Modified Methods of Estimating Space-Time Spectra from Polar-Orbiting Satellite Data

Part II: The Wavenumber Transform Method

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Abstract

In order to estimate space-time spectra correctly from uneven twice-daily data sampled by a polar-orbiting satellite, the wavenumber transform method (Hayashi, 1980) is modified by the use of a nonorthogonal Fourier inversion. The space-time spectra are obtained from the time-Fourier coefficients of the space-Fourier transforms of the asynoptic field with respect to its frequency-shifted wavenumber. Since this method requires the spatial interpolation of asynoptic data, it is effective only for ultralong waves. The wavenumber-frequency aliasing characteristics are examined and the computer code is exemplified.

1. Introduction

In Part I (Hayashi, 1983), the non-orthogonal Fourier transform method (Salby, 1982) is simplified by transforming frequency only. In the present paper (Part II), the wavenumber transform method (Hayashi, 1980) is also modified by use of the nonorthogonal Fourier inversion in order to eliminate the aliasing errors due to uneven twice-daily data. Also, a more efficient computational scheme is given. Section 2 modifies the wavenumber transform method, while Section 3 tests the method. Summary and remarks are given in Section 4. Appendix A gives the derivation of the nonorthogonal Fourier inversion, while Appendix B exemplifies the computer code of the modified method.

2. The modified wavenumber transform method

2.1 Space-time Fourier series

The space-time Fourier series for a continuous longitude λ and time t is given by

$$w(\lambda, t)=R_e \sum_{m=0}^{\infty} \sum_{f=-\infty}^{\infty} \delta_m \hat{w}_{m,f} \exp(im\lambda+i2\pi ft), \tag{2.1}$$

where $\hat{w}_{m,f}$ are the space-time Fourier coefficients and

$$\delta_m=1 \text{ except for } \delta_0=0.5. \tag{2.2}$$

2.2 Time coordinate transformation

The standard time t and local time t_l of a satellite at the longitude λ are related by

$$t=t_l-\lambda/\Omega, \tag{2.3}$$

where Ω is the zonal angular velocity (2π day^{-1}) of a sun-synchronous satellite.[1)]

Inserting (2.3) into (2.1) gives

$$w(\lambda, t_l-\lambda/\Omega) =\frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{f=-\infty}^{\infty} \hat{w}_{m,f} \exp(im_f\lambda+i2\pi ft_l), \tag{2.4}$$

where m_f is the frequency-shifted wavenumber of the asynoptic field and is given by

$$m_f=m-f. \tag{2.5}$$

2.3 Space-time Fourier inversion

The above space-time Fourier series (2.4) is not orthogonal in the space-time domain when t_l is not of equal time intervals. Nevertheless, its coefficients can be determined explicitly as

$$2\hat{w}_{m,f}=\hat{F}_{m,f}, \tag{2.6}$$

where $F_{m,f}$ is the space-Fourier transform of the

1) For a non-sunsynchronous satellite which drifts around the earth once per δ day ($\delta \neq 1$), time and period must be measured in units of δ days to apply the present method.

asynoptic field with respect to m_f as defined by

$$F_{m,f}=\pi^{-1}\int_0^{2\pi} w\exp(-im_f\lambda)d\lambda \qquad (2.7a)$$

$$=\pi^{-1}\int_0^{2\pi}[w\exp(if\lambda)]e^{-im\lambda}d\lambda\,. \qquad (2.7b)$$

and $\hat{F}_{m,f}$ is the time-Fourier coefficient of $F_{m,f}$ and is determined by the nonorthogonal time-Fourier inversion described in the following.

2.4 *Nonorthogonal time-Fourier inversion*

The twice-daily time series $w(t)$ with increments of $0.5(1+\varepsilon)$ and $0.5(1-\varepsilon)$ days consists of two sets of once-daily time series as

$$X_p\equiv w(p)\,,\quad p=0,1,2,\cdots,N-1\,, \qquad (2.8)$$

$$Y_p\equiv w[p+0.5(1+\varepsilon)],\quad p=0,1,2,\cdots,N-1\,. \qquad (2.9)$$

The twice-daily series is expanded into a Fourier series as

$$w(t)=R_e\sum_{n=0}^{N}\delta_n\hat{w}_n\exp\left(i2\pi\cdot\frac{n}{N}\cdot t\right), \qquad (2.10)$$

where

$$f=n/N\,, \qquad (2.11)$$

and

$$\delta_n=1 \text{ except for } \delta_0=\delta_N=0.5\,. \qquad (2.12)$$

Due to the unevenness of the increments, the Fourier series is not orthogonal. Nevertheless (2.10) can be inverted, as derived in Appendix A as

$$\hat{w}_n=\frac{\hat{X}_nE(n-N)-\hat{Y}_n}{E(n-N)-E(n)}\,, \qquad (2.13)$$

and

$$\hat{w}^*_{N-n}=\frac{-\hat{X}_nE(n)+\hat{Y}_n}{E(n-N)-E(n)}\,, \qquad (2.14)$$

where the asterisk denotes the complex conjugate and

$$E(n)\equiv\exp[i\pi n(1+\varepsilon)/N]\,. \qquad (2.15)$$

In the above, $\hat{X}_n$ and $\hat{Y}_n$ are the Fourier transforms of X_p and Y_p, and are given by

$$\hat{X}_n=\frac{4}{N}\sum_{p=0}^{N/2}X_p\exp\left(-i2\pi\cdot\frac{n}{N}\cdot p\right), \qquad (2.16)$$

and

$$\hat{Y}_n=\frac{4}{N}\sum_{p=0}^{N/2}Y_p\exp\left(-i2\pi\cdot\frac{n}{N}\cdot p\right). \qquad (2.17)$$

2.5 *Computational procedure*

The computational procedure of the generalized wavenumber transform method is summarized as follows:

1) Compute the space-Fourier trasnform (2.7) of the given asynoptic data set with respect to a specified wavenumber and frequency.

2) The space-time Fourier coefficients (2.6) are then obtained by computing the time Fourier coefficients of this space-Fourier transform with respect to the specified frequency.

3) The space-time cross spectra can be obtained from the space-time Fourier coefficients by the use of formulas given in the Appendix C of Part I.

It should be noted that day and night asynoptic data must be on the same equatorial local date (see Fig. 3a of Hayashi, 1980) rather than on the same standard date (see Fig. 2 of Hayashi, 1980). In order to reduce leakage, the original time series data should be tapered at each end (see Section 2.6 of Part I). However, longitudinal data should not be tapered, although the data are discontinuous from one end to the other.

3. Test of the method

3.1 *Interpolation error*

Since the wavenumber transform method assumes that spatial data are somehow interpolated to regular grid points, it is of importance to examine the resulting interpolation error. Table 1 shows the wavenumber (n) distribution of the amplitude of a cosine function with wavenumber m, which is linearly interpolated from 13.6 to 14 points. The correct wavenumber amplitude is 1 for $n=m$ and 0 for $n\neq m$. It is seen that wavenumber 4–6 are greatly distorted. It is expected, however, that a more general objective analysis scheme has less distortion than a linear interpolation.

3.2 *Time spectra*

Since the generalized method involves a non-

Table 1 The wavenumber (n) distribution of the amplitude of a cosine function with wavenumbers m which are linearly interpolated from 13.6 to 14 points. The correct wavenumber amplitude should be 1 for $n=m$ and 0 for $n\neq m$.

n \ m	1	2	3	4	5	6
1	0.987	0.015	0.022	0.033	0.046	0.060
2	0.006	0.947	0.036	0.040	0.052	0.065
3	0.003	0.022	0.886	0.064	0.065	0.076
4	0.003	0.012	0.046	0.811	0.100	0.098
5	0.002	0.009	0.023	0.075	0.733	0.152
6	0.002	0.008	0.017	0.033	0.099	0.680

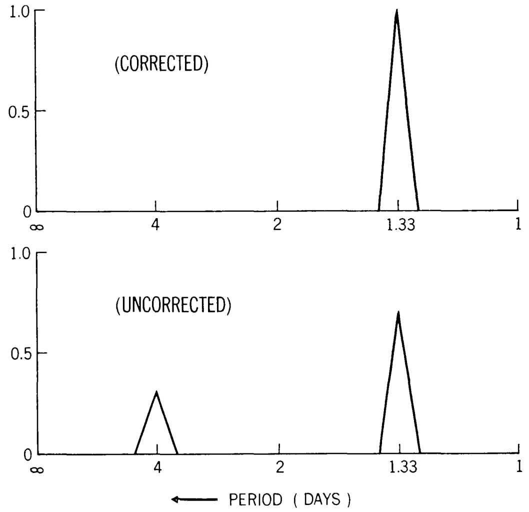

Fig. 1 Frequency distribution of the power spectra of a cosine function (period=1.33 days) estimated from uneven twice-daily data with (upper) and without (lower) corrections for uneven time increments.

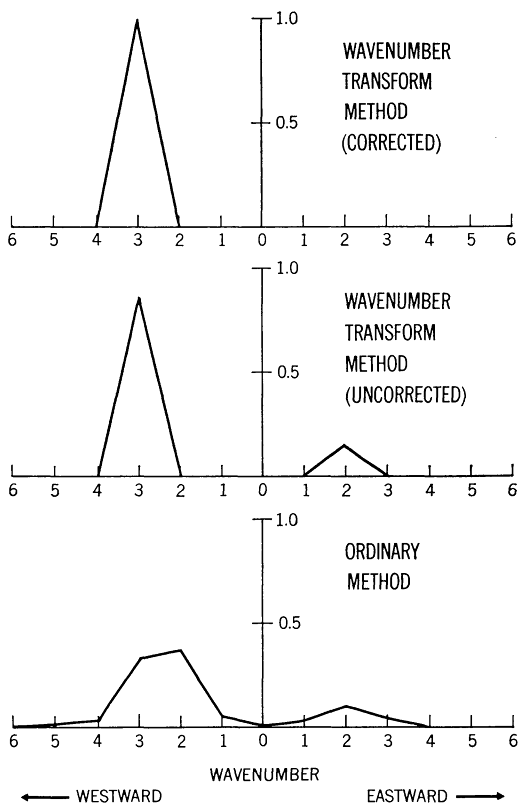

Fig. 2 Wavenumber distributions (period=2 days) of the space-time power spectra of sinusoidal waves (wavenumber=3, period=2 days, westward moving) estimated from asynoptic data with time increments of 18 and 6 hours (ε=0.5). The wavenumber transform methods with (top) and without (middle) a correction for uneven time increments. The ordinary method (bottom).

orthogonal time-Fourier inversion, time spectra given by this inversion are tested. Fig. 1 shows the time-power spectra of a cosine function (period=1.33 days) which are estimated from uneven interval time series data with increments of 18 and 6 hours (ε=0.5). These spectra are correctly estimated by the nonorthogonal inversion method with a correction for the uneven time increments. Without this correction (ordinary method) the 1.33-day period peak is aliased to a 4-day period peak, as is the case with once-daily data.

When the input is given at the Nyquist frequency (1 day^{-1}), it is aliased to the zero frequency even when a correction for the uneven time increments is included (not illustrated).

3.3 Space-time spectra

Fig. 2 shows the wavenumber distribution (period=2 days) of the space-time power spectra of a given sinusoidal wave which are estimated from asynoptic data hypothetically sampled at regular spatial grid points at uneven time intervals of 18 and 6 hours. These spectra are correctly estimated by the wavenumber transform method with a correction for the uneven time increments (top), whereas they are aliased without this correction (middle). They are severely aliased by the ordinary method (bottom).

It turns out (not illustrated) that the aliasing configurations of the generalized wavenumber transform method are identical to those (see Part I) of the generalized frequency transform method for $m_N=F_N$, provided that there is no distortion due to the spatial interpolation.

4. Summary and remarks

In order to estimate space-time spectra from uneven twice-daily data sampled by a polar-orbiting satellite, the wavenumber transform method of Hayashi (1980) is modified by the use of nonorthogonal Fourier inversion. The modified method is summarized as follows:

1) The space-time spectra are obtained from the time-Fourier coefficients of the space-Fourier transforms of the asynoptic field which is associated with the frequency-shifted wavenumbers.

2) The nonorthogonal Fourier series of uneven twice-daily data is inverted by splitting them into two sets of once-daily data. Their coefficients are reduced to a linear combination of the Fourier transforms of the day and night data.

3) The wavenumber-frequency aliasing configurations of the generalized wavenumber transform method are identical to those of the modified frequency transform method, provided that there is no distortion due to the spatial interpolation.

The modified wavenumber transform method is convenient when objectively analyzed satellite asynoptic data are available. However, this method is effective only for ultralong waves, since the high wavenumber components are seriously distorted by the spatial interpolation. This method is less efficient than the frequency transform method in computing wavenumber-frequency distribution of spectra, since different space-Fourier transforms must be computed for different frequencies. However, this inefficiency is not a serious problem, since these spectra are computed for only a few wavenumbers. Moreover, the present computational scheme is more efficient and can more easily be automated than that of the wavenumber transform method of Hayashi (1980) which must discard all the frequencies except for the tuning frequency of the frequency-shifted wavenumbers.

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Appendix A

Derivation of Nonorthogonal Fourier Inversion

The Fourier series of a real time series is written as

$$w(0.5r)=R_e\sum_{n=0}^{N}\delta_n\hat{w}_n\exp(i\pi nr/N), \qquad (\mathrm{A.1})$$

where

$$\delta_n=1 \text{ except for } \delta_0=\delta_N=0.5. \qquad (\mathrm{A.2})$$

This series can be rewritten as

$$w(0.5r)=R_e\sum_{n=0}^{N/2}\sigma_n\{\hat{w}_n\exp(i\pi nr/N)+\hat{w}_{N-n}\exp[i\pi(N-n)r/N]\} \qquad (\mathrm{A.3a})$$

$$w(0.5r)=R_e\sum_{n=0}^{N/2}\sigma_n\{\hat{w}_n\exp(i\pi nr/N)+\hat{w}^*_{N-n}\exp^*[i\pi(N-n)r/N]\} \qquad (\mathrm{A.3b})$$

where the asterisk denotes the complex conjugate and

$$\sigma_n=1 \text{ except for } \sigma_0=\sigma_{N/2}=0.5, \qquad (\mathrm{A.4})$$

since the two terms in (A.3a) coincide for $n=N/2$.

Following the principle of the fast Fourier transform, the above Fourier series is reduced to

$$X_p=w(p) \qquad (\mathrm{A.5a})$$

$$=R_e\sum_{n=0}^{N/2}\sigma_n(\hat{w}_n+\hat{w}^*_{N-n})\exp(i2\pi p/N), \qquad (\mathrm{A.5b})$$

and

$$Y_p=w[p+0.5(1+\varepsilon)]$$

$$=R_e\sum_{n=0}^{N/2}\sigma_n[\hat{w}_nE(n)+\hat{w}^*_{N-n}E(n-N)]\times\exp(i2\pi np/N), \qquad (\mathrm{A.6b})$$

where

$$E(n)\equiv\exp[i\pi n(1+\varepsilon)/N], \qquad (\mathrm{A.7})$$

and the identity $\exp(i2p\pi)=1$ has been used.

The Fourier series (A.5) and (A.6) can be inverted by virtue of the orthogonality of the Fourier series with an equal increment as

$$\hat{w}_n+\hat{w}^*_{N-n}=\hat{X}_n, \qquad (\mathrm{A.8})$$

and

$$\hat{w}_nE(n)+\hat{w}^*_{N-n}E(n-N)=\hat{Y}_n, \qquad (\mathrm{A.9})$$

where $\hat{X}_n$ and $\hat{Y}_n$ are the Fourier transform of X_p and Y_p and are given by (2.16) and (2.17).

Solving (A.8) and (A.9) for $\hat{w}_n$ and $\hat{w}^*_{N-n}$ gives (2.13) and (2.14).

Appendix B

Computer Code of the Wavenumber Transform Method

SUBROUTINE WNT (List 1) computes the space-time Fourier coefficients from twice-daily or once-daily sunsynchronous satellite data (see footnote[1]) for non-sunsynchronous satellite data) with respect to a specified wavenumber and frequency. This program calls SUBROUTINE SFT (List 2) which computes the space-Fourier transform (2.7b) and SUBROUTINE NFI (List 3) which computes the time Fourier coefficients (2.13) and (2.14).

SUBROUTINE SFT calls SUBROUTINE SFC which is exactly the same as SUBROUTINE TFC

List 1

```
      SUBROUTINE WNT(ITIME,MTIME,NX,N,EPS,WA,WD,
     *M,NF,F,WMN)
      PARAMETER ID=N,JD=NX
      DIMENSION WA(ID,JD),WD(ID,JD),WAX(JD),WDX(JD),
     *CMNA(ID),SMNA(ID),CMND(ID),SMND(ID)
      COMPLEX CI,CN,SN,WMN
      PI=3.141592653
      CI=(0.0,1.0)
      DO 20 I=1,N
      DO 10 J=1,NX
      WAX(J)=WA(I,J)
   10 WDX(J)=WD(I,J)
      CALL SFT(MTIME,PI,CI,NX,N,WAX,M,NF,F,CMN,SMN)
      CMNA(I)=CMN
      SMNA(I)=SMN
      CALL SFT(MTIME,PI,CI,NX,N,WDX,M,NF,F,CMN,SMN)
      CMND(I)=CMN
   20 SMND(I)=SMN
      CALL NFI(ITIME,PI,CI,N,EPS,CMNA,CMND,NF,CN)
      CALL NFI(ITIME,PI,CI,N,EPS,SMNA,SMND,NF,SN)
      WMN=(CN-CI*SN)/2.0
      RETURN
      END
```

List 2

```
      SUBROUTINE SFT(MTIME,PI,CI,NX,N,WX,
     *M,NF,F,CMN,SMN)
      PARAMETER JD=NX
      COMPLEX CI,WFC,WFS
      DIMENSION WX(JD),WC(JD),WS(JD)
      DO 10 J=1,NX
      ARG=F*2.0*PI*FLOAT(J-1)/FLOAT(NX)
      WC(J)=WX(J)*COS(ARG)
   10 WS(J)=WX(J)*SIN(ARG)
      CALL SFC (MTIME,PI,CI,NX,WC,M,WFC)
      CALL SFC (MTIME,PI,CI,NX,WS,M,WFS)
      CMN=REAL(WFC+CI*WFS)
      SMN=-AIMAG(WFC+CI*WFS)
      RETURN
      END
```

(List 2 in the Appendix in Part I) except that TFC and its dimension ID are replaced by SFC and JD, respectively. SUBROUTINE NFI calls SUBROUTINE TFC.

When SUBROUTINE WNT is called for the first time, set ITIME$=1$ and MTIME$=1$ to compute and store cosine and sine functions in the subroutines SFC and TFC.

The input arguments are given by

NX = even number of space points,

N = even number of half time point,

List 3

```
      SUBROUTINE NFI(ITIME,PI,CI,N,EPS,X,Y,NF,WN)
      PARAMETER ID=N
      DIMENSION X(ID),Y(ID)
      COMPLEX CI,WN,EX1,EX2,XF,YF
      KF=IABS(NF)
      IF(KF.GT.N/2) KF=N-KF
      CALL TFC(ITIME,PI,CI,N,X,KF,XF)
      CALL TFC(ITIME,PI,CI,N,Y,KF,YF)
      EX1=CI*PI*FLOAT(KF)*(1.0+EPS)/FLOAT(N)
      EX1=CEXP(EX1)
      EX2=CI*PI*FLOAT(KF-N)*(1.0+EPS)/FLOAT(N)
      EX2=CEXP(EX2)
      WN=(XF*EX2-YF)/(EX2-EX1)
      IF(IABS(NF).GT.N/2)
     *WN=CONJG((-XF*EX1+YF)/(EX2-EX1))
      IF(NF.LT.0) WN=CONJG(WN)
      RETURN
      END
```

$EPS=\varepsilon$ for twice-daily data

$=0$ for once-daily data

$WA(I,J)=w(t_l,\lambda)$ for daytime or odd day,

$WD(I,J)=w(t_l,\lambda)$ for nighttime or even day,

$M=m(0\leqq M\leqq NX/2)$,

$NF=\pm n(-N\leqq NF\leqq N)$,

and $F=f=\pm n/N$ for twice-daily data

$=\pm n/(2N)$ for once-daily data.

The output argument is given by

$WMN=\hat{w}_{m,\pm n}$ (complex).

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時空間スペクトルを極軌道衛星データから求める修正方法

第2部：波数変換法

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極軌道衛星により採集された1日2回の不等時間隔データから時空間スペクトルを正確に求められるように波数変換法 (Hayashi, 1980) を非直交フーリエ変換により修正した。時空間スペクトルは変換された波数についての非綜観場の空間フーリエ変換の時間フーリエ係数から求まる。この方法は非綜観場の空間補間を必要とするので超長波の解析のみに効果的である。波数—振動数 aliasing の性質を調べ，計算機プログラムも例示した。