NATIONAL CENTER FOR EDUCATION STATISTICS

Essential Skills in Mathematics

A Comparative Analysis of American and Japanese Assessments of Eighth-Graders

John A. Dossey
Department of Mathematics
Illinois State University

Lois Peak and Dawn Nelson, Project Officers
National Center for Education Statistics

U.S. Department of Education
Office of Educational Research and Improvement

NCES 97-885

For sale by the U.S. Government Riming Office
Superintendent of Documents, Mail Stop SSOP, Washington, DC 20402-9328

U.S. Department of Education
Richard W. Riley
Secretary

Office of Educational Research and Improvement
Marshall S. Smith
Acting Assistant Secretary

National Center for Education Statistics
Pascal D. Forgione, Jr.
Commissioner

The National Center for Education Statistics (NCES) is the primary federal entity for collecting, analyzing, and reporting data related to education in the United States and other nations. It fulfills a congressional mandate to collect, collate, analyze, and report full and complete statistics on the condition of education in the United States; conduct and publish reports and specialized analyses of the meaning and significance of such statistics; assist state and local education agencies in improving their statistical systems; and review and report on education activities in foreign countries.

NCES activities are designed to address high priority education data needs; provide consistent, reliable, complete, and accurate indicators of education status and trends; and report timely, useful, and high quality data to the U.S. Department of Education, the Congress, the states, other education policymakers, practitioners, data users, and the general public.

We strive to make our products available in a variety of formats and in language that is appropriate to a variety of audiences. You, as our customer, are the best judge of our success in communicating information effectively. If you have any comments or suggestions about this or any other NCES product or report, we would like to hear from you. Please direct your comments to:

National Center for Education Statistics
Office of Educational Research and Improvement
U.S. Department of Education
555 New Jersey Avenue NW
Washington, DC 20208–5574

April 1997

The NCES World Wide Web Home Page is
http://www.ed.gov/NC ES/

Suggested Citation
U.S. Department of Education. National Center for Education Statistics. *Essential Skills in Mathematics: A Comparative Analysis of American and Japanese Assessments of Eighth-Graders*, NCES 97–885, by John A. Dossey. Lois Peak and Dawn Nelson, Project Officers. National Household Education Survey, Washington, DC: 1997.

Each year the National Center for Education Statistics (NCES) commissions papers by recognized experts on topics of interest. These papers are published by NCES to preserve the information contained in these documents and to promote the sharing of valuable work experience, unique expertise, and knowledge. However, these papers do not undergo vigorous NCES publication review and editing prior to their publication. The conclusions, opinions, and/or recommendations contained in the papers are those of the authors and do not necessarily reflect the views of NCES or the U.S. Department of Education. Consequently, we encourage readers of the papers to contact the individual authors regarding questions and for citations.

Contact:
Dawn Nelson
(202) 219-1740

Acknowledgments

The present report was completed in 1996 and is the result of significant input from a number of individuals. Lois Peak at the National Center for Education Statistics (NCES) initiated the study and suggested important frames of reference throughout the early stages of its development.

Along the way, I profited by continued input from Sayuri Takahira who carefully translated the Japanese results and provided interpretive guidance on exact meanings of several terms. Colleagues, including Eizo Nagasaki, Toshio Swada, and Hanako Senuma, at the National Institute for Educational Research at the Ministry of Education in Tokyo, Japan, also provided input and helpful assistance at points in the analysis of results and interpretations of the curriculum in Japan. Similar input was received from Steve German and Sharif Shakrani who reviewed the document at NCES and from Mary M. Lindquist, Callaway Professor of Math Education at Columbus State University.

Ina Mull is (formerly at the National Assessment of Educational Progress Center at Educational Testing Service (ETS) and now at Boston College) and Chan Jones and Jeff Haberstroh at ETS provided support in identifying and classifying comparable items from the NAEP item pool for comparative analysis. Pat Kenney of the University of Pittsburgh's Learning Research and Development Center's NCTM/NAEP Interpretive Project provided helpful assistance in tracking down elusive p-values for several items.

Dawn Nelson at NCES carefully guided the report through its final development and publication process at NCES. And last, but not least, my wife Anne assisted through data analysis, proof-reading, and provided other forms of support across the span of the project.

Contents

Acknowledgments iii

Chapter 1: Background and Purpose 1

- Introduction 1
- Data Sources 2
- Format of the Studies 2

Chapter 2: The Mathematics Assessments 7

- Japanese Content Tests 8
- NAEP Content Tests 9

Chapter 3: The Items 13

- Numbers and Operations 13
- Measurement 14
- Geometry 15
- Data Analysis, Statistics, and Probability 24
- Algebra and Functions 26

Chapter 4: A Retrospective View 37

- The Assessments 37
- The Curriculum 38
- Focus and Intensity 39

References 41

Appendix A: Content Matrix for the 1992 NAEP Mathematics Assessment 43

Appendix B: Mathematical Abilities for the 1992 NAEP Mathematics Assessment . . . 53

Appendix C: Framework for the Japanese Essential Mathematics Assessment 57

Chapter 1: Background and Purpose

Introduction

The definition of essential skills in mathematics has long been a goal of mathematics educators and others interested in the school mathematics curriculum. From the late 1970s forward, attempts have been made in the United States to provide a framework defining the basic essentials of mathematics that all students should know and be able to apply (National Council of Supervisors of Mathematics, 1977; National Council of Teachers of Mathematics, 1980, 1981). Such recommendations for new directions in school mathematics called for a broader view of content and, in general, an increased emphasis on the development of student abilities to solve non-routine problems. These changes brought incremental changes in classrooms where the overall focus remained on number work and teacher dominated discussion of mathematics. In the late 1980s, the National Council of Teachers of Mathematics (NCTM) produced its *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). This document was designed to help strengthen the efforts in the United States toward developing foci for the mathematics curriculum. While its goals have yet to be implemented on a wide scale basis (Weiss, 1995), it has influenced changes made in the definition of mathematics used for the Department of Education's National Assessment of Education Progress (NAEP) mathematics assessments (NAEP, 1987, 1988).

Over this same period of time, from the 1970s through the 1980s, there was increasing evidence that the traditional mathematics curriculum was not preparing U.S. students to the same level of understanding as the Japanese curriculum was preparing students in Japanese schools (Husén, 1967; McKnight, et al., 1987; Travers & Westbury, 1989; Robitaille & Garden, 1989). Although the Second International Mathematics Study (SIMS) was unable to locate specific factors accounting for the differences in U.S. and Japanese students' performance by the junior high school level, there was strong circumstantial evidence that the outmoded and unfocused mathematics curricular program of the early grades combined with the ways in which mathematics is taught in the elementary and middle grades was a probable cause (Leestma & Walberg, 1992; Stevenson, et al., 1990; Stigler, et al, 1990: Stigler & Perry, 1988). Specific evidence indicated that students in Japanese classrooms had greater opportunities to learn and were often expected to master mathematical topics one- to two-years ahead of the time that American students first encountered similar topics. However, great emphasis was also placed on the differences in child-rearing and in the early educational experiences of children in the two societies as significant factors in creating the differences in mathematical performance.

Studies comparing the mathematical achievement, or proficiency, of the two nations' students have been hard to interpret. Studies by the International Association for the Evaluation of Educational Achievement (IEA) have provided one vantage point. However, many critics contend that the IEA analyses are based on tests that contain items which not all students have had equal opportunities to learn in their schooling. The IEA tests are aimed to measure across a broad framework of mathematical skills and abilities, but invariably measure items included in neither country's curriculum. Analyses indicated that the IEA tests were probably equally disadvantageous to each group of students. In the Second International Mathematics Study, the comparison groups of students were Japan's seventh-graders and the United States' eighth-graders. Studies carried out by Stevenson and Stigler and their associates measured student achievement with tests better fitted to the curricula studied by Japanese and American students, but their work only included students through the fifth grade level. In the early 1990s, new sets of data became available that made another form of comparison possible between Japanese and American eighth-graders. This was data from assessments given in each country, using tests developed via governmental agencies. In Japan, the test was developed by the Japanese National Institute of Educational Research and administered to a sample of eighth-graders in schools chosen according to some criteria. In the United States, the test administered to a random sample of eighth-graders was developed for the National Assessment of Educational Progress (NAEP). In both cases, these tests were developed to mirror the basic curricula currently being offered to students in the two countries. Unlike other comparative studies, these tests were developed to assess the essential components of the taught curriculum in each country. During this period of time, both countries were undergoing significant changes in their school mathematics programs (Ministry of Education, n.d.; NIER, 1992; National Council of Teachers of Mathematics, 1989; NAEP, 1988).

In each country, students were assessed using examinations designed specifically for students in their country. The present paper compares the nature of these examinations, the expectations based on the curriculum, and student performances on the included items. This comparison, combined with an accompanying analysis of the curricular intents for students of grade two in the lower secondary school in Japan, the equivalent of grade eight in American schools, and students in grade eight in the United States provides a rich picture of the differences in student performance and curricular emphases and expectations that mark this study. From this point forward, we shall refer to both groups as eighth-graders.

Data Sources

The study makes use of data drawn from the Japanese National Institute of Educational Research's Special Study on Essential Skills in Mathematics (NIER, 1992) and data from U.S. student performance on the 1990 and 1992 NAEP mathematics assessment (Mullis, et al., 1991, 1993). These data provide the first comparison of Japanese eighth-graders and American eighth-graders since the data collected during 1964 in the First International Mathematics Study (Husén, 1967). However, in this case, the performance of each group of students is measured on a different set of items developed in their own country.

Format of the Studies

Each of the studies described in the following sections examined content achievement assessments, students' and teachers' beliefs and attitudes, as well as environmental characteristics of the classroom, school, and community. While the full nature of the studies is briefly detailed to understand the context of the data collection, the following analyses are limited to the:

- comparison of mathematics items tested on the Japanese NIER and American NAEP mathematics examinations, and
- comparison of items to the espoused curriculum that each student was to have studied.

In doing so, the NAEP item and ability classifications will be used to describe items of content and their intent (NAEP, 1988).

Japanese Study. The Japanese data result from a study conducted by the National Institute of Educational Research to develop a definition of "essential achievement level" in mathematics and to determine the variables which may influence the formation of essential achievement. To accomplish this purpose, the study investigated the relationships between mathematics achievement and a number of variables, including:

- Student attitudes: Motivation, interest, self assessment of performance or achievement, plans for the future, purpose of the studying, etc.
- School environment: Teacher attitudes (teaching, teacher training, and specialization), school characteristics (number of teachers in each subject, number of students, number of classrooms, and frequency of use).
- Environment outside school: Population of the town or city, distinction between urban and rural areas (NIER, 1992, p. 1).

This study encompassed the areas of mathematics, Japanese language, social studies, science, and English. The data examined in the present paper result from the 1990 testing of eighth-graders in mathematics. The actual assessment instruments were constructed by teachers from public schools, university faculty, and investigators from the ministry of education.

The resulting examinations, to be described later, were administered to a nationally representative sample of students selected from schools in eight prefectures and two cities, see figure 1. The prefectures were Hokkaido, Iwate Ken, Chiba Ken, Toyama Ken, Shiga Ken, Okayama Ken, Kagawa Ken, and Kagoshima Ken. The cities were Tokyo and Nagoya. Four schools were selected in each of the cities and four schools were selected in each prefecture. In each prefecture, one school was selected from each of the following areas:

- capital city of prefecture;
- suburbs of each capital city;
- city or town with population less than 100,000; and
- areas neighboring city or town with population less than 100,000 (NIER, 1992, p. 2).

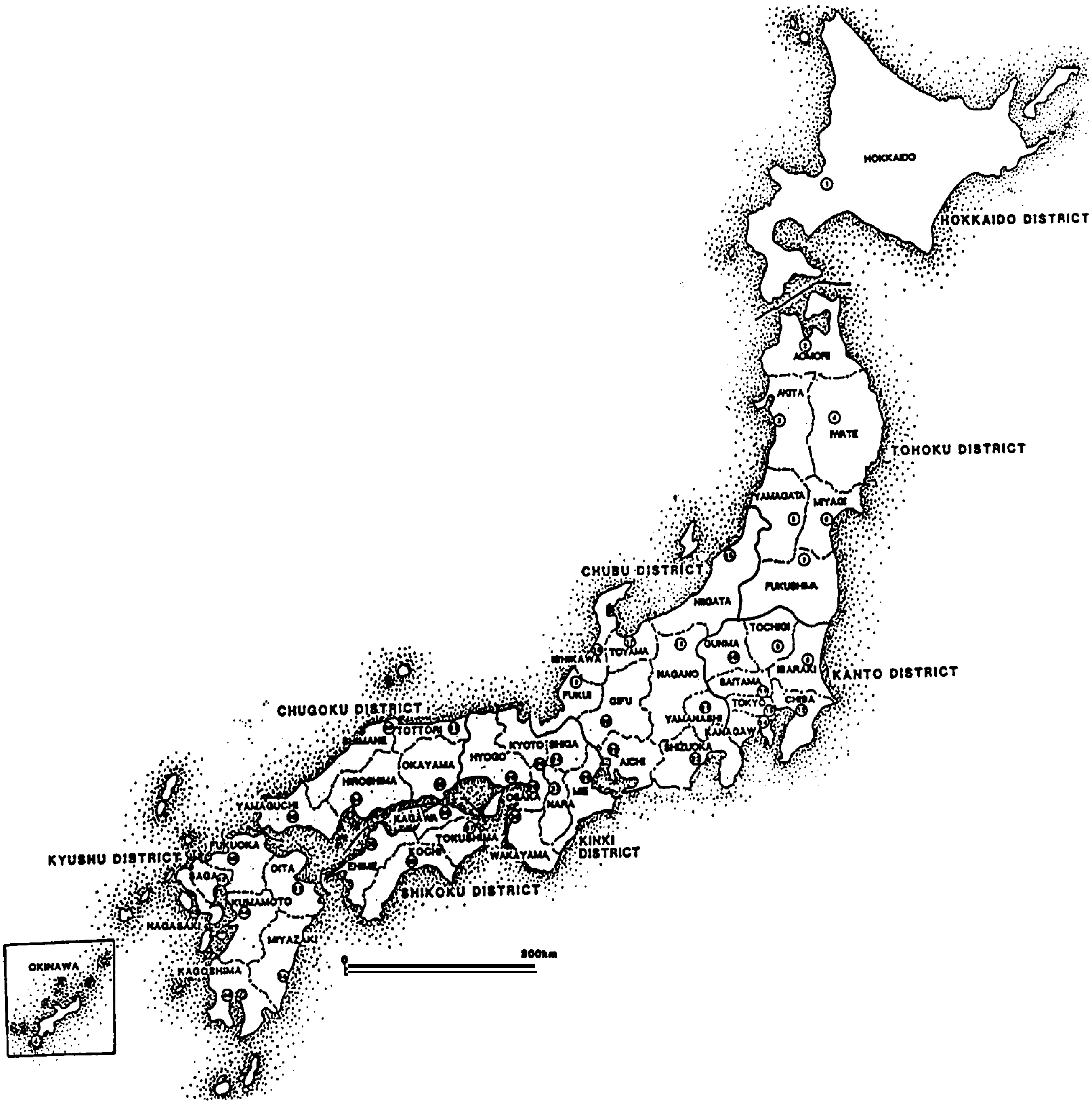
In each school selected, one eighth-grade classroom was selected and all students in that class served as subjects in the study for that school site. This resulted in 40 classrooms of students with 1,441 students for an average class size of 36 students.

The administration of assessment instruments was done by the students' classroom teachers. For the eighth-grade mathematics assessment, there were two forms of the mathematics achievement instrument and a questionnaire for both students and teachers to acquire background information on student attitudes and environmental variables, as described above. These instruments were completed in two successive class periods. During the first session, students were given 45 minutes to complete the 22 mathematics achievement items on their tests. This gave Japanese students about twice the amount of time per mathematics item as given American students on the NAEP examination. During the following session, students were given 40 minutes to complete the student background questionnaires concerning their attitudes and other relevant family and environmental information. This assessment took place in a time period between January 21, 1990 and March 20, 1990.

United States Study. The NAEP mathematics assessments have been given on a somewhat regular schedule since 1973, with assessments in 1973, 1978, 1982, 1986, 1990, and 1992. These assessments have measured the performance of U.S. students at both ages 9, 13, and 17, and grades 4, 8, and 12. Assessments are given in a wide variety of areas including mathematics, science, social studies, reading, and writing. The data collected are used to provide a rich mosaic picture of the current status of schooling in the United States. Achievement tests in the subject areas provide a picture of the proficiency levels of students at the respective ages or grades. They also provide information about long-term trends in what students know and are able to do in particular subject-matter areas in the curriculum. The assessments also collect background data on student attitudes and beliefs, as well as demographic information about their homes, families, and mathematical backgrounds. Additional information is gathered from teachers and administrators about the classroom and school environments. This data contains background information on teachers, their teaching styles, the school curricula, and the community.

The NAEP mathematics assessment instruments are developed by a committee of mathematics teachers, university faculty, and mathematics professionals on the staff of Educational Testing Service, the contractor responsible to the government for conduct of the study. Prior to usage, the tests and questionnaires go through an approval process that includes representatives from both the policy and mathematics branches of state education agencies.

Figure 1.—Map Location of Japanese Study Prefectures and Cities



The actual sampling of students and data collection for both the 1990 and 1992 NAEP eighth-grade mathematics assessments was conducted by Westat, Inc. The three-stage sampling design for NAEP assessments consists of the selection of geographical sampling units, the random selection of schools within that geographical region, and the random selection of eighth-graders from within the chosen schools. Some students, less than 6 percent, are excluded due to limited English proficiency or severe disabilities. In both 1990 and 1992, 406 schools containing eighth-graders participated in the NAEP study. This number of schools represented an 84 percent participation rate in 1990 and an 87 percent participation rate in 1992 from among the schools randomly chosen to participate in the study. Within these schools, lists of students were again randomly sampled. In 1990, 8,888 students participated and in 1992, 9,432 students participated. In each year, the percentage of students participating represented 89 percent of the students random ly selected for participation.

Each student received a booklet containing a set of questions about his/her mathematics background, three 15-minute blocks of mathematics items, and a set of questions about his/her motivation. Students were allowed 50 minutes to respond to the three blocks of mathematics items and the background information. The mathematics items in the three blocks to which an individual student would respond were portions of a broader set of mathematics items. This broader set of items consisted of 191 items in 10 different blocks in 1990 and 235 items in 16 blocks in 1992. These items were spiraled into students' test booklets according to a balanced incomplete block design which allows for broader coverage of the mathematics content while minimizing the burden placed on any individual student. Thus, during the 45 minute mathematics examination period, an individual eighth-grader in the U.S. had approximately 57 items to respond to in 1990 and 44 items to respond to in 1992. This decrease in the average number of items per student between the assessments was accompanied by an increase in the number of items requiring students to construct their own response to the item, rather than select a response from a multiple choice question. In these years, the individual eighth-grader in the U.S. had approximately 2 to 2.5 times as many items as Japanese students were asked to do in a similar time period.

Trained field staff provided by Westat collected all of the data for the 1992 mathematics assessment. The data was collected from January, 1992 through mid-March, 1992. The materials collected were scored by National Computer Systems. This scoring included the open-ended student constructed response items included as part of the assessment. (Mullis, et al., 1991, 1993).

Chapter 2: The Mathematics Assessments

The mathematics content tests given students as part of both nations' examinations were intended to be reflective of content that was deemed appropriate for eighth-grade students in Japan. In the United States, items were chosen according to the content matrix established for the 1990 and 1992 NAEP mathematics assessments. This matrix was developed in late fall 1988 by a committee of teachers of mathematics, state supervisors of mathematics, and university faculty in mathematics education and mathematics. In doing so, the NAEP mathematics framework committee, chosen by the Council of Chief State School Officers, used a draft of the NCTM *Standards* as a guide to item specification and grade level emphasis. A copy of th is content matrix is included in appendix A. This framework divides school mathematics into five major content areas: Numbers and Operations; Measurement; Geometry; Data Analysis, Statistics, and Probability; and Algebra and Functions. The content to be included in each of these areas is rather self-explanatory. A more detailed definition of each of the areas is given in appendix A. For each subarea of content within the five groups, a darkened bullet is displayed to the right in the appendix materials indicating if that material is appropriate for assessment purposes at grade 4, 8, or 12. These judgments were developed by the NAEP mathematics framework committee and were open to comment by mathematics educators nationwide before final revisions and final adoption.

The Japanese mathematics assessment was constructed based on a framework developed specifically for the research study of which it was a part (NIER, 1992). The material accompanying the assessment materials detai ls attempts to develop a position on essent ial mathematics achievement. In attempting to move to such a definition, the researchers established the following tenets:

- Mathematics achievement extends beyond the acquisition of concepts and related skills, to include individuals' attitudes and motivations in mathematical situations.

- While mathematics problems worked in class often have little relationship to reality, individuals should be able to successfully deal with mathematics problems in their everyday lives and the classroom, by solving them and verifying their solutions.

- Individuals should be able to productively transfer their mathematical concepts and skills, as well as problem-solving abilities, from mathematics classrooms to other situations in their lives.

Using these tenets, the researchers at the Japanese National Institute of Educational Research (NIER, 1992) defined essential achievement in mathematics as:

> Essential mathematical achievement is a necessary level of achievement in order to solve problems mathematically in academic institutions and society. It includes the possibility of being able to cope with new situations. From the viewpoint of the development of students, the contents of the basic achievement level ought to make it possible for teachers to guide and teach students. The essential achievement level should be learned effectively in the school system (p.11).

In many ways, this movement to define essential achievement in mathematics for Japanese eighth-graders is similar to the attempts to define achievement levels for American students of the same age for the NAEP assessment analyses.

In structuring the mathematics assessments, the Japanese developers created a three-dimensional matrix framework. The dimensions of the matrix were behavior types, contents of mathematics, and mathematical processes. The subcategories of each are similar in some ways to the contents of the NAEP framework. The behavior types dimension was further divided into the behaviors of knowledge, understanding, thoughts, skit ls, and attitudes. The contents of mathematics dimension was divided into three branches: numerical content, geometrical content, and relational content. The third dimension, mathematical processes, was broken into the subcategories of mathematicalization, mathematical transaction, and mathematical verification. These three process categories define the dynamic use of mathematics by students as they apply it to translating problems into mathematics, using mathematical knowledge to solve the problems, and verifying that the results obtained fit the problem and answer the original questions. A further description of the Japanese framework is contained in appendix C.

Japanese Content Tests

The 40 items chosen by the research group designing the assessments for the Japanese study were placed into two forms, C and D, for ease of administration. Each form contained 22 items. Four items were included on both forms of the examination as a cross-check on the comparability of the two forms. Thus each form had 18 unique items and the set of four items used as a cross-check between Forms C and D. Students in each of the classes were divided into two groups and the members of each group received one of the forms of the test. They were given 45 minutes to complete the 22 items on the form. All, but two, of the items were multiple-choice in format. The two exceptions were an item which required students to sketch the planar net associated with a spatial perception item and another item which required students to draw a line having a special property onto a figure.

The items included in the 40-item pool reflected current thinking about the nature of mathematical essentials all students at the grade level should have. However, eight items were chosen from earlier studies: the survey of achievement level (1981-82) and the academic achievement survey which was conducted before 1981 (NIER, 1992).

Using the Japanese mathematics framework described above and in appendix C, the 40 items were categorized as shown in table 1.

Table 1.— Categorization and Percent of Items on the Japanese Test Using the Japanese Essential Mathematics Framework

Mathematical Behavior Type	n*	Mathematical Content Type	n	Mathematical Process Type	n
Skills	9				
Knowledge	14	Numerical	15	Mathematicalization	8
Understanding	15	Geometrical	14	Mathematical Transaction	32
Thoughts	11	Relational	11	Mathematical Verification	0

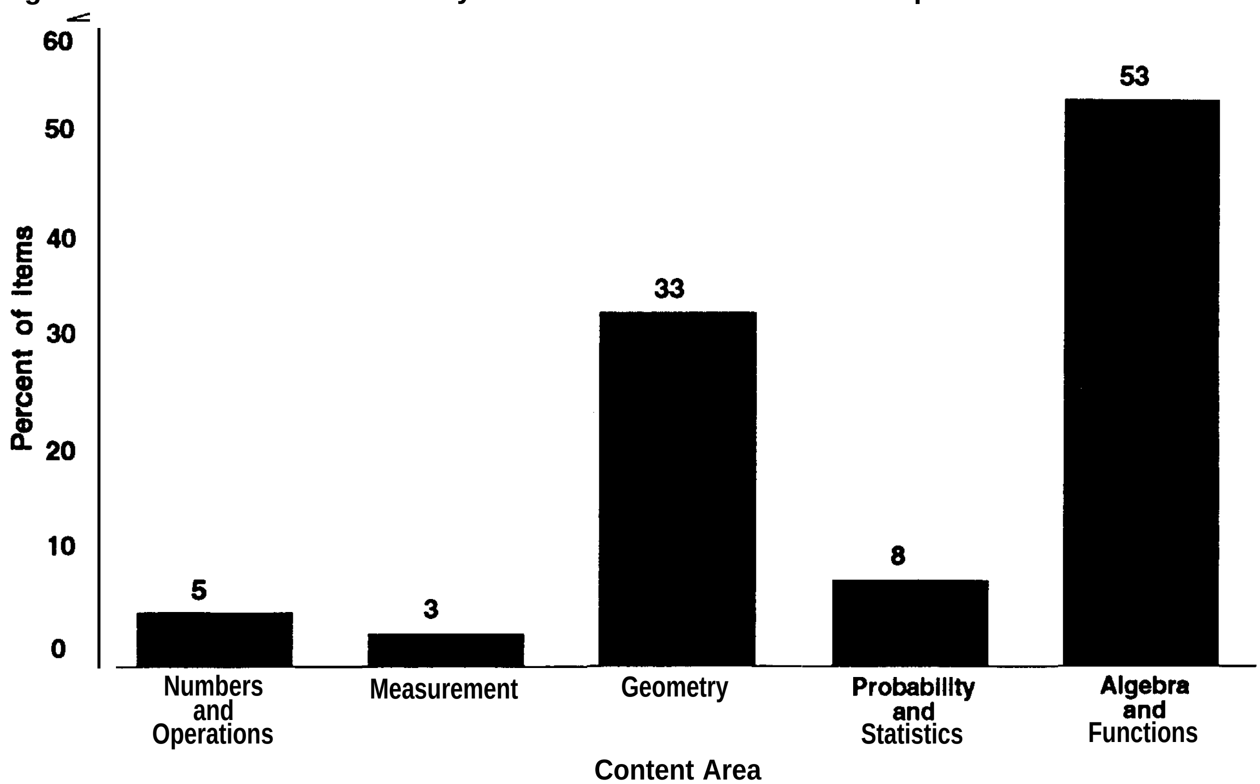
*Some items are categorized in more than one behavior type category.

An analysis of the items reveals that many skills were required in order for the students to correctly respond to some of the items measuring knowledge, understanding, and thoughts. The behavior type called "Thoughts" dealt with students ability to interpret new problem types using previously known facts, concepts, and principles. In the Mathematical Process Type dimension, no item was categorized as measuring the process of verification.

The 40 items on the Japanese examination were also categorized using the 1992 NAEP framework. This categorization was done independently by two mathematics education researchers at the NAEP offices at Educational Testing Service and the author. There was a 95 percent agreement on the categorization of items into the five major content categories and a 90 percent agreement on the categorization of the items according to mathematical ability categories.

The 40 items spread across the NAEP categories is illustrated in figure 2. One can see the heavy emphasis in the examination on items measuring geometry and algebra and functions content. This "spiked" emphasis in two content areas with a relatively low number of items in the other content areas is a curricular emphasis pattern similar to that noted in the SIMS study. Some have argued that a targeted emphasis within a year on fewer topics will potentially lead to greater growth than a broader smorgasbord of topics with smaller expectations of growth (McKnight, et al., 1987). A content emphasis comparison of Forms C and D showed that the two forms were well balanced in the distribution of items from the five NAEP content areas.

Figure 2.— Item Distribution by NAEP Content Areas for Japanese Test



NAEP Content Tests

The NAEP content tests for 1990 and 1992 were developed in accordance with the framework and item distributional requirements set out by the design developed in 1988 (NAEP, 1988). This resulted in a set of 191 items distributed across 10 test blocks in 1990 and 235 items distributed across 16 test blocks in 1992. These blocks were then reassembled into test booklets of three blocks each.

In 1990, grade eight students were allowed to use calculators on two of these blocks and a ruler and protractor on another block of items. In 1992, eighth-grade students were allowed to use calculators on three of the blocks and a ruler and protractor on one other block For another block in 1992, students were given a packet of geometric shapes to use in responding to a number of items. In both years, three blocks of items were administered through the use of a paced-audio-tape format to move students through a series of items dealing with estimation, data analysis, and algebra. The use of the paced-tape approach was chosen to forestall computations on the estimation items and to ease the reading requirements while assisting in time management on more complex items.

In 1990, 149 of the 191 items at grade 8 were multiple choice items (47 from the paced-tape blocks) and 42 items required students to construct a computed answer or short response to an item. In 1992, the 235 items at grade 8 included 164 multiple choice items (46 from the paced-tape section), 65 items requiring students to construct a computed answer or write a short response to the item, and 6 items that required an extended written response on the part of the student.

Independent of which set of three blocks an individual student received in an individual student test booklet, they had 45 minutes to respond to approximately 57 questions in 1990 and 44 questions in 1992. This was a much higher level of average number of items per student for a comparable time period than was observed in the Japanese tests. This placed from 2.5 to 2 times as much item response load per time period on individual U.S. students, plus 22 percent of the NAEP items in 1990 and 30 percent in 1992 required U.S. students to produce some form of self-constructed written response.

Figure 3 illustrates the balance of items across the five content categories for both the 1990 and 1992 NAEP mathematics assessments. The allocation of items to categories exhibits a more rectangular distribution of items across the five content categories than was observed for the Japanese tests. No "spikes" were observed, unless one would say that the Number and Operations category is perhaps a "spike."

Figure 3.— Percent of Items by NAEP Content Areas for the 1990 and 1992 NAEP Tests

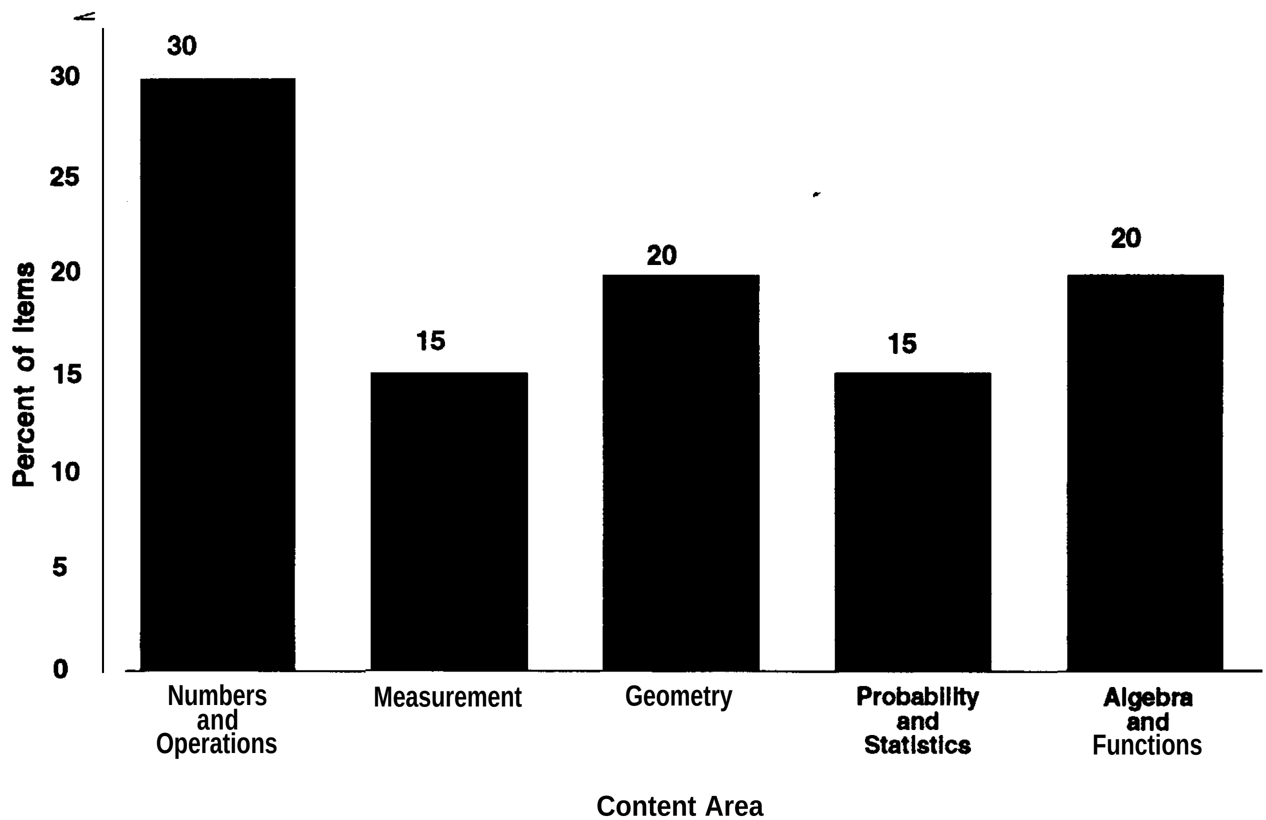


Figure 4 provides a direct comparison of the allocation of items to the five NAEP content categories across the two examinations. Here one sees immediately the pattern of a "spiked" approach which gives a heavy emphasis to a few areas in deference to others. The Japanese model provides heavy coverage of student achievement in the areas of geometry and algebra, but provides little coverage for the other three areas. This undoubtedl y signals what is important to the students and teachers when the examinations come from the ministry offices. In a like manner, the U.S. examinations may be signaling that numbers and operations should be the main focus of our curriculum for eighth-graders. This appeared to be one of the findings of the SIMS study (McKnight, et al, 1987).

Figure 4.— Comparison of the Distribution of Items to Content Areas in the Japanese Essential Skills and the U.S. NAEP Examination

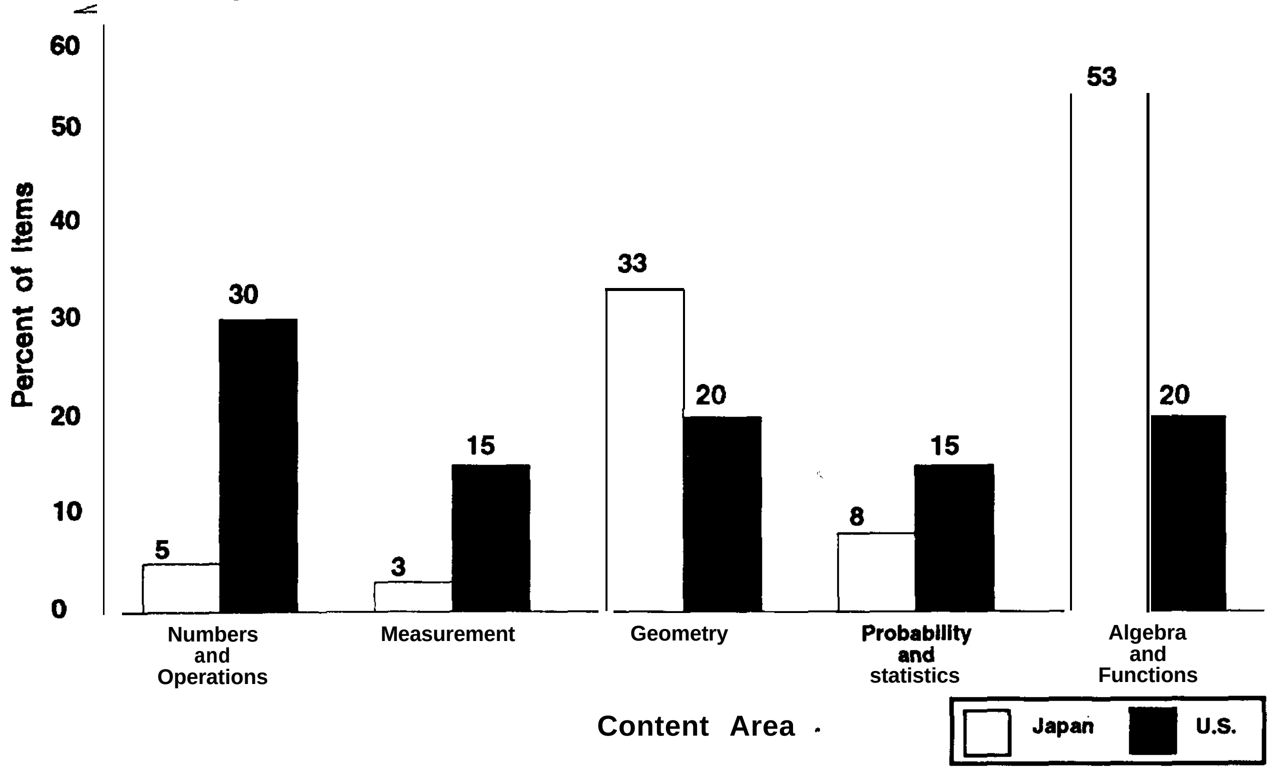
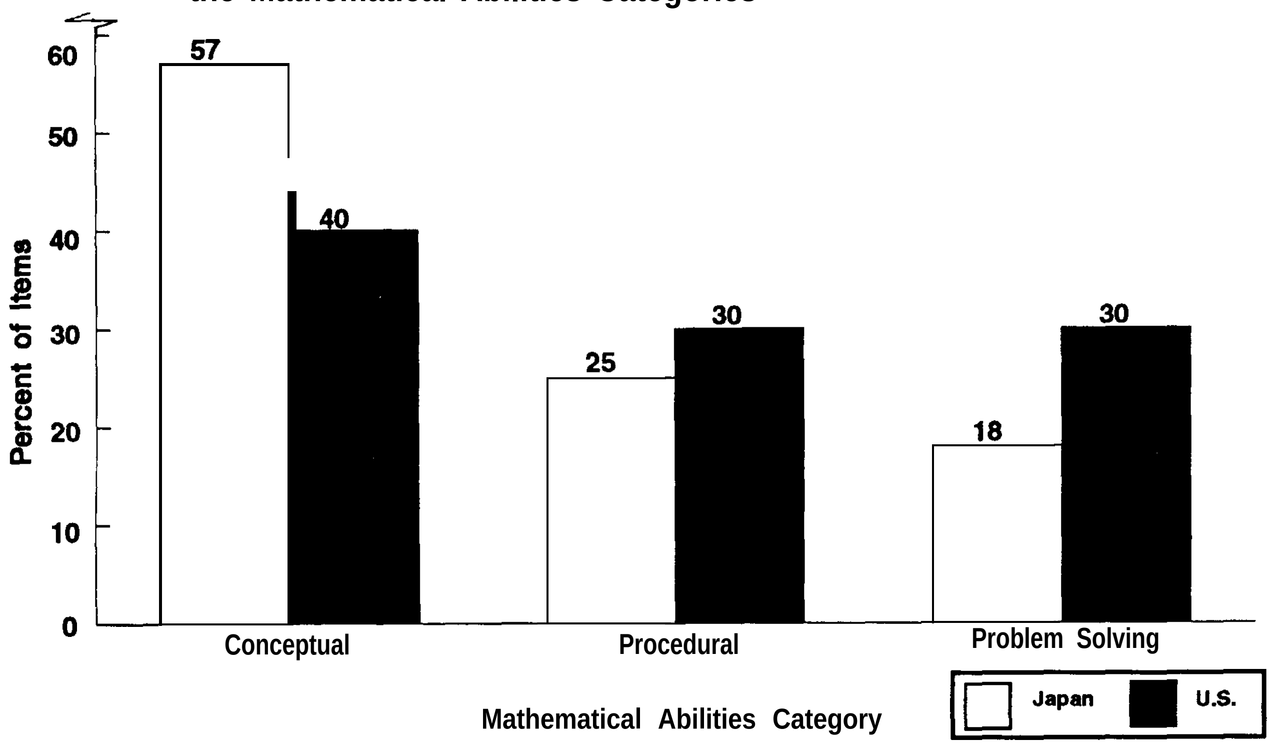


Figure 5 shows the distribution of the items to the three NAEP mathematical abilities categories described in appendix B. These data indicate that the Japanese may have placed somewhat more emphasis on conceptual understanding and slightly less on straight problem solving abilities in designing their assessment items.

Figure 5.— Percentage Distribution of Items on the NAEP and Japanese Tests to the Mathematical Abilities Categories



Chapter 3: The Items

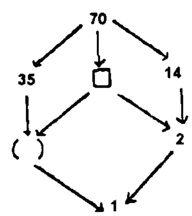
In the following sections, each devoted to a content area, examples of the items from each assessment will be provided to compare the nature of the items and the cognitive load each presents for the students taking the examinations. In a direct comparison of items from the NAEP pool, 38 items were classified in the same cells as those from the Japanese assessment. Of these, 21 are still in the secure pool for use in later NAEP assessments for the determination of proficiency, trends in the content areas. These items will not be displayed, but performance on those from the grade eight examinations and general descriptions of their content and format will be provided to give some idea of the comparability of the assessments and their results. More in-depth analysis of the 1990 and 1992 NAEP assessments and items can be found in the national reports describing these assessments (Mull is, et al., 1991, 1993).

Numbers and Operations

There were two items categorized as Numbers and Operations items on the Japanese assessment. These items were number C 16 (designator indicates form and item number) and D2. These items are illustrated below, along with a strip indicating the percentage of students responding to each alternative response. The asterisk indicates correct response.

C16 In the figure below, two numbers are connected by an arrow. For example, $70 \longrightarrow 14$ indicates that 70 can be divided by 14. When you can use any numbers in ( ) and □, except those already used in the figure, what number will fit into ( )? Select one answer from (1) to (5).

(A) 3
(B) 5
(C) 7
(D) 10
(E) 15



1) 1.9 2) *60.7 3) 20.7 4) 12.2 5) 3.0 (no answer) 1.4

Item C 16 required students to deal with the relation of "is divided by" as signified by the arrow. Students had additional needed information presented in the form of a directed graph. The number theory required in the item is contained in the middle school mathematics programs at most U.S. schools. However, few U.S. children would have seen the directed graph representation for the "is divided by" relation. Such graph theory content is only now beginning to enter the secondary school curriculum under the guise of "discrete mathematics."

There was one item on the 1990 grade eight NAEP examination corresponding to item C 16. Item 15 shown below was administered to both grade 8 and grade 12 students in 1990. It measures the related number theory concept of least common multiple. The fact that the third number is unknown makes this problem slightly more difficult than item C 16. Eighteen percent of the grade 8 students and 30 percent of the grade 12 students correctly answered this item in 1990.

NAEP 15. The least common multiple of 8, 12, and a third number is 120. Which of the following could be the third number?

(A) 15
(B) 16
(C) 24
(D) 32
(E) 48

A) *18 B) 24 C) 39 D) 8 E) 10

Item D2 of the Japanese examination required students to compare five numerical expressions where operations involving signed numbers were performed with an unknown number a and to decide which of the five expressions represented the "biggest number." While not required, no qualifications were placed on the values that a itself could be.

Like its predecessor, Item D2 had no direct counterpart on the 1992 NAEP examination. The content of the item was judged as appropriate for assessing at the grade eight level, but no specific item measuring the same content was included on the 1992 assessment.

D2. Which one is the biggest number among the following? Select one answer from (1) to (5).

(1) $a-(-2)$

(2) $a-(-1/3)$

(3) $a-0$

(4) $a+(-1/3)$

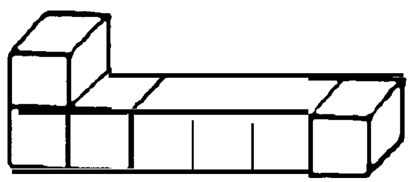
(5) $a+(-2)$

1) *86.5 2) 2.9 3) 5.2 4) 2.2 5) 2.9 (no answer) 0.1

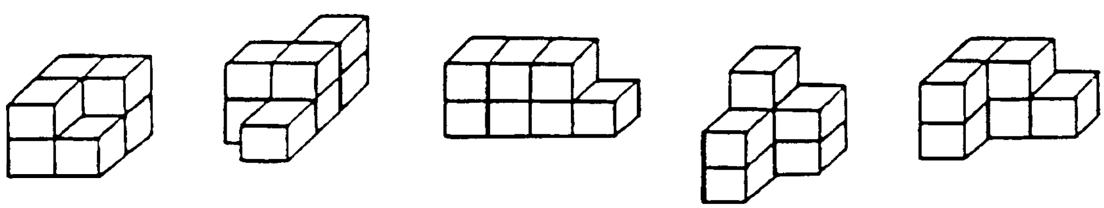
Measurement

There was only one Measurement item included on the Japanese essential mathematics assessment. This was item C 11. This item dealt with the concept of surface area. The presentation of the item included a graphic that could be employed by the student taking the examination to recall what "surface area" refers to if they had forgotten. Students then had to search through at least three of the possible responses before they could identify the correct answer. This item was difficult for the Japanese students as only 36 percent of the students answered it correctly.

C11. There are 7 cubic blocks with a side length of 1 cm. Consider the surface area as we make a solid by using all blocks. For example, the surface area of the figure at the right is 30 square cm. In the following 5 solids, there is one solid which has a surface area different from the others. Select one answer from (1) to (5).



(1) (2) (3) (4) (5)



1) * 36.3 2) 15.1 3) 14.8 4) 25.3 5) 6.6 (no answer) 1.9

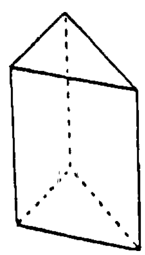
There were two unreleased (secure) items on the 1990 NAEP that correspond to item C 11. The first of these required students to identify which of five possible alternatives gave the surface area for a rectangular solid of blocks when provided with a graphic illustration of a pile of blocks similar to that shown in C 11. Only 20 percent of the students taking this item completed it correctly. The cognitive load for this item was less, since there were no indentations in the surface. The derivation of the answer was very straightforward. There was little a student could do with the item stem to discover the meaning of "surface area" had they forgotten, as was possible in item C 11.

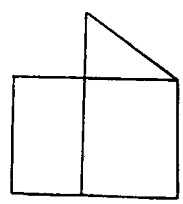
The other corresponding item on the 1990 NAEP examination was an item which presented students with a planar net for a cube and asked them to identify a pair of opposite surfaces. Performance was considerable y higher on this item, with 51 percent of the students correctly answering the item. This item is considered to correspond to C 11 because some spatial representation skills are needed to correctly complete either item.

Geometry

There were 13 geometry items in the pool of items comprising the Japanese examinations. The following analysis treats them in the order that they appear in the NAEP content framework. Item C21 presents a pair of figures showing a triangular prism being unfolded and students are asked to use a compass and ruler to complete a representation of the fully unfolded prism. Although 41 percent of Japanese eighth-graders correctly answered this item, little information was given as to how a response was coded as correct with regard to the use of the ruler and compass representations of the student's response. This was one of the two items that were not multiple choice items.

C21. There is a triangular solid shown below on the left. We draw the figure unfolded halfway as shown below on the right. Draw a completely unfolded figure using a compass and a ruler.





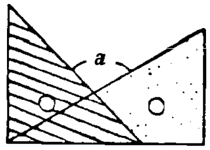
1) *41.1 2) 9.1 3) 22.3 4) 1.8 5) 6.0 (6) 9.8 (no answer) 10.0

The 1990 NAEP contained a counterpart item to C21. This secure item asked students to imagine cutting a three dimensional paper shape along an indicated dotted line. They were then asked to write a short description of what the shape of the "flattened" paper would be. Seventy-eight percent of U.S. eighth-graders were able to correctly respond to this item.

Item C 14 asked students to give the size of an angle formed when two triangle rulers are placed atop one another in a manner shown in the accompanying illustration. Students had to assume that one of the drafting triangles was a 45°-45°-90° triangle and the other was a 30°-60°-90° triangle. There are a number of ways of solving the problem posed. One could use the angle sum theorem for a triangle and either measures of exterior or vertical angles. Those with good estimation skills could have recognized that the angle formed was greater than 90° and less than 120° and have selected the correct answer on the basis of that information. Sixty-six percent of the Japanese eighth-graders answered the item correctly.

C14. In the figure shown below, when we put a pair of triangular rulers atop of one another, what is the degree measure of angle a? Select one answer from (1) to (5).

(1) 60 degree
(2) 75 degree
(3) 90 degree
(4) 105 degree
(5) 120 degree



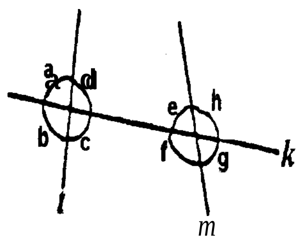
1) 1.6 2) 8.1 3) 10.3 4) * 65.9 5) 13.0 (no answer) 1.0

The 1992 NAEP contained a secure item that is somewhat similar to C 14. This item asked students to find the measure of an interior angle of a triangle given the measures of another interior angle and a remote exterior angle. Hence, the exterior angle theorem could be applied. Only 31 percent of U.S. students were able to correctly answer this multiple choice item. The correct answer to this item was not easily estimated from visual inspection.

Item D 12 asked Japanese students to identify which of a number of angles has to have the same measure as $\angle a$ so that lines l and m will be parallel. This item asks students to recall some geometric principles concerning parallel lines and the measures of corresponding and vertical angles to correctly answer the question. The data show that 70 percent of Japanese eighth-graders were able to successfully select the correct answer. There was no directly comparable NAEP item on the 1992 assessment for item D 12.

D12. In the figure shown, a line k intersects two lines l and m. Which angle has to be equal to $\angle a$ in order for the line l and line m to be parallel?

(1) $\angle b$
(2) $\angle c$
(3) $\angle d$
(4) $\angle g$
(5) $\angle h$

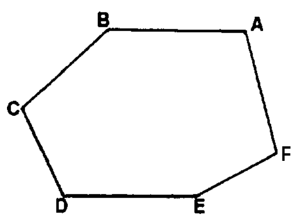


1) 0.7 2) 13.2 3) 9.1 4) *70.3 5) 6.6 (no answer) 0.1

Item D14 presented Japanese students with another geometric situation intending to require knowledge of polygons, their sides, congruent triangles, and related angles and their measures for a correct solution. Forty-five percent of Japanese students were able to respond correctly to the item.

D14. In the hexagon $ABCDEF$ at the right, $AB \parallel DE$, $AB=DE$, $BC=FA$, $CD=EF$. In this case, which one is correct? Select one answer from (1) to (5).

(1) $\angle BCD = \angle AFE$
(2) $\angle CBA = \angle FED$
(3) $BC \parallel EF$
(4) $CD \parallel FA$
(5) BE and CF intersect each other at the midpoint.

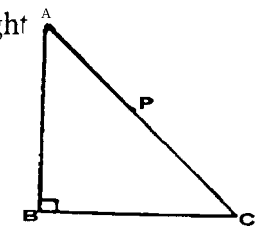


1) *44.6 2) 27.6 3) 4.5 4) 2.1 5) 20.6 (no answer) 0.6

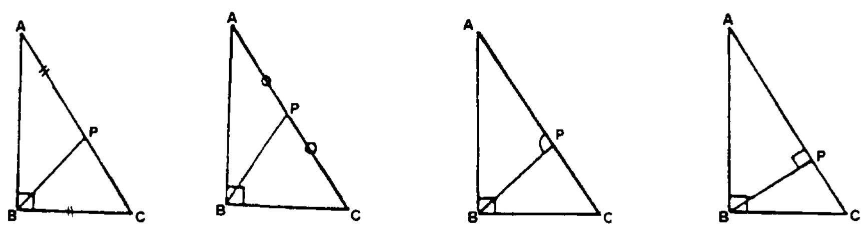
Like the preceding item, there was no direct counterpart to this item on either the 1990 or 1992 NAEP examinations. This item called for several applications of geometric properties usually taught in either the ninth- or tenth-grade in the U.S. It seems, based on students' opportunity to learn, that such an item would be inappropriate at the present for the great majority of U.S. eighth-graders. Visually the item is much easier than actually having to construct the location of point P.

Item D15 asked Japanese students to select one of five possible locations for a point P on the hypotenuse of a right triangle ABC so that the two resulting triangles ABP and BCP would be similar. Students familiar with the family of theorems about the altitude to the hypotenuse of a right triangle would be able to answer the question quite rapidly. Many students may have seen this content in a verification of the Pythagorean theorem. Other students, familiar with similar triangle relations, could solve the problem working from basic principles. In any case, 58 percent of Japanese students correctly answered the item.

D15. There is a right triangle as it is shown at right. Take point P on the hypotenuse AC and connect it with the vertex B. In order for $\triangle ABP$ and $\triangle BCP$ to be similar, where do you place point P? Select one answer from (1) to (5).



(1) $AP=BC$ (2) $AP=PC$ (3) $\angle APB=100^\circ$. (4) $\angle APB=90^\circ$ (5) $\triangle ABP$ and $\triangle BCP$ can be never similar



1) 8.0 2) 16.0 3) 4.5 4) *58.3 5) 12.8 (no answer) 0.4

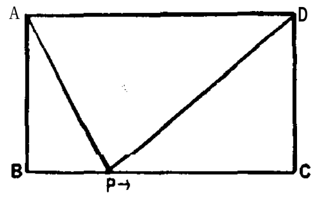
The only corresponding item on the 1990 NAEP assessment for eighth-grade students was a secure item asking students to find the length of one side of a triangle given that the triangle is similar to another triangle. All of the necessary measures of sides are given and the two triangles given are positionally arranged so that corresponding sides are easy to note in setting up the required correspondence of sides and their measures. While considerably easier to answer, only 48 percent of the U.S. sample correctly responded to this item dealing with similar triangles.

Item C18 asked students to imagine a dynamic situation as one vertex of a triangle moves along a line parallel to the opposite side of the triangle. Students are asked to identify which of a group of alternatives captures all of the possible changing measures associated with the changing triangle APD. The important thing for students to note is that the area of triangle APD does not change. Sixty-eight percent of Japanese students were able to identify that the length of segment BP, the length of segment PC, the area of triangle ABP, and the area of triangle APD would all change under the movement shown.

C18. Rectangle $ABCD$ is shown below. A point P moves from the vertex B to the vertex C on the side BC with a speed of 1 cm per second. When we connect the point P, the vertex A, and the vertex D, which of the following answers includes all of the parts of the diagram that change as point P moves? Select one answer from (1) to (5).

(A) The length of BC	(B) The length of BP
(C) The length of PC	(D) The area of $\triangle ABP$
(E) The area of $\triangle APD$	(F) The area of $\triangle PCD$

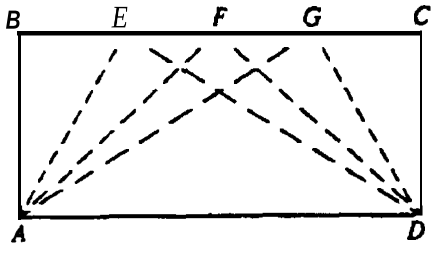
(1) (A), (B), & (C)
(2) (D), (E), & (F)
(3) (A), (B), (D), & (E)
(4) (B), (C), (D), & (E)
(5) (B), (C), (D), & (F)



1) 6.9 2) 6.6 3) 5.8 4) 9.9 5) *68.3 (no answer) 2.6

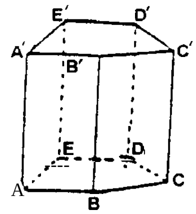
The only item in the pool of NAEP items resembling the content contained in C 18 was an item given to 12th-graders in the 1992 NAEP. This slightly more difficult item, illustrated below, asks students to explain why three triangles having one side as a side of the same rectangle and their third vertices located on the opposite side of the rectangle all have the same area. This item calls on content they should be able to discern from the drawing, that is, they all have the same base and altitude, hence the same area. The resulting performance, 10 percent answering correct, shows that U.S. grade 12 students were not able to extrapolate from the drawing and their knowledge of the area formula for a triangle to develop a correct explanation. This is consistent with past NAEP findings about student knowledge of measurement formulas.

NAEP 18. In the rectangle $ABCD$ below, explain why triangles AED, AFD, and AGD have equal areas.



Item C 10 asked Japanese students to select which approach to a problem might yield a desired result. To establish the desired relationship, the student must establish that the segment AA' is perpendicular to the base of the prism. To do this, segment AA' must be perpendicular to two intersecting segments intersecting at A in the base $ABCDE$. As the data indicate, 44 percent of Japanese students answered correctly. Like the previous items, this question requires students to have a considerable command of basic relations from elementary plane and solid geometry.

C10. In the figure to the right, we want to find whether or not the edge AA' is vertical to the base $ABCDE$. How can we find out? Select one answer from (1) to (5).



(1) find whether the edge AA' and the edge AB are vertical.
(2) find whether the edge AA' and the edge AB are parallel.
(3) find whether the edge AA' and the 2 edges AB and AE are vertical.
(4) find whether the edge AA' and the line segment AD are vertical.
(5) none of the above.

1) 17.4 2) 9.8 3) * 43.8 4) 15.6 5) 12.1 (no answer) 1.4

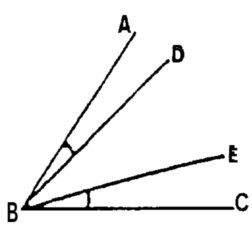
While there was no direct companion item for C 10 on the eighth-grade portion of the 1990 NAEP examination, there was one item on the grade 12 1990 NAEP examination that paralleled some of the ideas of C 10. This secure item asked students to select the shape of an intersection of a plane with an illustrated right pentagonal prism. Students needed to visualize the intersection, which includes points on both bases of the prism, to answer the question. Twenty-seven percent of U. S. grade 12 students were able to correctly answer this item. This item differed from the previous item on the Japanese examination in that students had to visualize the solution and then select the choice describing it, rather than recall a definition of what perpendicularity means for a line and plane in space.

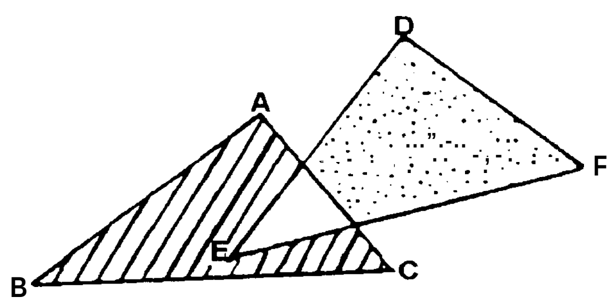
Item C 12 presented Japanese students with two examples of the equal additions property embedded in the angle addition and area addition properties of elementary plane geometry. The equal additions property was stated in both geometric representations and then students were asked to deal with the decoding of the property into its algebraic representation. As the data show, 45 percent of the students were able to make this transition correctly.

C12. In the figure below, there are two characteristics.

When $\angle ABE = \angle DBC$,
then $\angle ABD = \angle EBC$.

When $\triangle ABC = \triangle DEF$,
then the area of the diagonal lines (///)
= area of the dots (:: ::).





We can prove these 2 characteristics based on the same rule. Find the rule from (1) to (5).

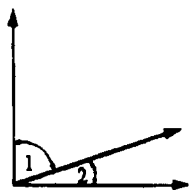
(1) When $a = b, a + c = b + c$ (a, b, c are positive value)
(2) When $a = b, a - c = b - c$ (a, b, c are positive value)
(3) When $a = b, a \times c = b \times c$ (a, b, c are positive value)
(4) When $a = b$, $a/c = b/c$ (a, b, c are positive value)
(5) When $a = b$ and $b = c$, $a = c$ (a, b, c are positive value)

1) 12.9 2) *44.6 3) 11.0 4) 8.2 5) 20.7 (no answer) 2.5

One item on the 1992 NAEP which dealt with the angle addition property in a different way is shown below. This item presented eighth-grade students with two complementary angles and the information that they each were bisected. Students were asked to give the measure of the angle formed by the two bisecting rays. NAEP data show that 23 percent of U.S. students were able to select the correct answer to the item. These items differ in that the Japanese item is more symbolic and the NAEP item depends more on a visual interpretation.

NAEP13. The sum of the measures of angles 1 and 2 in the figure below is 90°. What is the measure of the angle formed by the bisectors of these two angles?

(A) 60°
(B) 45°
(C) 30°
(D) 20°
(E) 15°

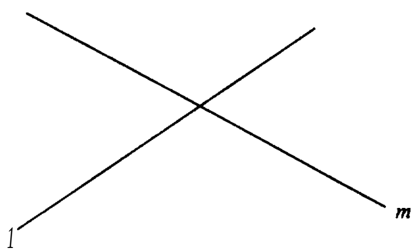


A) 21 B) *23 C) 27 D) 17 E) 8

Item D10 asked students to describe the number of points in the plane that satisfy the locus of being one centimeter from both lines *l* and *m* as shown in the accompanying illustration. Only 36 percent of Japanese students were able to correctly respond to this item. Locus problems are not a part of the standard curriculum at this level of education in Japan.

D10. In the figure below, two lines *l* and *m* intersect each other. How many points are there that are 1 cm distant from both line *l* and line *m*? Select one answer from (1) to (5).

(1) 1 point
(2) 2 points
(3) 3 points
(4) 4 points
(5) infinite points

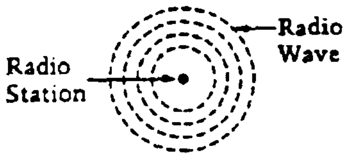


1) 7.9 2) 9.8 3) 3.1 4) *36.2 5) 42.1 (no answer) 1.0

On the 1992 U.S. examination there was only one item that directly dealt with a locus problem. This was one of the extended student constructed response items. These items required students to spend a minimum of five minutes working on the item and then to write a paragraph, along with accompanying illustrations, to fully respond to the requirements of the problem. This item was graded according to a scoring rubric that allowed for students to be judged along a five-point scale with major levels of incorrect, minimal, partial, satisfactory, and extended responses. The percentage of students reaching these levels of response were 45, 22, 13, 4, and 1 respectively. Thus, only 18 percent of the students received a score of partial or better, 5 percent received a score of satisfactory or better. Sixteen percent of the students did not even respond to the item (Dossey, et al., 1993).

NAEP27. Radio station KMAT in Math City is 200 miles from radio station KGEO in Geometry City. Highway 7, a straight road, connects the two cities.

KMAT broadcasts can be received up to 150 miles in all directions from the station and KGEO broadcasts can be received up to 125 miles in all directions. Radio waves travel from each radio station through the air, as represented below.



On the next page, draw a diagram that shows the following.

- Highway 7
- The location of the two radio stations
- The part of Highway 7 where both radio stations can be received

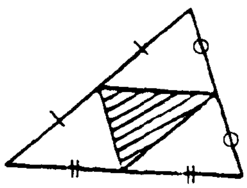
Be sure to label the distances along the highway and the length in miles of the part of the highway where both stations can be received.

Item C17 involved Japanese students in determining the validity of an assertion. In this case, the students were **involved** in considering a proposition about the length of a circuit formed by connecting, in order, the midpoints of the sides of a closed convex polygon. They are introduced to a conjecture made by a hypothetical student Taro and asked whether Taro is correct or not. Some of the choices allow the student to further qualify their answers. The results show that 46 percent of the students were able to correctly complete the item.

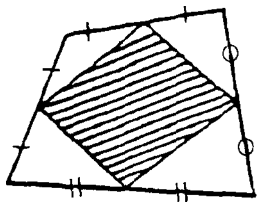
C17. Taro is happy because he discovered the following characteristics when he saw two figures (a) and (b).

"In both cases, circumferences of the figures which are created by connecting midpoints of each side are one half of the circumferences of original figure."

(a)



b)



Is Taro right? Select one answer from (1) to (5).

(1) Right
(2) (a) is right, but (b) is wrong
(3) (a) is wrong, but (b) is right
(4) both (a) and (b) are wrong
(5) None of the above

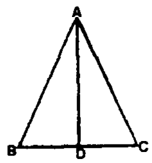
1) 23.1 2) *45.9 3) 15.0 4) 10.7 5) 4.0 (no answer) 1.4

There were no comparable geometry items for U.S. grade eight students included in the 1992 NAEP assessment, as verification of geometric conjectures receives low priority in the current mathematics curriculum at the eighth-grade level.

Item D 13, like the previous item C 10, asked students to make rather extensive use of their knowledge of geometric modes of argument to answer the question proposed. Here students had to read through a problem outlining a given statement to prove. Then they had to consider Hanako's reasoning presented in the stem of the problem. Based on the flow of the argument in the reasoning, the students then had to select which pair of reasons best fit into the flow of the argument being formed by Hanako in response to the problem presented. This problem samples students' understanding of the nature of a mathematical argument, as well as their knowledge of geometrical principles. Students are required to bring together an assumption and complete a side—side—side triangle congruence argument. The form of question is very different from ones used as part of the NAEP mathematics assessment. Data from eighth-grade students in Japan indicate that 52 percent of them were able to correctly answer the item as presented.

D13. In order to prove the following problem, Hanako solved it in the following way.

Problem
In the figure at the right, D is the midpoint of base BC of isosceles triangle ABC. Prove that a line segment AD divides vertex angle A into one half.



Hanako's way of thinking

In order to prove $\angle BAD = \angle CAD$, I should prove that $\triangle ABD$ and $\triangle ACD$ *are* congruent. I examined corresponding sides and angles of $\triangle ABD$ and $\triangle ACD$. Since $\triangle ABC$ is an isosceles triangle, $AB = AC$. Also, AD is common to both triangles. Moreover, from the assumption, (a).
Therefore, from (b), $\triangle ABD$ and $\triangle ACD$ *are* congruent.

In Hanako's way of thinking, what equation and sentence should be in (a) and (b)? Select one answer from (1) to (5).

(1) (a) $\angle BAD = \angle CAD$
(b) Two pairs of corresponding sides are equal and the angles that are in between these two lines are equal.

(2) (a) $\angle ADB = \angle ADC$
(b) A pair of corresponding sides is equal and the angles which are located at both edges of the sides are equal.

(3) (a) $\angle ADB = \angle ADC$
(b) Two pairs of corresponding sides are equal and the angles in between these two sides are equal.

(4) (a) $BD = CD$
(b) A pair of corresponding sides is equal and the angles which are located at both edges of the sides are equal.

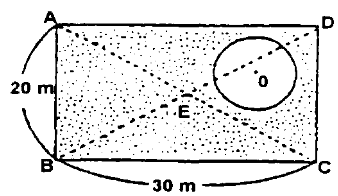
(5) (a) $BD = CD$
(b) Three pairs of corresponding sides are equal.

1) 25.2 2) 6.2 3) 9.1 4) 7.6 5) "51.6 (no answer) 0.3

There are no comparable items for eighth-grade students in the U.S. on the 1992 NAEP assessment. Such forms of verification are not a major portion of the grade eight geometry strand in most schools and most students do not have an opportunity to study the various methods for proving a pair of triangles congruent.

The Japanese item D22 asks students to consider a given rectangle, representing a tract with a circular pond included in the tract. Students are asked to draw a line on the figure through the center of the circle that will bisect the shaded region in the interior of the rectangle. This item calls on student understanding of symmetry and its relationship to area, as well as to their overall knowledge of area as a concept. The data indicate that 65 percent of the students were able to correctly indicate a line running through points O and *E* as the correct solution. This item was the second item that was not given in multiple choice format. No information was presented on the rubric used to assess student responses or what the percents associated with the other response values represented.

D22. In the figure at the right, within rectangle *ABCD* (the intersection of diagonal lines is@, is a circular pond with a radius of 5 m (the center is 0). Draw a line which goes through the center O of the circle and bisects the area of the rest of the figure (shaded area).

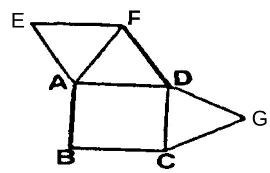


1) *64.7 2) 0.6 3) 7.6 4) 2.5 5) 1.1 6) 3.1 7) 5.2 (no answer) 15.3

There was one secure item on the 1992 NAEP that had some similar properties to item D22. This item was one of the secure items from the 1992 assessment that made use of the geometric pieces that students were provided for one of the blocks of items. Students were given the statement that one student said that the two pieces had the same area, another student disagreed. The student taking the test then had to take a position backing one of the students and explain why that particular position was correct. They were explicitly asked to "use words or pictures (or both) to explain why. " Only 22 percent of the U.S. eighth-graders could successfully complete this task, which required a much easier dissection than the corresponding Japanese item D22.

Item D11 is another question dealing with solid geometry. In this item, students are presented with a planar net for a square pyramid and asked to indicate the number of ways that a fourth triangular side could be added to the net so that, when folded, the four triangles and base will come together to form a pyramid. The data indicate that 52 percent of the Japanese students were able to correctly determine that there are three different locations where the fourth triangle could be correctly added to complete the desired figure.

D11. We can make an unfolded figure of a square pyramid if we add one more triangle in the figure at the right. How many ways of adding a triangle are there? Select one answer from (1) to (5).



(1) 1 way
(2) 2 ways
(3) 3 ways
(4) 4 ways
(5) 5 ways

1) 9.8 2) 20.6 3) *51.5 4) 13.2 5) 4.1 (no answer) 0.8

An analysis of the NAEP items from 1990 revealed one secure item that was somewhat similar to D 11. This item required students to identify which, of a number of potential nets, that when folded, would *not* form a particular geometric solid. This item lacked the combinatorial aspect of asking how many ways an additional needed piece could be added to a partially completed net, but required that more information be processed. Fifty-eight percent of U.S. eighth-graders were able to answer this easier item. Analysis of upcoming NAEP assessment blocks for mathematics indicates that another similar item will, most probably, be included in the next mathematics assessment.

Data Analysis, Statistics, and Probability

The Japanese essential mathematics assessment contained three items measuring content that would be classified in the NAEP content category of Data Analysis, Statistics, and Probability. This content is an area of some interest in the curriculum for Japanese eighth-graders (Ministry of Education, n.d.). This is especially true for content dealing with collecting real world data, making tables and graphs, and considering the nature of the resulting distributions, patterns of frequencies, and central tendencies.

Item C20 provides students with a combinatorial question equivalent to the proverbial "handshake" counting problem. In this case, a student's solution to a counting problem for games in a tournament is presented and the students taking the examination are asked which of four other problems can be solved using the same solution paths employed by the student in solving the tournament problem. The data indicate that 36 percent of the Japanese students were able to complete this problem correctly.

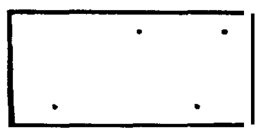
C20. There are 4 baseball teams. In order to find out how many games are possible when every team plays with every other team, Isao thought of the following: $3+2+1=6$, so 6 games. Which one of the following problems can be solved using the same method that Isao used to solve the problem. Select one answer from (1) to (5).

(a) Finding how many ways of numbers are possible when we make 2–digit numbers using 4 cards that have a number 1, 2, 3, or 4.

(b) Finding how many way of putting fruits into a basket are possible when we choose 2 fruits out of 4 different fruits.

(c) Finding how many ways of lining up are possible, when 4 children line up vertically.

(d) Based on the 4 points in the figure shown, find how many lines we can draw when we draw lines which go through 2 points.



(1) (a) & (c)
(2) (a) & (d)
(3) (b) & (c)
(4) (b) & (d)
(5) (c) & (d)

1) 13.0 2) 15.9 3) 12.9 4) * 36.0 5) 16.6 (no answer) 5.5

An analysis of problems on the grade eight portion of the 1992 NAEP located one secure item testing similar material. This item was one of the extended student constructed response items. It required students to consider various combinations possible when two coins are split into two groupings. In particular, the item extended the situation to ask for the probability that the student described would have sufficient money in one pocket to make a particular type of purchase. Data shows that 18 percent of U.S. eighth-graders were able to give a satisfactory or better response to this extended response item.

The second Data Analysis item contained in the Japanese essential mathematics assessment dealt with averages. Item D20 presented students with a frequency distribution for an experiment of throwing a ball for distance. Students are asked to select a statement about the mean of one group, given some information about the mean of the other group of students taking part in the experiment. Students should be able to solve the problem by considering the relative positions of data points in the two distributions. Fifty-three percent of the students in the Japanese classrooms responding to this item selected the correct answer.

D20. The table below shows the frequency distribution of the results of throwing a ball for 8th–grade female students in the classes A and B. The average for class A is 13.3m. From the table, what fact can you find about the average of class B? Select one answer (1) to (5).

Distance of throwing ball (m)		Number of students Class A	Class B
more than 20	less than 22	0	2
more than 18	less than 20	1	0
more than 16	less than 18	3	4
more than 14	less than 16	2	5
more than 12	less than 14	8	5
more than 10	less than 12	4	1
more than 8	less than 10	2	2
Total		**20**	**19**

(1) The average of class B is smaller than the average of class A.
(2) The average of class B is equal to the average of class A.
(3) The average of class B is bigger than the average of class A.
(4) The average of class B is 2 m smaller than the average of class A.
(5) We cannot say either one of the classes is bigger than the other class.

1) 20.9 2) 4.9 3) *53.3 4) 5.8 5) 13.9 (no answer) 1.2

By contrast, U.S. students were asked, in a secure item from the 1992 assessment, to give the mean examination score for a small group of students whose test scores were presented in a frequency diagram. Only 16 percent of the U.S. students were able to solve this item requiring the use of a weighted average, an item that is directly related to curricular objectives at the grade eight level.

The third item from this content category was a probability item (D21) that sampled students' knowledge about long-run behavior of outcomes in an experiment based on drawing an object, with replacement, from a known distribution of objects. Only 27 percent of the Japanese students were able to correctly answer this item.

D21. There are two red balls and eight blue balls in a bag. From this bag, take one ball at a time and examine the color of the ball and then return it to the bag. When we repeat this, which one of the following is right? Select one answer from (1) to (5).

(1) When you take a ball only once, the ball is blue.
(2) If the first ball is blue, the second ball will be red.
(3) If we repeat this 10 times, 8 times you will certainly get blue balls.
(4) If we repeat this 100 times, about 80 times, we may get blue balls.
(5) No matter how many times we repeat this, we cannot predict how many times we may get blue balls.

1) 3.5 2) 2.9 3) 12.6 4) *27.2 5) 52.6 (no answer) 1.1

There was one somewhat related probability item in the secure 1990 NAEP items for grade eight. It dealt, however, with sampling objects from a large set to estimate the number of defective objects in the set. Thirty-six percent of the U.S. students were able to correctly answer this item.

Algebra and Functions

The content area of Algebra and Functions contains nearly half of the items presented to students on the Japanese tests. Algebra and the study of variation in the form of direct proportion is at the heart of the grade one and grade two (U.S. eighth-grade equivalent) mathematics curricula for lower secondary schools in Japan. This emphasis is strongly represented in the items chosen for this portion of the essential mathematics assessment.

Item D 16 requires students to abstract a pattern from an arrangement of matchsticks and evaluate their pattern for the case of 20 squares. Students performed well on this item with 69 percent of them correctly answering it.

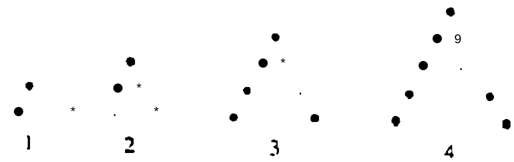
D16. Like the figure shown, we made squares with matches. When we lined up squares and made a rectangle, we used 7 matches to make 2 squares and 10 matches to make 3 squares. How many matches do we need to make 20 squares? Select one answer from (1) to (5).

(1) 60 matches
(2) 61 matches
(3) 70 matches
(4) 79 matches
(5) 80 matches

1) 10.4 2) *68.6 3) 14.2 4) 3.4 5) 3.2 (no answer) 0.3

A similar item on the 1990 NAEP examination, set in a similar but slightly more abstract context, asked for the evaluation of the pattern for a larger case value. The resulting achievement level was 32 percent correct.

NAEP 32. If this pattern of dot–figures is continued, how many dots will be in the 100th figure?



(A) 100
(B) 101
(C) 199
(D) 200
(E) 201

A) 6 B) 12 C) 17 D) 29 E) *34 omit) 2

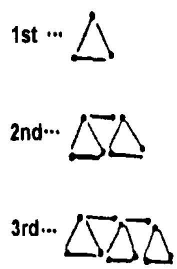
A similar pattern-based problem C9 asked students to evaluate the meaning of an expression, containing a variable, that was used to describe the a^{th} figure in the pattern. This item proved more difficult than just describing the pattern and its values as in the previous problem D 16. Only 45 percent of the students were able to note that $(a-1)$ represents the number of horizontal, right-headed matches.

C9. We make the following figures with matches. We found how many matches were needed to make ath figure from the following equation.

$$3 \times a + (a-1)$$

In the above equation, what does $(a-1)$ express?

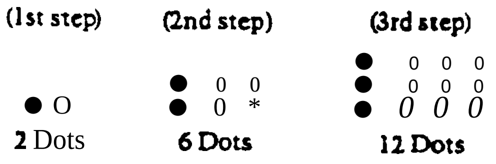
(1) Number of matches which are
(2) Number of matches which are
(3) Number of matches which are
(4) Number of triangles which are
(5) Number of triangles which are



1) *45.1 2) 10.7 3) 20.6 4) 16.5 5) 5.9 (no answer) 1.2

This problem has a similar counterpart in one of the extended student constructed response items from the 1992 NAEP. This item, shown below, asked students to formulate an expression describing the relationship between the case number for the pattern and the number of dots involved and then evaluate it for the 20^{th} step in the process. This is a slightly harder item than the corresponding C9. Data from the assessment showed that 62 percent of the students answered the item incorrectly, 9 percent gave a minimal response, 6 percent a partial response, 1 percent a satisfactory response, and 4 percent an extended response. Thus only 5 percent of U. S. eighth-graders could give a satisfactory or better response to this item (Dossey, et al., 1993).

NAEP 9. A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.



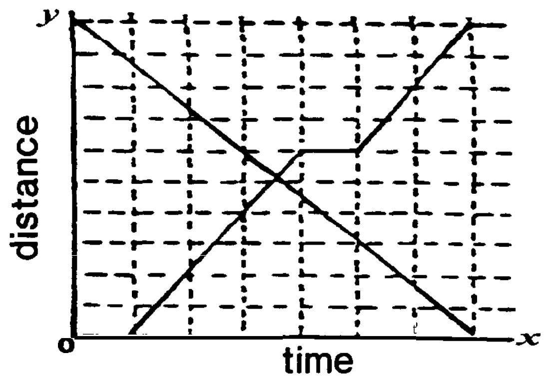
Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots.

Explain or show how she could do this *and* give the answer that Marcy should get for the number of dots.

Another item type which assesses student understanding of variation is one which presents a graphical representation and then asks students to interpret the relation shown between the two variables involved. In item D 17, Japanese students had to carefully examine the graphs relating the distances people are from point Pat time x. They were then asked to find which of the five descriptions provided was incorrect. Sixty percent of the Japanese eighth-grade students were able to correctly note that neither person changed their speed for the times that they were in motion. Some might contend that the incorrect response (1) which the students were asked to find could have been a bit more explicit about this. This type of item had no counterpart in either the 1990 or 1992 NAEP examinations, but an item of this type is planned for the next NAEP mathematics assessment. Several of the newest American textbooks are including items of this type in teaching about functional relationships and their representations.

D17. The graph below shows the traces by which two people, A and B, walked a street that connects point P and point Q. It indicates a relationship between time and location of the two people, time on x axis and distance from point P on y axis. Which of the following *is not right* among the information we get from this graph? Select one answer from (1) to (5).

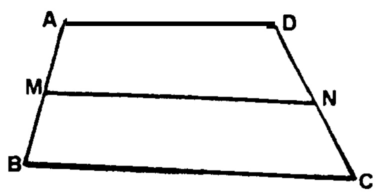
(1) one person changed his speed on his way.
(2) one person took a rest on his way.
(3) two people did not start at same time.
(4) two people met each other on their way.
(5) two people arrived at same time.



1) *60.3 2) 12.6 3) 10.0 4) 4.8 5) 11.5 (no answer) 0.8

Item C15 could be classified in either the geometry or algebraic content area. It has no direct counterpart in the NAEP items and is difficult to place in the NAEP content categories. Its closest description is that of an item that requires students to translate geometric and numerical representations into symbolic form. Part of the translation required that the students represent the relationship between the length of segment *MN* and the lengths of segments *AD* and *BC*. For that reason, it was classified in the Algebra and Functions category. This item was relatively easy for students to complete correctly, as 62 percent of the students answered it correctly.

C15. Read the following sentences. "In a trapezoid *ABCD* in which *AD* and *BC* are parallel, the midpoints of *AB* and *CD* are *M* and *N*. In this case, the line segment *MN* is parallel to the line segment *BC* and the length of *MN* is equal to half the length of the sum of the length of *AD* and *BC*."



How can you express this conclusion? Select one answer from (1) to (5).

(1) $MN \parallel BC$
(2) $MN=(AD + BC) / 2$
(3) $MN \parallel BC,\ MN = (AD + BC) / 2$
(4) $DN=NC,\ MN = (AD + BC) / 2$
(5) None of the above

1) 5.5 2) 15.1 3) * 61.8 4) 8.7 5) 6.9 (no answer) 2.1

Item C4 required students to translate a problem that was described verbally into mathematical terms and solve it in terms of a specified time. Then the problem was shifted to a slightly more abstract form and a related question about the round-trip time was asked, which required a symbolic expression for an answer. The shift from the first part, where 67 percent of the students gave the correct response, to the second part, where 59 percent of the students gave the correct response, shows the decrease in performance that might be attributed to the shift from concrete to abstract settings.

C4. We make a round trip to the top of a mountain. It is 6km from here to the top of the mountain.

(A) Go by foot with a speed of 3km per hour, take a rest for 30 minutes at the top of the mountain, and return by foot with a speed of 4km per hour. How many hours does it take for a round trip. Select one answer from (1) to (5).

(1) 3 hours and 30 minutes
(2) 4 hours
(3) 5 hours 30 minutes
(4) 6 hours
(5) 7 hours 30 minutes

1) 13.2 2) * 67.0 3) 8.5 4) 4.9 5) 4.9 (no answer) 1.4

(B) Go by foot with a speed of *a* km per hour, rest 30 minutes at the top of the mountain, and return by foot, with a speed of *b* km per hour. How many hours does it take for a round trip. Select one answer from (1) to (5).

(1) $3a + 4b + 1/2$
(2) $6a + 6b + 1/2$
(3) $ab + 1/2$
(4) $6/a + 6/b + 1/2$
(5) $a/6 + b/6 + 1/2$

1) 8.8 2) 6.7 3) 5.6 4) * 58.9 5) 18.3 (no answer) 1.6

There are no comparable NAEP items at the grade eight level. A similar item was field tested in 1993, but so few students were able to perform on it that it was omitted from the test form in favor of an item that would at least be able to discern differences in student performance.

Item C19 measured students' ability to select an ordinate for point C so that the points A, B, and C all fall on a straight line. Students could approach this by considering the relationship existing between the abacuses and ordinates in the other two ordered pairs given, or simply graph the first two and extrapolate the pattern to reach point C, noting its y-value. Given the ease of answering this item, only 31 percent of the students were able to correctly select the answer to this item. There was no directly comparable item in the NAEP items that have been administered.

C19. Enter a number into the [ ] so that the 3 points A, B, and C *are* on one line.

A (2,1), B (5,7), C (18, [ ])

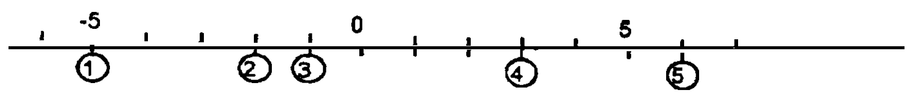
Select one answer from (1) to (5).

(1) 9
(2) 15
(3) 27
(4) 33
(5) 36

1) 16.2 2) 16.2 3) 22.3 4) *30.9 5) 10.7 (no answer) 3.6

Item D1 was a straightforward item dealing with the location of points and their coordinates on an integer number line. This item had the highest percent of students responding correctly, 94 percent. There is no similar item on the present NAEP examinations, but one is planned for the next assessment.

D1. Find a point which is larger than -3 by 2 on the line below. Select one answer from (1) to (5).



1) 3.5 2) 1.1 3) *93.8 4) 0.7 5) 0.6 (no answer) 0.3

The next item was one of the items that was used to consider the equivalence of the two Japanese forms given. Item C5, which required students to select an equation that represents the problem described, also appeared as item D6. Student performance data on this item showed 70 percent of the students taking Form C getting the item correct, while 67 percent of those taking Form D got the item correct. There again is no directly comparable NAEP item, although a similar item is planned for the 1996 assessment.

C5/D6. Mr. A planned to read a book within a certain number of days. At first he planned to read 30 pages a day; he noticed that 50 pages would be left at the end of the period. Then he changed his plan and decided to read 35 pages a day. However, 15 pages would still be left at the end of the period. In order to know the total number of days and total number of pages of the book which Mr. A is trying to read, we set up an equation, where x is the total number of days Mr. A would have to spend to read the book. Select one answer from (1) to (5).

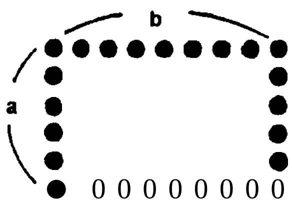
(1) $30(x + 50) = 35(x + 15)$
(2) $30x + 50 = 35x + 15$
(3) $30x - 50 = 35x - 15$
(4) $30 + x + 50 = 35 + x + 15$
(5) $30/x + 50 = 35/x + 15$

FORM C: 1) 9.5 2) *69.5 3) 10.0 4) 4.5 5) 5.1 (no answer) 1.4

FORM D: 1) 9.0 2) *66.6 3) 10.9 4) 4.8 5) 8.1 (no answer) 0.6

Item D4 asks students to select an equation that describes a physical situation involving the arranging of marbles. The situation is the classic context where students have to take care not to double count marbles in the corners of the arrangement. Both ploys (1) and (4), which contain this error explicitly, with no attempts to adjust, account for 34 percent of the responses. Forty-eight percent of the students answered the question correctly. No NAEP item directly measures the same content as measured by item D4.

D4. We lined up a pieces of marbles vertically and b pieces of marbles horizontally. Which one of the following equations expresses the total number of marbles? Select one answer from (1) to (5).



(1) $2(a + b)$
(2) $2(a + b) - 4$
(3) $2(a + b) - 2$
(4) ab
(5) $ab - (a - 1)(b - 1)$

1) 22.7 2) *48.0 3) 13.7 4) 11.6 5) 3.6 (no answer) 0.3

Item D9 shifts the interest to the solution of inequalities, as it asks the Japanese youth to find the inequality equivalent to $x/2<7$. This was a relatively easy item for students, as 68 percent of the students got it correct.

D9. Solve the following inequality and select one answer from (1) to (5).
$x/2 < 7$

(1) $x > 5$
(2) $x > 14$
(3) $x < 7/2$
(4) $x < 5$
(5) $x < 14$

1) 2.0 2) 9.8 3) 17.3 4) 2.5 5) *68.0 (no answer) 0.4

Two similar items appeared on the grade eight portions of NAEP examinations. In 1990 students were asked to select the least whole number satisfying $2x > 11$ and in 1992 to supply two whole numbers that would satisfy the open sentence $54 < 3 \times \square$. Performance on these two items was 45 percent correct and 49 percent correct respectively. This is somewhat disappointing in that the Japanese item is harder than either of the corresponding NAEP items as it asks students to make a transformation of the inequality and then identify an equivalent inequality.

NAEP 25. What is the least whole number x for which $2x > 11$?

(A) 5
(B) 6
(C) 9
(D) 22
(E) 23

A) 5 B) *46 C) 10 D) 20 E) 1 omit) 1

NAEP26. Write two numbers that could be put in the □ to make the number sentence below true.

$54 < 3 \times \square$

Item C 1 tests the order of operations. Many students make the error of adding first, obtaining a 13 and then finding the product of 13 and -5. That this error pattern is universal is shown by the 11 percent of the students selecting the incorrect answer of –65. As this item is a target of instruction, and apparently learned, at this grade level, 85 percent of the students answered it correctly.

Cl. Compute the following.

$9 + (+4) \times (-5)$

Select one answer from (1) to (5).

(1) 8
(2) –9
(3) –11
(4) –25
(5) –65

1) 1.8 2) 0.4 3) *84.5 4) 1.2 5) 10.9 (no answer) 1.2

Two NAEP items also measure students' knowledge of order of operations. The first item, a secure item in 1990, is very similar to C 1, but it doesn't involve multiplication by a negative number. Ninety-four percent of U.S. grade eight students answered this item correctly. The second item from 1992, illustrated below, combines five operations, counting the exponentiation. This level of complexity brought performance of eighth-graders in the U.S. down to 22 percent correct.

NAEP 3. $3^3 + 4(8 - 5) \div 6 =$

(A) 6.5
(B) 11
(C) 27.5
(D) 29
(E) 34.16

A) 26 B) 10 C) 15 D) *22 E) 22 omit) 5

Question C2 asked students to select which one of a number of statements about operations with integers was not always true. The first statement that says that a positive number plus a negative number is always a positive number stands out quickly as the obviously correct answer. However, it was not quite so obvious to Japanese students, as only 66 percent of them answered the item correctly. This is still a relatively high level of performance, but beneath what might be expected. There was no directly comparable NAEP item in the pool.

C 2. Which one of the following mathematics statements *is not always true*. Select one answer from (1) to (5).

(1) (positive number) + (negative number) = (positive number)
(2) (negative number) + (negative number) = (negative number)
(3) (positive number) – (negative number) = (positive number)
(4) (negative number) – (positive number) = (negative number)
(5) (negative number) x (positive number) = (negative number)

1) *66.3 2) 8.1 3) 8.4 4) 11.0 5) 5.1 (no answer) 1.1

Item C3 required students to place an expression involving the difference of an expression and twice a second expression into a simpler form. This item also appeared as item D5 on the second form of the test. This was one of the easier items for Japanese students. Eighty-one percent of the students got it correct on form C and 79 percent got it correct on Form D. The remaining students basically had one of two common difficulties in dealing with the –2 coefficient on the second expression. This item has no direct counterpart on the present NAEP assessments for eighth-graders, but a similar item is planned for the next assessment.

C3/D5. Compute the following. $(4a - 6) - 2(a - 3)$

Select one answer from (1) to (5).

(1) $2a$
(2) $2a - 3$
(3) $2a - 12$
(4) $3a - 9$
(5) $3a - 11$

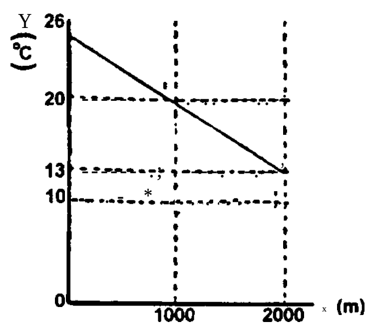
FORM C: 1) *80.5 2) 9.1 3) 7.8 4) 1.0 5) 0.8 (no answer) 0.8

FORM D: 1) *78.5 2) 14.2 3) 4.9 4) 1.3 5) 0.8 (no answer) 0.3

Item D19 asks students to either extrapolate a pattern given in a graph to find the altitude at which the temperature first reaches 0°C or to set up an equation representing the relationship between temperature and altitude illustrated and then solve for the altitude, given the temperature of 0°C. As shown, 66 percent of the students were able to correctly solve the problem posed and represented by the graph relating the two variables. This item is similar to items beginning to appear in U.S. algebra textbooks and teaching materials aimed at helping students develop a meaning of variable. However, there is no current NAEP item similar to this item.

D19. The graph below indicates a relationship between x and y, the temperature of y°C at the altitude of x m. At what altitude (m), does the temperature become 0°C. Select one answer from (1) to (5).

(1) 3000 m
(2) 4000 m
(3) 5000 m
(4) 10000m
(5) 26000 m



1) 8.0 2) *66.3 3) 10.9 4) 7.0 5) 6.9 (no answer) 0.8

Item C7 is repeated, in a slightly different form, as item D8. This item asks students which one of a number of pairs of values for the variables x and y satisfy a given pair of linear equations in two variables. This problem could be answered by either solving for the values or substituting the various choices given. An analysis of student responses shows that 76 percent of the students answered the item correctly on Form C and 79 percent of the students answered the item correctly on Form D.

C7/D8. We solved a simultaneous equation:

$$\begin{cases} 5x + 7y = 3 \\ 2x + 3y = 1 \end{cases}$$

Select a solution from (1) to (5).

(1) $\begin{cases} x = -5 \\ y = 4 \end{cases}$ (2) $\begin{cases} x = 5 \\ y = -3 \end{cases}$ (3) $\begin{cases} x = -2 \\ y = 3 \end{cases}$ (4) $\begin{cases} x = 2 \\ y = 1 \end{cases}$ (5) $\begin{cases} x = 2 \\ y = -1 \end{cases}$

FORM C: 1) 3.3 2) 3.3 3) 4.9 4) 11.5 5) * 76.0 (no answer) 1.0

FORM D: 1) 3.6 2) 4.1 3) 3.1 4) 9.7 5) * 79.4 (no answer) 0.1

Solving a system of equations is considered an appropriate topic for inclusion at the present on the grade eight NAEP assessment in mathematics. However, there are not any comparable items in the present grade eight pool.

Item C6, and matching item D7, present students with a sequence of steps that portray one method of solving the equation $0.3x - 0.15 = 0.9 - 0.2x$. Students are asked to examine a set of potential reasons that could be used to justify the transition from one form of equivalencies to another. This tests students' knowledge of properties and equation solving procedures. Seventy-two percent of Japanese eighth-graders selected the correct statement on Form C and 70 percent did the same on Form D. At present there are no similar items in the NAEP pool for grade eight students. Similar items have been field tested for the NAEP examination, but they have been rejected because the level of student performance was exceedingly low.

C6/D7. We solved an equation as follows:

$$0.3x - 0.15 = 0.9 - 0.2x$$
$$30x - 15 = 90 - 20x$$
$$20x + 30x = 90 + 15$$
$$50x = 105 \text{ ——— (A)}$$
$$x = 105/50 \text{ ——— (B)}$$
$$x = 21/10$$

In this case, what rule did we use to modify from (A) to (B)? Select one answer from (1) to (5).

(1) Added the same number to both sides of the equation.
(2) Subtracted the same number from both sides of the equation.
(3) Divided both sides of the equation by the same number.
(4) Combined similar terms.
(5) Transposed letters and numbers.

FORM C: 1) 1.2 2) 1.0 3) *71.6 4) 11.3 5) 13.7 (no answer) 1.2

FORM D: 1) 2.4 2) 0.6 3) *70.3 4) 11.5 5) 15.0 (no answer) 0.3

Item D3 deals with the properties of signed numbers by positing a generalization about the value of $-a$ under different substitutions for the value of a. Students are asked to select a position and supporting reason for their position. Fifty-nine percent of Japanese youth correctly answered this item. Nearly 21 percent said that one cannot say whether the statement is right or wrong. They may have been thinking that the answer could be either positive or negative.

D3. Taro said. "The value of–a will always be negative." Is Taro right? Select one answer from (1) to (5).

(1) Right, because there is a negative symbol.
(2) Right, because $(0 - a)$ will be a negative number.
(3) Right, because if a is substituted by 1, 2, or 3, it will be a negative number.
(4) Wrong, because if a *is* substituted by -5, it becomes a positive number.
(5) We cannot say whether it is right or wrong.

1) 6.7 2) 6.0 3) 7.9 4) * 58.6 5) 20.5 (no answer) 0.3

A similar item from the 1992 NAEP assessment is shown below. This item also posits statements made by students about the magnitude of a product involving a given number and another number, which could be a negative integer or a positive rational number less than one. Forty-nine percent of U. S. students were able to answer this student constructed response item correctly.

NAEP23. Tracy said, "I can multiply 6 by another number and get an answer that is smaller than 6."

Pat said, "No, you can't. Multiplying 6 by another number always makes the answer 6 or larger."

Who is correct? Give a reason for your answer.

Item C8 also dealt with the comparison of the magnitudes of numbers. In this case the comparison was between the potential values of $(a + 2)$ and a. Sixty-nine percent of the students were able to see that choice (5) was the correct alternative. No comparable item exists in the NAEP pool from past assessments.

C8. When we compare the sizes between $(a + 2)$ and $(2a)$, which one of the following is right?

(1) always $a + 2 < 2a$
(2) always $a + 2 > 2a$
(3) always $a + 2 <= 2a$
(4) always $a + 2 >= 2a$
(5) depends on the value of a, sometimes $a + 2 >= 2a$ and some other times $2 + a <= 2a$.

1) 14.8 2) 2.6 3) 10.4 4) 2.5 5) *68.5 (no answer) 1.2

Item D 18 asks students to identify which of five groupings of situations contains all of the context descriptions which might be represented as linear functions of the independent variable x. The item is worded in an obtuse fashion and that may be responsible for the low level of performance, 21 percent correct. There is no directly comparable item about functions in the present NAEP item pool for grade eight.

D18. Which of the following answers include all the cases in the box where y is a linear function of x? Select one answer from (1) to (5).

(a) When we cut x cm from a 180 cm stick, the length of the rest is y cm.
(b) When we enclose a rectangle with a 30 cm string, if the width is x cm, the area is y square cm.
(c) When we buy an eraser, costing 50 yen, and x notebooks, costing 120 yen each, the total cost is y yen.
(d) The sum of inner angles of x polygon is y degrees.

(1) a & c

(2) b & d

(3) c & d

(4) a, b, & c

(5) a, c, & d

1) 21.7 2) 14.2 3) 25.2 4) 17.5 5) *20. 5 (no answer) 0.8

The remaining algebra item on the Japanese test is item C 13. This item also deals with functions and asks students to respond to the change in output associated with an increase of 2 in the input. Students familiar with the meaning of slope, or describing changes, would quickly indicate that this would be twice the value of the slope, or unitary change rate. This would quickly give the answer of 6. Only 35 percent of the students were able to correctly determine this value. An almost equal percentage, 34 percent, answered with 4, the value obtained by evaluating the functional expression for an input of 2.

C13. In the linear function, $y = 3x - 2$, when the value of x increases by 2, how much does the value of y increase? Select one answer from (1) to (5).

(1) -2
(2) 2
(3) 3
(4) 4
(5) 6

1) 4.8 2) 11.4 3) 13.2 4) 34.3 5) * 35.0 (no answer) 1 . 2

As in many other cases, there was no NAEP item measuring something comparable to this item from the Japanese examination.

Chapter 4: A Retrospective View

After an examination of the intent of the essential mathematics study and the items given students, along with comparable NAEP items, it is important to pause and consider the differences and what they might tell about the mathematics education of the two groups of students involved. in doing so, it is important to remember the difficulty and dangers involved in drawing simple conclusions about cross-cultural differences from a small sample of items reflecting learning that has taken place in two different cultures' homes and educational systems over a period of nearly thirteen years for the students involved (Stigler, et al., 1990: Leestma and Walberg, 1992; Medrich and Griffith, 1992: Robitaille and Travers, 1992). Even in the face of such obstacles, it is important to consider the patterns observed and try to fit them with other observed work and form inferences from the data.

The Assessments

A consideration of the items on the Japanese essential mathematics assessment forms rather clearly shows the greater cognitive demand the items place on students than the items drawn from the same content areas in NAEP. This is almost uniformly true, from the first item examined in Numbers and Operations through the final question examined in Algebra and Functions. In format, the Japanese items were essentially all multiple choice, with the exception of the paper unfolding item (C21) and area bisection item (D22) in Geometry. The NAEP examination had a broader venue of response forms, including both regular and extended student constructed response items. Given the same amount of time to respond, the average number of items asked of Japanese students was about half the number given to U.S. students. However, the Japanese items tended to present the mathematics assessed in forms that were:

- more verbally intensive. The item stems were longer and provided more context or information. As such, they placed greater reading and, in many cases, organizational loads on the students in the study.
- more focused on strategies for problem solving. Several items asked students to either side with one or another of two potential avenues for dealing with a contextual situation, often by agreeing or disagreeing with a potential solution path. Other items asked what information needed to be established in order to either establish a claim or move forward from a given position. As a particular example, consider item D 13 in Geometry.
- more involved. While items on the NAEP established whether or not students knew given "factoids" of information or could correctly implement an algorithm or procedure, items on the Japanese examination often started with such an assumption and then asked an additional step, requiring some insightful analysis, of the students. For example, consider the first item in Numbers and Operations, C 16, or D11 and D22 in Geometry, or item C 13 in Algebra and Functions.

These differences are, no doubt, reflective of even greater differences in the two groups of students' opportunity to learn in classroom settings, the nature of the learning environments in their classes, and the expectations that their parents, teachers, and cultures place on them. These differences are well developed and analyzed elsewhere in the literature.

However, these differences notwithstanding, an objective analysis of the comparability of the items, at an item-by-item level, would indicate that the items on the Japanese eighth-grade assessment are more difficult than their content-area matches on the NAEP tests. That said, it is difficult to, as directly, compare the difficulty of the assessment forms and contexts themselves. Students working on the NAEP examination have to complete almost twice the number of items in the same time period. The press placed on U.S. students is indicated by the increase in non-completion of items near the end of several of the fifteen minute blocks of items. Thus the NAEP test has three rushes to completion in the 45 minute testing period. Analysis of the performance data on the last few items of the Japanese assessment did not indicate an increase in non-com - pletion.

Beyond form differences, there is evidence in the cross-cultural literature of differences in student motivation associated with taking such examinations. Some of this comes from the perceived importance of doing well on the assessment (Lapointe, et al., 1992), other influences may come from the perceived relationship of the match between the assessment's contents and the curriculum–coverage that the students taking the assessment have experienced in their current school year (Robitaille and Travers, 1992).

The Curriculum

Probably the greatest difference in the assessments, and the program of studies preparing the students in mathematics, is the form and substance of the mathematics curriculum itself. The nature of these differences has been covered by global, and some local, analyses in the past (McKnight, et al., 1987; Travers and Westbury, 1989; Robitaille and Garden, 1989; and Robitaille and Travers, 1992). However, the opportunity to examine the differences from the actual items on a Japanese examination purporting to measure the "basic essentials" in mathematics that students should have garnered from their learning experiences and the opportunity to match those observations to the nature of the age/grade level curriculum provides a venue of comparison not afforded by the nature of most cross-cultural studies focusing on achievement or proficiency differences based from a common examination.

The Japanese mathematics curriculum for students in lower secondary school is quite explicit in its goals and expectations (Ministry of Education, n.d.). At grade one of the lower secondary school (equivalent to the U.S. seventh grade), the grade preceding the grade in which the assessment was given, the curriculum focuses on the system of integers and their operations; use of variables to represent physical situations involving linear equations; construction of basic geometric figures using straightedge and compass; consideration of the properties of elementary translations, reflections, and rotations of basic geometric figures; and graphing basic functional relationships in the Cartesian plane.

Grade two mathematics of the lower secondary school (equivalent to the U.S. eighth grade) has the objectives (Ministry of Education, n.d.) of:

- To help students develop their abilities to compute and transform algebraic expressions using letter symbols, according to their purposes, and to help them understand linear inequalities and simultaneous equations, and to foster their abilities to use them.

- To help students deepen their understanding of the properties of the fundamental figures in a plane, and thereby understand the significance and methods of mathematical inference with reference to consideration of the properties of figures, and to foster their abilities to precisely represent the process of inference.

- To help students further deepen their way of viewing and thinking variation and correspondence and understand the characteristics of linear functions, and foster their abilities to use them. Furthermore, to help students adequately represent numbers according to their purposes and develop their abilities to grasp the tendencies of statistical phenomena (p. 27).

These objectives of study for the year are followed by a careful curriculum content breakdown that shows a close correlation with the content found on the essential mathematics assessment. A section following the listing of the content provides direction on the comparison of the importance of the three objectives. In the teaching of content related to the first objective, teachers should represent the procedures through the use of flow-charting methods and see that students really cover the work on systems of equations in two variables; in the teaching of the second objective, teachers should ensure that students see the applications of similarity to the measurement of height and distance; in the consideration of content related to the third objective, focus should be given to the $a \times 10^n + \ldots$ standard form for showing decimal expansions of real numbers and to daily-life phenomena for data analysis situations.

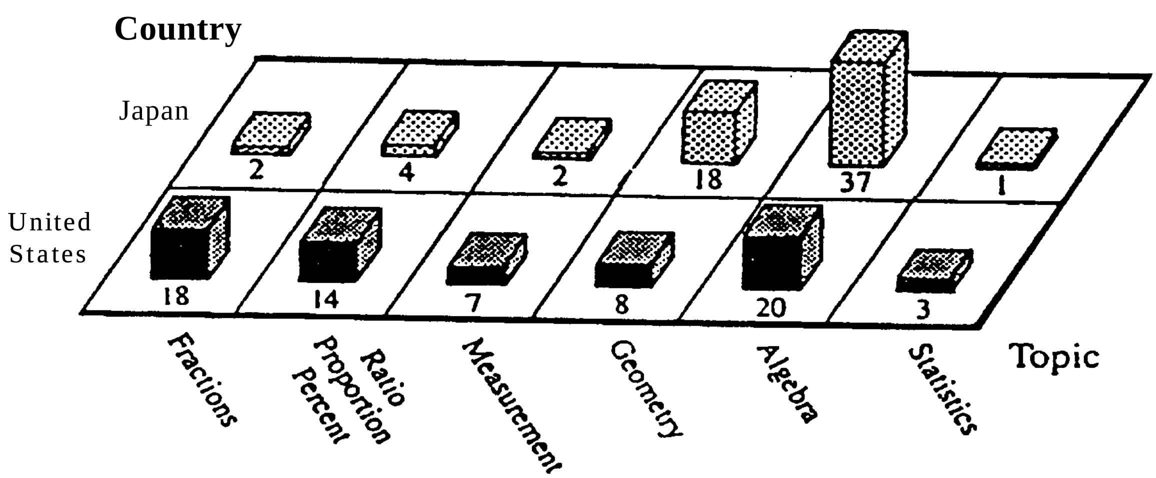
Thus the curriculum objectives for the year are focused on three basic areas of mathematics, specific content listing are provided for these, and additional comments on coverage of that content are given. This is akin to having curriculum standards and, a beginning to, delivery standards. It is clear from the curricular document what students are to have an opportunity to learn.

The closest comparison document, at the national level, for U.S. teachers at the same level is probably the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989). However, many middle school mathematics teachers are unaware of its existence (Weiss, 1995), as it has no legal authority in setting curricular content for the nation's schools. At the school level, there are stated goals for the grade eight mathematics curriculum. However, these goals may be tied to the NCTM recommendations, to a state or local curriculum guide, or to the sequence of pages in the book provided for instruction in mathematics. This leads to a great deal of diversity in the curriculum and a lack of national focus for mathematics education for students of this age (Mullis, et al., 1993; Dossey, et al., 1994). These differences of intentions, actions, and outcomes for grade eight mathematics are well covered in Robitaille's analysis (1989) of the outcomes of the SIMS study for age 13 youth.

Focus and Intensity

The above observations, supported by the distribution of expectation outlined by the profile of items included on the test, i Illustrated in figure 3, show a carefully focused curricular plan for achieving student growth in the areas of algebra and geometry, with lesser expectations in data analysis, at grade two of the lower secondary school. This plan is directly supported by the intended learning's outlined for the previous year. McKnight (1987) and Robitaille (1989) clearly outlined the impact that such intensity might have on student achievement in mathematics. The present curricular anal ysis and examination of the assessment items shows that both the message to teachers about the intentions in the national curricular plan and the intentions as communicated to teachers and students through the items they see on the assessment are careful ly coupled and mutually supporting. Further, consider the minor changes in this plan from the data reported on curricular intensity in 1981–82 for the eighth grade level curriculum in the SIMS study. These data, illustrated in figure 6 show teachers' reports of anticipated percentages of time they would be spending on five areas of the curriculum in their school year. These differences are also noted in analyses of textbooks for the respective grades (Stevenson and Bartsch, 1992).

Figure 6.— Teacher Reported Percents of Curricular Content Intensity for U.S. Grade 8 Classrooms and Japanese Level 1 Lower Secondary School Classrooms in 1981.



Source: adapted from McKnight, et al., 1992 p. 94.

These differences in intensity, or narrowed content focus, during a school year are directly related to the philosophical differences in the actual construction of the curricula. American textbooks, which define classroom activities in mathematics more than any other source, present a broad homogeneous content introduction for students, attempting to address in each year all of the potential topics outlined in the *Curriculum and Evaluation Standards for School Mathematics* (1989) appropriate for the intended grade level. There is ample review and some extension at each grade level, based on a "spiral" model for content introduction and coverage. This same pattern of broad coverage of possible topics of instruction with little change from year to year is modeled by the ubiquitous achievement tests used in schools and state assessments. Japanese textbooks, and the essential mathematics examination, paint a much narrower focus for the school curriculum.

This is a focus of few major objectives for a year's study. The ensuing intensity has a number of potentially strong outcomes related to observed proficiency and the supporting personal construction of knowledge in students. A prolonged focus on a smaller set of content topics in a given grade may allow for teachers to observe students' knowledge of the related concepts, procedures, understandings, and problem-solving skills in greater depth. This narrowed responsibility in coverage allows the teachers to become a bit more specialized in their teaching, allowing more preparation aimed at the connections necessary for real student growth in these few areas. The greater period of time devoted to these topics allows for teachers to observe more than the quick review and minimal extension of a topic before rotating to the next content area in a spirally designed curriculum.

The greater opportunity to observe student knowledge and ability to apply that knowledge allows for greater opportunity for the teacher to assess, plan, implement, and assess again relevant aspects of the enabling knowledges that students need to master the targeted objectives. The rapid coverage and rotation between topics in a spirally organized curriculum with many more grade level objectives and minimal growth expectations per grade does not afford teachers with such an opportunity to use their teaching skills.

Beyond the ability to actually implement good teaching aimed at bringing all students to a minimal performance level, as was evidenced by the performance of Japanese students on the items analyzed earlier, the narrowed focus potentially communicates both expectation and accountability to Japanese students and teachers. It is clear what they are to accomplish in a given school year. NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) presents a first step towards such a possible change in U.S. school mathematics. However, the broad statement of individual process and content standards for problem solving, communication, reasoning, connections, number and number relationships, number systems and number theory, computation and estimation, patterns and functions, algebra, statistics, probability, geometry, and measurement for grades five through eight does not sufficiently guide either a school, materials producer, or teacher to develop the focus observed in the Japanese curriculum and the related examinations. To achieve the coherence and focus observed in the Japanese materials, the *Curriculum and Evaluation Standards for School Mathematics* need to be further extended to provide grade level guidance about focus and primary activities for given years. This step to achievement and delivery standards for school mathematics *is curricularly achievable* within the framework outlined by the NCTM content standards. Whether it is *politically acceptable* or *systemically implementable are* larger and more volatile questions.

References

Dossey, J. A., Mullis, I. V. S., and Jones, C. O. (1993). *Can students do mathematical problem solving? Results from constructed-response questions in NAEP's 1992 mathematics assessment.* Washington, DC: National Center for Education Statistics.

Dossey, J. A., Mullis, I., Gorman, S., and Latham, A. (1994). *How school mathematics functions: Perspectives from the NAEP 1990 and 1992 Assessments.* Washington, DC: National Center for Education Statistics.

Grouws, D. A. (Ed.). (1992). *Handbook of research on mathematics teaching and learning.* New York, NY: Macmillan.

Husén, T. (Ed.). (1967). *International study of achievement in mathematics: A comparison of twelve countries.* (2 vols.). New York, NY: John Wiley and Sons.

Lapointe, A., Mead, N., and Askew, J. (1992). *Learning mathematics.* Princeton, NJ: Educational Testing Service.

Leestma, R., and Walberg, H. (Eds.). (1992). *Japanese educational productivity.* Ann Arbor, MI: Center for Japanese Studies, The University of Michigan.

McKnight, C. C., Crosswhite, F. J., Dossey, J. A., Kifer, E., Swafford, J. O., Travers, K. J., and Cooney, T. J. (1987). *The underachieving curriculum: Assessing U.S. School mathematics from an international perspective.* Champaign, IL: Stipes Publishing.

Medrich, E., and Griffith, J. (1992). *International mathematics; and science assessments: What have we learned?* Washington, D. C.: National Center for Education Statistics.

Ministry of Education. (n.d.). *Excerpt from the National Courses of Study: Revised by the Ministry o fEducation.* (Mathematics Sections). (Translated by Nagasaki, E., Sawada, T., and Senuma, H.) Tokyo, Japan: Japan Society of Mathematical Education.

Mullis, I., Dossey, J., Owen, E., and Phillips, G. (1993). *NAEP 1992 mathematics report card for the nation and states: Data from the national and trial state assessments.* Washington, D. C.: National Center for Education Statistics.

Mullis, I., Dossey, J., Owen, E., and Phillips, G. (1991). *The State of Mathematics Achievement: NAEPs 1990 Assessment of the Nation and the Trial Assessment of the States.* Washington, D. C.: National Center for Education Statistics.

National Assessment of Educational Progress. (1988). *Mathematics objectives: 1990 assessment.* Princeton, NJ: Educational Testing Service.

National Assessment of Educational Progress. (1987). *Mathematics objectives: 1985-86 assessment.* Princeton, NJ: Educational Testing Service.

National Council of Supervisors of Mathematics. (1977). *National Council of Supervisors of Mathematics Position Paper on Basic Mathematics Skills.* Minneapolis, MN: National Council of Supervisors of Mathematics.

National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics of the 1980s.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1981). *Priorities in school mathematics: Executive summary of the PRISM Project.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Institute of Educational Research. (1992). *Special Study on "Essential Skills" Statistical Report No. 1:1990 data*. (Translated by S. Takahira). Tokyo, Japan: National Institute of Educational Research.

Robitaille, D., and Garden, R. (1989). *The IEA Study of Mathematics II: Contexts and outcomes of school mathematics*. *New* York, NY: Pergamon Press.

Robitaille, D. (1989). Students' achievements: Population A. In D. Robitaille and R. Garden. *The IEA Study of Mathematics II: Contexts and outcomes of school mathematics*. *New* York, NY: Pergamon Press. (pp. 102–125).

Robitaille, D., and Travers, K. (1992). International studies of achievement in mathematics. In D. A. Grouws, (Ed.). *Handbook of research on mathematics teaching and learning*. *New* York, NY: Macmillan. (pp. 687-709)

Travers, K., and Westbury, I. (1989). *The IEA Study of Mathematics I: Analysis of mathematics curricula*. *New* York, NY: Pergamon Press.

Sawada, T. (1983). *Mathematics achievement and teaching practice in lower secondary schools (Grade 7)*. Tokyo, Japan: National Institute for Educational Research.

Stevenson, H., and Bartsch, K. (1992). An analysis of Japanese and American textbooks in mathematics. In R. Leestma and H. Walberg. (Eds.). *Japanese educational productivity*. Ann Arbor, MI: Center for Japanese Studies, University of Michigan. (pp.103–134).

Stevenson, H., Lummins, M., Lee, S., and Stigler, J. W. (1990). *Making the grade in mathematics: Elementary school mathematics in the United States, Taiwan, and Japan*. Reston, VA: National Council of Teachers of Mathematics.

Stigler, J., and Perry, M. (1988). Cross-cultural studies of mathematics teaching and learning: Recent findings and new directions. In D. Grouws, T. Cooney, and D. Jones. (Eds.). *Perspectives on research on effective mathematics teaching*. Reston, VA: National Council of Teachers of Mathematics.

Stigler, J., Lee, S., and Stevenson, H. (1990). *Mathematical knowledge of Japanese, Chinese, and American elementary school children*. Reston, VA: National Council of Teachers of Mathematics.

Westbury, I., Ethington, C., Sosniak, L., and Baker, D. P. (Eds.). (1994). *In search of more effective mathematics education: Examining data from the IEA Second International Mathematics Study*. Norwood, NJ: Ablex Publishing Company.

Weiss, I. (1995). *A profile of science and mathematics in the United States: 1993*. Chapel Hill, NC: Horizon Research, Inc.

Appendix A:

Content Matrix for the

1992 NAEP Mathematics Assessment

Taken from:

National Assessment of Educational Progress. (1988). ***Mathematics objectives: 1990 assessment.*** **Princeton, NJ: Educational Testing Service.**

Note: These objectives were also used for the 1992 NAEP assessment.

Chapter four

Content Areas

To conduct a meaningful assessment of mathematics proficiency, it is necessary to measure students' abilities in various content areas. Classification of topics into these content areas cannot be exact, however, and inevitably involves some overlap. For example, some topics appearing under Data Analysis, Statistics, and Probability may be closely related to others that appear under Algebra and Functions. Context can also determine content area; for example, a question asking students to compute the area of a geometric figure may be considered either Measurement or Geometry, depending on the representation of the problem.

The following sections of this chapter provide a brief description of each content area with a list of topics and subtopics illustrative of those to be included in the assessment. Using the topics provided in the CCSSO report, the NAEP Item Development Panel generated lists of subtopics. This level of specificity was needed to guide item writers and ensure adequate coverage of the content areas and abilities to be assessed.

Numbers and Operations

This content area focuses on students' understanding of numbers (whole numbers, fractions, decimals, integers) and their application to real-world situations, as well as computational and estimation situations. Understanding numerical relationships as expressed in ratios, proportions, and percents is emphasized. Students' abilities in estimation, mental computation, use of calculators, generalization of numerical patterns, and verification of results are also included.

The grade 4 assessment should include questions requiring the manipulation of whole numbers, simple fractions, and decimals, using the operations of addition, subtraction, multiplication, and division. The grade 8 assessment should include questions using whole numbers, fractions, decimals, signed numbers, and numbers expressed in scientific notation. In addition to the operations included in the grade 4 assessment, students at grade 8 should be asked to demonstrate their ability to work with elementary powers and roots.

Students participating in the grade 12 assessment should demonstrate a detailed understanding of real numbers — including whole numbers, fractions, decimals, signed numbers, rational and irrational numbers, and numbers expressed in scientific notation — and a general understanding of complex numbers. The operations assessed at this grade level include addition, subtraction, multiplication, division, powers, and roots.

Topic: Numbers and Operations	Grade 4	Grade 8	Grade 12
1. Relate counting, grouping, and place value.			
a. Whole number place value	•	•	•
b. Rounding whole numbers	•	•	•
c. Decimal place value	•	•	•
d. Rounding decimals	•	•	•
e. Order of magnitude (estimation related to place value)	•	•	•
f. Scientific notation		•	•

Topic: Numbers and Operations	Grade 4	Grade 8	Grade 12
2. Represent numbers and operations using models, diagrams, and symbols.			
a. Set models such as counters	•	m.	
b. Number line models	•	09	
c. Region models (two- and three-dimensional)	•	•	•
d. Other models (e.g., draw diagrams to represent a number or an operation; write a number sentence to fit a situation or describe a situation to fit a number sentence)	•		--
3. Read, write, rename, and compare numbers	•		--
4. Compute with numbers.			
a. Basic properties of operations	•	00	
b. Effect of operations on size and order of numbers	•		--
c. Features of algorithms (e.g., regrouping and partial products)	•	•	\
d. Selection of procedure (e.g., pencil and paper, calculator, mental arithmetic)	•	*o	
e. Applications	•	**	
5. Make estimates appropriate to a given situation.			
a. When to estimate	•	00	
b. What form to use			
i. Overestimate	•		--
ii. Underestimate	•	00	
iii. Range of estimate	•	0	0
c. Applications	•	*	*
d. Order of magnitude (scientific notation)	○	•	•
6. Verify solutions and determine the reasonableness of a result.			
a. Abstract settings	•	*	6
b. Real-world situations	•	**	
7. Apply ratios, proportions, and percents in a variety of situations.			
a. Ratio and proportion			
i. Meaning of ratio and proportion	○	•	•
ii. Simple ratio	○	•	•
iii. Proportion	○	•	•
iv. Scale	○	•	•
v. Rate	○	•	•

Topic: Numbers and Operations	Grade 4	Grade 8	Grade 12
b. Percent			
i. Meaning of percent	○	•	•
ii. p% of q = r (find one, given the other two)	○	•	•
iii. Percent change	○	•	•
v. Percents greater than 100	○	•	•
v. Percents less than 1	○	•	•
vi. Applications such as interest, discounts, prices, rates	○	•	•
8. Use elementary number theory.			
a. Odd and even	•	**	
b. Multiples including LCM and divisors including GCD	○	•	•
c. Prime numbers	○	•	•
d. Factorization (includes prime factorization)	○	•	•
e. Divisibility	○	•	•
f. Remainders	○	•	•
g. Number patterns	○	•	•

Measurement

This content area focuses on students' ability to describe real-world objects using numbers. Students should be asked to identify attributes, select appropriate units, apply measurement concepts, and communicate measurement-related ideas to others. Questions should be included that require an ability to read instruments using metric, customary, or nonstandard units, with emphasis on precision and accuracy. Questions requiring estimates, measurements, and applications of measurements of length, time, money, temperature, mass/weight, area, volume, capacity, and angles are also included under this content area.

The measurement concepts to be considered in the grade 4 assessment are length (perimeter), area, capacity, weight and mass, angle measure, time, money, and temperature. At grades 8 and 12, these measurement concepts are length (perimeter and circumference), area and surface area, volume and capacity, weight and mass, angle measure, time, money, and temperature. At all three grades, students are asked to work with customary, metric, and nonstandard units.

Topic: Measurement	Grade 4	8	12
1. Compare objects with respect to a given attribute.	●		‑‑
2. Select and use appropriate measurement instruments.			
a. Ruler, meter stick, etc. (distance)	●		‑‑
b. protractor	●	00	
c. Thermometer	●	**	
d. Scales for weight or mass	●	●	●
e. Gauges	●	*	9
3. Select and use appropriate units of measurement.			
a. Type	●	0*	
b. Size	●	00	
4. Determine perimeter, area, volume, and surface area.			
a. Perimeter			
i. Triangles	●	●	●
ii. Squares	●	●	●
iii. Rectangles	●	●	●
iv. Parallelograms	●	●	●
v. Trapezoids	●	●	●
vi. Other quadrilaterals	●	*	O
vii. Combinations	●	●	●
viii. Other polygons	●	●	●
ix. Circles	o	●	●
b. Area			
i. Squares	●	●	●
ii. Rectangles	●	●	●
iii. Triangles	0	●	●
iv. Parallelograms	o	●	●
v. Trapezoids	0	●	●
vi. Other quadrilaterals	0	●	●
vii. Circles	o	●	●
viii. Combinations	o	●	●
ix. Other polygons	o	●	●

Topic: Measurement	Grade 4	8	12
c. Volume			
i. Rectangular solids	●	*o	
ii. Cylinders	o	●	●
iii. Cones	o	o	●
iv. Pyramids	o	o	●
v. Prisms	o	o	●
vi. Combinations	o	o	●
d. Surface area			
i. Rectangular solids	o	●	●
ii. Cylinders	o	●	●
iii. Cones	o	o	●
iv. pyramids	o	o	●
v. Prisms	o	o	●
vi. Combinations	o	o	●
5. Estimate the size of an object or a measurement.	●	●	●
6. Apply common measurement formulas.	o	●	●
7. Convert from one measurement to another within the same system.	o	●	●
8. Determine precision accuracy, and error.			
a. Significant digits	o	●	●
b. Size of unit of measurement	o	●	●
c. Accuracy of measurement	o	●	●
d. Absolute and relative error	o	o	●
9. Make and read scale drawings.			
a. Convert from scale to actual measurement	o	●	●
b. Convert from actual measurement to scale	o	●	●

Geometry

This content area focuses on students' knowledge of geometric figures and relationships and on their skills in working with this knowledge. These skills are important at all levels of schooling as well as in practical applications. Students need to be able to model and Visualize geometric figures in one, two, and three dimensions and to communicate geometric ideas. In addition, students should be able to use informal reasoning to establish geometric relationships.

	Grade 4	Grade 8	Grade 12
1. Describe, compare, and classify geometric figures.			
a. Points, lines, segments, and rays in a plane and in space.			
i. Parallel lines	●	●	●
ii. Perpendicular lines	●	●	●
iii. Skew lines	○	●	●
iv. Diagonals	○	●	●
v. Bisectors	○	●	●
vi. Radius	○	●	●
vii. Diameter	○	●	●
viii. Altitudes	○	○	●
ix. Medians	○	○	●
b. Angles in a plane			
i. In triangles and other polygons	●	●	●
ii. Supplementary	○	●	●
iii. Complementary	○	●	●
iv. En circles	○	●	●
v. Right angles	○	●	●
vi. Angle bisector	○	○	●
vii. Alternate interior and corresponding	○	○	●
viii. Vertical	○	○	●
c. Triangles			
i. General properties of triangles	●	●	●
ii. Acute, right, or obtuse	○	●	●
iii. Equilateral	○	●	●
iv. Isosceles	○	●	●
v. Scalene	○	●	●
d. Quadrilaterals			
i. Square	●	●	●
ii. Rectangle	●	●	●
iii. Parallelogram	○	●	●
iv. Trapezoid	○	●	●
v. Rhombus	○	●	●
e. Other polygons			
i. Regular, not regular	○	●	●
ii. Convex, concave	○	○	●
iii. Interior and exterior angle measures	○	○	●
f. Three-dimensional solids			
i. Rectangular solid	●	●	●
ii. Prism	●	●	●
iii. Pyramid	●	●	●
iv. Cylinder	●	●	●
v. Cone	○	●	●
vi. Sphere	○	●	●

	Grade 4	Grade 8	Grade 12
g. Circles			
i. General properties of circles	●	mm	
ii. Secants, tangents, chords, arcs, circumscribed and inscribed circles	○	○	●
2. Given descriptive information, visualize, draw, and construct geometric figures.			
a. Draw or sketch a figure given a verbal description	●	**	
b. Straightedge and compass constructions			
i. Angle bisector	○	●	●
ii. A line perpendicular to a given line that passes through a given point	○	●	●
iii. A line parallel to a given line that passes through a given point	○	●	●
c. Given a figure, Write a verbal description of its geometric qualities	○	●	●
3. Investigate and predict results of combining, subdividing, and changing shapes [e.g., paper folding, dissecting, tiling, and rearranging pieces of solids).	●	**	
4. Identify the relationship between a figure and Its image under a transformation.			
a. Motion geometry (informal: lines of symmetry, flips, turns, and slides)	●	**	
b. Transformations (translations, rotations, reflections, dilations, symmetry)	○	○	●
5. Describe the intersection of two or more geometric figures.			
a. Two-dimensional	○	●	●
b. Three-dimensional	○	●	●
6. Classify figures in terms of congruence and similarity, and informally apply these relationships.	○	●	●
7. Apply geometric properties and relationships in solving problems.			
a. Between, inside, on, and outside	○	●	●
b. Pythagorean relationship			
i. Special right triangles (e.g., 3-4-5, 30° 60°-90° 45°-45°-90°)	○	●	●
c. Properties of similarity			
i. Ratio and proportion..	○	○	●

Topic: Geometry	Grade 4	8	12
d. Prove congruence of triangles	○	○	●
e. Others	○	●	●
Establish and explain relationships involving geometric concepts.			
a. Logic	●	●	●
b. Informal induction and deduction	○	●	●
9. Represent problem situations with geometric models and apply properties of figures.	●	●	●
10. Represent geometric figures and properties algebraically using coordinates and vectors.			
a. Distance formula	○	○	●
b. Slope	○	○	●
c. Parallel, perpendicular lines	○	○	●
d. Midpoint formula	○	○	●
e. Conic sections	○	○	●
f. Vectors			
i. Addition, subtraction	○	○	●
ii. Scalar multiplication, dot product	○	○	○

Data Analysis, Statistics, and Probability

This content area focuses on data representation and analysis across all disciplines, and reflects the importance and prevalence of these activities in our society. Statistical knowledge and the ability to interpret data are necessary skills in the contemporary world. Questions should emphasize appropriate methods for gathering data, the visual exploration of data, and the development and evaluation of arguments based on data analysis. For grade 4, students can be asked to make predictions from given results and explain their reasoning.

Topic: Data Analysis, Statistics, and Probability	Grade 4	8	12
1. Read, interpret, and make predictions using tables and graphs.			
a. Read data	●	●	●
b. Interpret data	●	●	●
c. Solve problems and compute with data	●	●	●
d. Estimate using data	●	●	●
e. Interpolate or extrapolate	○	●	●
2. Organize and display data and make inferences.			
a. Tables	●	●	●
b. Bar graphs	*	●	●
c. Pictograms	*	●	●
d. Line graphs	○	●	●
e. Circle graphs	○	●	●
f. Scattergrams	○	●	●
g. Other			
i. Stem and leaf plots	○	●	●
ii. Box and whisker plots	○	*	*
iii. Outliers	○	*	m
3. Determine the probability of a simple event.			
a. Sample space	●	●	●
b. Definition of probability	●	●	●
c. Odds	●	●	●
d. Expected value	●	●	●
e. Counting principle (permutations and combinations)	○	●	●
f. Independent/dependent events	○	●	●
4. Compute measures of central tendency and dispersion			
a. Mean	●	●	●
b. Range	○	○	●
c. Median	○	●	●
d. Mode	○	●	○
5. Recognize sampling, randomness, and bias in data collection.			
a. Given a situation, identify sources of sampling error	○	●	●
b. Describe a procedure for selecting an unbiased sample	○	○	●

Topic: Algebra and Functions

	Grade 4	Grade 8	Grade 12
3. Use number lines and rectangular coordinate systems.			
a. Plot or identify points on a number line or in a rectangular coordinate system	●	●	●
b. Graph solution sets on the number line	○	●	●
c. Work with elementary applications using coordinates	○	●	●
4. Solve linear equations and inequalities. (Note: The complexity of equations and inequalities will vary depending on the coefficients, number of terms, operations, and solution set.)			
a. Solution sets of whole numbers	●	●	●
b. Solution sets of rational numbers	○	●	●
c. Solution sets of ordered pairs	○	●	●
d. Solution sets of real and imaginary numbers	○	○	●
5. Perform algebraic operations with real numbers and algebraic expressions.			
a. Addition, subtraction, multiplication, division	○	●	●
b. Powers and roots	○	●	●
c. Multiple operations (grouping and order of operations)	○	●	●
d. Substitution in expressions and formulas	○	●	●
e. Equivalent forms (simplify, combine, expand, and factor)	○	●	●
f. Solving a formula for one variable	○	●	●
6. Represent functions and relations by number sentences, verbal statements, models, tables, graphs, variables, algebraic expressions, and equations, and translate among modes. (Note: At the grade 4 level, algebraic and function concepts are treated in more informal, exploratory ways.)	●	●	●
7. Solve systems of equations and inequalities algebraically and graphically.	○	●	●
8. Use mathematical methods.			
a. Logic	○	●	●
b. informal induction and deduction	○	●	●

Topic: Algebra and Functions

	Grade 4	Grade 8	Grade 12
9. Represent problem situations with discrete structures.			
a. Finite graphs	○	○	●
b. Matrices	○	○	●
c. Sequences	○	○	●
d. Series	○	○	●
e. Recursive relations	○	○	●
10. Solve polynomial equations with real and complex roots algebraically and graphically.			
a. Factoring	0	0	●
b. Graphing	0	0	●
c. Factor Theorem	0	0	●
d. Synthetic and long division	0	0	●
e. Estimation of roots....	0	0	●
f. Special techniques for quadratic equations (quadratic formula, completing squares)	○	○	●
11. Apply function notation and terminology.			
a. Domain and range	○	○	●
b. Composite functions	0	0	●
c. inverse	○	○	●
12. Compare and apply the numerical, algebraic, and graphical properties of functions.			
a. Absolute value	○	○	●
b. Linear	○	○	●
c. Polynomial	○	○	●
d. Exponential	0	0	●
e. Logarithmic	○	○	●
f. Trigonometric	○	○	●
13. Apply trigonometric concepts.			
a. Circular functions and their inverses	○	○	●
b. Radian measure	○	○	●
c. Trigonometric identities.	○	○	●
d. Applications			
i. Geometric problems	○	○	●
ii. Periodic real-world phenomena	○	○	●

Appendix B:

Mathematical Abilities for the 1992 NAEP Mathematics Assessment

Taken from:

National Assessment of Educational Progress. (1988). *Mathematics objectives: 1990 assessment.* Princeton, NJ: Educational Testing Service.

Note: These ability definitions were also used in the 1992 assessment.

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples and counterexamples of concepts; can use and interrelate models, diagrams, and varied representations of concepts; can identify and apply principles; know and can apply facts and definitions; can compare, contrast, and integrate related concepts and principles; can recognize, interpret, and apply the signs, symbols, and terms used to represent concepts; and can interpret the assumptions and relations involving concepts in mathematical settings. Such understandings are essential to performing procedures in a meaningful way and applying them in problem-solving situations.

Abilities:

1. Recognize, label, and generate examples and counterexamples of concepts.
2. Use models, diagrams, and symbols to represent concepts.
3. Identify and apply principles.
4. Know and apply facts and definitions.
5. Make connections among different modes of representation of concepts.
6. Compare, contrast, and integrate concepts and principles.
7. Recognize, interpret, and apply symbols to represent concepts.
8. Interpret assumptions and relations involving concepts.

Procedural Knowledge

Students demonstrate procedural knowledge in mathematics when they provide evidence of their ability to select and apply appropriate procedures correctly; verify and justify the correctness of a procedure using concrete models or symbolic methods; and extend or modify procedures to deal with factors inherent in problem settings.

Procedural knowledge includes the various numerical algorithms in mathematics that have been created as tools to meet specific needs in an efficient manner. It also encompasses the abilities to read and produce graphs and tables, execute geometric constructions, and perform noncomputational skills such as rounding and ordering.

Abilities:

1. Select and apply appropriate procedures correctly.
2. Verify and justify the correctness of applications of procedures.

Problem Solving

In problem solving, students are required to use their reasoning and analytic abilities when they encounter new situations. Problem solving includes the ability to recognize and formulate problems; determine the sufficiency and consistency of data; use strategies, data, models, and relevant mathematics; generate, extend, and modify procedures; use reasoning (i. e., spatial, inductive, deductive, statistical. and proportional); and judge the reasonableness and correctness of solutions.

Abilities:

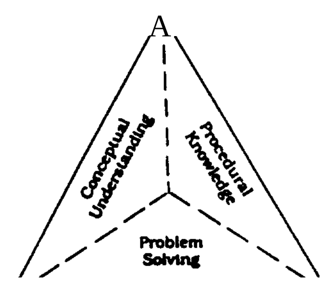
1. Recognize and formulate problems.
2. Understand data sufficiency and consistency.
3. Use strategies, data, models, and relevant mathematics.
4. Generate, extend, and modify procedures.
5. Reason (spatially, inductively, deductively, statistically, and proportionally).
6. Judge the reasonableness and correctness of solutions.

Chapter three

Mathematical Abilities

Students' mathematical abilities can be classified into three categories: conceptual understanding, procedural knowledge, and problem solving (see Figure 2). This classification is not meant to be hierarchical, in that questions within any of the three categories may be relatively complex or simple. Problem solving involves interactions between conceptual knowledge and procedural skills at any grade level, but what is considered complex problem solving at one grade level may be considered conceptual understanding or procedural knowledge at a different grade level. The same concept or skill can be assessed in a variety of representations, with tables, pictures, verbal descriptions, or other cues. The context of a question thus helps to determine its categorization.

Figure 2
Mathematical Abilities



Appendix C:

Framework for the Japanese Essential Mathematics Assessment

Taken from:

National Institute of Educational Research. (1992). ***Special Study on "Essential Skills" Statistical Report No. 1:1990 data.*** **(Translated by Takahira, S.). Tokyo, Japan: Japanese National Institute of Educational Research.**

The following materials provide NIER's (1992) translation of the Japanese National Institute of Educational Research framework for the essential mathematics assessment.

The First Dimension: Behavior Types.

It is a general misinterpretation that the acquisition of mathematical knowledge and skills are [sic] the most important objectives [sic] of education, and that mathematical ways of thinking or creative thinking are at levels somewhat higher than the essential mathematical achievement level. Rather, in the present study, we emphasize the importance of both acquisition of knowledge and skills and the acquisition of mathematical ways of thinking. In order to make this point clear, we looked at the behavioral components of the purpose of education and identified five behavior types.

A) Knowledge:

> Knowledge is the contents of memory wherein one is able to understand meaning and is able to adapt and apply those meanings as they are needed. Examples include declarative knowledge (symbols and terms), procedural knowledge (how to do things) and so on.

B) Understanding:

> Understanding is a state of mind in which one is able to see underlying relationships among variables, such as the relationships between a part and a whole, or subordinate relationships between variables. For example, one understands meanings, concepts, principles, and laws.

C) Thoughts:

> In a new problem situation, one has the ability to be able to interpret and solve new problems by using knowledge and principles that were already known. For example, thought is the ability to discover or interpret the characteristics of problems.

D) Skills:

> Skills are the behavior styles that are formalized in order to accomplish certain goals utilizing one's knowledge and understanding. For example, there are computational skills and drawing skills.

E) Attitudes:

> Attitudes are the tendencies to think and to view things. Also, attitude has emotional and cognitive aspects that direct a person's own behavior (e.g. tendency to view things uniform ly)....

The Second Dimension: Contents of Mathematics

In order to be consistent with the entire school system (from elementary school to high school), mathematical content for essential mathematical achievement was divided into three types: numerical content, geometrical content, and relational content. The details of each content follow.

P) Numerical content:

> number, computation, formulas, and algebra

Q) Geometrical content:

> figure, figure and measurement, analytical geometry, and trigonometric ratio

R) Relational content:

> set, measurement, ratio and proportion, function, probability, and statistics

The Third Dimension: Mathematical Processes

In mathematics classes, the materials for mathematical learning are often limited to artificial logical content which is not practical and totally separated from the students' everyday lives. On the other hand, it is known that understanding the importance of applicability of mathematics to real life more effectively improves the mathematical activity of students. We emphasized not only the static aspect of mathematical activity but also the dynamic aspect (processes) of mathematical activity. The mathematical processes are divided into three areas: mathematicalization, mathematical transaction, and mathematical verification.

X) Mathematicalization:

> mathematicalization is the process in which a phenomenon is translated into mathematical structures (e.g. designing hypotheses, designing functions, expressing phenomena by symbols, and applying mathematical expressions to real-life situations)

Y) Mathematical transaction:

> mathematical transaction is the process in which a mathematical operation is carried out based on a mathematical structure. (e.g. carrying out calculation and operation, logical inference, and selection of an axiom)

Z) Mathematical verification:

> Mathematical verification is the process which confirms whether or not the mathematical operation was right. (e.g. confirmation of results of computations, solutions, and data)

There are two processes within each of these three processes. The first process in mathematical ization is to mathematicalize the phenomenon in real life. The second process is the process that modifies the problem in order to make it easier to solve it.

The first process in mathematical transaction is the process in which each computation is carried out and propositions are proven. Generally, almost all mathematics classes are spent on this activity. The second situation in mathematical transaction is the process in which the propositions are being synthesized into mathematical theory and systematized.

The first process of the mathematical verification is the process which confirms whether or not the results of the mathematical transaction fits [sic] into a real life situation. The second process is the process which verifies whether or not the process of mathematical operation is right. The confirmation of whether or not the resu its or proof processes are right is an example of the second process (pp.12-14).