Anatomy of price change

Basic components of the CPI: estimation of price changes

The use of a Laspeyres type of formula in calculating the Consumer Price Index may be improved by incorporating an alternative formula that combines the geometric mean index with the Laspeyres formula

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The U.S. Consumer Price Index (CPI) is constructed from basic component indexes. The goods and services that consumers purchase are classified into 207 strata of items, and the urban areas in the United States are divided into 44 areas, each with its own index. Thus, there are 9,108 (207 times 44) basic CPI components into which expenditures are classified. These item-area strata represent the whole population of goods and services priced by the CPI, so the combination of their component price indexes into indexes at higher levels of aggregation represents a choice of aggregation formula, rather than an issue in statistical estimation.

Calculating the price indexes for the basic components, however, involves estimating the indexes on the basis of samples used from all the items that consumers buy. (The term *item* is construed narrowly in this article, referring to a specific brand, product, outlet, or service.) Also, the available items themselves are constantly changing as outlets enter and exit the market, new products or brands are introduced, and old products are modified, improved, or dropped.

Prior to 1978, the CPI used a nonprobability approach to selecting sample items. Very detailed product specifications were set a priori for each item in the CPI "market basket," and prices of a sample of items meeting those specifications were collected from outlets. The average prices of the items were then used in determining the basic component indexes. A similar method is still employed in calculating price indexes in most other countries.[1]

The 1978 revision of the CPI instituted a probability approach to sampling, with the purpose of making the CPI sample more representative of the items consumers buy and the outlets at which they shop.[2] To take account of changes in those items and outlets, the sample for each urban area in the CPI, known as a *primary sampling unit*, is replaced at approximately 5-year intervals. Thus, both the higher level aggregation of the CPI (from item-stratum-area indexes up to higher level aggregates) and the basic component indexes can be thought of as chained Laspeyres indexes, with chaining at roughly 10-year intervals for the higher level aggregates and at rotating 5-year intervals for basic components. The idea underlying the Laspeyres formula is that it calculates the cost during period t of buying the same quantities (Q_{ib}) of each item i that were consumed during the base period b. Mathematically,

$$(1) \qquad I_t = \frac{\sum_i P_{it} Q_{ib}}{\sum_i P_{ib} Q_{ib}},$$

where P_{it} and P_{ib} are the prices of item i during periods t and b, respectively.

The index's basic components, the price indexes of the aforesaid strata of items, are not di-

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rectly observable and must be estimated using observations from a random sample. The index is updated each month by an estimated price *relative*,

$$(2) \qquad I_t = I_{t-1} \times R_{t,t-1},$$

where I_t denotes the index in period t for a given area and a given stratum of items, and $R_{t,t-1}$ denotes an estimate of the average relative price change from period $t-1$ to period t in that area and stratum. For most strata of commodities and services, base-period quantities are estimated indirectly, using the ratio of base-period expenditures to base-period prices. The Laspeyres-type estimator of relative price change used by BLS for these strata is

$$(3) \qquad R_{t,t-1} = \frac{\sum_i W_{ib} \times P_{it} / P_{ib}}{\sum_i W_{ib} \times P_{i,t-1} / P_{ib}},$$

where W_{ib} is a weight that estimates the base-period total expenditures represented by item i. The procedures used to estimate the base-period prices and expenditure weights are discussed below.

Recently, some research findings by BLS economist Marshall Reinsdorf have led to a reexamination of the methods used for sampling and estimating the basic-component indexes of the CPI.[3] In particular, Reinsdorf found that over more than a decade (January 1980 to September 1992), the average prices for some food items and gasoline systematically grew at slower rates than did closely related CPI component indexes. This observation has led to concern that the CPI may be overstating rates of changes in prices and has resulted in additional research on CPI procedures.

F. G. Forsyth and R. F. Fowler, Bohdan J. Szulc, and Jorgen Dalén have studied the chaining of Laspeyres indexes.[4] They show that when Laspeyres indexes are chained together, they can be subject to "drift"; that is, a chained Laspeyres index may grow at a faster rate than an unchained index. The drift tends to be largest when prices oscillate or bounce, as is common for fresh fruits and vegetables and other grocery items.

Problems with the chained Laspeyres index have led researchers to study alternative formulas for estimating relative price changes. The weighted geometric mean is defined as

$$(4) \qquad R^G_{t,t-1} = \prod_i (P_{it} / P_{i,t-1})^{S_{ib}}$$
$$= \exp[\sum_i S_{ib} \log(P_{it} / P_{i,t-1})],$$

where S_{ib} is the base-period expenditure share for the item—that is, $S_{ib} = W_{ib}/\sum W_{ib}$. Note that the current and lagged prices do not need to be divided by base-period prices, as in (3). (If they were divided by base-period prices, the base-period prices would cancel out of the numerator and denominator of the formula.) Thus, the base-period price affects the geometric mean index only to the extent that it affects the estimated expenditure share S_{ib}.

An intuitive explanation of the geometric mean index is that it holds the *share of expenditures* on an item constant, whereas the Laspeyres index holds the *quantities* of the item constant. Also, the geometric mean index treats price increases and price decreases symmetrically. For example, if item i undergoes a price increase from \$4 in base period b to \$5 in period t, its relative price change is 1.25, but if item j decreases in price from \$5 to \$4, its relative price change is 0.80. If the base-period expenditures W_{ib} are the same for the two items, then the Laspeyres index, applied to formula (3), shows the price increase as more important than the decrease, producing a value of $(1.25 + 0.80)/2 = 1.025$. The geometric mean index, treating the increase and decrease symmetrically, produces a value of $1.25 \times 0.80 = 1$.

Other formulas for calculating basic component indexes have also been proposed, but the geometric mean has been shown to possess the best statistical properties.[5]

The next section examines how price oscillations, in conjunction with consumer behavior that substitutes purchases of less expensive items for purchases of more expensive items, can cause the Laspeyres index formula to overstate the rate of growth of prices. The section following that discusses the procedures used by BLS in periodically replacing its samples of items and considers some of the effects these procedures have on the performance of the CPI. The subsequent two sections make empirical comparisons between the Laspeyres formula and the alternative geometric mean formula for basic index components. The comparisons demonstrate that the formula used in constructing these components can have a significant impact on the measured rate of inflation.

Laspeyres index

The Laspeyres formula measures changes in price by measuring the cost of a fixed "market basket," that is, fixed quantities of goods and services. How is this basket defined? Prior to 1978, the basket consisted of several hundred narrowly defined items, and within those narrow definitions, items were assumed interchangeable. For example, an item might be a 20-inch color television set with a remote control produced by a major manufacturer. The basic component price relatives were then calculated as ratios of average prices for items in the sample meeting the said specification.

Since 1978, however, the adoption of the *entry-level item* approach allows any television set to be selected. The selection of the item is implemented using the principles of probability sampling, setting the probability of selection of any item proportional to the expenditures on the item. This process of selecting items is described below.

One implication of the entry-level item (or probability sampling) approach is that the items within any stratum are often quite heterogeneous. For example, a single CPI stratum of items contains video cassette recorders, video cameras, video cassettes, and video game hardware and software. Clearly, prices of a random sample of goods from this stratum cannot simply be averaged—any such average would be meaningless. Instead, some sort of index number must be formed to measure inflation for strata characterized by this kind of heterogeneity. In the 1978 CPI revision, BLS adopted the Laspeyres index number formula, already in use for aggregation across strata of items, for the calculation of the basic component indexes under the entry-level item method of sampling.

The characteristics that are desirable for an index have been much discussed by economists.[6] One characteristic that seems reasonable for a basic component index is that it should measure inflation correctly when a stratum of items is nearly homogeneous. By "nearly homogeneous," I mean that prices within the stratum are characterized by having a *common trend*, although individual prices per se may deviate from that trend either permanently or temporarily.[7] Thus, over medium to long time intervals, the prices within a nearly homogeneous stratum of items would move in the same direction and by about the same amount.

The following simple formal model of economic behavior that captures this idea was developed by Reinsdorf.[8] Suppose that the common price trend is multiplicative. In other words, suppose that the price P_{it} of item i in time period t is a random variable distributed with a density $f(P_{it})$, and that the density of the price in time period s is the same as in t, except for a multiplicative constant; that is, $P_{is} = cP_{it}$, so that $g(P_{is}) = f(P_{is}/c)/c$. Then the rate of price change between periods t and s is the multiplicative constant c.

This multiplicative constant for the distribution of prices is an additive constant for the distribution of the logarithm of prices. In terms of the latter, suppose

$$(5) \qquad \log P_{it} = \pi_t + u_i + e_{it},$$

where π_t is the logarithm of the common price trend, u_i is a permanent component of variation in an item's price (which might represent the effects of permanent differences in the quality or the marketing of a product or outlet), and e_{it} is a temporary component of variation in prices. It is assumed that the e_{it} have mean zero, are stationary, and are independent of the u_i. Under this model of prices, the inflation between periods t and s is $\exp(\pi_s - \pi_t)$, a consequence of the assumption of multiplicative inflation between periods, which does not imply, for example, that $E(P_{it})$ is equal to e^{π_t}. (In fact, they are not equal.) Also, individual prices oscillate under such a model, and the amount of oscillation can be measured by the variance of differences in e_{it}, that is, $\mathrm{Var}(e_{is} - e_{it})$.

Treating prices as outcomes of a random process does not suggest that the sellers who set the prices are behaving randomly. Presumably, the setting of prices for individual items at outlets is based on a variety of considerations, including supply conditions for a particular item and marketing strategy. The point is simply that, from the point of view of the consumer, prices for some items appear to be oscillating randomly due to sales and other temporary price changes.

Suppose that the demand for the items within the stratum can be approximated by the constant-elasticity demand function

$$(6) \qquad \log Q_{it} = -\eta \log P_{it} + \delta_t + v_i + w_{it},$$

where η is the elasticity of demand for the narrowly defined item; δ_t is the effect on the level of demand of the business cycle, seasonality, and other factors relating to or occurring in the period t; v_i is an item-specific permanent component of variation in the quantity demanded; and w_{it} is a transitory component of variation in the quantity demanded. Both v_i and w_{it} are assumed to be independent of e_{it}. Economic theory and empirical evidence suggest that the price elasticities of demand for narrowly defined brands and outlets are much larger than for more aggregated commodities and are probably greater than 1 for most items.[9]

Consider the special case of the model given by (5) and (6) in which e_{it} is normally distributed with variance, $\mathrm{Var}(e_{it}) = \sigma^2$, and correlations $\mathrm{Corr}(e_{it}, e_{ib}) = \rho_{t-b}$. The appendix describes Reinsdorf's derivation of the following expression for the large-sample limiting value of the Laspeyres index:[10]

$$(7) \qquad \frac{E(P_{it}Q_{ib})}{E(P_{ib}Q_{ib})} = \exp(\pi_t - \pi_b)\exp[\eta(1 - \rho_{t-b})\sigma^2]$$

$$= \exp(\pi_t - \pi_b)\exp\{(\eta/2)\mathrm{Var}[\log(P_{it}/P_{ib})]\}.$$

The true inflation rate is the first exponential factor on the right, $\exp(\pi_t - \pi_b)$, and the exponential factor following it is always greater than or equal to 1, so the Laspeyres index is an upper bound on the true inflation rate. The index measures inflation correctly when (a) demand is perfectly inelastic, that is, $\eta = 0$, or (b) there is no transitory variation in individual prices, so that all prices move by exactly the same proportion and $\sigma^2 = 0$. In other cases, that is, $\eta > 0$ and $\sigma^2 > 0$, the Laspeyres index overestimates the inflation, with the magnitude of the overestimate depending on the price elasticity of demand, η, and the magnitude of oscillation in the prices of individual items around their common trend.

For example, suppose that $\eta = 1.5$, $\sigma^2 = 0.01$, and $\rho_{t-b} = 0.6^{t-b}$. Then for the first period ($t = b + 1$), the Laspeyres index overstates inflation by $e^{0.006} - 1$, or 0.6 percent. In this example, the index continues to overestimate inflation during subsequent periods, but the magnitude of the overestimate declines. For the first six periods, the cumulative overestimate in the example amounts to 1.44 percent. After the sixth period, very little additional overstatement of inflation occurs, and the index eventually converges to a value that is too large by 1.51 percent.

Because the overstatement is proportional to the variability of individual prices, the Laspeyres formula is especially likely to overstate inflation for goods that have highly variable prices or are frequently put on sale at large discounts, such as fresh fruits and vegetables and apparel. In addition, if the correlations ρ_{t-b} are assumed to decline rapidly to zero, as they would under an autoregressive-moving average model, then the overstatement of inflation would occur principally during the first few periods. Essentially, there would be a one-time overstatement of the price increase, following which the Laspeyres index would correctly measure the rate of change of prices.

Let us next consider the problem of estimating a component index when items are heterogeneous, that is, when they are not characterized as having a common trend.

For heterogeneous commodities, the theory of the cost-of-living index can serve as a guide. For example, if the goods in a component index are separable from other goods in the consumer utility function, then a category utility function can be defined on the basis of the consumers' preferences within that category. The *partial cost-of-living index* for that component can then be defined in the usual way, as the ratio of the cost of reaching a given utility level under prices prevailing during period t to the cost under prices prevailing during the base period b.[11] If the category utility functions do not allow for substitution (that is, if they have a Leontief functional form, with constant relative quantities as prices change), then the partial cost-of-living index is the Laspeyres index. If the category utility functions have constant relative expenditure shares as relative prices change (that is, if they have unitary elasticities of substitution, or the Cobb-Douglas functional form), then the partial cost-of-living index is the weighted geometric mean.

The assumption of constant relative expenditure shares is likely to be a more accurate approximation for many heterogeneous strata of items than the assumption of no substitution would be, although for some such strata (for example, prescription drugs), the assumption of no substitution may be more nearly appropriate. If data on quantities or expenditures over time were available, the degree of substitutability could be estimated directly, or the Laspeyres index could be compared with the superlative indexes, such as the Tornqvist or the Fisher ideal indexes, which accommodate substitution flexibly.

Item and outlet sampling and rotation

Because the market is constantly shifting, with new brands, shifting outlet shares, entries and exits of firms, the introduction of new items, and other changes over time, a pure Laspeyres index would soon have an outdated sample. In recognition of this problem, since 1978 the CPI samples of items and outlets for most commodities and services have been rotated at 5-year intervals, with the samples in roughly 20 percent of the primary sampling units being replaced during any given year.[12] With each rotation, a new sample is selected, the selection consisting of two stages: selecting outlets and initiating the item sample within the outlets. At each stage, the probability of selection of an outlet or item is set proportional to the estimated expenditures by consumers at that outlet or on that item.

The sample of outlets is drawn from a sampling frame that is created from a special household survey known as the Point-of-Purchase Survey. This survey is conducted by the Bureau of the Census for the Bureau of Labor Statistics in each of the rotating primary sampling units about 2 years prior to the rotation of the samples. It provides data on spending patterns of households and the specific outlets at which the commodities and services were purchased. The results of the survey are used to estimate expenditures on items at specific outlets, so that a sample of outlets can be drawn with probability proportional to expenditure.[13]

Based on a concordance of CPI entry-level items with Point-of-Purchase Survey categories, a number of price quotes are assigned to each outlet for collection. About 3 to 6 months prior to the "link month," when the old and new samples for the primary sampling unit are rotated, a BLS field representative visits the outlets to initiate the item sample. During initiation, the field representative first identifies all items sold at the outlet that fall within the entry-level item definition. Then, with the assistance of the store manager or some other respondent for the outlet, the field representative tries to determine the proportions of total sales for the items within the entry-level item, to enable a particular item to be selected with probability proportional to sales. This process may require a series of iterations, as groups of products get broken down into successively smaller categories. If the respondent does not provide direct information on sales, alternative methods for estimating the proportion of sales of each item are used. The specific item that is finally selected is then described in detail on a checklist that will be used to identify the item during subsequent visits to collect the item's price.[14]

The weight W_{ib} carried by the item in the calculation of the index is an estimate of the base-period expenditures represented by the sample item. The weight is calculated as the expenditure for the Point-of-Purchase Survey category in the primary sampling unit times adjustments for the following: (a) the ratio of the sales in an entry-level item category to the sales in the corresponding Point-of-Purchase Survey category in the specific outlet, (b) a geographic factor that adjusts for boundary changes in metropolitan areas between revisions, (c) a factor that adjusts for the upper limit on the number of quotes that can be collected from any single outlet, (d) the probability of selection for the entry-level item in the stratum of items, and (e) an adjustment for the number of usable quotes, when quotes drop out of the sample or a substitution is required. The purpose of all of these adjustments is to keep an item's weight equal to the expenditures represented by the item.

When the primary sampling unit rotates, the base price for a specific item is estimated by taking the item's price during the link month, P_{il}, and deflating it by a long-term index relative, namely, the ratio of the index for the area-item stratum for the link month to the index for the base month—that is, $P_{ib} = P_{il}/(I_l/I_b)$. The largest urban areas are known as A-size primary sampling units, and most of them undergo rotation all at once. In that case, the same long-term relative adjustment is applied to the link-month price of each item in the stratum to obtain the base-month price. Since the base price appears in both the numerator and denominator of (3), the adjustments from link-month to base-month price will cancel and can be ignored.

The CPI estimator, when applied at the lowest level of aggregation, differs from the pure Laspeyres concept in several important respects. The base periods for the basic component indexes are not the same base period (currently 1982–84) that is used in the aggregation of basic component indexes to higher level aggregates. Rather, the base period for the basic components is the middle of the year in which the Point-of-Purchase Survey was taken, so there is a change in the base period every 5 years with each new sample replacement. Furthermore, a single index can include price quotes with several base years, because the various primary sampling units rotate their samples on a staggered schedule. More importantly, the base price is derived from the item's price during a period different from the Point-of-Purchase Survey and item selection periods, which establish the expenditure weight. Consequently, the formula is systematically different from a pure Laspeyres formula, which would use the actual base-period prices to deflate base-period expenditures to form implicit quantity weights.

If we examine how the elasticity of demand for an item affects the expenditure weights in the CPI formula, we see that a sample item's weight depends on the probabilities involved in selecting outlets during the Point-of-Purchase Survey period and selecting items during the initiation period. For many categories of items, such as food, it is likely that the expenditure, and thus the probability of selection, is influenced more by the item's price during initiation of the sample of items than by the variation in the general level of prices at the outlet. Hence, for the following example, I assume that the weight is determined by the price of the item during initiation of the outlet, rather than during the Point-of-Purchase Survey period. The model presented below can be easily adapted, however, to allow for alternative assumptions about the determination of expenditure weights.

Under the aforesaid assumption, and supposing that the sample does not change between the link month and the period t, the CPI formula's estimate of the relative price change between $t-1$ and t, for an A-size primary sampling unit, can be written as

$$(8) \qquad R_{t,t-1} = \frac{\sum_i W_{is} \times P_{it}/P_{il}}{\sum_i W_{is} \times P_{i,t-1}/P_{il}},$$

where $W_{is} = P_{is}Q_{is}$ represents expenditures during the item's sampling or initiation period, which occurs 3 to 6 months prior to the link month.

Following the model developed in equations (5) through (7), the relative price change measured by the CPI between the link month l and a subsequent period t will converge to

$$(9) \qquad \frac{E(P_{is}Q_{is}P_{it}/P_{il})}{E(P_{is}Q_{is})} = \exp(\pi_t - \pi_l) \times \exp\{[1 - \rho_{t-l} + (1-\eta)(\rho_{t-s} - \rho_{l-s})]\sigma^2\}.$$

Examining this expression, we see that, in contrast to the pure Laspeyres index in (7), it entails that the CPI is likely to overstate inflation immediately following the link month, even if the price elastic-

Table 1. **Rates of change of simulated Consumer Price Index for all Urban Consumers, U.S. city average, by expenditure category, with basic components computed using Laspeyres and geometric mean index formulas, June 1992 to June 1993**

Expenditure category	Laspeyres index	Geometric mean index	Difference
All available items			
(70.3 percent of all items)	2.95	2.48	0.47
Food and beverages	2.11	1.56	.55
Food	2.18	1.59	.59
Food at home	2.37	1.52	.85
Cereals and bakery products	3.36	2.78	.58
Meats, poultry, fish, and eggs	3.90	3.28	.62
Dairy products	1.61	1.29	.33
Fruits and vegetables	1.58	–.70	2.28
Fresh fruits and vegetables	4.09	1.09	3.00
Processed fruits and			
vegetables	–3.03	–3.98	.96
Other food at home	.86	.39	.48
Sugar and sweets	–.09	–.59	.50
Fats and oils	–.02	–.46	.44
Nonalcoholic beverages	–.27	–.64	.38
Other prepared foods	2.21	1.67	.55
Food away from home	1.85	1.70	.14
Alcoholic beverages	1.48	1.32	.16
Housing	—	—	—
Shelter	—	—	—
Renters' costs	—	—	—
Homeowners' costs	—	—	—
Maintenance and repairs	1.90	1.84	.06
Fuel and other utilities	—	—	—
Fuels	3.55	3.54	.01
Fuel oil and other			
household fuel commodities	.38	.31	.07
Gas (piped) and electricity			
(energy services)	3.88	3.87	.01
Other utilities and public services	—	—	—
Household furnishings and operation	—	—	—
Housefurnishings	.14	–.53	.67
Housekeeping supplies	1.22	.59	.63
Housekeeping services	—	—	—
Apparel and upkeep	.59	–1.21	1.80
Apparel commodities	.47	–1.52	1.98
Men's and boys' apparel	.19	–1.31	1.50
Women's and girls' apparel	.44	–2.43	2.87
Infants' and toddlers' apparel	–.13	–.01	–.13
Footwear	.21	.03	.17
Other apparel commodities	1.83	–.84	2.67
Apparel services	1.79	1.71	.08
Transportation	—	—	—
Private transportation	—	—	—
New vehicles	2.46	2.30	.16
New cars	2.23	2.09	.15
Used cars	—	—	—
Motor fuel	–3.03	–3.04	.01
Maintenance and repairs	3.19	2.93	.26
Other private transportation	—	—	—
Other private transportation			
commodities	–1.75	–1.85	.10
Other private transportation			
services	—	—	—
Automobile insurance	5.39	5.33	.06
Automobile finance charges	—	—	—
Automobile fees	5.21	5.19	.02
Public transportation	13.19	12.86	.33
Medical care	—	—	—
Medical care commodities	3.58	3.19	.38
Medical care services	—	—	—
Professional medical services	5.37	4.99	.38
Hospital and related services	8.74	8.24	.49

See footnote at end of table.

ity of demand $\eta = 0$. If the price elasticity of demand is greater than 1, however, as is likely, then the overstatement is smaller than it would be if expenditure weights were actually drawn from the link month (in which case the formula would behave like a true Laspeyres index with base period l). The overstatement, however, is larger than it would be if prices and expenditure weights were actually observed during the nominal base period, when the Point-of-Purchase Survey occurs. The cases in which the CPI would not overstate inflation immediately following linking are those in which the variance of individual price quotes, σ^2, is near 0. As with the pure Laspeyres index, we might expect the overstatement of inflation to occur primarily during the first few months after rotation.

To understand how the CPI index method of linking may work in practice, consider again the example in which $\eta = 1.5$, $\sigma^2 = 0.01$, and $\rho_k = 0.6^k$, and assume that $l - s = 5$ months. Then in the first period after linking, $t = l + 1$, the CPI could be expected to overstate inflation by 0.42 percent. As in the example of the pure Laspeyres index, the CPI index could be expected to overestimate inflation during subsequent periods, but at a declining rate. The cumulative expected overestimate for the first six periods would amount to 1 percent, and the CPI would eventually converge to a value that is too large by 1.04 percent. We find that the cumulative overstatement of price change in the CPI formula is relatively insensitive to η and usually is close to σ^2.

In contrast, the weighted geometric mean index tends to track inflation more accurately under most conditions. Under the same assumptions made in deriving equation (9), the weighted geometric mean converges to

$$(10)\quad \frac{E[P_{is}Q_{is}\log(P_{it}/P_{il})]}{E(P_{is}Q_{is})} = \exp(\pi_t - \pi_l) \times \exp[(1-\eta)(\rho_{t-s} - \rho_{l-s})\sigma^2].$$

The second exponential factor on the right-hand side of this equation can be larger or smaller than 1, showing that the weighted geometric mean index can overstate or understate inflation. If $\eta = 1$ or $\sigma^2 = 0$, the geometric mean index converges to the correct relative price change. Even when η is different from 1, however, the geometric mean formula often converges to an estimate near the actual change in prices. For the example in the preceding paragraph, the weighted geometric mean formula converges to a value that is too large by 0.04 percent. Because this overstatement would occur at 5-year intervals as samples are periodically replaced, the average overstatement would be less than 0.01 percent per year. This result suggests that the weighted geometric mean formula may be more accurate than the Laspeyres formula, especially during the first months after a sample is replaced.[15]

In sum, the sample rotation procedures used by BLS in the CPI commodities and services sample serve many useful purposes, including keeping the sample up to date and allowing it to incorporate changes in available products and the mix of outlets. The use of the Laspeyres formula, combined with the procedures for linking the price changes in old and new samples, however, may cause the CPI to overstate price change, especially for items with highly volatile prices and during the months immediately following a sample rotation and linking.

Empirical comparison of indexes

This section empirically compares the Laspeyres and geometric mean formulas for basic CPI components. The basic component indexes have been computed for those strata of items that use the Point-of-Purchase Survey/outlet rotation sampling method (these strata carry approximately 70 percent of the weight, or relative importance, of the CPI), using both the current Laspeyres-type formula and the alternative geometric mean formula over the period from June 1992 to June 1993. The data base was reconstituted from archived data, and two sets of basic component indexes were calculated using exactly the same price quotes. The computer program used to simulate the CPI Laspeyres index is not identical to the program used in actual production, so there are slight differences between the Laspeyres indexes calculated for this research and the published CPI. In most cases, however, the simulated CPI Laspeyres indexes are very close to the published indexes.[16]

In aggregating the basic components to higher levels, I have used the usual CPI Laspeyres formula and Consumer Expenditure Survey-based aggregation weights. There is a difference between the two formulas only at the lowest level of aggregation.

Table 1 compares the annual percent changes for the two sets of indexes for various items over the period. Note that the geometric mean indexes almost always exhibit lower rates of price growth than the Laspeyres-type indexes do, a result that is not surprising in view of the known properties of the two types of averages.[17] More importantly, the size of the difference between the two indexes varies substantially between classes of items. For fresh fruits and vegetables and for apparel, the Laspeyres indexes showed rates of change 2 to 3 percentage points higher than the geometric mean indexes. These differences are comparable in magnitude to the large differences in rates of change between CPI and average price series for food that have been noted by Reinsdorf.[18] The large differences in annual rates of change for these expenditure classes are consistent with the model derived in equations (9) and (10). Fresh fruits and vegetables and apparel are characterized as having highly variable prices at the level of the outlet, due to either perishable food items or the use of frequent sales with substantial discounting.

For other expenditure categories, however, the differences tend to be smaller, in most cases less than 1 percent a year. For some expenditure categories that tend not to rely on sale pricing, such as automobile parts and equipment and apparel services, there is little difference between the Laspeyres and geometric mean indexes. The difference is also small for motor fuel, which is typically thought to have volatile prices. Most of the price variability for motor fuel, however, is common to all outlets at which it is sold, and the transitory, outlet-specific price variability that is measured by σ^2 is fairly small.

Another implication of the model sketched out above is that the largest differences in measured rates of change between the Laspeyres and geometric mean indexes should occur immediately following sample rotation. Table 2 compares the rates of change of local area indexes based on the simulated Laspeyres and geometric mean estimators for the basic components.

From June 1992 to June 1993, three of the local areas listed in table 2—New York City, Detroit, and San Francisco—had new samples introduced.

Table 1. **Continued—Rates of change of simulated Consumer Price Index for all Urban Consumers, U.S. city average, by expenditure category, with basic components computed using Laspeyres and geometric mean index formulas, June 1992 to June 1993**

Expenditure category	Laspeyres index	Geometric mean index	Difference
Entertainment	—	—	—
Entertainment commodities	—	—	—
Reading materials	3.60	3.22	.38
Sporting goods and equipment	—	—	—
Toys, hobbies, and other entertainment	1.07	.37	.70
Entertainment services	3.32	2.57	.74
Other goods and services	6.41	6.05	.35
Tobacco and smoking products	7.80	7.21	.60
Personal care	2.43	2.05	.38
Toilet goods and personal care appliances	2.40	1.89	.51
Personal care services	2.48	2.23	.24
Personal and educational expenses	7.06	6.83	.23
School books and supplies	3.76	3.79	-.03
Personal and educational services	7.27	7.03	.24

Note: Dash indicates data not available (usually because the index includes some strata that are not part of the Point-of-Purchase Survey and sample rotation). Rates of change of the simulated Laspeyres indexes are not identical to the published rates of change of the CPI, because of differences between the index simulation and the actual index calculation and because the simulated indexes were not rounded prior to computing rates of change. For both indexes, aggregation above the level of the basic components (that is, indexes of strata of items and areas) was based on the usual Laspeyres formula and weights that were in turn based on the Consumer Expenditure Survey.

It is interesting to note that San Francisco has the largest difference in rates of change for the Laspeyres and geometric mean component indexes for both all available items and food at home. New York has the second largest difference for food at home. For all three areas, the Laspeyres component indexes showed a larger rate of change than the geometric mean indexes did. In most of the areas that did not introduce a new sample during the June 1992–June 1993 period, the Laspeyres component indexes also showed a larger rate of change than the geometric mean component indexes did, but the differences were smaller than those for the cities that rotated their samples. The effect of rotation is particularly noticeable when one examines the month-to-month differences. For San Francisco, the Laspeyres component index for food at home produced a rate of change 1.11 percentage points larger than the geometric mean component index during the month after the new sample was introduced. For New York, the difference during the month following the introduction of the new sample was 1.49 percentage points.

To summarize the potential differences between the two formulas on the all-items CPI, consider the differences calculated for all of the 170 available strata of items. These strata carry 70 percent of the weight in the CPI. For all of the available strata of items, the index based on geometric mean components grew 0.47 percentage point less than the index based on Laspeyres components grew. Most of the remaining 30 percent of the weight in the CPI consists of shelter. Because the shelter sample rotates less frequently than does the commodities-and-services sample, it seems likely that the differences between the two formulas for shelter would be small. For example, if there were no difference between the Laspeyres and geometric mean formulas for the unavailable strata, the differential for the all-items index would be about 0.7 $\times$ 0.47 = 0.33 percentage point. By contrast, if the difference for the unavailable strata were 0.2 percentage point, then the differential for the all-items index would be about 0.39 percentage point. Thus, if the period from June 1992 to June 1993 is representative of the differential that might result from using the geometric mean formula to estimate the basic component indexes, the all-items inflation rate might be lowered by 0.3 to 0.4 percentage point.

Table 2. **Rates of change of simulated Consumer Price Index for All Urban Consumers, selected local areas, all available items and food at home, with basic components computed using Laspeyres and geometric mean index formulas, June 1992 to June 1993**

Local area	Laspeyres index	Geometric mean index	Difference
All available items (70.3 percent of all items)			
Chicago-Gary-Lake County, IL-IN-WI	3.57	2.81	0.76
Los Angeles-Anaheim-Riverside, CA	2.83	2.34	.49
N.Y.-Northern N.J.-Long Island, NY-NJ-CT	3.11	2.54	.57
Philadelphia-Wilmington-Trenton, PA-NJ-DE-MD	2.22	1.78	.44
San Francisco-Oakland-San Jose, CA	2.66	1.77	.89
Baltimore, MD[1]	2.02	1.63	.40
Cleveland-Akron-Lorain, OH[1]	2.17	1.66	.51
Miami-Fort Lauderdale, FL[1]	4.22	3.95	.27
St. Louis-East St. Louis, MO-IL[1]	.56	.45	.11
Washington, DC-MD-VA[1]	3.52	3.16	.36
Dallas-Fort Worth, TX	1.96	1.32	.64
Detroit-Ann Arbor, MI	2.95	2.37	.58
Houston-Galveston-Brazoria, TX	2.18	2.35	–.17
Pittsburgh-Beaver Valley, PA	3.21	2.66	.55
Food at home			
Chicago-Gary-Lake County, IL-IN-WI	2.62	2.00	.62
Los Angeles-Anaheim-Riverside, CA	4.21	3.61	.60
N.Y.-Northern N.J.- Long Island, NY-NJ-CT	1.65	.35	1.30
Philadelphia-Wilmington-Trenton, PA-NJ-DE-MD	1.69	2.13	–.44
San Francisco-Oakland-San Jose, CA	2.62	.06	2.56
Baltimore, MD	2.12	2.05	.06
Boston-Lawrence-Salem, MA-NH	3.55	3.46	.09
Cleveland-Akron-Lorain, OH	2.81	2.47	.34
Miami-Fort Lauderdale, FL	5.81	5.34	.47
St. Louis-East St. Louis, MO-IL	–2.55	–2.69	.14
Washington, DC-MD-VA	1.86	2.03	–.17
Dallas-Fort Worth, TX	2.32	1.81	.51
Detroit-Ann Arbor, MI	1.32	1.04	.28
Houston-Galveston-Brazoria, TX	–.59	–1.68	1.09
Pittsburgh-Beaver Valley, PA	3.38	2.72	.66

[1]Because of the bimonthly sampling for nonfood items, the period for these indexes is July 1992 to May 1993.

NOTE: Rates of change of these simulated Laspeyres indexes are not identical to the published rates of change of the CPI, because of differences between the index simulation and the actual index calculation and because the simulated indexes were not rounded prior to computing rates of change. For both indexes, aggregation above the level of the basic components (that is, indexes of strata of items and areas) was based on the usual Laspeyres formula and weights that were in turn based on the Consumer Expenditure Survey.

Other empirical evidence

If the application of a Laspeyres type of formula causes an index to overstate significantly the inflation rate immediately following sample rotation, evidence of the effect should appear in the historical behavior of the indexes. Because the samples in the smaller urban areas do not all rotate at the same time, I examined the price changes for large urban areas (A-size primary sampling units) immediately following rotation. Rotation schedules designating the link month for the two samples were obtained for the years 1980 to 1985 and 1988 to the present. The link months are listed in the footnote to table 3.

Table 3 presents the mean difference between the measured inflation rate for the rotated area *a* and the U.S. average inflation rate during two separate periods: the 2-month period and the 6-month period after the rotated samples are introduced. If introducing the new sample induces a positive shock to the inflation rate, it should result in positive values for the mean difference.

The results shown in the table are generally consistent with the above model. There are significant positive differences between the area inflation rates and the U.S. average inflation rates for food, especially fruits and vegetables and meat. The numerical magnitude of these differences, however, appears to be too small to explain the entire difference between the geometric mean indexes and the Laspeyres indexes. For example, if the Laspeyres index overstates the inflation rate for fruits and vegetables by about 2 percent a year, as suggested by the comparison with the geometric mean index, and most of the overstatement occurs shortly after each 5-year rotation, then we might expect a 10-percentage-point differential in the inflation rate immediately following each rotation. The observed differentials for fruits and vegetables in table 3 are 2.3 percent for the 2-month period and 2.0 percent for the 6-month period after new samples are introduced. One possible explanation is that the autocorrelation of the individual transitory component of prices may diminish slowly, rather than rapidly, as has been assumed.[19] Another possible explanation is that between rotations items drop out of the sample and are replaced with substitutes, so that sample initiation effects reappear.

It is also important to try to verify some of the assumptions that guided the theoretical model. For example, from equation (7), we can directly measure the degree of price oscillation as $\mathrm{Var}[\log(P_{it}/P_{ib})]$. I selected 10 strata of food-at-home items that, in my opinion, are likely to be nearly homogeneous, as explained earlier. Table 4 presents a comparison of the Laspeyres and geometric mean indexes, as well as $\mathrm{Var}[\log(P_{\mathrm{Jun}\ '93}/P_{\mathrm{Jun}\ '92})]$, for these strata of items. The theory predicts that the Laspeyres index will have the greatest tendency to overstate inflation when price oscillation, measured by the variance in the logarithm of the price differences, is largest. This implication of the theory is confirmed in table 4: the three strata with the largest variances—oranges, lettuce, and tomatoes—also have the largest differences between the Laspeyres and geometric mean indexes, more than 1 percentage point in each case. At least for these relatively homogeneous food-at-home strata, some of the assumptions implicit in equations (9) and (10) are consistent with the data.

Table 3. **Mean differences in measured inflation between Consumer Price Indexes for A-size primary sampling units and U.S. average Consumer Price Indexes 2 months and 6 months after rotation of samples[1]**

Expenditure category	2-month difference	6-month difference
All items	0.08 (0.10)	−0.12 (−0.17)
All items less shelter	.02 (.10)	−.11 (.15)
Food and beverages	[2].47 (.13)	[2].43 (.23)
Food	[2].50 (.13)	[2].45 (.24)
Food at home	[2].75 (.19)	[2].68 (.32)
Cereals and bakery products	.25 (.23)	−.21 (.30)
Meats, poultry, fish, and eggs	[2]1.07 (.23)	[2]1.05 (.40)
Dairy products	−.25 (.30)	−.29 (.48)
Fruits and vegetables	[2]2.30 (.61)	[2]1.99 (.89)
Other food at home	.22 (.26)	.47 (.34)
Food away from home	.01 (.13)	−.03 (.22)
Alcoholic beverages	−.20 (.21)	−.17 (.28)
Transportation	.08 (.16)	−.31 (.23)
Motor fuel	−.30 (.43)	−.40 (.51)
Medical care	.08 (.21)	.31 (.32)
Entertainment	−.19 (.25)	−.36 (.47)
Other goods and services	.14 (.19)	.01 (.28)

[1] Numbers in parentheses are standard errors of the means.

[2] Significant at the 5-percent level in one-sided test of H_0: Diff = 0 vs. H_a: Diff > 0, where Diff is defined as $\mathrm{Diff}_2 = 100 \times [(I^a_{l+2}/I^a_l) - (I^{us}_{l+2}/I^{us}_l)]$ and $\mathrm{Diff}_6 = 100 \times [(I^a_{l+6}/I^a_l) - (I^{us}_{l+6}/I^{us}_l)]$, in which I^a_l is the CPI for area a in link month l, and I^{us}_l is the same month's U.S. CPI.

NOTE: The sample size for this table is $N = 35$, and the indexes were taken from the BLS LABSTAT program. The rotation link months for A-size primary sampling units used in the analysis are as follows: Philadelphia—January 1980, January 1985, February 1989; Boston—July 1983, January 1989; Pittsburgh—October 1982, October 1991; Buffalo—August 1980, February 1985; Chicago—February 1980; Detroit—February 1981, October 1992; St. Louis—July 1980, September 1990; Cleveland—October 1982; Minneapolis—October 1983; Milwaukee—January 1984; Cincinnati—September 1984; Kansas City—August 1984; Washington—January 1981, July 1991; Dallas—June 1981, October 1990; Baltimore—September 1983, July 1989; Houston—June 1982; Atlanta—August 1982; Miami—July 1983; Los Angeles—February 1982; San Francisco—August 1982, November 1992; Seattle—July 1981; San Diego—September 1983; Honolulu—August 1984; Anchorage—January 1980.

Conclusion

This article has discussed two important aspects of the CPI: estimators used for basic component indexes and the ongoing process for replenishment of the CPI sample of commodities and services. Regular sample rotation provides important advantages, keeping the sample current within a rapidly changing economic environment. Recent research at BLS and the empirical results reported in this article, however, have identified possible problems with the CPI's Laspeyres-type formula that may cause it to overstate inflation, especially immediately following sample rotation.

The basic problem that has been identified is yet another manifestation of the effects of substitution by consumers as relative prices change. The research presented herein has explored the effects of substitution within strata of items, whereas previous research has focused on substitution between strata.[20] The current research suggests that for some goods and services, the substitution effects within strata of items may be larger than the effects between strata. Several types of empirical evidence indicate that this class of substitution has led to overestimation of price change for food at home. Apparel, because of its price volatility, clearly has the potential for such overestimation as well, although other factors may have offset these effects for the apparel indexes.[21]

A Laspeyres type of index will always be prob-

Table 4. **Rates of change of simulated Consumer Price Index, selected items, with basic components computed using Laspeyres and geometric mean index formulas, and variances of logarithm of differences of individual price quotes, June 1992 to June 1993**

Item	Laspeyres index	Geometric mean index	Difference	$Var[\log(\frac{P_{Jun '93}}{P_{Jun '92}})]$
White bread	2.70	1.86	0.84	.0290
Round roast	4.40	4.48	–.08	.0624
Round steak	4.49	4.12	.37	.0523
Bacon	7.36	7.43	–.06	.0403
Pork chops	3.79	3.45	.34	.0397
Fresh whole chicken	5.82	5.00	.82	.0497
Bananas	–3.22	–3.89	.67	.0976
Oranges	–4.82	–7.82	3.00	.1108
Lettuce	3.84	2.12	1.72	.1509
Tomatoes	60.00	55.69	4.31	.1603

NOTE: Rates of change of the simulated Laspeyres indexes are not identical to the published rates of change of the CPI, because of differences between the index simulation and the actual index calculation and because the simulated indexes were not rounded prior to computing rates of change. For both indexes, aggregation above the level of the basic components (that is, indexes of strata of items and areas) was based on the usual Laspeyres formula and weights that were in turn based on the Consumer Expenditure Survey.

lematic for estimating basic index components, but the research presented in this article suggests that such a measure would work better if the base-period price were derived from the price observed at the time of sample initiation, rather than from the link month price, as most of the overstatement of inflation probably occurs during the months immediately following the observation of the base price.

The geometric mean index represents an alternative formula for estimating basic index components. This formula has many beneficial features. The assumption underlying it is that expenditure shares are constant, so that consumer substitution reduces quantities purchased proportionately with any increase in price. This concept would appear to be most useful within narrowly defined strata whose component items are known to be quite close substitutes for each other. The geometric mean index approach would support redefining the classification of strata to ensure that, whenever possible, they consist of items that are close substitutes for one another. For example, in the case of prescription drugs, it might make sense to use the geometric mean index to aggregate across drugs that are aimed at treating a single ailment, such as hypertension, and then use the Laspeyres index to aggregate across treatment groups, where little consumer substitution takes place. In general, the geometric mean index could be used to calculate the basic index components, while the usual Laspeyres formula, with its standard fixed-basket interpretation, could be used for the higher level aggregation.

One possible objection to the use of the geometric mean formula is that such use would lead to a lack of consistency—a different formula for different levels of aggregation. In response, we merely need observe the following: (a) The Laspeyres-type formula currently used for estimating the basic components is not defined consistently across levels of aggregation, for reasons described earlier. (b) The use of a Laspeyres-type formula at the lowest level of aggregation has been a feature of the U.S. CPI only since the 1978 revision and is not commonly used in the CPI's of foreign countries. (c) Specifying the consumer's "market basket" as containing fixed quantities of specific brands purchased from specific outlets, regardless of each item's price at the outlet at which it is purchased, is a qualitatively different and stronger assumption than the assumption at higher levels of aggregation that quantities of broad categories of expenditures are fixed.

Additional research is needed to examine the empirical and conceptual implications of alternative estimators to the Laspeyres formula. The measurement objectives of such estimators need to be clarified, and criteria for assessing the performance of the proposed price measures need to be established. Although the literature on economic and statistical approaches to price measurement provides some guidance, in many respects the implementation of price measurement for constantly changing commodities and services remains a difficult and imperfectly understood problem. □

Footnotes

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of the computations. Paul Armknecht, Ralph Bradley, Timothy Erickson, Dennis Fixler, Patrick Jackman, Mary Kokoski, Marshall Reinsdorf, and David Richardson provided useful discussion and comments.

[1] For a description of BLS practices prior to the 1978 revision, see *BLS Handbook of Methods for Surveys and Studies*, Bulletin 1910 (Bureau of Labor Statistics, 1976). For a discussion of CPI practices in other countries, see Ralph Turvey, *Consumer Price Indices: An ILO Manual* (Geneva, International Labour Office, 1989); and United Nations, *Guidelines on Principles of a System of Price and Quantity Statistics*, Statistical Papers, Series M, No. 59 (New York, United Nations, 1977).

[2] For information on the 1978 revision, see W. John Layng, "The Revision of the Consumer Price Index," *American Statistical Association 1977 Proceedings of the Business and Economic Statistics Section, Part I* (Washington, American Statistical Association, 1978), pp. 195–203; and William L. Weber and Frank R. Lambrecht, "Within Outlet Item Selection Techniques for the Consumer Price Index Revision," *American Statistical Association 1978 Proceedings of the Section on Survey Research Methods* (Washington, American Statistical Association, 1979), pp. 305–10. Current BLS sampling procedures are described in detail in *BLS Handbook of Methods*, Bulletin 2414 (Bureau of Labor Statistics, 1992). The CPI also selects a probability sample of the urban areas, or primary sampling units, for geographic coverage, a subject that will not be touched on in this article, but is discussed in the *BLS Handbook of Methods*.

[3] Marshall Reinsdorf, "The Effect of Outlet Price Differentials on the U.S. Consumer Price Index," in Murray F. Foss, Marilyn E. Manser, and Allan H. Young, eds., *Price Measurements and Their Uses* (Chicago, University of Chicago Press, 1993); and "Price Dispersion, Seller Substitution, and the U.S. CPI," paper presented at Statistics Canada, Ottawa, Ontario, Canada, April 1993.

[4] F. G. Forsyth and R. F. Fowler, "The Theory and Practice of Chain Price Index Numbers," *Journal of the Royal Statistical Society*, Series A, 1981, pp. 224–46; Bohdan J. Szulc, "Linking Price Index Numbers," in W. E. Diewert and C. Montmarquette, eds., *Price Level Measurement: Proceedings from a Conference Sponsored by Statistics Canada* (Ottawa, Ontario, Canada, Minister of Supply and Services Canada, 1983), pp. 537–66; and Jörgen Dalén, "Computing Elementary Aggregates in the Swedish Consumer Price Index," *Journal of Official Statistics*, 1992, pp. 129–47.

[5] For example, Dalén discusses the ratio of mean prices, the ratio of harmonic means, the harmonic mean of ratios, a Fisher ideal approximation, and a ratio of normed mean prices. (See "Computing Elementary Aggregates," pp. 137–43.) The ratio of normed mean prices, which is the formula that was adopted by the Swedish CPI, replaces P_{ib} in (3) with the average of P_{ib} and P_{it}. Dalén proposes six statistical tests that a basic component index should satisfy, and the geometric mean is the only one of the indexes considered that satisfies all of the tests. (See also Bohdan J. Szulc, "Price Indices below the Basic Aggregation Level," in Turvey, *Consumer Price Indices*, pp. 167–78.)

[6] See W. Erwin Diewert, "Index Numbers," in John Eatwell, Murray Milgate, and Peter Newman, eds., *The New Palgrave: A Dictionary of Economics*, vol. 2 (London, Macmillan, 1987); Wolfgang Eichhorn and Joachim Voeller, "Axiomatic Foundation of Price Indexes and Purchasing Power Parities," in Diewert and Montmarquette, *Price Level Measurement*, pp. 411–50; and Robert A. Pollak, *The Theory of the Cost-of-Living Index* (New York, Oxford University Press, 1989).

[7] This definition is similar to that of a *market* proposed in George J. Stigler and Robert A. Sherwin, "The Extent of the Market," *Journal of Law and Economics*, October 1985, pp. 555–85.

[8] Reinsdorf, "Price Dispersion."

[9] See Gerard J. Tellis, "The Price Elasticity of Selective Demand: A Meta-Analysis of Econometric Models of Sales," *Journal of Marketing Research*, November 1988, pp. 331–41. Tellis surveyed 367 estimated price elasticities of demand for particular brands and found that the average estimated elasticity was 1.76.

[10] Reinsdorf, "Price Dispersion." The case where e_{it} is not assumed to be normally distributed is discussed briefly in the appendix.

[11] See Robert A. Pollak, "Subindexes in the Cost-of-Living Index," *International Economic Review*, February 1975, pp. 135–50.

[12] The most important strata that do not follow the rotating sample design are rent and homeowners' equivalent rent, which together are responsible for about 25 percent of the weight of the CPI. These indexes are estimated separately, from a housing survey using different methods. Several other strata of items (for example, postage, various utilities, and used cars) also do not use the sampling methods described here, usually because data on prices are readily available from centralized data bases or administrative sources. See *BLS Handbook of Methods*, pp. 187–90, 233–35.

[13] *BLS Handbook of Methods*, pp. 185–88.

[14] *Ibid.*, p. 188.

[15] It has been suggested that an unweighted geometric mean would converge to the correct price change in this model. This suggestion overlooks the fact that the weights appear because of the influence of expenditures on the probability of selecting an item for the sample. The selection of the sample by using probability proportional to expenditures has many desirable consequences and should not be abandoned simply because it may result in a small correlation between the expenditure weights and any measured price change.

[16] Ken Stewart, Claire Gallagher, and Karin Smedley of the Division of Consumer Prices and Price Indexes developed the computer programs and estimates for the two sets of indexes.

[17] A well-known mathematical result is that the geometric mean of a group of positive numbers must be less than the arithmetic mean. In the case of index numbers, this result is applicable only during the first period after linking, when the base prices cancel out of the formula, so that the Laspeyres formula (8) is a simple weighted average of price ratios. During subsequent periods it is possible for the geometric mean index to exhibit higher estimated inflation than the Laspeyres index does, although that result seldom seems to occur in practice.

[18] Reinsdorf, "Outlet Price Differentials"; and "Price Dispersion."

[19] The autocorrelation would die down slowly, for example, if the transitory component followed a fractionally integrated time series process. See, for instance, Jan Beran, "Statistical Methods for Data with Long-Range Dependence," *Statistical Science*, November 1992, pp. 404–27.

[20] See, for example, Marilyn E. Manser and Richard J. McDonald, "An Analysis of Substitution Bias in Measuring Inflation, 1959–85," *Econometrica*, July 1988, pp. 909–30.

[21] For a discussion of some of the price measurement problems relating to apparel, see Paul R. Liegey, Jr., "Adjusting Apparel Indexes in the Consumer Price Index for Quality Differences," in Foss, Manser, and Young, eds., *Price Measurements*.

APPENDIX: Mathematical derivations

The Laspeyres index estimator (1) for a basic component can be rewritten as

$$(A1) \qquad I_t = \frac{(1/n)\sum_{i=1}^{n} P_{it}Q_{ib}}{(1/n)\sum_{i=1}^{n} P_{ib}Q_{ib}}.$$

In large samples, $(1/n)\,\Sigma\, P_{it}Q_{ib}$ converges in probability to $E(P_{it}Q_{ib})$, and $(1/n)\,\Sigma\, P_{ib}Q_{ib}$ converges in probability to $E(P_{ib}Q_{ib})$, so I_t converges in probability to $E(P_{it}Q_{ib})/E(P_{ib}Q_{ib})$.[1]

Under the model set out in equations (5) and (6), we then solve for the two expectations and obtain

$$(A2) \qquad \frac{E(P_{it}Q_{ib})}{E(P_{ib}Q_{ib})} = \exp(\pi_t - \pi_b)\,\frac{E[\exp(e_{it} - \eta e_{ib})]}{E[\exp((1-\eta)e_{ib})]}.$$

This result is obtained by simply taking the expectation of the antilogarithm of the sum of the right-hand sides of equations (5) and (6) for the appropriate periods, substituting the expression in equation (5) for log P_{it} in equation (6), and noting that if two random variables x and y are independent, then $E[f(x)g(y)] = E[f(x)]E[g(y)]$. This allows $E\{\exp[(1-\eta)u_i + v_i + w_{ib}]\}$ to be factored out of both the numerator and denominator and "cancel out."[2]

In the special case where demand is perfectly inelastic, so that $\eta = 0$, the expression in equation (2) for the Laspeyres index converges to the correct inflation measure, because the marginal distributions of e_{it} and e_{ib} are assumed to be the same if the error is strongly stationary. This is consistent with the theoretical optimality of the Laspeyres index with perfectly inelastic demand.[3]

In the case where e_{ib} and e_{it} are normally distributed, we can solve explicitly for the two expectations. If z is normally distributed with mean μ and variance δ^2, then e^z has a lognormal distribution, and its mean is $\exp(\mu + \delta^2/2)$.[4] Thus,

$$E(e_{it} - \eta e_{ib}) = E((1-\eta)e_{ib}) = 0,$$

$$\mathrm{Var}(e_{it} - \eta e_{ib}) = (1 + \eta^2 - 2\eta\rho_{t-b})\sigma^2,$$

$$(A3) \qquad \mathrm{Var}((1-\eta)e_{ib}) = (1-\eta)^2\sigma^2,$$

$$\frac{E[\exp(e_{it} - \eta e_{ib})]}{E[\exp((1-\eta)e_{ib})]} = \frac{\exp[(1 + \eta^2 - 2\eta\rho_{t-b})\sigma^2/2]}{\exp[(1-\eta)^2\sigma^2/2]}$$

$$= \exp[\eta(1 - \rho_{t-b})\sigma^2].$$

The same methods are used to derive equation (9) for the CPI formula, and similar steps were used in the derivation of equation (10) for the geometric mean formula. The latter derivation uses the result that if x and y have a bivariate normal distribution with means μ_x and μ_y, standard deviations σ_x and σ_y, and correlation ρ, then $E(xe^y) = (\sigma_x\sigma_y\rho + \mu_x)\exp(\mu_y + \sigma^2_y/2)$.

I am pursuing additional research on the properties of the alternative index formulas when e_{it} is drawn from a nonnormal probability distribution.

Footnotes to the appendix

[1] Halbert White, *Asymptotic Theory for Econometricians* (San Diego, Academic Press, 1984), pp. 22–24.

[2] Alexander M. Mood, Franklin A. Graybill, and Duane C. Boes, *Introduction to the Theory of Statistics,* 3rd ed. (New York, McGraw-Hill, 1974), p. 160.

[3] Robert A. Pollak, *The Theory of the Cost-of-Living Index* (New York, Oxford University Press, 1989), pp. 13–15.

[4] Mood, Graybill, and Boes, *Theory of Statistics*, p. 117.