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TECHNIQUES FOR DETERMINING IF A SEASONAL
TIME SERIES CAN BE SEASONALLY ADJUSTED RELIABLY
BY A GIVEN SEASONAL ADJUSTMENT METHODOLOGY

by

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Abstract. Some new techniques are presented for analyzing the revisions which occur when the seasonal (and/or calendar) adjustment of a given month of a "seasonal" time series is recalculated based on additional data for the series. By analyzing these revisions, the analyst can see how stable the estimates are of the seasonal factors and, therefore, gain information about how reliable the seasonal adjustment method is in estimating the factors. These new techniques are applied to a Census Bureau series, and the conclusions are compared with those drawn from more traditional techniques.

INTRODUCTION

Deciding when a series should be seasonally adjusted can be a very difficult problem. There are situations where a series may show evidence of seasonality, but because of a strong irregular component, for example, or a volatile seasonal component, the seasonal factors can not be estimated reliably. In these circumstances, the estimates of a given month's seasonal factor may change dramatically when more data are added to the series, or when earlier data are deleted.

The diagnostics which are widely used at the present time for determining if a series can be reliably adjusted are sometimes inadequate. In order to provide the analyst with more tools, we have developed two new

techniques for deciding when a series can be seasonally adjusted reliably. One technique involves using the revisions history of a series to give measures of (a) how much the initial seasonal adjustments get revised and (b) how rapidly these adjustments converge to a final value. The other uses adjustments performed on sliding spans of data to enable the analyst to see how stable the estimates are of seasonal factors and month-to-month changes in the seasonally adjusted data (and also, if desired, trading day factors). If too many months have unstable estimates, it is an indication that the seasonal adjustment method being used cannot reliably adjust the series being examined.

In this paper, we will use these methods in conjunction with others to show that a Census Bureau series called XU3 (exports of mineral fuels, lubricants and related materials) should probably not be seasonally adjusted using X-11 (or X-11-ARIMA without the ARIMA forecasts), despite some evidence that there is seasonality present in the data and some suggestions from other diagnostic statistics that this series can be successfully adjusted. Although X-11 and X-11-ARIMA are the only methods discussed in this study [1], the techniques presented can be adapted for use with other seasonal adjustment methods.

The graph of the series XU3, given in Figure 1, does not reveal a stable or persistent seasonal pattern. Also, it suggests that the series undergoes a significant change around 1974. The careful analyst should seriously consider the question of what data span to use. We will begin our analysis on the full series (January, 1966 to December, 1983), and later give a summary of an analysis performed on the shortened series (January, 1974 to December, 1983).

X-11-ARIMA RESULTS

X-11-ARIMA was used, without forecasting, to seasonally adjust the full series with 3x9 seasonal filters. Using diagnostics from X-11-ARIMA, we find some evidence to support seasonally adjusting the series. A summary of these diagnostics is given in Table 1. The F-tests used to test for stable seasonality tentatively suggest that there is significant seasonal variation in this series. The F-test for moving seasonality indicates that the pattern of seasonality is not changing in a deleterious manner [2]. However, there are other indications that this series may not be a good candidate for seasonal adjustment. The series is very irregular (58.98 percent of the variation is attributed to the irregular component, after first differences are taken), and this can cause problems in the estimation of the seasonal factors. An examination of the SI ratios [3] for the last six years of data (given in Figure 2) shows how this irregular can spread out the values of the SI ratios for a given month. This spread can cause problems in the estimation of seasonal factors.

The quality control statistics of X-11-ARIMA are not encouraging. X-11-ARIMA provides eleven quality control statistics to help the user to evaluate the acceptability of a seasonal adjustment performed by X-11-ARIMA. These eleven statistics are used in a weighted average to derive Q, an overall measure of the acceptability of the seasonal adjustment (see [4] for more details). If Q is less than one, the adjustment is deemed acceptable by X-11-ARIMA's criterion; if Q is greater than one, the adjustment is unacceptable. For XU3, the value of Q is 1.08, casting some doubt on the adjustability of

the series. (We have revised this Q measure, due to an anomaly found in one of the eleven quality control statistics. Our revision of Q is described in the Appendix. The original Q's value for XU3 is .87.)

REVISION HISTORY

We will now explain in detail the first of our new techniques. This procedure uses the revision histories of a span of individual months of a series to compute two measures, which we call CPREV and CONRAT, to help the user evaluate the reliability of the seasonal adjustment.

To make it possible to produce a history of seasonal adjustment revisions, the series being investigated must be long enough that a several year span of (nearly) final seasonal adjustments can be calculated. Such adjustments are only available for months far enough away from the ends of the series that their seasonal adjustments are obtained by the use of the symmetric versions of the type of moving average (filter) specified by the adjustor. Thus, in particular, a "start up" period is required. For XU3, we chose the first seven years of the series. Thereafter, for each month within a span of past this start-up period, a set of successive seasonal adjustments is calculated as later data are added to the series, a month at a time.

Let $X_{i,t}$ be the seasonally adjusted estimate for month i obtained by running the seasonal adjustment program with data up through month $i+t$. Therefore, $X_{i,0}$ is the concurrent estimate for month i , and $X_{i,17}$ is the seasonally adjusted estimate for month i from the seasonal adjustment calculated with data

up through month $i+17$. Because of the finite length of the filters used these $X_{i,t}$ converge to a final value which is nearly reached as soon as the estimate of the seasonal factor is obtained using the symmetric version of the X-11 seasonal filters, rather than the unsymmetric versions used initially. The number of months N until this final adjustment X_i is reached depends upon the length of the seasonal filter used to adjust the data.

For months falling within a chosen span of months after the start-up period, we will track values of the successive X-11 adjustments, $X_{i,0}, \dots, X_{i,N}$. This span is called the experimental period.

The first diagnostic quantity we discuss measures the cumulative amount of revision undergone by the seasonal adjustments of a given month in the experimental period. This is expressed as a percentage of the concurrently adjusted value for that month. Let NOBS be the number of observations in the experimental period. Then

$$\text{CPREV}(i) = \frac{1}{X_{i,0}} \sum_{t=0}^{N-1} |X_{i,t+1} - X_{i,t}| (60/N), \quad (1)$$

for $i = 1, \dots, \text{NOBS}$.

The factor $(60/N)$ in (1) was added to make it easier for the user to compare values of CPREV generated for seasonal adjustments derived from different seasonal filters having different lengths. Longer filters take longer to produce a final seasonal estimate, so if no compensating factor were used in CPREV, adjustments obtained from shorter filters would always have smaller CPREV values.

If $CPREV(i)$ is large for a particular month i , this usually means that the seasonally adjusted value for month i undergoes frequent substantial revisions, as more data become available. This is what one observes when seasonal adjustment is performed on a nonseasonal or erratically seasonal series, and is usually a sign that none of the adjustments can be regarded as reliable. If enough months have large values of $CPREV(i)$, we conclude that the series cannot be adequately adjusted by the methodology being used.

Some indication of the reliability of the final (presumably best) seasonal adjustment of a given month's datum can be obtained by assessing how erratically the preliminary seasonal adjustments converge to the final value. The intuitive reasoning goes as follows: The finite length of X-11's filters ensures that the adjustments obtained for a given month will always converge to a final value as future data are added to the series. This occurs even if (a) there is no seasonality in the series, (b) there is a seasonal pattern which is changing too rapidly to be estimable, or (c) there is a regular seasonal pattern which is too weak to measure relative to the "noise background" (irregular). Now in each of these three situations, the final adjustment is merely an artifact of the X-11 procedure and the manner in which preliminary adjustments converge to it should be more erratic than in a strongly and regularly seasonal series.

The measure $CONRAT$, which we use to assess the rate (or manner) of convergence to the final seasonal adjustment, is defined as follows. Let $NOBS$ and N be the same as in (1). Then

$$\text{CONRAT}(i) = \frac{\sum_{t=0}^{N-1} \beta^{N-1-t} \left| \frac{X_{i,t} - X_{i,N}}{X_{i,N}} \right|}{\sum_{t=0}^{N-1} \beta^t}, \quad (2)$$

where $i = 1, \dots, \text{NOBS}$, and $0 < \beta < 1$.

We use the weights β^{N-1-t} in (2) to give more weight to deviations from the final value of seasonal adjustments occurring closer in time to the final value.

One other quantity of interest can be computed from these revisions histories, the relative size of the total revision. This is measured by

$$\text{TOTREV}(i) = |X_{i,N} - X_{i,0}| / X_{i,N}. \quad (3)$$

Table 2 contains summary statistics for CPREV, CONRAT, and TOTREV taken from a revisions analysis of XU3. The means of CPREV and CONRAT are used to give an overall indication of the adjustability of the series. For CONRAT, β was selected so that

$$\beta^{N/2} = 1/2.$$

which ensures that the middle term in (2) is weighted half as much as the final term. Since 3x9 seasonal filters were selected for the X-11 concurrent adjustments used in this analysis, we set the number of months until the final adjustment for a given month is reached (denoted by N) to be equal to 60,

this being the length of the filter. Therefore, we solved $\beta^{30} = 0.5$, so that $\beta = (0.5)^{1/30} = 0.97716$.

The authors have examined the values of these statistics from a number of series regarded as candidates for seasonal and trading day adjustment and, based on this experience, have derived empirical guidelines for interpreting the results of a revisions history run. We consider values of CPREV < 0.2 and CONRAT < 0.01 to be signs of a series which one can reliably seasonally adjust with X-11's procedure. We see that the averages for CPREV and CONRAT for XU3 are higher than these empirically derived limits, so we are inclined to conclude that X-11's seasonal adjustment of XU3 is not acceptable. (The analysis of sliding spans below supports this conclusion.)

SLIDING SPANS ANALYSIS

Another method of testing the adequacy of a seasonal adjustment is to examine the results of a given method of seasonal adjustment for months common to a sequence of "sliding spans" of data within a series. We then can see how the seasonal adjustments change according to which span is used. We also use this method to check the stability of the trading day adjustment being done on the series.

To obtain these sliding spans, a first span is selected. Then the second span is obtained by deleting the earliest year of data from the first span and appending the year of data following the latest year in the first span. The third span is obtained from the second in like manner, and the process continues until there is no "future" data with which to create a new span. For example, for a series starting in January, 1974 and ending in December, 1983, three eight year sliding spans can be formed: one using data

from 1974 to 1981, another with data from 1975 to 1982, and a third with data from 1976 to 1983. The number and length of these sliding spans will depend on how much data is available, and on which seasonal filter lengths the analyst wishes to use in the analysis.

In this procedure, each month common to more than one span is examined to see if the seasonal adjustments obtained using different spans vary excessively. To show how we make such a distinction, let

$S_t(k)$ = the seasonal factor estimated at month t for span k ;

$CI_t(k)$ = the seasonally adjusted value at month t for span k ;

N_t = number of sliding spans containing month t .

We flag a month t as having an unreliable seasonal factor if

$$\frac{\max_{1 < i < N_t} (S_t(i)) - \min_{1 < i < N_t} (S_t(i))}{\min_{1 < i < N_t} (S_t(i))} > 0.03, \quad (4)$$

and as having an unreliable estimate of the month-to-month percentage change in the seasonally adjusted data if

$$\max_{1 < i < N_t} \left(\frac{CI_t(i) - CI_{t-1}(i)}{CI_{t-1}(i)} \right) - \min_{1 < i < N_t} \left(\frac{CI_t(i) - CI_{t-1}(i)}{CI_{t-1}(i)} \right) > 0.03. \quad (5)$$

Equation (4) tests if the maximum percentage difference in the seasonal factors for a month t is greater than 3 percent. Equation (5) tests if the largest difference in the month-to-month change in the seasonally adjusted data is greater than 3 percent for a month t .

Based on our somewhat limited experience, the threshold values (0.03, 0.03) seem adequate for use with X-11 or X-11-ARIMA. Other seasonal adjustment methods can be analyzed using this method, but it may be appropriate to change the threshold values depending on the method used. Also, specific user requirements for reliability might dictate different values.

Our software flags the individual months for which a threshold value is exceeded. It also produces a series of summary tables, making it easier for the analyst to understand the results of the analysis. One table gives a summary of how many months were flagged for each category (seasonal factors, month-to-month). Another shows how many times each calendar month (January, February, etc.) was flagged and how many months in each calendar year were flagged for each category.

If too many (see below) of these months are flagged, it means that enough of the seasonal adjustments are unreliable to cast doubt upon the wisdom of seasonally adjusting the series. Note that an unreliable estimate of a month's seasonal factor can give rise to two unreliable estimates in the month-to-month changes for the neighboring months. For this reason, there are almost always more months flagged for differences in the month-to-month changes than in the seasonal factors. Also, one should look for problems in certain calendar months or certain years as well. For example, problems in early years may sometimes be a sign that seasonal adjustments should be calculated from a segment of the series which does not include these earlier years.

Trading day factors can also be analyzed in this manner. Let

$TD_t(i)$ = the trading day factor estimated for month t in span i ;

N_t = number of spans containing month t .

We will flag a month t as having an unreliable trading day factor if:

$$\frac{\max_{1 \leq i \leq N_t} (TD_t(i)) - \min_{1 \leq i \leq N_t} (TD_t(i))}{\min_{1 \leq i \leq N_t} (TD_t(i))} > 0.02. \quad (6)$$

Equation (6) tests to see if the maximum percentage difference in the trading day factors for a given month t is greater than two percent. Again, summaries of the months flagged can be produced, broken down by year and calendar month.

If a large number of months have trading day factors which have been deemed unreliable, the results of the trading day regression must be considered suspect. Another sign of a troublesome trading day adjustment is a series which has significantly more unacceptable month-to-month changes than unacceptable seasonal factors. This is because instabilities in the trading day factors can be reflected in the month-to-month changes, but not in the seasonal factors.

The sliding spans analysis performed on XU3 consisted of an analysis of four eleven-year spans. X-11-ARIMA was used, with 3x9 seasonal factors.

The results of this analysis are shown in the first run of Table 3. Note the large number of trading day factors flagged, as well as the month-to-month changes. This immediately suggests a problem with the trading day factors.

An examination of the X-11-ARIMA program's F-test for trading day in each of the four spans examined showed that the trading day variation in two of these spans (1971-1981 and 1973-1983) was not judged to be significant enough for adjustment. A spectral analysis was done of the final modified irregular from an X-11-ARIMA run of XU3 without trading day adjustment. This is a technique used in the SABL seasonal adjustment program [5]. The spectrum is shown in Figure 3. We can see that there are no strong peaks at either of the trading day frequencies. Therefore, we conclude that there is not enough trading day variation present in the series for X-11-ARIMA to estimate trading day factors reliably.

The sliding spans analysis described above was repeated, this time without incorporating a trading day adjustment into the seasonal adjustment procedure. This result is also given in the second row of Table 3. Note that although the seasonal factors improve, they do not improve very much. We consider these numbers too high, indicating again that X-11 does not give reliable seasonal factors for XU3.

In our investigations, we found that series which seemed to have good characteristics for seasonal adjustment usually had less than 15 percent of their months flagged for erratic seasonal factors, while series which could not be reliably adjusted had more than 25 percent of these months flagged. We found a "grey area" between 15 and 25 percent in which the some of the series in question probably could be adequately adjusted. These same limits held for the percentage of months flagged for erratic trading day factors.

The user may have to raise these limits depending upon his or her own sense of how much variability should be allowed in the adjustment. (We do not have a recommendation concerning the tolerable percentage of erratic estimates of month-to-month change.)

ADJUSTING A SHORTER SERIES

In examining Figure 1, we noted that an alternative to adjusting the full series would be adjusting the series from January, 1974 to December, 1983. We adjusted this subset of XU3 using X-11-ARIMA (using 3x5 seasonal filters). Some diagnostics for this run are reproduced in the second column of Table 1.

Note how the F-statistics give a different indication of the seasonality in this instance. The stable seasonality F-tests show less indication of a stable seasonal pattern, and the moving seasonality F-test shows a prohibitive amount of moving seasonality in the unmodified SI ratios. The series still exhibits a high degree of irregularity. Therefore, the shortened series does not appear to be a good candidate for seasonal adjustment.

We will now see if the new techniques support this conclusion. We cannot do a complete analysis of revisions histories because the shortened series is not long enough to get an experimental period in which final adjustments are available. A sliding spans analysis was done with three eight-year spans, using X-11-ARIMA (with 3x5 seasonal filters). The results are presented in the third row of Table 3. The results of this analysis do not differ greatly from the analysis done before. There is still a problem with unstable trading day factors, despite a high value for the F-statistic for trading day. The spectrum plot of the modified irregular (shown in Figure 4) shows little power at

either trading day frequency. We conclude therefore that what trading day variation there is in the series cannot be reliably estimated by X-11's procedure.

Finally, a sliding spans analysis was performed on the abridged series without adjusting for trading day. These results are shown in the last row of Table 3. The seasonal factors do not show very much improvement, and do not appear to be stable enough to warrant seasonally adjusting the abridged series.

FINAL REMARKS

In this paper, we have sought to demonstrate some new techniques for determining the seasonal adjustability of a series. There is still a great deal of work to be done in this area. We will indicate some areas for future study in this section.

The revisions history of a series cannot be calculated if the series is too short. We are experimenting with using shorter start-up periods with these series.

There are some seasonal adjustment methods that do not have seasonal factors which converge to a final value in any practical sense [6]. In this case, the value for CONRAT becomes meaningless, as there is no final value to converge to.

Other criteria for the selection of β , the weight in CONRAT, as well as other weighting schemes, could be explored. How sensitive CONRAT is to the selection of β is another topic of interest.

Finally, the acceptance criteria for both the revisions history measures and sliding spans measures should be examined further. There is some evidence that more stringent criteria should be used when no trading day variation is present in the series being examined. For example, if seasonal adjustments are calculated for the full series XU3, without use of X-11's trading day adjustment option, then we obtain $CPREV = .18$ and $CONRAT = 0.016$.

NOTES AND REFERENCES

- [1] The reader should be aware that even if ARIMA forecasts are not used in the adjustment process, X-11-ARIMA uses a modified X-11 procedure which leads to different seasonally adjusted values than are obtained from Census X-11. For more information see :

Monseil, B. C. (1984), "The Substantive Changes in the X-11 Procedure of X-11-ARIMA," Bureau of the Census, Statistical Research Division Report No. CENSUS/SRD/RR-84/10.

- [2] Lothian, J. and Morry, M. (1978), "A Test for the Presence of Identifiable Seasonality When Using the X-11 Program," Research Paper, Seasonal Adjustment and Time Series Staff, Statistics Canada.

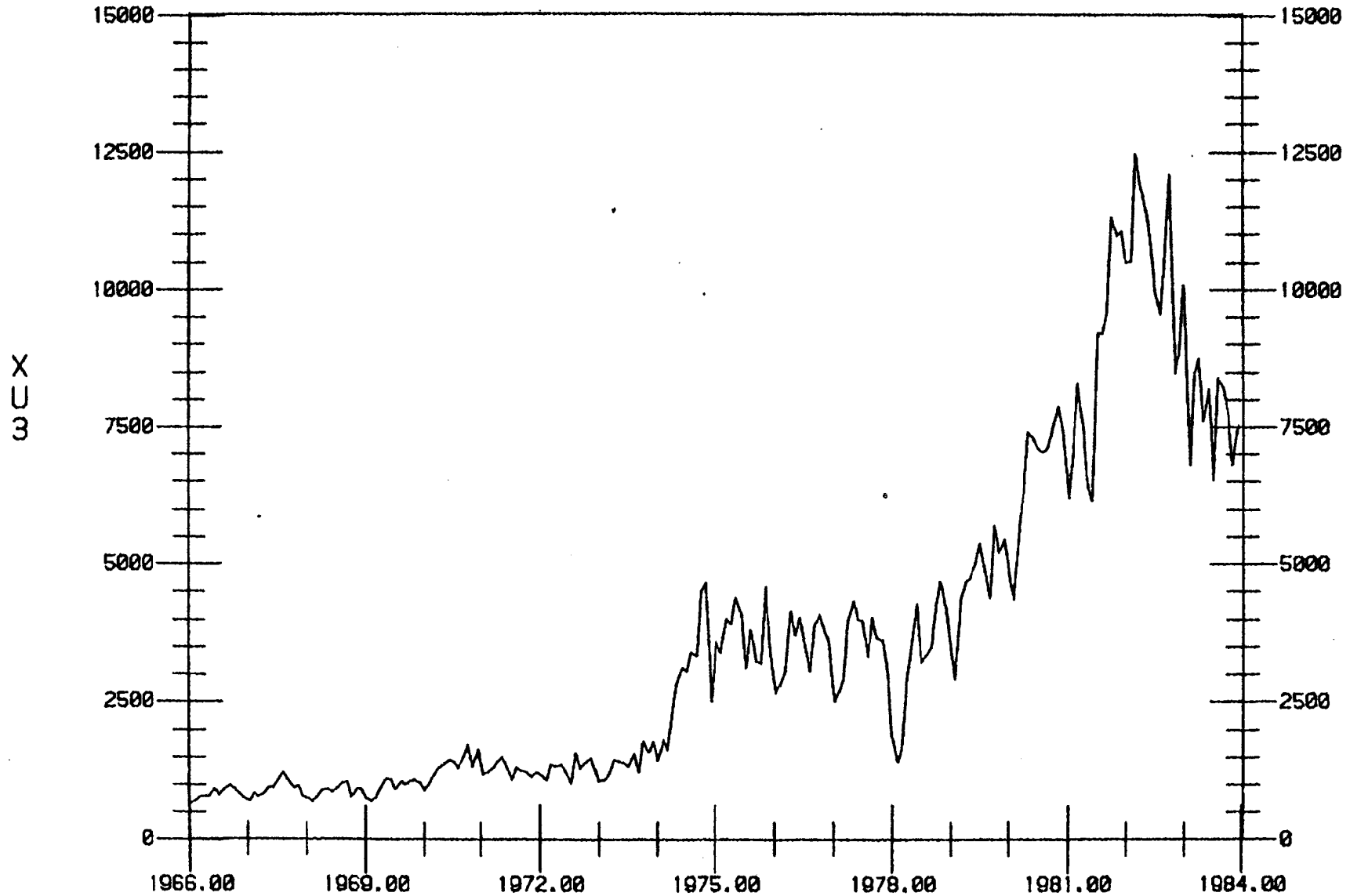
- [3] The SI ratios for a given X-11 (or X-11-ARIMA) seasonal adjustment are the combined seasonal and irregular components. They are called "ratios" because, in the multiplicative model (series = trend x seasonal x irregular), these values can be derived by dividing the original data (or the original data adjusted for extremes) by an estimate of the trend. For more information, see:

Dagum, Estela B. (1983) : The X-11-ARIMA Seasonal Adjustment Method, Ottawa, Statistics Canada;

Shiskin, Julius; Young, Alan H. and Musgrave, John C. (1967) : The X-11 Variant of the Census Method II Seasonal Adjustment Program, Technical Paper No. 15, Bureau of the Census, U.S. Department of Commerce.

- [4] Lothian, J. and Morry, M. (1978), "A Set of Quality Control Statistics for the X-11-ARIMA Seasonal Adjustment Program," Research Paper, Seasonal Adjustment and Time Series Staff, Statistics Canada.
- [5] Cleveland, W. S. and Devlin, S. J. (1980), "Calendar Effects in Monthly Time Series : Detection by Spectrum Analysis and Graphical Methods," Journal of the American Statistician, 75, 487-496.
- [6] Bell, W. and Hillmer, S. (1984), "Issues Involved with the Seasonal Adjustment of Economic Time Series," to appear in the October issue of the Journal of Business and Economic Statistics.

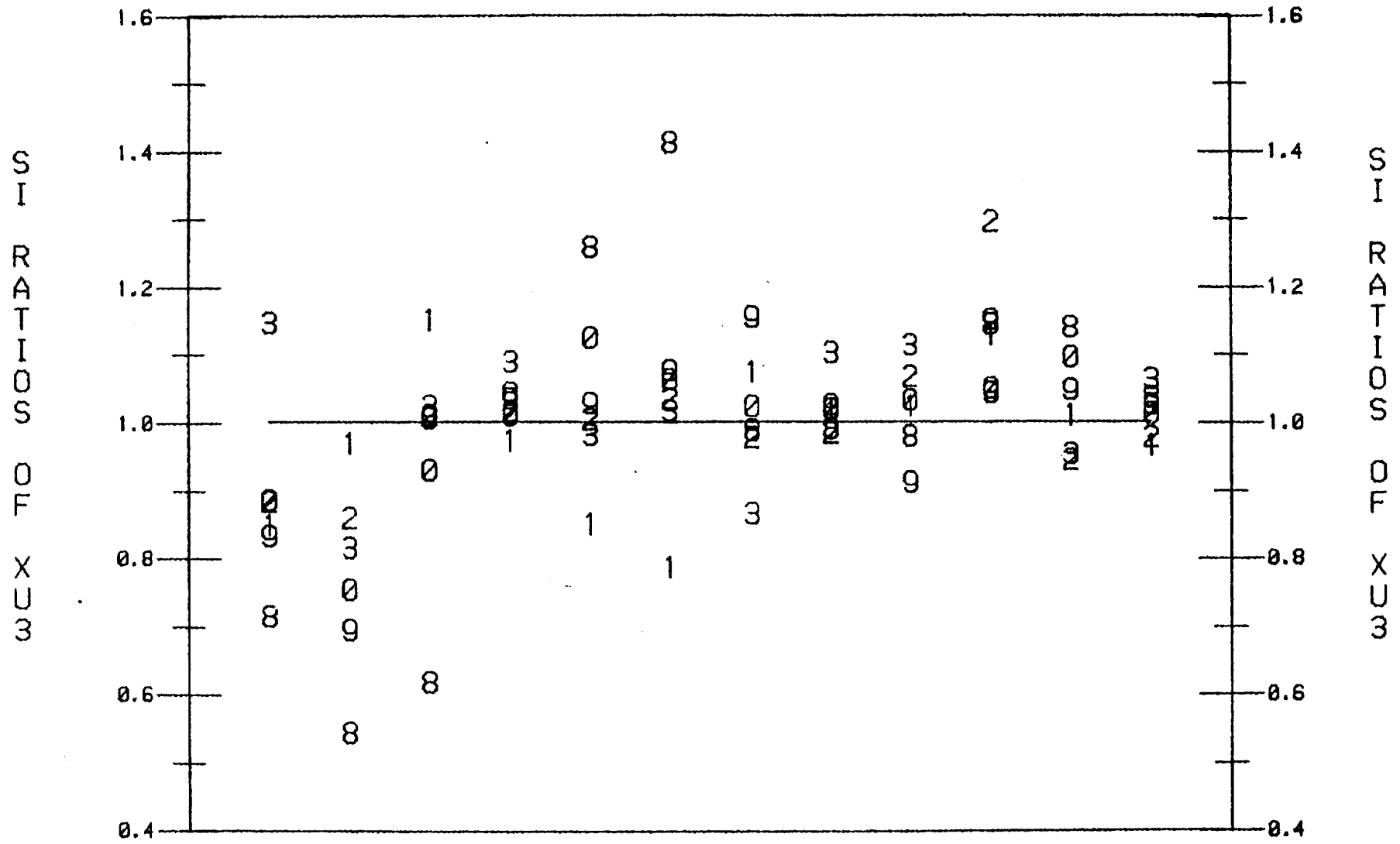
PLOT OF XU3



EXPORTS OF MINERAL FUELS, LUBRICANTS AND RELATED MATERIALS
FIGURE 1

YEAR-OVER-YEAR PLOT FOR SI RATIOS OF XU3

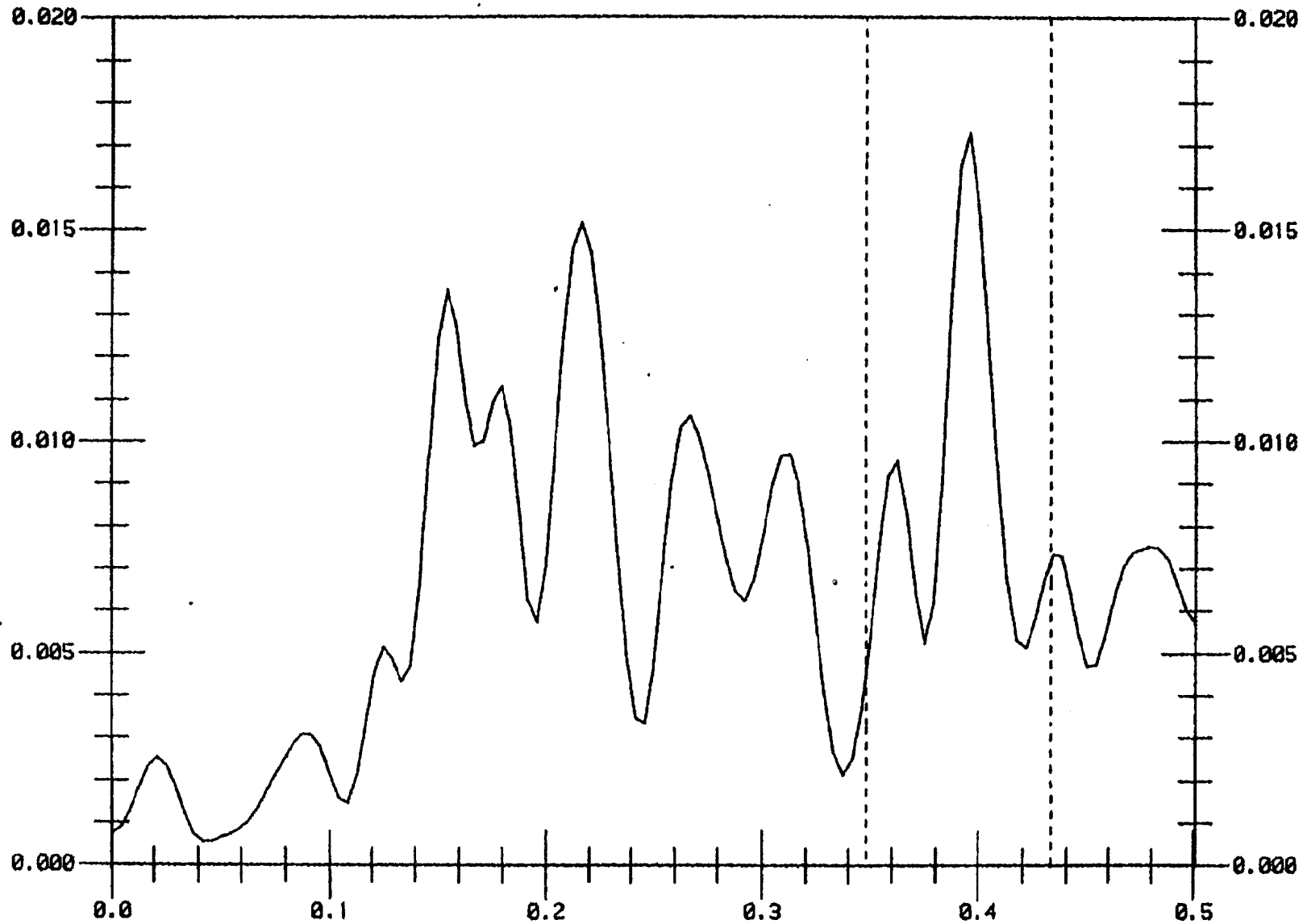
1966-1982



8 = 1978, 9 = 1979, 0 = 1980
 1 = 1981, 2 = 1982, 3 = 1983

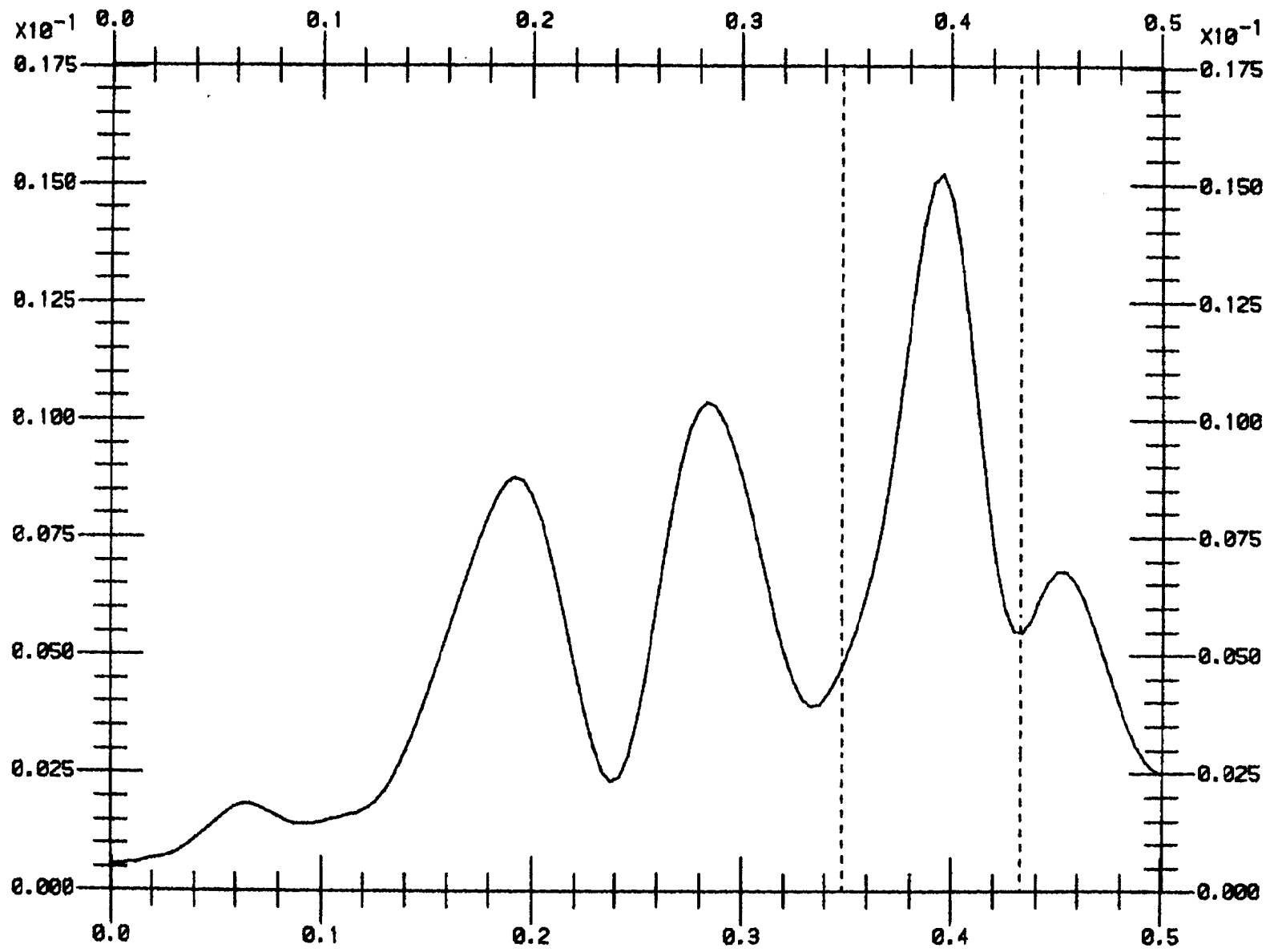
FIGURE 2

SPECTRUM PLOT OF THE MODIFIED IRREGULAR OF XU3



FREQUENCY (CYCLES PER MONTH)
DOTTED LINE = TRADING DAY FREQUENCY
FIGURE 3

SPECTRUM PLOT OF THE MODIFIED IRREGULAR OF XU3 (FROM 1974)



FREQUENCY (CYCLES PER MONTH)
DOTTED LINE = TRADING DAY FREQUENCY
FIGURE 4

TABLE 1 : X-11-ARIMA DIAGNOSTICS FOR XU3

| | full series (1966-83) | abridged series (1974-83) |
|-----------------------------------|-----------------------------|---------------------------------|
| F-test for stable sesonality, B1 | 11.081 | 5.792 |
| F-test for stable seasonality, D8 | 15.240 | 7.695 |
| F-test for moving seasonality, D8 | 1.629 | 2.465 |
| F-test for trading day, C16B | 4.069 | 7.695 |

TABLE 2 : RESULT OF REVISION HISTORY ANALYSIS FOR XU3

| | AVE | MAX | MIN |
|--------|-------|-------|-------|
| CPREV | .2851 | .4178 | .1879 |
| TOTREV | .0370 | .0970 | .0000 |
| CONRAT | .0176 | .0682 | .0044 |

Note: For CONRAT, $\beta = 0.977160$

TABLE 3 : RESULT OF SLIDING SPANS ANALYSIS FOR XU3

| # of spans | length of spans | first year | seasonal filters | 3 percent difference in S_t (*) | 3 percent difference in month-to-month change in CI_t (*) | 2 percent difference in TD_t (*) |
|------------|-----------------|------------|------------------|-----------------------------------|---|------------------------------------|
| 4 | 11 | 70 | 3x9 | 53/144 (.368) | 117/143 (.818) | 78/135 (.578) |
| 4 | 11 | 70 | 3x9 | 47/144 (.326) | 65/143 (.455) | ----- |
| 3 | 8 | 74 | 3x5 | 34/96 (.354) | 66/95 (.695) | 43/90 (.478) |
| 3 | 8 | 74 | 3x5 | 31/96 (.323) | 43/95 (.453) | ----- |

(*) - expressed as total number of months deemed extreme over total number of months tested.

APPENDIX

The quality control statistic M2 uses information from table F2.F of the X-11-ARIMA output to test the relative contribution of the irregular component to the variation of the series about a fitted mean function (referred to as the "stationary portion of the variance" in X-11-ARIMA). In X-11-ARIMA, the mean function used is a straight line fit to the final trend estimates given in table D12 of the X-11-ARIMA program (or the logarithm of this table if the series is being adjusted multiplicatively). This mean function is then subtracted from the original data given in table B1 (we will call the result B1') and from the final trend cycle stored in D12 (we will call the result D12'). Then, the entries in table F2.F are calculated as follows:

$$RS_I = \frac{\text{Var}(\text{final irregular})}{\text{Var}(B1')},$$

$$RS_S = \frac{\text{Var}(\text{final seasonal})}{\text{Var}(B1')},$$

$$RS_C = \frac{\text{Var}(D12')}{\text{Var}(B1')},$$

$$RS_P = \frac{\text{Var}(\text{monthly prior factors})}{\text{Var}(B1')},$$

$$RS_{TD} = \frac{\text{Var}(\text{final trading day factors})}{\text{Var}(B1')}.$$

Using the values given above,

$$M2 = (RS_I / (100 - RSp)) / 0.10 . \quad (a)$$

If M2 is greater than 1, the variation of the irregular component is deemed excessive. If the result of (a) is greater than 3, M2 is set equal to 3.

Lothian and Morry [4] state that "the average series adjusted had a cycle which contributes about 5 to 10% to the stationary portion of the variance. The threshold level for the M1 and M2 statistics are based on this assumption." However, many Census Bureau series which can be reliably adjusted for seasonal variation show much higher contributions by the trend cycle than are allowed for by the criteria of Lothian and Morry. After examining graphs of the series and the X-11-ARIMA output for such series, we felt that a straight line was not an adequate mean function for the purposes of the M2 statistic.

In our revised procedure, a linear spline (a continuous piecewise linear function) is fit by least squares to the final trend cycle estimates, instead of a straight line. The spline is constrained to be a linear function for each January through December period.

This "spline trend" is used instead of the X-11-ARIMA's linear mean function to produce new B1' and D12' series and the calculation of the F2.F table and the revised M2 then proceeds as before. A new value of Q is calculated, using the revised M2.

In the first two columns of Table A we compare the results of these two methods for a Census Bureau series WFURN (wholesale furniture sales from January, 1967 to December, 1979). Note how the X-11-ARIMA method does not eliminate the trend cycle variation in the stationary series as well as the "spline trend" does (RS_C is 28.97 versus the new value of 7.74). However, since this series shows a strong degree of seasonality (high RS_S values), the irregular component's contribution is not affected very much. Therefore, the impact on the value of Q is minimal.

The last two columns of Table A also compare the two methods as they are applied to XU3. There are large differences in the contribution of the trend cycle (73.53 versus 13.46 for the new method) and irregular (12.67 versus 35.80 for the new method) components. This has a significant effect on the values of $M2$ and Q , with Q going from 0.87 (a sign that the seasonal adjustment is acceptable) for the standard X-11-ARIMA method to 1.08 (a sign of an unacceptable adjustment) for the revised method.

TABLE A : COMPARISON OF M2 RESULTS

| | WFURN | | XU3 | |
|------------------|---------------------------------|-----------------------|---------------------------------|-----------------------|
| | <u>X-11-ARIMA procedure</u> | <u>Revised M2</u> | <u>X-11-ARIMA procedure</u> | <u>Revised M2</u> |
| RS _I | 3.69 | 4.73 | 12.67 | 35.80 |
| RS _C | 28.97 | 7.74 | 73.53 | 13.46 |
| RS _S | 58.69 | 76.65 | 15.19 | 42.93 |
| RS _p | 0.00 | 0.00 | 0.00 | 0.00 |
| RS _{TD} | 12.45 | 15.99 | 0.32 | 0.90 |
| Total | 104.80 | 105.11 | 101.71 | 93.10 |
| M2 | 0.369 | 0.473 | 1.267 | 3.000 |
| Q | 0.25 | 0.26 | 0.87 | 1.08 |