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ANALYSIS OF FIELD TENSION DISC INFILTROMETER DATA BY PARAMETER ESTIMATION

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A b s t r a c t. Using numerically generated data, we previously showed that it is theoretically possible to estimate the soil hydraulic functions from the cumulative infiltration curve measured with a tension disc infiltrometer at several consecutive tensions, provided that estimates of the initial and final water contents are available. In this study we used two field data sets to obtain the soil hydraulic functions by parameter estimation. Our inverse procedure combined the Levenberg-Marquardt nonlinear parameter optimization method with a numerical solution of the axisymmetric variably-saturated flow equation. We used a disc permeameter with a radius of 10 cm and applied consecutive tensions of -20, -10, and -3 cm. The average initial water content of the soil was 0.077 cm^3cm^{-3} and the final water contents below the disc were approximately 0.24 and 0.27 cm^3cm^{-3} for two runs. This is about 0.11 and 0.08 cm^3cm^{-3} lower than the saturated water content as measured in the laboratory. The objective function for parameter estimation was defined in terms of the cumulative infiltration curve and the final water content. Alternatively, we added into the objective function two values of the unsaturated conductivity obtained using Wooding's analytical solution. Unsaturated soil hydraulic conductivities obtained using the inverse solution compared closely with those resulting from Wooding's analysis. However, relatively large differences were found between retention parameters obtained with the inverse solution and those measured independently in the laboratory. Simulations using soil hydraulic parameters determined in the laboratory did not accurately reproduce the field infiltration experiment.

K e y w o r d s: soil hydraulic properties, tension disc infiltrometer, parameter estimation, numerical modeling

INTRODUCTION

Estimation of the in situ soil hydraulic properties is an important step in understanding field-scale water flow and solute transport processes. The effective use of increasingly sophisticated numerical models to predict fate of pollutants also requires knowledge of the soil hydraulic parameters. While many laboratory and field methods exist to determine the soil water retention and unsaturated hydraulic conductivity curves [11,16], most methods remain relatively expensive and too cumber-some for applications to relatively large areas of land.

Tension disc infiltrometry has recently become a popular method for in situ measurement of the unsaturated soil hydraulic properties [1,19,24,25]. Thus far, tension infiltration data have been used primarily for evaluating saturated and unsaturated hydraulic conductivities, and for quantifying the effects of macropores and preferential flow paths on infiltration. Recently several studies [29,35] have suggested the use of tension infiltration data in combination with parameter estimation techniques to estimate additional soil hydraulic parameters.

Šimůnek and van Genuchten [29,30] showed that infiltration curves measured with a tension disc infiltrometer at one or several tensions do not provide enough information to estimate van Genuchten's soil-hydraulic parameters [32] by numerical inversion of the Richards equation for unsaturated flow. They showed that additional information about the flow processes, such as water contents or pressure heads in the soil profile or the water content corresponding to the final supply pressure head, are needed to successfully employ inverse methods. Their studies indicated that the best practical scenario is to use the cumulative infiltration curve measured at several consecutive tensions, in conjunction with the initial and final water contents. Šimůnek and van Genuchten [30] also showed that the classical Wooding's [38] analysis of tension infiltrometer data can be used successfully in combination with the parameter estimation procedure. These results suggest that one should be able to use information typically being collected with a tension disc infiltrometer not only to estimate unsaturated hydraulic conductivity function, but without further experiments also the soil water retention curve.

In this paper we will use the methodology of Šimůnek and van Genuchten [30] to estimate the soil hydraulic properties from field tension disc infiltrometer data. We will compare soil hydraulic parameters estimated from field measurements with parameters derived from independent laboratory analyses, and with results based on Wooding's analytical solution.

THEORY

Wooding's analysis

The traditional analysis of tension disc infiltration data based on Wooding's analytical solution [38] requires two steady-state fluxes at different tensions [1] to yield estimates of the saturated hydraulic conductivity, K_s(LT^{-1}), and the sorptive number, α^* (L^{-1}), in Gardner's exponential model of the unsaturated hydraulic conductivity [9]:

$$K(h) = K_s \exp(\alpha^* h). \tag{1}$$

Wooding's solution for infiltration from a circular source with a constant pressure head at the soil surface, and with the unsaturated hydraulic conductivity described by Eq. (1), is given by:

$$Q(h_o) = \left(\pi r_o^2 + \frac{4r_o}{\alpha^*}\right) K(h_o) \tag{2}$$

where Q is the steady-state infiltration rate (L^3T^{-1}), r_0 is the radius of the disc (L), h_o is the wetting pressure head (L), and $K(h_o)$ is the unsaturated hydraulic conductivity (LT^{-1}) at pressure head h_o. The first term on the right side represents the effect of gravitational forces and the second term the effect of capillary forces. Methods for obtaining the unsaturated hydraulic conductivity in the middle of an interval between two successively applied pressure heads were described previously by Ankeny *et al.* [1], Reynolds and Elrick [25] and Jarvis and Messing [13], among others. The approach assumes that the sorptive number α^* in Eq. (1) is constant over the interval between two adjacent supply pressure heads such that:

$$\alpha^*_{i+1/2} = \frac{\ln \dfrac{Q_i}{Q_{i+1}}}{h_i - h_{i+1}} \qquad i = 1, \ldots, n-1 \tag{3}$$

where n is the number of infiltration tensions used. The infiltration rate, $Q_{i+1/2}$, at the half-way point between two adjacent supply pressure heads $h_{i+1/2} = (h_i + h_{i+1})/2$ is calculated as a geometric mean of the actual infiltration rates Q_i and Q_{i+1}:

$$Q_{i+1/2} = \exp \frac{\ln Q_i + \ln Q_{i+1}}{2}. \tag{4}$$

With this information, the unsaturated hydraulic conductivity at pressure head $h_{i+1/2}$ can then be calculated as:

$$K_{i+1/2} = \frac{Q_{i+1/2}}{\pi r^2 + \dfrac{4r}{\alpha^*_{i+1/2}}}. \tag{5}$$

Finally, K_s is calculated from Eq. (1) using known values of $h_{i+1/2}$, $K_{i+1/2}$ and $\alpha^*_{i+1/2}$ as follows:

$$K_s = \frac{K_{i+1/2}}{\exp\left(\alpha^*_{i+1/2}\, h_{i+1/2}\right)}. \qquad (6)$$

Numerical model

In our analysis of tension disc infiltrometer data we will use a numerical solution of the Richards' equation [28] coupled with the Levenberg-Marquardt nonlinear minimization method [21]. The governing flow equation for radially symmetric isothermal Darcian flow in a variably-saturated isotropic rigid porous medium is given by the following modified form of the Richards' equation [37]:

$$\frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial h}{\partial r}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right) + \frac{\partial K}{\partial z} \qquad (7)$$

where θ is the volumetric water content (L^3L^{-3}), h is the pressure head (L), K is the hydraulic conductivity (LT^{-1}), r is a radial coordinate (L), z is the vertical coordinate (L) positive upward, and t is time (T). Equation (7) was solved numerically for the following initial and boundary conditions applicable to a disc tension infiltrometer experiment:

$$\theta(r,z,t) = \theta_i \qquad t=0 \qquad (8)$$

$$h(r,z,t) = h_o(t) \qquad 0 < r < r_o, z = 0 \qquad (9)$$

$$\frac{\partial h(r,z,t)}{\partial z} = -1 \qquad r > r_o, z = 0 \qquad (10)$$

$$h(r,z,t) = h_i \qquad r^2 + z^2 \to \infty \qquad (11)$$

where θ_i is the initial water content (L^3L^{-3}), h_o is the time-variable supply pressure head imposed by the tension disc infiltrometer (L) and r_o is the disc radius (L). Equation (8) specifies the initial condition in terms of the water content. Boundary condition Eq. (9) prescribes the time-variable pressure head under the tension disc permeameter, while Eq. (10) assumes a zero flux at the remainder of the soil surface (evaporation is hence neglected). Equation (11) states that the other boundaries are sufficiently distant from the infiltration source so that they do not influence the infiltration experiment. The boundary condition at the axis of symmetry (r=0) is a no-flow condition. Equation (7), subject to the above initial and boundary conditions, was solved using the finite element code, HYDRUS-2D, as documented by Šimůnek *et al.* [29]. The numerical solution was based on a mass-conservative iterative scheme proposed by Celia *et al.* [7].

A model of the unsaturated soil hydraulic properties must be selected prior to application of the numerical solution of the Richards' equation. In this study we will limit ourselves to the unsaturated soil hydraulic functions [32]:

$$S_e(h) = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left(1 + |\alpha h|^n\right)^m} \qquad h<0$$

$$\theta(h) = \theta_s \qquad h \geq 0 \qquad (12)$$

and

$$K(\theta) = K_s S_e^l \left[1 - \left(1 - S_e^{1/m}\right)^m\right]^2 \qquad h < 0$$

$$K(\theta) = K_s \qquad h \geq 0 \qquad (13)$$

where S_e is the effective fluid saturation (-), K_s is the saturated hydraulic conductivity (LT^{-1}), θ_r and θ_s denote the residual and saturated water contents (L^3L^{-3}), respectively; l is the pore-connectivity parameter (-), and α (L^{-1}), n (-), and m (= 1 - 1/n) (-) are empirical shape parameters. The pore-connectivity parameter l in $K(\theta)$ was estimated by Mualem [22] to be 0.5 as an average for many soils. Taking l=0.5, the above hydraulic functions contain 5 unknown parameters: θ_r, θ_s, α, n, and K_s. In this study we treat both K_s and θ_s as fitting parameters in Eqs (12) and (13). Direct measurement of K_s and θ_s is sometimes substantially different from the fitted values because of the effects of macropores that may saturate only after a zero or positive pressure head is applied [19].

Since saturation will never be reached during tension infiltration experiments, K_s and θ_s in this study are in essence extrapolated, empirical parameters outside the range of the disc experiment [29]. Also, tension disc infiltration in general is a wetting process (assuming that one can neglect internal drainage at the initial pressure head), which means that the hydraulic parameters in Eqs (12) and (13) should represent wetting branches of the unsaturated hydraulic properties. We will refer furtheron in this paper to Eqs (12) and (13) as the van Genuchten - Mualem model.

Formulation of the inverse problem

The objective function ϕ to be minimized during the parameter estimation process can be formulated either in terms of only cumulative infiltration data, or cumulative infiltration data in combination with additional information such as the water content corresponding with the final supply pressure head, or the unsaturated hydraulic conductivities obtained using Wooding's analysis. The objective function is defined as [28]:

$$\phi(\beta, q_m) = \sum_{j=1}^{m} \left(v_j \sum_{i=1}^{n_j} w_{ij} \left[q_j^*(t_i) - q_j(t_i, \beta) \right]^2 \right) \tag{14}$$

where m represents the number of different sets of measurements such as cumulative infiltration data, in situ determined pressure heads, or additional information; n_j is the number of measurements in a particular set, $q_j^*(t_i)$ is the specific measurement at time t_i for the jth measurement set, β is the vector of optimized parameters (e.g., θ_r, θ_s, α, n, and K_s), $q_j(t_i, \beta)$ represents the corresponding model predictions for parameter vector β, and v_j and w_{ij} are weights associated with a particular measurement set j or a measurement i within set j, respectively. We assume for now that the weighting coefficients w_{ij} in Eq. (13) are equal to one, that is, the variances of the errors inside a particular measurement set are all the same. The weighting coefficients v_j for sets with more than one data member are given by:

$$v_j = \frac{1}{n_j \sigma_j^2}. \tag{15}$$

The above approach views the objective function as the average weighted squared deviation normalized by measurement variances σ_j^2. The different measurement sets could represent cumulative infiltration data, unsaturated hydraulic conductivities obtained by Wooding's analysis, in situ determined pressure heads, or the final water content. Since the final water content is only one number and the variance can not be defined, the weight for this data point is assumed to be one. Minimization of the objective function ϕ is accomplished by using the Levenberg-Marquardt nonlinear minimization method [21].

MATERIALS AND METHODS

The field tension infiltration experiment was carried out in Riverside, California, on an Arlington fine sandy loam (coarse-loamy, mixed, thermic, Haplic Durixeralf) [36]. The soil contained 71.4% sand, 19.8% silt, and 8.8% clay, i.e., a sandy loam according to the textural triangle of U.S. Department of Agriculture [31]. The particle-size analysis was carried out with the hydrometer method [10]; wet sieving was used. The average bulk density from all samples was 1.53 g/cm^3. The average organic carbon and calcium carbonate contents were 0.54 and 0.18%, respectively [36].

Infiltration measurements were performed using a tension infiltrometer (Soil Measurement Systems) with the infiltrometer disc detached from the supply and tension control tubes. A level was used to assure that the disc and the infiltrometer base were always at the same level (zero relative distance), so that the head between the bubbling outlet at the bottom of the water supply tube and the disc membrane was constant. A 21X datalogger (Campbell Scientific, Inc.) and two pressure transducers (MICRO SWITCH, Honeywell, Inc.) were used to record transient infiltration rates in a setup similar to that described by Ankeny *et al.* [2]. We used a layer of approximately

1 mm of no. 60 silica sand (diameter =0.25 mm) between the disc membrane and the leveled smoothed soil surface to improve hydraulic contact. The measurements were made with a 10 cm disc radius and three supply tensions. We started infiltrating at a tension of 20 cm for about 60 min, after which we decreased the tension consecutively to 10 and 3 cm. These two supply tensions each were also maintained for about one hour. Pressure transducer readings were recorded every 5 s for about 5 min after a new tension was implemented, and later every 30 s. Three soil samples (diameter 7.3 cm, height 10.3 cm) were taken immediately before the infiltration experiment in the vicinity of the experimental site to measure the initial and saturated water contents in the laboratory. Immediately after the experiment four small samples were taken (diameter 5.4 cm, height 3 cm) directly below the infiltration disc to obtain the water content corresponding with the final supply pressure head. We took relatively shallow samples in view of possible large moisture gradients directly below the tension infiltrometer [26,36]. The entire procedure was repeated twice. The second run was carried out at a distance of about 1 m from the first site. Figures 1a and 2a show the measured cumulative infiltration volumes versus time for the first and second runs, respectively. The actual final infiltration rates at particular supply pressure heads, the water contents corresponding to the final supply pressure head (h=-3 cm), and the initial water

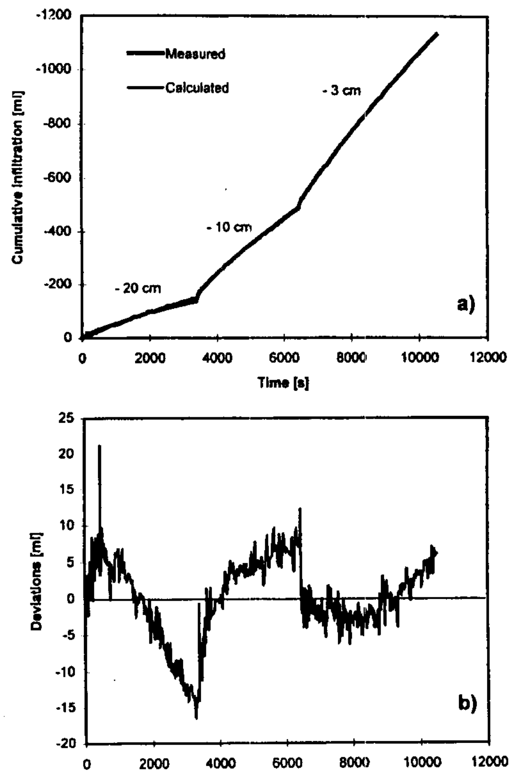

Fig. 1. Tension disc infiltrometer experiment; Run 1: a) measured and fitted cumulative infiltration curves versus time, b) deviations between measured and fitted cumulative infiltration versus time. The fitted curve was obtained by minimizing the objective function $\phi(I,\theta_f)$.

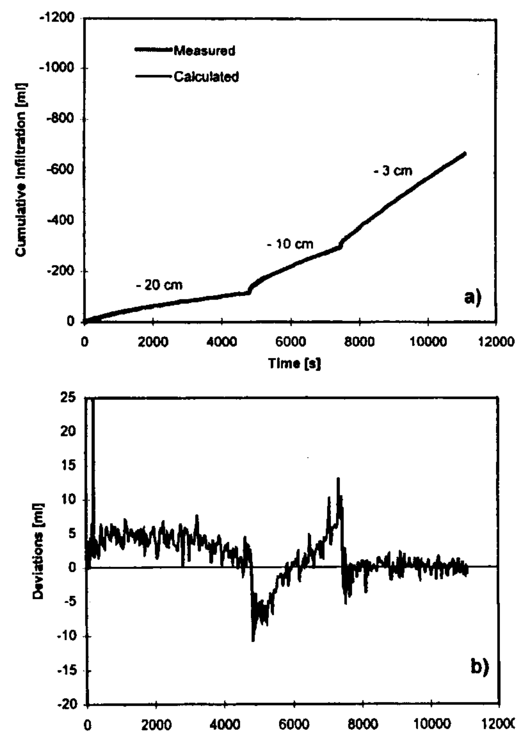

Fig. 2. Tension disc infiltrometer experiment; Run 2: a) measured and fitted cumulative infiltration curves versus time, b) deviations between measured and fitted cumulative infiltration versus time. The fitted curve was obtained by minimizing the objective function $\phi(I,\theta_f)$.

contents for both runs are given in Table 1. Note that the infiltration rates for the first run were from 50 up to 100% higher than for the second run. Also, the measured final water content was higher for the first run as compared to the second run.

Five soil samples (diameter 5.36 cm, height 5.97 cm) were taken for measurement of the retention curve and the saturated hydraulic conductivity in the laboratory. The saturated hydraulic conductivity was measured with the falling-head method [16]. The retention curve was measured with the Tempe Cell (Soilmoisture Equipment Corp.) method for pressure heads from saturation down to -8 m, and a pressure chamber (Soilmoisture Equipment Corp.) for lower pressure heads (down to -37 m). The soil hydraulic parameters obtained by fitting Eq. (12) to the laboratory data using the RETC code [33] are given in Table 2 for all five samples. The mean parameter values in the last column of Table 2 are simple arithmetic averages of the parameters of the five samples. Geometric averages of K_s, and especially α and n, did not differ much from the arithmetic values (Table 2).

RESULTS AND DISCUSSION

Wooding's analysis

First we used for each of the two runs the final infiltration rates for particular supply pressure heads as given in Table 1, in combination with the Wooding's analysis (Eqs (1) through (6)). From this analysis we obtained unsaturated hydraulic conductivities K and sorptivity numbers α^* in the middle of the interval between two supply tensions, i.e., for pressure heads of -6.5 and -15 cm (see Table 3). Hydraulic conductivities estimated in this way were always higher for the first run than for the second one. For both runs, the extrapolated values of K_s (32.03 and 20.42 cm d^{-1} for the first and second run, respectively) as calculated from the lower supply pressure head interval (-20 to -10 cm) were higher than K_s-values (14.00 and 12.33 cm d^{-1}) for the higher supply pressure head interval (-10 to -3 cm). Values of α^* calculated from the infiltration rates at supply pressure heads of -20 and -10 cm were 0.103 and 0.0814 cm^{-1} for the first and second run, respectively. Corresponding values calculated for supply pressure heads of

Table 1. Results of the tension infiltrometer experiments

Variable	Run 1	Run 2
Final infiltration rate at h=20 cm, Q (-20) (mLs^{-1})	0.0330	0.0164
Final infiltration rate at h=-10 cm, Q (-10) (mLs^{-1})	0.0927	0.0509
Final infiltration rate at h=-3 cm, Q (-3) (mLs^{-1})	0.1380	0.0900
Final volumetric water content for h=-3 cm; θ_f (-3) (m^{3}m^{-3})	0.2740	0.2370
Standard deviation of final water content (m^{3}m^{-3})	0.0413	0.0153
Initial volumetric water content, θ_i (m^{3}m^{-3})	0.0768	0.0768
Standard deviation of initial water content (m^{3}m^{-3})	0.0106	0.0106

Table 2. Soil hydraulic parameters determined from laboratory analysis

Parameter	1st sample	2nd sample	3rd sample	4th sample	5th sample	Mean value
θ_r (m^{3}m^{-3})	0.107	0.101	0.102	0.107	0.121	0.107
θ_s (m^{3}m^{-3})	0.357	0.331	0.359	0.375	0.314	0.347
α (cm^{-1})	0.0810	0.0340	0.0394	0.0543	0.0176	0.0453(0.0401)*
n (-)	1.37	1.48	1.50	1.44	1.96	1.55 (1.54)*
K_s (cm d^{-1})	232.0	202.0	735.0	356.0	121.0	329.0 (272.0)*

*Geometric mean.

Table 3. Comparison of results obtained with Wooding's analysis and using parameter estimation

Parameter	Wooding's analysis		Parameter estimation	
	(-10, -20 cm)	(-3, -10 cm)	$\phi(I, \theta_f)$	$\phi(I, K, \theta_f)$
			Run 1	
α (cm^{-1})			0.0412	0.0471
n (-)			3.61	6.19
θ_s (m^{3}m^{-3})			0.274	0.273
K_s (cm d^{-1})	32.03	14.00	9.24 (-34.1%)*	9.59 (-31.5)
K(-6.5 cm) (cm d^{-1})		9.65	8.80 (-8.81%)	9.54 (-1.14)
K(-15 cm) (cm d^{-1})	6.81		4.95 (-27.31%)	6.56 (-3.67)
α^* (-6.5 cm) (cm^{-1})		0.0572		
α^* (-15 cm) (cm^{-1})	0.103			
ϕ_f			0.000250	0.00841
			Run 2	
α (cm^{-1})			0.0406	0.0456
n (-)			2.76	3.32
θ_s (m^{3}m^{-3})			0.238	.236
K_s (cm d^{-1})	20.42	12.33	6.57 (-46.72%)	7.78 (-36.9)
K(-6.5 cm) (cm d^{-1})		7.26	5.34 (-26.44%)	6.84 (-5.78)
K(-15 cm) (cm d^{-1})	3.74		2.49 (-33.42%)	3.03 (-18.98)
α^* (-6.5 cm) (cm^{-1})		0.113		
α^* (-15 cm) (cm^{-1})	0.0814			
ϕ_f			0.000480	0.0318

*The number in parentheses shows by how many percent the hydraulic conductivity obtained by parameter estimation differs from corresponding values obtained using Wooding's analysis.

-10 and -3 cm were 0.0572 and 0.113 cm^{-1}, respectively. The relatively large difference in α^* between the two intervals of the first run resulted also in large difference between the extrapolated K_s values. Nevertheless, the saturated and unsaturated hydraulic conductivities both showed fairly close agreement between the two runs.

Inverse solutions

We next carried out numerical inversions using for each of the two experiments the entire cumulative infiltration curve and the water content, θ_f, corresponding to the final supply pressure head of -3 cm (Table 1). Figures 1a and 2a show the experimental and fitted cumulative infiltration curves for the two runs. The estimated soil hydraulic parameters are given in Table 3. The residual water content, θ_r, was found to be zero for all inverse solutions. We used the symbol $\phi(I, \theta_f)$ in Table 3 to indicate that the objective function was defined in terms of the cumulative infiltration curve, I, and the final water content, θ_f. Figures 1b and 2b show very small differences between the best-fit and measured values versus time. Deviations range from about -15 to +10 ml for the first experiment, and from -7 to about +8 ml for the second experiment, with most deviations being much smaller. Figures 1b and 2b illustrate the type of small-scale fluctuations (noise) that are typically recorded when transducers are used to measure the infiltration rate. We emphasize that the optimized solutions were in most or all cases very stable. We started with several different initial estimates and obtained always identical soil hydraulic parameters.

The fitted saturated water contents for both runs were very close to the water contents corresponding to the final supply tensions. The values of α in Eq. (12) were also essentially identical for both runs (0.0412 and 0.0406). The relatively high n-values (3.61

and 2.76) are typical for coarse-textured soils. Our values were slightly above the mean n value for sands (n = 2.68) and loamy sands (n = 2.28) as reported by Carsel and Parrish [6] for these two textural classes. On the other hand, the mean value of K_s determined in the laboratory was closer to that for loamy sand (3.5 md^{-1}) as reported by Carsel and Parrish [6].

In the second set of optimization runs we included into the objective function, in addition to the cumulative infiltration and final water content, also two values of the unsaturated hydraulic conductivity as obtained by Wooding's analysis. Figures 3 and 4 show the resulting best fit using this approach involving objective function $\phi(I,K,\theta_f)$. The fit is now considerably worse because of constraints imposed by the unsaturated hydraulic conductivities in the objective function. The $K(h)$ function is forced to remain closer to Wooding's results, thus allowing for less freedom in terms of fitting the cumulative infiltration data. Notice from Table 3 that α and θ_s have very similar values as in the previous analysis. The main effect of including Wooding's estimates of the unsaturated hydraulic conductivity in the objective function was a considerably higher n value, especially for the first run.

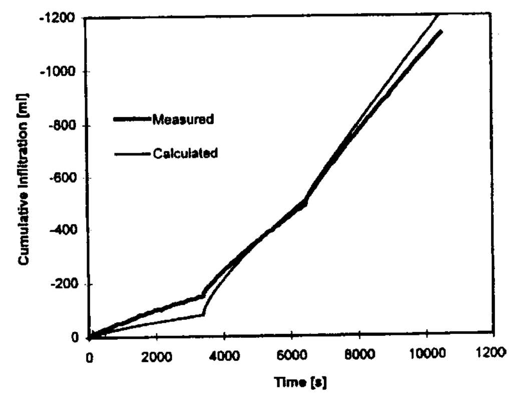

Fig. 3. Tension disc infiltrometer experiment; Run 1: measured and fitted cumulative infiltration curves versus time. The fitted curve was obtained by minimizing the objective function $\phi(I,K,\theta_f)$.

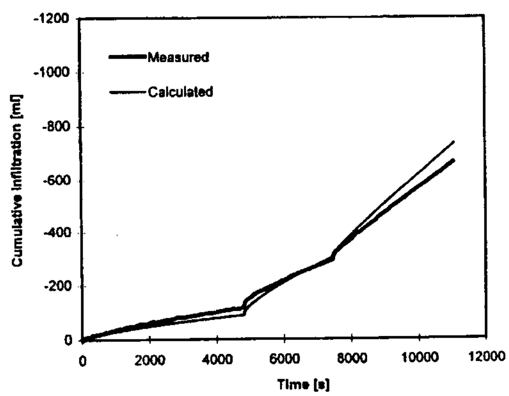

Fig. 4. Tension disc infiltrometer experiment; Run 2: measured and fitted cumulative infiltration curves versus time. The fitted curve was obtained by minimizing the objective function $\phi(I,K,\theta_f)$.

Comparison of inverse solutions and Wooding's analysis

Unsaturated hydraulic conductivities obtained by parameter estimation using the $\phi(I,\theta_f)$ objective function corresponded closely with those obtained with Wooding's analysis. For the first experimental run the differences in K were only 8.8 and 27% at pressure heads of -6.5 and -15 cm, respectively. The two sets of unsaturated hydraulic conductivities differed by only about 30% for the second experiment. The differences were higher for the extrapolated values of K_s (34 and 47%), but even those appear acceptable for most practical applications.

Table 3 shows that, as expected, the unsaturated hydraulic conductivities resulting from minimization of the objective function $\phi(I,K,\theta_f)$ were closer to Wooding's values than when the objective function $\phi(I,\theta_f)$ was optimized. However, as noted before, the fit of the cumulative infiltration curve was less successful due to constraints imposed on the final solution by the inclusion of additional information in the objective function.

Comparison of inverse solutions and laboratory analysis

Saturated hydraulic conductivities measured on small samples in the laboratory (Table 2) were about one order of magnitude higher than those obtained with the inverse solution

(Table 3). Note that the mean value of the residual water content, θ_r, fitted to the laboratory data, was 0.1074, while the optimized values for the field experiments were equal to zero. The initial condition in the field (θ_i= 0.077) was actually lower than the average laboratory θ_r value (Table 1). Also, the estimates of θ_s for the field experiment (0.274 and 0.238 for the first and second runs, respectively) were 0.07 and 0.11 lower than the mean laboratory value. These results are consistent with our experience, and that of others (e.g., [15,33]), that field-measured θ_s-values are generally about 10-20% lower than the porosity, while laboratory values are usually somewhere in between the field-measured values and porosity. The higher laboratory values for θ_s are likely due to permitting more time for equilibration (yielding steady-state values with less entrapped air), the use of de-aired water, and forcing water to be constrained one-dimensionally within the Tempe cells. The water content range θ_s - θ_r for the laboratory data was 0.24, which is similar to values obtained with the inverse method. The mean laboratory value of α was also very similar to the one obtained by parameter estimation. However, large differences existed in the values of n (1.55 for the laboratory analysis, and 3.61 and 2.76 for the inverse solutions when minimizing the objective function $\phi(I,\theta_f)$ for both runs), which caused the retention curves to acquire rather different shapes. This is shown in Fig. 5 which compares retention curves obtained bv inverse solution and independent laboratory analyses. The figure shows two different sets of curves: one set for which the water content decreases quickly with increasing tension (high n values) as obtained by numerical inversion of the field data, and a second set of curves having a much more gradual appearance (the laboratory curves).

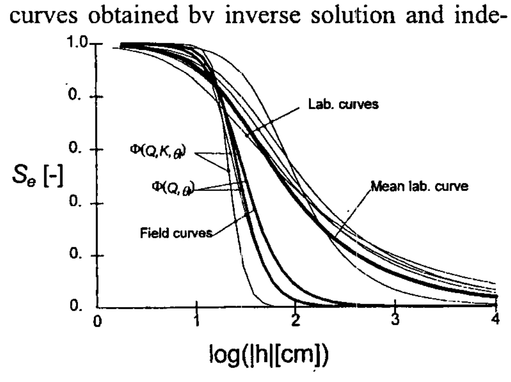

Fig. 5. Soil retention curves as obtained from the laboratory analysis and by numerical inversion of the field data using objective functions $\phi(I,\theta_f)$ and $\phi(I,K,\theta_f)$.

The relatively large differences in water retentions shown in Fig. 5 were somewhat surprising since both the laboratory and field experiments were carried out with great care and without experiencing major experimental problems. Several explanations appear possible. First, any model, including the Richards' equation, is based on a number of assumptions and simplifications which will make the simulated system different from the system it is intended to represent [12]. Our assumptions, in addition to those included in Darcy's Law, were that the field site was homogeneous, isotropic, and with a uniform initial condition. Another assumption deals with the analytical description of the soil hydraulic properties [27]. A large number of analytical models is available to describe both the retention curve and the unsaturated hydraulic conductivity function [18]. In this study we limited ourselves to the closely coupled van Genuchten-Mualem model given by Eqs. (12) and (13) and assuming a fixed pore-connectivity parameter l of 0.5. This parameter, however, can have widely different values as shown by Wösten and van Genuchten [39], Kaveh and van Genuchten [14], and de Vos *et al.* [8], among others.

Figure 6 shows the effects of allowing the pore-connectivity parameter l to be an additional fitting parameter. Deviations between measured and calculated cumulative infiltrations for both runs are plotted for the case when the objective function $\phi(I,\theta_f)$ was minimized, and with l being an additional unknown parameter. The fitted cumulative infiltration curve now closely fitted the measured curve, similarly as shown in Figs 1a and 2a. Final values of the objective function ϕ_f (Table 4) were about half of those when l was assumed equal to 0.5 (Table 3). The largest deviations were now about -10 and +5 ml for the first experiment, and -6 and +5 ml for the second experiment, with the majority of deviations again being much smaller. Note that both runs yielded l-values that are significantly different from the average value of 0.5 [22]. Although n now is somewhat lower than in the previous analysis (Table 4), the values are still much higher than the laboratory-derived estimates.

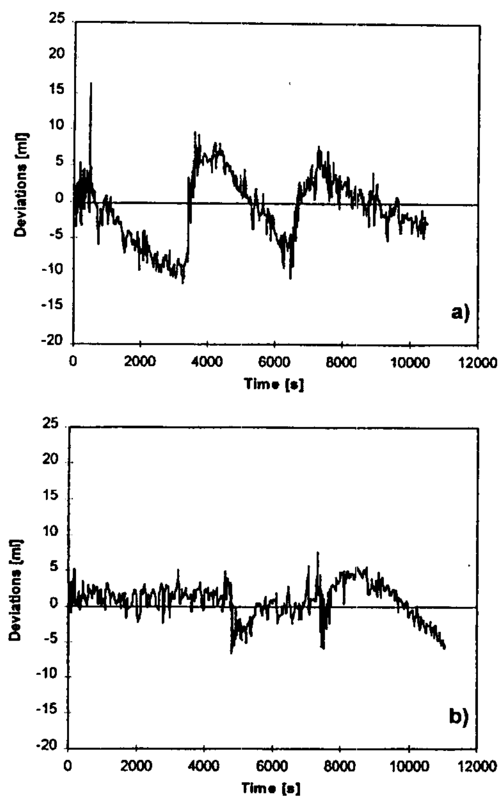

Fig. 6. Deviations between measured and calculated cumulative infiltration curves versus time for: a) the first and b) the second field experiments. The pore-connectivity parameter l was also optimized. The fitted curves were obtained by minimizing the objective function $\phi(l, \theta_p)$.

T a b l e 4. Soil hydraulic parameters obtained by minimization of the objective function $\phi(l, \theta_p)$ assuming l to be an uknown parameter

Parameter	Run 1	Run 2
α (cm^{-1})	0.0704	0.0654
n (-)	2.94	2.45
θ_s (m^{3}m^{-3})	0.275	0.240
K_s (cm d^{-1})	10.8	7.86
l (-)	-2.58	-2.24
ϕ_f	0.000125	0.000211

Parameters in the retention curve can be closely coupled with those in the hydraulic conductivity function as is the case with the van Genuchten-Mualem model [32] or the traditional Brooks and Corey-Burdine model [4,5]. They can also be taken partially or completely independent of each other, such as was the case above with independent l, or by assuming independent α and n values for the retention and conductivity curves [23]. Based on our experience with numerically generated data [29,30] we believe that tension disc measurements do not provide enough information to allow a complete decoupling of the van Genuchten-Mualem model by determining the soil water retention and hydraulic conductivity parameters independently. Such decoupling probably leads to a nonunique solution where the final values of optimized parameters will depend on their initial estimates.

We now address the question whether or not the soil hydraulic parameters determined from laboratory measurements will reproduce our field infiltration experiment. Figure 7 compares the measured cumulative infiltration curve for the first experiment with the calculated curve using the soil hydraulic parameters

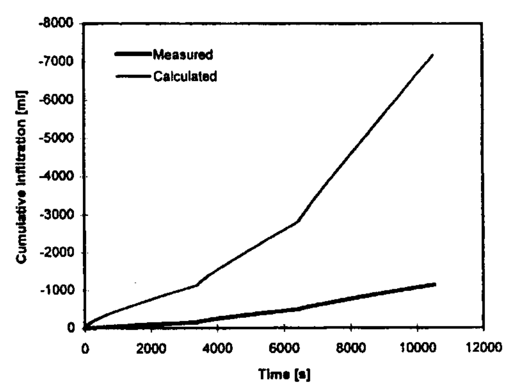

Fig. 7. Measured (first run) and calculated cumulative infiltration curves. Mean soil hydraulic parameters determined by laboratory analysis were used for numerical solution.

obtained from the laboratory analysis (Table 2). The geometric mean was used for K_s and arithmetic means for α and n. Figure 7 demonstrates that, at least for our experiments, soil hydraulic parameters determined in the laboratory cannot be immediately used to predict field-scale flow processes. This result was somewhat expected for K_s for a variety of reasons (e.g., [20]), including the use of de-aired water in the laboratory, but not necessarily for α and n. To further investigate this problem, we next used the mean laboratory retention parameters and independently fitted l, n, and K_s according to several scenarios. In a first set of simulations we fitted only K_s (obtaining 33.2 and 18.1 cm d^{-1} for the first and second field experiments, respectively; Table 5) while assuming applicability of the closed-form van Genuchten-Mualem model and keeping l at 0.5. In a second set of simulations we allowed l to be an additional fitting parameter, while in a third set of inversions we completely decoupled the van Genuchten-Mualem retention and conductivity functions by allowing n to be an unknown in Eq. (12). The latter case assumes that the retention curve is given by the mean parameters determined in the laboratory, while n and K_s in the unsaturated hydraulic conductivity function are optimized. We carried out the simulations in terms of effective water contents to avoid problems with undefined water contents (i.e., for the initial condition).

Table 5 summarizes the results of these additional numerical inversions using two options for α. In one option we set α equal to the mean value determined by the laboratory analysis, while the other option assumed α to be twice that value. The latter option was also studied since field tension infiltration is an imbibition process to be represented by wetting branches of the soil hydraulic properties, whereas in the laboratory the soil samples were sequentially drained starting at saturation, thus yielding draining branches of the water retention curve. The value of α for the main drying branch is often assumed to be about half of its value for wetting [17,20]. We briefly note here that hysteretic phenomena could also have contributed to some of the differences between the retention curves shown in Fig. 5.

Figure 8 shows the measured and optimized cumulative infiltration curves for the first set of inversions using the laboratory derived retention parameters and assuming K_s to be an unknown. The results were found to be worse than those presented in Figs 1 and 2. The final value of the objective function ϕ_f (Table 5) was about 40 times larger than when all five soil hydraulic parameters were fitted independently to the field data (Table 3). Although the values of ϕ_f decreased when l was also fitted to the data, they were still much larger than those reported in Table 3. Note that

Table 5. Parameters of the hydraulic conductivity function obtained by minimization of the objective function $\phi(I,\theta_f)$ and using independently measured laboratory retention data

Parameter	Run 1			Run 2		
	a) Retention parameters: α = 0.0453, n=1.55					
K_s (cm d^{-1})	33.2	41.6	52.9	18.1	24.5	34.0
l (-)	(0.5)*	1.89	(0.5)	(0.5)	2.19	(0.5)
n (-)	(1.55)	(1.55)	1.44	(1.55)	(1.55)	1.40
ϕ_f	0.00891	0.00623	0.00651	0.0153	0.00625	0.00787
	b) Retention parameters: α = 0.0906, n=1.55					
K_s (cm d^{-1})	116.3	64.9	44.6	69.6	37.3	28.3
l (-)	(0.5)	-1.65	(0.5)	(0.5)	-1.64	(0.5)
n (-)	(1.55)	(1.55)	1.87	(1.55)	(1.55)	(1.78)
ϕ_f	0.0291	0.00495	0.00735	0.0588	0.00458	0.00915

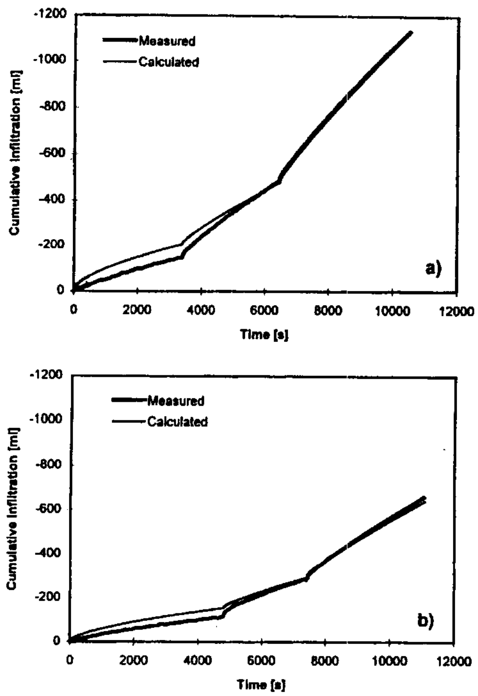

Fig. 8. Measured and optimized cumulative infiltration versus time for: a) the first and b) the second field experiments. Mean laboratory retention function parameters were used and only the saturated hydraulic conductivity was optimized. The fitted curves were obtained by minimizing the objective function $\phi(I,\theta_f)$.

l was positive for α =0.0453 and negative for α =0.0906. When weddecoupling of the van Genuchten-Mualem model by viewing n in $K(\theta)$ as an additional independent parameter unrelated to the measured value in the retention curve (while keeping l at 0.5), the value of ϕ_f and the final fit were essentially identical to the inversion results with a floating l. Also, the fitted values of n in the hydraulic conductivity function (1.435 and 1.395 for the first and second run, respectively) were not too different from the mean laboratory- derived (retention) value of n. For most cases the value of ϕ_f was higher when α was set equal to twice the mean value determined by the laboratory analysis. The estimated values of K_s were in close agreement with those estimated by Wooding's analytical solution, especially when α=0.0453. All numerical inversions using one or more unknown parameters in the hydraulic conductivity function, but with the retention curve fixed by the laboratory analysis, were substantially less successful than those cases where the retention parameters and K_s were estimated simultaneously from the field experimental data.

The above results raise several issues. One important advantage of parameter estimation methods is the possibility to simultaneously estimate parameters in the retention and hydraulic conductivity functions [29]. This approach requires the selection of an analytical model for the soil hydraulic properties prior to the numerical inversion process. Selection of an appropriate soil hydraulic model especially for $K(h)$ may still be an open question. When the model for $\theta(h)$ or $K(h)$, or both, is not adequate, the resulting fitted parameters may not be applicable to different types of water flow processes. Tension infiltration experiments provide information from which one can estimate with reasonable accuracy the $K(h)$ function, and to a somewhat lesser degree also $\theta(h)$. Good estimates of $\theta(h)$ are possible only when the invoked model for the unsaturated hydraulic characteristics accurately describes the properties of a particular soil. We showed earlier [30] using numerically generated data for an ideal soil, a tension infiltrometer experiment should provide enough information to permit accurate inversion when a closely coupled hydraulic property model is used (i.e., van Genuchten-Mualem formulation). However, if the selected hydraulic model does not properly describe the coupling between the retention and hydraulic conductivity functions, the unsaturated hydraulic conductivity function may become dominant in the optimization process and could significantly alter the shape of the estimated retention curve. The numerical algorithm then finds parameters which represent a compromise between those applicable to $K(h)$ and those for $\theta(h)$. This issue also pertains to other transient flow experiments to which parameter estimation methods are applied. The degree of compromise between parameters for the retention and conductivity functions can be different for different experiments with possible dominance of retention properties for some (e.g., multistep outflow) and conductivity properties for other (e.g., tension disc infiltration) experimental procedures. Additional studies, preferably on numerically generated data, should be carried out to evaluate the relative importance of either retention or conductivity function parameters for particular experiments.

CONCLUSIONS

Excellent agreement between measured and fitted cumulative field infiltration curves was obtained when we used as the objective function $\phi(I,\theta_f)$ and optimized the five major soil hydraulic parameters in van Genuchten's model (Figs 1 and 2). An even better fit was obtained when the pore-connectivity parameter l was also optimized. Hydraulic conductivities (both saturated and unsaturated) determined by the numerical inversion compared well with those obtained using Wooding's analytical solution.

Agreement between retention curves obtained by numerical inversion of the field infiltration experiment and by laboratory analysis was relatively poor. The entire laboratory retention curve shifted by about 0.10 units towards higher water contents, while also the parameter n had different values (α was much closer for both analyses). We could not closely reproduce the infiltration experiment when we used the mean laboratory soil hydraulic parameters and assumed validity of the closed-form van Genuchten-Mualem model. A somewhat better fit was obtained when we allowed K_s and l to be estimated independently. The field cumulative infiltration data could not be closely replicated with mean laboratory retention parameters even when we allowed for complete decoupling of the van Genuchten-Mualem model and optimized n in the hydraulic conductivity function (while assuming $l = 0.5$). Therefore, overall, we believe that results obtained for the first set of simulations involving objective function $\phi(I,\theta_f)$ and using no additional information (i.e., Wooding's K - values, or mean laboratory-measured retention parameters) are the most expedient and relevant for application to field-scale flow and transport processes.

This study indicates that similar estimate of the soil hydraulic properties can be obtained when different methods of analysis (e.g., Wooding's analytical solution, parameter estimation) are employed for the same set of data. However, similar results may not necessarily be obtained when analyzing the same soil using different experimental methods. Different laboratory approaches (e.g., evaporation methods, one- or multi-step outflow methods, steady-state experiments) and field methods (e.g., tension disc infiltration, unit gradient methods) may well give different results for the unsaturated soil-hydraulic properties.

REFERENCES

1. **Ankeny M.D., Ahmed M., Kaspar T.C., Horton R.:** Simple field method for determining unsaturated hydraulic conductivity. Soil Sci. Soc. Am. J, 55, 467-470, 1991. .
2. **Ankeny M.D., Kaspar T.C., Horton R.:** Design for an automated tension infiltrometer. Soil Sci. Soc. Am. J., 52, 893-896, 1988.
3. **Baumer O.W.:** Predicting unsaturated hydraulic parameters. In: Indirect Methods for Estimating the Hydraulic Properties of Unsaturated Soils, (Eds M. Th. van Genuchten, F. J. Leij, and L. J. Lund). Proc. Int. Workshop, Univ. California, Riverside, CA, 341-354, 1992.
4. **Brooks R.H., Corey A.T.:** Properties of porous media affecting fluid flow. J. Irrig. Drain. Div. Amer. Soc. Civ. Eng., 92, 61-88, 1966.
5. **Burdine N.T.:** Relative permeability calculations from pore-size distribution data. Petrol Trans., Am. Inst. Min. Eng., 198, 71-77, 1953.
6. **Carsel R.F., Parrish R.S.:** Developing joint probability distributions of soil water retention characteristics. Water Resour. Res., 24, 755-769, 1988.
7. **Celia M.A., Bouloutas E.T., Zarba R.L.:** A general mass-conservative numerical solution for the unsaturated flow equation. Water Resour. Res., 26, 1483-1496, 1990.
8. **de Vos J.A., Inckel M.S., Raats P.A.C.:** The water balance of the unsaturated zone considered within the context of sustainable agriculture. In: Indirect Methods for Estimating the Hydraulic Properties of Unsaturated Soils, (Eds M. Th. van Genuchten, F. J. Leij, and L. J. Lund). Proc. Int. Workshop, Univ. California, Riverside, CA, 585-599, 1992.
9. **Gardner W.R.:** Some steady-state solutions of the unsaturated moisture flow equation with application to evaporation from a water table, Soil Sci., 85, 228-232, 1958.

10. **Gee G.W., Bauder J.W.:** Particle-size analysis. In: Methods of Soil Analysis. Part 1 - Physical and Mineralogical Methods. (Ed. A. Klute). 2nd ed. Agronomy Monograph, 9:383-411, 1986.
11. **Green R.E., Ahuja L.R., Chong S.K.:** Hydraulic conductivity, diffusivity, and sorptivity of unsaturated soils: Field methods. In: Methods of Soil Analysis, Part 1 - Physical and Mineralogical Methods. (Ed. A. Klute). 2nd ed. Agronomy Monograph, 9:771-798, 1986.
12. **Hornung U.:** Mathematical aspects of inverse problem, model calibration, and parameter identification. Sci. Total Environ., 183, 17-23, 1996.
13. **Jarvis N.J., Messing I.:** Near-saturated hydraulic conductivity in soils of contrasting texture measured by tension infiltrometers. Soil Sci. Soc. Am. J., 59, 27-34, 1995.
14. **Kaveh F., van Genuchten M.Th.:** A further look at a new unsaturated hydraulic conductivity equation. Iranian J. Agri. Sci., 23, 24-32, 1992.
15. **Klute A.:** Water retention: Laboratory methods. In: Methods of Soil Analysis, Part 1 - Physical and Mineralogical Methods. (Ed. A. Klute). 2nd ed. Agronomy Monograph, 9:635-662, 1986.
16. **Klute A., Dirksen C.:** Hydraulic conductivity and diffusivity: Laboratory methods. In: Methods of Soil Analysis, Part 1 - Physical and Mi-neralogical Methods. (Ed. A. Klute). 2nd ed. Agronomy Monograph, 9:687-734, 1986.
17. **Kool J.B., Parker J.C.:** Development and evaluation of closed-form expressions for hysteretic soil hydraulic properties. Water Resour. Res., 23, 105-114, 1987.
18. **Leij F.J., Russell W.R., Lesch S.M.:** Closed-form expressions for water retention and conductivity data. Ground Water, 35(5), 848-858, 1997.
19. **Logsdon S.D., McCoy E.L., Allmaras R.R., Linden D.R.:** Macropore characterization by indirect methods. Soil Sci., 155, 316-324, 1993.
20. **Luckner L., van Genuchten M.Th., Nielsen D.R.:** A consistent set of parametric models for the two-phase flow of immiscible fluids in the subsurface. Water Resour. Res., 25, 2187-2193, 1989.
21. **Marquardt D.W.:** An algorithm for least-squares estimation of nonlinear parameters. SIAM J. Appl. Math., 11, 431-441, 1963.
22. **Mualem Y.:** A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour. Res., 12, 513-522, 1976.
23. **Nielsen D.R., Luckner L.:** Theoretical aspects to estimate reasonable initial parameters and range limits in identification procedure for soil hydraulic properties. In: Indirect Methods for Estimating the Hydraulic Properties of Unsaturated Soils, (Eds M. Th. van Genuchten, F.J. Leij, and L.J. Lund). Proc. Int. Workshop, Univ. California, Riverside, CA, 147-160, 1992.
24. **Perroux K.M., White I.:** Design for disc permeameters. Soil Sci. Soc. Am. J., 52, 1205-1215, 1988.
25. **Reynolds W.D., Elrick D.E.:** Determination of hydraulic conductivity using a tension infiltrometer. Soil Sci. Soc. Am. J., 55, 633-639, 1991.
26. **Reynolds W.D., Zebchuk W.D.:** Use of contact material in tension infiltrometer measurements. Soil Technology, 9, 141-159, 1996.
27. **Russo D.:** Determining soil hydraulic properties by parameter estimation: On the selection of a model for the hydraulic properties. Water Resour. Res., 24, 453-459, 1988.
28. **Šimůnek J., Šejna M., van Genuchten M. Th.:** The HYDRUS-2D software package for simulating water flow and solute transport in two-dimensional variably saturated media. Version 1.0. IGWMC - TPS - 53. International Ground Water Modeling Center, Colorado School of Mines, Golden, CO, 1996.
29. **Šimůnek J., van Genuchten M.Th.:** Estimating unsaturated soil hydraulic properties from tension disc infiltrometer data by numerical inversion. Water Resour. Res., 32, 2683-2696, 1996.
30. **Šimůnek J., van Genuchten M.Th.:** Parameter estimation of soil hydraulic properties from multiple tension disc infiltrometer data. Soil Science, 6, 383-398, 1997.
31. Soil Survey Staff: Soil taxonomy: A basic system of soil classification for making and interpreting soil surveys. USDA-SCS Agric. Handb. 436. U.S. Government Printing Office. Washington, DC, 1975.
32. **van Genuchten M.Th.:** A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J., 44, 892-898, 1980.
33. **van Genuchten M.Th., Leij F.J., Yates S.R.:** The RETC code for quantifying the hydraulic functions of unsaturated soils. EPA/600/2-91-065, US EPA, Office of Research and Development, Washington, DC, 1991.
34. **van Genuchten M.Th., Šimůnek J.:** Evaluation of pollutant transport in the unsaturated zone. In: Regional Approaches to Water Pollution in the Environment. (Eds P.E. Rijtema and V. Eliáš). NATO ASI Series: 2. Environment. 139-172, Kluwer, Dordrecht, The Netherlands, 1996.
35. **Vogeler I., Clothier B.E., Green S.R., Scotter D.R., Tillman R.W.:** Characterizing water and solute movement by time domain reflectometry and disk permeametry. Soil Sci. Soc. Am. J., 60, 5-12, 1996.
36. **Wang D., Yates S.R., Ernst F.F.:** Determining soil hydraulic properties using tension infiltrometers, time domain reflectometry and tensiometers. Soil Sci. Soc. Am. J., (in press) 1998.
37. **Warrick A.W.:** Model for disc infiltrometers. Water Resour. Res., 28, 1319-1327, 1992.
38. **Wooding R.A.:** Steady infiltration from large shallow circular pond. Water Resour. Res., 4, 1259-1273, 1968.
39. **Wösten J.H.M., van Genuchten M.Th.:** Using texture and other soil properties to predict the unsaturated soil hydraulic functions. Soil Sci. Soc. Am. J., 52, 1762-1770, 1988.