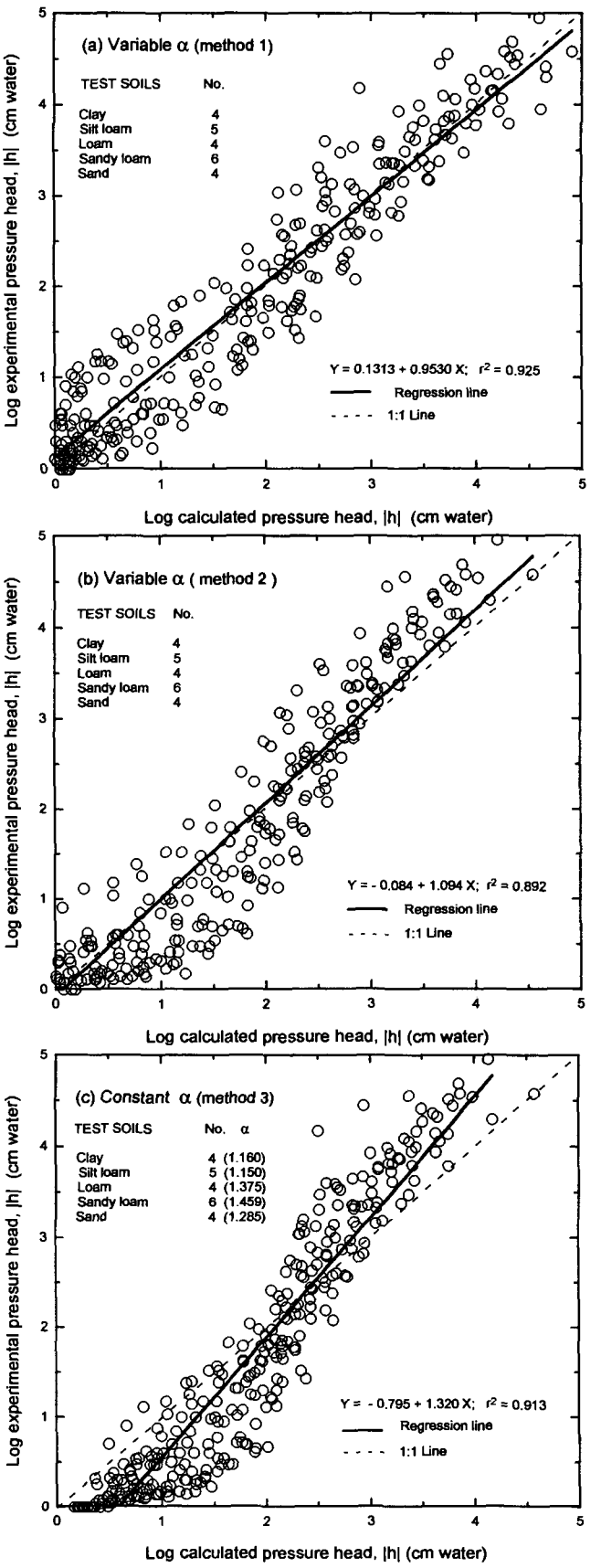

Fig. 7. Comparison of calculated and experimental pressures. Test results for 23 soils are pooled: (a) Method 1, (b) Method 2, and (c) Method 3.

of the soil water characteristic, which translates a particle-size distribution curve into a corresponding soil water characteristic curve. The scaling parameter is used to estimate pore lengths in natural structure samples. If the solid mass, w_i, in a given particle-size fraction with mean particle radius R_i is equivalent to n_i uniform size spherical particles, the pore length generated by these particles, when arranged in a cubic, close-packed assemblage, can be approximated by $2n_i R_i$. However, tracing the pore length generated by the same amount of solid mass in a natural structure assemblage would require N_i spherical particles of radius R_i, where $N_i = n_i^{\alpha}$.

The new formulations express N_i as a continuous function of particle-size distribution parameters. In one formulation α is predicted according to Eq. [11], and log N_i is related nonlinearly to log n_i using a logistic growth equation, while in another formulation, α is defined by Eq. [13], and log N_i is related linearly to log (w_i/R_i^3) based on the principle of similarity.

The predictive ability of the new formulations and that of the original constant α approach were compared. Predictions of soil water characteristics for a range of soil textures with varying bulk densities and organic matter contents showed reasonable to excellent agreement with experimental data. Use of a constant α usually led to under-predictions in the dry range and over-predictions in the wet range. The predictions appeared unbiased if the formulation for α was based on the logistic growth equation. On the other hand, the formulation based strictly on the principle of similarity usually produced bias similar to that of the constant α approach. Numerous problems exist with experimental data, including random and systematic errors, and it may not be possible to critically differentiate between the formulations based on tests with a heterogeneous data set. Uniform data sets, free of bias and errors, are needed to more critically evaluate indirect methods for estimating soil hydraulic properties.

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