Nonlinear Energy Transfer between Stationary and Transient Waves Simulated by a GFDL Spectral General Circulation Model

Y. HAYASHI AND D. G. GOLDER

Geophysical Fluid Dynamics Laboratory/NOAA, Princeton University, Princeton, New Jersey 08542

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ABSTRACT

A wavenumber spectral analysis has been made of the nonlinear energy transfer between the tropospheric stationary (January mean) and transient waves in the midlatitudes simulated by a GFDL 9-level spectral general circulation model with 30 zonal wavenumbers.

It is shown that the wavenumber energy spectra are fairly well simulated, although the kinetic energy of simulated ultralong waves is about 60% of that observed. In particular, both simulated and observed ultralong waves are maintained primarily by energy transfer from zonal available potential energy. The model's energy spectra are then partitioned into stationary and transient wave parts. It is found that stationary ultralong waves gain kinetic energy but lose available potential energy through the nonlinear interaction with transient waves. Since this loss is much larger than the gain, transient waves act to destroy stationary wave energy which is maintained primarily by conversion from the zonal available potential energy, being consistent with observations. On the other hand, transient ultralong waves gain both kinetic and available potential energy through wave–wave interactions. This gain is comparable to the gain from the zonal available potential energy.

1. Introduction

Nonlinear wave–wave interaction distributes energy among different wavenumber components. It transfers energy not only from one wavenumber to another but also from one frequency to another. According to observational analysis (Saltzman, 1970) tropospheric ultralong waves in the midlatitude gain kinetic energy and lose available potential energy by wave-wave interaction. However, this analysis did not isolate transient waves from stationary waves. According to Steinberg *et al.* (1971), not only stationary (seasonal mean) but also transient ultralong waves gain kinetic energy and lose their available potential energy by wave-wave interaction. However, as pointed out by Hayashi and Golder (1983b), this conclusion can be erroneous, since their calculation of wave-wave energy transfer of transient waves implicitly includes stationary-transient wave interactions which probably dominate their estimate.[1] According to Holopainen (1970) and Lau and Oort (1982), stationary waves transfer their available potential energy to transient waves.

The present paper studies the energy transfer between stationary (January mean) and transient waves simulated by a 9-level GFDL 30-wavenumber spectral general circulation model with the biharmonic viscosity coefficient of 0.25×10^{24} cm^4 s^{-1} (see Manabe *et al.*, 1979 for the model's detail), and is an extension of Hayashi and Golder (1983a,b) which analyzed only *transient* planetary waves simulated by 9-level GFDL 15-wavenumber models with and without topography.

2. Wavenumber energetics

Although there is no unique formulation of energy cycles and the conventional formulation does not describe the zonal-wave energy transfer in the simplest manner, it still serves as a diagnostic tool for comparing observed and simulated energy cycles. Unfortunately, the transformed energy cycle proposed by Plumb (1983) and Kanzawa (1984) is not applicable to the present problem, since it does not describe wave–wave interactions.

a. Wavenumber equations

The conventional wavenumber energy equations (Saltzman, 1957) are written as

$$\partial K_n/\partial t = \langle K_m \cdot K_n \rangle + \langle K_0 \cdot K_n \rangle + \langle A_n \cdot K_n \rangle + \cdots, \tag{1}$$

$$\partial A_n/\partial t = \langle A_m \cdot A_n \rangle + \langle A_0 \cdot A_n \rangle - \langle A_n \cdot K_n \rangle + \cdots, \tag{2}$$

where energy flux convergence, diffusion and heating terms have been omitted.

[1] Steinberg *et al.* (1971) subtracted energy transfer among stationary waves from the total wave–wave energy transfer to obtain the estimate.

Here K_n and A_n are the kinetic and available potential energies for the zonal wavenumber n component. In (1), $\langle K_m \cdot K_n \rangle$ represents the "wave–wave transfer" of kinetic energy to the wavenumber n component by interaction among all the different (m) wavenumber components excluding zero and n, while $\langle K_0 \cdot K_n \rangle$ represents the "zonal–wave transfer" of kinetic energy to the wavenumber n component by interaction between the zonal flow and the wavenumber n component. Similarly, $\langle A_m \cdot A_n \rangle$ and $\langle A_0 \cdot A_n \rangle$ in (2) represent the wave–wave and zonal–wave transfers of available potential energy, respectively. In (1) and (2), $\langle A_n \cdot K_n \rangle$ represents the conversion of A_n into K_n. These energy spectra are formulated from the equations of motion in the advection form and the explicit expressions are summarized in Appendix B of Hayashi and Golder (1983b). These spectra are computed by the cross spectral method proposed by Hayashi (1980).

The time-averaged energy spectra and energy transfer spectra can be further partitioned into stationary (time mean) and transient (deviation from the time mean) parts as

$$\bar{K}_n = K_n^s + K_n^t, \quad (3)$$

$$\overline{\langle K_m \cdot K_n \rangle} = \langle K_m \cdot K_n^s \rangle + \langle K_m \cdot K_n^t \rangle, \quad (4)$$

$$\overline{\langle K_0 \cdot K_n \rangle} = \langle K_0 \cdot K_n^s \rangle + \langle K_0 \cdot K_n^t \rangle, \quad (5)$$

$$\overline{\langle A_n \cdot K_n \rangle} = \langle A_n^s \cdot K_n^s \rangle + \langle A_n^t \cdot K_n^t \rangle, \quad (6)$$

where the overbar denotes the time mean. The explicit expression for these partitions can be found in Hayashi (1980).

Then, (1) is partitioned into stationary and transient parts as

$$\partial K_n^s/\partial t = \langle K_m \cdot K_n^s \rangle + \langle K_0 \cdot K_n^s \rangle + \langle A_n^s \cdot K_n^s \rangle + \cdots, \quad (7)$$

$$\partial K_n^t/\partial t = \langle K_m \cdot K_n^t \rangle + \langle K_0 \cdot K_n^t \rangle + \langle A_n^t \cdot K_n^t \rangle + \cdots. \quad (8)$$

The above nonlinear energy spectra are interpreted as follows:

$\langle K_m \cdot K_n^s \rangle$ Transfer of kinetic energy into the stationary wavenumber n component (K_n^s) by stationary–stationary interaction or by transient–transient interaction among different wavenumber components.

$\langle K_m \cdot K_n^t \rangle$ Transfer of kinetic energy into the transient wavenumber n component (K_n^t) by transient–transient interaction or stationary–transient interaction among different wavenumber components.

$\langle K_0 \cdot K_n^s \rangle$ Transfer of kinetic energy into the stationary wavenumber n component (K_n^s) by interaction between the stationary zonal mean flow and the stationary wavenumber n component or between the transient zonal mean flow and the transient wavenumber n component.

$\langle K_0 \cdot K_n^t \rangle$ Transfer of kinetic energy into the transient wavenumber n component (K_n^t) by interaction between the stationary zonal mean flow and the transient wavenumber n component or between the transient zonal mean flow and the stationary (or transient) wavenumber n component.

$\langle A_n^s \cdot K_n^s \rangle$ Conversion of stationary A_n into stationary K_n.

$\langle A_n^t \cdot K_n^t \rangle$ Conversion of transient A_n into transient K_n.

b. Results

Table 1 compares the wavenumber energy spectra (stationary plus transient) of observed data (December 1964–February 1965, 25–75°N, 100–925 mb, based on Tomatsu, 1979), to that of the model data (one January, 26–75°N, 95–940 mb). The energy spectra are partitioned into low (1–3), and high (4–30) wavenumber categories; observational spectra are resolvable up to wavenumber 15. It should be noted that the energy spectra are fairly well simulated, although the low wavenumber kinetic energy is about 60% of that observed. The energy transfer spectra have the same sign as those observed, although there are substantial disagreements in the absolute values.

Figure 1 schematically illustrates the simulated and observed energy flow among the kinetic energy (K_n) and available potential energy (A_n) of wavenumber 1–30 and 4–30 components. It is seen that both the simulated and observed wavenumber 1–3 components are maintained by the energy transfer from the zonal available potential energy (A_0) to the eddy available potential energy (A_{1-3}) and its conversion into the eddy kinetic energy (K_{1-3}). These wavenumber 1–3 components lose available potential energy and gain kinetic energy by wave–wave interactions. It should be mentioned that this loss and gain are not equal to the gain and loss in A_{4-30} and K_{4-30} by wave–wave interactions. This means that some of the wave energy is nonlinearly transferred away from the domain of integration. There is also a loss of wave energy due to wave energy flux divergence. There is

TABLE 1. Wavenumber energy spectra (stationary plus transient) of observed data (December 1964–February 1965, 25–75°N, 100–925 mb, based on Tomatsu, 1979) and the model (January, 26–75°N, 95–940 mb). Energy spectra (10^3 J m^{-2}), energy transfer spectra (10^{-3} W m^{-2}).

n	1–3 Simulated	1–3 Observed	4–30 Simulated	4–15 Observed
K_n	372	606	481	560
A_n	795	810	316	338
$\langle K_m \cdot K_n \rangle$	101	425	−181	−481
$\langle K_0 \cdot K_n \rangle$	−510	−310	−386	−198
$\langle A_n \cdot K_n \rangle$	2076	1216	2603	1957
$\langle A_m \cdot A_n \rangle$	−423	−446	237	528
$\langle A_0 \cdot A_n \rangle$	2023	2258	2506	2314

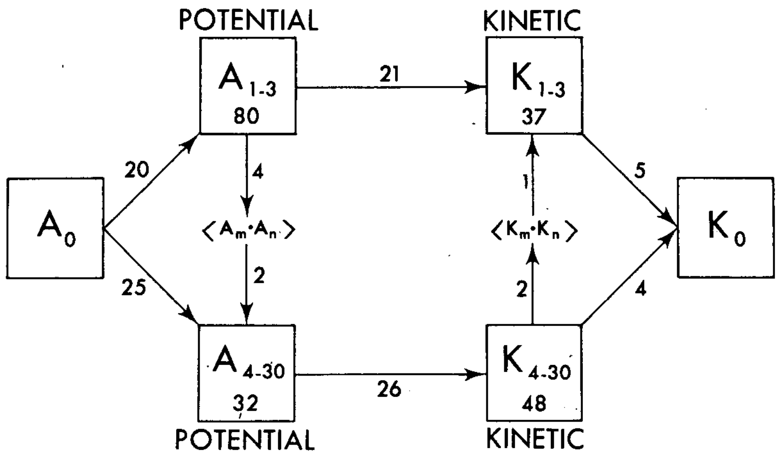

FIG. 1a. Schematic energy flow (10^{-1} W m^{-2}) among K_n (kinetic energy, 10^3 J m^{-2}) and A_n (available potential energy, 10^3 J m^{-2}) of wavenumbers 1-3 and 4-30 components of the model (January, 26-75°N, 95-940 mb); $\langle A_m \cdot A_n \rangle$ and $\langle K_m \cdot K_n \rangle$ denote transfer of A_n and K_n by wave-wave interactions.

also an additional source of eddy available potential energy (A_n) due to diabatic heating.

Table 2 shows the wavenumber energy spectra of simulated stationary and transient waves appearing in the midlatitude troposphere (January, 26–75°N, 95–940 mb). According to this table, the stationary wavenumber 1–3 components gain kinetic energy by wave–wave interaction ($\langle K_m \cdot K_n \rangle > 0$) and lose available potential energy by wave–wave interaction ($\langle A_m \cdot A_n \rangle < 0$). This loss is consistent with the observational analysis (Lau, 1979) that the heat transport by transient eddies in the lower troposphere tends to destroy the zonally asymmetric component of the time mean temperature field. This loss is much larger than the gain, being consistent with the observational analysis (Lau and Oort, 1982) of stationary–transient wave energy transfer. On the other hand, the transient wavenumber 1–3 components gain both kinetic energy and available potential energy by wave–wave interactions ($\langle K_m \cdot K_n \rangle > 0$, $\langle A_m \cdot A_n \rangle > 0$).

Figure 2 schematically illustrates the energy flow of total energy E_n which is a sum of kinetic energy and available potential energy. It is seen that the stationary wavenumber 1–3 components ($E_{1\text{-}3}{}^s$) lose total energy by wave–wave interaction but are compensated for by energy transfer from the total energy of the zonal mean (E_0). This transfer is essentially from the zonal available potential energy (A_0), according to Table 2. On the other hand, the transient wavenumber 1–3 components ($E_{1\text{-}3}{}^t$) gain total energy by wave–wave interaction. This gain is mainly associated with westward moving components according

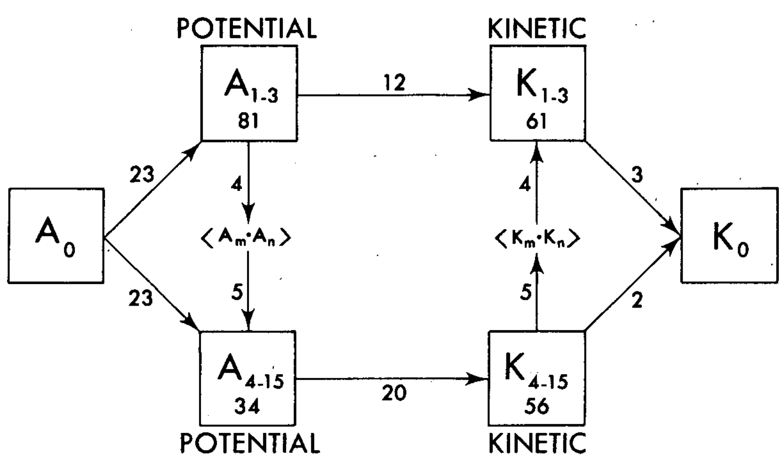

FIG. 1b. As in Fig. 1a, except for observed data (December 1964–January 1965, 25–75°N, 100–925 mb, based on Tomatsu, 1979).

TABLE 2. Wavenumber energy spectra of stationary and transient waves simulated by the model (January, 26–75°N, 95–940 mb). Energy spectra (10^3 J m^{-2}), energy transfer spectra (10^{-3} W m^{-2}).

	1–3		4–30	
n	Stationary	Transient	Stationary	Transient
K_n	199	173	98	382
A_n	571	225	77	240
$\langle K_m \cdot K_n \rangle$	38	63	−104	−76
$\langle K_0 \cdot K_n \rangle$	−519	9	−203	−182
$\langle A_n \cdot K_n \rangle$	1630	446	552	2052
$\langle A_m \cdot A_n \rangle$	−674	251	−176	414
$\langle A_0 \cdot A_n \rangle$	1602	421	639	1866

to Hayashi and Golder (1983b). This gain is comparable to the energy transfer from the total energy (E_0) of the zonal mean. This transfer is essentially from the zonal available potential energy (A_0) according to Table 2, and mainly associated with quasi-stationary or eastward moving components according to Hayashi and Golder (1983b).

According to Plumb (1983), $\langle A_0 \cdot A_n \rangle$ of topographically forced planetary waves are almost balanced by $\langle K_0 \cdot A_0 \rangle$ which transfers energy from K_0 to A_0 due to the indirect mean meridional circulations induced by eddy heat fluxes. This energy cycle should be interpreted as how the waves modify the zonal kinetic energy in the *absence* of zonal mean heating rather than how the waves are maintained. In the presence of mean heating, these indirect circulations are almost canceled by the direct circulations induced by the mean heating. As illustrated schematically in Fig. 3, theoretical planetary waves lose their K_n and A_n at the rates a and b due to dissipation and cooling, respectively. These energy losses are ultimately compensated for by the generation (a + b) of A_0 due to mean heating. This energy input is transferred to eddies by $\langle K_0 \cdot K_n \rangle + \langle A_0 \cdot A_n \rangle$ = (a + b). According to observations (Oort and Peixóto, 1979) and simulations (Manabe *et al.*, 1979), $\langle K_0 \cdot K_n \rangle$ and $\langle A_0 \cdot K_0 \rangle$ are much smaller than $\langle A_0 \cdot A_n \rangle$. Thus, the energy input is primarily transferred to A_n by $\langle A_0 \cdot A_n \rangle$ as is the case with the simulations (Manabe *et al.*, 1970).

3. Conclusions

A wavenumber spectral analysis has been made of the nonlinear energy transfer between tropospheric stationary and transient waves in the midlatitudes during January simulated by a GFDL spectral general circulation model with 30 wavenumbers. The main conclusions are summarized as follows:

1) The wavenumber energy spectra are fairly well simulated, although the kinetic energy of simulated ultralong waves is about 60% of that observed. In particular, both simulated and observed ultralong waves are primarily maintained by energy transfer from zonal available potential energy.

2) Simulated stationary ultralong waves gain kinetic energy but lose available potential energy by nonlinear interaction with transient waves. Since this loss is much larger than the gain, transient waves act to dissipate stationary wave energy which is maintained primarily by conversion from the zonal available potential energy, being consistent with observations.

3) Simulated transient ultralong waves gain both kinetic and available potential energy by wave–wave interactions. This gain is comparable to the gain from the zonal available potential energy.

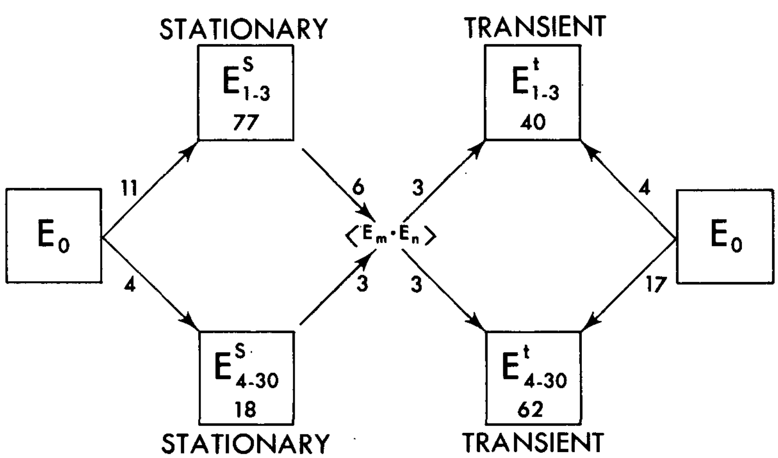

FIG. 2. Schematic energy flow (10^{-1} W m^{-2}) of E_n (kinetic and available potential energy, 10^4 J m^{-2}) among stationary (E_n^s) and transient (E_n^t) wavenumber n components (n = 1–3 and 4–30) simulated by the model (January, 26–75°N, 95–940 mb); $\langle E_m \cdot E_n \rangle$ denotes transfer of E_n by wave–wave interactions.

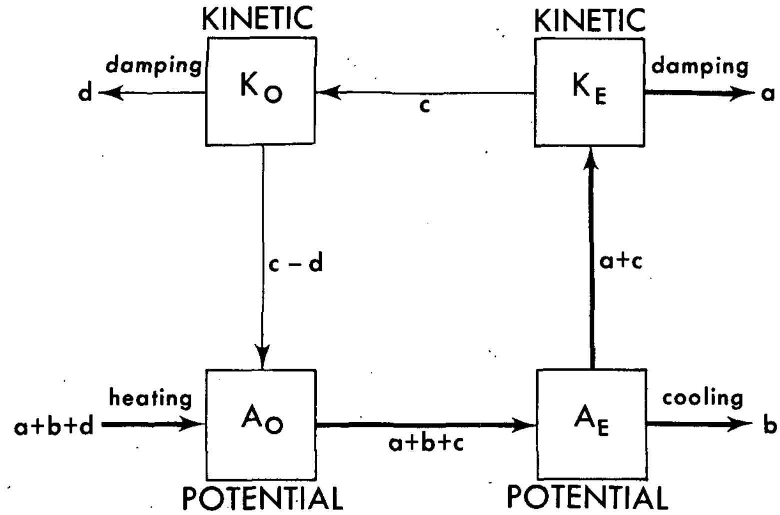

FIG. 3. Schematic theoretical energy flow (integrated over the entire atmosphere) between zonal mean and eddies in the steady state. In the case of topographically forced stationary waves, $\langle K_0 \cdot K_n \rangle$ is not only due to eddy momentum fluxes but also includes zonal-wave energy transfer of kinetic energy due to topographically induced Eulerian energy flux at the lower boundary. There is no energy flux at the infinite height, since wave energy is dissipated above the level of forcing.

The conclusion 3) should be observationally verified. These conclusions are based on the time-averaged energy spectra. There is the possibility that wave–wave interaction contributes to the amplification of quasi-stationary waves, since enhanced wave–wave energy transfer enhances the energy of these waves. According to Hansen and Chen (1982) and Itoh (1983), observed ultralong waves can gain available potential energy by wave–wave energy transfer when they are being amplified. The observed and simulated atmospheric blocking is sometimes associated primarily with wave–wave energy transfer of kinetic energy (Hansen and Chen, 1982; Fischer, 1984).

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