Simple Method for Predicting Drainage from Field Plots[1]

J. B. Sisson, A. H. Ferguson, and M. Th. van Genuchten

ABSTRACT

When the one-dimensional moisture flow equation is simplified by applying the unit gradient approximation, a first-order partial differential equation results. The first-order equation is hyperbolic and easily solved by the method of P. D. Lax. Three published $K(\theta)$ relationships were used to generate three analytical solutions for the drainage phase following infiltration. All three solutions produced straight lines or nearly straight lines when log of total water above a depth was plotted versus log of time. Several suggestions for obtaining the required parameters are presented and two example problems are included to demonstrate the accuracy and applicability of the method.

Additional Index Words: **Cauchy problem, redistribution, characteristic value problem, hydraulic conductivity, infiltration.**

Sisson, J. B., A. H. Ferguson, and M. Th. van Genuchten. 1980. Simple method for predicting drainage from field plots. Soil Sci. Soc. Am. J. 44:1147–1152.

Water draining from soil profiles is an important factor in many contemporary and environmental problems. In the Northern Great Plains this water is responsible for the annual destruction of thousands of hectares of cropland by contributing to the formation and growth of saline seeps (Brown and Ferguson, 1973; Ferguson et al., 1972). In some irrigated areas, subsurface drainage water contributes to river pollution (Wierenga and Patterson, 1972).

Estimation of the rate and quantity of drainage water contributing to these problems is essential to finding feasible solutions. But estimating these variables requires predicting the hydrologic behavior of large areas and frequently the characterization of many soils under field conditions. This necessitates the use of simple, yet accurate, models which contain parameters that can be obtained on site as quickly as possible.

This paper considers a special class of models based on the assumption of a unit gradient of the total potential head. Several studies (cf. Black et al., 1969, and Davidson et al., 1969, among others) have shown that a unit gradient often exists during the redistribution and drainage phases when a uniform profile is draining freely in the absence of a shallow water table. Three solutions are presented, each based upon a different conductivity equation. Two example problems are further included to demonstrate the use of the present approach.

THEORETICAL CONSIDERATIONS

The equation for predicting the one-dimensional flow of water in porous materials is (Taylor and Ashcroft, 1972):

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K\frac{\partial H}{\partial z}\right) \qquad [1]$$

where

θ = $\theta(z,t)$ is the volumetric moisture content,
t = time,
z = depth (positive downward),
K = $K(\theta)$ is the hydraulic conductivity, and
H = $H(\theta,z) = h(\theta)\text{-}z$; i.e., H(hydraulic head) = h(pressure head) − z(gravitational head).

When a unit gradient in the total head H is assumed, $\partial H/\partial z = -1$, Eq. [1] becomes

$$\frac{\partial \theta}{\partial t} = -\frac{\partial K}{\partial z} = -\frac{dK}{d\theta}\frac{\partial \theta}{\partial z}. \qquad [2]$$

When only the drainage phase is considered, Eq. [2] may be solved subject to the conditions

$$\theta(z,0) = \theta_i(z) = \begin{cases} \theta_c \text{ if } z < 0 \\ \theta_m \text{ if } z > 0 \end{cases} \qquad [3]$$

where the subscripts c and m denote minimum and maximum obtainable values, respectively.

When the profile is saturated initially, θ_m equals the moisture content at saturation, θ_s. In its general case, however, θ_m may be less than θ_s; for example, following an irrigation with a flux less than the saturated hydraulic conductivity.

The initial value problem given by Eq. [2] and [3], also known as a Cauchy or a characteristic value problem, has been the subject of many studies in mathematical and engineering literature (cf. Lax, 1972; Aris and Amundson, 1973, where Aris and Amundson present examples and are an excellent introduction to this subject). The characteristics of Eq. [2] are obtained by solving the following system of ordinary differential equations written in standard form.

$$\frac{dt}{1} = \frac{dz}{dK/d\theta} = \frac{d\theta}{0}. \qquad [4]$$

The right hand expression implies that $d\theta$ must be zero or θ remains constant for certain values of z and t. Since θ is constant, $dK/d\theta$ will be constant and the first two terms in Eq. [4] can be integrated to give

$$z = \left.\frac{dK}{d\theta}\right|_{\theta_i} \cdot t = At \qquad [5]$$

where

$$A = \left.\frac{dK}{d\theta}\right|_{\theta_i}.$$

These results imply that a set of curves (i.e. characteristics) propogate from the initial condition θ_i. If

[1] Contribution from Rockwell International, Rockwell Hanford Operations, Richland, Wash. Document RHO-SA-138. Part of a thesis submitted by senior author in partial fulfillment of the M.S. Degree at Montana State University. Received 13 Nov. 1979. Approved 6 Aug. 1980.

[2] Senior Soil Physicist, Rockwell International, P.O. Box 800, Richland, WA 99352; Professor, Agronomy Department, Montana State Univ.; and Soil Physicist, U.S. Salinity Lab., Riverside, Calif., respectively.

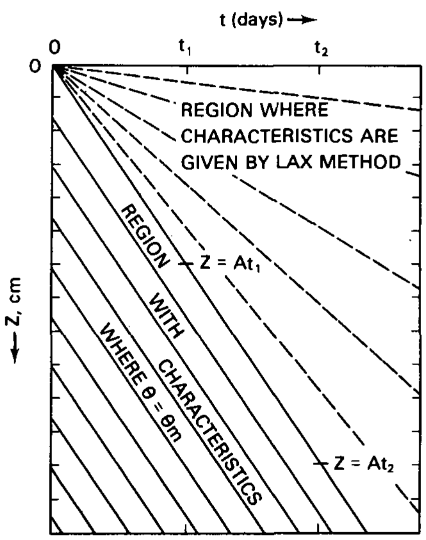

Fig. 1—Characteristics defined by Eq. [5]. Solid lines fill region where water content is unchanging and dashed lines fill region where desorbtion is taking place.

one could travel along one of these curves with an instrument that sensed moisture content, the instrument readings would not change.

The characteristics of Eq. [2] subject to [3] are shown in Fig. 1 as parallel solid lines propagating from $t=0$ and lie in the region given by $z > At$. The wedge-shaped region defined by $0 < z < At$ and shown in Fig. 1 as being filled with dashed lines is devoid of characteristics when the usual mathematical methods are applied. Lax (1972) studied Eq. [2] and showed that unless $\theta_i(z)$ is a monotonically increasing function of z then no continuous solution exists in this region $(0 < z < At)$. If discontinuous solutions were allowed Lax demonstrated that several distinct solutions exist to Eq. [2] with the same initial condition [3]. Lax noted that only one solution exists in the region $(0 < z < At)$ such that the characteristics do not intersect (Note: Intersecting characteristics indicate the development of a shock wave or a mathematical discontinuity). The Lax solution is given by

$$\theta(z,t) = \begin{cases} f(z/t) & \text{for } 0 < z < At \\ \theta_m & \text{for } z > At \end{cases} \qquad [6]$$

where the function f is defined such that

$$f\left(\frac{dK}{d\theta}\right) \equiv \theta \qquad [7]$$

provided that

$$d^2K/d\theta^2 \neq 0 \text{ for all } \theta. \qquad [8]$$

Note here that θ defined by Eq. [6] is continuous throughout z but that the derivative of θ with respect to z is two valued at $z = 0$ and $z = At$, depending whether the limit is taken from the "left" or the "right." In general, if Eq. [8] is violated Eq. [6] becomes multivalued, or imaginary, imaginary solutions are easily obtained by choosing $K(\theta)$ as a cubic polynomial, solving Eq. [6] and determining values of the polynomial coefficients where Eq. [8] is violated.

The behavior of the Lax solution for monotonically increasing functions $K(\theta)$ is schematically shown in Fig. 2 where the desorption of a uniform profile during the early drainage phase is considered (the desorption zone is still confined to the upper part of the profile, i.e., $z < At$). The lower boundary of the desorption zone is given by Eq. (5), with $\theta_i = \theta_m$. This boundary propagates downward at a constant rate $A = dK/d\theta|_{\theta_m}$. The method for obtaining solutions can be summarized in the following three-step algorithm based on the Lax solution scheme:

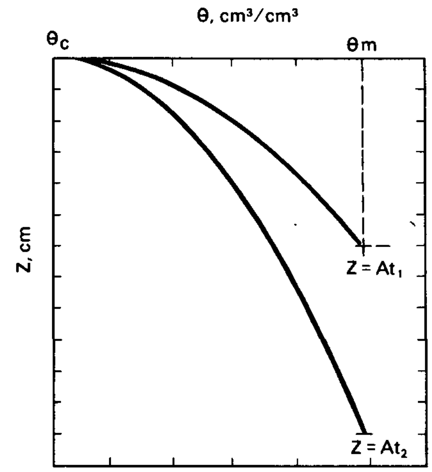

Fig. 2—Typical behavior of Lax solutions with a monotonically increasing $K(\theta)$. Points denoted by t_1 and t_2 are also denoted in Fig. 1.

1) Differentiate a chosen $K(\theta)$ with respect to θ obtaining $dK/d\theta$.
2) Replace the resulting $dK/d\theta$ term with z/t.
3) Solve the resulting expression explicitly for $\theta = f(z/t) = \theta(z,t)$.

Three frequently used $K(\theta)$ relationships found in current literature are presented in Table 1 with all steps in the Lax algorithm. Similar solutions have been published previously for the $K(\theta)$ relationships referenced as Davidson et al. (1969) and Watson (1967). [See Davidson et al. (1969) and Gardner et al. (1970), respectively.] But unlike the previous solutions, found by methods apparently analogous to the separation of variables, the Lax solutions have no arbitrary constants arising from integration. More important, however, the Lax solutions are valid for all times $t \geqslant 0$, i.e., also for the early drainage phase, a phase which has to date been largely ignored.

It should be noted that the values of θ_c (i.e., the minimum obtainable value for the moisture content) in models of Watson (1967) and Davison et al. (1969) (Table 1) are assumed to be zero. Although this does not present any problems for the Watson model, the model based on the exponential conductivity curve will exhibit a mathematical peculiarity in this respect. Because both K and its slope $dK/d\theta$ are non-zero when $\theta = 0$, another characteristic will emanate from the region $(z \leqslant 0,\ t = 0)$ describing the downward propagation of the zero water content. The position of zero moisture content is again given by Eq. [5], whereby it is understood that θ_i is now zero. Although this particular case is physically unobtainable (since no one has observed a dry layer of measurable depth immediately following an irrigation), the complete and correct solution is

Table 1—Algorithm steps for obtaining Lax's solution for the given hydraulic conductivity (K) water content (θ) relations.

		Steps to the Lax solution		
Source of $K(\theta)$	$K(\theta)$†	Step 1‡	Step 2	Step 3
Watson (1967)	$K = K_m\left(\frac{\theta}{\theta_m}\right)^{1/\beta}$	$\frac{dK}{d\theta} = A\left(\frac{\theta}{\theta_m}\right)^{(1-\beta)/\beta}$	$\frac{z}{t} = A\left(\frac{\theta}{\theta_m}\right)^{(1-\beta)/\beta}$	$\theta = \theta_m\left(\frac{z}{At}\right)^{\beta/(1-\beta)}$
Davidson et al (1969)	$K = K_m \exp \alpha(\theta-\theta_m)$	$\frac{dK}{d\theta} = A \exp \alpha(\theta-\theta_m)$	$\frac{z}{t} = A \exp \alpha(\theta-\theta_m)$	$\theta = \theta_m + \frac{1}{\alpha}\ln\left(\frac{z}{At}\right)$
Brooks & Corey (1964)	$K = K_m\left(\frac{\theta-\theta_c}{\theta_m-\theta_c}\right)^{1/n}$	$\frac{dK}{d\theta} = A\left(\frac{\theta-\theta_c}{\theta_m-\theta_c}\right)^{(1-n)/n}$	$\frac{z}{t} = A\left(\frac{\theta-\theta_c}{\theta_m-\theta_c}\right)^{(1-n)/n}$	$\theta = \theta_c + (\theta_m-\theta_c)\left(\frac{z}{At}\right)^{n/(1-n)}$

† Subscripts m and c denote maximum and minimum obtainable values, respectively.
‡ Parameter A defined with Eq. [5].

$$\theta(z,t) = \begin{cases} 0 & \text{for } z < dK/d\theta|_{\theta=0}\cdot t \\ \theta_m + \frac{1}{\alpha}\ln(z/At) & \text{for } dK/d\theta|_{\theta=0}\cdot t < z < At. \\ \theta_m & \text{otherwise} \end{cases} \quad [9]$$

PARAMETER ESTIMATION

Any solution of Eq. [2] involves parameters that must be estimated from experimental data. While θ_m and K_m can be estimated in principle from direct observation during the latter stages of infiltration (in the case of ponding), or from measurements on soil cores, the exponential parameters are probably most easily obtained from nonlinear regression of field data taken sometime after the entire profile is undergoing desorption. Although this procedure holds strictly only for uniform profiles, it may be extended to situations wherein the profile is reasonably uniform or exhibits some weak layering. This may be done by considering the total water, W, above a depth, z:

$$W = W(z,t) = \int_0^z \theta dz. \quad [10]$$

This function is generally smoother with depth and hence more amenable to curve fitting than the moisture contents.

Results of forming W from the solutions in Table 1 (see Table 2) are rather surprising since they all yield nearly straight lines when W vs. t is plotted on log-log paper (provided θ_c/θ_s is small). This straight line relationship was apparently first noted by Ogata and Richards (1957). Wilcox (1959) fitted parameters to a wide range of soils and also found that the log W-log t relationship provided a reasonable approximation of the drainage process. However, Wilcox also reported that the exponential parameter generally decreased with depth. It is easily verified that the total water equations in Table 2 predict the Wilcox anomaly exactly by ignoring the early drainage phase and using times 1 and 10 to obtain the exponents in the Watson model. The resulting exponents will appear to decrease with depth.

Wilcox (1959) modified the original Ogata-Richards model by writing the flux at a depth explicitly as a function of the total water above that depth, and used the resulting expression to estimate deep drainage losses. For the conductivity equations presented here, the Wilcox flux model results in expressions of the form $K = K(W)$ (see Table 2), where $K(W)$ was obtained by eliminating the term z/At from $W(z,t)$ and $K(z,t)$.

To estimate the exponents required by the models in Table 1, z/At was eliminated from $W(z,t)$ and $\theta(z,t)$ and the results given in Table 2. When total water above a depth is treated as a function of θ, it is ap-

Table 2—Total water (W) above a depth as $W(z,t)$ and $W(z,\theta)$ and hydraulic conductivity (K) as $K(z,t)$ and $K(W)$.

	Source of $K(\theta)$		
	Watson	Davidson et al.	Brooks & Corey
$W = W(z,t) =$	$(1-\beta)\theta_m z\left(\frac{z}{At}\right)^{\beta/(1-\beta)}$	$\theta_m z + \frac{z}{\alpha}\left[\ln\left(\frac{z}{At}\right)-1\right]$	$\theta_c z + (1-n)z(\theta_m-\theta_c)\left(\frac{z}{At}\right)^{n/(1-n)}$
Exponent parameter in terms of discernible quantities†	$\theta = \frac{\Delta W}{W_m}$	$\alpha = \frac{z}{\Delta W}$	$n = \frac{\Delta W}{W_m - W_c}$
$W = W(z,\theta) =$	$(1-\beta)z\theta$	$z(\theta - 1/\alpha)$	$nz\theta_c + (1-n)z\theta$
$K = K(z,t) =$	$K_m\left(\frac{z}{At}\right)^{1/(1-\beta)}$	$K_m\left(\frac{z}{At}\right)$	$K_m\left(\frac{z}{At}\right)^{1/(1-n)}$
$K = K(W) =$	$K_m\left(\frac{W}{(1-\beta)W_m}\right)^{1/\beta}$	$K_m \exp(1-\alpha(W_m-W)/z)$	$K_m\left(\frac{W-W_c}{(1-n)(W_m-W_c)}\right)^{1/n}$

† See text for definition of ΔW.

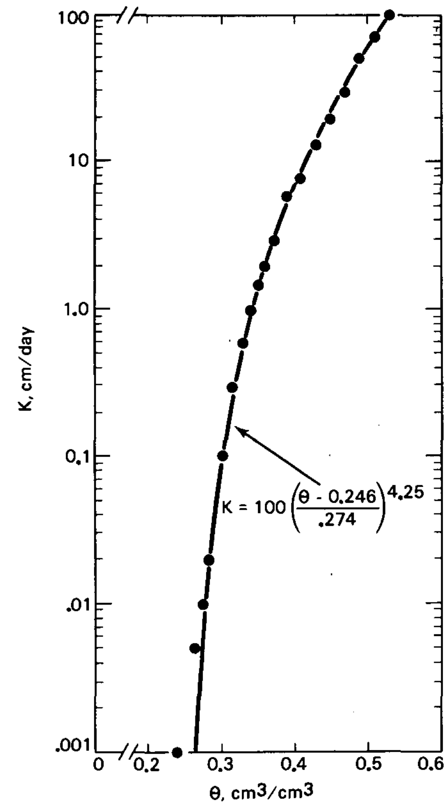

Fig. 3—Hydraulic conductivity $K(\theta)$ for Glendale clay loam.

parent that the exponents can be estimated from simple regression of average total water above a depth versus water content at that depth. The true implication is that parameters can be obtained while conducting the usual evapotranspiration studies without doing an actual infiltration drainage study. Once the exponent is estimated and K_m estimated from infiltration rates, drainage from cropped profiles can be estimated. The added advantage of this type of analysis is that variation among soils from plot to plot can be compensated for by using a drainage estimate for each individual plot.

To obtain a check on the value of exponents computed, the total water equations in Table 2 were evaluated at $z = At$ and exponents solved for explicitly in terms of physical measurements. The quantity of water lost between $t = 0$ and the time when desorption starts at a depth, i.e., $W_m - W(z = At, t)$, is denoted as ΔW and the results given in Table 2.

APPLICATIONS

Two example problems are now considered to demonstrate the use and accuracy of the unit gradient method. The first example compares the unit gradient method with a numerical solution of the complete flow equation [1], while the second example demonstrates the application of the method to an actual field experiment.

Example 1

In this example a large (150 cm long) soil column, uniformly filled with Glendale clay loam and initially saturated ($\theta_s = 0.52$), is allowed to drain freely in the

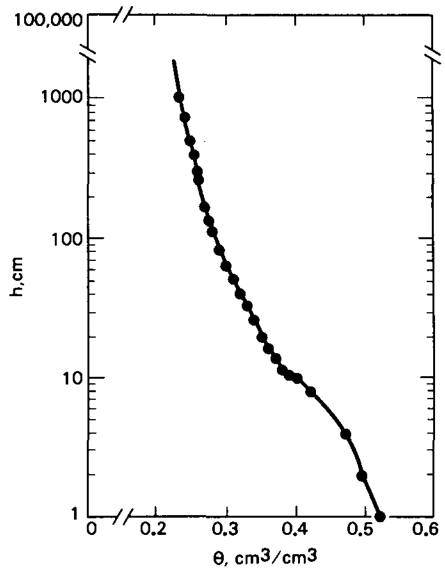

Fig. 4—Moisture release curve for Glendale clay loam.

absence of evaporation. The experimental setup of the column and its soil is described by Dane and Wierenga (1975). The initial and boundary conditions for the numerical model are:

$$\theta(z,0) = \theta_m \quad 0 \leq z \leq 150 \quad t = 0$$

$$K\frac{\partial H}{\partial z} = 0 \quad z = 0 \quad t > 0$$

$$\frac{\partial H}{\partial z} = -1 \quad z = 150 \quad t \geq 0$$

The hydraulic functions K(θ) and h(θ) of the soil are given in Fig. 3 and 4, respectively. The hydraulic conductivity function, $K(\theta)$, is well described by the Brooks and Corey model (Table 1), i.e.:

$$K(\theta) = 100\left(\frac{\theta - 0.246}{0.274}\right)^{4.25} \quad [12]$$

($\theta_m = 0.52$; $\theta_c = 0.246$; $n = 0.2353$). Figure 5 presents results obtained with the numerical (finite difference CSMP) solution of Eq. [1] and [11] (Dane and Wierenga, 1975) with results obtained with the unit gradient method. The moisture distributions based upon the latter method are obtained directly by following the three-step algorithm as outlined previously (see also Table 1):

1. Obtain dK/dθ. Hence from [12]:

$$\frac{dK}{d\theta} = 1551\left(\frac{\theta - 0.246}{0.274}\right)^{3.25} \quad [13]$$

2. Equate the resulting expression to z/t,

$$\frac{z}{t} = 1551\left(\frac{\theta - 0.246}{0.274}\right)^{3.25} \quad [14]$$

3. Solve for θ,

$$\theta = 0.246 + 0.02858\,(z/t)^{0.3077} \; (0 < z < 1551t). \quad [15]$$

Figure 5 demonstrates that Eq. [15] generates results which compare well with those obtained with the numerical solution of Eq. [1], except near the

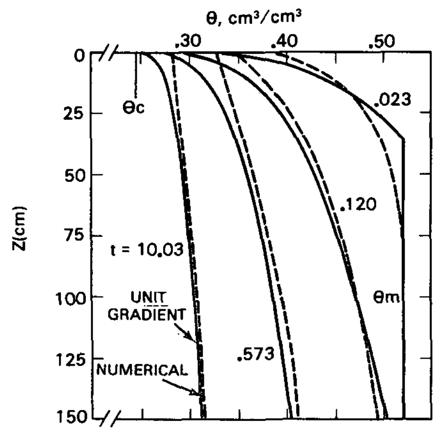

Fig. 5—Comparison of unit gradient solution with numerical solution for predicting water contents.

surface where the approximate solution approaches $\theta_c = 0.246$ at $z = 0$. The largest differences between the two methods occur at intermediate drainage times (between 0.3 and 2 days), the maximum deviation (for $z > 25$ cm) being about 0.01 cm³/cm³. It is doubtful whether such differences are important when drainage from deep profiles is being estimated.

Figure 6 further compares the amount of drainable water ($W - 150 \cdot \theta_c$) as obtained with the numerical and the unit gradient solutions. The solid line was obtained by integrating Eq. [15] over the entire profile, leading to the expression (see also Table 2):

$$W(150,t) = 36.90 + 15.318\, t^{-0.3077} \quad (t > 150/1551) \qquad [16]$$

Because of the restriction $z < At$ on Eq. [15], Eq. [16] does not hold for the early drainage phase when part of the profile is still saturated. For this period $W(150,t)$ is given by

$$W(150,t) = \int_0^{z_1} \theta dz + \int_{z_1}^{150} \theta_m dz = \int_0^{150} \theta_m dz - K_m t = 78 - 100\,t \quad (t < 150/1551). \qquad [17]$$

The use of Eq. [17] constitutes an important improvement over earlier drainage models in that it accurately describes drainage at small times, as demonstrated in Fig. 6. Overall, the unit gradient method provides results which approach the numerical predictions; although, the method seems to overpredict the drainage rate somewhat, especially at intermediate times ($0.10 < t < 0.30$).

Thus far in this example, the unit gradient method was used to predict moisture distributions and amounts of drainable water in the profile. A demonstration of how the method can also be used to analyze drainage equations like those shown in Table 2 will now be carried out. Suppose the total water curve in Fig. 6 given by the CSMP solution is given and one knows θ_c. For illustrative purposes, θ_m and θ_c are assumed to be the same as before (0.52 and 0.246, respectively). Hence the drainage data can be plotted in the same way as before (log ($W - 36.9$) vs. log t). Ignoring the early drainage phase, the dashed line in Fig. 6 is given by

$$W(150,t) = 36.9 + 16.60\, t^{-0.303} \quad (t > 0.3 \text{ days}). \qquad [18]$$

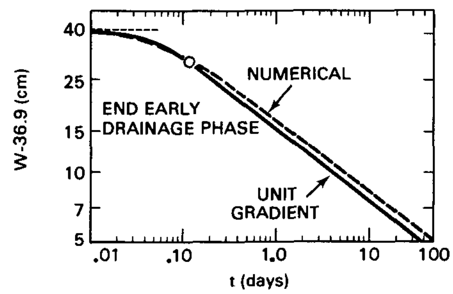

Fig. 6—Comaprison of numerically predicted total drainable water above 150 cm with unit gradient predictions. During early drainage phase (times less than value shown with open circle) only part of the profile is undergoing desorption.

By comparing Eq. [18] with the total water equation based on the Brooks and Corey model (Table 2), it is easily shown that n equals 0.2353 and A equals 1249.2. Hence from the definition of $A = K_s/n(\theta_m - \theta_c)$, one obtains $K_s = 79.6$. This value for the saturated hydraulic conductivity is certainly a reasonable approximation of the "correct" value, i.e., of $K_s = 100$ as originally employed in the numerical solution of Eq. [1]. Using the value of m, the complete conductivity equation is (see Table 1)

$$K(\theta) = 79.6 \left(\frac{\theta - 0.246}{0.274}\right)^{4.30} \qquad [19]$$

which is very close to the original curve shown in Fig. 3. Equation [19] in turn can now be used to predict moisture distributions within the 150-cm profile, as discussed before.

Example 2

This example considers drainage from a field site containing Gerber silty clay loam, one of the dominant soils of a saline seeped area on the Northern Great Plains. Some physical characteristics of this soil are given in Table 3. The values given are representative of the glacial soils on which many of the saline seeps occur. The experimental conditions of this example are extensively discussed by Sisson (J. B. Sisson, 1972. Hydraulic properties of the Gerber soil. M.S. Thesis, Montana State Univ., Bozeman) and are briefly summarized in Table 3.

In the fall of 1970, two field plots on the Gerber soil were established, and neutron access tubes were installed to allow for an over-winter stabilization period. The plots were irrigated in the spring of 1971, one for 4 hours and the other for 24 hours, and covered with plastic to prevent evaporation. Moisture contents were determined at 15-cm intervals, at first

Table 3—Physical properties of Gerber silty clay loam.

Depth	Texture	Bulk density of peds	Water content 0.3 bar	Water content 15 bars
cm		g/cm³	g/g	g/g
0–20	sicl	1.70	31.5	14.0
20–36	sic	1.86	33.7	20.2
36–53	sicl	1.59	28.2	15.0
53–74	sicl	1.59	32.1	14.7
74–94	sicl	1.59	28.0	13.2
94–122	cl	1.62	26.7	12.3
122–198	cl	1.86	24.8	11.4
198–244	cl	1.86	26.0	10.3

on a daily basis, later every 2 or 3 days. The observed values were assumed to be representative for each 15-cm depth increment. Depth to the water table was > 13.8 m.

The drainage equations for this particular experiment were based on the Watson model (see Tables 1 and 2), Table 4 presents observed and calculated total water contents, $W(z,t)$, for two times. The drainage equations used are:

$$W(z,t) = 0.320\ z^{1.026}\ t^{-0.026} \quad [20]$$

and

$$W(z,t) = 0.312\ z^{1.033}\ t^{-0.033}. \quad [21]$$

Both equations were obtained by fitting data to the Watson model, using a multiple regression program. The coefficients in Eq. [20] and [21] were based on soil water content data for $z < 105$ and $z < 180$ cm, respectively, and utilizing data for days 2 through 10 after irrigation only. The greatest difference between observed and calculated values is about 1 cm. Equation [20] appears to give somewhat better results than Eq. [21], especially in the upper part of the profile. This is probably a result of the textural changes which occur at about 100 cm (Table 3). It is emphasized here that the predictions assume a uniform profile, so that some inaccuracies may ge expected when soil horizons with different hydraulic properties are lumped into one profile, which is then assumed to be uniform. Notwithstanding the apparent nonuniformity of the profile, the correspondence between measured and predicted values after 54 days in Table 4 is certainly acceptable for most purposes where drainage has to be estimated for large areas using a simple mathematical model.

Table 4—Measured and predicted total water above a given depth in Gerber silty clay loam.

	Total water above a depth			
	Measured		Predicted day 54	
Depth	Initial day 2	Day 54	Based on 180 cm†	Based on 105 cm‡
	cm			
75	26.8	23.4	24.2	23.6
90	32.0	28.1	29.2	28.6
105	37.4	33.0	34.2	33.5
120	42.6	38.0	39.2	38.5
135	47.9	43.1	44.2	43.4
150	53.4	48.2	49.3	48.4
165	59.0	53.8	54.3	53.4
180	64.8	59.3	59.4	58.5

† Prediction equation: $W(z,t) = 0.320\ z^{1.026}\ t^{-.026}$.
‡ Prediction equation: $W(z,t) = 0.312\ z^{1.033}\ t^{-.033}$.

SUMMARY

Although only two examples were used for assessing the accuracy and applicability of the prediction equations, it does appear that the unit gradient method holds considerable promise as a means for estimating long-term drainage from soils. The unknown parameters are easily determined, whether by direct observation or by regression analysis on observed data, and the determinations based on shallower depths can be used to predict drainage from deeper depths, provided the profile is reasonably uniform.

LITERATURE CITED

1. Aris, R., and N. R. Amundson. 1973. Mathematical methods in chemical engineering, 2. First-order partial differential equations with applications. Prentice-Hall, Englewood Cliffs, N.J. 369 p.
2. Black, T. A., W. R. Gardner, and G. W. Thurtell. 1969. The prediction of evaporation, drainage, and soil water storage for a bare soil. Soil Sci. Soc. Amer. Proc. 33:655–660.
3. Brooks, R. H., and A. T. Corey. 1964. Hydraulic properties of porous media. Colorado State Univ., Fort Collins. Hydrol. Paper No. 3 27 p.
4. Brown, P. L., and H. Ferguson. 1973. Crop management in Montana for control of dryland salinity. p. 77–92. *In* Proc., Alberta Dryland Salinity Workshop, Lethbridge, Alberta. 18–19 Jan. 1973. Plant Ind. Div., Alberta Dep. of Agric., Edmonton, Can.
5. Dane, J. H., and P. J. Wierenga. 1975. Effects of hysteresis on the prediction of infiltration, redistribution and drainage of water in a layered soil. J. Hydrol. 25:229–242.
6. Davidson, J. M., L. R. Stone, D. R. Nielsen, and M. E. Larue. 1969. Field measurement and use of soil-water properties. Water Resour. Res. 5:1312–1321.
7. Ferguson, H., P. L. Brown, and M. R. Miller. 1972. Saline seeps on non-irrigated lands of the Northern Plains. Seminar on Control of Agriculture — Related Polution, Lincoln, Nebr., 24–25 July 1972. Great Plains Agri. Coun. Publ. no. 60:169–191.
8. Gardner, W. R., D. Hillel, and Y. Benyamini. 1970. Post irrigation movement of soil water: I. Redistribution. Water Resour. Res. 6(3):851–861.
9. Lax, P. D. 1972. The formation and decay of shock waves. Am. Math Monthly. 79:227–241.
10. Ogata, Gen, and L. A. Richards. 1957. Water content changes following irrigation of bare-field soil that is protected from evaporation. Soil Sci. Soc. Am. Proc. 21:355–356.
11. Taylor, S. A., and G. S. Ashcroft. 1972. Physical edaphology. W. H. Freeman & Company, San Francisco.
12. Watson, K. K. 1967. The measurement of the hydraulic conductivity of unsaturated porous materials utilizing a zone of entrapped air. Soil Sci. Soc. Am. Proc. 32:716–720.
13. Wierenga, P. J., and T. C. Patterson. 1972. Irrigation return flow studies in the Mesilla Valley. p. 173–179. *In* Proc. National Conf. on Managing Irrigated Agriculture to Improve Water Quality, Colorado State Univ.. 16–18 May 1972.
14. Wilcox, J. C. 1959. Rate of soil drainage following an irrigation. Can. J. Soil Sci. 40:15–27.