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ON CONCURRENT SEASONAL ADJUSTMENT

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Summary

Concurrent seasonal adjustment utilizes all information up to and including the current month's figure in forming seasonally adjusted data, and thus should provide more accurate estimates of final seasonally adjusted data than the prevalent official method where the seasonal component is forecasted from data through the preceding December. This paper evaluates the expected gain, in terms of the reduction in RMSE of seasonal revisions, from employing concurrent seasonal adjustment.

The framework of the paper is then extended to the case where the data contain nonseasonal as well as seasonal revisions, the former resulting from preliminary-data error in the first-published, not seasonally adjusted (NSA) data. It is found that the gain from concurrent adjustment is usually reduced, often substantially, by noise in preliminary NSA data. However an offset to this effect also occurs since the forecasted seasonal component must also be derived from preliminary data.

Some of the paper's results are applied to a linearized X-11-ARIMA procedure, using a common seasonal ARIMA model. An analysis of actual series containing preliminary-data error provides confirmation of the main results.

1. INTRODUCTION

Most seasonally adjusted series published in the U.S. and elsewhere are formed using a procedure [usually a variant of Census X-11 (Shiskin, Young, and Musgrave, 1967) or of X-11-ARIMA (Dagum, 1975)] which, for historical data, depends equally on the future and past of the series relative to the figure being adjusted. This is natural, as both future and past observations generally contain comparable information concerning seasonality at a given point in the series. However, this means that for seasonally adjusting data at current and recent time periods, not all relevant information is available. Thus initial or preliminary seasonal estimates are first determined, which are subsequently revised as more series values are observed, until the unobserved future is sufficiently distant to be no longer relevant.

For the initial publication of seasonally adjusted data, the traditional practice at most institutions has been to determine, at the end of each calendar year, a set of twelve forecasts of seasonal factors[1/] for the following year. These forecasted factors are then applied to the incoming data during the year to form preliminary seasonally adjusted figures, which can later be revised. Thus, in projected-factor adjustment the observations from after the preceding December through the current month are disregarded in forming the current seasonal factor and seasonally adjusted value.

In <u>concurrent</u> seasonal adjustment, by contrast, the adjustment procedure is applied each month to the entire available series up to and including the most recent month's figure. Thus the first-published figure

1/ More generally, a forecast of the seasonal "component" is provided. However, a multiplicative model is often assumed for post-war economic time series—y_t = (Trend) x (Seasonal) x (Irregular) = $C_tS_tI_t$—so that the seasonal S_t is a "factor" of the series y_t. The adjustment provided is thus X_t/S_t.

is based on more information than for the projected-factor method, so that in principle concurrent adjustment should be more accurate with on-average smaller revisions. Several empirical studies, such as Kenny and Durbin (1982) and McKenzie (1984), have in fact found this to be the case, and the merits of this procedure are becoming sufficiently recognized that it is finding increasing use. The Census Bureau, for example, has adopted concurrent seasonal adjustment for several of its Construction Statistics series, with plans to proceed with more series in 1985.

The purpose of the present paper is twofold. First, assuming that the series to be seasonally adjusted are stationary or homogeneously nonstationary time series, Sections 2 and 3 present a theoretical analysis of the expected gain, in terms of reduction of the RMSE of the seasonal revisions, from the use of concurrent seasonal adjustment. This time domain analysis complements the frequency domain study of Dagum (1983) and is in general consistent with the findings of the empirical studies cited above: that significant reductions in revision mean square can be expected from concurrent adjustment.

Second, this framework is extended in Section 4 to the case where the data contain nonseasonal as well as seasonal revisions, that is, where there is preliminary-data error in the first-released, not seasonally adjusted (NSA) data. We assume that the nonseasonal revision has mean zero and is independent of the preliminary NSA figure—assumptions which if violated would imply the error could be in part anticipated and thereby reduced. We then measure the degree to which both projected and concurrent seasonal component estimates are worsened. Frequently the concurrent estimate undergoes a greater deterioration, so that the gain from concurrent adjustment is reduced by error in preliminary NSA data. However, at least for X-11 and typical model-based seasonal filters, this reduction in gain may not occur if data are still noisy 12 or more months prior to the projected seasonal (e.g., for the December projection made as of the previous December when NSA data for that month are preliminary).

Section 5 then applies these results to a linear-filter approximation to the X-11 ARIMA procedure, using the "Airline" model of Box and Jenkins which is commonly found to characterize economic and social time series. For given parameter values (including NSA revision variance), the accuracies of concurrent and projected-component adjustment are compared with and without preliminary-data error.

Section 6 contains an analysis of a component of the Industrial Production Index, with results essentially as expected from Sections 3 through 5. A summary and conclusions comprise Section 7.

2. PROJECTED-COMPONENT AND CONCURRENT SEASONAL ADJUSTMENT

The choice between concurrent versus projected-factor seasonal adjustment can occur with a wide variety of adjustment procedures, and we now restrict the procedures considered to those which can be characterized as symmetric moving averages (linear filters) applied to forecast-augmented series. Also we shall be working with additive representations, assumed appropriate for the logarithm x_t of a multiplicative series y_t, so that $s_t = \log S_t$ is the seasonal component of $x_t = \log y_t$. Thus we henceforth speak of "components" rather than "factors," the former being the logarithms of the latter for multiplicatively generated series. Appropriate substitutions in terminology can be made for series which are additive in original form, or for which an other-than-logarithmic transformation is appropriate.

To seasonally adjust historical values of the series x_t, it is assumed that a symmetric filter is applied to the series; thus the resulting "final" seasonal component determined by the adjustment procedure is

$$\begin{aligned} s_t^f &= \sum_{j=-M}^{M} \lambda_j x_{t-j} \quad (\lambda_{-j} = \lambda_j) \\ &= \lambda(B) x_t \end{aligned} \tag{2.1}$$

where

$$\lambda(z) = \sum_{j=-M}^{M} \lambda_j z^j \quad .$$

The seasonally adjusted series is then

$$x_t^a = x_t - s_t^f = [1 - \lambda(B)]x_t \quad . \tag{2.2}$$

The Census X-11 procedure is of this form, as are most model-based procedures (with M often infinite for the latter).

Suppose now that it is desired to seasonally adjust x_t based only on data x_{t-m}, $x_{t-m-1}, \ldots$ for some m. If m = 0 this would be concurrent adjustment; for m = 1, ..., 12 this would be projected-component adjustment. That is, the projected-component adjustment for x_t = January is based on data x_{t-1}, x_{t-2}, ...; the adjustment for February is based on data x_{t-2}, x_{t-3}, ...; and so on, up through the projected component adjustment for x_t = December, which is based on data x_{t-12}, x_{t-13}, ... For $- M < m < 0$ some but not all of the relevant future ($x_{t+1}, \ldots, x_{t+M}$) is available.

It is assumed that the seasonal adjustment procedure forms an estimate

$$s_t^{(m)} = \sum_{j=m}^{M} \lambda_j^{(m)} x_{t-j} = \lambda^{(m)}(B)x_t \tag{2.3}$$

of the final seasonal component s_t^f. In particular, if m = 0, then

$$s_t^c = s_t^{(o)} = \lambda^{(o)}(B)x_t = \lambda_o^{(o)}x_t + \lambda_1^{(o)}x_{t-1} + \ldots \tag{2.4}$$

is the concurrent seasonal estimate, and for $1 \leq m \leq 12$, if x_t falls in the m^{th} month of the year the projected-component estimate is

$$s_t^p = s_t^{(m)} = \lambda^{(m)}(B)x_t = \lambda_m^{(m)}x_{t-m} + \lambda_{m+1}^{(m)}x_{t-m-1} + \ldots \quad . \tag{2.5}$$

Note that in (2.5), since $\lambda_m^{(m)}B^m$ is the leading coefficient in $\lambda^{(m)}(B)$, the value x_{t-m} is the most recent value to enter this calculation. As noted between (2.2) and (2.3), this value is the previous December's observation if $1 \leq m \leq 12$. Also, setting $m = -M \geq -\infty$,

$$s_t^f = s_t^{(-M)} \tag{2.6}$$

is the final seasonal component.

For any two times t-m and t-n, two estimates $s_t^{(m)}$ and $s_t^{(n)}$ of the final component s_t^f can be calculated. Suppose $n < m$, so that x_{t-n} is a more recent observation than x_{t-m}. Then the revision in the seasonal component estimate $s_t^{(m)}$ is

$$r_t^{(m,n)} = s_t^{(n)} - s_t^{(m)} \tag{2.7}$$

and reflects the availability of the additional information $x_{t-m+1},\ldots,x_{t-n}$.

The total revision in the projected-component estimate $s_t^{(m)}$, $m = 1,2,\ldots,12$, is

$$r_t^p = r_t^{(m,-M)} = s_t^{(-M)} - s_t^{(m)} = s_t^f - s_t^p \quad . \tag{2.8}$$

This quantity can be represented as the sum of two revisions, one (denoted r_t^c) occurring even under concurrent adjustment and the other (denoted r_t^*) occurring explicitly because of the failure to use information available on s_t at the time (t) of the initial seasonal adjustment of x_t. That is,

$$r_t^p = r_t^c + r_t^* \quad , \tag{2.9}$$

where

$$r_t^c = r_t^{(o,-M)} = s_t^f - s_t^c \quad , \tag{2.10}$$

is the total revision that would occur with a concurrent estimate, and

$$r_t^* = r_t^{(m,o)} = s_t^c - s_t^p \quad . \tag{2.11}$$

is the revision due to "the failure to employ concurrent adjustment." From (2.11), the concurrent estimate is

$$s_t^c = s_t^p + r_t^* \quad . \tag{2.12}$$

Intuitively, concurrent seasonal adjustment should involve smaller subsequent revisions since it is based on more information, an idea which is strengthened by relations such as (2.9) and (2.12). We now show formally that this is in fact the case for "forecast augmented" seasonal adjustment procedures.

3. GAIN FROM CONCURRENT ADJUSTMENT WITH ARIMA TIME SERIES

The foregoing development is in terms of a given seasonal adjustment procedure such as (the linear filter version of additive) X-11, i.e., for given weights λ_j and $\lambda_j^{(m)}$ in (2.1) and (2.3); but nothing has yet been said about x_t or its "true" seasonality. In particular s_t^f has been defined by the procedure, whereas in model-based procedures s_t^f is an estimate of an unobserved true seasonal component, say s_t. We shall continue not to require a definition of a true seasonal; however, to investigate further the properties of concurrent adjustment it is necessary to know something about x_t.

It is supposed that the observable series x_t has the representation

$$\Delta(B)x_t = \psi(B)a_t \tag{3.1}$$

where

$$\psi(z) = \sum_{j=0}^{\infty} \psi_j z^j$$

is nonzero and absolutely convergent for $|z| \leq 1$, the zeros of $\Delta(z)$ are on the unit circle, and $\{a_t\}$ is a white noise sequence. The series

$$w_t = \Delta(B)x_t$$

is thus a stationary nondeterministic time series. A common example of (3.1) is the "Airline" model

$$(1-B)(1-B^{12})x_t = (1-\theta B)(1-\Theta B^{12})a_t \tag{3.2}$$

with $|\theta| < 1$, $|\Theta| < 1$.

Revisions can depend in general on the entire past history of the series; however, it is natural for a revision $r_t^{(m,n)}$ to be a function only of the new information contained in the values $x_{t-m+1},\ldots,x_{t-n}$, that is, of the innovations $a_{t-m+1},\ldots,a_{t-n}$ (lest it be partially anticipated from information already available). This will be true if (and only if) the seasonal estimate $s_t^{(m)}$ in (2.3) can be obtained from the application of the central (symmetric) filter $\lambda(B)$ to the series x_t extended by a set of forecasts from its model; that is, if

$$s_t^{(m)} = \lambda(B)x_{t-m}^{(t-m)} \tag{3.3}$$

where the extended series $x_t^{(\tau)}$ consists of actual values x_τ, $x_{\tau-1}$, ... prior to and including time $t=\tau$ and minimum MSE forecasts (at origin τ) based on the model (3.1) for times $\tau+1$, $\tau+2$, ... (here $\tau=t-m$). Assuming this to be the case, it was shown in Pierce (1980) that the revision $r_t^{(m,n)}$ in (2.7) follows a moving average process of order $m-n-1$,

$$r_t^{(m,n)} = \sum_{j=n}^{m-1} \mu_j a_{t-j} \tag{3.4}$$

where the $\{\mu\text{'s}\}$ are obtained by equating coefficients in the formal expansion of

$$\mu(B) = \lambda(B)\Delta^{-1}(B)\psi(B) \quad . \tag{3.5}$$

We therefore have the following results:

(1) The revision r_t^* in (2.11) avoided by employing concurrent adjustment follows the stochastic process

$$r_t^* = \sum_{j=0}^{m-1} \mu_j \, a_{t-j} \quad ; \tag{3.6}$$

(2) the three quantities r_t^* (the revision avoided by concurrent adjustment), r_t^c (the revision with concurrent adjustment), and s_t^p (the initial estimate) are all orthogonal (they are based on nonoverlapping segments of the series $\{a_t\}$); and thus

(3) the reduction in the revision MSE from employing concurrent adjustment is given directly by

$$\sigma_*^2 = \mathrm{Var}(r_t^*) = \sigma_a^2 \sum_{j=0}^{m-1} \mu_j^2 \quad . \tag{3.7}$$

As noted, m can take on values 1, ..., 12 depending on whether x_t falls in the month January, ..., December. Thus $\{r_t^*\}$ is a <u>periodically correlated</u> process, being marginally white noise in January, MA(1) in February, ..., MA(11) in December. The variance (3.7) is correspondingly periodic, being

$$\sigma_a^2\mu_o^2 \quad ,$$
$$\sigma_a^2(\mu_o^2 + \mu_1^2) \quad ,$$
$$\ldots,$$
$$\sigma_a^2(\mu_o^2 + \mu_1^2 + \ldots + \mu_{11}^2) \tag{3.8}$$

for January, February, ..., December.

For X-11 and other seasonal adjustment procedures, the "center weight" μ_o is typically much larger than the neighboring weights μ_1, μ_2, ... The variances in (3.8) are thus of comparable magnitude, and this supports two earlier empirical findings: (i) that there are comparable gains from concurrent adjustment in all months of the year, and (ii) the gain from decreasing the length of projection (increasing the frequency of revision) to (say) six or three months is far less than the gain from moving to a fully concurrent adjustment.

The presence of the "$\mu_o a_t$" term in r_t^*, or equivalently the "$\lambda_o x_t$" term in s_t^c, also explains to a large extent why data appear to be smoothed after seasonal factor revision. Under projected-component adjustment this term, though known, is not removed from the first-published SA figure, and thus an amount $\mu_o^2\sigma_a^2$ is added to the variance. Therefore, when the term $\mu_o a_t$ is removed (for the first annual revision at the end of the year) an amount $\mu_o^2\sigma_a^2$ is taken from the adjusted series' variance and added to the seasonal component variance, thus smoothing any aberrant movements (unusually large a_t values) in the series. This phenomenon is solely an artifact of the failure to employ concurrent adjustment: when preliminary data are optimally (i.e., concurrently) estimated the term $\mu_o a_t$ (and earlier terms) are absent from both the preliminary SA data and the revised data, giving the preliminary figures a smoother appearance.

It is also of interest to determine the gain from concurrent seasonal adjustment in relative terms. In Sections 5 and 6 we give examples estimating the ratio of the population root mean square revisions with concurrent and projected-factor adjustment, which is

$$\frac{SD(r_t^c)}{SD(r_t^p)} = \left[\frac{\sigma_p^2 - \sigma_*^2}{\sigma_p^2}\right]^{1/2} = \left[\frac{\sum_{j=-M}^{-1} \mu_j^2}{\sum_{j=-M}^{m-1} \mu_j^2}\right]^{1/2} \tag{3.9}$$

The gain from concurrent adjustment under these circumstances is entirely realized in the first year. That is, assuming that under concurrent seasonal adjustment annual revisions continue to be made after December of each year, and only at those times, the annual revisions after the first-year revision are the same whether concurrent or projected-component adjustment is used. But for the initial annual revision, since t-m and t+12-m are the times of the Decembers preceding and following observation t (occurring at month m), this first-year revision is

$$r_t^{(o,m-12)} = \sum_{j=m-12}^{-1} \mu_j a_{t-j} \tag{3.10}$$

(which is 0, for example, for a December) for concurrent adjustment and

$$r_t^{(m,m-12)} = \sum_{j=m-12}^{m-1} \mu_j a_{t-j} = r_t^* + r_t^{(o,m-12)} \tag{3.11}$$

for projected-component adjustment. Therefore the ratio of the two expected first-year RMS revisions is

$$\left[\sum_{m-12}^{-1} \mu_j^2 \Big/ \sum_{m-12}^{m-1} \mu_j^2 \right]^{1/2} , \tag{3.12}$$

which is the right member of (3.9) with m-12 replacing -M.

4. EFFECT OF ERROR IN PRELIMINARY NSA DATA

The gain from concurrent adjustment was derived in the preceding section under the assumption that actual (final) NSA data were available through the month being adjusted. However, the availability of the final NSA value is frequently delayed, and thus a preliminary estimate of the NSA figure is first constructed. The question then arises of whether concurrent adjustment applied to this figure (or figures, as several months' values may be not yet final), containing an as-yet unremoved error, is still preferable to forming the preliminary seasonal estimate nonconcurrently (which might still not be from error-free data, depending on whether the most recent values used at the time of projection were preliminary). Under certain assumptions on the NSA revisions, this section measures the effect of error in preliminary NSA data on the gain from concurrent adjustment.

Suppose that the NSA series x_t can be represented as

$$x_t = X_t + R_t \tag{4.1}$$

where X_t is the preliminary NSA value and R_t is the revision, which is unknown at time t. It is assumed that the revision R_t is independent of all quantities known at time t—which include X_t, X_{t-1}, ... as well as x_{t-d}, x_{t-d-1}, ... and R_{t-d}, R_{t-d-1}, ..., where d is the number of periods following time t before the final figure (equivalently the revision) is known. It is moreover assumed that R_t is identically distributed and stationary over time, and in particular has zero mean and constant variance. While this may represent an ideal situation, it hopefully will be at least approximated in practice. For example, if R_t were serially correlated at lag d or greater, or cross correlated with X_{t-j} for $j \geq 0$, then this revision could be in part predicted and the preliminary figure X_t thereby made more accurate.

4.1 Single Revision with One-Month Delay

Suppose first that d=1, that is, the final figure x_t is known one month later. Then R_t is white noise. Under concurrent adjustment the filter $\lambda^{(o)}(B)$ as in (2.4) is applied to the series X_t, x_{t-1}, x_{t-2}, ... to produce an estimate

$$\tilde{s}_t^c = \lambda_o^{(o)} X_t + \lambda_1^{(o)} x_{t-1} + \dots \tag{4.2}$$

$$= \lambda^{(o)}(B)\tilde{x}_t$$

where $\tilde{x}_{t-j} = X_t$ if $j = 0$ and x_t if $j \geq 1$. The difference between this and the concurrent estimate if x_t were known is

$$s_t^c - \tilde{s}_t^c = \lambda_o^{(o)} R_t \tag{4.3}$$

The analogous situation under projected-component adjustment depends on how the presence of preliminary data error affects the timing of the annual projection. In the present (d=1) case we can distinguish three possibilities:

(i) assuming the current month t is a December, project the following year's factors based on X_t, x_{t-1}, ..., acting as if X_t were x_t;

(ii) wait a month and base the projected factors (which will include a concurrent factor as in (4.2) for January) on X_{t+1}, x_t, x_{t-1}, ...; or

(iii) wait a month, ignore the preliminary January figure X_{t+1} and base the projection on x_t, x_{t-1}, ... (exactly as could have been done a month earlier were there no preliminary data error).

The alternative (iii) requires the projection of thirteen months' seasonal components (12+d months in general). Choosing (ii) rather than (i) simply shifts the annual projection by one time period. Thus we assume that the alternative (i) is employed.

Therefore, letting t denote the m^{th} month ($1 \leq m \leq 12$) of the forthcoming year, to estimate the component s_t^f the filter $\lambda^{(m)}(B)$ in (2.5) is applied to the series X_{t-m}, x_{t-m-1}, ..., yielding

$$\begin{aligned} \tilde{s}_t^p &= \lambda_m^{(m)} X_{t-m} + \lambda_{m+1}^{(m)} x_{t-m-1} + \cdots \\ &= \lambda^{(m)}(B)\tilde{x}_{t-m} \quad . \end{aligned} \tag{4.4}$$

The difference between this and the projected estimate (2.5), representing the effect of error in X_{t-m}, is

$$s_t^p - \tilde{s}_t^p = \lambda_m^{(m)} R_{t-m} \quad . \tag{4.5}$$

Subtracting (4.5) from the analogous expression (4.3) for the concurrent estimate, we obtain

$$s_t^c - s_t^p = \lambda_o^{(o)} R_t - \lambda_m^{(m)} R_{t-m} + \tilde{s}_t^c - \tilde{s}_t^p \quad ,$$

or

$$r_t^* + \lambda_m^{(m)} R_{t-m} = \tilde{r}_t^* + \lambda_o^{(o)} R_t \quad , \tag{4.6}$$

where

$$\tilde{r}_t^* = \tilde{s}_t^c - \tilde{s}_t^p \tag{4.7}$$

$$= \sum_{j=0}^{m-1} \mu_j a_{t-j} + [\lambda_m^{(m)} R_{t-m} - \lambda_o^{(o)} R_t] \tag{4.8}$$

is the revision avoided by concurrent adjustment with error in the preliminary NSA data.

Since r_t^* is a function only of $a_t, \ldots, a_{t-m+1}$, r_t^* is uncorrelated with R_{t-m}; and since R_t is by assumption independent of $X_t, X_{t-1}, X_{t-2}, \ldots$ and $R_{t-1}, R_{t-2}, \ldots$, it follows that R_t is uncorrelated with $\tilde{r}_t^*$, which depends only on X_t, $(X_{t-1}+R_{t-1}), \ldots$, i.e., on quantities known at time t. Consequently the pairs of terms on each side of (4.6) are uncorrelated, and taking variances we therefore have

$$\sigma_*^2 + (\lambda_m^{(m)})^2\sigma_R^2 = \tilde{\sigma}_*^2 + (\lambda_o^{(o)})^2\sigma_R^2$$

or

$$\tilde{\sigma}_*^2 = \sigma_*^2 + [(\lambda_m^{(m)})^2 - (\lambda_o^{(o)})^2]\sigma_R^2 \quad . \tag{4.9}$$

Thus the gain from concurrent adjustment for noise-free initial series, σ_*^2, is decreased by an amount $(\lambda_o^{(o)})^2\sigma_R^2$ reflecting error in the value X_t (causing a deterioration in the concurrent adjustment) and increased by an amount $(\lambda_m^{(m)})^2\sigma_R^2$ reflecting error in the value X_{t-m} (causing a deterioration in the projected-component adjustment).

Analogous to equation (3.9), it is of interest to determine the relative magnitudes of the projected and concurrent revisions in the present case of NSA preliminary data error. Since the variances of the concurrent and projected-component revisions $\tilde{r}_t^c$ and $\tilde{r}_t^p$ are, respectively,

$$\sigma_a^2 \sum_{j=-M}^{-1} \mu_j^2 + \sigma_R^2(\lambda_o^{(o)})^2$$

and

$$\sigma_a^2 \sum_{j=-M}^{m-1} \mu_j^2 + \sigma_R^2(\lambda_m^{(m)})^2 \quad ,$$

the ratio of the standard deviations of these two revisions is

$$\frac{SD(\tilde{r}_t^c)}{SD(\tilde{r}_t^p)} = \left[\frac{\sum_{j=-M}^{-1} \mu_j^2 + v(\lambda_o^{(o)})^2}{\sum_{j=-M}^{m-1} \mu_j^2 + v(\lambda_m^{(m)})^2} \right]^{1/2} \tag{4.10}$$

where

$$v = \sigma_R^2/\sigma_a^2$$

is a measure of the size of the revisions relative to the innovations in the error-free series. Note that $v \leq 1$, since the innovation variance of x, σ_a^2, is at least as big as the sum of the innovation variances of X and R, the latter being σ_R^2 itself.

Equations (4.9) and (4.10) show the effects of preliminary-data error on the gain from concurrent adjustment, the gain being measured by the reduction in variance of the seasonal revisions. As expected, error in the current NSA figure X_t reduces the gain from concurrent adjustment; however, error in X_{t-m} (the previous December's value) increases $\tilde{\sigma}_*^2$ since the information ignored by the projected-factor method includes the December NSA revision R_{t-m} in addition to the available current-year data.

For projecting the months January through November, for X-11 and generally for model-based procedures the weights $\lambda_m^{(m)}$ are small relative to the center weight $\lambda_o^{(o)}$, so that $\tilde{\sigma}_*^2$ is less than σ_*^2. On the other hand, the December projected value $s_t^{(12)}$ in general depends more heavily on the previous December's value, since $\lambda_{12}^{(12)}$ is typically larger than $\lambda_1^{(1)}, \ldots, \lambda_{11}^{(11)}$ —see Table 4 of Section 5, for example. Therefore the presence of NSA data error would be expected to cause a greater deterioration in the projected-component estimate for December than for the other months. This deterioration may in fact exceed the deterioration in the concurrent estimate for December, as seen for example by the occurrence of $\lambda_{12}^{(12)} > \lambda_o^{(o)}$ in Table 4.

4.2 Several Successive Revisions

When error in NSA data persists for more than one month, with possibly more than one successive revision leading to the final NSA data, the situation is more complex. For example, in some of the retail trade series published by the Census Bureau, an "advance" figure is revised one month later to a "preliminary" figure which after another month is again revised to the final figure x_t. And the Federal Reserve's money supply and industrial production series typically undergo several revisions.

Thus, suppose that there are k successive revisions (some may be zero), so that the data are final after k additional months. Let the successive revised values for month t be X_{0t} (the initial NSA figure), X_{1t}, ..., $X_{kt} = x_t$ (the final NSA figure). Define E_{it}, R_{it}, $1 \leq i \leq k$, by the relations

$$\begin{aligned} x_t &= X_{k-1,t} + E_{kt} &&= X_{k-1,t} + R_{kt} \\ &= X_{k-2,t} + E_{k-1,t} + E_{kt} &&= X_{k-2,t} + R_{k-1,t} \\ &\;\;\vdots \\ &= X_{1t} + E_{2t} + \cdots + E_{kt} &&= X_{1t} + R_{2t} \\ &= X_{0t} + E_{1t} + E_{2t} + \cdots + E_{kt} &&= X_{0t} + R_{1t} \quad . \end{aligned} \tag{4.11}$$

The quantities

$$E_{it} = X_{it} - X_{i-1,t}$$

are the incremental revisions made in each time period; X_{0t} is observed at time t, adjusted by E_{1t} (revised to X_{1t}) at time t+1, and so on until at time t+k the time-(t+k-1) figure $X_{k-1,t}$ is revised by the amount E_{kt} to produce the "final" figure x_t. The quantity

$$R_{it} = \sum_{j=i}^{k} E_{jt} \tag{4.12}$$

represents the revision yet remaining in $X_{i-1,t}$ required to produce the final NSA figure x_t.

There is an interesting analogy between (i) X_{it} and a (k-i)-period forecast of x_t, (ii) R_{it} and the (k-i)-step-ahead forecast error, and (iii) $\{E_{it}\}$ and updates to forecasts. The terminology used for each set of quantities reflects primarily the time origin: a constructed figure X_t is a forecast (with corresponding updates, forecast errors, and actual value) if we are at time $t'<t$, while X_t is an estimate (with corresponding revised values, revisions, and final value) if the current time is $t''>t$. Also, in the latter case the final value may still be an estimate.

To compare concurrent and projected-component adjustment, it is useful to consider information known and unknown at a given time t, which is summarized in Table 1. Part (b) of this table shows the updates or "elementary" revisions $E_{i,t-j}$ added to each as-yet preliminary figure at each time period. The updates $E_{1,t-1}, \ldots, E_{k,t-k}$ along the main diagonal are the quantities which become known at time t. (In the ARIMA forecasting analogy they would correspond respectively to the forecast updates $\Psi_k a_t, \ldots, \Psi_1 a_t$ of what previously were k-step, ..., 1-step forecasts of $x_{t-1}, \ldots, x_{t-k}$).

As in Section 4.1 (where d=1, $X_{0t} = X_t$, and $E_{1t} = R_{1t} = R_t$), we assume that the information at any given time t is "optimally" used in calculating the preliminary values $X_{0t}, X_{1,t-1}, \ldots, X_{k-1,t-k+1}$, in the sense that the remaining revisions at that time (i.e., everything in part (c) of Table 1) are uncorrelated with any existing data (parts (a) and (b)). Thus R_{it} is independent of everything known at time t+i, and the $\{E_{it}\}$ across a row in (b) and (c) of Table 1 are a univariate white noise sequence. However, along a northwest/southeast diagonal the E_{it} are correlated as they are all functions of (and only of) the information which becomes available at time t.

Under concurrent adjustment, therefore, the series to which the filter $\lambda^{(o)}(B)$ is applied is $X_{0t}, X_{1,t-1}, \ldots, X_{k-1,t-k+1}, x_{t-k}, \ldots,$ resulting in the seasonal component estimate

$$\tilde{s}_t^c = \lambda_o^{(o)} X_{0t} + \ldots + \lambda_{k-1}^{(o)} X_{k-1,t-k+1} + \lambda_k^{(o)} x_{t-k} + \ldots \quad , \tag{4.13}$$

and the difference between this and the estimate which could be determined if there were no preliminary-data error is

$$s_t^c - \tilde{s}_t^c = \lambda_o^{(o)} R_{1t} + \ldots + \lambda_{k-1}^{(o)} R_{k,t-k+1} \quad .$$

Similarly, with projected-component adjustment

$$\tilde{s}_t^p = \lambda_m^{(m)} X_{0,t-m} + \ldots + \lambda_{m+k-1}^{(m)} X_{k-1,t-m-k+1} + \lambda_{m+k}^{(m)} x_{t-m-k} + \ldots \tag{4.14}$$

and

$$s_t^p - \tilde{s}_t^p = \lambda_m^{(m)} R_{1,t-m} + \ldots + \lambda_{m+k-1}^{(m)} R_{k,t-m-k+1} \quad .$$

Therefore, analogous to equation (4.6),

$$r_t^* + \lambda_m^{(m)} R_{1,t-m} + \ldots + \lambda_{m+k-1}^{(m)} R_{k,t-m-k+1}$$

$$= \tilde{r}_t^* + \lambda_o^{(o)} R_{1t} + \ldots + \lambda_{k-1}^{(o)} R_{k,t-k+1} \tag{4.15}$$

or

$$r_t^* + \underset{\sim}{\lambda}^{(m)'} \underset{\sim}{R}_{t-m} = \tilde{r}_t^* + \underset{\sim}{\lambda}^{(o)'} \underset{\sim}{R}_t \qquad (4.16)$$

where the k-dimensional vectors in (4.16) are defined in an obvious way from (4.15). By arguments directly analogous to those in the paragraph following (4.8), it follows that

$$\sigma_*^2 + \underset{\sim}{\lambda}^{(m)'} \Sigma \underset{\sim}{\lambda}^{(m)} = \tilde{\sigma}_*^2 + \underset{\sim}{\lambda}^{(o)'} \Sigma \underset{\sim}{\lambda}^{(o)}$$

or

$$\tilde{\sigma}_*^2 = \sigma_*^2 + \lambda^{(m)'} \Sigma \underset{\sim}{\lambda}^{(m)} - \underset{\sim}{\lambda}^{(o)'} \Sigma \underset{\sim}{\lambda}^{(o)} \qquad (4.17)$$

where Σ is the covariance matrix of $\underset{\sim}{R}_t$.

For example, if k=2 a preliminary figure X_{0t} is revised twice to produce

$$X_{1t} = X_{0t} + E_{1t}$$

and

$$X_{2t} = X_{0t} + E_{1t} + E_{2t} = X_{0t} + R_{1t} \quad ,$$

so that $R_{2t} = E_{2t}$. Thus $\underset{\sim}{R}_t = (R_{1t}, R_{2,t-1})'$ and the elements σ_{ij} of Σ are, with $\sigma_i^2 = \mathrm{Var}(E_{it})$,

$$\sigma_{11} = \mathrm{Var}(R_{1t}) = \sigma_1^2 + \sigma_2^2$$

$$\sigma_{22} = \mathrm{Var}(R_{2,t-1}) = \sigma_2^2$$

$$\sigma_{12} = \mathrm{Cov}(R_{1t}, R_{2,t-1}) = E(E_{1t}\, E_{2,t-1}) \quad .$$

Therefore, from (4.17) and assuming as before that the distributions are stationary, the expression analogous to (4.9) for the variance of $\tilde{r}_t^*$ is

$$\tilde{\sigma}_*^2 = \sigma_*^2 + [(\lambda_m^{(m)})^2 - (\lambda_o^{(o)})^2](\sigma_1^2 + \sigma_2^2)$$
$$+ [(\lambda_{m+1}^{(m)})^2 - (\lambda_1^{(o)})^2]\sigma_2^2$$
$$+ 2[\lambda_m^{(m)}\lambda_{m+1}^{(m)} - \lambda_o^{(o)}\lambda_1^{(o)}]\sigma_{12} \quad . \tag{4.18}$$

In terms of the variance effects, the gain from concurrent adjustment (for k=2) is seen from (4.18) to be reduced from the presence of $\lambda_o^{(o)}$ and $\lambda_o^{(1)}$, that is from a deterioration of the concurrent adjustment resulting from the noisy current and previous months' data. On the other hand the noise or error in NSA data also causes a deterioration in the projected values which is seen (from the $\lambda_m^{(m)}$ and $\lambda_{m+1}^{(m)}$ terms) to increase the gain from concurrent adjustment. This effect will be strong relative to the current effect for both the November and December seasonal projections; it is the term λ_{m+1} in the filter $\lambda^{(m)}(B)$ which multiplies the November value.

Finally, generalizing equation (4.10), the ratio of expected revision mean squares under concurrent and projected-component seasonal adjustment is

$$\frac{RMS(\tilde{r}_t^c)}{RMS(\tilde{r}_t^p)} = \left[\frac{\sum_{-M}^{-1} \mu_j^2 + \sigma_a^{-2} \underset{\sim}{\lambda}^{(o)\prime} \Sigma \underset{\sim}{\lambda}^{(o)}}{\sum_{-M}^{m-1} \mu_j^2 + \sigma_a^{-2} \underset{\sim}{\lambda}^{(m)\prime} \Sigma \underset{\sim}{\lambda}^{(m)}} \right]^{1/2} . \tag{4.19}$$

5. APPLICATIONS TO X-11

We now apply some of the previous results to the Census X-11 program for seasonal adjustment (Shiskin, Young, and Musgrave 1967) or more precisely to a variant of the X-11 ARIMA procedure (Dagum 1975), confining ourselves to the cases of no error in preliminary NSA data (Section 3) and of a single-month revision in the NSA data (Section 4.1). We obtain in this section some predicted or expected effects of concurrent seasonal adjustment, by substituting for $\lambda(B)$ a linear filter approximation to the X-11 program, for $\Psi(B)$ a common ARIMA model for economic and other time series, and for v a range of plausible variance ratios.

5.1 Linear Filter Approximation to X-11

For historical data the X-11 and X-11-ARIMA procedures consist largely of a set of symmetric linear filters applied to the series to estimate its seasonal component, which can be written in the form

$$s_t^f = \sum_{-M}^{M} \lambda_{|j|} x_{t-j} = \lambda(B)x_t \quad . \tag{5.1}$$

For the standard-option seasonal filter the value of M is 82, 84, or 89 according to the detrending filter chosen; however as shown by Young (1968), to a very close approximation the symmetric moving average in (5.1) is given by

$$\lambda(B) = \sum_{-42}^{42} \lambda_k B^k = \ell(B)[1 - g(B)] \quad , \tag{5.2}$$

where

$$g(B) = \frac{1}{24} B^{-6} (1 + B)(1 + B + \ldots + B^{11})$$

is the filter for computing a centered 12-month moving average, and

$$\ell(B) = \frac{1}{15} B^{-36} (1 + B^{12} + B^{24})(1 + B^{12} + B^{24} + B^{36} + B^{48})$$

corresponds to a centered "3 x 5" moving average of like months in adjacent years. Because of this result, equations (5.1) and (5.2) will be used here as the linear filter approximation to X-11.

Figure 1 presents a graph of the coefficients λ_k, $-42 \leq k \leq 42$. The largest values are at 0 and at multiples of 12, and the negative values in between result in $\Sigma\lambda_k = 0$.

Similarly, for the seasonal adjustment of data near the end of the sample period, and for projected-component seasonal adjustment, the filtering procedure in X-11 can be expressed in the form (2.3); see Wallis (1982) for a fuller description. However, for computing $s_t^{(m)}$, X-11 uses a different set of weights $\{\lambda_k^{(m)}\}$ for each m, weights which are determined à priori rather than in accordance with the model fox x_t, and thus the condition (3.3) is not met. By contrast, the "X-11-ARIMA" variant proceeds by forecasting (and backcasting) the series and applying the central X-11 filter to the extended series as in (3.3), and it is this procedure to which the present section's results apply, although as a less accurate approximation they may also indicate what might be expected from X-11 itself. The filters $\lambda^{(m)}(B)$ therefore depend on the model chosen; as an illustration, Figures 2 and 3 present the concurrent and 6-month-ahead filters $\lambda^{(o)}(B)$ and $\lambda^{(m)}(B)$ for the Airline model following.

5.2 Airline Model

The model

$$(1-B)(1-B^{12})x_t = (1-\theta B)(1-\Theta B^{12})a_t \qquad (5.3)$$

has been found to effectively represent a large number of practically occurring time series. It was first fit by Box and Jenkins (1970, Chapter 9) to a series of logged monthly passenger totals in international air travel and has thus become known as the Airline model. Additionally, with suitable values of the parameters θ and Θ, it is close to the observable-series models found to be implied by the X-11 and X-11 ARIMA procedures (Burridge and Wallis 1984, Cleveland and Tiao 1976). We shall examine in greatest detail this model with parameter values $\theta = .4$, $\Theta = .6$, corresponding to those obtained by Box and Jenkins for the Airline data, though in Section 5.5 we also consider other sets of parameter values. The values $\theta = .4$, $\Theta = .6$ are close to the range of those implied by the Cleveland-Tiao and Burridge-Wallace models, and while a somewhat smaller Θ-value may give a slightly improved X-11 approximation (Bell and Hillmer 1984), somewhat larger values are often observed in practice, so that the original airline-data value of $\Theta = .6$ seems like a useful compromise; moreover Burridge and Wallace observe a robustness in that modestly different models are all capable of approximating the filtering characteristics of the X-11 procedure.

Given the model (5.3) with $\theta = .4$, $\Theta = .6$, we can calculate the leading coefficients in $\mu(B)$ in (3.5) [with $\Delta(B) = (1 - B)(1 - B^{12})$] and the various quantities which depend on m such as the variances σ_*^2 in (3.7), the ratios in (3.9), and the analogous measures in Section 4. Additionally we shall make use of the coefficients $\lambda_j^{(m)}$ in

$$\lambda^{(m)}(B) = \frac{\Delta(B)[\mu(B)]_m}{\psi(B)} \tag{5.4}$$

where

$$[h(B)]_m = h_m B^m + h_{m+1} B^{m+1} + \ldots$$

denotes the operator whose coefficients h_j of B^j are identical to those of $h(B)$ if $j \geq m$ and are 0 if $j < m$. For comparison of projected-component and concurrent adjustment our interest is largely in $m = 1,\ldots,12$.

In analyzing economic time series the changes, or rates of change in the case of logged data, are frequently of at least as great an interest as the levels. Thus our comparisons are given for both levels and changes of the series. To the extent that the linear filter version of X-11 is an accurate characterization, the seasonal component for the change in a series is the change in the seasonal component for the levels series. Thus $\lambda^{(m)}(B)$, $\mu(B)$ and the ensuing quantities can be determined for changes by removing $(1 - B)$ from the model, that is using $\Delta(B) = 1 - B^{12}$ in (3.5).

5.3 No Revision Error in NSA Data

Equations (3.9) and (3.12), based respectively on total revisions and first-year revisions, show the ratio of the revision standard deviation from concurrent adjustment to that from projected-component adjustment. These ratios lie between 0 and 1; the smaller a ratio, the greater the gain from concurrent adjustment. The total and first-year revision measures are respectively of the form $[k/t_m]^{1/2}$ and $[k_m/f_m]^{1/2}$, where the quantities

$$k = \sum_{-42}^{-1} \mu_j^2 \tag{5.5}$$

$$= .110 \text{ (levels)}, .125 \text{ (changes)} \quad ,$$

$$t_m = \sum_{-42}^{m-1} \mu_j^2 = k + c_m \quad , \tag{5.6}$$

$$k_m = \sum_{m-12}^{-1} \mu_j^2 \quad , \tag{5.7}$$

and

$$f_m = \sum_{m-12}^{m-1} \mu_j^2 = k_m + c_m \tag{5.8}$$

are $(1/\sigma_a^2)$ times the revision variances under concurrent [(5.5) and (5.7)] and projected-component [(5.6) and (5.8)] adjustment, for total [(5.5) and (5.6)] and first-year [(5.7) and (5.8)] revisions.

Table 2 shows these variances, and also gives the values

$$c_m = (1/\sigma_a^2) \text{ Var}(r_t^*) = \sum_{j=0}^{m-1} \mu_j^2$$

as in (3.7) and (3.8). Regarding the series levels, note that the variance c_m, of the revision r_t^* due to the failure to concurrently adjust, increases dramatically at $m = 11, 12$ as does the variance of the total revision, t_m. The variance of the first-year revision with concurrent adjustment, k_m, is less than that from projected-component adjustment, f_m, for all twelve values of m. For series changes, the variance c_m increases monotonically, but less dramatically at $m = 11, 12$ than is the case for levels. However,

c_m for changes jumps quite dramatically at $m = 2$, as does the variance of the total revision, t_m. The variance of the first-year revision with concurrent adjustment, k_m, is substantially less than that of the projected-component adjustment, f_m, for $m = 1, \ldots, 12$. Also, note that the first-year revision variance with concurrent adjustment drops off by at least two thirds for $m>1$.

Table 3 shows the ratios (3.9) and (3.12) of the revision standard deviations under concurrent adjustment to those under projected-component adjustment. In all cases the ratios are smaller for m near 12 than for m near 1, reflecting the greater amount of information ignored by the projected-component method later in the calendar year. The ratio of the standard deviations is smaller for the first-year revisions than for the total revisions, which is expected since the subsequent revisions are unchanged under either procedure. The zero value for December is because the first-year revision utilizes no further information than is already available for the December concurrent value. Table 3 also reveals that the gain from concurrent adjustment is greater for changes than for levels, a phenomenon which has also been observed in empirical studies (e.g., McKenzie 1984).

5.4 Single-Month Revision in NSA Data

Continuing with the linear-filter approximation to X-11-ARIMA and the Airline model (5.3), we wish to determine the effects of preliminary-data error on measures of the gain from concurrent adjustment, such as those shown in Table 3. We confine attention to the case in Section 4.1 where the preliminary NSA data are revised once one month later; and we further consider only the total seasonal revisions. As seen in Section 4.1, two effects in offsetting directions are present: the current month's value is known less precisely, increasing the revision from concurrent adjustment; and last December's value was known less precisely at the time of the seasonal projections, increasing the projected-component revision. From (4.10) these effects depend quantitatively on the relative magnitudes of the leading coefficients $\lambda_o^{(o)}$ and $\lambda_m^{(m)}$ of the filters $\lambda^{(o)}(B)$ and $\lambda^{(m)}(B)$ used in concurrent and projected-component adjustment (of the series value for the m^{th} month), and on the ratio v of the variances of the revisions R_t and series innovations a_t. Table 4 gives the first of these, the coefficients $\lambda_o^{(o)}$ and $\lambda_m^{(m)}$.

Table 5 presents the values of the ratio of standard deviations (4.10), for levels and changes of the series and for a range of variance ratios v. Several observations can be made about these results. As one progresses downward (as m increases), the entries generally decrease, although they are quite stable for the majority of "central" m values (generally excepting January, November, and December). As was the case without NSA data error (Table 3), the revision RMS ratios for December and to a lesser extent November exhibited the greatest deteriorations relative to the preceding months, resulting in the largest payoffs from the concurrent procedure.

To assess the effects of preliminary-data error, it is of interest to compare Table 5(a) with the first column of Table 3 and Table 5(b) with the second column. As perhaps expected, the effects of small amounts of NSA data error, say for v = .01 or .1, are very slight; the gain from concurrent adjustment is about what it would have been had the initial NSA figures been final. However, as we move across Table 5 the situation deteriorates markedly. When the variance of the revisions R_t approaches that of the revised NSA series' innovations (when v is near 1), there is a much reduced gain from concurrent adjustment for all months except December. In terms of equation

(4.10), since $\lambda_o^{(o)}$ is much larger than $\lambda_m^{(m)}$ for $1 \leq m < 12$ (Table 4), increasing v results in a faster increase in the numerator than in the denominator of this equation.

5.5 Effects of Parameter Changes

The foregoing results were all for a given model and parameter values, and it is of interest to see how variations in the model can affect the performance of concurrent adjustment. Table 6 gives the same information as in Table 3, for four additional configurations of Airline-model parameter values. The basic pattern is the same: all ratios are less than 1 (concurrent seasonal adjustment is superior) and decrease as m goes from 1 to 12 (the superiority of concurrent adjustment increases as one progresses through the months of the year).

The effects of error in the preliminary NSA data for these four models were also computed, and were found to be comparable to those for the Airline-data parameter values in Table 5.

6. EMPIRICAL EXAMPLE

We illustrate the foregoing results by analyzing one of the components of the Federal Reserve's Industrial Production Index, namely the Index of Nonelectrical Machinery, over the period 1970-1983. The series of final NSA values (in logarithmic form) is shown in Figure 4. The preliminary data for a given month are first released on or near the 14th of the following month and then undergo three successive monthly revisions, as shown in Figure 5 for the year 1980. Thus, in the notation of Section 4.3, k=3 and the historical record consists of the four series $\{X_{0t}\}$, $\{X_{1t}\}$, $\{X_{2t}\}$, and $\{x_t\}$. However, to obtain a more direct comparison with the numerical X-11 calculations in Section 5 for a single month's delay, we took X_{2t} as the preliminary-data series (so that k=1); thus at a given time t (from April 1970 onward) the available data are

$$\tilde{x}_t = X_{2t}, X_{t-1}, X_{t-2}, \ldots \quad .$$

We chose a sample period of January 1977 through December 1980, consisting of 48 months. This gave six initial years with which to begin the ARIMA modelling and the X-11 ARIMA seasonal adjustment (moreover a major re-benchmarking of the NSA data occurred in 1976), and three subsequent years from which to compute final seasonal component estimates.

For the eight sample periods ending in December for the years 1976 through 1983 we made the following basic calculations, illustrated for the period ending in 1979:

1. An ARIMA model was fitted to the series as of the end of 1978 (to the series $\tilde{x}_t$ with t denoting December 1978). This model was of the form

$$(1-\phi_1 B-\phi_2 B^2)\nabla\nabla_{12}x_t = (1-\Theta B^{12})a_t \qquad (6.1)$$

and Table 7 gives parameter values for this period and the other seven sample periods used.

2. An additive X-11 ARIMA procedure was used to obtain projected seasonal component estimates, $\tilde{s}_t^p$ as in (4.14), for the year 1979.

3. This same procedure (and with the same ARIMA model) was applied twelve times, on data ending in January 1979, ..., and in December 1979 to obtain concurrent seasonal component estimates, $\tilde{s}_t^c$ as in (4.13), for each month of 1979.

4. From the X-11 ARIMA run in (2), "first-revised" seasonal component estimates were obtained for the year 1978.

5. Steps 2 and 3 were also run using final historical NSA data (x_t rather than $\tilde{x}_t$) for comparison purposes.

In addition, based on the entire sample period, final seasonal component estimates were obtained for the years 1980 and earlier. From these figures the revisions r_t^c, r_t^p, $\tilde{r}_t^c$, and $\tilde{r}_t^p$, for t ranging over the 48 months January 1977 through December 1980, were obtained, their empirical mean squares calculated, e.g.,

$$RMS(r_t^c) = [(1/48) \sum_{1-77}^{12-80} (\tilde{r}_t^c)^2]^{1/2} \quad , \qquad (6.2)$$

and the appropriate RMS ratios (the empirical analogues of (3.9) and (4.19)) determined. All of this was done for changes as well as levels of the series.

The results of this analysis, summarized in Table 8, are generally in agreement with the findings of Section 5. For example, the gain from concurrent adjustment is reduced by the presence of preliminary-data error (RMS ratios of .96 rather than .84 for levels and .71 rather than .57 for changes); but there still is a gain (the RMS ratios .96 and .71 are less than 1). Moreover, as expected the biggest payoff from using concurrent adjustment is in measuring the changes (growth rates) of the series; a 29 percent reduction in RMS revision for changes versus a 4 percent reduction for levels. (These are for total revisions; as in Tables 3 and 5 the proportionate reduction in first-year RMS revisions would be still greater).

7. CONCLUSIONS

This study has shown theoretically the expected gain from concurrent seasonal adjustment, including the case of preliminary not-seasonally-adjusted (NSA) data error. The expected gain is calculated in terms of the reduction in root mean square error of revisions. The gain from concurrent adjustment is found to be reduced by the presence of error in the NSA data, but not eliminated.

Some specific findings are of interest.

1) The seasonal component estimate at time t-m, $s_t^{(m)}$, and the revision in that estimate between times t-m and t-n, $r_t^{(m,n)}$, are expressed as linear combinations of, respectively, the historical data and the innovations in the incoming series values. This leads to expressions of the MSE of the "penalty revision" of projected-component adjustment. The MSE of the penalty revision is found to be periodic: $\sigma_a^2\mu_o^2$ for January, $\sigma_a^2(\mu_o^2+\mu_1^2)$ for February, and so on, where the μ_i are coefficients in the expression of the revisions as linear combinations of the series innovations. These expressions for revision MSE are dominated by the term containing the center-weight term μ_0, which supports the earlier empirical findings (i) that there are comparable gains from concurrent adjustment in all months of the year, and (ii) that the gain from increasing the frequency of projected-component estimation (say, to every six months or even to every two or three months) is far less than the gain from moving to a fully concurrent adjustment.

2) The revision procedure (for already-published seasonally adjusted data) which is currently in widespread use is revising initially-published estimates only once a year, after every December. If concurrent seasonal adjustment is employed under such a revision scheme, then the gain from concurrent seasonal adjustment is entirely realized in the first year. However, other revision schemes have been advocated—such as every month revising the one-month-ago and twelve-months-ago figures—and the gain from concurrent adjustment under such alternative revision schemes may well extend past the first year.

3) When preliminary-data error exists, the uncertainty in the concurrently determined value is increased. However, deterioration is also observed in the projected-component estimates for December and other end-of-year months for which non-seasonally-adjusted data are still preliminary when projections are made. The deterioration in the projected-component estimate may exceed that of the concurrent estimate for December, so that even with preliminary-data error a substantial gain from concurrent adjustment is realizable, especially for the latter months of the year.

4) Using typical forecast-augmented series (from the "airline" ARIMA model) with Young's linear approximation to Census X-11, theoretical root mean square revisions under both projected-component and concurrent adjustment are calculated, for both the level of the series and the month-to-month change (or growth rate). The ratio of the revision standard deviation under

concurrent to that under projected-component adjustment is calculated; the gain from concurrent adjustment is expressed as 1 minus that ratio. A gain is always observed, but to varying degrees. The gain from concurrent adjustment is greater for changes than for levels. Without preliminary-data error, the theoretical gain with concurrent adjustment for total revisions is 18 to 30 percent for levels and 28 to 32 percent for changes, with a markedly increased gain in the latter months of the year.

In the case of one month of preliminary-data error, calculations show the decreasing gain from concurrent adjustment as the degree of this error increases (measured by the variance of the revisions due to preliminary-data error relative to the variance of the revised NSA series' innovations). For example, when the variance of the revision is half that of the revised NSA series' innovations, the gain from concurrent adjustment for total revisions is 9 to 27 percent for levels and 15 to 29 percent for changes. Substantial gains from concurrent adjustment are still present for the latter months of the year. These results hold for a range of values of the parameters in the ARIMA model used to produce the forecasts which augment the series.

5) The theoretical results are illustrated with an empirical example, the Federal Reserve Board's Industrial Production Index, a monthly series which undergoes three NSA preliminary data revisions. As expected from the preceding theoretical results, concurrent adjustment exhibits a gain, but one which

is reduced by the preliminary data error. However, the gain (for total revisions) from concurrent adjustment for month-to-month changes (i.e., the growth rate) in the series remains substantial.

Table 1. Information Known and Unknown at Time t for a Series x_t with Preliminary-Data Error

(a) Known Observation	(b) Representation in Terms of Initial Data and Incremental Revisions	(c) Unknown Revision Remaining
X_{0t}	X_{0t}	$R_{1t} = E_{1t} + E_{2t} + \ldots + E_{k-1,t} + E_{kt}$
$X_{1,t-1}$	$X_{0,t-1} + E_{1,t-1}$	$R_{2,t-1} = E_{2,t-1} + \ldots + E_{k-1,t-1} + E_{k,t-1}$
$\vdots$	$\vdots$	$\vdots$
$X_{k-1,t-k+1}$	$X_{0,t-k+1} + E_{1,t-k+1} + \ldots + E_{k-1,t-k+1}$	$R_{k,t-k+1} = E_{k,t-k+1}$
$X_{k,t-k} = x_{t-k}$	$X_{0,t-k} + E_{1,t-k} + \ldots + E_{k-1,t-k} + E_{k,t-k}$	0

Table 2. Revision Variances (Normalized by σ_a^2), Airline Model

	(a) Levels				(b) Changes			
m	c_m	t_m	k_m	f_m	c_m	t_m	k_m	f_m
1	.053	.164	.037	.090	.117	.242	.019	.136
2	.058	.168	.034	.092	.143	.268	.003	.146
3	.061	.171	.033	.093	.144	.268	.003	.146
4	.062	.172	.032	.094	.144	.268	.002	.146
5	.062	.172	.032	.094	.144	.269	.002	.146
6	.062	.172	.032	.094	.145	.269	.002	.146
7	.063	.173	.031	.094	.145	.270	.002	.147
8	.065	.175	.030	.095	.146	.270	.0014	.147
9	.070	.180	.027	.096	.146	.271	.0010	.147
10	.078	.188	.021	.099	.146	.271	.0006	.147
11	.091	.201	.012	.103	.147	.272	.0003	.147
12	.109	.220	.000	.109	.147	.272	.000	.147
		k = .110				k = .125		

Legend

c_m: variance of r_t^*, revision avoided by concurrent adjustment.

t_m: variance of total revision, projected-component adjustment.

k_m: variance of first-year revision, concurrent adjustment.

f_m: variance of first-year revision, projected-component adjustment.

k: variance of total revision, concurrent adjustment.

Table 3. Ratio of Revision Standard Deviations Under Concurrent and Projected-Component Seasonal Adjustment, Airline Model with θ = .4, Θ = .6

	Total Revisions		First-Year Revisions	
m	Levels	Changes	Levels	Changes
1	.821	.718	.640	.371
2	.809	.682	.608	.137
3	.803	.682	.591	.132
4	.800	.681	.583	.127
5	.800	.681	.582	.122
6	.800	.680	.582	.116
7	.798	.680	.578	.107
8	.793	.679	.561	.094
9	.783	.679	.525	.081
10	.765	.678	.460	.066
11	.740	.677	.345	.047
12	.708	.677	.000	.000

Table 4. Leading Coefficients in Concurrent and Projected-Component Seasonal Filters $\lambda^{(m)}(B)$

m	$\lambda_m^{(m)}$ Levels	Changes
0	.23	.34
1	.07	-.16
2	.05	-.02
3	.03	-.02
4	.02	-.02
5	-.00	-.02
6	-.02	-.02
7	-.05	-.02
8	-.07	-.02
9	-.09	-.02
10	-.11	-.02
11	-.14	-.02
12	.28	.42

m = 0: Concurrent adjustment

m = 1,...,12: Projected-component adjustment, January, ..., December

Table 5. Ratio of Revision Standard Deviations Under Concurrent and Projected-Component Seasonal Adjustment, for Given Ratio v of NSA Revision Variance to Innovation Variance

(a) Levels

m v→	.01	.1	.25	.5	.75	1
1	.823	.839	.866	.908	.948	.986
2	.811	.828	.855	.898	.939	.978
3	.805	.822	.849	.893	.935	.975
4	.802	.819	.847	.892	.934	.974
5	.802	.819	.847	.891	.934	.974
6	.802	.819	.846	.891	.933	.973
7	.800	.817	.844	.887	.928	.966
8	.795	.811	.837	.878	.917	.953
9	.784	.799	.824	.862	.898	.932
10	.766	.780	.803	.838	.871	.901
11	.741	.754	.775	.806	.836	.863
12	.709	.712	.718	.726	.733	.739

(b) Changes

m v→	.01	.1	.25	.5	.75	1
1	.721	.747	.787	.848	.902	.950
2	.685	.713	.758	.827	.890	.949
3	.685	.713	.757	.826	.889	.949
4	.684	.712	.757	.826	.889	.948
5	.684	.712	.756	.825	.888	.947
6	.684	.712	.756	.824	.888	.947
7	.683	.711	.755	.824	.887	.946
8	.682	.710	.754	.823	.886	.945
9	.682	.710	.754	.822	.885	.944
10	.681	.709	.753	.821	.884	.943
11	.680	.708	.752	.821	.884	.942
12	.678	.686	.698	.714	.726	.735

Table 6. **Ratio of Revision Standard Deviations Under Concurrent and Projected-Component Seasonal Adjustment, Alternative Airline-Model Parameter Values**

(a) $\theta = .4$, $\Theta = 0$				m	(b) $\theta = .4$, $\Theta = .9$			
Total Revisions		First-Year Revisions			Total Revisions		First-Year Revisions	
Levels	Changes	Levels	Changes		Levels	Changes	Levels	Changes
.784	.649	.633	.329	1	.859	.784	.650	.420
.771	.612	.605	.133	2	.849	.752	.614	.144
.764	.610	.589	.129	3	.843	.751	.593	.138
.761	.610	.583	.125	4	.841	.751	.584	.131
.760	.609	.582	.121	5	.841	.751	.582	.123
.760	.609	.582	.116	6	.841	.750	.582	.115
.758	.608	.577	.108	7	.840	.750	.578	.106
.752	.607	.560	.094	8	.836	.750	.563	.094
.738	.607	.521	.081	9	.829	.749	.530	.081
.715	.606	.453	.066	10	.817	.749	.468	.066
.684	.605	.336	.047	11	.799	.748	.356	.047
.646	.604	.000	.000	12	.775	.748	.000	.000

(c) $\theta = 0$, $\Theta = 0$				m	(d) $\theta = .9$, $\Theta = .9$			
Total Revisions		First-Year Revisions			Total Revisions		First-Year Revisions	
Levels	Changes	Levels	Changes		Levels	Changes	Levels	Changes
.914	.632	.865	.248	1	.770	.836	.298	.643
.867	.630	.785	.240	2	.770	.747	.295	.021
.842	.629	.739	.232	3	.770	.747	.291	.020
.830	.627	.716	.225	4	.769	.747	.286	.019
.826	.626	.708	.216	5	.768	.747	.279	.018
.825	.624	.707	.208	6	.767	.747	.270	.017
.824	.622	.705	.194	7	.766	.747	.256	.015
.817	.619	.690	.173	8	.764	.747	.238	.012
.798	.617	.647	.150	9	.762	.747	.213	.011
.764	.614	.563	.122	10	.759	.747	.180	.009
.716	.612	.416	.086	11	.757	.747	.132	.006
.659	.609	.000	.000	12	.753	.746	.000	.000

Table 7. ARIMA Models Fit to IP Index of Nonelectrical Machinery[a]

Sample Period	ϕ_1	ϕ_2	θ	σ_a	Q_{20}[b]
1970-76	.36	.36	.30	.0144	13.5
1970-77	.39	.25	.52	.0147	13.1
1970-78	.29	.33	.53	.0146	13.7
1970-79	.28	.28	.63	.0148	13.3
1971-80	.23	.26	.50	.0149	6.4
1972-81	.28	.19	.69	.0152	10.9
1973-82	.24	.27	.70	.0156	9.6
1974-83	.30	.25	.56	.0177	11.8

a/ Model: $(1-\phi_1 B-\phi_2 B^2)\nabla\nabla_{12}x_t = (1-\theta B^{12})a_t$

x_t = log of series.

b/ Box-Pierce-Ljung Q-Statistic [see (Box and Pierce 1970) and (Ljung and Box 1978)].

Table 8. Root Mean Square Revisions, Industrial Production Index of Non-electrical Machinery

	Concurrent	Projected	Ratio
(a) Levels			
No Preliminary Data Error $\{x_t\}$	.0021	.0025	.84
Preliminary Data Error $\{\tilde{x}_t\}$	.0049	.0051	.96
(b) Changes			
No Preliminary Data Error $\{x_t\}$	.0012	.0021	.57
Preliminary Data Error $\{\tilde{x}_t\}$	.0037	.0052	.71

Figure 1. Seasonal Weights λ_k in the Linear Filter Approximation to X-11

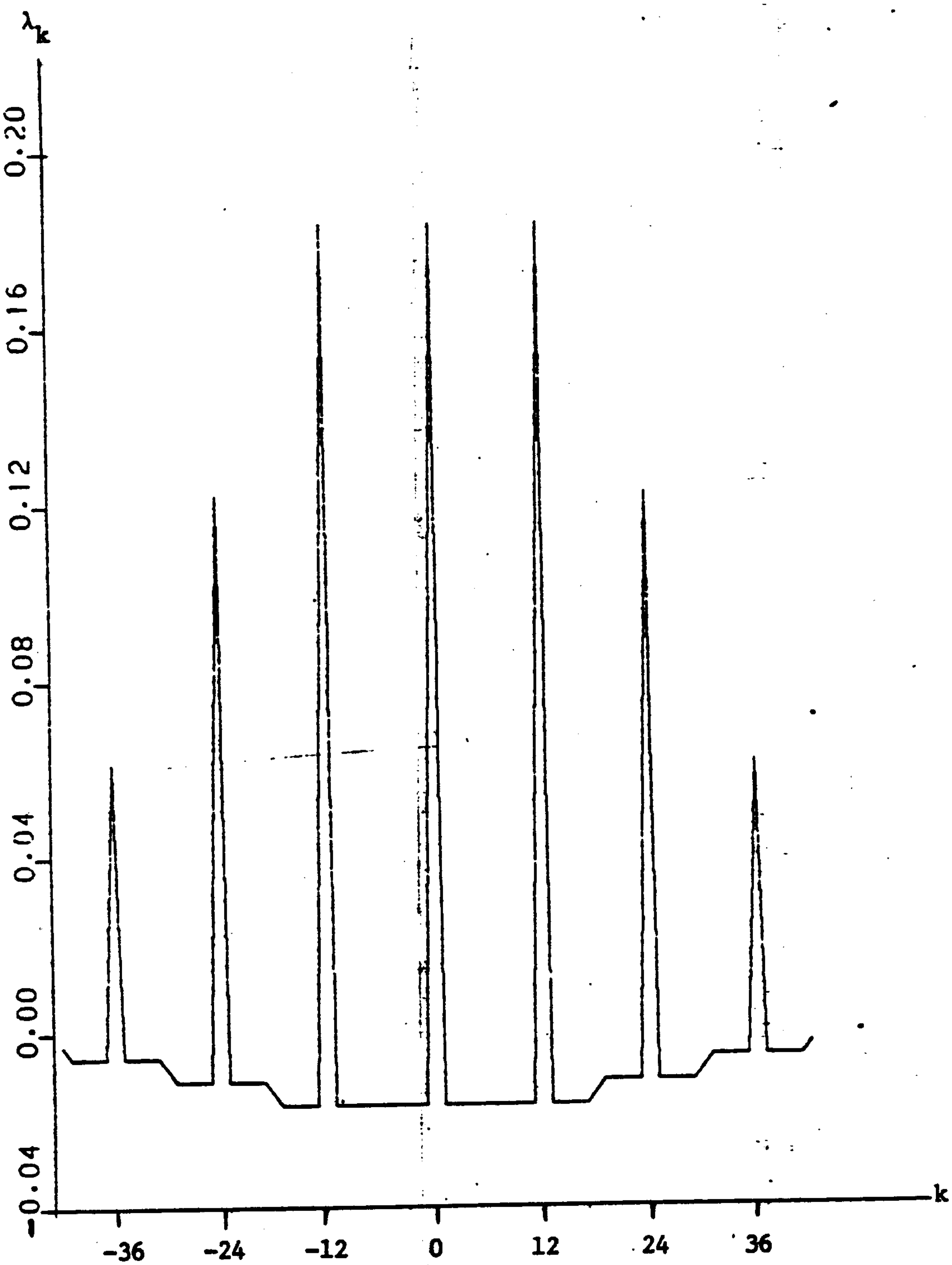


Figure 2. Concurrent Adjustment Filter Seasonal Weights

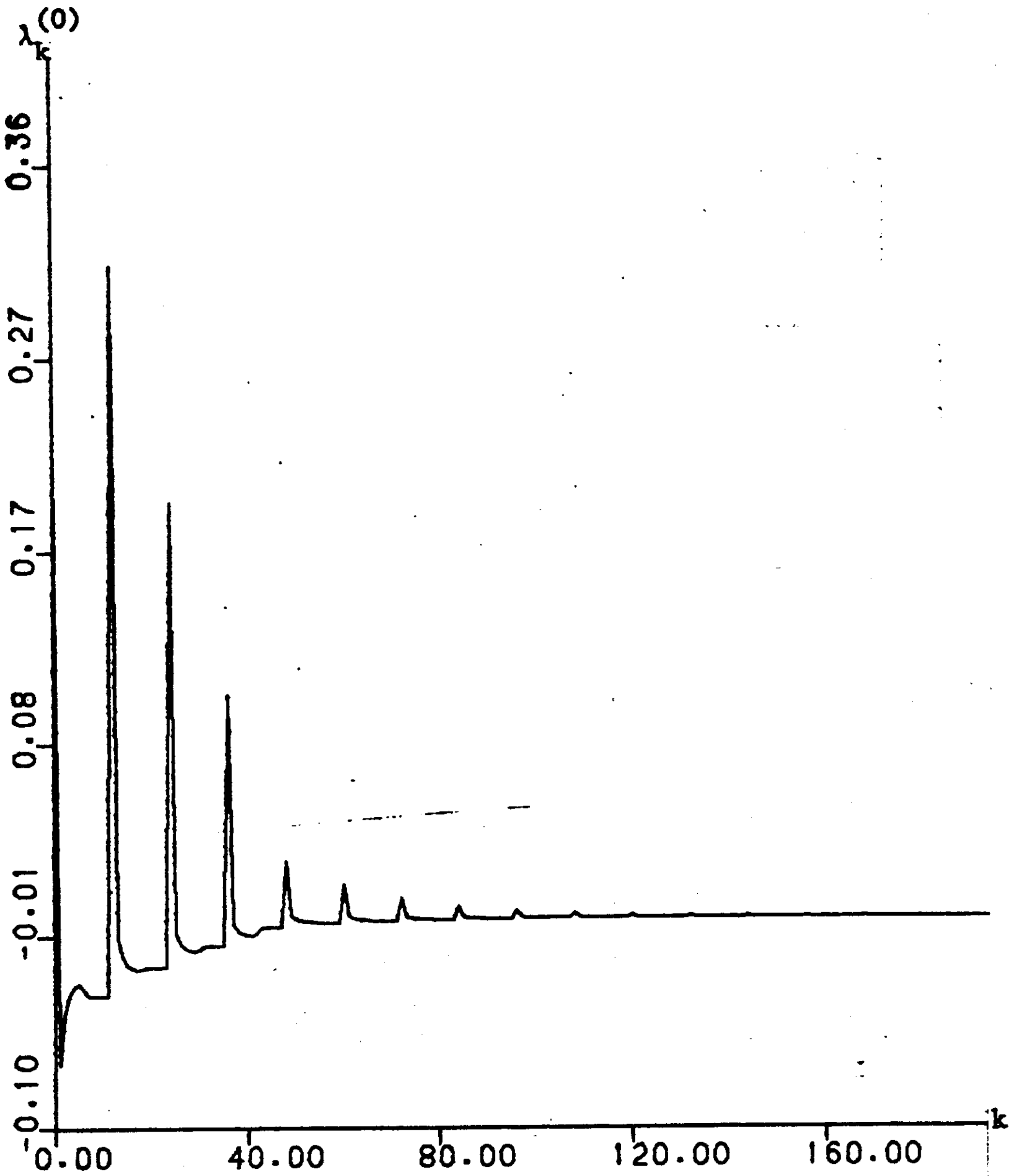


Figure 3. Six-Month-Ahead Adjustment Filter Seasonal Weights

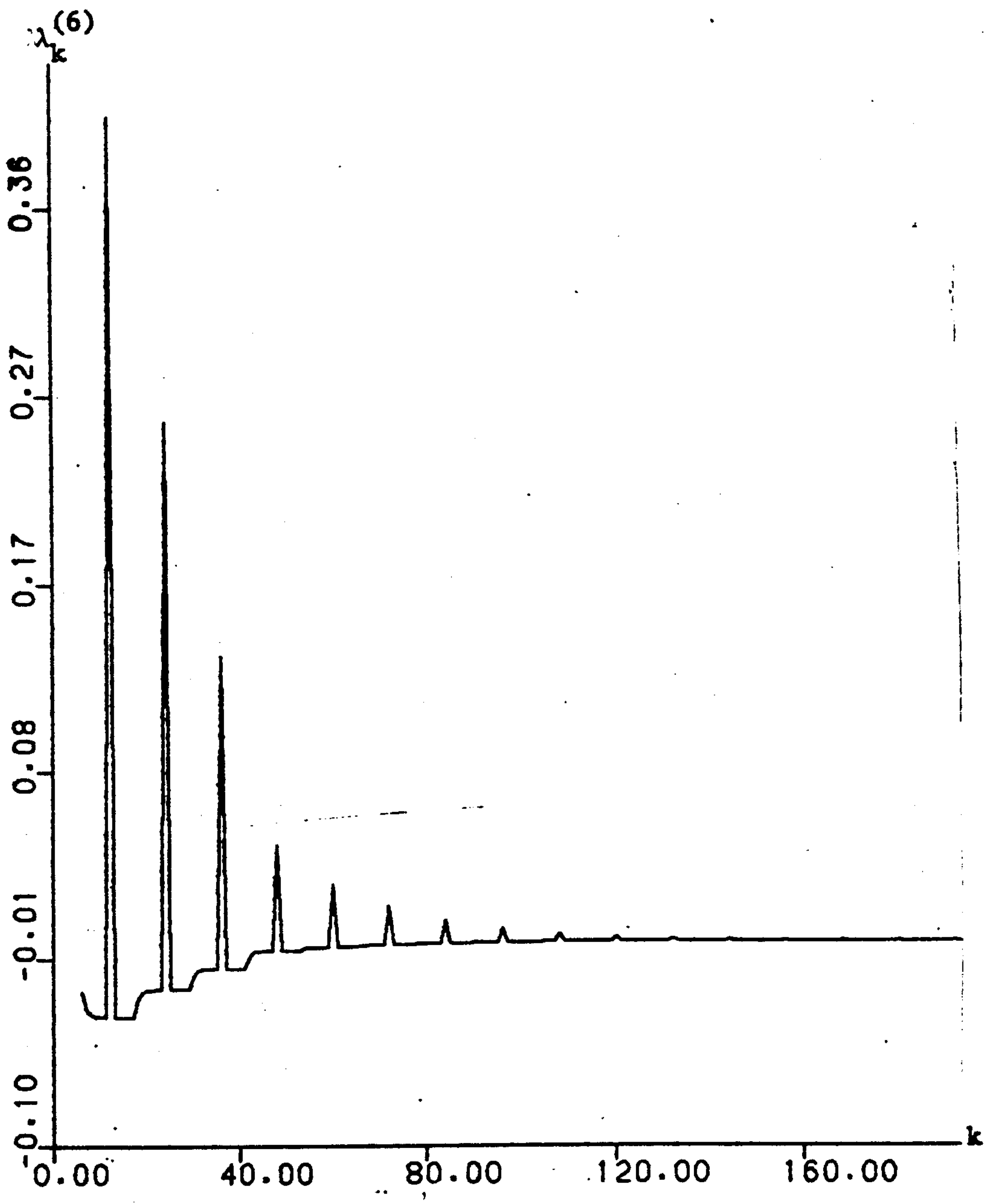


FIGURE 4. NATURAL LOGARITHM OF INDUS. PROD. INDEX, NON-ELEC. MACH.

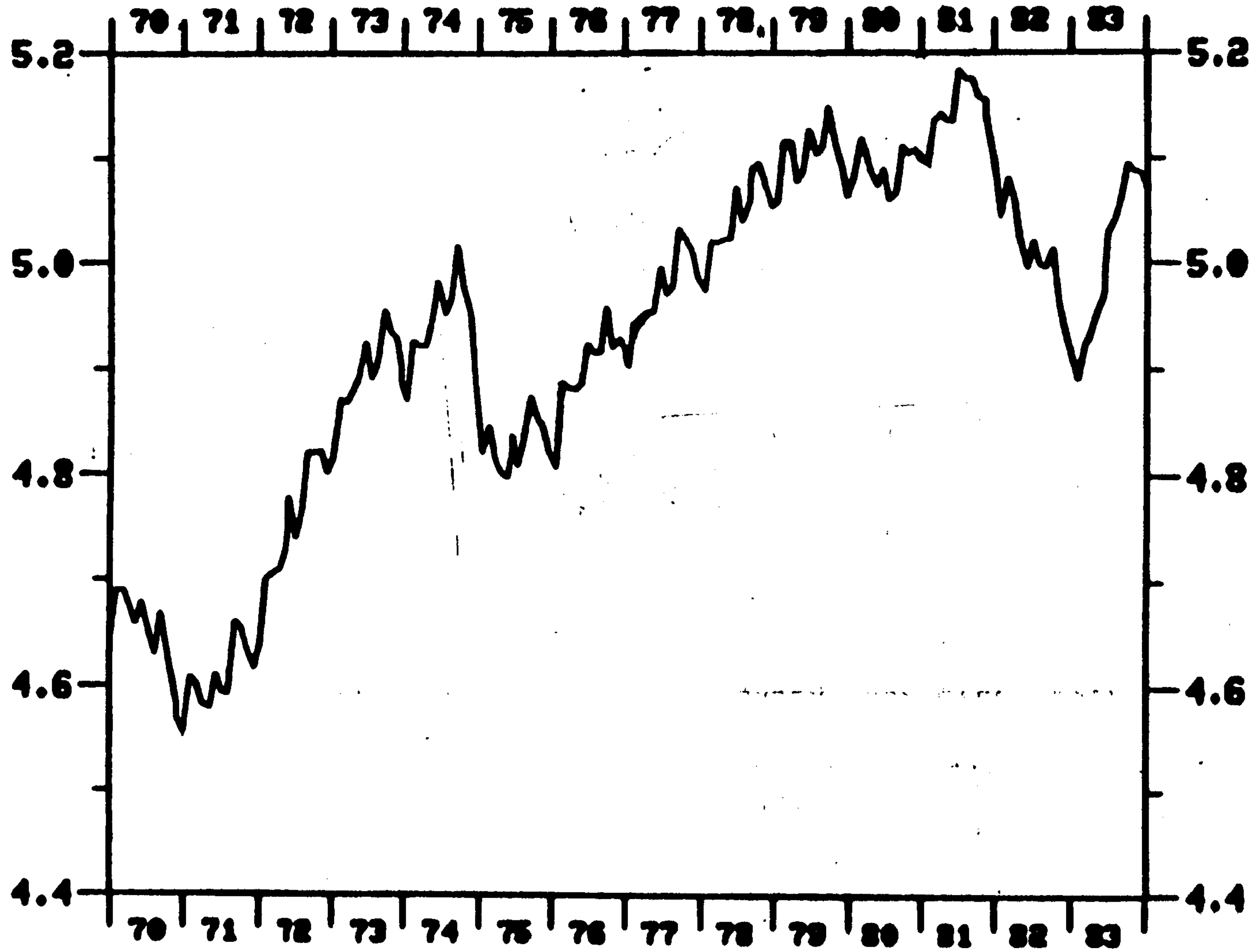
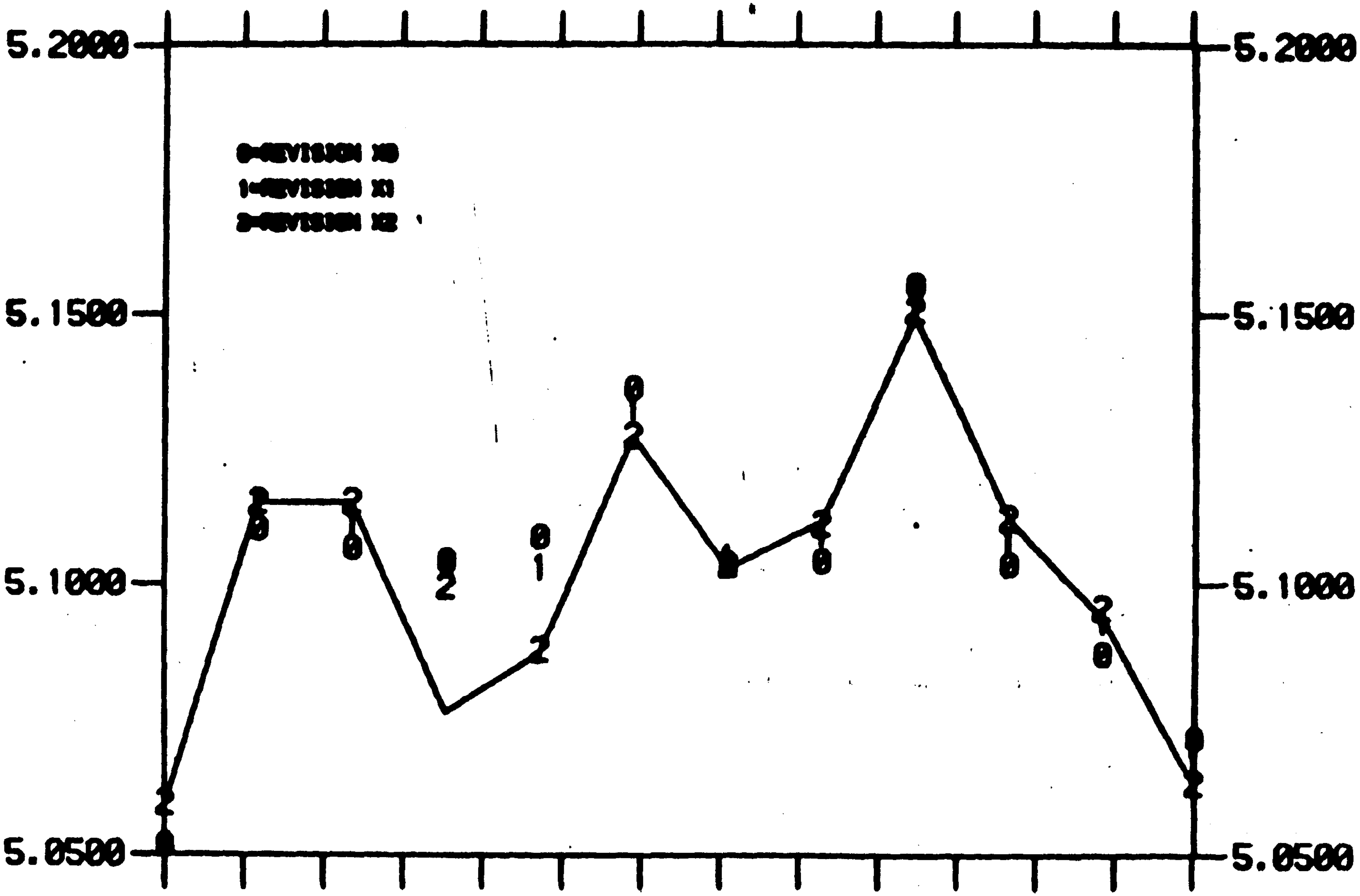


FIGURE 5. **FINAL, AND THREE PRELIMINARY REVISIONS (LN)**, 1980



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