Appendix A: Content Framework Summary Documents

This appendix presents information about the NAEP 2003 and TIMSS 2003 mathematics content frameworks used for item classifications at the expert panel meeting.

Exhibit A-1 is the summary document that was used by the expert panel for the classification of items to the *content strands*, *topics*, and *subtopics* in the NAEP 2003 mathematics framework.

Exhibit A-2 is the summary document that was used by the expert panel for the classification of items to the *content domains*, *topic areas*, and *objectives* in the TIMSS 2003 mathematics framework.

These summary documents are based on the NAEP 2003 and TIMSS 2003 framework and assessment specifications documents, but have been reformatted and adapted slightly to facilitate the classification process.

Framework summary documents were not prepared for PISA; expert panel content classifications based on the PISA framework were made only at the *overarching idea* level, and there are no topics or subtopics specified in the PISA framework. For more information about the PISA framework, see OECD (2003).

Exhibit A-1. NAEP mathematics framework and specifications summary: 2003

A	NUMBER SENSE, PROPERTIES AND OPERATIONS	Grade(s)		
A1	**Relate counting, grouping and place value**			
1a:	Use place value to model and describe whole numbers and decimals	4	8	12
1b:	Use scientific notation in meaningful contexts	•	8	12
A2	**Represent numbers and operations in a variety of equivalent forms using models, diagrams and symbols**			
2a:	Model numbers using set models such as counters	4	•	•
2b:	Model numbers using number lines	4	8	•
2c:	Use two- and three-dimensional region models to describe numbers	4	8	12
2d:	Use other models appropriate to a given situation (e.g., draw diagrams to represent a number or an operation; write a number sentence to fit a situation or describe a situation to fit a number sentence; interpret calculator or computer displays)	4	8	12
2e:	Read, write, rename, order, and compare numbers	4	8	12
A3	**Compute with numbers (i.e., add, subtract, multiply, divide)**			
3a:	Apply basic properties of operations	4	8	12
3b:	Describe effect of operations on size and order of numbers	4	8	12
3c:	Describe features of algorithms (e.g., regrouping with or without manipulative, partial products)	4	8	12
3d:	Select appropriate computation method (e.g., pencil and paper, calculator, mental arithmetic)	4	8	12
A4	**Use computations and estimation in applications**			
4a:	Round whole numbers, decimals, and fractions in meaningful contexts	4	8	12
4b:	Make estimates appropriate to a given situation			
	i. Know when to estimate	4	8	12
	ii. Select appropriate type of estimate (overestimate, underestimate, range of estimate)	4	8	12
	iii. Describe order of magnitude (estimation related to place value; scientific notation)	4	8	12
4c:	Select appropriate method of estimation (e.g., front-end, rounding)	4	8	12
4d:	Solve application problems involving numbers and operations, using exact answers or estimates, as appropriate	4	8	12
4e:	Interpret round-off errors using calculators/computers (i.e., truncation)	•	(8)	12
4f:	Verify solutions and determine the reasonableness of results			
	i. In real-world settings	4	8	12
	ii. In abstract settings	•	•	12

See notes at end of exhibit.

Exhibit A-1. NAEP mathematics framework and specifications summary: 2003—Continued

A	NUMBER SENSE, PROPERTIES AND OPERATIONS	Grade(s)		
A5	**Apply ratios and proportional thinking in a variety of situations**			
5a:	Use ratios to describe situations	(4)	8	12
5b:	Use proportions to model problems	•	8	12
5c:	Use proportional thinking to solve problems (including rates, scaling, and similarity)	•	8	12
5d:	Understand the meaning of percent (including percents greater than 100 and less than 1)	(4)	8	12
5e:	Solve problems involving percentages	•	8	12
A6	**Use elementary number theory**			
6a:	Describe odd and even numbers and their characteristics	4	8	12
6b:	Describe number patterns	(4)	8	12
6c:	Use factors and multiples to model and solve problems	•	8	12
6d:	Describe prime numbers	•	8	12
6e:	Use divisibility and remainders in problem settings (including simple modular arithmetic)	•	(8)	12

See notes at end of exhibit.

Exhibit A-1. NAEP mathematics framework and specifications summary: 2003—Continued

B	MEASUREMENT	Grade(s)		
B1	**Estimate the size of an object or compare objects with respect to a given attribute (e.g., length, area, capacity, volume, weight/mass)**			
		4	**8**	**12**
B2	**Select and use appropriate measurement instruments (e.g., manipulatives such as a ruler, meter stick, protractor, thermometer, scales for weight or mass, and gauges)**			
		4	**8**	**12**
B3	**Select and use appropriate units of measurement, according to**			
3a:	Type of unit	4	8	12
3b:	Size of unit	4	8	12
B4	**Estimate, calculate (using basic principles or formulas), or compare perimeter, area, volume, and surface area in meaningful contexts to solve mathematical and real-world problems.**			
4a:	Solve problems involving perimeter and area (e.g., triangles, quadrilaterals, other polygons, circles, and combined forms)	(4)	8	12
4b:	Solve problems involving volume and surface area (e.g., rectangular solids, cylinders, cones, pyramids, prisms, and combined forms)	(4)	(8)	12
B5	**Apply given measurement formulas for perimeter, area, volume, and surface area in problem settings**			
		•	**8**	**12**
B6	**Convert from one measurement to another within the same system (customary or metric)**			
		•	**8**	**12**
B7	**Determine precision, accuracy, and error**			
7a:	Apply significant digits in meaningful contexts	•	8	12
7b:	Determine appropriate size of unit of measurement in problem situations	•	8	12
7c:	Apply concepts of accuracy of measurement in problem situations	•	8	12
7d:	Apply absolute and relative error in problem situations	•	•	12
B8	**Make and read scale drawings**			
		•	**8**	**12**
B9	**Select appropriate methods of measurement (e.g., direct or indirect)**			
		4	**8**	**12**
B10	**Apply the concept of rate to measurement situations**			
		•	**8**	**12**

See notes at end of exhibit.

Exhibit A-1. NAEP mathematics framework and specifications summary: 2003—Continued

C	GEOMETRY AND SPATIAL SENSE	Grade(s)		
C1	**Describe, visualize, draw and construct geometric figures**			
1a:	Draw or sketch a figure given a verbal description	4	8	12
1b:	Given a figure, write a verbal description of its geometric qualities	•	8	12
C2	**Investigate and predict results of combining, subdividing, and changing shapes (e.g., paper folding, dissecting, tiling, and rearranging pieces of solids)**			
		4	**8**	**12**
C3	**Identify the relationship (congruence, similarity) between a figure and its image under a transformation**			
3a:	Use motion geometry (informal: lines of symmetry, flips, turns, and slides)	4	8	12
3b:	Use transformations (translations, rotations, reflections, dilations, symmetry)			
	i. synthetic	•	(8)	12
	ii. algebraic	•	•	12
C4	**Describe the intersection of two or more geometric figures**			
4a:	Two-dimensional	•	8	12
4b:	Planar cross-section of a solid	•	8	12
C5	**Classify figures in terms of congruence and similarity, and informally apply these relationships using proportional reasoning where appropriate**			
		•	**8**	**12**
C6	**Apply geometric properties and relationships in solving problems**			
6a:	Use concepts of 'between', 'inside', 'on' and 'outside'	4	8	•
6b:	Use the Pythagorean relationship to solve problems	•	8	12
6c:	Apply properties of ratio and proportion with respect to similarity	•	(8)	12
6d:	Solve problems involving right triangle trigonometric applications	•	•	12
C7	**Establish and explain relationships involving geometric concepts**			
7a:	Make conjectures	4	8	12
7b:	Validate and justify conclusions and generalizations	4	8	12
7c:	Use informal induction and deduction	(4)	8	12
C8	**Represent problem situations with geometric models and apply properties of figures in meaningful contexts to solve mathematical and real-world problems**			
		4	**8**	**12**
C9	**Represent geometric figures and properties algebraically using coordinates and vectors**			
9a:	Use properties of lines (including distance, midpoint, slope, parallelism, and perpendicularity) to describe figures algebraically	•	(8)	12
9b:	Algebraically describe conic sections and their properties	•	•	12
9c:	Use vectors in problem situations (addition, subtraction, scalar multiplication, dot product)	•	•	12

See notes at end of exhibit.

Exhibit A-1. NAEP mathematics framework and specifications summary: 2003—Continued

D	DATA ANALYSIS, STATISTICS AND PROBABILITY	Grade(s)		
D1	**Read, interpret, and make predictions using tables and graphs**			
1a:	Read and interpret data	4	8	12
1b:	Solve problems by estimating and computing with data	4	8	12
1c:	Interpolate and extrapolate from data	•	8	12
D2	**Organize and display data and make inferences**			
2a:	Use tables, histograms (bar graphs), pictograms, and line graphs	4	8	12
2b:	Use circle graphs and scattergrams	•	8	12
2c:	Use stem-and-leaf plots and box-and-whisker plots	•	8	12
2d:	Make decisions about outliers	•	8	12
D3	**Understand and apply sampling, randomness, and bias in data collection**			
3a:	Given a situation, identify sources of sampling error	•	8	12
3b:	Describe a procedure for selecting an unbiased sample	•	8	12
3c:	Make generalizations based on sample results	•	8	12
D4	**Describe measures of central tendency and dispersion in real-world situations**			
		(4)	**8**	**12**
D5	**Use measures of central tendency, correlation, dispersion, and shapes of distributions to describe statistical relationships**			
5a:	Use standard deviation and variance	•	•	12
5b:	Use the standard normal distribution	•	•	12
5c:	Make predictions and decisions involving correlation	•	•	12
D6	**Understand and reason about the use and misuse of statistics in our society**			
6a:	Given certain situations and reported results, identify faulty arguments or misleading presentations of the data	(4)	8	12
6b:	Appropriately apply statistics to real-world situations	(4)	8	12
D7	**Fit a line or curve to a set of data and use this line or curve to make predictions about the data, using frequency distributions where appropriate**			
		•	•	**12**
D8	**Design a statistical experiment to study a problem and communicate the outcomes**			
		•	**8**	**12**
D9	**Use basic concepts, trees, and formulas for combinations, permutations, and other counting techniques to determine the number of ways an event can occur**			
		•	**8**	**12**

See notes at end of exhibit.

Exhibit A-1. NAEP mathematics framework and specifications summary: 2003—Continued

D	DATA ANALYSIS, STATISTICS AND PROBABILITY	Grade(s)		
D10	**Determine the probability of a simple event**			
10a:	Estimate probabilities by use of simulation	•	8	12
10b:	Use sample spaces and the definition of probability to describe events	4	8	12
10c:	Describe and make predictions about expected outcomes	•	8	12
D11	**Apply the basic concept of probability to real-world situations**			
11a:	Informal use of probabilistic thinking	4	8	12
11b:	Use probability related to independent and dependent events	•	8	12
11c:	Use probability related to simple and compound events	•	•	12
11d:	Use conditional probability	•	•	12

See notes at end of exhibit.

Exhibit A-1. NAEP mathematics framework and specifications summary: 2003—Continued

E	ALGEBRA AND FUNCTIONS	Grade(s)		
E1	**Describe, extend, interpolate, transform and create a wide variety of patterns and functional relationships**			
1a:	Recognize patterns and sequences	4	8	12
1b:	Extend a pattern or functional relationship	4	8	12
1c:	Given a verbal description, extend or interpolate with a pattern (complete a missing term)	•	8	12
1d:	Translate patterns from one context to another	(4)	8	12
1e:	Create an example of a pattern or functional relationship	4	8	12
1f:	Understand and apply the concept of a variable	(4)	8	12
E2	**Use multiple representations for situations to translate among diagrams, models, and symbolic expressions**			
		4	**8**	**12**
E3	**Use number lines and rectangular coordinate systems as representational tools**			
3a:	Identify or graph sets of points on a number line or in a rectangular coordinate system	4	8	12
3b:	Identify or graph sets of points in a polar coordinate system	•	8	12
3c:	Work with applications using coordinates	•	8	12
3d:	Transform the graph of a function	•	(8)	12
E4	**Represent and describe solutions to linear equations and inequalities to solve mathematical and real-world problems**			
4a:	Solution sets of whole numbers	4	8	12
4b:	Solution sets of real numbers	(4)	8	12
E5	**Interpret contextual situations and perform algebraic operations on real numbers and algebraic expressions to solve mathematical and real-world problems**			
5a:	Perform basic operations, using appropriate tools, on real numbers in meaningful contexts (including grouping and order of multiple operations involving basic operations, exponents and roots)	•	8	12
5b:	Solve problems involving substitution in expressions and formulas	•	8	12
5c:	Solve meaningful problems involving a formula with one variable	•	8	12
5d:	Use equivalent forms to solve problems	•	8	12
E6	**Solve systems of equations and inequalities using appropriate methods**			
6a:	Solve systems graphically	•	8	12
6b:	Solve systems algebraically	•	•	12
6c:	Solve systems using matrices	•	•	12

See notes at end of exhibit.

Exhibit A-1. NAEP mathematics framework and specifications summary: 2003—Continued

E	ALGEBRA AND FUNCTIONS	Grade(s)		
E7	**Use mathematical reasoning**			
7a:	Make conjectures	4	8	12
7b:	Validate and justify conclusions and generalizations	4	8	12
7c:	Use informal induction and deduction	(4)	8	12
E8	**Represent problem situations with discrete structures**			
8a:	Use finite graphs and matrices	•	(8)	12
8b:	Use sequences and series	•	•	12
8c:	Use recursive relations (including numerical and graphical iteration and finite differences)	•	•	12
E9	**Solve polynomial equations with real and complex roots using a variety of algebraic and graphical methods and using appropriate tools**			
		•	•	**12**
E10	**Approximate solutions of equations (bisection, sign changes, successive approximations)**			
		•	**(8)**	**12**
E11	**Use appropriate notation and terminology to describe functions and their properties (includes domain, range, function composition, inverses)**			
		•	•	**12**
E12	**Compare and apply the numerical, symbolic, and graphical properties of a variety of functions and families of functions, examining general parameters and their effect on curve shape**			
		•	**(8)**	**12**
E13	**Apply function concepts to model and deal with real-world situations**			
		•	**(8)**	**12**
E14	**Use trigonometry**			
14a:	Use triangle trigonometry to model problem situations	•	•	12
14b:	Use trigonometric and circular functions to model real-world phenomena	•	•	12
14c:	Apply concepts of trigonometry to model solve real-world problems	•	•	12

NOTE: *Content strands* are identified by capital letters (A, B, C,…), *topics* are identified by numbers (1, 2, 3,…), and *subtopics* are identified by lowercase letters (a, b, c, …). Topics and subtopics can be assessed at those grade levels indicated by 4, 8, and 12 on the right side of the exhibit. Parentheses around a grade level indicate that a topic may be introduced at a simple level at that grade. If a topic or subtopic should not be addressed at a specific grade level, it is indicated by a dot (•).
SOURCE: U.S. Department of Education, National Assessment Governing Board, *Mathematics Framework for the 2003 National Assessment of Educational Progress*, 2002.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003

A Number		Grade(s)	
A1	Whole Numbers	4	8
A2	Fractions and Decimals	4	8
A3	Integers	•	8
A4	Ratio, Proportion, and Percent	4	8

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

A NUMBER	
A1 Whole Numbers	
Grade 4	**Grade 8**
1a: Represent whole numbers using words, diagrams, or symbols, including recognizing and writing numbers in expanded form. 1b: Demonstrate knowledge of place value. 1c: Compare and order whole numbers. 1d: Identify sets of numbers according to common properties such as odd and even, multiples, or factors. 1e: Compute with whole numbers. 1f: Estimate computations by approximating the numbers involved. 1g: Solve routine and non-routine problems, including real-life problems.	1a: Demonstrate knowledge of place value and of the four operations. 1b: Find and use factors or multiples of numbers, and identify prime numbers. 1c: Express in general terms and use the principles of commutativity, associativity, and distributivity. 1d: Evaluate powers of numbers, and square roots of perfect squares to 144. 1e: Solve problems by computing, estimating, or approximating.
A2 Fractions and Decimals	
Grade 4	**Grade 8**
2a: Recognize fractions as parts of unit wholes, parts of a collection, locations on number lines, divisions of whole numbers. 2b: Identify equivalent fractions. 2c: Compare and order fractions. 2d: Show understanding of decimals. 2e: Represent fractions or decimals using words, numbers, or models. 2f: Add and subtract fractions with the same denominator. 2g: Add and subtract with decimals. Notes: Grade 4 fractions items will involve denominators of 2, 3, 4, 5, 6, 8, 10, or 12. Grade 4 decimals items will involve decimals to tenths and/or hundredths.	2a: Compare and order fractions. 2b: Compare and order decimals. 2c: Demonstrate knowledge of place value for decimals. 2d: Represent decimals and fractions using words, numbers, or models (including number lines). 2e: Recognize and write equivalent fractions. 2f: Convert fractions to decimals and vice versa. 2g: Relate operations with fractions or decimals to situations and models. 2h: Compute with fractions and decimals, including use of commutativity, associativity, and distributivity. 2i: Approximate decimals to estimate computations. 2j: Solve problems involving fractions. 2k: Solve problems involving decimals.

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

A NUMBER	
A3 Integers	
Grade 4	**Grade 8**
Not assessed at this level.	3a: Represent integers using words, numbers, or models (including number lines). 3b: Compare and order integers. 3c: Show an understanding of addition, subtraction, multiplication, and division with integers. 3d: Compute with integers. 3e: Solve problems using integers.
A4 Ratio, Proportion, and Percent	
Grade 4	**Grade 8**
4a: Solve problems involving simple proportional reasoning.	4a: Identify and find equivalent ratios. 4b: Divide a quantity in a given ratio. 4c: Convert percents to fractions or decimals, and vice versa. 4d: Solve problems involving percents. 4e: Solve problems involving proportions.

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

B Measurement		Grade(s)	
B1	Attributes and Units	4	8
B2	Tools, Techniques and Formulas	4	8

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

B MEASUREMENT	
B1 Attributes and Units	
Grade 4	**Grade 8**
1a: Use given non-standard units to measure length, area, volume, and time (e.g., paper clips for length, tiles for area, sugar cubes for volume). 1b: Select appropriate standard units to measure length, area, mass/weight,* angle, and time (e.g., kilometers for car trips, centimeters for human height). 1c: Use conversion factors between standard units (e.g., hours to minutes, grams to kilograms). 1d: Recognize that total measures of length, area, volume, angle, and time do not change with position, decomposition into parts, or division. *More properly mass, but weight expressed in grams or kilograms is the common usage at these levels. Countries in which mass is the common usage for grades 4 and/or 8 will frame items accordingly.	1a: Select and use appropriate standard units to find measures of length, area, volume, perimeter, circumference, time, speed, density, angle, mass/weight.* 1b: Use relationships among units for conversions within systems of units, and for rates. * More properly mass, but weight expressed in grams or kilograms is the common usage at these levels. Countries in which mass is the common usage for grades 4 and/or 8 will frame items accordingly.

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

B MEASUREMENT	
B2 Tools, Techniques, and Formulas	
Grade 4	**Grade 8**
2a: Use instruments with linear or circular scales to measure length, weight, time, and temperature in problem situations (e.g., dimensions of a window, weight of a parcel). 2b: Estimate length, area, volume, weight, and time in problem situations (e.g., height of a building, volume of a block of material). 2c: Calculate areas and perimeters of squares and rectangles of given dimensions. 2d: Compute measurements in simple problem situations (e.g., elapsed time, change in temperature, difference in height or weight).	2a: Use standard tools to measure length, weight, time, speed, angle, and temperature in problem situations and to draw line segments, angles, and circles of a given size. 2b: Estimate length, circumference, area, volume, weight, time, angle, and speed in problem situations (e.g., circumference of a wheel, speed of a runner). 2c: Compute with measurements in problem situations (e.g., add measures, find average speed on a trip, find population density). 2d: Select and use appropriate measurement formulas for perimeter of a rectangle, circumference of a circle, areas of plane figures (including circles), surface area and volume of rectangular solids, and rates. 2e: Find measures of irregular or compound areas by covering with grids or dissecting and rearranging pieces. 2f: Give and interpret information about the precision of measurements (e.g., upper and lower bounds of a length reported as 8 centimeters to the nearest centimeter).

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

C Geometry		Grade(s)	
C1	Lines and Angles	4	8
C2	Two- and Three-Dimensional Shapes	4	8
C3	Congruence and Similarity	4	8
C4	Locations and Spatial Relationships	4	8
C5	Symmetry and Transformations	4	8

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

C GEOMETRY	
C1 Lines and Angles	
Grade 4	**Grade 8**
1a: Classify angles as greater than, equal to, or less than a right angle (or 90°). 1b: Identify and describe parallel and perpendicular lines. 1c: Compare given angles and place them in order of size.	1a: Classify angles as acute, right, straight, obtuse, reflex, complementary, and supplementary. 1b: Recall the relationships for angles at a point, angles on a line, vertically opposite angles, angles associated with a transversal cutting parallel lines, and perpendicularity. 1c: Know and use the properties of angle bisectors and perpendicular bisectors of lines.
C2 Two- and Three-Dimensional Shapes	
Grade 4	**Grade 8**
2a: Know and use vocabulary associated with familiar two- and three-dimensional shapes. 2b: Identify common geometric shapes in the environment. 2c: Classify two- and three-dimensional shapes according to their properties. 2d: Know properties of geometric figures and use them to solve routine problems. 2e: Decompose shapes and rearrange the parts to form simpler shapes.	2a: Recall properties of geometric shapes: triangles (scalene, isosceles, equilateral, right) and quadrilaterals (scalene, trapezoid, parallelogram rectangle, rhombus, square). 2b: Use properties of familiar geometric shapes in a compound figure to make conjectures about properties of the compound figure. 2c: Recall properties of other polygons (regular pentagon, hexagon, octagon, decagon). 2d: Construct or draw triangles and rectangles of given dimensions. 2e: Apply geometric properties to solve routine and non-routine problems 2f: Use Pythagorean theorem (not proof) to solve problems (e.g., find the length of a side of a right-angled triangle given the lengths of the other two sides; or, given the lengths of three sides of a triangle, determine whether the triangle is right-angled).

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

C GEOMETRY	
C3 Congruence and Similarity	
Grade 4	**Grade 8**
3a: Identify triangles that have the same size and shape (congruent). 3b: Identify triangles that have the same shape but different sizes (similar).	3a: Identify congruent triangles and their corresponding measures. 3b: Identify congruent quadrilaterals and their corresponding measures. 3c: Consider the conditions of congruence to determine whether triangles with given corresponding measures (at least three) are congruent. 3d: Identify similar triangles and recall their properties. 3e: Use properties of congruence in mathematical and practical problem situations. 3f: Use properties of similarity in mathematical and practical problem situations.
C4 Locations and Spatial Relationships	
Grade 4	**Grade 8**
4a: Use informal coordinate systems to locate points in a plane. 4b: Relate a net to the shape it will make. 4c: Recognize relationships between two-dimensional and three-dimensional shapes when shown nets and different two-dimensional views of three-dimensional objects.	1a: Locate points using number lines, coordinate grids, and maps. 1b: Use ordered pairs, equations, intercepts, intersections, and gradients to locate points and lines in the Cartesian plane. 1c: Recognize relationships between two-dimensional and three-dimensional shapes when shown nets and different two-dimensional views of three-dimensional objects.
C5 Symmetry and Transformations	
Grade 4	**Grade 8**
5a: Recognize line symmetry. 5b: Draw two-dimensional symmetrical figures. 5c: Recognize translation, reflection, and rotation.	5a: Recognize line and rotational symmetry for two-dimensional shapes. 5b: Draw two-dimensional symmetrical figures. 5c: Recognize, or demonstrate by sketching, translation, reflection, rotation, and enlargement. 5d: Use transformations to explain or establish geometric properties.

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

D Data		Grade(s)	
D1	Data Collection and Organization	4	8
D2	Data Representation	4	8
D3	Data Interpretation	4	8
D4	Uncertainty and Probability	•	8

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

D DATA	
D1 Data Collection and Organization	
Grade 4	**Grade 8**
1a: Match a set of data with appropriate characteristics of situations or contexts (e.g., outcomes from rolling a die). 1b: Organize a set of data by one characteristic (e.g., height, color, age, shape).	1a: Match a set of data, or a data display, with appropriate characteristics of situations or contexts (e.g., monthly sales of a product for a year). 1b: Organize a set of data by one or more characteristics using a tally chart, table, or graph. 1c: Recognize and describe possible sources of error in collecting and organizing data (e.g., bias, inappropriate grouping). 1d: Select the most appropriate data collection method (e.g., survey, experiment, questionnaire) to answer a given question, and justify the choice.
D2 Data Representation	
Grade 4	**Grade 8**
2a: Read data directly from tables, pictographs, bar graphs, and pie charts. 2b: Display data using tables, pictographs, and bar graphs. 2c: Compare and match different representations of the same data.	2a: Read data from charts, tables, pictographs, bar graphs, pie charts, and line graphs. 2b: Display data using charts, tables, pictographs, bar graphs, pie charts, and line graphs. 2c: Compare and match different representations of the same data.

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

D DATA	
D3 Data Interpretation	
Grade 4	**Grade 8**
3a: Compare characteristics of related data sets (e.g., given data or representations of data on student heights in two classes, identify the class with the shortest/tallest person). 3b: Draw conclusions from data displays.	3a: Compare characteristics of data sets, using mean, median, range, and shape of distribution (in general terms). 3b: Interpret data sets (e.g., draw conclusions, make predictions, and estimate values between and beyond given data points). 3c: Evaluate interpretations of data with respect to correctness and completeness of interpretation. 3d: Use and interpret data sets to answer questions.
D4 Uncertainty and Probability	
Grade 4	**Grade 8**
Not assessed at this grade.	4a: Judge the likelihood of an event as certain, more likely, equally likely, less likely, or impossible. 4b: Use data from experiments to estimate probabilities for favorable outcomes. 4c: Use problem conditions to calculate theoretical probabilities for possible outcomes.

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

E Algebra		Grade(s)	
E1	Patterns	4	8
E2	Algebraic Expressions	•	8
E3	Equations and Formulas	4	8
E4	Relationships	4	8

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

E ALGEBRA	
E1 Patterns	
Grade 4	**Grade 8**
1a: Extend and find missing terms of numeric and geometric patterns. 1b: Match numeric and geometric patterns with descriptions. 1c: Describe relationships between adjacent terms in a sequence or between the number of the term and the term.	1a: Extend numeric, algebraic, and geometric patterns or sequences using words, symbols, or diagrams; find missing terms. 1b: Generalize pattern relationships in a sequence, or between adjacent terms, or between the number of the term and the term, using words or symbols.
E2 Algebraic Expressions	
Grade 4	**Grade 8**
Not assessed at this grade.	2a: Find sums, products, and powers of expressions containing variables. 2b: Evaluate expressions for given numeric values of the variable(s). 2c: Simplify or compare algebraic expressions to determine equivalence. 2d: Model situations using expressions.
E3 Equations and Formulas	
Grade 4	**Grade 8**
3a: Show understanding of equality using equations, areas, volumes, masses/weights. 3b: Find the missing number in an equation (e.g., if 17 + __ = 29, what number would go in the blank to make the equation true?). 3c: Model simple situations involving unknowns with an equation. 3d: Solve problems involving unknowns.	3a: Evaluate formulas given the values of the variables. 3b: Use formulas to answer questions about given situations. 3c: Indicate whether a value (or values) satisfies a given equation. 3d: Solve simple linear equations and inequalities, and simultaneous (two variables) equations. 3e: Write linear equations, inequalities, or simultaneous equations that model given situations. 3f: Solve problems using equations or formulas.

See notes at end of exhibit.

Exhibit A-2. TIMSS mathematics framework and specifications summary: 2003—Continued

E ALGEBRA	
E4 Relationships	
Grade 4	**Grade 8**
4a: Generate pairs of numbers following a given rule (e.g., multiply the first number by 3 and add 2 to get the second number). 4b: Write, or select, a rule for a relationship given some pairs of numbers satisfying the relationship. 4c: Graph pairs of numbers following a given rule. 4d: Show why a pair of numbers follows a given rule. (E.g., a rule for a relation between two numbers is "multiply the first number by 5 and subtract 4 to get the second number." Show that when the first number is 2 and the second number is 6 the rule is followed.)	4a: Recognize equivalent representations of functions as ordered pairs, tables, graphs, words, or equations. 4b: Given a function in one representation, generate a different but equivalent representation. 4c: Recognize and interpret proportional, linear, and nonlinear relationships (travel graphs and simple piecewise functions included). 4d: Write or select a function to model a given situation. 4e: Given a graph of a function, identify attributes such as intercepts on axes and intervals where the function increases, decreases, or is constant.

NOTE: *Content domains* are identified by capital letters (A, B, C,…), *topic areas* are identified by numbers (1, 2, 3,…), and *objectives* are identified by lowercase letters (a, b, c, …). Topic areas can be assessed at those grade levels indicated by 4 and 8 on the right side of the table. If a topic area should not be addressed at a specific grade level, it is indicated by a dot (•).
SOURCE: International Study Center, Lynch School of Education, Boston College, *TIMSS Assessment Frameworks and Specifications 2003, 2nd ed.,* 2003.

Appendix B: Levels of Mathematical Complexity

Exhibit B-1 is the document that was used by the expert panel for the *mathematical complexity* classifications. The summary document is based on the prepublication version of the NAEP 2005 framework that was available at the time of the expert panel meeting but has been reformatted and adapted slightly to facilitate the classification process.

Exhibit B-1. Levels of mathematical complexity adapted from the NAEP 2005 mathematics framework

Low Complexity	Moderate Complexity	High Complexity
This category relies heavily on the recall and recognition of previously learned concepts and principles. Items typically specify what the student is to do, which is often to carry out some procedure that can be performed mechanically. It is not left to the student to come up with an original method or solution.	Items in the moderate-complexity category involve more flexibility of thinking and choice among alternatives than do those in the low-complexity category. They require a response that goes beyond the habitual, is not specified, and ordinarily has more than a single step. The student is expected to decide what to do, using informal methods of reasoning and problem-solving strategies, and to bring together skill and knowledge from various domains.	High-complexity items make heavy demands on students, who must engage in more abstract reasoning, planning, analysis, judgment, and creative thought. A satisfactory response to the item requires that the student think in an abstract and sophisticated way.
The following are some, but not all, of the demands that items in the low-complexity category might make: • Recall or recognize a fact, term, or property; • Recognize and example of a concept; • Compute a sum, difference, product, or quotient; • Recognize an equivalent representation; • Perform a specified procedure; • Evaluate an expression in an equation or formula for a given variable; • Solve a one-step word problem; • Draw or measure simple geometric figures; or • Retrieve information from a graph, table, or figure.	The following illustrate some of the demands that items of moderate complexity might make: • Represent a situation mathematically in more than one way; • Select and use different representations, depending on situation and purpose; • Solve a word problem requiring multiple steps; • Compare figures or statements; • Provide a justification for steps in a solution process; • Interpret a visual representation; • Extend a pattern; • Retrieve information from a graph, table, or figure and use it to solve a problem requiring multiple steps; • Formulate a routine problem, given data and conditions; or • Interpret a simple argument.	Items at the level of high complexity may ask the student to do any of the following: • Describe how different representations can be used for different purposes; • Perform a procedure having multiple steps and multiple decision points; • Analyze similarities and differences between procedures and concepts; • Generalize a pattern; • Formulate an original problem, given a situation; • Solve a novel problem; • Solve a problem in more than one way; • Explain and justify a solution to a problem; • Describe, compare, and contrast solution methods; • Formulate a mathematical model for a complex situation; • Analyze the assumptions made in a mathematical model; • Analyze or produce a deductive argument; or • Provide a mathematical justification.

SOURCE: U.S. Department of Education, National Assessment Governing Board, *Mathematics Framework for the 2005 National Assessment of Educational Progress*, 2004.

Appendix C: Expert Panel

Members and Staff

Expert Panel Members

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Independent Project Consultant

Appendix D: Methodological Notes and Supplementary Data

Considerations in Selecting Classification Methods

The cross-classification approach (classification of items in one assessment to the other assessment framework) was selected for the examination of content and grade match so that there would be multiple content profiles for each assessment. This method also prevents each assessment from being evaluated through only one perspective, which may or may not be reflective of its purposes. This approach takes advantage of the multiple ways of describing content found in the frameworks and enables direct comparisons across the assessments.

For the classifications based on cognitive processes and skills, a common classification system was chosen—levels of *mathematical complexity* from the NAEP 2005 framework. There were several reasons for deciding, first, to use only a single classification system and, second, for selecting this NAEP 2005 dimension for the common system. With regard to the first decision, the study organizers recognized that classifications in these dimensions likely would require more discretionary judgment than those in content areas, and thought a single rubric would be the most realistic to implement under the time constraints. With regard to the second decision, although all three 2003 mathematics assessment frameworks compared in this study include a dimension related to cognitive skills and processes, the classification system from the NAEP 2005 framework focuses on the characteristics of items rather than on inferred cognitive abilities of students, which may vary widely from student to student. Furthermore, it is expected to be in use in NAEP for several years to come, making it a potentially valuable link to similar comparison studies in the future.

Reliability Analyses

For the classification of items, the expert panel was divided into three groups to review items by content area (as described in section 3). To measure the extent to which the different content area groups were interpreting the common rubric in similar ways, a common set of items was classified by all three groups with respect to *mathematical complexity* level. The degree to which the three groups classified these items in the same categories on this dimension serves as a measure of the reliability of these classifications. The set of 60 items (30 from NAEP and 30 from TIMSS), which reflects approximately 9 percent of the total item classifications across both assessments, was taken from across the mathematics content areas and grade levels. This was not a random sample, but a representative set chosen to cover the main categories addressed in the study (content area and grade level). Some effort was also made to ensure that there were at least some items from each of the cognitive categories based on the original assessment developers' classifications (*cognitive domains* in TIMSS and *mathematical abilities* in NAEP). Reliability items were classified at regular intervals throughout the classification process. Given the limited time available for the expert panel meeting, the 30 items from each assessment was the maximum number of items that could be included in the reliability set.

The reliability set of 30 items from each assessment (NAEP and TIMSS) was composed of 10 items from the *number* category and 5 items from each of *measurement, geometry, data*, and *algebra*. This reflects 10 items from each of the three primary content groups into which the expert panel was divided (*number, measurement/geometry*, and *data/algebra*). For the reliability results, each group contributed 10 classifications from their primary content areas and another 20 secondary classifications for reliability items from the other content areas. Due to time constraints, one of the

three groups only provided secondary classifications for 5 of the 20 items outside their primary content area. This group contributed primary classifications for items in *data* and *algebra*, and 5 secondary classifications from the *number* category. Therefore, the set of 15 items that have classifications from all three groups do not reflect any items from *measurement* and *geometry*; for those items, reliability data are based on only two classifications. With respect to other item characteristics (e.g., grade level, item format, and NAEP *mathematical ability* category), the set is still balanced and representative of the full reliability set.

The multiple classification data for the reliability set were analyzed based on the percentage of classifications where there was agreement. Classification reliability statistics were computed in two ways, as follows:

- The percentage of total comparisons: based on the number of comparisons where there was agreement between any two groups (i.e., groups 1 and 2, groups 2 and 3 and groups 1 and 3) across ALL items; and

- The percentage of items: based on the number of items where there was agreement across ALL three groups.

The results from these two types of analyses are shown in tables D-1 and D-2.

The results of the reliability analyses shown in table D-1 (based on number of comparisons across any two groups) were checked to evaluate any impact of removing the 5 *number* and 10 *measurement/geometry* NAEP items with data from only two groups. The results showed no change in the reported percentage agreement (78 percent). Thus, the full set of reliability data were used for the analyses shown.

There was reasonably high agreement across groups on classification to the three levels of *mathematical complexity* (low, moderate, and high). The results indicate 79 percent agreement for all comparisons between any two groups across all items (table D-1). These results reflect agreement across all groups for 69 percent of all items (table D-2). When broken down into the NAEP and TIMSS items, the results are similar. For items where there was not total agreement across groups, disagreement was always to "adjacent" categories (i.e., low/moderate and moderate/high). There were no instances of disagreement between low and high complexity.

Table D-1. Reliability of mathematical complexity level classifications for mathematics items in NAEP 2003 and TIMSS 2003, by number of comparisons and percentage agreement

Number of comparisons and percentage agreement	NAEP 2003	TIMSS 2003	Overall
Total number of comparisons across items	60	90	150
Number of comparisons with agreement between groups	47	72	119
Percentage agreement	78	80	79

NOTE: Data are based on 30 NAEP items and 30 TIMSS items that were classified by three expert panel groups and reflect all comparisons between any two groups (i.e., groups 1 and 2; groups 2 and 3; and groups 1 and 3). One group classified only 15 of 30 NAEP items in the reliability set, meaning that agreement on the classification for 15 NAEP items was based on the classifications of two groups instead of three. This results in a total number of comparisons of 60 for NAEP items (instead of 90) and 150 overall.
SOURCE: Expert panel classifications of selected fourth- and eighth-grade mathematics items from the National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment and the Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

Table D-2. Reliability of mathematical complexity level classifications for mathematics items in NAEP 2003 and TIMSS 2003, by number of items and percentage agreement

Number of items and percentage agreement	NAEP 2003	TIMSS 2003	Overall
Total number of items	15	30	45
Number of items with agreement across all groups	10	21	31
Percentage agreement	67	70	69

NOTE: Data are based on 30 NAEP items and 30 TIMSS items that were classified by three expert panel groups and reflect comparisons across all three groups for each item. One group classified only 15 of 30 NAEP items in the reliability set; the percentage agreement for NAEP items includes only the items that were classified by all three groups.
SOURCE: Expert panel classifications of selected fourth- and eighth-grade mathematics items from the National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment and the Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

In sum, the main focus of the present study is a content comparison—classification of items to the content framework of the other assessment—which was done by the separate content-area subpanels. The reliability tables are included only to provide some indication of the extent to which the expert panelists agreed on the other metric that used a common rubric (*mathematical complexity* level). Expert panelists typically spent less time reviewing and classifying the items in the reliability set that were outside of their primary content areas, and the results from these secondary classifications should not be viewed as a complete replication of the process used by the primary group which was most familiar with the items in the respective content areas. Therefore, only the primary group classifications were used in the reporting of results for *mathematical complexity* level.

Data Processing

After the expert panel meeting, the facilitators of each group met to review the methods used and the data collected to ensure consistency. In some cases, methods or reporting conventions were slightly different between groups. For these cases, the facilitators reviewed their notes and the notes of individual panel members to standardize the data. Datasets were produced that included the standardized expert panel classifications for all items from each assessment (including multiple classifications on the reliability set) as well as original classification information for each item provided by the assessment developers. The raw data containing all original panelist classifications and comments from each subgroup were also available for analysts and were consulted in the writing of this report.

Supplementary Data on NAEP/PISA Comparisons

Some supplementary data to inform the discussion of the NAEP/PISA content comparisons in Section 6.1 are shown in Table D-3. This table shows the distribution of PISA items classified across topics in the NAEP *content strands*, indicating the degree of overlap between PISA items from each *overarching idea* and NAEP topics.

Table D-3. Distribution of PISA 2003 mathematics items across topics within the NAEP 2003 mathematics framework content strands, by PISA overarching idea category

NAEP content strand and topic	PISA overarching idea: Change and Relationships	Quantity	Space and shape	Uncertainty
Number sense, properties, and operations				
Represent numbers and operations using models, diagrams, and symbols	0	2	0	0
Compute with numbers	1	0	1	0
Use computations and estimation in applications	1	9	1	0
Apply ratios and proportional thinking	1	0	0	1
Use elementary number theory	0	2	0	0
Measurement				
Estimate the size of an object or compare objects with respect to a given attribute	0	0	1	0
Estimate, calculate or compare perimeter, area, volume, and surface area	0	0	7	0
Convert from one measurement to another within the same system	0	2	0	0
Make and read scale drawings	0	0	1	0
Apply the concept of rate to measurement situations	1	1	0	0
Geometry and spatial sense				
Describe, visualize, draw and construct geometric figures	0	0	1	0
Investigate and predict results of combining, subdividing, and changing shapes	0	0	2	0
Identify the relationship (congruence, similarity) between a figure and its image under a transformation	0	0	1	0
Apply geometric properties and relationships in solving problems	0	0	1	0
Represent situations with geometric models and apply properties of figures	0	0	3	0
Data analysis, statistics, and probability				
Read, interpret, and make predictions using tables and graphs	9	3	0	4
Organize and display data and make inferences	1	0	0	0
Understand and apply sampling, randomness, and bias in data collection	0	0	0	1
Describe measures of central tendency and dispersion	0	0	0	5
Understand and reason about the use and misuse of statistics in our society	0	0	0	4
Use basic concepts, trees, and formulas for combinations, permutations, and other counting techniques	0	2	0	0
Determine the probability of a simple event	0	0	0	2
Apply the basic concept of probability	0	0	0	3
Algebra and functions				
Describe, extend, interpolate, transform and create patterns and functional relationships	0	2	0	0
Use multiple representations to translate among diagrams, models, and symbolic expressions	1	0	0	0
Interpret situations and perform algebraic operations	3	0	0	0
Compare and apply the numerical, symbolic, and graphical properties of a variety of functions	1	0	0	0
Apply function concepts to model real-world situations	1	0	0	0

NOTE: Data reflect the number of PISA items classified by the panel to a topic or subtopic at any grade level in the NAEP 2003 framework. Items classified to multiple topics were counted in all relevant topics. Four PISA items that were classified to a NAEP strand but not to a topic are not reflected in this table: three items from *change and relationships* classified as *measurement*, *algebra* or *geometry* and one item from *space and shape* classified as *geometry*.

SOURCE: Organization for Economic Cooperation and Development, Program for International Student Assessment (PISA) 2003 Mathematical Literacy Assessment; Organization for Economic Cooperation and Development (OECD), Program for International Student Assessment (PISA), *The PISA 2003 Assessment Framework: Mathematics, Reading, Science and Problem Solving Knowledge and Skills*, 2003; and U.S. Department of Education, National Assessment Governing Board, *Mathematics Framework for the 2003 National Assessment of Educational Progress*, 2002.

Appendix E: Example Items

Exhibit E-1. Index of example items from NAEP 2003, TIMSS 2003, and PISA 2003

Example Number	Description of characteristics illustrated in text
1	NAEP *conceptual understanding* item (low *mathematical complexity*)
2	NAEP *problem solving* item (low *mathematical complexity*)
3	TIMSS *reasoning* item (low *mathematical complexity*)
4	PISA *reproduction* item (moderate *mathematical complexity*)
5	NAEP short constructed-response item (high *mathematical complexity*)
6	NAEP extended constructed-response item (high *mathematical complexity*)
7	TIMSS extended constructed-response item (moderate *mathematical complexity*)
8	PISA complex multiple-choice item (moderate *mathematical complexity*)
9	PISA open constructed-response item (low *mathematical complexity*)
10	TIMSS item described as "basic computation" by the expert panel (low *mathematical complexity*)
11	TIMSS item described as "select an operation or procedure to solve a problem" by the expert panel (low *mathematical complexity*)
12	NAEP item described as "basic computation" by the expert panel (low *mathematical complexity*)
13	NAEP fourth-grade *number* item classified at eighth-grade level on TIMSS mathematics framework (moderate *mathematical complexity*)
14	TIMSS eighth-grade *measurement* item classified at fourth-grade level on NAEP mathematics framework (moderate *mathematical complexity*)
15	TIMSS fourth-grade *geometry* item not classified at the topic level on the NAEP mathematics framework (low *mathematical complexity*)
16	NAEP fourth-grade *geometry* item with a general match to the TIMSS mathematics framework—classified at the topic level but not to a specific subtopic (moderate *mathematical complexity*)
17	NAEP cross-grade *geometry* item (grades 4, 8, and 12) classified at the fourth-grade level on TIMSS mathematics framework (low *mathematical complexity*)
18	TIMSS eighth-grade *geometry* item classified at the twelfth-grade level on the NAEP mathematics framework (low *mathematical complexity*)
19	TIMSS eighth-grade *geometry* item classified to NAEP topic of *apply geometric properties and relationships in solving problems* (low *mathematical complexity*)
20	TIMSS fourth-grade *data* item classified to NAEP topic of *read, interpret, and make predictions using tables and graphs* (low *mathematical complexity*)
21	NAEP eighth-grade *algebra* item classified to TIMSS *number* content domain (low *mathematical complexity*)
22	NAEP *problem solving* item classified to PISA *reproduction competency cluster* (moderate *mathematical complexity*)
23	NAEP *problem solving* item classified to PISA *connections competency cluster* and judged as appropriate for PISA (moderate *mathematical complexity*)
24	NAEP *problem solving* item judged as not likely to appear on the PISA mathematics assessment (moderate *mathematical complexity*)
25	NAEP *problem solving* item judged as requiring revision to appear on the PISA mathematics assessment *(*moderate *mathematical complexity*)
26	PISA task requiring graphical interpretation (low-moderate *mathematical complexity*)

EXAMPLE 1

NAEP multiple-choice item – grade 4

Which of the following has only 3 angles?

A) A triangle
B) A square
C) A rectangle
D) A cube

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Answer Key: A

Framework classifications

NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Geometry and spatial sense Describe, visualize, draw, and construct geometric figures Grade 4 Conceptual understanding	Geometry Two- and three-dimensional shapes Grade 4
Mathematical complexity level:[3] low	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 2

NAEP multiple-choice item – grades 4, 8, and 12

The perimeter of a square is 36 inches. What is the length of one side of the square?

A) 4 inches
B) 6 inches
C) 9 inches
D) 18 inches

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Answer Key: C

Framework classifications

NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Measurement Estimate, calculate (using basic principles or formulas), or compare perimeter, area, volume, and surface area in meaningful contexts to solve mathematical and real-world problems Grade 4 Problem solving	Measurement Tools, techniques, and formulas Grade 4
Mathematical complexity level:[3] low	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 3

TIMSS multiple-choice item – grade 8

(3, 6), (6, 15), (8, 21)

Which of these describes how to get the second number from the first number in every ordered pair above?

A) Add 3
B) Subtract 3
C) Multiply by 2
D) Multiply by 2 and then add 3
E) Multiply by 3 and then subtract 3

SOURCE: International Association for the Evaluation of Educational Achievement, Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

Answer Key: E

Framework classifications

TIMSS 2003 framework[1]	NAEP 2003 framework[2]
Algebra	Algebra and functions
Relationships	Describe, extend, interpolate, transform, and create a wide variety of patterns and functional relationships
Grade 8	Grade 8
Reasoning	
Mathematical complexity level:[3] low	

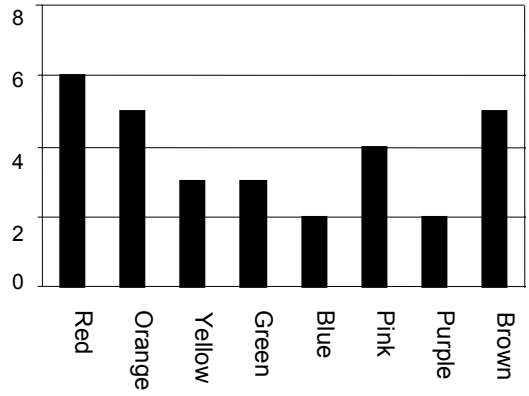
[1] Classified by TIMSS assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 4

PISA multiple-choice item

Colored Candies

Robert's mother lets him pick one candy from a bag. He can't see the candies. The number of candies of each color in the bag is shown in the following graph.



What is the probability that Robert will pick a red candy?

A) 10%
B) 20%
C) 25%
D) 50%

SOURCE: Organization for Economic Cooperation and Development, Program for International Student Assessment (PISA) 2003 Mathematical Literacy Assessment.

Answer Key: B

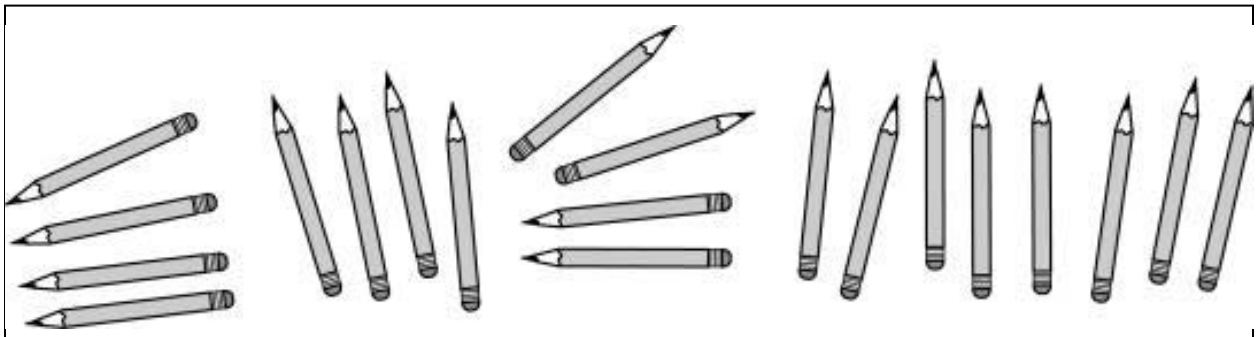
Framework classifications

PISA 2003 framework[1]		NAEP 2003 framework[2]	
Overarching idea	Uncertainty	Content strand	Data analysis, statistics, and probability
Competency cluster	Reproduction	Topic	Determine the probability of a simple event
Situation or context	Personal	Grade level	8
Mathematical complexity level:[3] moderate			

[1] Classified by PISA assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 5

NAEP short constructed-response item – grade 4



Together, Sara and Brendan have 20 pencils. Sara says 1/4 of the pencils are hers. Brendan says 15 of the pencils belong to him. Explain how they both could be right. Use words or drawings.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Scoring guide

In this question the student was given information in two different ways—a fractional part and a number of items—and the student needed to justify that these two interpretations of the same situation were consistent. To answer the question, the student needed to observe that the fractional part has meaning in terms of the number of items, or that the number of items can be represented as a fractional part of the whole amount. Students were permitted to use a calculator.

Correct

They can both be right because ¼ of 20 = 5 and 20 – 5 = 15

OR

¼ is 5 and ¾ is 15

OR

Sara ¼ or 5	Brendan ¾ or 15
|||||	||||| ||||| |||||

Partial

¼ is 5

OR

¾ is 15

OR

"Sara has 5" (5 must be connected to Sara; if states Sara has 5 because 20 – 15 = 5, item is scored as incorrect).

Incorrect

Incorrect response (includes 20 – 15 = 5, 5 + 15 = 20, and switching names)

EXAMPLE 5—continued

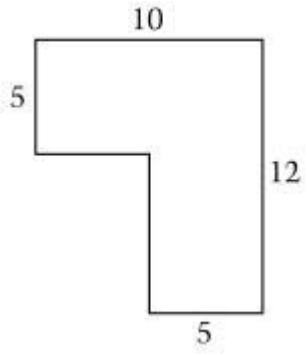
Framework classifications

NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Number sense, properties, and operations	Number
Apply ratios and proportional thinking in a variety of situations	Fractions and decimals
Grade 4	Grade 4
Conceptual understanding	
Mathematical complexity level:[3] high	

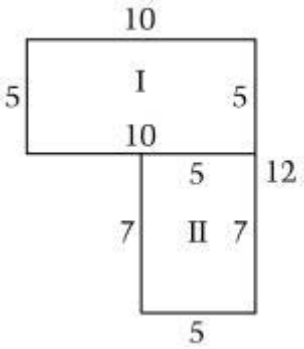
[1] Classified by NAEP assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 6

NAEP extended constructed-response item – grade 8

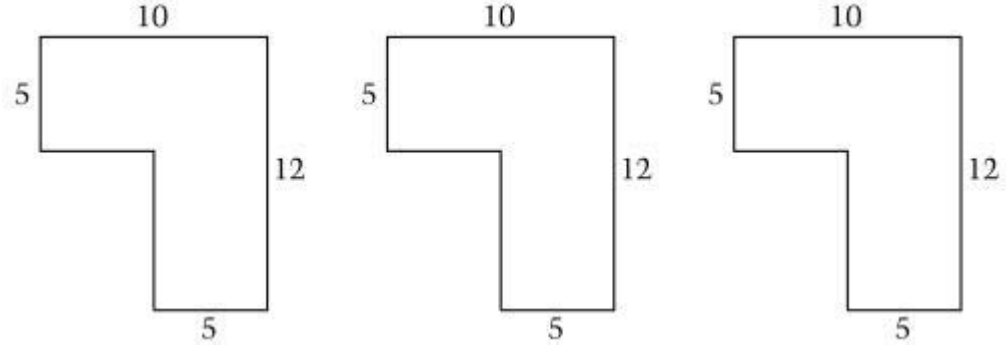


Ted wants to purchase floor covering for the hallway shown above. He knows there are many ways to find the area of the hallway. One way is to divide the hallway into the sections shown below and then add together the area of each section.



Area of Hallway = Area of Region I + Area of Region II
Area = (5 x 10) + (7 x 5)

Use the figures below to show 3 other ways that Ted can divide the hallway to find its area. Below each figure explain what numbers and operations Ted could use to calculate the area.

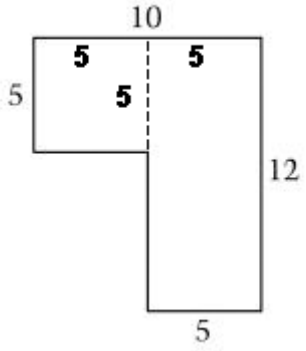


SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

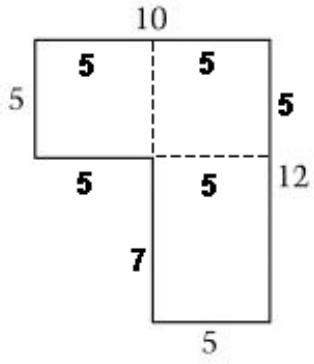
EXAMPLE 6—continued

Solution:

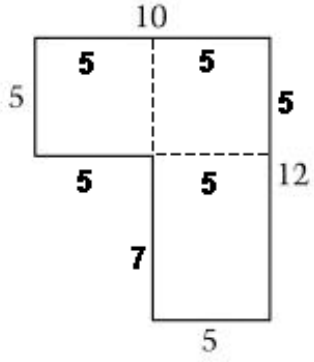
Possible answers include:



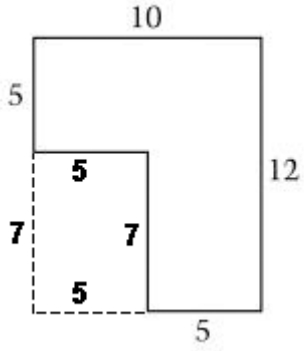
Area = (5 × 5) + (5 × 12)



Area = (5 × 5) + (5 × 5) + (7 × 5)



Area = $5\left(\frac{7+12}{2}\right)+5\left(\frac{5+10}{2}\right)$



Area = (10 × 12) + (7 × 5)
For this method the missing piece must be indicated

EXAMPLE 6—continued

Scoring guide

In this question the student was asked to draw and explain three different ways to divide an L-shaped region to determine the area. The student was also required to give an expression representing the area for each of the different divisions of the region (however, the student was not asked to calculate the area of the region). There are many possible ways to do this but to earn full credit the student needed to show three different divisions of the region, label the lengths in each figure correctly, and write an expression for the area consistent with each figure. This question requires visualization and knowledge of one or more formulas for finding area. Any division of the figures into rectangles, triangles, trapezoids, or parallelograms is acceptable. Student either needs to show (5 x 5) + (5 x 12), for example OR label all appropriate dimensions in the figure to give credit for 25 + 60. However, "25 + 60" is not acceptable if dimensions are not labeled.
Extended Three figures divided correctly with no incorrect labels and three correct expressions for area.
Satisfactory Three figures divided correctly with no incorrect labels and two correct expressions for area.
Partial Two figures divided correctly with no incorrect labels and one or two correct expressions for area of those figures. OR Three figures divided correctly with no incorrect labels and one correct expression for area.
Minimal One figure divided correctly with no incorrect labels and correct expression for area of that figure. OR Two or three figures divided correctly with no incorrect labels and no correct (or missing) expressions for area of figures.
Incorrect Incorrect response.

Framework classifications

NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Measurement Estimate, calculate (using basic principles or formulas), or compare perimeter, area, volume, and surface area in meaningful contexts or solve mathematical and real-world problems Grade 8 Problem solving	Measurement Tools, techniques, and formulas Grade 8

PISA 2003 framework[2]	
Overarching idea	Space and shape
Competency cluster	Reproduction
Situation or context	Educational/occupational

Mathematical complexity level:[3] high

[1] Classified by NAEP assessment developers

[2] Classified by expert panel. NAEP grade 8 *problem solving* items were classified to both the TIMSS and PISA frameworks.

[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 7

TIMSS extended constructed-response item – grade 8

Betty, Frank, and Darlene have just moved to Zedland. They each need to get phone service. They received the following information from the telephone company about the two different phone plans it offers.

They must pay a set fee each month and there are different rates for each minute they talk. These rates depend on the time of the day or night they use the phone, and on which payment plan they choose. Both plans include time for which phone calls are free. Details of the two plans are shown in the table below.

Plan	Monthly Fee	Rate per minute		Free minutes per month
		Day (8 am – 6 pm)	Night (6 pm – 8 am)	
Plan A	20 zeds	3 zeds	1 zed	180
Plan B	15 zeds	2 zeds	2 zeds	120

Betty talks for less than 2 hours per month. Which plan would be less expensive for her?

Less expensive plan _____

Explain your answer in terms of both the monthly fee and free minutes.

SOURCE: International Association for the Evaluation of Educational Achievement, Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

Scoring guide

Correct response Plan B with explanation that includes free minutes and explicit reference to lower monthly fee for Plan B.
Partial response Plan B with explicit reference to lower monthly fee and no reference to free minutes.
Incorrect response Plan B with inadequate (only free minutes) or no explanation. OR Plan A with or without explanation.

EXAMPLE 7—continued

Framework classifications

TIMSS 2003 framework[1]	NAEP 2003 framework[2]
Data	Data analysis, statistics, and probability
Data interpretation	Read, interpret, and make predictions using tables and graphs
Grade 8	Grade 8
Reasoning	
Mathematical complexity level:[3] moderate	

[1] Classified by TIMSS assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

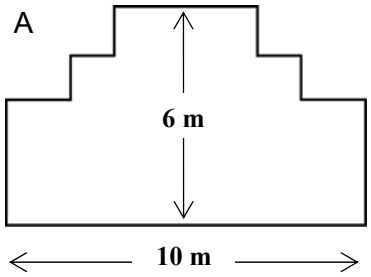
EXAMPLE 8

PISA complex multiple-choice item

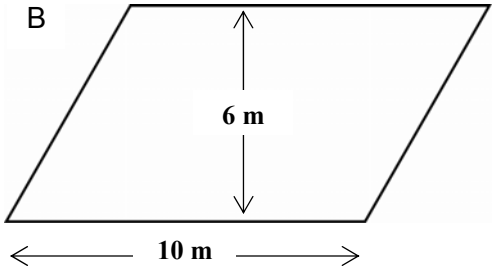
Carpenter

A carpenter has 32 meters of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.

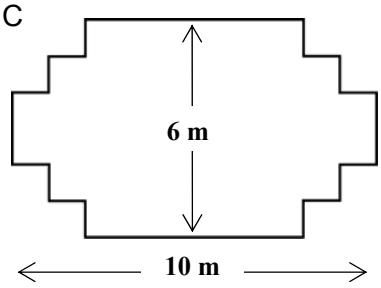
A



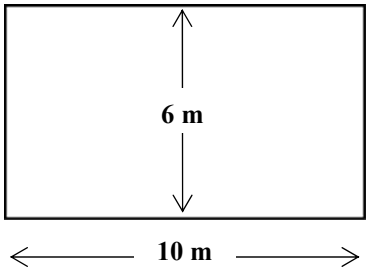
B



C



D



Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 meters of timber.

Garden bed design	Using this design, can the garden bed be made with 32 meters of timber?
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

SOURCE: Organization for Economic Cooperation and Development, Program for International Student Assessment (PISA) 2003 Mathematical Literacy Assessment.

EXAMPLE 8—continued

Scoring guide

Full credit Exactly four correct. Design A Yes Design B No Design C Yes Design D Yes
Partial credit Exactly three correct.
No credit Two or fewer correct.

Framework classifications

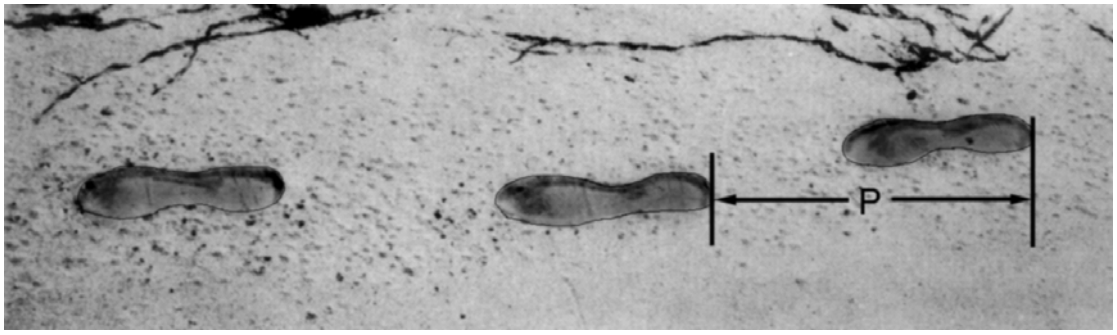
PISA 2003 framework[1]		NAEP 2003 framework[2]	
Overarching idea	Space and shape	Content strand	Measurement
Competency cluster	Connections	Topic	Estimate, calculate (using basic principles or formulas), or compare perimeter, area, volume, and surface area in meaningful contexts to solve mathematical and real-world problems.
Situation or context	Educational/occupational		
		Grade level	8
Mathematical complexity level:[3] moderate			

[1] Classified by PISA assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 9

PISA open constructed-response item

Walking



The picture shows the footprints of a man walking. The pacelength *P* is the distance between the rear of two consecutive footprints.

For men, the formula, $\frac{n}{P} = 140$, gives an approximate relationship between *n* and *P* where,

n = number of steps per minute, and

P = pacelength in meters.

Question 1: **WALKING**

If the formula applies to Heiko's walking and Heiko takes 70 steps per minute, what is Heiko's pacelength? Show your work.

SOURCE: Organization for Economic Cooperation and Development, Program for International Student Assessment (PISA) 2003 Mathematical Literacy Assessment.

EXAMPLE 9—continued

Scoring guide

Full credit 0.5 m OR 50 cm OR ½ (unit not required) • 70/ P = 140 [substitute numbers in the formula only]. 70 = 140 P P = 0.5 • 70/140
Partial credit Correct substitution of numbers in the formula, but incorrect answer, or no answer. • 70/ P = 140 [substitute numbers in formula only]. • 70/ P = 140 70 = 140 P P = 2 [correct substitution, but working out is incorrect]. OR Correctly manipulated the formula into $P = n/140$, but no further correct working.
No credit Other responses

Framework classifications

PISA 2003 framework[1]		NAEP 2003 framework[2]	
Overarching idea	Change and relationships	Content strand	Algebra and functions
Competency cluster	Reproduction	Topic	Interpret contextual situations and perform algebraic operations on real numbers and algebraic expressions to solve mathematical and real-world problems
Situation or context	Personal		
		Grade level	8
Mathematical complexity level:[3] low			

[1] Classified by PISA assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 10

TIMSS short constructed-response item – grade 4

204 ÷ 4 =

Answer: _____

SOURCE: International Association for the Evaluation of Educational Achievement, Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

Scoring guide

Correct response 51
Incorrect response Incorrect

Framework classifications

TIMSS 2003 framework[1]	NAEP 2003 framework[2]
Number Whole numbers Grade 4 Knowing facts and procedures	Number sense, properties, and operations Compute with numbers (i.e., add, subtract, multiply, divide) Grade 4
Mathematical complexity level:[3] low	

[1] Classified by TIMSS assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 11

TIMSS multiple choice item – grade 4

It takes Chris 4 minutes to wash a window. He wants to know how many minutes it will take him to wash 8 windows at this rate. He should

A) multiply 4 x 8
B) divide 8 by 4
C) subtract 4 from 8
D) add 8 and 4

SOURCE: International Association for the Evaluation of Educational Achievement, Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

Answer Key: A

Framework classifications

TIMSS 2003 framework[1]	NAEP 2003 framework[2]
Number Whole numbers Grade 4 Solving routine problems	Number sense, properties, and operations Compute with numbers (i.e., add, subtract, multiply, divide) Grade 4
Mathematical complexity level:[3] low	

[1] Classified by TIMSS assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 12

NAEP multiple-choice item – grades 4, 8, and 12

Add:

$$\begin{array}{r} 238 \\ +\ 462 \\ \hline \end{array}$$

A) 600
B) 690
C) 700
D) 790

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Key: C

Framework classifications

NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Number sense, properties, and operations Compute with numbers (i.e., add, subtract, multiply, divide) Grade 4 Procedural knowledge	Number Whole numbers Grade 4
Mathematical complexity level:[3] low	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 13

NAEP multiple-choice item – grade 4

Estela wants to buy 2 notebooks that cost $2.79 each, including tax. If she has one-dollar bills and no coins, how many one-dollar bills does she need?

A) 3
B) 4
C) 5
D) 6

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Answer Key: D

Framework classifications

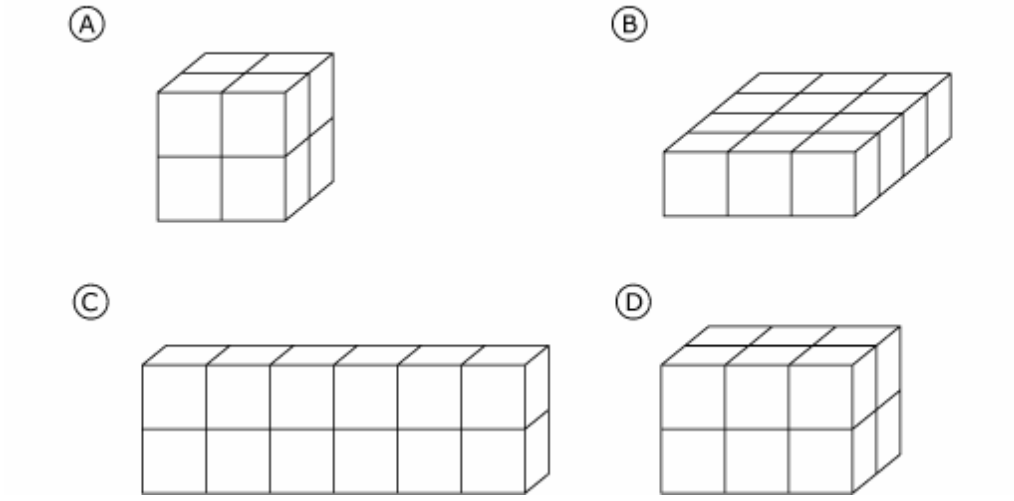
NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Number sense, properties, and operations	Number
Use computations and estimation in applications	Fractions and decimals
Grade 4	Grade 8
Problem solving	
Mathematical complexity level:[3] moderate	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 14

TIMSS multiple-choice item – grade 8

All the small blocks are the same size. Which stack of blocks has a different volume from the others?



SOURCE: International Association for the Evaluation of Educational Achievement, Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

Answer Key: A

Framework classifications

TIMSS 2003 framework[1]	NAEP 2003 framework[2]
Measurement	Measurement
Tools, techniques, and formulas	Estimate, calculate (using basic principles or formulas), or compare perimeter, area, volume, and surface area in meaningful contexts to solve mathematical and real-world problems.
Grade 8	Grade 4
Using concepts	
Mathematical complexity level:[3] moderate	

[1] Classified by TIMSS assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 15

TIMSS multiple-choice item – grade 4

All of the pupils in a class cut out paper shapes. The teacher picked one out and said, "This shape is a triangle." Which of these statements MUST be correct?

A) The shape has three sides.
B) The shape has a right angle.
C) The shape has equal sides.
D) The shape has equal angles.

SOURCE: International Association for the Evaluation of Educational Achievement, Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

Answer Key: A

Framework classifications

TIMSS 2003 framework[1]	NAEP 2003 framework[2]
Geometry Two- and three-dimensional shapes Grade 4 Knowing facts and procedures	Geometry and spatial sense No match to topic Grade 4
Mathematical complexity level:[3] low	

[1] Classified by TIMSS assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 16

NAEP short constructed-response item – grade 4

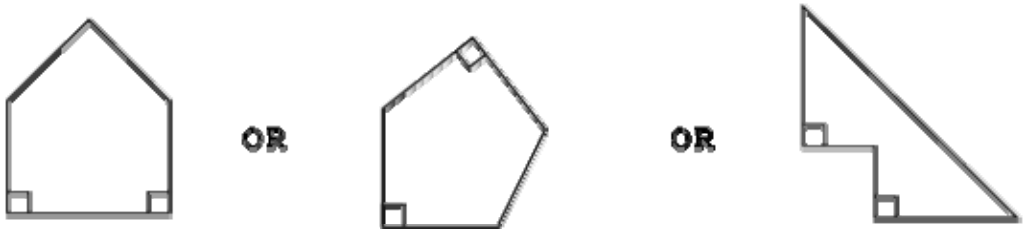
In the space below, draw a closed figure with 5 sides. Make 2 of the angles right angles.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Scoring guide

Solution:
Figure must be closed and have 5 sides and 2 or more right angles.

Right angles do not have to be marked, but should appear to be right angles. Two right angles must be on the inside of the figure.



In this question the student needed to show geometric understanding by drawing a closed figure with 5 sides and at least 2 right angles. Students did not have a ruler or protractor.
Correct Correct response.
Incorrect Figure drawn is a five-pointed star with a pentagon shown in the interior. The pentagon may or may not have right angles. OR Figure has 5 sides and only 1 right angle. OR No right angles in the figure drawn. OR Figure is not 5-sided or is not closed.

Framework classifications

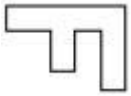
NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Geometry and spatial sense Describe, visualize, draw, and construct geometric figures Grade 4 Conceptual understanding	Geometry Two- and three-dimensional shapes Grade 4
Mathematical complexity level:[3] moderate	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 17

NAEP multiple-choice item – grades 4, 8, and 12

The figure above is shaded on the top side and white on the under side. If the figure were flipped over, its white side could look like which of the following figures?

A) 

B) 

C) 

D) 

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Answer Key: D

Framework classifications

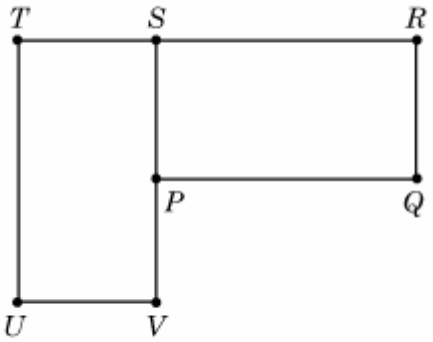
NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Geometry and spatial sense Identify the relationship (congruence, similarity) between a figure and its image under a transformation Grade 4 Conceptual understanding	Geometry Symmetry and transformations Grade 4
Mathematical complexity level:[3] low	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 18

TIMSS multiple-choice item – grade 8

Rectangle *PQRS* can be rotated (turned) onto rectangle *UVST*.



What point is the center of rotation?

A) *P*
B) *R*
C) *S*
D) *T*
E) *V*

SOURCE: International Association for the Evaluation of Educational Achievement, Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

Answer Key: C

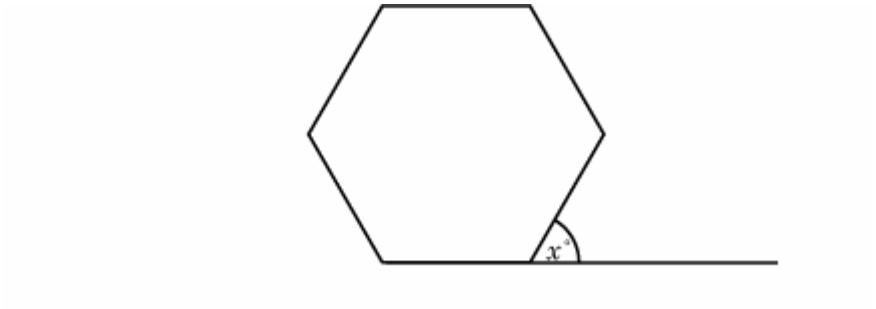
Framework classifications

TIMSS 2003 framework[1]	NAEP 2003 framework[2]
Geometry	Geometry and spatial sense
Symmetry and transformations	Identify the relationship (congruence, similarity) between a figure and its image under a transformation
Grade 8	Grade 12
Reasoning	
Mathematical complexity level:[3] low	

[1] Classified by TIMSS assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 19

TIMSS short constructed-response item – grade 8



The figure above is a regular hexagon. What is the value of x?

Answer: _____

SOURCE: International Association for the Evaluation of Educational Achievement, Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

Scoring guide

Correct response 60 degrees
Incorrect response 120 degrees OR Other incorrect

Framework classifications

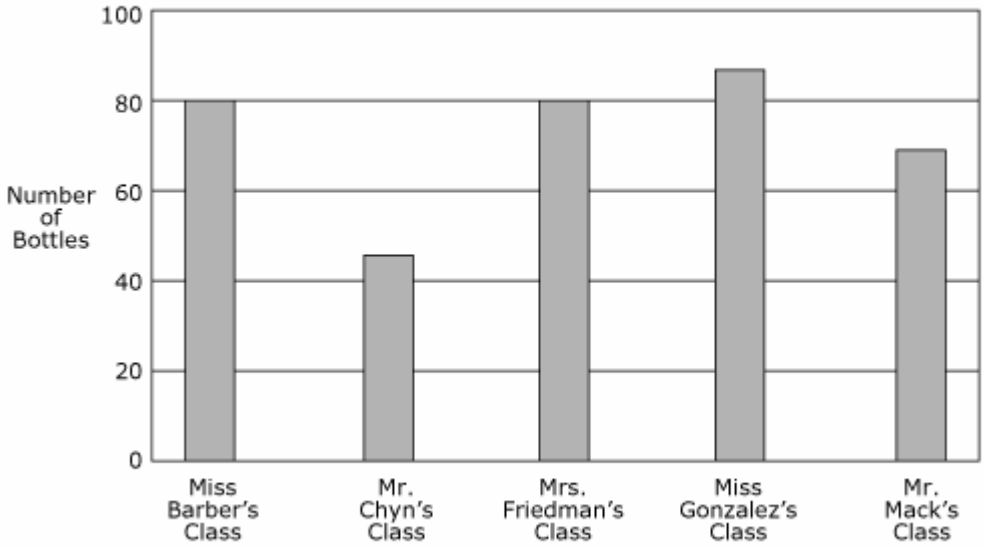
TIMSS 2003 framework[1]	NAEP 2003 framework[2]
Geometry Two- and three-dimensional shapes Grade 8 Solving routine problems	Geometry and spatial sense Apply geometric properties and relationships in solving problems Grade 8
Mathematical complexity level:[3] low	

[1] Classified by TIMSS assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 20

TIMSS multiple-choice item – grade 4

Central School had a bottle collection. Children in each class brought empty bottles to school. The principal made a bar graph of the number of bottles from five classes.



Which two classes collected exactly 80 bottles?

A) Miss Barber's and Mrs. Friedman's classes
B) Miss Barber's and Mr. Mack's classes
C) Mrs. Friedman's and Miss Gonzalez's classes
D) Miss Gonzalez's and Mr. Mack's classes

SOURCE: International Association for the Evaluation of Educational Achievement, Trends in International Mathematics and Science Study (TIMSS) 2003 Assessment.

Key: A

Framework classifications

TIMSS 2003 framework[1]	NAEP 2003 framework[2]
Data	Data analysis, statistics, and probability
Data representation	Read, interpret, and make predictions using tables and graphs
Grade 4	Grade 4
Solving routine problems	
Mathematical complexity level:[3] low	

[1] Classified by TIMSS assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 21

NAEP multiple-choice item – grade 8

$3 + 15 \div 3 - 4 \times 2 =$

A) -9
B) -2
C) 0
D) 4
E) 5

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Key: C

Framework classifications

NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Algebra and functions Interpret contextual situations and perform algebraic operations on real numbers and algebraic expressions to solve mathematical and real-world problems Grade 8 Procedural knowledge	Number Whole numbers Grade 8
Mathematical complexity level:[3] low	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 22

NAEP multiple choice item – grades 8 and 12

Fifteen boxes each containing 8 radios can be repacked in 10 larger boxes each containing how many radios?

A) 8
B) 10
C) 12
D) 80
E) 120

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Answer key: C

Framework classifications

NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Number sense, properties, and operations Use computations and estimation in applications Grade 8 Problem solving	Number Whole numbers Grade 8
PISA 2003 framework[2]	
Overarching idea	Quantity
Competency cluster	Reproduction
Situation or context	Not classified
Mathematical complexity level:[3] moderate	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel. NAEP grade 8 *problem solving* items were classified to both the TIMSS and PISA frameworks.
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 23

NAEP short constructed-response item – grades 8 and 12

One store, Price Pleasers, reduces the price each week of a $100 stereo by 10 percent of the original price.

Another store, Bargains Plus, reduces the price each week of the same $100 stereo by 10 percent of the previous week's price.

After 2 weeks, how will the prices at the two stores compare?

(A) The price will be cheaper at Price Pleasers.
(B) The price will be the same at both stores.
(C) The price will be cheaper at Bargain Plus.

Explain your reasoning.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Scoring guide

In this question the student was asked to compare the sale price of a stereo, after 3 weeks, based on two different ways for reducing the price. In one store, the price was reduced each week by a fixed amount (10% of $100, or $10). In the other store the price was reduced each week by a varying amount (10% of the current price, which is less each week). To earn full credit, the student needed to indicate that the price would be less at the first store after 3 weeks and explain how the solution was obtained. Students were permitted to use a calculator.
Solution: A. Cheaper at Price Pleasers At Price Pleasers the stereo would be $80 after 2 weeks. At Bargain Plus, it would cost $81. OR Successive 10% reductions of the original price will yield greater savings than successive reductions of 10% of the reduced price.
Correct Correct response—cheaper at Price Pleasers with an explanation that compares price at each store after 2 weeks ($80 vs. $81). OR Cheaper at Price Pleasers with an explanation that generalizes as described in solution above. NOTE: Score CORRECT if incorrect answer is B or C with a clear statement that Price Pleasers is cheaper and explanation is correct and complete.
Partial Cheaper at Price Pleasers with anything less than a complete explanation. OR Computes the correct amount for at least 2 weeks for either Price Pleasers or Bargain Plus, but conclusion is missing, incomplete, or incorrect (if the store is not identified the score is still a 2)
Incorrect Incorrect response.

EXAMPLE 23—continued

Framework classifications

NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Number sense, properties, and operations Apply ratios and proportional thinking in a variety of situations Grade 8 Problem solving	Number Ratio, proportion, and percent Grade 8
PISA 2003 framework[2]	
Overarching idea	Change and relationships
Competency cluster	Connections
Situation or context	Public/personal
Mathematical complexity level:[3] moderate	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel. NAEP grade 8 *problem solving* items were classified to both the TIMSS and PISA frameworks.
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 24

NAEP extended constructed-response item – grade 8

While she was on vacation, Tara sent 14 friends either a letter or a postcard. She spent $3.84 on postage. If it costs $0.20 to mail a postcard and $0.33 to mail a letter, how many letters did Tara send?
Show what you did to get your answer.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

Scoring guide

This question was a word problem that asked students to consider two values—the number of letters and the number of postcards—even though the student was only asked for the number of letters. This question could be solved in several ways. A student could reason numerically to find the number of letters and the number of postcards, possible by using a guess-and-check strategy or by creating a table. Another possibility was to set up and solve a system of two linear equations in two unknowns. To earn full credit, students needed to show how they obtained the answer. Students were permitted to use a calculator.

Solution:
8 letters
.20 (6) + .33(8) = $3.84
Students may use a variety of strategies to solve this, including guess and check, formal algebra, or others. For example,

# postcards	# letters	Total cost
1	13	4.49
2	12	4.36
3	11	4.23
4	10	4.10
5	9	3.97
6	8	**3.84**
7	7	3.71
8	6	3.58

OR
$x + y = 14$
$.20x + .33y = 3.84$
Therefore $.20x + .33(14-x) = 3.84$
So $x = 6$ and y = 8

Extended
Correct response.

Satisfactory
Correct, complete process is indicated, but answer is not 8 and has a minor computational error.
OR
Shows correct, complete process but does not indicate answer.

Partial
Correct, complete process in indicated, but answer is not 8 and there are several computational errors. (Process must clearly illustrate a correct strategy, such as a table or equations.)
OR
Correct response of 8 but shows no work or incomplete work.

Minimal
Process is incorrect because it ignores one or more pieces of given information.
OR
Process is correct but incomplete (process may be guess and check or another process which may lead to the correct answer, i.e., a chart but no equation, but goal is not clearly defined) and answer is not 8

Incorrect
Incorrect response.

EXAMPLE 24—continued

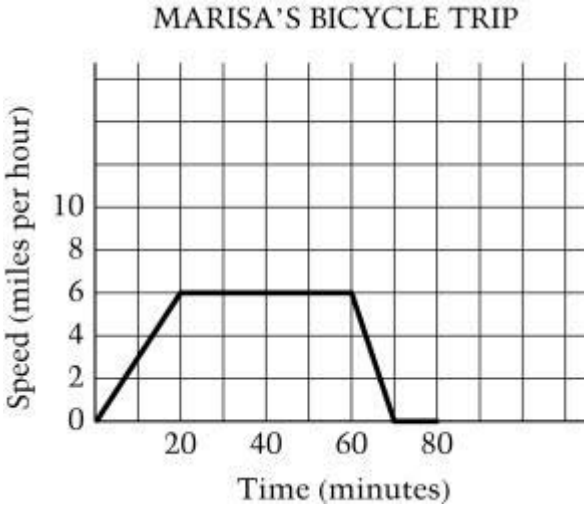
Framework classifications

NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Algebra and functions Solve systems of equations and inequalities using appropriate methods Grade 8 Problem solving	Algebra Equations and formulas Grade 8
PISA 2003 framework[2]	
Overarching idea	Change and relationships
Competency cluster	Connections
Situation or context	Public/personal
Mathematical complexity level:[3] moderate	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel. NAEP grade 8 *problem solving* items were classified to both the TIMSS and PISA frameworks.
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 25

NAEP extended constructed-response item – grade 8



The graph above represents Marisa's riding speed throughout her 80-minute bicycle trip. Use the information in the graph to describe what could have happened on the trip, including her speed throughout the trip.

During the first 20 minutes, Marisa

From 20 minutes to 60 minutes, she

From 60 minutes to 80 minutes, she

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment.

EXAMPLE 25—continued

Scoring guide

In this question the student was asked to translate across representations by interpreting information presented graphically and giving a verbal description (story) of Marisa's bicycle trip. The given graph presented Marisa's speed on the trip as a function of time. To earn full credit, the student needed to give both quantitative and qualitative information about the situation, including the observation that Marisa was at a stop during the last time interval.

Solution:

During the first 20 minutes, Marisa increased her speed from 0 to 6 mph.

From 20 to 60 minutes, she remained at 6 mph.

From 60 to 80 minutes, she decreased her speed from 6 mph to 0 mph and stopped. (Must have both decrease and stop from this interval).

Responses may be presented in the following ways.

Category A	Category B	Category C
Speed from 0 to 6	Increase	Downhill
Speed at 6	Remained the same	Float road (level)
Speed from 6 to 0; stop	Decrease	Uphill

- Responses may mix parts of more than one category
- Speed from 0 to 6 may be expressed as 0-6 (likewise for speed from 6 to 0 as 6-0)
- Responses may include extraneous correct information.
- "Stop" after 70 minutes may be expressed as "had no speed" or "maintained speed of 0," "stayed at that speed" having stated 0 mph.

Extended

All of Category A and all of Category B or Category C.

Satisfactory

All of Category A and incomplete Category B or Category C.
OR
Category A without stop and all of Category B or Category C.

Partial

All of Category A and no Category B or Category C.
OR
All of Category B and Category C, with stop.

Minimal

Category A without stop.
OR
Category B only.
OR
Category C only.
OR
Response that accounts for each of three parts of trip.

Incorrect

Incorrect response.

EXAMPLE 25—continued

Framework classifications

NAEP 2003 framework[1]	TIMSS 2003 framework[2]
Data analysis, statistics, and probability Read, interpret, and make predictions using tables and graphs Grade 8 Problem solving	Data Data interpretation Grade 8
PISA framework[2]	
Overarching idea	Change and relationships
Competency cluster	Connections
Situation or context	Educational/occupational
Mathematical complexity level:[3] moderate	

[1] Classified by NAEP assessment developers
[2] Classified by expert panel. NAEP grade 8 *problem solving* items were classified to both the TIMSS and PISA frameworks.
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.

EXAMPLE 26

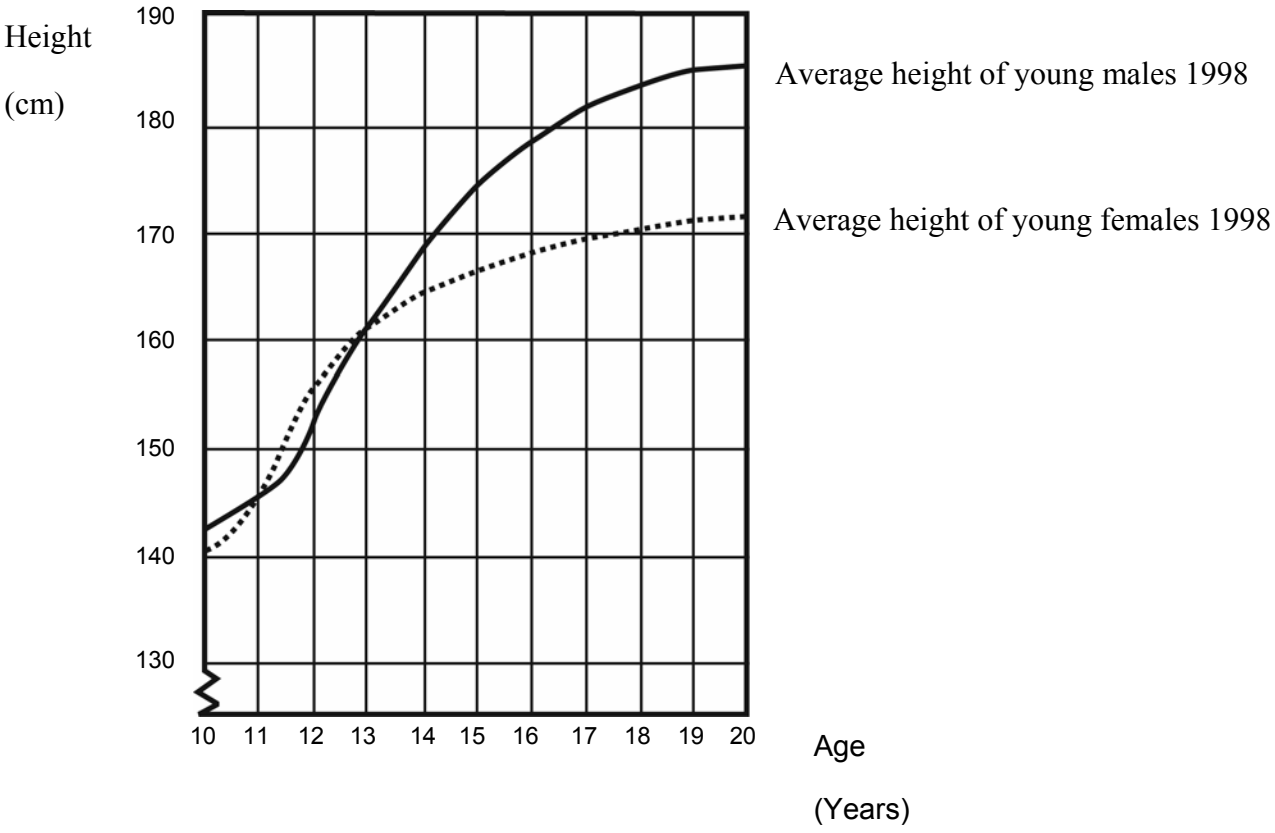
PISA task

Question 1: closed constructed-response item
Question 2: closed constructed-response item
Question 3: open constructed-response item

Growing Up

YOUTH GROWS TALLER

In 1998 the average height of both young males and young females in the Netherlands is represented in this graph.



EXAMPLE 26—continued

Question 1

Since 1980 the average height of 20-year-old females has increased by 2.3 cm, to 170.6 cm. What was the average height of a 20-year-old female in 1980?

Answer: _____cm

Question 2

According to this graph, on average, during which period in their life are females taller than males of the same age?

_____

_____

Question 3

Explain how the graph shows that on average the growth rate for girls slows down after 12 years of age.

_____

_____

_____

SOURCE: Organization for Economic Cooperation and Development, Program for International Student Assessment (PISA), 2003 Mathematical Literacy Assessment.

EXAMPLE 26—continued

Scoring guide

Question 1

Full credit 168.3 cm (unit already given).
No credit Other responses.

Question 2

Full credit Gives the correct interval, from 11-13 years. OR States that girls are taller than boys when they are 11 and 12 years old. (This answer is correct in daily-life language because it means the interval from 11-13).
Partial credit Other subsets of (11, 12, 13), not included in the full credit section.
No credit Other responses.

Question 3

Full credit The key here is that the response should refer to the "change" of the gradient of the graph for female. This can be done explicitly or implicitly. Full credit is for explicitly mentioning about the steepness of the curve of the graph or for implicit comparison using the actual amount of growth before 12 years and after 12 years of age. Refers to the reduced steepness of the curve from 12 years onwards, using daily-life language, not mathematical language. OR Refers to the reduced steepness of the curve from 12 years onwards, using mathematical language. OR Comparing actual growth (comparison can be implicit).
No credit Student indicates that female height drops below male height, but does NOT mention the steepness of the female graph or a comparison of the female growth rate before and after 12 years. OR Other incorrect responses. For example, the response does not refer to the characteristics of the graph, as the question clearly asks about how the graph shows the answer.

EXAMPLE 26—continued

Framework classifications

Question 1

PISA 2003 framework[1]		NAEP 2003 framework[2]	
Overarching idea	Change and relationships	Content strand	Number sense, properties, and operations
Competency cluster	Reproduction	Topic	Use computations and estimation in applications
Situation or context	Scientific	Grade level	8
Mathematical complexity level:[3] low			

Question 2

PISA 2003 framework[1]		NAEP 2003 framework[2]	
Overarching idea	Change and relationships	Content strand	Data analysis, statistics, and probability
Competency cluster	Reproduction	Topic	Read, interpret, and make predictions using tables and graphs
Situation or context	Scientific	Grade level	8
Mathematical complexity level:[3] moderate			

Question 3

PISA 2003 framework[1]		NAEP 2003 framework[2]	
Overarching idea	Change and relationships	Content strand	Data analysis, statistics, and probability
Competency cluster	Connections	Topic	Read, interpret, and make predictions using tables and graphs
Situation or context	Scientific	Grade level	8
Mathematical complexity level:[3] low			

[1] Classified by PISA assessment developers
[2] Classified by expert panel
[3] Mathematical complexity level classifications were made by the expert panel based on the definitions in the NAEP 2005 framework.