Vertical-Zonal Propagation of a Stationary Planetary Wave Packet

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ABSTRACT

In order to explain why the Aleutian high stands out in the winter stratosphere, a complex Fourier analysis is made of simulated and observed stationary waves. It is found that in the troposphere the envelope of the time mean geopotential height consisting of wavenumbers 1 ~ 3 attains its major and minor maxima in the Pacific and Atlantic, respectively. The major maximum is dominated by wavenumbers 1 ~ 2 and shifts eastward with height in the stratosphere in the approximate direction of the group velocity and strengthens the Aleutian high. The minor maximum is dominated by wavenumber 3 and is confined in the troposphere.

1. Introduction

The pressure pattern in the winter stratosphere is characterized by the so-called "Aleutian high" as observed by Boville (1960), Teweles *et al.*, 1960, Hare (1960), Wilson and Godson (1963) and Sawyer (1964). As illustrated in Fig. 1, this anticyclone is well simulated by a linear model (Matsuno, 1970) with a realistic zonal mean state and observed pressure pattern specified at a tropospheric (500 mb) level. The intense Aleutian high in the linear model results from a constructive interference between wavenumber 1 and 2 components which are forced from below. It will be of interest to interpret this phenomenon in terms of the vertical-zonal propagation of a wave group.

The vertical propagation of planetary waves was first discussed by Charney and Drazin (1961). According to their linear theory, the energy of stationary planetary waves is trapped in easterlies or strong westerlies and only large-scale waves can propagate upward into weak westerlies. Subsequently, Dickinson (1969) showed that vertically propagating planetary waves also attenuate with height by the effect of Newtonian cooling.

The zonal group velocity of planetary waves was first discussed by Rossby (1945, 1949), who demonstrated that the amplitude of nondivergent planetary waves propagates eastward. Subsequently, Yeh (1949) showed that the group velocity of divergent Rossby waves with large wavelength is westward relative to the basic flow. Charney (1949) showed by use of an influence function that a numerical prediction at some point is influenced by initial values over some limited regions due to the maximum and minimum values of horizontal and vertical group velocities.

The zonal-meridional ray paths of barotropic and baroclinic Rossby waves were discussed by Longuet-Higgins (1964a, 1965) and Schopf *et al.* (1981), respectively. Hoskins *et al.* (1977) numerically demonstrated the zonal-meridional propagation of a barotropic Rossby wave group.

The vertical-zonal propagation of equatorial mixed Rossby-gravity wave packets forced by a localized tropospheric heat source has been demonstrated by Holton (1972). Unlike mixed Rossby-gravity waves with wavenumbers 3 ~ 5, the amplitude modulation of midlatitude planetary waves consisting of wavenumbers 1 ~ 3 cannot be easily distinguished from their phase variation and the conventional ray tracing method (see Bretherton, 1971) based on the WKB approximation is not well applicable. Nevertheless, it will be shown that their envelope can be visually traced by use of a complex Fourier analysis.

In Section 2 we analyze the envelope of a simulated and an observed stationary planetary wave packet, and in Section 3 we give a theoretical interpretation. Conclusions and remarks are given in Section 4. The Appendix gives a simple example of a two-dimensional wave group with a single frequency to show that a stationary wave group attains its largest amplitude modulation in the approximate direction of the group velocity.

2. Complex Fourier analysis of stationary planetary waves

The space series $w(x)$ can be represented by a Fourier series as

$$w(x) = \mathrm{Re} \sum_k W_k \exp(ikx). \quad (1)$$

Selecting some wavenumber range Δk, the com-

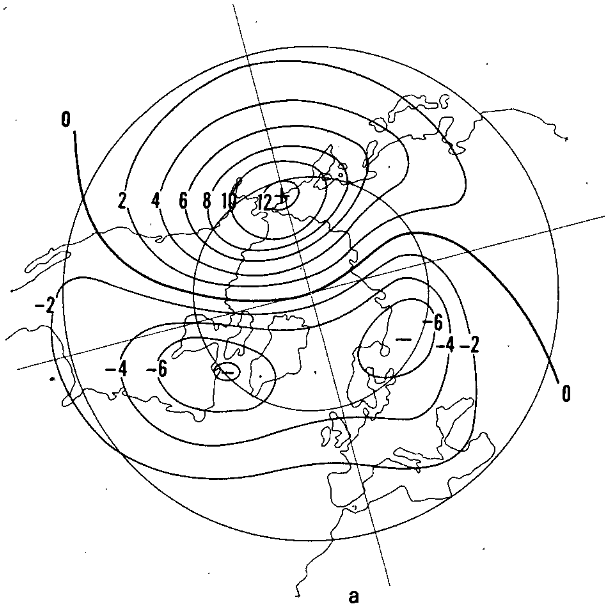

FIG. 1a. Observed distribution (after Matsuno, 1970) of disturbed height of the 10 mb surface, in January 1967, with 200 m intervals.

plex Fourier series can be rewritten as

$$\sum_{\Delta k} W_k \exp(ikx) = a(x) \exp[i\phi(x)], \quad (2)$$

where

$$a(x) = \left|\sum_{\Delta k} W_k \exp(ikx)\right|, \quad (3)$$

$$\phi(x) = \arg[\sum_{\Delta k} W_k \exp(ikx)]. \quad (4)$$

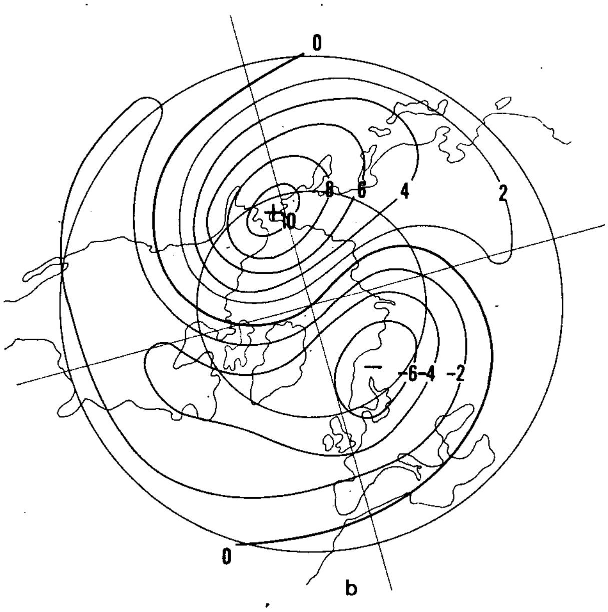

FIG. 1b. Computed distribution (after Matsuno, 1970) of disturbed height of the pressure surface approximately at the 30 km level, with 200 m intervals. The observed distribution is specified at the 500 mb level of a linear model.

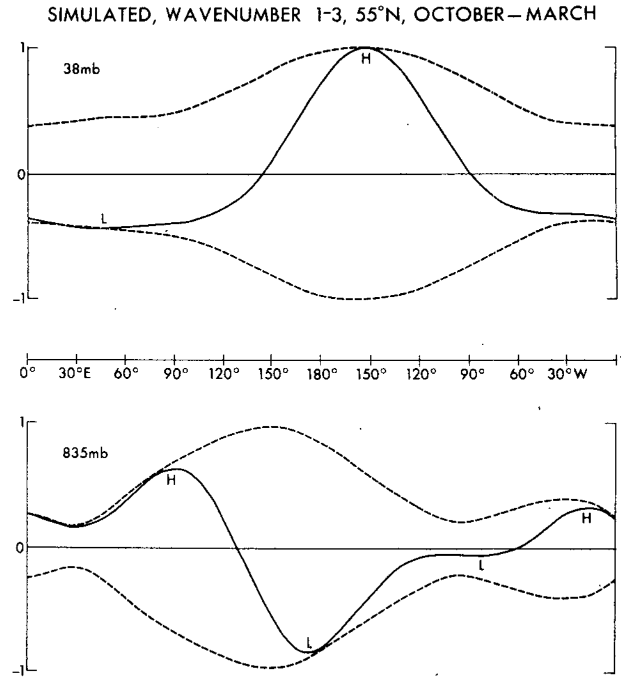

FIG. 2. Longitude distribution of the normalized time-mean geopotential height (solid) and its envelope (dashed) consisting of wavenumbers 1 ~ 3 during October–March simulated by a GFDL general circulation model. Upper (38 mb), lower (835 mb).

Here $a(x)$ is the envelope function (see Longuet-Higgins, 1964b) and $\phi(x)$ is the phase function. The envelope agrees with the local amplitude defined as the square root of the local spatial variance and $d\phi(x)/dx$ gives the local wavenumber, provided that $k \gg \Delta k$.

Fig. 2 shows the time mean geopotential height consisting of wavenumbers 1 ~ 3 and its envelope during the period October–March, simulated by an 11-layer GFDL general circulation model (Manabe and Mahlman, 1976). A detailed analysis of these planetary waves was given by Hayashi and Golder (1977). In the stratosphere (38 mb), the dominating Aleutian high coincides with the maximum of the envelope. In the lower troposphere (835 mb) the envelope attains its major maximum in the region (150°E) between the Siberian high and Aleutian low and its minor maximum in the Atlantic (30°W). These envelopes, however, do not agree with the local amplitude due to the fact that planetary waves with wavenumbers 1 ~ 3 are not associated with large k ($k/\Delta k = 2$).

Fig. 3 shows a longitude-height distribution (55°N) of the time mean geopotential height (wavenumbers 1 ~ 3) and its envelope. The major maximum in the envelope shifts eastward with height (0.3 km/deg. longitude) in the stratosphere and strengthens the stratospheric Aleutian high. On the other hand, the minor maximum over the Atlantic is confined in the troposphere.

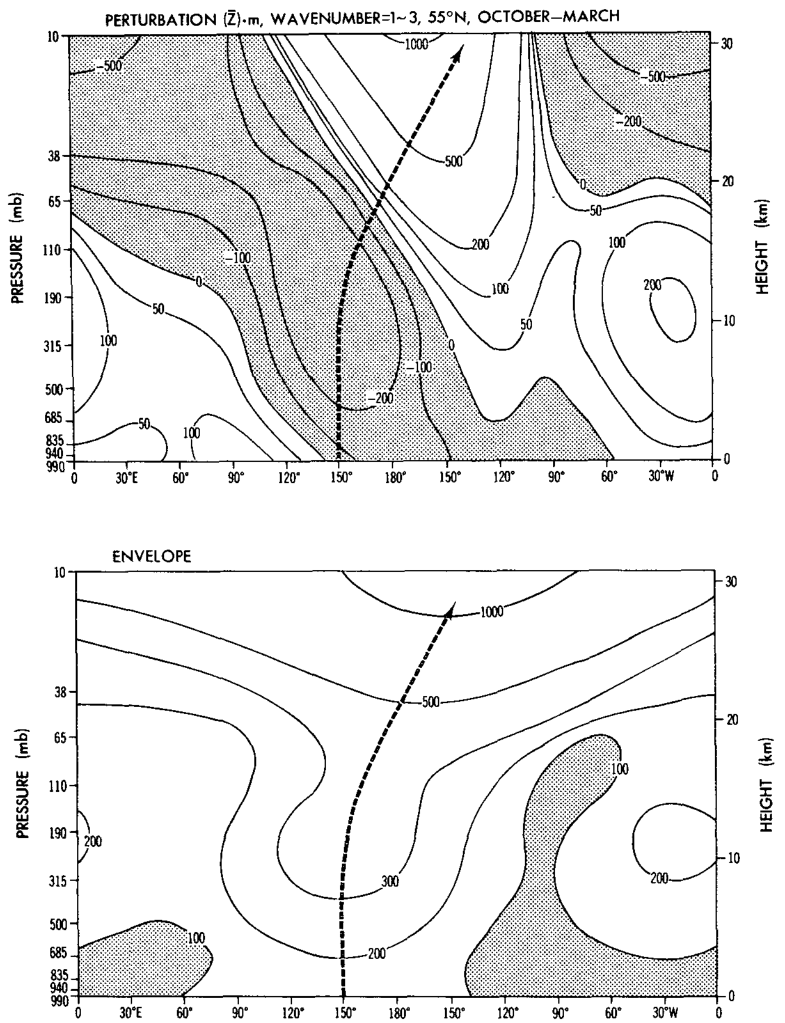

FIG. 3. Longitude-height distribution (55°N) of the simulated time mean geopotential height (upper) and its envelope (lower) consisting of wavenumbers 1 ~ 3 during October–March. Dashed line indicates the position of the zonal maximum of the envelope.

In order to isolate the vertically propagating wavenumber 1 ~ 2 components, the wavenumber 3 component was eliminated as shown in Fig. 4. The comparison between Fig. 3 and Fig. 4 shows that the effect of the wavenumber 3 component is indeed confined to the troposphere. In the absence of this component the minor maximum is no longer isolated from the major maximum and the maximum shifts slightly westward with height in the lower troposphere. The maximum and minimum lines slope in exactly the same direction due to the fact that the wave packet consists of only two wavenumbers.

In order to corroborate the above findings based on the general circulation model, Fig. 5 shows observed planetary waves (wavenumbers 1 ~ 3) based on a 6 years winter average of the NMC geopotential analysis data used in Lau (1979). Considering the year-to-year variability (not illustrated) of the observed planetary waves, the simulated planetary waves (Fig. 3) compare favorably with the observed waves.

3. Theoretical interpretation

It will be of interest to interpret the above results from the viewpoint of wave propagation theories.

The quasi-geostrophic dispersion relation of Rossby waves on a beta plane in a hydrostatic atmosphere is given by

$$\sigma = kU - \frac{\beta k}{k^2 + l^2 + f^2/(gh)}, \tag{5}$$

where σ is the frequency which is positive for eastward-moving waves, while k and l are the zonal and meridional wavenumbers, respectively; f is the

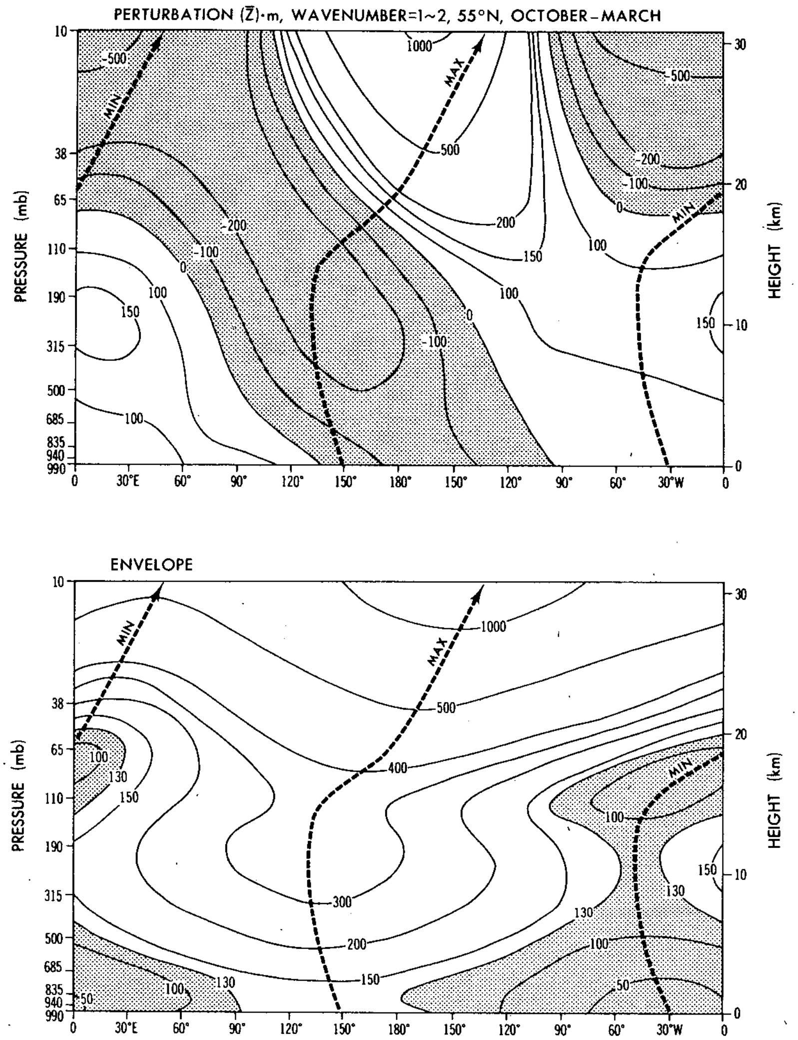

FIG. 4. As in Fig. 3 except for wavenumbers 1 ~ 2. Dashed lines indicate the position of the zonal maximum and minimum of the envelope.

Coriolis frequency, β its latitudinal derivative, U the constant zonal flow and g the gravitational acceleration.

In the absence of thermal forcing the equivalent depth h is related to the vertical wavenumber m in an isothermal atmosphere as

$$m^2 = \kappa/(Hh) - 1/(4H^2), \tag{6}$$

where H is the scale height and $\kappa = (C_p - C_v)/C_p$.

The zonal (G_x) and vertical (G_z) group velocities of Rossby waves are given by

$$G_x = \partial\sigma/\partial k \tag{7a}$$

$$= U - \beta \cdot \frac{-k^2 + l^2 + f^2/(gh)}{[k^2 + l^2 + f^2/(gh)]^2} \tag{7b}$$

and

$$G_z = \partial\sigma/\partial m \tag{8a}$$

$$= \frac{2f^2N^{-2}\beta km}{[k^2 + l^2 + f^2/(gh)]^2}, \tag{8b}$$

where N is the Brunt-Väisälä frequency for an isothermal atmosphere given by $N^2 = g\kappa/H$.

For stationary Rossby waves ($\sigma = 0$), the dispersion relation (5) gives

$$f^2/(gh) = \beta/U - (k^2 + l^2). \tag{9}$$

Insertion of (9) into (7b) and (8b) gives

$$G_x = 2(kU)^2\beta^{-1}, \tag{10}$$

$$G_z = 2kmU^2f^2N^{-2}\beta^{-1}, \tag{11}$$

with

$$G_z/G_x = f^2N^{-2}(m/k), \tag{12}$$

where m is related to k, l, U as

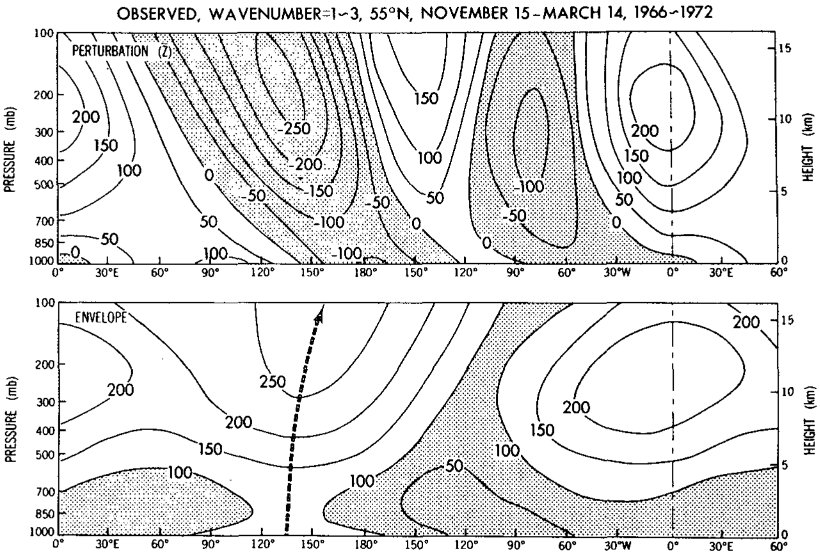

FIG. 5. Longitude-height distribution (55°N) of the observed time-mean geopotential height (upper) and its envelope (lower) consisting of wavenumber 1 ~ 3 during 15 November–14 March averaged over the years 1966–72 (NMC analysis). Dashed line indicates the position of the zonal maximum of the envelope.

$$(f^2/N^2)[m^2 + 1/(4H^2)] = \beta/U - (k^2 + l^2). \quad (13)$$

Fig. 6 illustrates G_x, G_z, and the ratio G_z/G_x of stationary Rossby waves as a function of k and U for a meridional wavelength $L = 10^4$ km, $f = 1.0 \times 10^{-4}$ s^{-1}, $\beta = 1.6 \times 10^{-11}$m^{-1} s^{-1}, at 45° and $H = 7.07$ km. The zonal group velocity (Fig. 6a) is always eastward[1] and increases with k and U. The vertical group velocity (Fig. 6b) attains its maximum (3.3 km day^{-1}) at wavenumber 2.5 and $U = 14$ m s^{-1}. Nevertheless, the direction of the group velocity (Fig. 6c) becomes monotonically more horizontal as U increases. It should be noted that the wavenumber 3 component is vertically trapped for $U > 17$ m s^{-1}, since m^2 in (13) becomes negative.

As will be discussed in the Appendix, the amplitude modulation (envelope) of a stationary wave packet consisting only of two wavenumbers (k_1 and k_2) is largest or smallest in the direction given exactly by

$$-(k_2 - k_1)/(m_2 - m_1) = 0.44 \text{ km deg}^{-1}, \quad (14)$$

for wavenumbers 1 and 2 and $U = 20$ m s^{-1}.

This value is in good agreement with the direction of the group velocity given by (12) as

$$G_z/G_x = 0.53, 0.47, 0.42 \text{ km deg}^{-1}, \quad (15)$$

for wavenumbers 1.4, 1.5, 1.6, respectively.

[1] The zonal group velocity of traveling Rossby waves can be westward.

The above agreement is due to the fact that the following finite-difference approximation of $\partial m/\partial k$ holds well for planetary waves if $\Delta k \ll 2\partial m/\partial k (\partial m^2/\partial k^2)^{-1}$, even if the WKB approximation ($k \gg \Delta k$) may not hold well for wavenumbers 1–2 ($k/\Delta k = 3$):

$$G_z/G_x = [\partial\sigma/\partial m]_k/[\partial\sigma/\partial k]_m = -[\partial k/\partial m]_\sigma$$
$$\approx -(k_2 - k_1)/(m_2 - m_1), \quad (16)$$

provided that the two wavenumbers are associated with the same frequency.

The upward-eastward group velocity of the theoretical planetary waves may explain why the maximum of the envelope of simulated and observed stationary waves consisting of wavenumbers 1 and 2 shifts eastward with height in the stratosphere. In the lower troposphere, however, this maximum shifts slightly westward with height, contrary to the group velocity. This meandering may be due to the interference with the downward propagating component which is partially reflected by the stratospheric shear [see Sato (1974) and Bates (1977) for observational and theoretical evidence of this reflection]. It also may be due to the thermal excitation in the troposphere.

It should be remarked that the zonal-vertical refractive index cannot be defined, whereas the meridional-vertical refractive index can be defined as in Matsuno (1970). This is because the wave equation [see Eq. (9) of Matsuno (1970)] is not elliptic with respect to the zonal and vertical coordinates.

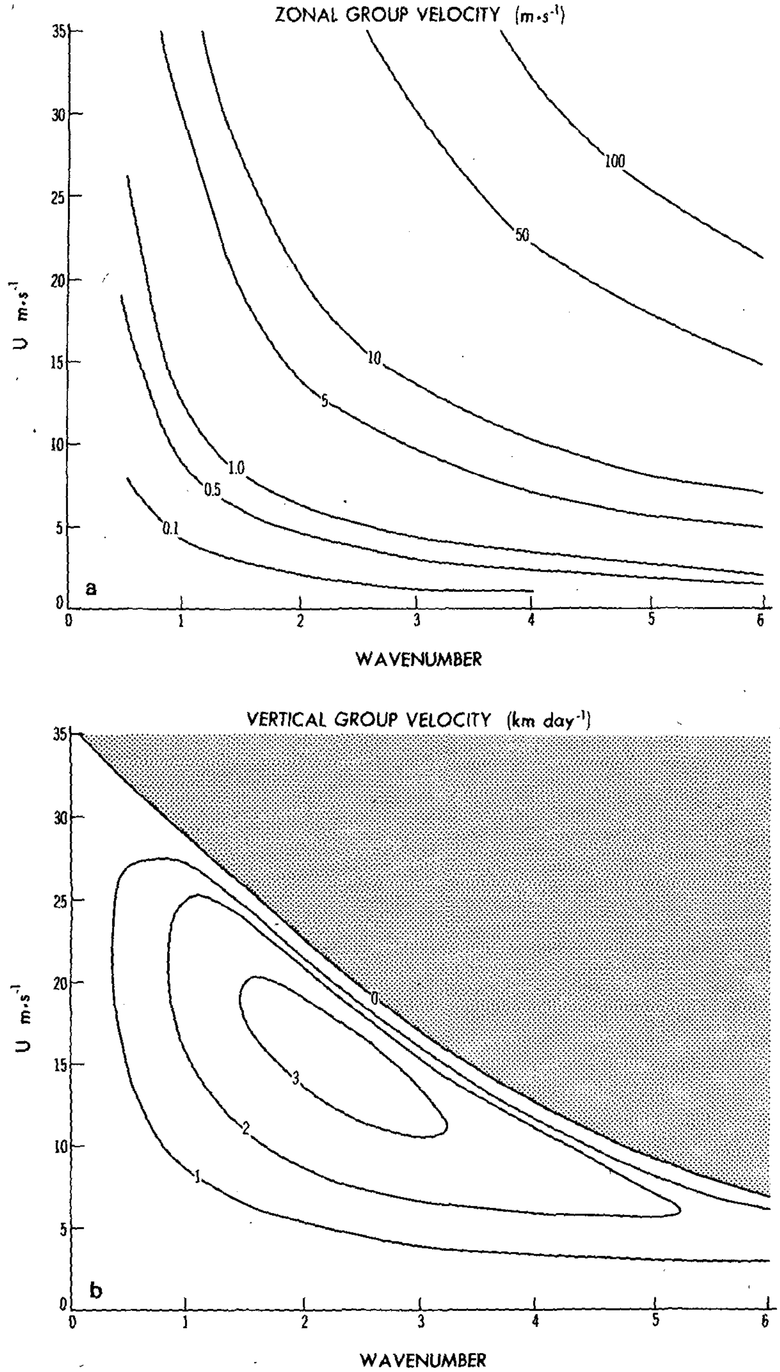

FIG. 6. Isolines of zonal (a), vertical (b) group velocities and the ratio (c) for stationary Rossby waves as a function of zonal wavenumber and zonal flow U (m s^{-1}) for a meridional wavelength of 10^4 km. Units of the isopleths are m s^{-1}, km day^{-1} and km deg^{-1}, respectively. The shading indicates vertical trapping.

4. Conclusions and remarks

Based on a complex Fourier analysis of stationary planetary waves simulated by a GFDL general circulation model and partly corroborated by observed data, the following conclusions have been obtained:

1) In the troposphere the envelope of the time-mean geopotential height consisting of wavenumbers $1 \sim 3$ attains its major and minor maxima in the Pacific and Atlantic, respectively. The major maxi-

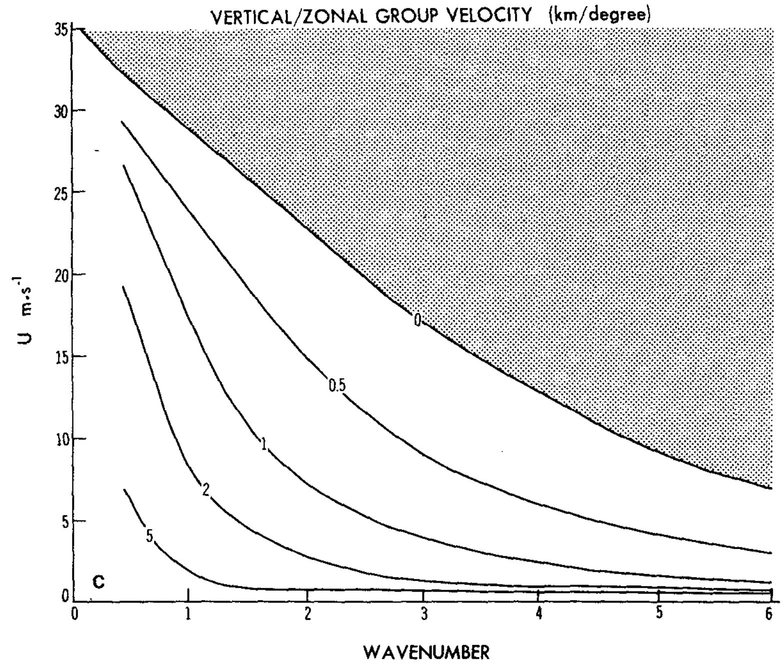

FIG. 6. (*Continued*)

mum extends from the tropopause into the stratosphere and strengthens the Aleutian high.

2) In the stratosphere the major maximum is dominated by wavenumbers 1 ~ 2 and shifts eastward with height in the approximate direction of the group velocity. The minor maximum is dominated by the wavenumber 3 component and is confined in the troposphere.

3) The maximum of the envelope consisting only of wavenumbers 1 and 2 shifts slightly westward with height in the troposphere and reverses its direction in the stratosphere. This meandering is probably due to the interference with the vertically reflected wave component.

The above conclusions should be confirmed by use of theoretical models with a realistic mean state, general circulation models with high resolution in the upper stratosphere and observed data of the upper atmosphere. The waveguide of midlatitude planetary waves may incline equatorward with height in the upper stratosphere as inferred from linear models (Dickinson, 1968; Matsuno, 1970; Simmons, 1974; Schoeberl *et al.*, 1979).

It was difficult to make a clear identification of the lateral ray path of stationary planetary waves toward the subtropical jet, although this ray path is inferred from the equatorward energy flux which was found by Hayashi and Golder (1977) in the same GFDL model and corroborated observationally by Lau (1977), Sato (1980) and Edmon *et al.* (1980). This is probably due to the fact that tropospheric planetary waves take the form of a quasi-normal meridional mode as discussed theoretically by Tung (1979).

It would be of interest to theoretically study whether the meandering of the zonal-vertical "ray path" is due to the reflection from stratospheric shear or due to tropospheric thermal forcing and to

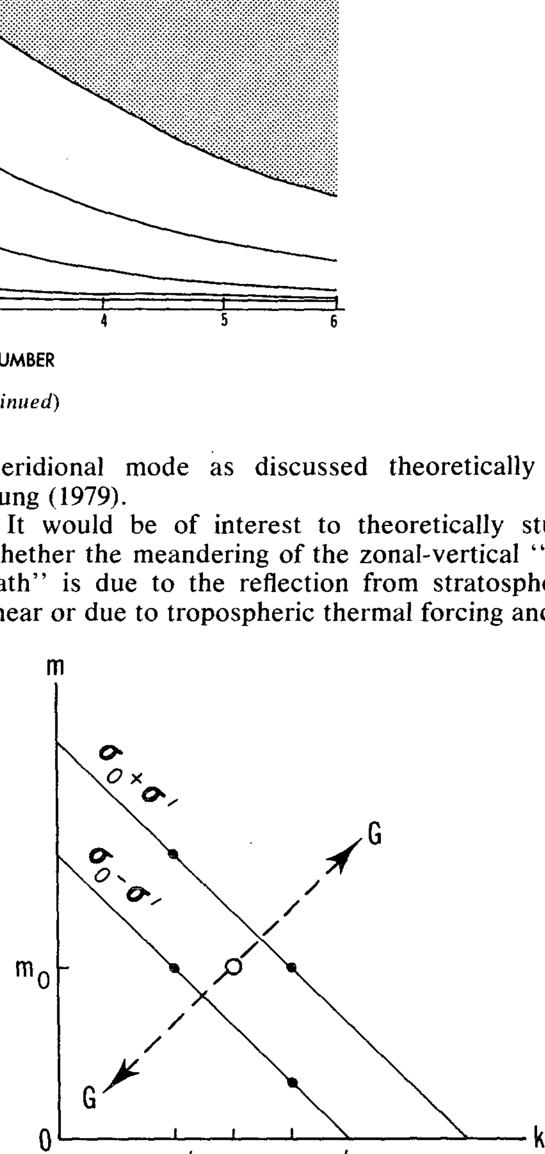

FIG. 7. Two-dimensional wavenumber distribution of a wave group consisting of four combinations of wavenumber-frequency components ($k_0 \pm k'$, $\sigma_0 \pm \sigma'$). Solid circles indicate two-dimensional wavenumber vectors, while the open circle indicates the central wavenumber vector. The slanted lines represent isopleths of frequency. The arrow shows the direction (perpendicular to the isopleths) of group velocity when the wavenumber axes are regarded as spatial axes.

examine observationally whether this meandering is affected by the upper westerlies which vary from year to year.

It also would be of interest to examine both theoretically and observationally how the stratospheric pressure pattern is affected by the tropospheric blocking anticyclone. According to Austin (1980) the blocking is accompanied by an anomalous increase in the amplitude of wavenumbers $1 \sim 4$, while their phases remain normal. The anomalous wavenumber $1 \sim 2$ components should influence the stratospheric pressure patterns as is the case with the sudden warming phenomena discussed theoretically by Matsuno (1971).

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APPENDIX

Two-Dimensional Wave Group with a Single Frequency

This appendix proves that a wave group with a single frequency attains its largest amplitude modulation in the approximate direction of the group velocity.

A simple example of a two-dimensional wave group w is given by four combinations of discrete wavenumber-frequency components ($k_0 \pm k'$, $\sigma_0 \pm \sigma'$) illustrated by Fig. 7 as

$$w = \mathrm{Re}[\sum_{k',\sigma'} W_{k,\sigma} \exp[i(k'x + m'z - \sigma't)] \times \exp(i\psi_0)], \quad \text{(A1)}$$

where (k', m', σ') is the deviation from the central wavenumber-frequency (k_0, m_0, σ_0) and

$$\psi_0 = k_0x + m_0z - \sigma_0t. \quad \text{(A2)}$$

The above example differs from an example given by Eckart (1961) in that m is not specified but determined by given k and σ through the dispersion relation. m' can be approximately determined through the first-order Taylor expansion[2] of the dispersion relation given by

$$\sigma' \approx G_xk' + G_zm', \quad \text{(A3)}$$

where G_x and G_z are the zonal and vertical components of group velocity defined by

$$(G_x, G_z) = (\partial\sigma/\partial k, \partial\sigma/\partial m). \quad \text{(A4)}$$

Elimination of m' between (A1) and (A3) gives

$$w \approx \mathrm{Re}[\sum_{k',\sigma'} W_{k,\sigma} \exp[ik'(x - zG_x/G_z) + i\sigma'(z/G_z - t)] \exp(i\psi_0)]. \quad \text{(A5)}$$

Thus the modulation is approximately constant on a ray point moving with the group velocity as[3]

$$(x, z) = (G_x, G_z)t + (x_0, z_0). \quad \text{(A6)}$$

If a wave group is forced with a single frequency, it will ultimately be associated with the same single frequency in a steady state. In this limit ($\sigma' \to 0$), Eqs. (A1) and (A5) are reduced to

$$w = \mathrm{Re}[\sum_{k'} W_{k,\sigma} \exp[i(k'x + m'z)] \exp(i\psi_0)], \quad \text{(A7)}$$

$$w \approx \mathrm{Re}[\sum_{k'} W_{k,\sigma} \exp[ik'(x - zG_x/G_z)] \exp(i\psi_0)]. \quad \text{(A8)}$$

Thus the modulation of a wave group with a single frequency is *exactly* constant for *two* wavenumber components along

$$z = -(k'/m')x + \text{constant}, \quad \text{(A9)}$$

and is *approximately* constant for *multiple* wavenumber components in the direction of the group velocity given by

$$z = (G_z/G_x)x + \text{constant}, \quad \text{(A10)}$$

since $-k'/m'$ is not exactly the same for different components but is approximately equal to G_z/G_x provided that σ is fixed.

In a *nonuniform medium the amplitude modulation is not constant but is largest or smallest in the approximate direction of the group velocity.*

For a wave group with multiple frequencies the time variance of (A1) averaged over a long time interval is equal to the frequency integration of the time variance of a wave group with a single frequency given by

$$\overline{w^2} = \frac{1}{2} \sum_{\sigma'} \left|\sum_{k'} W_{k,\sigma} \exp[i(k'x + m'z)]\right|^2. \quad \text{(A11)}$$

Thus the time variance of a traveling wave group is

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[2] This expansion can be rewritten as follows:

$$\sigma'/k' \approx G_x + G_z \tan\theta,$$

$$\sigma'/(k'^2 + m'^2) \approx G_x \cos\theta + G_y \sin\theta,$$

where $\tan\theta = m'/k'$.

[3] In particular, the modulation of two wavenumber-frequency components is exactly constant on a point moving in any direction with the velocity $[(\sigma'/2 + \alpha)/k', (\sigma'/2 - \alpha)/m']$, where α is an arbitrary constant.

approximately constant in the direction of the group velocity (A10) provided that the frequency band is narrow.

The time power spectrum of transient disturbance consisting of multiple wavenumbers can be partitioned into standing and traveling wave parts by use of a space-time cross spectral method developed by Hayashi (1979).

REFERENCES

Austin, J. F., 1980: The blocking of middle latitude westerly winds by planetary waves. *Quart. J. Roy. Meteor. Soc.*, **106**, 327–350.

Bates, J. R., 1977: Dynamics of stationary ultra-long waves in middle latitudes. *Quart. J. Roy. Meteor. Soc.*, **103**, 397–430.

Boville, B. W., 1960: The Aleutian stratospheric anticyclone. *J. Meteor.*, **17**, 329–336.

Bretherton, F. P., 1971: The general linear theory of wave propagation. *Lectures in Applied Mathematics*, Vol. 13, Amer. Math. Soc., 61–102.

Charney, J. G., 1949: On a physical basis for numerical prediction of large-scale motions in the atmosphere. *J. Meteor.*, **6**, 371–385.

——, and P. G. Drazin, 1961: Propagation of planetary-scale disturbances from the lower into the upper atmosphere. *J. Geophys. Res.*, **66**, 83–109.

Dickinson, R. E., 1968: Planetary Rossby waves propagating vertically through weak westerly wind waveguides. *J. Atmos. Sci.*, **25**, 984–1002.

——, 1969: Vertical propagation of planetary Rossby waves through an atmosphere with Newtonian cooling. *J. Geophys. Res.*, **74**, 929–938.

Eckart, C., 1960: *Hydrodynamics of Oceans and Atmospheres.* Pergamon Press, 290 pp.

Edmon, H. J., Jr., B. J. Hoskins and M. E. McIntyre, 1980: Eliassen-Palm cross sections for the troposphere. *J. Atmos. Sci.*, **37**, 2600–2616.

Hare, F. K., 1960: The disturbed circulation of the Arctic stratosphere. *J. Meteor.*, **17**, 36–51.

Hayashi, Y., 1979: A generalized method of resolving transient disturbances into standing and traveling waves by space-time spectral analysis. *J. Atmos. Sci.*, **36**, 1017–1029.

——, and D. G. Golder, 1977: Space-time spectral analysis of midlatitude disturbances appearing in a GFDL general circulation model. *J. Atmos. Sci.*, **34**, 237–262.

Holton, J. R., 1972: Waves in the equatorial stratopshere generated by tropospheric heat sources. *J. Atmos. Sci.*, **29**, 368–375.

Hoskins, B. J., A. J. Simmons and D. G. Andrews, 1977: Energy dispersion in a barotropic atmosphere. *Quart. J. Roy. Meteor. Soc.*, **103**, 553–567.

Lau, N.-C., 1979: The observed structure of tropospheric waves and the local balance of vorticity and heat. *J. Atmos. Sci.*, **36**, 996–1016.

Longuet-Higgins, M. S., 1964: Planetary waves on a rotating sphere I. *Proc. Roy. Soc. London*, **279**, 446–473.

——, 1964b: On group velocity and energy flux in planetary wave motions. *Deep-Sea Res.*, **11**, 35–42.

——, 1965: Planetary waves on a rotating sphere II. *Proc. Roy. Soc. London*, **284**, 40–68.

Manabe, S., and J. D. Mahlman, 1976: Simulation of seasonal and interhemispheric variations in the stratospheric circulation. *J. Atmos. Sci.*, **33**, 2185–2217.

Matsuno, T., 1970: Vertical propagation of stationary planetary waves in the winter Northern Hemisphere. *J. Atmos. Sci.*, **27**, 871–883.

——, 1971: A dynamical model of the stratospheric sudden warming. *J. Atmos. Sci.*, **28**, 1479–1494.

Rossby, C. G., 1945: On the propagation of frequencies and energy in certain types of oceanic and atmospheric waves. *J. Meteor.*, **2**, 187–203.

——, 1949: On the dispersion of planetary waves in a barotropic atmosphere. *Tellus*, **1**, 54–58.

Sato, Y., 1974: Vertical structure of quasi-stationary planetary waves in several winters. *J. Meteor. Soc. Japan*, **52**, 272–282.

——, 1980: Observational estimates of Eliassen and Palm flux due to quasi-stationary planetary waves. *J. Meteor. Soc. Japan*, **58**, 430–435.

Sawyer, J. S., 1964: Dynamical aspects of the bi-polar stratospheric circulation. *Quart. J. Roy. Meteor. Soc.*, **90**, 395–404.

Schoeberl, M. R., M. A. Geller and S. K. Avery, 1979: The structure of stationary planetary waves in winter: A correction. *J. Atmos. Sci.*, **36**, 365–369.

Schopf, P. S., D. L. T. Andersen and R. Smith, 1981: Beta-dispersion of low-frequency Rossby waves. *Dyn. Atmos. Oceans.*, **5**, 187–214.

Simmons, A. J., 1974: Planetary-scale disturbances in the polar winter stratosphere. *Quart. J. Roy. Meteor. Soc.*, **100**, 76–108.

Teweles, S., L. Rothenberg and F. G. Finger, 1960: The circulation at the 10-millibar constant pressure surface over North America and adjacent ocean areas: July 1957 through June 1958. *Mon. Wea. Rev.*, **88**, 137–150.

Tung, K. K., 1979: A theory of stationary long waves. Part III: Quasi-normal modes in a singular waveguide. *Mon. Wea. Rev.*, **107**, 751–774.

Wilson, C. V., and W. L. Godson, 1963: The structure of the Arctic winter stratosphere over a 10-year period. *Quart. J. Roy. Meteor. Soc.*, **89**, 205–224.

Yeh, T. C., 1949: On energy dispersion in the atmosphere. *J. Meteor.*, **6**, 1–16.