PISA USA

MATHEMATICS CONCEPTS & MATHEMATICS ITEMS

The *Mathematics Concepts and Mathematics Items* section contains four mathematics assessment units and 11 questions related to these units. These are the released items from the 2000 assessment (they are distinct from the secure items, which are kept confidential so that they may be used in subsequent cycles to monitor trends). In addition, an excerpt from the mathematics curriculum framework is included at the back of this volume.

Turn the page for instructions and an illustrative example.

5.1

Guide to the Content and Layout of This Book

A unit is made up of

- stimulus material, and
- questions relating to this material.

Apples is the name given to the first unit you will see. The three questions that follow ask questions about the Apples stimulus material—for example, *Complete the table*.

Process and content descriptors appear directly under the question heading:

- **Process** identifies the class of mathematical processes required. For *Apples* Question 1, the process is *Connections and Integration for Problem Solving*; and
- **Content** refers to the broad mathematical category. For *Apples* Question 1, content is *Change and Relationships*.

Each unit may use as many as three different **question-and-response formats.** All three formats are described below.

- **Multiple-choice response formats** ask the student to choose among several alternatives. In the *Speed of a Racing Car* unit, Question 1 is a multiple-choice item.
- **Short-answer response formats** ask the student to write down a short answer to the question. In the *Apples* example, Questions 1 and 2 ask for short-answer responses.
- **Extended-response formats** ask the student to write an somewhat extended answer to the question. In the *Apples* example, Question 3 asks for an extended response.

Scoring of student responses takes two forms:

- **Correct/incorrect**—some items are simply scored as correct/incorrect. In the *Apples* example, Question 1 is scored this way.
- **Correct/partly correct/incorrect**—the scoring for some items allows partial credit for the response in addition to full credit and no credit. In the *Apples* example, Question 3 is scored this way.

Scoring guides are provided for each question. In this kit, only the general instructions are provided. Illustrative examples presented in the original version of the scoring guide have been deleted in the interest of conserving space. The full version of these scoring guides can be found in the Organization for Economic Cooperation and Development (OECD) publication *Sample Tasks from the PISA 2000 Assessment* (see the publications guide in the *Readme First* book).

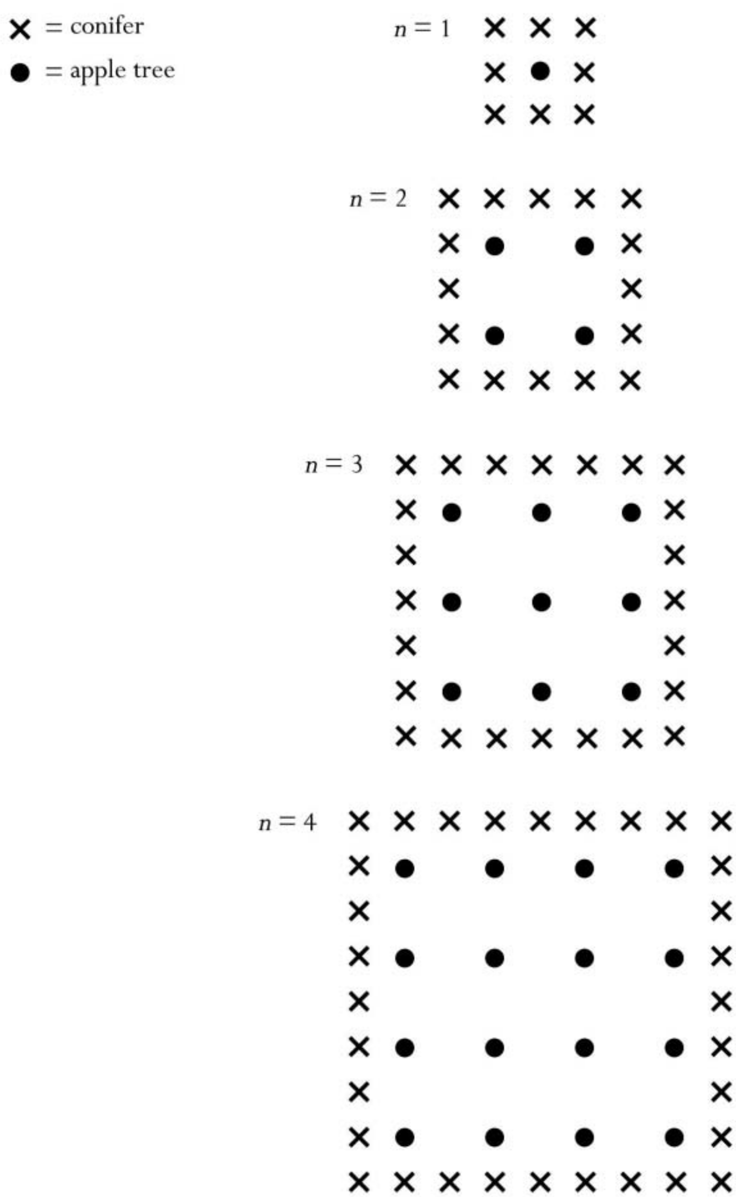
International benchmarks are provided next to each question. These consist of statistics on the percentage of students in each country who answered the question correctly. The countries are ordered in terms of this percentage. The OECD average is included as well. This display also indicates which countries scored significantly higher, significantly lower and no differently from this OECD average.

Mathematics Unit 1

Apples

A farmer plants apple trees in a square pattern. In order to protect the trees against the wind he plants conifers all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifers for any number (n) of rows of apple trees:



Students are given a hypothetical scenario involving planting an orchard of apple trees in a square pattern, with a row of protective conifer trees around the square. They are asked to complete a table of values generated by the functions that describe the number of trees as the size or orchard is increased. This task requires students to interpret a written description of a problem situation, to link this to a tabular representation of some of the information, to recognize a pattern and then to extend this pattern. Students need to work with given models and to relate two different representations (pictorial and tabular) of two relationships (one quadratic and one linear) in order to extend the pattern.

Question 1: APPLES

Process: Competency class 2 (Connections and integration for problem solving)
Content: Change and relationships

Complete the table:

n	Number of apple trees	Number of conifers
1	1	8
2	4	*16*
3	*9*	*24*
4	*16*	*32*
5	*25*	*40*

Scoring – Question 1: APPLES

Correct: Answers which show all 7 entries correct. Correct entries shown in italics.

Incorrect: Other answers.

Overall Percent Correct

Country	%	
Japan	83	▲
Korea, Republic of	76	▲
United Kingdom	69	▲
New Zealand	67	▲
Canada	63	▲
Australia	63	▲
Denmark	60	▲
Czech Republic	57	▲
Belgium	56	▲
United States	55	O
Russian Federation	53	O
Finland	53	O
Switzerland	53	O
Hungary	52	O
OECD average	**51**	
Sweden	48	O
Germany	47	O
Austria	47	O
Iceland	46	▼
Spain	45	▼
Ireland	44	▼
France	42	▼
Poland	42	▼
Italy	37	▼
Norway	37	▼
Luxembourg	37	▼
Latvia	36	▼
Greece	35	▼
Portugal	30	▼
Mexico	28	▼
Brazil	20	▼

Country average vs. OECD average:

Higher	▲
Not different	O
Lower	▼

The OECD average is the average of 27 of the 32 national averages. Brazil, Latvia, Liechtenstein, and the Russian Federation are not OECD countries. The Netherlands is omitted for technical reasons.

This task requires students to interpret expressions containing words and symbols, and to link different representations (pictorial, verbal and algebraic) of two relationships (one quadratic and one linear). Students have to find a strategy for determining when the two functions will have the same solution (for example, by trial and error, or by algebraic means), and to communicate the result by explaining the reasoning and calculation steps involved.

Question 2: APPLES

Process: Competency class 2 (Connections and integration for problem solving)
Content: Change and relationships

There are two formulae you can use to calculate the number of apple trees and the number of conifers for the pattern described above:

Number of apple trees = n^2

Number of conifers = $8n$

where n is the number of rows of apple trees.

There is a value of n for which the number of apple trees equals the number of conifers. Find the value of n and show your method of calculating this.

Overall Percent Correct

Country	Percent	
Korea, Republic of	85	▲
Japan	82	▲
Russian Federation	68	▲
Hungary	62	▲
Latvia	61	▲
Austria	60	▲
France	59	▲
Italy	59	▲
Czech Republic	57	O
Denmark	54	O
Belgium	54	O
Germany	53	O
Spain	52	O
Switzerland	51	O
Greece	51	O
OECD average	**51**	
Poland	50	O
Australia	49	O
New Zealand	49	O
Canada	47	▼
United Kingdom	45	O
Finland	44	O
Ireland	43	O
Sweden	41	▼
Norway	40	O
United States	38	▼
Portugal	36	▼
Iceland	31	▼
Luxembourg	31	▼
Brazil	23	▼
Mexico	17	▼

Country average vs. OECD average:

Higher	▲
Not different	O
Lower	▼

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Scoring – Question 2: APPLES

These scores are for the correct answer, n=8, using different approaches.

Correct:

1. Answers which give n=8, with the algebraic method explicitly shown.
2. Answers which give n=8, but no clear algebra is presented, or no work shown.
3. Answers which give n=8 using other methods, *e.g.*, using pattern expansion or drawing.
4. Answers which are similar to those given under Correct(1) (clear algebra), but give both answer n=8 AND n=0.
5. Answers which are similar to those given under Correct(2) (no clear algebra), but give both answer n=8 AND n=0.

Incorrect: Other answers, including the answer n=0.

This task requires students to show insight into mathematical functions by comparing the growth of a linear function with that of a quadratic function. Students are required to construct a verbal description of a generalized pattern, and to create an argument using algebra. Students need to understand both the algebraic expressions used to describe the pattern and the underlying functional relationships, in such a way that they can see and explain the generalization of these relationships in an unfamiliar context. A chain of reasoning is required, and communication of this in a written explanation.

Question 3: APPLES

Process: Competency class 3 (Mathematization, mathematical thinking, generalization and insight)
Content: Change and relationships

Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifers?

Explain how you found your answer.

Scoring – Question 3: APPLES

Fully Correct: Answers which are correct (apple trees) AND which give some algebraic explanations based on the formulae n^2 and $8n$.

Partially Correct: Answers which are correct (apple trees) AND are based on specific examples or on extending the table.

OR Answers which are correct (apple trees) and show SOME evidence that the relationship between n^2 and $8n$ is understood, but not so clearly expressed as in Fully Correct.

Incorrect: Answers which are correct (apple trees) but give an insufficient or wrong explanation, or no explanation.

Overall Percent Correct

Korea, Republic of	28	▲
Japan	19	▲
New Zealand	18	▲
Australia	17	O
Belgium	16	▲
Czech Republic	15	O
United Kingdom	14	O
Germany	14	O
Hungary	13	O
Canada	13	O
Ireland	12	O
United States	12	O
Latvia	12	O
Austria	11	O
OECD average	**11**	
Norway	10	O
Poland	10	O
Greece	10	O
Switzerland	9	O
France	8	▼
Spain	8	▼
Denmark	8	▼
Russian Federation	6	▼
Luxembourg	6	▼
Italy	6	▼
Iceland	6	▼
Finland	5	▼
Portugal	4	▼
Sweden	4	▼
Brazil	3	▼
Mexico	2	▼

Country average vs. OECD average:

Higher	▲
Not different	O
Lower	▼

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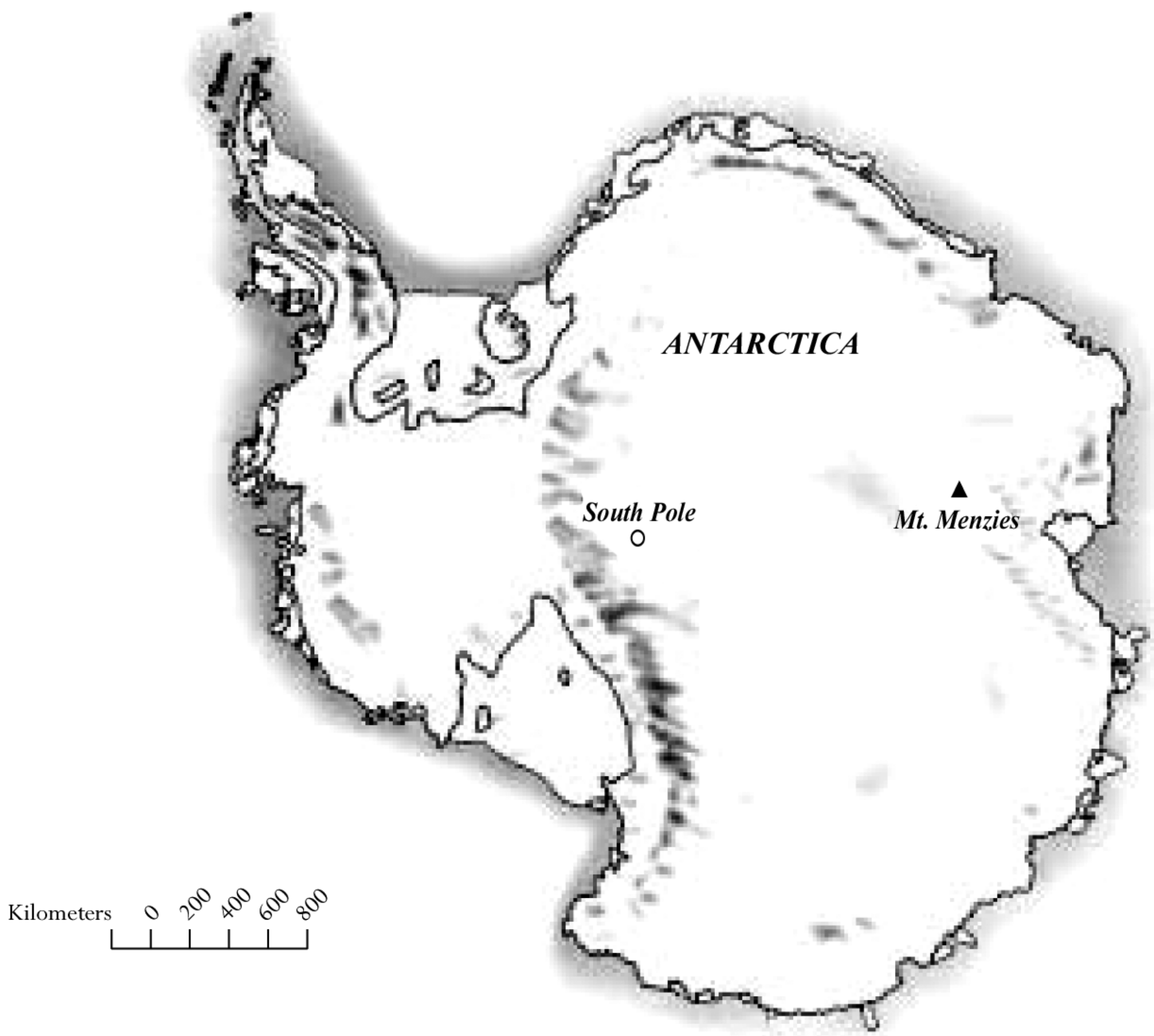
MATHEMATICS UNIT 2

Continent area

Here is a map of Antarctica.



This task requires students to identify an appropriate strategy and method for estimating the area of an irregular and unfamiliar shape, and to select and apply the appropriate mathematical tools in an unfamiliar context. Students need to choose a suitable shape or shapes with which to model the irregular area [for example, approximating parts of the map with rectangle(s), circle(s), triangle(s)]. Students need to know and apply the appropriate formulae for the shapes they use; to work with scale; to estimate length; and to carry out a computation involving a few steps.

Question 1: CONTINENT AREA

Process: Competency class 2 (Connections and integration for problem solving)
Content: Space and shape

Estimate the area of Antarctica using the map scale.

Show your working out and explain how you made your estimate. (You can draw over the map if it helps you with your estimation)

Scoring – Question 1: CONTINENT AREA

These scores are for answers that use the correct method AND give the correct result. The digit indicates the different approaches.

Fully Correct:

1. Answers which are estimated by drawing a square or rectangle – between 12,000,000 sq kms and 18,000,000 sq kms (units not required).

2. Answers which are estimated by drawing a circle – between 12,000,000 sq kms and 18,000,000 sq kms.

3. Answers which are estimated by adding areas of several regular geometric figures – between 12,000,000 and 18,000,000 sq kms.

4. Answers which are estimated by other correct methods – between 12,000,000 sq kms and 18,000,000 sq kms.

5. Answers which are correct (between 12,000,000 sq kms and 18,000,000 sq kms) but no working out is shown.

Overall Percent Correct

Liechtenstein	32	O
Finland	30	▲
Czech Republic	30	▲
Russian Federation	27	O
Austria	25	▲
Denmark	25	O
Switzerland	24	O
Hungary	24	O
Japan	23	O
New Zealand	23	O
Germany	21	O
United Kingdom	21	O
Iceland	21	O
Australia	21	O
Belgium	20	O
France	20	O
OECD average	**20**	
Canada	20	O
Sweden	19	O
Italy	19	O
Poland	18	O
Korea, Republic of	18	O
Luxembourg	15	O
Norway	15	▼
Spain	14	▼
Latvia	13	▼
United States	10	▼
Ireland	10	▼
Portugal	9	▼
Greece	9	▼
Mexico	2	▼
Brazil	2	▼

Country average vs. OECD average:

Higher	▲
Not different	O
Lower	▼

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These scores are for answers that use the correct method BUT give an incorrect or incomplete result. The digit indicates the different approaches, matching the digit of the Fully Correct scores.

Partially Correct

1. Answers which are estimated by drawing a square or rectangle – correct method but incorrect or incomplete answer.
2. Answers which are estimated by drawing a circle – correct method but incorrect or incomplete result.
3. Answers which are estimated by adding areas of several regular geometric figures – correct method but incorrect or incomplete result.
4. Answers which are estimated by other correct methods – but incorrect or incomplete result.

Incorrect:

1. Answers which show the perimeter instead of area.
2. Other incorrect.

Summary table

A summary table below shows the relationship between the scores:

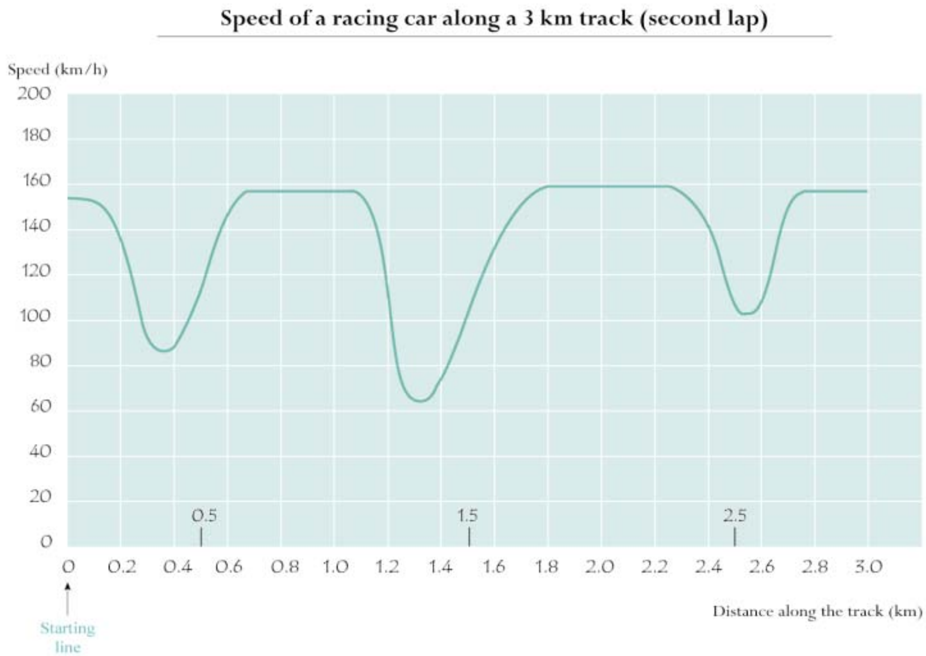
Estimation method	Score		
	Full credit – Answers which are correct: between 12,000,000 and 18,000,000 sq kms	Partial credit – Answers using the correct method but giving an incorrect or incomplete result.	No credit
Drawing a rectangle	1	1	–
Drawing a circle	2	2	–
Adding regular shapes	3	3	–
Other correct methods	4	4	–
No work shown	**5**	**–**	**–**
Perimeter	–	–	1
Other incorrect answers	–	–	2

Note: When coding this question, apart from reading what the student has written in words in the space provided, make sure that you look also at the map itself to see what drawings or marks the student made there. Very often, students find it difficult to explain in words exactly what they have done, but clues can be found from looking at the markings on the map itself. The aim is not to see whether students can express themselves well in words. The aim is to try to work out how they have arrived at their answers. Therefore, even if no written explanation is given, but you can tell what the student has done from sketches on the map or from the formulae used, the answer should be regarded as containing an adequate explanation.

MATHEMATICS UNIT 3

Speed of a racing car

This graph shows how the speed of a racing car varies along a flat 3 kilometer track during its second lap.



Source: In memory of Claude Janvier, who died in June 1998. Modified task after his ideas in Janvier, C. (1978): The interpretation of complex graphs – studies and teaching experiments. Accompanying brochure to the Dissertation. University of Nottingham, Shell Centre for Mathematical Education, Item C-2.

The pictures of the tracks are taken from Fischer, R & Malle, G. (1985): *Mensch und Mathematik*. Bibliographisches Institut: Mannheim-Wien-Zurich, 234-238.

This task requires students to interpret a graphical representation of a physical relationship (distance and speed of a car travelling on a track of unknown shape). Students need to interpret the graph by linking a verbal description with two particular features of the graph (one simple and straightforward, and one requiring a deeper understanding of several elements of the graph and what it represents), and then to identify and read the required information from the graph, selecting the best option from given alternatives.

Question 1: SPEED OF A RACING CAR

Process: Competency class 2 (Connections and integration for problem solving)
Content: Change and relationships

What is the approximate distance from the starting line to the beginning of the longest straight section of the track?

A 0.5 km.

B 1.5 km.

C 2.3 km.

D 2.6 km.

Scoring – Question 1: SPEED OF A RACING CAR

Correct: Answer B – 1.5 km.

Incorrect: Other answers.

Overall Percent Correct

Country	%	vs. OECD
Iceland	84	▲
Japan	83	▲
France	82	▲
Finland	82	▲
Liechtenstein	77	O
Korea, Republic of	77	▲
Australia	76	▲
United Kingdom	75	▲
New Zealand	74	O
Canada	73	▲
Czech Republic	73	O
Russian Federation	73	O
Norway	72	O
Belgium	72	O
Sweden	71	O
Denmark	70	O
Latvia	70	O
Austria	70	O
OECD average	**69**	
Switzerland	68	O
Spain	68	O
Ireland	67	O
Germany	66	O
Luxembourg	66	O
Portugal	63	O
United States	63	O
Poland	60	▼
Italy	58	▼
Hungary	57	▼
Brazil	56	▼
Greece	50	▼
Mexico	37	▼

Country average vs. OECD average:

Higher ▲
Not different O
Lower ▼

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This task requires students to read information from a graph representing a physical relationship (speed and distance of a car). Students need to identify one specified feature of the graph (the display of speed); to read directly from the graph a value that minimizes the feature; and then to select the best match from among given alternatives.

Question 2: SPEED OF A RACING CAR

Process: Competency class 1 (Reproduction, definitions and computations)
Content: Change and relationships

Where was the lowest speed recorded during the second lap?

A At the starting line.

B At about 0.8 km.

C At about 1.3 km.

D Halfway around the track.

Scoring – Question 2: SPEED OF A RACING CAR

Correct: Answer C – at about 1.3 km.

Incorrect: Other answers.

Overall Percent Correct

Country	Percent	vs. OECD
Finland	93	▲
Denmark	92	▲
France	92	▲
Korea, Republic of	92	▲
Japan	91	▲
Australia	91	▲
Norway	91	▲
Spain	90	▲
Canada	90	▲
New Zealand	90	▲
United Kingdom	88	▲
Portugal	87	O
Ireland	87	O
Belgium	86	O
Austria	85	O
Iceland	85	O
OECD average	**85**	
Hungary	85	O
United States	84	O
Czech Republic	84	O
Poland	83	O
Latvia	83	O
Switzerland	83	O
Liechtenstein	80	O
Russian Federation	80	O
Sweden	79	▼
Germany	78	▼
Italy	77	▼
Luxembourg	74	▼
Greece	73	▼
Mexico	57	▼
Brazil	55	▼

Country average vs. OECD average:

Higher ▲
Not different O
Lower ▼

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This task requires students to read information from a graph representing a physical relationship (speed and distance of a car). Students need to identify the place in the graph referred to in a verbal description in order to recognize what is happening to the speed of the vehicle at that point, and then to select the best matching option from among given alternatives.

Question 3: SPEED OF A RACING CAR

Process: Competency class 1 (Reproduction, definitions and computations)
Content: Change and relationships

What can you say about the speed of the car between the 2.6 km and 2.8 km marks?

A The speed of the car remains constant.

B The speed of the car is increasing.

C The speed of the car is decreasing.

D The speed of the car cannot be determined from the graph.

Scoring – Question 3: SPEED OF A RACING CAR

Correct: Answer B – the speed of the car is increasing.

Incorrect: Other answers.

Overall Percent Correct

Denmark	91	▲
Finland	90	▲
New Zealand	89	▲
Australia	89	▲
France	89	▲
Japan	88	▲
Korea, Republic of	88	▲
Poland	87	O
Sweden	87	O
Canada	87	▲
Norway	87	O
United Kingdom	86	O
Czech Republic	85	O
Iceland	84	O
Austria	84	O
OECD average	**84**	
Belgium	84	O
Hungary	83	O
United States	83	O
Italy	83	O
Spain	82	O
Ireland	82	O
Portugal	82	O
Germany	81	O
Liechtenstein	81	O
Switzerland	80	O
Latvia	80	O
Greece	79	O
Luxembourg	76	▼
Russian Federation	68	▼
Brazil	61	▼
Mexico	57	▼

Country average vs. OECD average:

Higher	▲
Not different	O
Lower	▼

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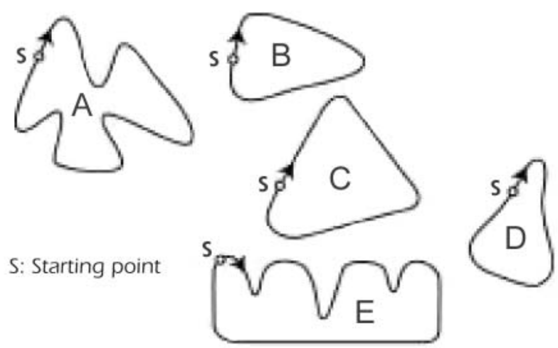
This task requires students to understand and interpret a graphical representation of a physical relationship (speed and distance of a car) and relate it to the physical world. Students need to link and integrate two very different visual representations of the progress of a car around a racetrack. Students have to identify and select the correct option from among given challenging alternatives.

Question 4: SPEED OF A RACING CAR

Process: Competency class 2 (Connections and integration for problem solving)
Content: Change and relationships

Here are pictures of five tracks:

Along which one of these tracks was the car driven to produce the speed graph shown earlier?



Scoring – Question 4: SPEED OF A RACING CAR

Correct: Answer B.

Incorrect: Other answers.

Overall Percent Correct

Japan	55	▲
Finland	40	▲
France	38	▲
Denmark	38	▲
New Zealand	37	▲
Australia	37	▲
Czech Republic	36	▲
Belgium	36	▲
Austria	34	○
Iceland	33	○
Canada	33	▲
Korea, Republic of	33	○
Sweden	32	○
Liechtenstein	32	○
United Kingdom	31	○
Norway	31	○
OECD average	**30**	
Germany	29	○
Luxembourg	29	○
Switzerland	29	○
Hungary	27	○
Spain	24	▼
United States	23	▼
Poland	22	▼
Latvia	21	▼
Portugal	19	▼
Ireland	19	▼
Russian Federation	19	▼
Italy	16	▼
Greece	15	▼
Brazil	6	▼
Mexico	5	▼

Country average vs. OECD average:

Higher	▲
Not different	○
Lower	▼

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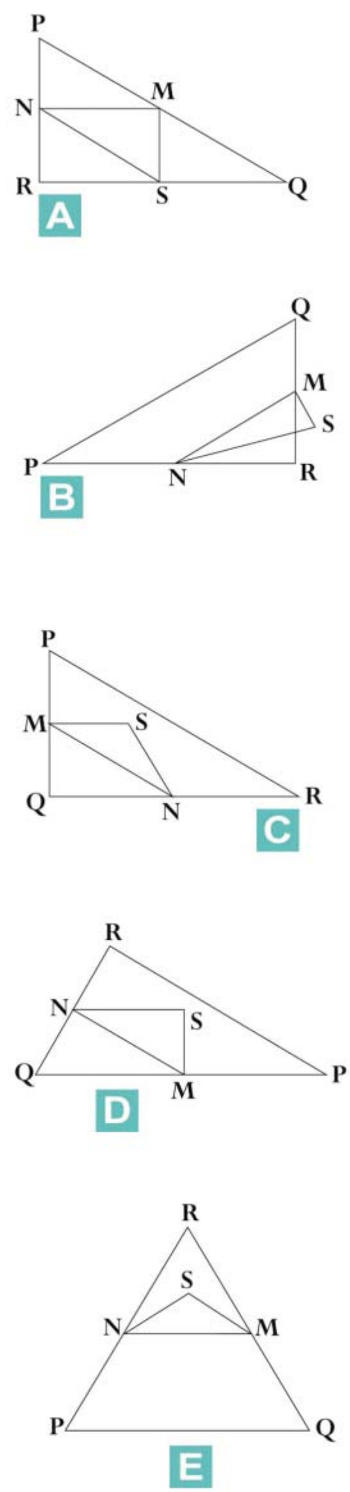
Mathematics Unit 4

Triangles



Students are given a written mathematical description of geometric objects, and are asked to select from given options a diagram that fits the description. This is an intramathematical task that requires students to link several pieces of information in text containing mathematical terms to standard geometric representations. Students need to link elements of one representation in words and symbols with corresponding elements of a representation in diagram form, selecting the appropriate matching representation from a number of options.

Question 1: TRIANGLES

Process: Competency class 1 (Reproduction, definitions and computations)
Content: Space and shape

Circle the one figure on the previous page that fits the following description.

Triangle PQR is a right triangle with right angle at R. The line RQ is less than the line PR. M is the midpoint of the line PQ and N is the midpoint of the line QR. S is a point inside the triangle. The line MN is greater than the line MS.

Scoring – Question 1: TRIANGLES

Correct: Answer D.

Incorrect: Other answers.

Overall Percent Correct

France	83	▲
Japan	77	▲
Switzerland	76	▲
Czech Republic	72	▲
Hungary	72	▲
Belgium	71	▲
Poland	70	▲
Germany	70	▲
Liechtenstein	67	O
Russian Federation	65	O
Austria	65	O
Italy	64	O
Korea, Republic of	63	O
Australia	63	O
Latvia	63	O
OECD average	**62**	
Greece	61	O
Iceland	60	O
Luxembourg	60	O
Finland	60	O
Norway	59	O
New Zealand	59	O
United Kingdom	59	O
Canada	57	▼
Spain	56	O
Denmark	55	O
Portugal	55	▼
Ireland	54	▼
United States	46	▼
Sweden	46	▼
Brazil	40	▼
Mexico	29	▼

Country average vs. OECD average:

Higher	▲
Not different	O
Lower	▼

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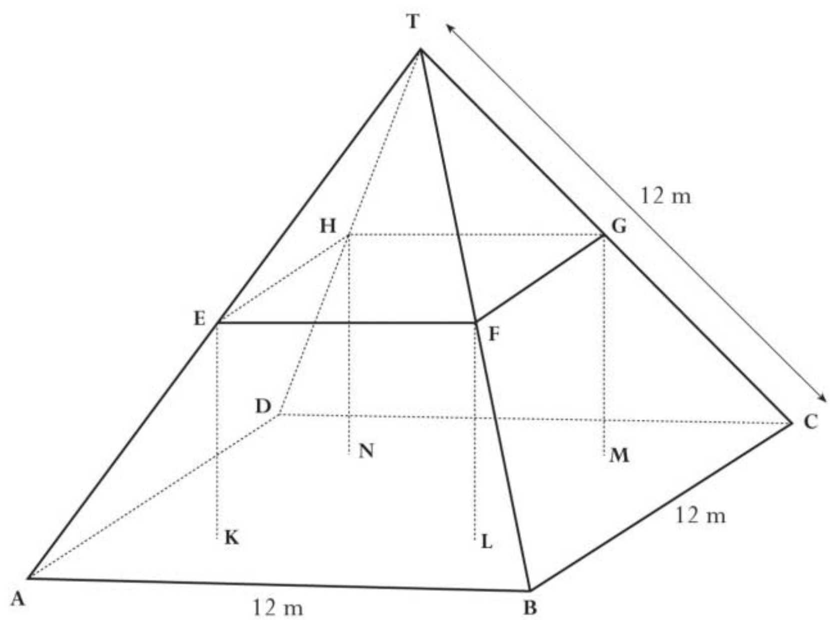
MATHEMATICS UNIT 5

Farms

Here you see a photograph of a farmhouse with a roof in the shape of a pyramid.

Below is a student's mathematical model of the farmhouse **roof** with measurements added.



The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a block (rectangular prism) EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT and H is the middle of DT. All the edges of the pyramid in the model have the length 12 m.

Students are given a mathematical model (in the form of a diagram) and a written mathematical description of a real-world object (a pyramid-shaped roof) and asked to calculate the area of the base. This task requires students to link a verbal description with an element of a diagram; to recall the area formula for a square with given sides; and to identify the required information in the diagram. Students then need to carry out a simple calculation in order to compute the required area.

Question 1: FARMS

Process: Competency class 1 (Reproduction, definitions and computations)
Content: Space and shape

Calculate the area of the attic floor ABCD.

The area of the attic floor ABCD = __________ m²

Scoring – Question 1: FARMS

Correct: 144 (unit already given).

Incorrect: Other answers.

Overall Percent Correct

Japan	88	▲
Czech Republic	84	▲
Finland	82	▲
Denmark	80	▲
Italy	78	▲
Belgium	77	▲
Switzerland	76	▲
Liechtenstein	76	O
Korea, Republic of	75	▲
France	75	▲
Sweden	74	▲
Austria	73	O
Iceland	73	O
Canada	72	O
Norway	72	O
Ireland	71	O
New Zealand	70	O
Australia	69	O
OECD average	**69**	
Poland	68	O
United Kingdom	67	O
Hungary	67	O
Russian Federation	65	O
Germany	61	O
Greece	61	O
Latvia	60	▼
Portugal	57	▼
Luxembourg	57	▼
United States	51	▼
Mexico	37	▼
Spain	35	▼
Brazil	24	▼

Country average vs. OECD average:

Higher	▲
Not different	O
Lower	▼

The OECD average is the average of 27 of the 32 national averages. Brazil, Latvia, Liechtenstein, and the Russian Federation are not OECD countries. The Netherlands is omitted for technical reasons.

Students are given a mathematical model (in the form of a diagram) and a written mathematical description of a real-world object (a pyramid-shaped roof) and asked to calculate one of the lengths in the diagram. This task requires students to work with a familiar geometric model and to link information in verbal and symbolic form to a diagram. Students need to visually "disembed" a triangle from a 2-dimensional representation of a 3-dimensional object; to select the appropriate information about side length relationships; and to use knowledge of similar triangles in order to solve the problem.

Question 2: FARMS

Process: Competency class 2 (Connections and integration for problem solving)
Content: Space and shape

Calculate the length of EF, one of the horizontal edges of the block.

The length of EF = ____________ m

Scoring – Question 2: FARMS

Correct: 6 (unit already given).

Incorrect: Other answers.

Overall Percent Correct

Japan	92	▲
Russian Federation	91	▲
Korea, Republic of	88	▲
France	85	▲
Poland	80	O
Greece	78	O
Hungary	78	O
New Zealand	77	O
Italy	77	O
Belgium	77	O
Canada	77	O
Denmark	77	O
Australia	76	O
Portugal	76	O
Spain	76	O
United Kingdom	76	O
Switzerland	75	O
Finland	75	O
Ireland	75	O
OECD average	**75**	
Latvia	74	O
Austria	72	O
Norway	71	O
Czech Republic	69	O
Iceland	68	▼
Sweden	67	▼
United States	67	▼
Luxembourg	65	▼
Germany	64	▼
Liechtenstein	61	O
Mexico	49	▼
Brazil	40	▼

Country average vs. OECD average:

Higher	▲
Not different	O
Lower	▼

The OECD average is the average of 27 of the 32 national averages. Brazil, Latvia, Liechtenstein, and the Russian Federation are not OECD countries. The Netherlands is omitted for technical reasons.

Basic Definitions from the Mathematics Curriculum Framework

(Excerpt from Sample Tasks from the PISA 2000 Assessment of Reading, Mathematical and Scientific Literacy-OECD, 2002)

The items contained in this package are sample tasks from the PISA 2000 assessment of mathematical literacy. PISA (Program for International Student Assessment) is a collaborative effort by members of the Organisation for Economic Co-operation and Development (OECD) to measure how well young adults at age 15, therefore approaching the end of compulsory schooling, are prepared to meet the challenges of today's knowledge societies. The assessment is forward looking, focusing on young people's ability to use their knowledge and skills to meet real-life challenges, rather than on the extent to which they have mastered a specific school curriculum.

PISA brings together mathematics expertise from the participating countries, steered jointly by their governments on the basis of shared, policy-driven interests. Experts from participating countries serve on working groups that are charged with linking the PISA policy objectives with the best available substantive and technical expertise in the field of international comparative assessment of educational outcomes. Through participating in these expert groups, countries ensure that the PISA assessment instruments are internationally valid and take into account the cultural and curricular contexts of OECD member countries, that they provide a realistic basis for measurement, and that they place an emphasis on authenticity and educational validity. The frameworks and assessment instruments for PISA 2000 are the product of a multi-year development process and were adopted by OECD countries in December 1999. The conceptual framework of PISA [described in its entirety in *Measuring Student Knowledge and Skills: A New Framework for Assessment* (OECD, 1999) - contained elsewhere in this package] is based on the content students need to acquire, processes that need to be performed, and the contexts in which knowledge and skills are applied.

The assessments are based on, and the items classified by, the following definitions of literacy:

> **Mathematical Literacy** - *The capacity to identify, to understand, and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual's current and future private life, occupational life, social life with peers and relatives and life as a constructive, concerned and reflective citizen.*

The PISA mathematics assessment framework is constructed of the following dimensions:

A. Processes: skills pertinent at all educational levels:

1. Mathematical thinking and reasoning
2. Mathematical argumentation
3. Mathematical communication
4. Modelling
5. Problem Posing and Solving
6. Representation
7. Using symbolic, formal, and technical language and operations
8. Use of aids and tools

B. Content: For the purpose of PISA, a selection of "overarching concepts" was made that would encompass sufficient variety and depth to reveal the essentials of mathematics and would at the same time represent or include the conventional mathematics curricular strands in an acceptable way.

1. Change and relationships:
    a. PISA examined students' ability to represent changes in a comprehensible form, to understand the fundamental types of change, to recognize particular types of change when they occur, to apply these changes to the outside world and to control a changing world to our best advantage.
    b. Functional thinking, i.e. thinking in terms of relationships, is one of the most fundamental disciplinary aims of the teaching of mathematics. Relationships may be given a variety of different representations, including symbolic, algebraic, graphical, tabular and geometrical. Different representations may serve different purposes and have different properties. Hence translation between representations is often of key importance in dealing with situations and tasks.
2. Space and shape:
    a. Shapes can be regarded as patterns: houses, office blocks, bridges, starfish, town plans, cloverleaves, crystals and shadows. Geometric patterns can serve as relatively simple models of many kinds of phenomena, and their study is possible and desirable at all levels.
    b. In understanding space and constructions, students need to look for similarities and differences as they analyze the components of form and recognize shapes in different representations and different dimensions. The study of shapes is closely connected to the concept of "grasping space." This means learning to know, explore and conquer, in order to live, breathe and move with more understanding in the space in which we live.
3. Quantity and uncertainty:
    a. Situations and contexts: An important aspect of the definition of mathematical literacy is using and doing mathematics in a variety of situations. One can think of a situation as being at a certain distance from the student. The closest is private life (daily life), next is school life, work and sports, followed by the local community and society as encountered in daily life, and furthest away are scientific concepts.

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PISA USA