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TECHNICAL ARTICLES

ESTIMATION OF WATER-RETENTION CHARACTERISTICS FROM THE BULK DENSITY AND PARTICLE-SIZE DISTRIBUTION OF SWEDISH SOILS

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A Swedish soils database containing soil-water retention data, particle-size fractions, dry bulk density, and organic matter content, was analyzed in order to find a relatively simple predictor of the soil-water retention curve (SWRC). As a SWRC model we chose a three-parameter function selected from nine van Genuchten-type retention models that were fitted to the measured retention data by nonlinear parameter optimization. Cumulative particle-size (CP) data were described by a logistic distribution model. Additionally, the mean and standard deviation of the CP distribution were estimated directly from the measured particle-size data. Regression equations were subsequently used to estimate the parameters in the selected SWRC model from available soil properties, particle-size data, and CP distribution parameters. Four alternative pedotransfer models were formulated to estimate the SWRC curve from the basic soil properties. These models predicted the SWRC curves from either the original soil data or from the CP distribution parameters. A mean estimation error of less than 2.5% was used to indicate a good estimation. The highest ratio (67%) of good estimations and the lowest ratio (12%) of poor estimations were obtained when both the original soil properties and the CP model parameters were used. Our study shows that the resulting simple SWRC model gives a good description and a usable prediction of the retention properties of Swedish soils for a wide range of soil textures.

RELIABLE estimates of the water-retention curve of soils are needed in order to model unsaturated flow accurately. Representative databases of soil hydraulic functions are unavailable in most countries. Direct sampling plus laboratory or field measurement of the hydraulic properties is generally too costly and time consuming and, therefore, impractical for most applications. Hence, methods are required for predicting the soil-water retention characteristic from more easily and routinely measured soil properties.

Many attempts have been made to quantify possible relationships between the water retention curve and other soil properties such as particle size distribution, organic matter content, bulk density, and, sometimes, cation exchange capacity and clay mineralogy. These efforts include studies by Gupta and Larson (1979). Bloemen (1980). Urakcnsiek et al. (1981). Rajkai et al. (1981). Rawls and Brakensiek (l982). De Jong (1983), Kritz (l983),William et al. (1983), Wollenhaupt (l984), and Puckett ct al. (1985). Some of these studies related specific retention values with available soil properties, whereas others estimated the retention curve by relating the unknown parameters in a specific retention function to the available data. Several empirical retention functions are available for this purpose

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(e.g., Brooks and Corey 1964; King 1965; Brutsaert 1966; Visser 1968; Laliberte 1969; Rogowski 1971; Ahuja and Swartzendruber 1972; van Genuchten 1980). Different retention models were tested for Belgian soil by Vereeken et al. (1989). McCuen et al. (1981) related parameters in the Brooks and Corey function in this manner to different textural classes. Madankumar (1985) correlated the parameters in the exponential function by Campbell (1974) with bulk density and sand, silt, and clay fractions. Rawls and Brakensiek (1989), and Vereecken et al. (1989) similarly correlated soil water retention parameters to soil properties. The applicability of these methods was tested by Tietje and Tapkenhinrichs (1993). Wösten et al (1995) derived class and continuous pedotransfer functions for generating soil hydraulic characteristics.

In a different approach, Arya and Paris (1981) predicted the soil water retention curve from particle-size distribution data, dry bulk density and particle density. Their model was modified later by Haverkamp and Parlange (1986), who applied the concept of shape similarly between the retention curve and the cumulative particle-size distribution for sandy soils without organic matter. Mishra et al. (1989), in addition, evaluated the uncertainty of the retention parameters obtained from particle-size data.

Sweden, like several other countries, has a history of analyzing available soil water retention data. For example, Andersson (1988) used a tangent function, Jonasson (199 1) employed different van Genuchten type functions, and Jansson (1992) applied a composite model built around the Brooks and Corey function.

In this paper we test the applicability of different functions for describing the water retention prop&ties of Swedish soils. A logistic distribution function is used for describing the cumulative particle-size fraction data of these soils. The selected soil water retention curve (SWRC) and cumulative particle size (CP) model parameters are then correlated to study possible interrelationships between the water-retention and particle-size distribution curves. Empirical methods are used to derive a number of mean and variance estimators from the particle-size data. Regression equations for the fitted SWRC and CP models, and empirical mean and variance estimators of the CP distribution, will also be formulated.

THE SWEDISH SOILS DATABASE

The Swedish soil physical database contains data from 2025 soil layers representing about 300 soil profiles. The data have been published by the Swedish University of Agricultural Sciences in Uppsala (Andersson and Wiklert 1967, 1972, 1977a, 1977b, 1977c; Wiklert et al. 1983) and are available in IBM PC compatible format together with a utility program for plotting and carrying out calculations (Jansson 1992).

Water retention data in the Swedish database were determined on 1 O-cm-long undisturbed soil cores having a diameter of 7 cm. The saturated water content was considered equal to total porosity as calculated from the dry bulk density and the particle density. Water contents at suctions of 5, SO, and 100 cm were obtained by means of a hanging water column. A vacuum pump connected to a pressure control system was used for retention measurements at 300 cm.

For the purposes of this study, we excluded organic soils having a porosity greater than 73% and a loss of ignition value exceeding 10%. Of the remaining 1604 soil samples, 1106 had six or more measured SWRC data. Whereas the maximum number of measured SWRC values was 14, relatively few, 184 soils, had water contents measured at the same pressure head. A subset of the 184 samples was used for deriving an appropriate SWRC model as discussed later. This subset involved soils having retention values at pressure heads of 1, 5, 50, 100, 300 and 15,000 cm. Once the SWRC model was selected, soils having relatively unusual properties (e.g., those having 80-cm-thick organic material accumulation, soils formed on glacial moraine material, or soils having a high swelling clay content) were excluded from the 184-sample database. The unusual nature of the SWRC curve for these soils resulted in relatively high fitting errors. As samples were excluded in the above manner, a few soils with relatively small SWRC fitting errors were also removed. The remaining database contained data of 156 soils. Table 1 lists the mean of several soil properties for the 156 Samples as arranged in three textural groups (TK 1-TK3).

Particle-size distributions in the database were determined using conventional methods (Ljung 1987) based on H_2O_2 pretreatment to eliminate organic matter. Size fractions up to 60 μm were determined by pipette sampling, whereas the coarse fractions (60-200, 200-600 and 600-2000 μm) were obtained by wet sieving. Swedish particle-size limits differ from the USDA standard and involve the loss of ignition percentage. For comparability, particle-size per-

TABLE 1
Average of soil properties of the data set used

Case	Sand	Silt	Clay	OM	ρ	Pressure head (cm) 1	300	15000	N
		%		%	Mg/m³	θ (m³/m³)			
All	6.0	65.0	29.1	3.0	1.42	46.9	36.2	15.6	156
TKl	10.8	80.8	8.4	2.5	1.44	45.8	28.4	6.6	40
TK2	5.3	67.8	27.0	2.9	1.43	46.5	37.2	14.1	72
TK3	2.8	46.0	51.2	3.5	1.40	48.5	41.5	26.2	44

TKl - TK3 are textural groups: for TK1 clay ≤ 15%; for TK2 clay > 15% and clay <40%; for TK3 clay ≤ 40%.

Soils of the database

Sample No.	Profile name	Sample No.	Profile name
1-10	Nordvik 1	11-19	Nordvik 2
20-29	Vassbo 1	30-39	Alvgarden 1
40-49	Uddeholm 1	50-61	Apertin 1
62-71	Nuntorp 1	72-81	Nuntorp 2
82-88	Ryholm 2	89-102	Marsta 1
103-108	Kungsangen	109-1 18	Thorsatra 2
119-124	Thorsatra 3	125-129	Thorsatra 5
130-131	Tonnersa 1	132-138	Tonnersa 2
139-146	Aby 1	147-156	Ingelstorp 1

centages were recalculated by excluding the loss of ignition.The used particle-size limits were 2, 6, 20,60,200,600, and 2000 μm.

The 156-element subset of Swedish soils represents a wide range of soil textures, organic matter contents, and geological origins. Clay content varied between 3 and 70.8%, and sand content between 0.1 and 47.5%. Organic matter content was calculated from the loss of ignition values as given in Ljung (1987).The highest organic matter content was 9.9%, and dry bulk density ranged between 0.92 and 1.76 Mg/m³.

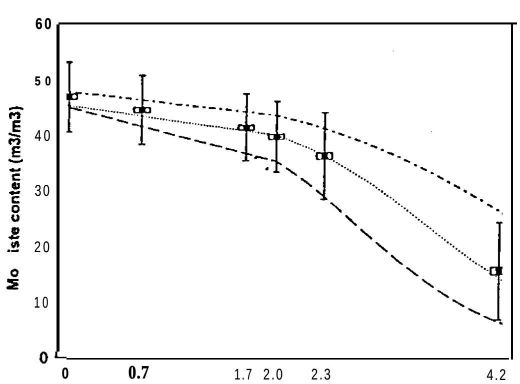

Fig. 1. Water retention characteristic curves of three soil texture groups as described by the M8 SWRC model.

Mean±SD
Mean±SE
Mean
TKl
TK2
TK3

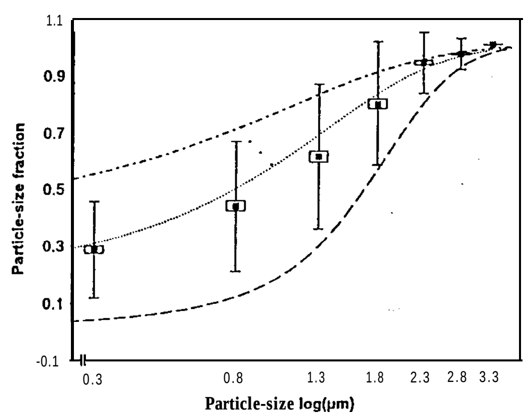

Fig. 2 Cumulative particle-size distribution curves of three texture groups as described by a logistic distribution function.

Mean±SD
Mean±SE
Mean
TK1
TK2
TK3

TABLE 2
Models fitted to the water retention data of soils

Model name	Equations	Parameters	Number of parameters
M1	$\theta = \theta_r + (\theta_s - \theta_r)/[1+(\alpha h)^n]^m$	θ_s, e,, α, n and m	5
M2	$\theta = \theta_s/[1+(\alpha h)^n]^m$	θ_s, α, n and m	4
M3	$\theta = \theta_r + (\theta_s - \theta_r)/[1+(\alpha h)^n]^m$ $n = 1/(1-m)$	θ_s, θ_r, α and m	4
M4	$\theta = \theta_r + (\theta_s - \theta_r)/[1+(\alpha h)^n]^m$ $n = 2/(1-m)$	θ_s, θ_r, α and m	4
M5	$\theta = \theta s/[1+(\alpha h)^n]^m$ $n = 1/(1-m)$	θ_s, α and m	3
M6	$\theta = \theta s/[1+(\alpha h)^n]^m$ $n = 2/(1-m)$	θ_s, a and m	3
M7	$\theta = \theta_r + (\theta_s - \theta_r)/[1+(\alpha h)^n]^m$ $m = 1$	θ_s, θ_r, α and n	4
M8	$\theta = \theta_s/[1+(\alpha h)^n]^m$ $m = 1$	θ_s, α and n	3
M9	$\theta = 1/[1+(\alpha h)^n]^m$ $m = 1$	α and n	2

θ = soil water content (m^3/m^3)
h = pressure head (cm)

Figure 1 shows the mean, standard error, and standard deviation of the mean of the six measured retention data points of the 156 Swedish soils. This figure also shows the fitted SWRC curves of the separate TK1-TK3 textural groups, and Fig. 2 illustrates the fitted CP curves for these textural groups.

SOIL-WATER RETENTION MODELS

The Swedish soils database was used to test nine different soil-water retention models, which are listed in Table 2. The models were fitted to the measured data using nonlinear least-squares optimization (Marquardt 1963). The sum of squared error (SSE) was calculated for each SWRC model, with the degree of freedom (DF) calculated as M(N-p), where M is the number of samples, N is the number of retention data points, and p is the number of model parameters (see also Table 2).The overall SSE of the nine SWRC models (Ml to M9 in Table 2) as fitted to the six retention data points of all soil samples are given in Table 3. Models M2 to M9 in Table 2 may be regarded as special forms of Ml, assuming specific restrictions (e.g., $\theta_r = 0$ for M2); therefore, the F-test can be carried out for model M_i (i = 2 to 9). Model M_i was accepted if the critical value of the F distribution having degrees of freedom (DF_i–DF_1, DF_1) exceeded the F-ratio:

$$F = \frac{(SSE_i - SSE_1)DF_1}{SSE_1(DF_i - DF_1)}$$

The fitting error of the five-parameter van Genuchten model (Ml) was compared with

TABLE 3
Comparison of fitted retention models by fitting errors and results of F-test

Fitted model		SSE	DF	F	P
M1	184	993.2	184		
M2	184	999.2	368	0.006	1 .000
M3	184	2207.2	368	1.222	0.087
M4	184	2984.9	368	2.005	0.000
M5	184	230 l.6	552	0.659	1 .000
M6	184	3039.8	552	1.030	0.413
M7	184	1214.2	368	0.223	1 .000
M8	184	1459.9	552	0.235	1 .000
M9	184	9148.7	736	2.737	0.000

TABLE 4
Parameter values of the fitted SWRC and CP models.

Sample group		Parameter value and standard error of the SWRC model						N
		θs*	α_{SWRC}	n_{SWRC}	SE of θ_s	SE α_{SWRC}	SE n_{SWRC}	
All	*	47.2	0.00029	0.4719	0.60	0.00003	0.022	156
	**	48.3	0.00196	0.5831	0.60	0.00073	0.021	
TKI	*	46.2	0.00143	0.5972	1.01	0.00022	0.047	40
	**	47.8	0.00646	0.7374	1.04	0.00273	0.046	
TK2	*	46.0	0.00029	0.5366	0.59	0.00003	0.027	72
	**	47.0	0.00~158	0.5805	0.74	0.0001 5	0.025	
TK3	*	48.8	0.00005	0.3925	1.04	0.00001	0.046	44
	**	51.0	0.001114	0.4473	1.73	0.00004	0.030	

		Parameter value and standard error of the CP model				N
		α_{CP}	n_{CP}	SE of α_{CP}	SE of n_{CP}	
Au	*	8.45	0.7120	0.391	0.024	156
	**	20.92	1.0715	2.690	0.044	
TKl	*	43.32	1.1 I44	2.251	0.05x	40
	**	59.67	1.5710	11.550	0.1 I7	
TK2	*	7.35	0.8056	0.289	0.0026	72
	**	10.65	(I.9704	I.180	0.037	
TK3	*	2.00	0.700')	0.1 17	0.020	44
	**	2.49	0.7829	0.290	0.043	

*Parameter values of group fitting.

**Mean of parameter values optimized for data of soil samples separately.

those of the other models (M2 to M9) by using the above F-test.

Results of the F-test (Table 3) show that the four-parameter M4 and the two-parameter M9 models were significantly worse than the other functions. Of the six functions that fitted the retention data as well as the five-parameter van Genuchten model (the M2, M3, M5, M6, M7, and M8), the three-parameter M8 function was selected for further study.This function will now be referred to as the SWRC model.

In order to determine one general SWKC curve for each soil textural group, the selected M8 SWKC model was fitted to all retention data in each TK group by nonlinear regression using the Statistica package (Statistica 1994).This fitting procedure, referred to as group fitting. produced the SWRC curves shown in Fig. 1. Parameter values of

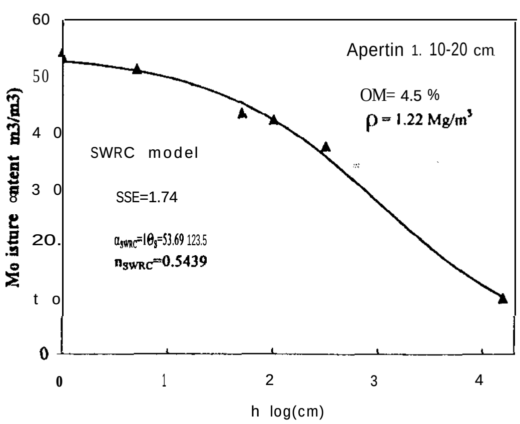

Fig. 3a Water retention curve of a typical Swedlsh soil.
—— Fitted SWRC model A Measured SWRC data

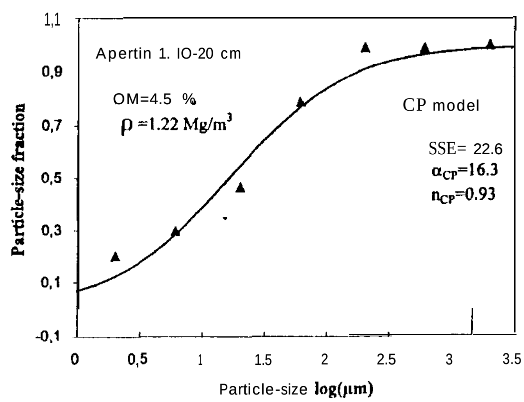

Fig. 3b Cumulative particle-size distribution curve of a typlcal Swedlsh soil.
—— Fitted CP model A Measured CP data

TABLE 5

Errors of the SWRC and pedotransfer models

Pressure head cm	Mean error ME	Estimation errors SE of ME	Mean SSE	$\sum_{i=1}^{156}$ SSE
		Fitted SWRC model		
θ_1	0.67	0.07	6.0	931.0
θ_5	-0.56	0.08		
θ_{50}	-0.29	0.08		
θ_{100}	-0.14	0.06		
θ_{300}	0.36	0.09		
θ_{15000}	-0.01	0.05		
		E1 *model*		
θ_1	-0.09	0.11	75.8	11819.9
θ_5	-1.10	0.20		
θ_{50}	-1.09	0.29		
θ_{100}	-0.76	0.32		
θ_{300}	0.11	0.37		
θ_{15000}	0.90	0.28		
		E2 model		
θ_1	0.02	0.10	57.6	8982.3
θ_5	-1.34	0.10		
θ_{50}	-1.00	0.26		
θ_{100}	-0.70	0.27		
θ_{300}	0.15	0.30		
θ_{15000}	0.47	0.27		
		E3 *model*		
θ_1	0.08	0.16	72.5	11303.1
θ_5	-1.24	0.21		
θ_{50}	-0.74	0.28		
θ_{100}	-0.42	0.30		
θ_{300}	0.36	0.30		
θ_{15000}	0.12	0.36		
		E4 model		
θ_1	-0.04	0.11	73.5	11471.1
θ_5	-1.41	0.19		
θ_{50}	-1.03	0.28		
θ_{100}	-0.72	0.31		
θ_{300}	0.11	0.37		
θ_{15000}	0.86	0.29		

the group-fitted SWRC curves, as well as the mean parameter values of the M8 SWRC model fitted to retention data of each soil sample separately are given in Table 4. The same technique will be used below for the cumulative particle-size curves.

Figure 3a shows a typical example of the observed and fitted retention curves. Fitted parameter values and their standard deviations obtained by group and individual fitting of the M8 SWRC model are listed in Table 4. Results indicate that the SWRC parameter α increases from clay to the sand texture group, whereas the opposite is true for the parameter n. Relatively large values for α and n are characteristic of the coarse-textured soils, whereas the opposite is the case for the clay category (fine-textured soils). Values for n and α for the silt category are in-between those of the sand and clay groups. These results reflect the fact that water retention values at 3 given pressure head are lower, and the SWRC curves are steeper, for coarse-textured compared with fine-textured soils.

Table 5 shows the mean error of the fitted SWRC model at each pressure head. The selected

M8 SWRC model generally overestimated water retention at saturation (h = 1 cm) and at field capacity (h = 300 cm), whereas it underestimated retention at 5, 50, and 100 cm pressure heads. Still, the suitability of the chosen three-parameter function for describing Swedish SWRC data is indicated by the relatively small values of the mean SSE in Table 5.

CUMULATIVE PARTICLE-SIZE DISTRIBUTION ANALYSIS

A two-parameter function was used to describe the cumulative particle-size (CP) data as follows:

$$F(d) = 1 / (1 + (\alpha_{CP}/d)^{n_{CP}} \quad (1)$$

where F is the cumulative particle-size fraction, d is the particle diameter (mm), and α_{CP} and n_{CP} are fitting parameters. The parameter α in the SWRC model (see Table 2) is marked as α_{SWRC}, and in the CP function as α_{CP}. Similar subscripts (SWRC and CP) are used for the parameter n. Notice that Eq. (1) can be rewritten as:

$$F(X) = 1 / (1 + e^{-((x-a)/b)}) \quad (2)$$

where $x = \ln(d)$. The function $F(x)$ defined by Eq. (2) is known as the logistic probability distribution function (e.g., Johnson and Kotz 1970). The parameters in Eq. (1) and Eq. (2) are related as follows:

$$a = \ln(\alpha_{CP}) \qquad b = 1/n_{CP}$$

The above relationships suggest that parameters in Eq. (1) carry information about the particle-size distribution. This connection was previously shown to hold by Campbell (1985) and later used by Haverkamp and Parlange (1986) in their analysis of shape-similar SWRC and CP distributions.

Without having to invoke a particular function, one can estimate the distribution parameters directly from the CP data. The idea stems from previous parameter estimation methods using the method of moments (e.g., Ledermann 1984) and is described in Appendix 1. The estimation procedure in Appendix 1 is ‘model-free since the method depends only on the observed distribution and not on the model used to describe the data. We also estimated the mean (DG) and the standard deviation (KG) of the particle-size distributions from the sand, silt, and clay fractions, as was done previously by Campbell (1985).

The logistic distribution (Eq. (1)) was previously considered suitable for about half of the soils in the USDA triangle (Buchan 1989). Figure 2 shows the distribution function fitted to particle-size data of the three TK groups. A typical example of the CP model fitted to soil texture data is given in Fig. 3b.The percentage of poor fits of the CP model to the total number of CP data sets was found to be 12%. From this we concluded that the logistic model is useful for the majority of Swedish soils. An advantage of using the logistic distribution model is that this model yields estimates directly for the parameters α_{CP}, which reflects the modal particle-size, and n_{CP}, which gives the standard deviation of the CP distribution. Table 4 indicates that the α_{CP} and n_{CP} parameter values are characteristic for the different TK groups.The α_{CP} and n_{CP} parameter values are becoming larger from clay to sand, again showing that the modal particle-size and the slope of the distribution are higher for coarse-textured materials (Fig. 2). The texture-dependent behavior of the SWKC model (Fig. 1) is used below for predicting SWRC parameters from selected soil properties and CP distribution parameters.

PEDOTRANSFER FUNCTIONS

Parameters in the M8 SWRC model were correlated with the original soil properties alone, and together with the particle-size distribution parameters, using step-wise linear regression analysis.The log transform of the α values were also used in the analysis.

In the regression analysis, the M8 SWRC model parameters were taken as the dependent variables. The independent variables were arranged in four groups.The first set of predictor variables (El) was formed by the original soil properties. The second set (E2) considered the parameters in the CP model in addition to those in El. The third set (E3) included the LTE and LTD empirical mean and variance estimators, in addition to those of El. Finally, the fourth set (E4) contained the DG and RG parameter values and those in El .The obtained pedotransfer functions are summarized. in Table 6.

The accuracy of the different pedotransfer, SWRC, and CP models was tested by calculating the SSE of the fit using the measured soil-water retention or CP data. Absolute values of the residual of the moisture content (denoted by $|E|$ and expressed in volumetric percentages) was used for evaluating the accuracy of the different pedotransfer models (El to E4). First, $|E|$ was calculated separately for each of the six retention data, and next the mean was taken of the six $|E|$

TABLE 6

Pedotransfer functions for parameters in thr M8 SWRC model

SWRC parameter	Regressed Variables	Pedotransfer functions	R^2	Predictive model	N
θ_S	ρ, S, I, OM	$\theta_S = 111.5-40.0*\rho-8.1*I+14.9*S -0.74*OM$	0.83	E1	156
	ρ, α_{CP}, OM C	$\theta_S = 107.6 -38.5*\rho-0.5*OM-3.7*\ln(C) +0.1*\alpha_{CP}$	0.85	E2	156
	ρ, LTE, OM C	$\theta_S = 106.9-39.6*\rho-0.68*OM+2.47*LTE +5.27*\ln(C)$	0.84	E3	156
	ρ, IX, OM IS	$\theta_S = 111.1-38.0*\rho-15.0*I+184.2*DG -0.48*OM-16.8*S$	0.85	E4	156
α_{SWRC}	ρ, I, C, S	$\ln(\alpha_{SWRC}) = -7.3-3.5*\rho-0.8*\ln(C)+10.2*S +3.7*I$	0.60	E1	156
	ρ, α_{CP}, C	$\ln(\alpha_{SWRC})=-2.27-3.5*\rho+0.027*\alpha_{CP}-4.9*C$	0.61	E2	156
	LTE, ρ	$\ln(\alpha_{SWRC})=-6.4+3.03*\ln(LTE)-3.2*\rho$	0.58	E3	156
	DG, ρ, C	$\ln(\alpha_{SWRC})=-1.05+0.82*\ln(DG)-3.5*\rho -0.7*\ln(C)+30.4*DG$	0.61	E4	156
n_{SWRC}	I	$n_{SWRC}=0.96*I$	0.41	E1	156
	α_{CP}, n_{CP} C	$n_{SWRC}=0.36+0.35*n_{CP}-0.002*\alpha_{CP} -0.37*C$	0.66	E2	156
	LTD, C	$n_{SWRC}=-0.2-0.35*C+0.37*LTD$	0.64	E3	156
	I	$n_{SWRC}=0.96*I$	0.41	E4	156

ρ = bulk density (Mg/m^3); OM = organic matter (%); s = sand(>50 μm).
I = silt (50-2 μm); C = clay (<2 μm); DG = mean estimator of CP by Campbell
LTE = mean estimator of CP by method of moments.
LTD = variance estimator of CP by method of moments.

values for one retention curve (ME). Estimation was considered good if the absolute or the mean absolute difference between the estimated and measured retention values was $\leq$2.5%. The SWRC prediction was judged to be poor when the estimation error was higher than the threshold value of the upper decile of the SSE of the M8 SWRC model fitted individually to SWRC data of the 156 soils (SSE > 12).The fit of the CP model was similarly considered poor whenever SSE exceeded 180. The estimation efficiency, as expressed by the ratio of good to total estimations, is shown in Fig. 4.

The E1 model (Table 6) for θ_S used as deterministic predictors of the bulk density, and the silt and clay fractions of the soil. The clay and silt fractions and the soil bulk density determined the logarithm of parameter α. The model parameter θ_s was determined within 83% and the parameter α within 60% by the basic soil properties. The parameter n was defined mostly by only the silt fraction. The coefficient of determination (R^2) was relatively low.

The E2 predictive model equations in Table 6 combined soil properties with the fitted CP parameters. In this way the information content of particle-size distribution parameters is potentially better preserved. The regression equation for θ_S includes, in addition to the clay fraction, the parameter α_{CP} as a substitute for the sand and silt fractions as used for the E1 model. In the regression equation for α_{SWRC}, α_{CP} substituted the silt fraction, while keeping the clay fraction and bulk density as predictors. The R^2 of the regression equation increased or remained almost the same as for E 1. The R^2 of the regression equations for

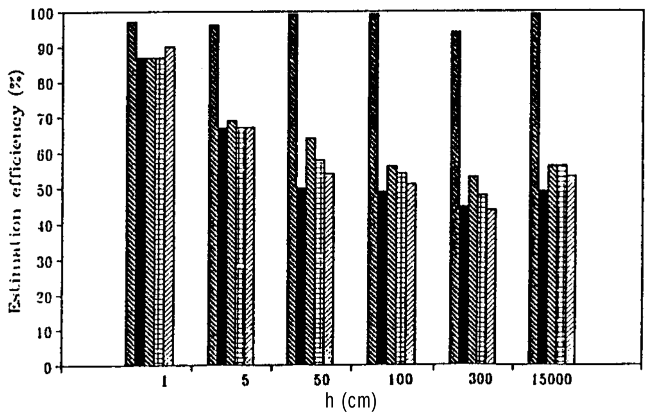

Fig. 4. Ratio of 'good' water retention estimations by pressure heads.

Fitted SWRC model; El model; E2 model; E3 model; E4 model

α_{SWRC} was 10% smaller, and for n_{SWRC} 1% smaller, when only the CP parameters were used as independent predictors in the regression analysis.

A modal pore diameter (r_α) of a soil may be obtained by considering the a parameter of the SWRC function as an equivalent soil water pressure value, i.e.:

$$r_\alpha = 0.1469 / \alpha_{SWRC} \quad (3)$$

where r_α is the pore diameter in cm, and α_{SWRC} is the pressure head in 1 cm of water at roughly the inflection point of the SWRC function. Because the SWRC model parameter α_{SWRC} and the pore diameter r_α calculated by Eq. (3) differ only by a constant, we performed a statistical analysis using the original parameters of the SWRC and CP models.

The significant relationship between α_{SWRC} and α_{CP} found in our study is consistent with the relationship between mean pore and mean particle diameter first described by Arya and Paris (1981) and later modified by Arya and Dierolf (1992). In our analysis we observed that fitted α values in the SWRC model showed an approximately lognormal distribution. Based on this fact α_{SWRC} values were transformed into logarithms in the regression analysis with basic soil properties and CP distribution parameters (see Table 6).

The regression equation established between the parameters n in the SWRC and CP models also showed a high coefficient of determination (R^2 = 0.60).The significant relationship between n_{SWRC} and n_{CP} experimentally verifies the shape similarity assumption of the SWRC and CP curves invoked by Brooks and Corey (1964) and Haverkamp and Parlange (1986).

The E3 and E4 predictive SWRC models in Table 6 used empirical particle-size distribution parameters together with the original soil properties.The E3 model used the LTE and LTD distributional parameters as its predictor variables instead of the fitted CP parameters.The results of the regression analysis illustrate the applicability of the LTE and LTD empirical parameters for representing the particle-size distribution of soils (see Table 6).

The E4 predictive SWRC model, using the DG and RG empirical parameters as its predictors, has a similar R^2 as the regression equation for α based on the original soil properties. However, the parameters DG and RG were not deterministic in the regression equation for parameter n. Therefore, the n in the E4 model was regressed only against the silt fraction, as was done for the El model. We believe that the statistically significant relationships found between the SWRC and CP model parameters are useful in practice for estimating SWRC curves via parameter estimation of the SWRC model.

Parameter estimation by regression is often considered to give less accurate results than methods based on regression estimates of individual SWRC data points (e.g., Thomasson and Carter 1992). However, simulation models in soil hydrology generally require continuous functions for the SWRC.To further evaluate the accuracy of **the El to E4** SWRC predictive models, calculated retention data were compared with the measured values for the 156 soils. The errors of estimation were evaluated by calculating the SSE of the different predictive models. Table 5 shows these errors of estimation. The results in this table indicate that the mean error of estimation at each pressure head is less than 2% for all predictive models. The sign of the deviation for all predictive models is mostly negative in the low pressure head range (h $\leq$100 cm) and positive at 300 and 15000 cm.

The predictive models reproduced the behavior of the selected SWRC model qualitatively, i.e., they generally underestimated retention values in the 5 to 100-cm range and overestimated them at the 300-cm pressure head. Of the four types of regression approaches, E3 was the only one that estimated the retention values at 15,000 cm as well as did the fitted SWRC model (see Table 5).

The mean SSE of the prediction was in most cases 10 to 12 times larger than the SSE of the fitted SWRC model. The E2-model had the lowest mean SSE, but it was still about 10 times higher than the SSE of the fitted SWRC model. We note here that the M8 SWRC function, when fitted individually by nonlinear regression to the 156 soils, used 468 free parameters, whereas the E2 predictive model contained only 13 free parameters for all cases.

Figure 5 illustrates one example of the types of predictions obtained with the El to E4 models. Notice the very similar behaviors of models E2 and E3, and of models E 1 and E4 in the 100 to 300-cm pressure head range. Also, the models using parameters based on detailed particle-size distribution (E2 and E3) gave better results than the others.

The invoked measure of having good predictions, i.e., when the absolute difference of estimated and measured retention value was $\leq$ 2.5% at a given pressure head, is shown in Fig. 4. The results show that the probability of estimating a water retention value within 2.5% is high in the low pressure head range (5 1-93%), lowest at field capacity 45-54%), and intermediate at the wilting point (50-56%). Also, the mean error of the predictive models increased with increasing pressure head values. This phenomenon is consistent with known behavior of retention data, i.e., decreasing values with increasing pressure heads but with their absolute error increasing.

When the mean |E| was used, i.e., as derived from all six retention data, the prediction efficiency varied from 57 to 67%. The El and E-l models showed almost the same estimation efficiency (57 and 58%), while E3 yielded 59%.The highest estimation efficiency (67%) was obtained with the E2 model. This predictive model resulted in the highest number of good estimations. An important advantage of using the DG and RG empirical parameters in the Ed model was its relatively high accuracy in the low pressure head range (< 100 cm).

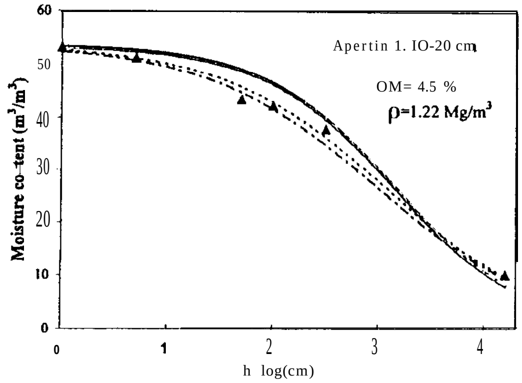

Fig. 5. Water retention curve as predicted by the different pedotransfer models.

▲ Measured SWRC data ——— El model
------E2 model —·—·— E3 model
. . . . E4 model

An evaluation of the mean SSE of the predictions, as given in Table 5 is also informative. The number of poor estimations was 22%, 12%, 17%, and 19% for the E 1, E2, E3, and E4 models, respectively. Altogether, 1 1 of 156 soils were qualified as poorly predictable by all four models. The fitting error of the SWRC model was also high (SSE > 12) for six of these 11 soils.The high fitting error of the CP model also seemed to be associated with poor estimations (6 of 11 soils). From this we conclude that poor predictions should be expected for soils that have different shapes of the SWRC curve as compared to the predictive modcl.

Errors in the predicted SWRC curves differed in the TK groups for models E2 and E3. The highest mean estimation error (SSE = 7 1 .0 for E2) was found for the coarse textured, and the lowest (SSE = 54.1 for E2) for the fine-textured TK group. However, the same tendency was apparent by the mean fitting errors of the SWRC and CP models in the TK groups.

For one soil profile (Märsta l), the frequency of poor estimations was high, i.e., 10 of the 14 horizons showed poor results. Four of the 11 most poorly predicted results pertained to the Märsta 1 soil profile. Overall, poor predictions generally resulted from having a high organic tnatter content (OM >5%). However, several of the poorly estimated SWRC curves did not have a high organic matter content. In this case, the type of organic matter or clay minerals present on the soil structure may have caused the extraordinary results. In general, the obtained SWRC estimation techniques seem to be acceptable for most non-site specific applications for Swedish soils. The wider applicability of the predictive procedure may need to be evaluated in future studies.

CONCLUSIONS

A three-parameter van Genuchten-type model was used to describe the water retention characteristic data of Swedish soils. A logistic distribution model was proposed for the particle-size data of soils. Fitted logistic CP parameters, and empirical mean and variance estimators (LTE and LTD), were used to represent the CP distribution of soils. SWRC model parameters were regressed with original soil properties and CP distribution parameters. A significant correlation was found between the SWRC and CP model parameters using linear regression.The parameter correlation was found to be consistent with earlier assumptions about the shape similarity of the SWRC and CP curves.The obtained regression equations allow prediction of the SWRC curve from soil properties and particle-size data. The ratio of good estimation generally increased when using CP distribution parameters, i.e., fitted CP model parameters, and the derived LTE and LTD empirical parameters.

The best predictions (67% of good estimations) were produced with the E2 model using fitted CP parameters, together with the clay and silt fractions, and the soil bulk density. Poor estimation is to be expected when the organic matter content is high (OM >5%).The mean error of the SWRC prediction decreased from the coarse-textured to the fine-textured soil group. The procedure for SWRC pedotransfer parameter estimation, but not the parameter values as estimated from basic soil properties and particle-size parameters, seems to be applicable to a wider array of soils than the Swedish soils database alone.

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APPENDIX 1

Properties of the logistic distribution used for the CP data motivated the calculation of CP parameters using a procedure other than the usual nonlinear least-squares methodology. The method consists of the following steps:

- Expressing moments in terms of parameters appearing in Eq. (2). In the present case: E = a and $D^2 = 1/3*b^2*\pi^2$, where E is the mean of the logistic distribution and D^2 the variance.
- Substituting moments by their corrected empirical equivalents (in the present case: LTE for E and LTD for D).
- Calculating the parameters by the moment equations (in the present case α_{CP} = exp(LTE) and $n_{CP} = (1/3)^{1/2}*\pi/(LTD)^{1/2}$).

Empirical means (LTE) and variances (LTD^2) were calculated as the corresponding theoretical moments of the empirical density function, which is derived from the empirical distribution function. The empirical distribution function, in turn, is calculated directly from the measured particle-size data.

The lower limit (0%) of the particle-size distribution was taken to be at 0.5 μm and denoted by x_0. The other particle-size limits (2,6,20,. . ., 2000 μm) were associated with N nodes denoted by $x_1, \ldots, x_n$. Let r_n be the relative frequency of particles with size between x_{n-1} and x_n, $n = 1..N$, and r_1 let the relative frequency of particles smaller than x_1. First, the density function is found which (1) has the support $[x_0, x_n]$, (2) is linear between two consecutive nodes, and (3) has an integral equal to r_n between two consecutive nodes, x_{n-1} and x_n.

A function satisfying these requirements is continuous except at the nodes. The function was chosen by minimizing the sum of absolute jumps at the nodes.

APPENDIX 2

List of Abbreviations and Symbols

α	parameter in the SWRC and CP models
α_{CP}	parameter in the CP model; modal particle-size
α_{SWRC}	paratneter in the selected M8 SWRC model
θ	soil moisture content (m^3/m^3)
θ_r	parameter in several SWRC models; residual water content
θ_s	parameter in the SWRC models; water content at saturation (m^3/m^3)
ρ	dry soil bulk density (Mg/m^3)
C	clay content (C < 2 μm) (%)
CP	cumulative particle-size distribution
d	particle-size (μm)
DF	degree of freedom
DG	mean estimator of CP by Campbell (1985)
|E|	absolute value of residual calculated as | measured-estimated |
E1	SWRC predictive model using basic soil properties
E2	SWRC predictive model using basic soil properties and CP model parameters
E3	SWRC predictive model using basic soil properties and the LTE, LTD distribution parameters
E4	SWRC predictive model using basic soil properties and the DG, RG distribution parameters
F(d)	cumulative particle-size distribution function
F(x)	logistic distribution function
F	value of F-ratio
h	suction head (cm)
I	silt content (2 μm ≤ I ≤ 50 μm) (%)
LTD	variance estimator of CP by method of moments
LTE	mean estimator of CP by method of moments
m	parameter in the SWRC models; exponent
ME	mean of |E|
SWRC	soil water retention characteristic
n	paranleter in the SWRC and CP models; exponent
n_{CP}	parameter in the CP model; variance of CP distribution

n_{SWRC}	parameter in the selected SWRC model; exponent
OM	organic matter content (%) calculated from loss of ignition by Ljung (1987)
r_α	pore diameter calculated from α_{SWRC} by Eq. (3)
$r_0, \ldots, r_n$	relative frequency of particles in particle-size classes
RG	variance estimator of CP by Campbell (1985)
S	sand content (>50 μm) (%)
SD	standard deviation
SE	standard error
SSE	sum of squared error
TK	textural category of soils
TK1	the sand category if C ≤ 15%
TK2	the silt category if C > 15% and C <40%
TK3	the clay category if C ≥40%
$x_0, \ldots, x_n$	cutpoints of particle-size classes

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