VOLUME 3

FOUNDATIONS

A monograph for professionals in science, mathematics, and technology education

Professional Development That Supports School Mathematics Reform

Division of Elementary, Secondary, and Informal Education

Directorate for Education and Human Resources

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Raffaella Borasi and Judith Fonzi

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About Foundations

Foundations is a monograph series published by the National Science Foundation's Division of Elementary, Secondary, and Informal Education (ESIE) in conjunction with the Division of Research, Evaluation and Communication (REC) to serve those working to better science, mathematics, and technology education in this nation. Foundations supports education reform by communicating lessons that have been learned from ESIE projects and activities to others in the field who may use and adapt them to build effective educational improvement strategies in their own classrooms and communities. Like the foundation of a schoolhouse, home, or other place of learning, the strength of what is above ground depends on the structural soundness of what lies below. Foundations will unearth the strategies that enable effective educational improvement at the K–12 level to take place. Welcome to Foundations…

In This Volume

Foundations examines opportunities and challenges for those at the front line of mathematics education in elementary and secondary schools. Designed as a resource for teachers and administrators who are interested in investigating inquiry-based mathematics education, this volume serves neither as a textbook nor as the final word on the subject. It is rather a short introduction for those beginning the complex and difficult journey of mathematics education reform based on the experiences of educators working in the inquiry field today.

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PREFACE

Purpose of This Monograph

We live in a time of great change in mathematics education. The National Council of Teachers of Mathematics (NCTM) Standards and other influential reports have called for radical reform in U.S. school mathematics in order to prepare *all* students to meet the mathematical demands of today's society (NCTM, 1989, 1991, 1995, 2000; National Research Council [NRC], 1989; U.S. Department of Education, 1996). These reports challenge the content and pedagogy of current mathematics instruction. Most importantly, they highlight the need for more student-centered and constructivist-based instruction so that problem-solving, meaning-making and conceptual understanding are emphasized, not rote memorization of facts and procedures.

This new focus departs radically from the way mathematics has traditionally been taught. Putting the reports' recommendations into practice will require teachers to rethink not only their teaching practices but also the very goals of teaching mathematics. Schools and districts will also have to revise their mathematics curricula and assessments and work together to redefine expectations for students' learning and teaching practices. These changes are not easy to accomplish; they demand not only supportive school structures but also high quality professional development programs to guide and support teachers' individual and collaborative efforts.

Such professional development programs present a new challenge for providers and consumers. As principal investigators of a few NSF-funded professional development projects in mathematics, we have learned that neither typical university courses nor traditional in-service workshops go far enough. Different kinds of experiences are needed if we wish to promote radically different beliefs and practices and create learning communities engaged in reform.

Fortunately, several examples of successful professional development initiatives that support school mathematics reform have emerged in the last twenty years (as reported, for example, in Friel & Bright [1997] and Eisenhower National Clearinghouse for Mathematics and Science Education [ENC] [2000]). These initiatives include not only a new set of goals and principles but also new kinds of professional development activities, such as innovative mathematical learning experiences for teachers, in-depth examinations of students' mathematical work, "case discussions" and various kinds of supported field experiences.

The main purpose of this monograph is to identify and critically examine these promising professional development practices, with the goal of enabling professional development providers and users to better evaluate how each can best be used to support school mathematics reform.

Because systematic research studies on effective mathematics teacher education are still limited, many questions about teacher change, school reform, and the effects of specific professional development strategies remain unanswered. At the same time, some theories and empirical data are emerging that may help providers and users evaluate the potential contributions of alternative professional development initiatives. In this monograph, we will synthesize and critique these theoretical and empirical contributions by looking at both the published literature and the "informal knowledge" shared among successful practitioners. As a main source for the latter, we draw primarily upon the results reported by the many Teacher Enhancement and Local Systemic Change projects funded by the National Science Foundation during the last twenty years.

We hope that the monograph will be useful for those who design and facilitate professional development for mathematics teachers ***of all grade levels***. Even more importantly, however, our goal is to support informed decisions on the part of a wide range of education professionals who are *consumers* of professional development. Consumers include school administrators who make decisions about the kind of professional development that should take place in their district, teachers who must choose professional development initiatives to participate in, and officials at government agencies and private foundations who fund teacher enhancement projects.

Assumptions informing this monograph

Before embarking on an in-depth analysis of specific professional development practices, we want to clarify some assumptions that inform our perspective.

Along with Susan Loucks-Horsley and her colleagues (1998), we believe that good professional development programs result from knowing the unique goals, needs and constraints of each audience:

> Professional development, like teaching, is about decision making – designing optimal opportunities tailored to the unique situation.
> (Loucks-Horsley, Hewson, Love & Stiles, 1998, p. xiii)

Thus, if professional development is audience-based, no single model of professional development will work for all. Rather, content, format and activities should be considered in light of each situation to determine which would be most appropriate and effective, and in which combination and sequence. At the same time, knowing about a variety of practices and their potential contributions to achieving specific goals will be critical to making informed decisions.

To accomplish this end, we suggest that professional development providers and consumers should know about, and take into consideration, the following elements:

- The needs of teachers engaging in school mathematics reform and how these needs may be effectively addressed,
- The principles informing the most successful professional development initiatives and the theoretical and empirical basis of those principles, and
- The strengths and limitations of different types of professional development activities that have been successfully developed and field-tested by the mathematics teacher education community.

We have created this monograph to help readers gain this knowledge base.

We are aware that there are other key logistical issues that professional development providers need to address, such as scheduling initiatives, organizing the space for them, grouping participants, and offering compensation or other incentives for participants, just to name a few. While we recognize that these issues are critical to the success of any professional development initiative, they are beyond the scope of this monograph.

How the monograph is organized

The next three chapters of the monograph are devoted to developing a framework to evaluate specific professional development initiatives and practices.

To this end, in Chapter 1 we examine what teachers may need in order to successfully engage in school mathematics reform. Based on what we have learned about teacher development and reform from research and exemplary practice, we identify and discuss nine categories of "teacher learning needs" that should be addressed by professional development that aims at supporting instructional innovation in mathematics.

Chapter 2 provides some images of effective professional development. Here we portray two of the several professional development programs in the literature that have documented success in supporting school mathematics reform. We selected these examples to illustrate significantly different approaches that can be used to address the teacher learning needs identified in Chapter 1 at different grade levels. We hope these descriptions will reveal the complexity of good professional development programs and show the many alternatives available.

In Chapter 3, we compare the programs described in Chapter 2 to identify both elements that are shared by most effective professional development programs and some options consumers and providers can choose from. Among these options, we identify the various *formats* that professional development can take on, the possible *areas of expertise* for professional development providers and, most importantly, a few categories of *professional development experiences* that represent quite different yet complementary approaches to address the needs of mathematics teachers engaging in reform.

In Chapters 4 to 8 we then examine in depth each of these types of professional development experiences, which we have characterized as engaging teachers in:

- Experiences-as-learners where they experience first-hand some innovative ways to learn and teach mathematics (Chapter 4).
- In-depth analysis of students' mathematical work (Chapter 5).
- "Case discussions" where a selected example of practice serves as the catalyst for reflecting on and discussing important issues related to school mathematics reform (Chapter 6).

- Structured and scaffolded attempts at instructional innovation (Chapter 7).
- Gathering and making sense of relevant information about various aspects of school mathematics reform, using a variety of tools (Chapter 8).

In each of these chapters, we use the framework developed in the first part of the monograph to examine what characterizes professional development activities within that given category, and how and why these activities can contribute to support teachers engaged in school mathematics reform. More specifically, we begin by discussing the *theoretical rationale and empirical evidence* that support the featured type of professional development experience. Next, we provide two *illustrations* to give a rich image of this type of experience in action, while also showing the many differences that could occur in its implementation. Referring to these examples, we then articulate the *characteristic elements* of this type of professional development experience and discuss some of its *main variations*. We follow this with an analysis of how each of the *teacher learning needs* identified in Chapter 1 can be addressed by variations within this kind of professional development experience (further summarized in Figure 11 in our concluding chapter). Each chapter concludes with suggestions for follow-up readings.

The monograph closes with a summary chapter in which we briefly review our major findings about what kinds of professional development can best support school mathematics reform. We also provide some suggestions about how providers and consumers can ensure that mathematics teachers are offered the high quality professional development they need to significantly improve mathematics instruction.

Part I.

Developing a Framework for Evaluating Professional Development

CHAPTER 1

What Are the Needs of Teachers Who Are Engaged in School Mathematics Reform?

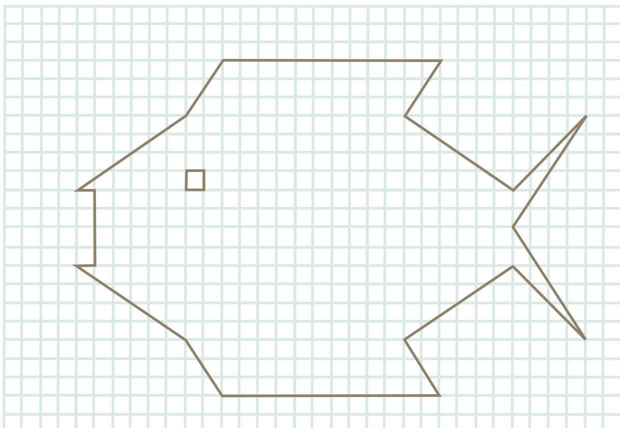
Knowing the needs that teachers engaging in school mathematics reform today are likely to experience is critical to help providers set goals for a specific professional development initiative and to evaluate the potential contributions of any given program. To highlight the extensive changes in teaching practices called for by the current reform and the challenges that these changes are likely to present teachers, we will first present an image of a reform-oriented mathematics classroom. The vignette that follows describes an actual classroom experience (see Callard, 2001, for a more detailed account of this experience). We will refer to this vignette throughout the chapter to illustrate key points about the learning needs of teachers engaged in school mathematics reform as identified in the research on teacher development and reform.

An image of a reform-oriented mathematics class

The instructional unit captured in this vignette was developed by Mrs. Callard, the classroom teacher, based on a set of instructional materials created to support an illustrative inquiry unit on area for middle school students (Borasi, 1994a). While these illustrative instructional materials provided an overall design for the unit, Mrs. Callard had to make a series of pedagogical decisions to adapt the unit to her own goals and to the constraints of her 8th grade mathematics class. For example, since she knew that her 6th grade colleagues had already worked with students on developing the concept of area and had introduced area formulas for rectangles and triangles, she decided to focus the unit on developing area formulas, drawing from the second part of the instructional resource materials.

Mrs. Callard began her four-week unit on developing area formulas with an activity that would invite students to review what they already knew about area with the goal of building on their prior knowledge and also identifying gaps and misconceptions in their understanding. The activity required students to find the area of a complex figure drawn on graph paper – the "fish" reproduced in Figure 1.

Figure 1
The "fish"

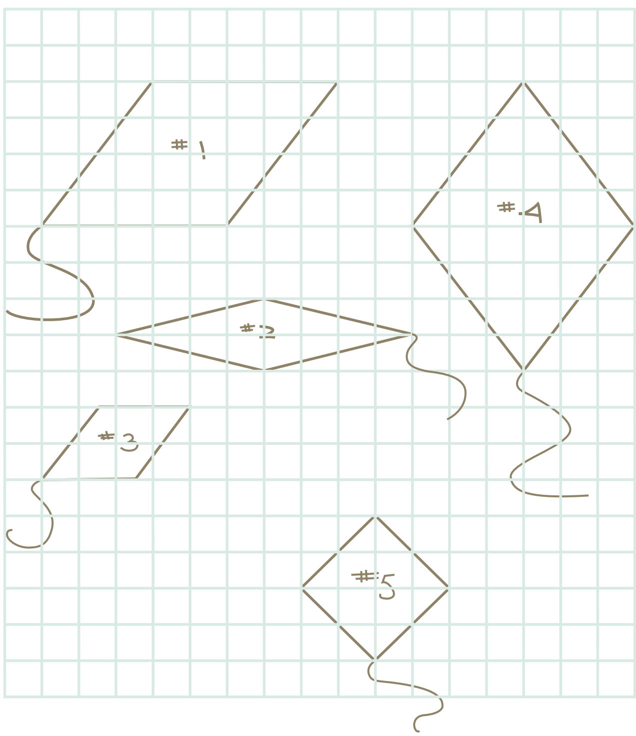


Mrs. Callard handed out a copy of this figure to each student, instructing the class to work on this task individually for a few minutes and then to share their preliminary results with a partner prior to a whole class discussion (so as to invite collaboration and scaffold their sharing in front of a large group). To further support the student's mathematical thinking and suggest alternative approaches, the teacher also made available a variety of tools, such as rulers, compasses, scissors, calculators, string, and tape, and even additional copies of the "fish." As the students worked, the teacher moved around the class for about 15 minutes observing, encouraging and supporting the students.

When most pairs reached solutions that satisfied them, the teacher asked volunteers to show their solution/strategies to the rest of the class. This sharing enabled students to appreciate the variety of approaches that could be used to solve this problem. These included strategies such as breaking the fish into rectangles and triangles and then adding the areas of these simpler figures, "boxing" the fish and then taking away the extra pieces, or simply counting the whole squares in the fish and approximating the partial ones. As each pair shared its solution, the teacher asked the students to articulate the strategy they used, and she recorded it on newsprint, so as to make each strategy explicit and to enable the class to later examine the strengths and weaknesses of the alternative strategies for computing the area of a complex figure. In this discussion, the teacher also pointed out the key role that the area formulas for rectangle and triangle played in several of the strategies identified.

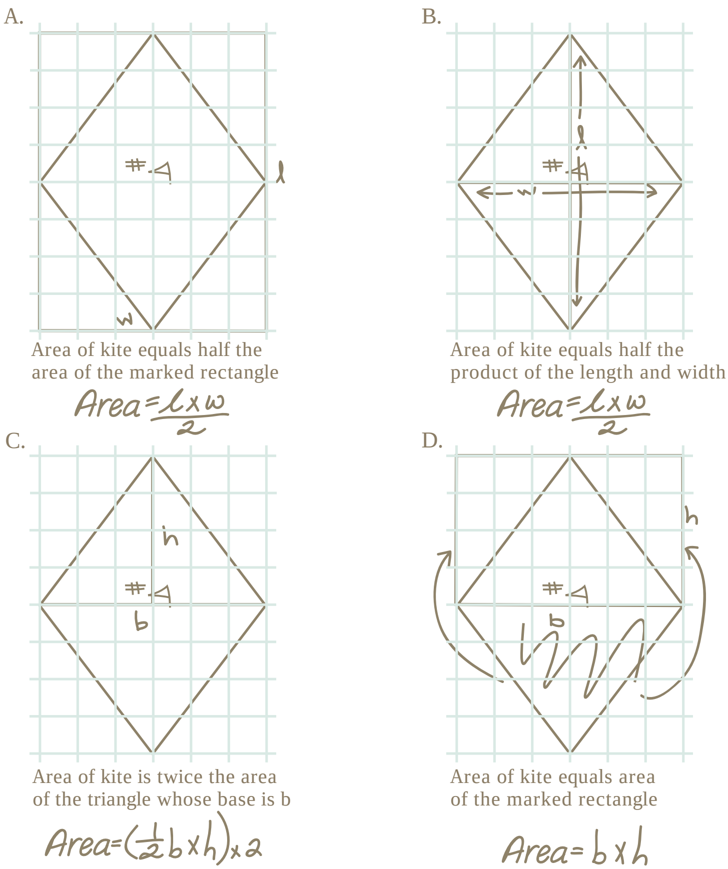
One of the teacher's main goals was to have her students appreciate that area formulas are not mysterious things to be memorized, but rather they are a short-hand notation that summarizes an effective strategy for computing the area of figures with certain common characteristics. This idea was further highlighted in the next activity, where students had to compute the area of different kinds of "kites" (see Figure 2).

Figure 2
"Kites"



Under the teacher's direction, the entire class attempted to develop an area formula that would work for all kites. Different students, by focusing on different characteristics of the kites in Figure 2, proposed the various procedures and formulas summarized in Figure 3.

Figure 3
Alternative area formulas for "kites" and their graphical explanation



As the class critically examined these potential solutions, the teacher carefully facilitated the discussion, making sure that nobody was left out and everybody's contribution was seriously considered. She also occasionally asked questions to highlight important mathematical points, noting, for example, that students developed different yet equally acceptable area formulas depending on what they chose to measure and how they named their variables.

Mrs. Callard also took advantage of the controversy that erupted when one student observed that formula D "may not work for all kites." Instead of resolving the student's concern, she asked the class how they could decide whether something was a kite or not. Since a "kite" is not one of the standard figures usually defined in mathematics textbooks, this apparently simple question led the class to grapple with the challenging task of *creating* their own definition for kite and then defending it! Eventually, the class voted to define a kite as "a quadrilateral with perpendicular diagonals." Based on this definition, the class concluded that formulas A and B were acceptable area formulas for kites, while formulas C and D worked only for special kinds of kites. This activity enabled the students to experience first-hand the power and excitement of "creating" mathematical formulas and definitions and also provided them with a deeper understanding of these fundamental mathematical concepts. To help the students reflect on and better appreciate the significance of what they had learned, the teacher then asked students to write answers to a few questions about mathematical definitions.

To help students synthesize and generalize what they had learned so far, Mrs. Callard led the class through a careful review of the steps they had followed over several class periods to come up with an area formula for a kite. She recorded each step on newsprint and later distributed this list (reproduced Figure 4) to the students as a reference for developing other area formulas in the future.

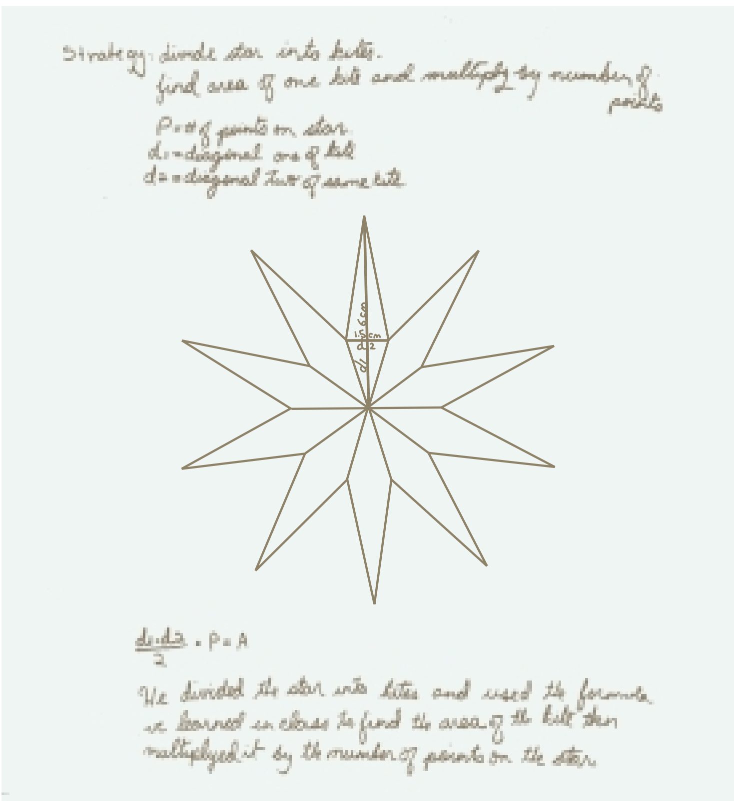
Figure 4
Key steps in developing an area formula

1. We started with examples of the figure and computed their area.
2. Shared strategies, ideas – discussed.
3. We checked to see if one strategy would work on all of the figures.
4. Tested the strategy.
5. Wrote a formula defining variables carefully.
6. Explained why the formula works.

As a culminating activity for the unit and as a form of performance assessment, Mrs. Callard asked students to create an area formula for a given "star." She carefully assigned partners for this project, taking into account students' different mathematical strengths, weaknesses and unique learning styles. Because students worked on part of the project in class, the teacher also provided more scaffolding for some pairs of students as needed. In the students' poster presentations at the end of the project, most of the pairs showed remarkable mathematical thinking abilities, and they communicated the results of their work effectively (see Figure 5 for an example).

To gather feedback about the learning achieved by individual students, the teacher also assigned two traditional take-home tests, one on developing area formulas and the other on applying known area formulas in practical situations. Students' grades for this unit also took into account their performance on homework and in-class assignments, so as to provide a comprehensive assessment of the students' learning based on data gathered from a variety of complementary tools.

Figure 5
Example of a "star" poster



Teachers' learning needs for implementing school mathematics reform

The previous vignette shows that the kind of school mathematics reform currently promoted by many constituencies involves much more than "superficial features" such as using manipulatives or introducing computers in the classroom. Rather, whenever we speak of "reform-oriented" practices in this monograph, we refer to a comprehensive approach to mathematics instruction that is centered on teaching for understanding and enabling students to engage with meaningful problems and "big ideas" in mathematics. This approach is characterized by a set of beliefs and theories about what counts as significant mathematics, how students learn and what conditions call such learning in a classroom environment, as articulated in the NCTM Standards (1989, 1991, 1995, 2000) and much of the current literature in mathematics education. At the same time, this does not mean that the most recent wave of school mathematics reform can be reduced to a prescriptive set of teaching strategies or "exemplary lessons." As argued throughout this monograph in the case of professional development, no single model of reform-oriented mathematics teaching will work for all, and all teachers will need to make decisions about what will be most appropriate and effective for their students.

Regardless of these differences, our vignette suggests that teaching mathematics in a reform-oriented way demands a lot more from teachers – even experienced teachers – than teaching a traditional mathematics lesson. However, teachers interested in reform should not be given the message that anything "traditional" is necessarily "bad" nor that they have done everything wrong so far and should abandon all their current practices. Teachers indeed bring valuable experience to reform, although they are asked to review their beliefs and practices critically in light of new instructional goals and pedagogical approaches. Identifying what teachers need to meet this enormous challenge, therefore, is a critical prerequisite to establishing worthwhile professional development goals and evaluating how specific professional development practices may contribute to achieving such goals.

Drawing from the literature on teacher development and reform (e.g., Friel & Bright, 1997; Fennema & Nelson, 1997; Darling-Hammond, 1997; Wilson & Berne, 1999), we grouped the main learning needs of teachers engaging in school mathematics reform into nine categories (see Figure 6), which we will examine in more depth in the rest of this chapter.

Figure 6
Main categories of teacher learning needs

1. Developing a vision and commitment to school mathematics reform.
2. Strengthening one's knowledge of mathematics.
3. Understanding the pedagogical theories that underlie school mathematics reform.
4. Understanding students' mathematical thinking.
5. Learning to use effective teaching and assessment strategies.
6. Becoming familiar with exemplary instructional materials and resources.
7. Understanding equity issues and their implications for the classroom.
8. Coping with the emotional aspects of engaging in reform.
9. Developing an attitude of inquiry toward one's practice.

Before engaging in this analysis, a few words about possible differences between elementary and secondary teachers of mathematics are warranted. Indeed, elementary and secondary teachers come to professional development experience with quite different preparation, background in mathematics and teaching experiences. Secondary teachers are usually specialists in their subject matter; most have completed the equivalent of a major in mathematics and teach only mathematics courses (often multiple sessions of the same two or three courses) to a total of 100 to 150 different students each year. Elementary teachers, instead, have been trained as generalists and usually teach all subjects to a class of 20 to 30 students; many of them have taken only one college-level mathematics course, although they may have had a wider exposure than their secondary colleagues to learning theories and innovative teaching practice as part of their training. It is also not uncommon for elementary teachers to express a greater interest and confidence in teaching language arts or almost any other subject matter! – than mathematics. These differences will undoubtedly play an important role in elementary and secondary teachers' expectations, responses and even attitudes toward professional development in mathematics, and it will be critical for every professional development provider to take them into serious consideration in their planning. At the same time, we believe that elementary and secondary teachers alike experience *all* of the learning needs identified in this chapter, although they may do so differently.

Developing a vision and commitment to school mathematics reform

Mathematical experiences such as the one described in the above vignette are not likely to happen unless teachers believe reform is important and understand what school mathematics reform calls for. Teachers interested in reform must thus become familiar with the new instructional goals and teaching practices proposed and understand their rationales.

Teachers need to develop a personal understanding of the reform recommendations articulated in the NCTM Standards (1989, 1991, 1995, and 2000) and other documents. Teachers also need “images” of reform classrooms in action, such as that offered in our vignette, because reform-oriented instruction is so different from the experiences of most teachers and students. By reading scenarios from actual mathematics classrooms, teachers can observe, in their mind’s eye, the learning environment, typical activities and tasks that are taking place, and students’ reactions. Several professional development projects have recently recognized this important need and responded to it by creating written and/or video images of reform-oriented mathematics lessons (e.g., Borasi, Fonzi, Smith & Rose, 1999; Ferrini-Mundy, 1997).

Because changing practices is not easy, teachers also need to be convinced that their students will benefit. Indeed, research on professional development efforts has shown that program outcomes, and teacher change in particular, correlate with the level of individual teachers’ participation, effort and identification with reform goals and agendas (e.g., Clarke, 1994; Loucks-Horsley, 1997). At the same time, participating teachers initially may have only a limited vision of their needs and goals in terms of instructional innovation (Ferrini-Mundy, 1997). Thus, a professional development program should strive to *create a felt need* for reform while also taking into consideration the participants’ *perceived needs* and actual constraints.

For some teachers, just witnessing students’ active engagement and enjoyment of reform activities and seeing the depth of the mathematics learned in those lessons may be reason enough to want to offer similar opportunities to their own students (Fennema, Carpenter & Franke, 1997). Others, however, may need further evidence of the need for change, such as data on student achievement in comparative studies.

Developing a vision and commitment to reform among mathematics teachers is an ongoing and long-term goal for any professional development project. It is clearly the most critical element of any professional development program aimed at *initiating* the process of reform, although it should also continue to be an ongoing goal for any professional development project.

Strengthening one's knowledge of mathematics

Shulman's research identified subject matter knowledge and pedagogical content knowledge as key variables influencing teachers' decisions in the classroom:

> Prior subject matter and background in a content area affect the ways in which teachers select and structure content for teaching, choose activities and assignments for students, and use textbook and other curriculum materials. (Shulman & Grossman, 1988, p.12).

While developing teachers' knowledge of mathematics has always been considered a desirable goal of professional development, what counts as desirable mathematical knowledge has changed with the reform agenda. Reform-based curricula are informed by a different set of instructional goals. These include areas of mathematics that have been neglected in the traditional K-12 curriculum, such as probability and statistics. Even more importantly, there is a new emphasis on understanding "big ideas" in mathematics and on apprenticing students to the ways of thinking practiced by mathematics professionals.

Given their limited preparation in mathematics, elementary teachers are the ones often feeling the greatest need for learning more mathematics and deepening their own understanding of and confidence in the subject. However, despite their more extensive preparation in mathematics, secondary teachers also experience this need, as illustrated in our classroom vignette. In order to conduct the lessons on area formulas reported in the vignette, Mrs. Callard needed to know a variety of strategies for computing the area of complex figures, not just how to apply known formulas. She had to know how to develop area formulas, when to apply them and where mathematical definitions come from. These are aspects of mathematics that even teachers certified to teach secondary mathematics have not learned in their previous training (Fennema & Franke, 1992; Sowder, Philipp, Armstrong & Schappelle, 1998).

Furthermore, research on teachers' beliefs about mathematics (Thompson, 1992) documents the impact on curricular decisions and instructional practices of teachers' views on the following key topics: the nature of mathematics as a discipline; what constitutes legitimate mathematical procedures, results and justifications; and what constitutes desirable goals and acceptable outcomes for school mathematics instruction. Most teachers, regardless of whether they are generalists or specialists, never had the opportunity to make their beliefs explicit in traditional teacher preparation. Readings and discussions about the discipline of mathematics are notably absent from school mathematics and even college-level mathematics courses. Nevertheless, because they studied in traditional mathematics classes, most teachers hold deep-seated beliefs that mathematics is a body of absolute truths with little room for creativity or personal judgment. This means that, as teachers, they are likely to value correct answers over tentative conjectures, standard procedures over personal approaches to solutions, and facts and algorithms over inductive problem solving and reasoning skills.

Since these views conflict with the most recent calls for school mathematics reform (Borasi, 1996; NCTM, 2000), professional development programs designed to promote reform must provide opportunities for participants to critically examine their views of mathematics as a discipline and offer alternative perspectives grounded in reform.

Understanding the pedagogical theories that underlie school mathematics reform

Research shows that most mathematics teachers, including prospective teachers, have strongly-held beliefs about student and teacher's roles, desirable instructional approaches, students' mathematical knowledge, how students learn and the purposes of schools (Thompson, 1992). These beliefs have mostly developed as a result of the teachers' own schooling. Although rarely made explicit, the following views of knowledge, learning and teaching lie behind what takes place in most traditional classrooms:

- ***Knowledge*** is a body of established facts and techniques that can be broken down and transmitted to novices by experts *(positivistic view of knowledge)*.

- ***Learning*** results from acquiring isolated bits of information and skills through listening, watching, memorizing and practicing *(behaviorist view of learning)*.

- ***Teaching*** is the direct transmission of knowledge from teacher to student; it takes place as long as the teacher provides clear explanations for the students to absorb *(direct instruction view of teaching)* (Borasi & Siegel, 1992, 2000).

In contrast, the teaching practices recommended by the NCTM Standards (NCTM, 2000) and illustrated in our classroom vignette are grounded in views of knowledge, learning and teaching informed by a constructivist perspective (e.g., Brooks & Brooks, 1999; Davis, Maher & Noddings, 1990; Fosnot, 1996). Although different interpretations of constructivism exist, current school mathematics reform efforts are generally characterized by the following constructivist assumptions:

- ***Knowledge*** is socially constructed through human activity, shaped by context and purposes, and validated through a process of negotiation within a community of practice. Thus, it is always tentative rather than absolute. However, although knowledge is provisional in this paradigm, it does not mean that "anything goes."
- ***Learning*** is a generative process of making meaning that builds on personal knowledge and social interactions. This process may be stimulated by perceived dissonance. Prior knowledge, context and purpose play critical roles in the shaping of learning situations.
- ***Teaching*** is facilitating students' learning by creating a learning environment conducive to inquiry, setting up problem-solving situations to stimulate both student interest and cognitive dissonance about important mathematical ideas, and supporting students' attempts to solve problems and make sense of mathematical concepts (Borasi & Siegel, 1992, 2000).

To fully appreciate the constructivist pedagogical approach recommended in the NCTM Standards, teachers need to identify and understand the non-traditional theories of teaching and learning mathematics and the research supporting such approaches.

Understanding students' mathematical thinking

One of the main challenges that the teacher in our vignette experienced during her inquiry on area was interpreting her students' thinking and responding appropriately, especially when students proposed new strategies or formulas for computing area and explained how they got their results. The teacher benefited considerably by having already investigated a range of possible strategies and solutions to the open-ended tasks she posed – although some of the students' strategies still took her by surprise! Indeed, understanding students' mathematical thinking is especially critical in any constructivist approach if teachers are to design instructional experiences that help students build on their existing knowledge (Confrey, 1991).

Research on Cognitive Guided Instruction (CGI) has provided both theoretical arguments and empirical evidence claiming that mathematics teachers benefit from knowing about their students' prior knowledge and ways of learning specific mathematical concepts (Carpenter & Fennema, 1992; Fennema, Carpenter & Franke, 1997). Knowledge of child-constructed procedures is a crucial prerequisite for designing learning experiences that capitalize on, rather than override, the informal mathematical knowledge children bring to school. For example, many elementary teachers are surprised to learn that children often develop their own procedures for solving simple arithmetic problems *before* they enter school. Knowing this fact can help teachers rethink how arithmetic operations might be introduced.

Further empirical support for the value of teachers' knowing how students think comes from the *Integrating Mathematics Assessment* (*IMA*) project. This project focused on making teachers aware of the key features of student thinking about fractions. As a result, students made significant gains in solving problems involving fractions (Gearhart, Saxe, & Stipek, 1995).

While the results of studies like CGI and *IMA* are compelling, it is reasonable to ask whether we should expect teachers to acquire research-based knowledge about student thinking in all the mathematical areas they will teach, especially when most topics taught in secondary school are not as well researched as basic arithmetic and rational numbers. Rhine (1998) suggests that rather than trying to create such a knowledge base among teachers, it may be more important to foster a new attitude, one that values analyzing student thinking as part of teachers' everyday practice and provides strategies to help them do so.

Learning to use effective teaching and assessment strategies

One element that most distinguished the inquiry on area in our classroom scenario was the extensive use of teaching practices that are usually absent from traditional mathematics instruction. These included, for example, orchestrating group work using a variety of techniques, such as the initial “think-pair-share” activity; facilitating class discussions in which students shared results and jointly constructed new knowledge; using effective questioning techniques to synthesize key mathematical ideas; and assessing students’ learning in multiple ways, such as the performance assessment in which students created an area formula for a star.

The pedagogical recommendations articulated in the NCTM Standards (NCTM, 1991, 2000) call specifically for teaching practices like these that are not currently used by many mathematics teachers, especially at the secondary level (for comprehensive lists of such practices, see Koehler & Grouws [1992] and Borasi & Fonzi [in preparation]). Non-traditional practices include not only facilitating what goes on in the classroom as lessons develop but also planning and assessing lessons effectively. Assessment has received special attention recently (e.g., Bright & Joyner, 1998; Lesh & Lamon, 1992; NCTM, 1995; Webb & Coxford, 1993) because determining what students know is necessary for teaching effectively within a constructivist paradigm. It is also critical for documenting the outcomes of reform efforts.

Learning to use novel teaching practices appropriately is not easy. Research on how people learn complex tasks may shed some light on what it takes teachers to adopt a new teaching practice. For example, Collins and his colleagues (1989) have suggested the following three-phase process for learning a complex task:

1. ***Modeling*** – The learner observes and examines how an expert engages in the task.
2. ***Scaffolded practice*** – The learner engages in the task himself/herself, but with the help of an expert and/or of other supporting structures.
3. ***Independent practice*** – The learner engages in the task without support.

Clearly, using new teaching practices effectively goes far beyond simply knowing they exist. While mathematics teachers should learn about a variety of teaching strategies to enrich their repertoire of resources, they should also have the opportunity to personally experience these practices in supported situations in order to evaluate fully their pedagogical potential. It is also critical for teachers to learn not only to use specific practices well but also to appreciate their strengths and limitations so they can choose practices most appropriate to an audience and to unique instructional goals.

Becoming familiar with exemplary instructional materials and resources

When reading about a well-designed, complex experience such as the inquiry on area described in our vignette, teachers might feel daunted by the prospect of creating similar lessons on their own. Fortunately, today's mathematics teachers are not expected to always create innovative units on their own as they may take advantage of the many exemplary instructional materials informed by the NCTM Standards that have been produced in recent years. As argued by Russell, this by no means demeans the professionalism of teachers:

> Curriculum materials, when developed through careful, extended work with diverse students and teachers, when based on sound mathematics and on what we know about how people learn mathematics, are a tool that allows the teacher to do her best work with students... . It is not possible for most teachers to write a complete, coherent, mathematically sound curriculum. It is not insulting to teachers as professionals to admit this. (Russell, 1997, p. 248)

Exemplary instructional materials may consist of replacement units, which are individual units designed to replace parts of the traditional curriculum while expanding the instructional goals and introducing some effective teaching practices or of comprehensive curricula. These consist of a sequence of units intended to totally replace the current mathematics curriculum at either elementary, middle or high school. Among the latter group, a set of instructional materials consistent with the NCTM Standards has been recently developed with support from the National Science Foundation (NSF) (see Figure 7 for a complete list of these comprehensive curricula and their websites' addresses). Additional exemplary mathematics curricula have been identified in a study by the U.S. Department of Education (U.S. Department of Education's Mathematics and Science Education Expert Panel, 1999).

Figure 7
NSF-funded exemplary comprehensive mathematics curricula

Elementary school (K-5):

- Everyday Mathematics *(http://ars-www.uchicago.edu/ucsmp-el/)**
- Investigations in Number, Data and Space *(http://www.terc.edu/investigations)**
- Math Trailblazers *(http://www.math.uic.edu/IMSE/MTB/mtb.html)**

Middle school (5-8):

- Connected Mathematics Project (CMP) *(http://www.math.msu.edu/cmp)**
- Mathematics in Context (MiC) *(http://www.ebmic.com)**
- MathScape *(http://www.edc.org/mcc/cscape.htm)**
- Middle Grades Math Thematics *(http://www.math.umt.edu/~stem/)**
- Middle School Mathematics through Applications Project (MMAP) *(http://mmap.wested.org)**

High school (9-12):

- Contemporary Mathematics in Context (CORE-Plus) *(http://www.wmich.edu/cpmp)**
- Interactive Mathematics Program (IMP) *(http://www.mathimp.org)**
- Math Connections *(http://www.mathconnections.org)**
- Mathematics: Modeling our World (ARISE) *(http://www.comap.com)**
- SIMMS Integrated Mathematics *(http://www.montana.edu/~wwwsimms)**

* web addresses are current at time of publication

In order to be considered "exemplary," a unit or comprehensive curriculum must be consistent with the NCTM Standards, designed by groups of specialists in mathematical content and pedagogy, and revised based on field tests in various instructional settings.

Exemplary instructional materials are much more than a textbook for students. They usually include a rich collection of documents to support learning experiences. The documents may include suggestions for planning lessons and orchestrating class discussions, examples of student work, tools and rubrics for assessment, and opportunities for teachers to learn more about the mathematical concepts to be taught.

While there is certainly a value for teachers to create their own innovative lessons and units, the results of the multitude of Teacher Enhancement and Local Systemic Change projects supported by the NSF in the last two decades suggest that the use of exemplary comprehensive mathematics curricula is critical to the success of systemic reform. That is, if the goal is to reform the entire mathematics program within a given school or district, not just to improve the practices of a few committed teachers, it is very difficult to achieve significant success unless the system adopts a coherent curriculum that ensures that students engage in a well-constructed sequence of worthwhile mathematics experiences, and frees teachers to focus their energy on improving their instructional practices and evaluating their students' learning.

While exemplary instructional materials can revolutionize the way we approach school mathematics reform (Ball & Cohen, 1996; Russell, 1997), they also require considerable time (and, in some cases, special expertise) to be used efficiently. Therefore, professional development programs should include opportunities for teachers to become familiar with at least some exemplary instructional materials, selected so as to maximize the participants' opportunities to implement reform in their classes.

Teachers also need to learn about high quality software and other technological tools if they are to implement mathematical learning experiences consistent with the most recent calls for reform. Indeed, new technologies such as graphing calculators, spreadsheets, and programs like the "Geometer's Sketchpad" and statistical packages like "Fathom," have radically changed the way certain mathematical topics can be taught in school (e.g., Dunham & Dick, 1994; Rojano, 1996). Teachers need to become proficient users of these technologies and to learn to consider how using these tools could affect not only their teaching practices but also their instructional goals.

Understanding equity issues and their implications for the classroom

At the forefront of the current call for school mathematics reform is the directive that *all* students should have opportunities to learn mathematics (NCTM, 1989, 2000; Secada, Fennema & Adajian, 1995). The underachievement of some ethnic minorities and women has been the cause of serious concern and one of the reasons that led to the recent critical scrutiny of curricula and teaching practices (Chipman & Thomas, 1987; National Science Foundation, 1986; Oakes, 1990; Secada, 1992). Students with disabilities may also perform much better in mathematics if they have appropriate learning opportunities and support (Silver, Smith & Nelson, 1995; Thornton & Langrall, 1997).

Because the new instructional goals and teaching practices articulated in the NCTM (2000) Standards are meant to recognize and respond to student diversity, researchers and policy makers are confident they will help bridge the achievement gap. Our vignette is evidence of how mathematical tasks can be designed to provide access to students with diverse learning styles, strengths and background experiences. An open-ended task, such as finding the area of a "fish," offers many more opportunities for success for all students than traditional tasks that recognize only one correct solution and one way to achieve it. Multiple forms of assessment, as exemplified in our vignette by the combination of a group performance assessment and more traditional paper-and-pencil tests, may also help students with different strengths and learning styles to show more easily what they know.

However, taking on new instructional goals and teaching practices will not be enough for teachers to fully address equity issues in school mathematics. Each teacher must first gain a good understanding of the many issues related to equity and diversity and their implications for mathematics instruction (Darling-Hammond, 1998). Teachers must also become aware of their own biases and privileges and learn how these may affect their relationship with students who are different with respect to race, class, gender, primary language, sexual orientation, etc. (Weissglass, 1996). Teachers must also believe that all students can learn mathematics when they are provided with ample opportunities, conditions conducive to learning and high teacher expectations.

Teachers also need to know how to identify their students' unique needs and how to differentiate instruction to address those needs. For example, it was important for the teacher in our vignette to recognize the different strengths and abilities of her students in order to place them with an appropriate partner for the final project; the same knowledge enabled her to offer additional scaffolding for some students who needed it. To respond to students with specific learning disabilities, teachers may need knowledge that is even more specialized.

Coping with the emotional aspects of engaging in reform

Several reform projects have noted that emotions, both positive and negative, inevitably accompany efforts to change one's teaching practices (Clarke, 1994; Ferrini-Mundy, 1997). A participant in one of our professional development projects aptly described her initial experiences in instructional innovation as an "emotional roller-coaster"; at times she felt elated by her students' success and the depth of their mathematical thinking, but she could also sink into dejection from an unsuccessful instructional experience she had spent hours putting together or from the opposition presented by a parent or administrator. Some teachers may suddenly feel inadequate after years of perceiving themselves as successful teachers and may even blame themselves for "doing it wrong."

Studies of learning and problem solving show that behavioral changes often engender strong feelings of anxiety, frustration and elation (McLeod, 1992). Teachers need to know that conflicting feelings will inevitably arise and they need to find ways to cope with these feelings. If emotional needs are not directly addressed, teachers may even drop out of professional development programs and reform efforts. Weissglass (1993) has suggested that "any reform that does not provide methods for people to systematically and profoundly address their feelings, emotions and values related to reform will be inadequate." (p. 3)

For teachers to recognize and deal constructively with feelings, they need, among other things, to break the isolation that so often characterizes teachers' work. The need for teachers to share ideas and feelings with other teachers involved in research and reform has been long recognized in the teacher education literature (e.g., Clark, 1994). Quality professional development programs should strive to meet this need by creating opportunities for teacher collaboration.

Developing an attitude of inquiry toward one's practice

Several researchers have identified teacher reflection on their practice and student learning as critical to the success of school mathematics reform. Darling-Hammond (1998) writes:

> . . . teachers need to be able to analyze and reflect on their practice, to assess the effects of their teaching, and to refine and improve their instruction. They must continuously evaluate what students are thinking and understanding and reshape their plans to take account of what they have discovered. (p. 2)

Barnett (1998) echoes Darling-Hammond's call for teacher reflection and inquiry:

> Teacher inquiry plays a central role in many of the prevailing conceptions of teacher learning including critical reflection, reflection in and on action, personal and pedagogical theorizing, narrative inquiry, action research and teacher research. (p. 81)

Both researchers are building on the foundation laid by Schon (1983, 1987), who was one of the first to point out that teachers, just like professionals in other fields, need to become *reflective practitioners*. That is, they need to develop the habit of critically examining their practice to gain new insights on the teaching and learning of mathematics, which Schon calls *reflection-in-action*. In the complementary process called *reflection-on-action*, teachers learn to approach situations of uncertainty by bringing to bear all their professional knowledge, in addition to their understanding of the specific context, in order to make the best possible decisions.

Barnett (1998) also argues that teachers should engage in inquiry about their practice not only by themselves, in isolation, but also with others. This collective inquiry and critical reflection can provide teachers with opportunities to hear different perspectives. All participants benefit from public scrutiny of the hypotheses suggested by different individuals. Consequently, they collectively generate new ideas and draw more sophisticated conclusions than they might as individuals.

The ultimate goal of any professional development program supporting school mathematics reform should be to develop among teachers the mindset that they are lifelong inquirers. This means both developing the appropriate expectations and mindset, and providing teachers with strategies and skills to inquire effectively.

Summary

Our analysis so far has identified a complex set of teacher learning needs that professional development initiatives supporting school mathematics reform must consider seriously. This does not mean that any *single* professional development initiative should – or even could – try to address all of these needs at the same time. Rather, different needs may call for different kinds of professional development experiences, as we will discuss in more depth in Chapters 4 to 8. In the next chapter, we will show how some professional development programs have found non-traditional yet successful ways to meet this challenge.

CHAPTER 2

What Does Effective Professional Development Look Like?

Before analyzing in more depth *how* the teacher learning needs identified in the previous chapter might be addressed, we would like to provide some images of professional development projects that have been successful in supporting school mathematics reform.

It was difficult to select just a few out of the many creative professional development programs of the last two decades (as featured, for example, in Friel & Bright, 1997; Fennema & Nelson, 1997; Loucks-Horsley, Hewson, Love, & Stiles, 1998; Eisenhower National Clearinghouse [ENC], 2000). We eventually chose the two projects featured in this chapter because they differ considerably in terms of scope, goals, complexity, audience, context and grade levels. Therefore, we hope these examples will begin to show how the teacher learning needs described in Chapter 1 can be met in many diverse and viable ways.

In this chapter, we describe each project in some detail to convey a sense of its vision and complexity. Space constraints do not allow us for detailed descriptions of specific professional development activities within each project, although some of these will be described in more depth in vignettes reported in later chapters.

An implementation of the Cognitive Guided Instruction (CGI) program

We derived this first illustration from one of the many implementations of the CGI program as reported in Fennema, Carpenter, Franke, Levi, Jacobs and Empson (1996). The same article also provides evidence of the effectiveness of this specific professional development program in addressing teachers' beliefs, changing practices and increasing student achievement.

In this four-year project, a group of first-third grade teachers from four different schools volunteered to participate for minimal compensation and the option of receiving graduate credits for their work. The main goal of this program was to "help teachers develop an understanding of their own students' mathematical thinking and its development and how their students' thinking could form the basis for the development of more advanced mathematical ideas" (Fennema, Carpenter, Franke, Levi, Jacobs, and Empson, 1996, p. 406), as a main vehicle to improve mathematics instruction in their classes.

During the first two years, the teachers attended a series of workshops: A 2 1/2-day workshop in late spring of the first year, a 2-day workshop in the summer and 14 three-hour-long workshops during the following academic year. The workshops introduced the teachers to a research-based model of how young children understand basic number concepts and operations (for empirical research on this issue, see Carpenter, Fennema & Franke, 1994; Fuson, 1992; Greer, 1992). This approach is based on the assumption that increasing teachers' knowledge of students' thinking helps them design better instructional tasks, ask better questions during mathematics lessons and support individual students' learning more effectively.

The project purposefully made the decision NOT to provide teachers with any instructional materials or guidelines. Rather, they encouraged the teachers to use their growing knowledge of students' mathematical thinking to inform their instructional decisions.

Although the teachers read articles explaining the basis of the model in research, they primarily focused on analyzing students' mathematical thinking from samples of written works or videotapes of problem-solving sessions. Participants did not receive an explanation of each child's solution; rather, they examined the similarities and differences among different children's approaches and generated hypotheses about the mathematical concepts underlying them. Facilitators often asked participants to validate the research model by observing students in their own classes and discussing the results with the rest of the group.

The project purposefully made the decision *not* to provide teachers with any instructional materials or guidelines. Rather, they encouraged the teachers to use their growing knowledge of students' mathematical thinking to inform their instructional decisions. However, participants did receive support in translating their new knowledge into instructional practice from a project staff member and a mentor teacher assigned to each school. These teacher educators attended all workshops, visited each participant's classroom about once a week and worked individually with teachers to support their instruction.

In the following two years, teachers continued to participate in some workshops during the school year (four 2 1/2-hour workshops and a 2-day reflection workshop in year three, and one 3-hour reflection workshop and two 2 1/2-hour review workshops in year four). These workshops, however, did not introduce new information about the research model. They focused instead on helping teachers observe the mathematical thinking of their own students' and make instructional decisions based on what they had learned. Participants continued to receive on-site support, but the classroom visits were reduced gradually (once every two weeks in year three and only occasionally in year four).

Making mathematics reform a reality in middle schools

Making Mathematics Reform a Reality in Middle School (MMRR) was one of the Local Systemic Change projects that the National Science Foundation (NSF) funded to promote school mathematics reform in whole schools or districts. This three-year project was aimed at beginning the process of systemic reform in four suburban middle schools that had not adopted – nor yet decided to adopt – one of the new NSF-funded curricula for middle school mathematics. As such, the project involved *all* the teachers responsible for teaching mathematics at these school sites, which included teachers certified to teach secondary mathematics, special education teachers and even a few elementary teachers. Professional development, as the core of this project, consisted of several initiatives designed for teachers at different stages of development. In a recent national study (Killion, 1999), this program was cited as one of only eight in the country that have demonstrated a positive effect on students' mathematical learning in middle school.

Teachers joined the project by attending a one-week introductory Summer Institute and participating in related field experiences during the following year (as described in Borasi, Fonzi, Smith & Rose, 1999, and in even more detail in Borasi & Fonzi, in preparation). In the Summer Institute, teachers learned about an inquiry approach to teaching mathematics as a way to teach *all* students better. In the spirit of the NCTM Standards, the Summer Institute and its follow-up field experiences invited teachers to rethink their mathematical and pedagogical beliefs from a constructivist/inquiry perspective. It also enabled them to experience the power of learning mathematics themselves through inquiry activities and helped them actually begin the process of instructional innovation in their classes. Finally, it fostered a need to continue in the reform process.

Two illustrative inquiry units (i.e., the unit on area formulas informing our classroom vignette in Chapter 1 and another unit on tessellations) played a critical role in this program. These units modeled how middle school students could learn key ideas in geometry and measurement through inquiry. A team of mathematics education researchers and teachers had previously developed these units and successfully field-tested them in a variety of middle school settings (Borasi, Fonzi, Smith & Rose, 1999). They had also created a set of materials to support teachers in implementing each of these units (Borasi, 1994 a&b; Borasi & Smith, 1995; Fonzi & Rose, 1995 a&b). To participate in the Summer Institute, teachers had to commit to teaching one of the inquiry units in the following school year.

During the Summer Institute, teachers first participated, as learners, in two 5-hour mathematical inquiries on tessellations and area similar to those in the illustrative units. During these mathematical learning experiences, the facilitators modeled several inquiry-based teaching practices recommended by the NCTM Standards. These "experiences as learners of mathematics" served as the catalyst for teachers to reflect on the nature of mathematics and on teaching and learning, as each inquiry was followed by one or more sessions in which participants discussed these experiences from different perspectives. These inquiry-based experiences also introduced teachers to the unit they had committed to teach as part of their follow-up field experiences. The Summer Institute supported teachers in their first experience of instructional innovation in other ways. Teachers watched a video and read an accompanying narrative that documented the implementation of these units with middle school students. They were also introduced to the supporting materials accompanying each unit, and they participated in an initial planning session for their own unit.

As participants planned and implemented their chosen unit, they were supported by a lead teacher or a mathematics teacher educator assigned to be a facilitator in their schools. They could also participate in a follow-up meeting where other teachers who had implemented their first inquiry unit shared and discussed these experiences. Then, facilitators introduced teachers to some of the NSF-funded exemplary mathematics curricula for middle school as resources to support their planning of additional innovative instructional experiences. Teachers were encouraged to try at least a unit from one of these series in their classroom before the end of the school year.

Teachers who participated in this year-long component were then eligible to participate in a second, 5-day Advanced Summer Institute and its related field experiences. This second Summer Institute focused on the teaching and learning of algebra in middle school and on helping teachers become familiar with two of the NSF-funded curricula for middle school: the *Connected Mathematics Project* (*CMP*) and *Mathematics in Context* (*MiC*). As follow-up field experiences, teachers committed to implement at least one algebra unit from one of these curricula during the school year.

The first three days of the Advanced Summer Institute occurred at the beginning of the summer and the final two days near the end, as a follow-up. In the first part, teachers participated again as learners in mathematical experiences followed by focused reflective sessions. This time, however, the experiences focused on algebra rather than geometry and measurement, and they were designed around activities derived from *CMP* and *MiC* units. In analyzing these experiences, teachers focused mostly on the mathematical content and curricular implications. This activity invited a rethinking of the key ideas in algebra and, consequently, the main goals of teaching algebra in middle school. Participants then read articles on algebra and analyzed in depth at least one unit from either the *CMP* or the *MiC* curricula. During the last two days of the Advanced Summer Institute, participants presented their analyses of the assigned units and discussed each curriculum and the choices each represented in terms of mathematical content, learning priorities and sequencing of activities.

During the following school year, teachers implemented their chosen *CMP* or *MiC* unit. The instructional materials themselves provided the main support for these implementations. In most cases, a group of teachers chose to work together on the same unit and thus established a “study group” that met a few times after school. At first, a mathematics teacher educator facilitated these study groups, but the teachers eventually met independently. Later in the project, after one school had decided to adopt the *CMP*, its teachers continued to hold these collaborative sessions as a way to support the use of this curriculum.

Throughout the three years of the project, a subgroup of teachers who had taken leadership roles in their schools also participated in a monthly Leadership Seminar. The facilitators organized activities in this seminar in response to the needs of the participating lead teachers. The activities were designed to expand the lead teachers' personal understandings of school mathematics reform, to improve teaching practices and to develop leadership skills. For example, the group discussed a few cases of teaching mathematics through inquiry in order to develop a shared understanding of what characterizes such an instructional approach. Later on, teachers' need to rethink the teaching and learning of geometry in middle school led to a series of different group experiences, such as discussing several articles, analyzing the units developed by NSF-funded middle school curricula and hearing a presentation by a research mathematician.

Facilitators organized additional professional development opportunities in response to the needs of smaller subgroups. For example, some meetings were held for special education teachers only, in order to address issues they had raised about their unique role and responsibilities. New teachers were advised to observe their more experienced colleagues' classrooms regularly as a form of professional development. Curriculum writing groups and department meetings, often initiated and facilitated by the lead teachers themselves, also occasionally became sites for professional development.

Summary

The two examples of professional development reported in this chapter support the claim we made in the introduction to the monograph: There is no one model of professional development that works for all. Rather, professional development is about decision making in context. At the same time, the creative solutions generated by the projects described in this chapter suggest that professional development providers and consumers can make *informed decisions* about the kinds of experiences mathematics teachers need. Furthermore, those decisions should be made in light of what we know works best to address specific goals or teacher learning needs, however tentative that knowledge might be. The remainder of the monograph is dedicated to uncover and examine such knowledge.

CHAPTER 3

What Are Key Similarities and Differences in Successful Professional Development Programs?

We selected the projects described in the previous chapter based on their documented success in promoting school mathematics reform through professional development. Yet they are very diverse, not only in terms of the grade levels they address or the aspects of school mathematics reform they privilege but also in the methods and strategies they use to teach teachers.

In this chapter, we begin to examine similarities and differences among these, as well as other successful professional development projects documented in the literature. The goal of this analysis is the identification of some common principles that characterize high quality professional development, as well as some viable options within these parameters.

Characteristics of high quality professional development

Several scholars in teacher education (e.g., Clarke, 1994; Darling-Hammond, 1997, 1998; Friel & Bright, 1997; Wilson & Berne, 1999; Ball & Cohen, 1996) have recently tried to identify the characteristics of high quality professional development. Although not all characteristics proposed overlap, there is consensus that high quality professional development in support of school mathematics should contain the following elements:

- ***Be sustained and intensive.*** The changes in beliefs and practices called for by school mathematics reform require considerable time and multiple learning opportunities. The changes cannot be achieved with just a few workshops or readings. Rather, changes are likely to take several years, and teachers need to be supported appropriately throughout this undertaking.

- ***Be informed by how people learn best.*** The constructivist theories of learning that underlie school mathematics reform should be applied to structuring teachers' learning as well. Simon's (1994) model of "learning cycles" further explicates this principle. Simon suggests that teachers, just like other learners, learn in cycles by doing the following: (1) engaging actively in situations that provoke cognitive dissonance, thus initiating new constructions of meaning; (2) sharing and discussing these constructions with a group to arrive at consensus and generalizations; and (3) applying these generalizations to new situations to begin the learning cycle again at a higher level. Simon further notes that the focus of each learning cycle may be different at different points in time as teachers develop in the six following areas:

    1. Knowledge *of* mathematics
    2. Knowledge *about* mathematics
    3. Useful and personally meaningful theories of mathematics learning
    4. Knowledge of students' development of particular mathematical ideas
    5. Ability to plan instruction of this nature
    6. Ability to interact effectively with students (i.e., listening, questioning, monitoring and facilitating classroom discourse). (Simon, 1994, p.72)

- ***Focus on the critical activities of teaching and learning rather than abstractions and generalities.*** In the programs described in the previous chapters, teachers participated in activities close to their own practice. For example, they examined student work, analyzed videotaped classroom interactions, engaged as learners in innovative mathematical experiences and planned instruction to try out in their own classes. Theory and research have a role in professional development, but to be meaningful, they should be grounded in the practice of teaching and learning.

- ***Foster collaboration.*** A critical outcome of professional development should be a "community of learners" in which participants sustain each other as they undertake the challenge of school mathematics reform.

- ***Offer a rich set of diverse experiences.*** To meet the many teacher learning needs we identified in Chapter 1, professional development programs need to offer a variety of experiences. It is worth noting that, despite the different choices made by the two projects described in Chapter 2, they both offered multiple professional development experiences throughout the program.

The last points suggest the value of comparing not so much entire professional development programs, but rather the many specific professional development experiences that take place within high-quality programs.

Main differences within specific professional development experiences

As we look at the specific professional development experiences within the two projects described in Chapter 2, we see first of all that they are trying to achieve different goals. The process of reform is too complex to undertake at one time. Thus, it is important that teachers be helped to focus on different aspects of that process at different times. However, to ensure appropriate support for teachers, a project should eventually take into account *all* of the needs identified in Chapter 1.

It is worth noting that goals may differ not only between projects but also among the experiences that comprise one project. For example, the overall goal of the Cognitive Guided Instruction (CGI) project was to enable elementary teachers to understand children's thinking about basic arithmetic, operations concepts. The primary goal of the Making Mathematics Reform a Reality (MMRR) project, on the other hand, could be stated as to introduce mathematics teachers to an inquiry approach to teaching. Within the MMRR project itself, however, the goals for the first and second summer institute differed. The first institute focused mostly on changes in *pedagogy* while the second institute emphasized the need for a radical change in *mathematical content* and *goals*.

Thus, we suggest making a distinction between the *content* of specific professional development experiences (such as assessment, middle school algebra, early development of operations or teaching mathematics through inquiry) and the *roles* that such experiences will play within the broader agenda of promoting school mathematics reform (such as developing a need for school mathematics reform or learning to implement an exemplary curriculum). Professional development providers or consumers evaluating professional development experiences need to consider both.

What a program is trying to accomplish, combined with the constraints it has to deal with, influences choices about the overall *format* for the program, the kind of background and expertise needed by the professional development *providers*, and the *types of activities* teachers will engage in.

We can identify the following options for program *formats* by looking even just at the examples described in Chapter 2:

- ***Summer Institutes*** that engage teachers full time during the summer, for periods usually ranging from 1 to 3 weeks.
- ***A series of workshops*** taking place over the school year, during or after school hours.
- ***Study groups*** comprised of teachers who meet on a regular basis over the school year to work on their practice and/or discuss readings.
- ***One-to-one interactions*** between a teacher (or pair of teachers) and a mathematics teacher educator acting as consultant and/or mentor.
- ***Independent work*** done by a teacher, such as reading, planning and implementing innovative instruction, examining students' thinking or doing research.

The staff conducting professional development initiatives may also differ, even within the same project. For example, we find examples in the literature of sessions facilitated by the following personnel:

- ***Mathematics educators*** who are experts in mathematics education and mathematics teacher education. These professional are often, but not always, affiliated with a school of education within a higher education institution.
- ***Mathematicians*** who are experts in mathematics and are usually affiliated with a mathematics department in a college or university and who conduct mathematical research or teach advanced mathematics courses.

- ***Experts in related areas***, such as facilitators in leadership skills.
- ***Administrators*** who have responsibilities for staff development and supervision.
- ***Experienced teachers*** who have been implementing school mathematics reform for some time.
- Some of the ***participating teachers*** themselves.

Staffing professional development experiences appropriately is central to their success. The expertise that leaders need depends on the goals and content of a session. In the remaining chapters, we will examine what kind of expertise is needed and what it takes to effectively facilitate different kinds of professional development experiences.

The kind of activities that teachers engage in further distinguishes specific professional development experiences. Even just the two examples reported in Chapter 2 include a wide variety of activities: Teachers interpreted students' responses to a mathematical task, examined videotaped interviews or lessons, participated in mathematical inquiries, and conducted interviews with their students, among other things. Rather than trying to develop a comprehensive list of all possible activities, we have identified five main *types of professional development experiences* in which most professional development activities described in the literature fall:

- Mathematical experiences where teachers engage as genuine learners;
- In-depth analyses of student thinking based on their written work and or contributions to classroom discussions;
- The use of "cases," that is, examples of practice related to school mathematics reform that are presented as videotaped excerpts or written narratives to stimulate reflection and discussion on important issues;
- Supported field experiences in which teachers attempt instructional innovation; and

- Information gathering and interpretation through both traditional activities, such as reading articles and attending presentations, and conducting research on one's own practice.

In Chapters 4 to 8, we will examine in depth each of these five types of professional development experiences. We hope this analysis will help readers evaluate the quality and appropriateness of professional development initiatives they are considering.

Note that, although both projects described in Chapter 2 ask teachers to reflect on activities and discuss them, we decided not to consider these practices as a distinct type of professional development experience. Rather, consistent with constructivist theories of learning, we consider reflection and discussion as integral to *any* professional development experience.

Part II.

Analyzing Promising Professional Development Experiences for Mathematics Teachers

CHAPTER 4

Engaging in Mathematical Experiences-as-Learners

In the type of professional development experience we describe in this chapter, teachers engage as *genuine learners* in mathematical learning experiences. While the nature, content and duration of these learning experiences may vary considerably, they all model effective instructional and/or learning practices promoted by school mathematics reform. Reflection is a critical part of these activities because it helps teachers analyze the experiences in light of their own beliefs and practices.

Theoretical rationale and empirical support

The benefits of teachers experiencing mathematics as learners go well beyond the important, rather obvious one, that teachers learn more mathematics. Research shows that teachers' beliefs about mathematics and about teaching mathematics are formed mostly as a result of having been students in traditional mathematics classrooms (Thompson, 1992). Since traditional mathematics is informed by pedagogical beliefs and practices that are radically different from those promoted by the current reform efforts, many teacher educators argue that before classroom teachers can change their beliefs, they must have personal experience of alternative pedagogical approaches (Brown, 1982; Schifter & Fosnot, 1993).

Further support for the value of experiences-as-learners for teachers comes from research on the learning of complex tasks. As we discussed in Chapter 1, Collins and his colleagues (1989) identified *modeling* as the first of three phases in the process of learning a complex task. When the complex task is learning a novel approach to teaching mathematics, we believe that facilitated "experiences as learners" activities offer an especially effective vehicle for such modeling. First, teachers observe an expert mathematics teacher educator teach mathematics in a non-traditional way. Second, because teachers participate in this instructional experience as learners themselves, they are in a unique position to examine how their students may *feel* about the new approach. As a result, they are in a better position to evaluate its potential advantages and drawbacks.

Simon's "learning cycles" model of teacher learning, which we described in Chapter 3, clarifies further the multiple roles that this type of activity can play in a professional development program. In Simon's first phase of the learning cycle, teachers must participate in situations that engage them actively as learners and that evoke cognitive dissonance. In this way, they are stimulated to construct new meanings. In the second phase, through sharing and discussing these constructions with a group, teachers come to consensus and make generalizations. This model suggests to us that good mathematical learning experiences for teachers need to invite active engagement, provoke cognitive dissonance, and encourage social as well as individual construction of meaning. Simon's model further claims that what is learned in one cycle can be used to stimulate another cycle of learning. We suggest that reflecting on these mathematical learning experiences can become the catalyst for teachers to begin yet another "learning cycle," this time focusing on the nature of mathematics as a discipline, how people learn and what can best support such learning.

Research corroborates the benefits of teachers experiencing mathematics as learners articulated above. This type of professional development experience plays a central role in several professional development programs with documented success (Simon & Schifter, 1991; Schifter & Fosnot, 1993; Borasi, Fonzi, Smith & Rose, 1999). A systematic study conducted by Simon and Schifter (1991) in the context of one of these programs has specifically shown changes in teachers' beliefs and practices toward a more constructivist approach to teaching mathematics. Since mathematical experiences-as-learners were not the *only* kind of professional development experience employed in these professional development programs, the results may not be considered conclusive. However, case studies and anecdotal evidence (Schifter & Fosnot, 1993; Borasi, Fonzi, Smith, & Rose, 1999) further confirm that experiences-as-learners were a critical element in changing the beliefs and practices of several participants in these programs.

Illustration 1: A facilitated inquiry on area for teachers

We derive the illustration in this section from one of the Introductory Summer Institutes in the Making Mathematics Reform a Reality in Middle

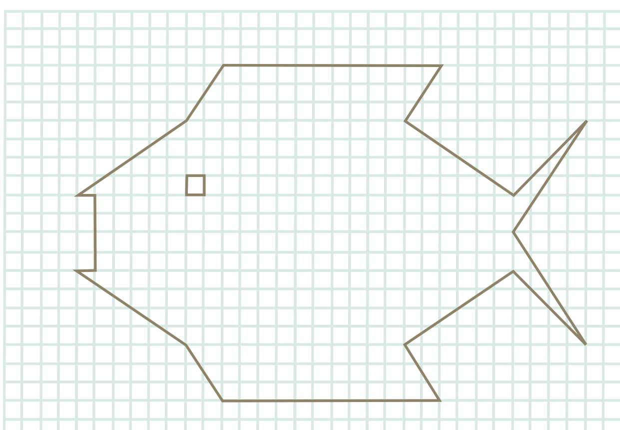
School (MMRR) project described in Chapter 2. This experience-as-learners was designed to help teachers analyze how an inquiry approach to teaching mathematics involves a radical rethinking of both mathematical content and pedagogical practices. It was also intended to introduce teachers to an "illustrative inquiry unit" they might be teaching in their own classes later -- a unit on area formulas designed for middle school students (the same unit featured in the classroom vignette included in Chapter 1). This experience-as-learners thus engaged participants in an inquiry similar to one they might be using with students.

The participants in the implementation described here included elementary teachers, secondary mathematics teachers, and special education teachers at the middle school level. It took about seven hours over three consecutive days to complete.

The instructor began by asking participants to take off their "teachers' hats" and become learners in a series of activities about the concept of area. The instructor warned participants that this was *not* going to be a simulation in which they should pretend to be elementary or secondary students. Rather, the content would challenge everyone at their own level of expertise, so they should participate as genuine learners and use all they knew to deal with the tasks presented to them.

The first task was to find the area of a "fish" similar to the one middle school students worked with in the classroom vignette (see Figure 8).

Figure 8
The "fish"



Each teacher worked on this task first individually, then with a partner. The pairs then shared their results with the whole class. Most secondary mathematics teachers broke the fish into simpler figures, computed their areas using formulas they knew, and then added up those areas. A special education teacher had used a similar approach, yet made more efficient by using the symmetry of the fish and folding the figure in half. An elementary teacher showed instead how she had “boxed” the fish and then subtracted the area of the “extra pieces.” Another elementary teacher “admitted” that she had simply “counted the squares,” matching partial squares as best as she could to form whole squares.

Everybody was surprised by the variety of these approaches and by the fact that non-mathematics specialists had proposed the most creative solutions. A lively discussion surrounded this sharing, and participants came to appreciate the value of alternative strategies for finding the area of complex figures and the role that area formulas played in some of these strategies.

Next, the instructor challenged the participants to develop some area formulas on their own. First, she modeled this novel process by creating, together with the participants, an area formula for “diamonds.” Later in the activity, she defined a diamond as “a quadrilateral with perpendicular diagonals.” This task, and the reflection that followed it, highlighted important elements in the process of developing area formulas.

Participants then worked independently in small groups to develop area formulas for “regular” stars. The next day, they shared the area formulas they had created and explained the process they had used to derive them. Once again, everyone was amazed by the variety of area formulas thus created and by the creativity shown by several class members who had little mathematical background.

To help participants further appreciate the complexity of the mathematical concept of area, the instructor asked them to grapple with some thought-provoking questions for homework:

- Why are squares chosen as the “unit” to measure areas? Could other shapes be used? Why or why not?

- How do we choose the "size" of the squares to be used as units? Can this choice affect the value of the area of a given figure?
- Area formulas essentially enable us to compute the area of a two-dimensional figure by taking only linear measures (i.e., the length of the height, base, radius, etc.). How is this possible? Does this mean that you can measure area with a ruler?
- Can we ever find the area of a curved figure EXACTLY? For example, does $A=\pi r^2$ give us the exact value for the area of a circle or just a good approximation?

The difficulty they encountered responding to these apparently simple questions astounded the teachers. In all of their years as students of mathematics, not even the secondary mathematics teachers had been asked to think about questions like these, because learning about area had been reduced to memorizing and applying area formulas.

In the next session, the group discussed these questions in depth. At the end of this discussion, the facilitator handed out a mathematical essay on area as a follow-up reading assignment, to both validate some of the conclusions the group had reached and expand them further.

A number of follow-up activities encouraged the participants to reflect on this unusual learning experience and to analyze it from different perspectives. Participants listed "what they had learned" about area from this experience. This list was quite detailed and complex. Interestingly, although the teachers included a few technical facts, such as learning area formulas for diamonds and stars, they primarily identified elements related to mathematical processes and the nature of mathematics. For example, they highlighted the importance of learning to develop area formulas on their own, of understanding the role played by the choice of unit in measuring area, and of recognizing that mathematical problems could have more than one acceptable solution. Several participants also mentioned gaining increased confidence as learners of mathematics as a result of this experience.

The facilitator then began a discussion on the instructional goals that should inform a unit on area *for their students*. Not surprisingly, the group established quite different goals for their students than it is traditionally the case, such as: Students should understand the concept of area (how it is useful, what is actually measured); students should understand the concept of scale; students should discover that there is more than one formula for a given figure; students should be able to derive formulas.

A day later, as a culminating experience for the whole Summer Institute, the facilitator asked participants to reflect on this experience-as-learners on area and another experience-as-learners on tessellations they had engaged in a few days earlier. This time, the participants were asked to identify the *teaching practices* that the institute instructors had modeled in these experiences. As individuals shared their reflections with the whole group, the facilitator probed responses to elicit the rationale for, and potential effects of, these practices in *their own classroom instruction*.

For example, when someone identified the think/pair/share technique for the "fish" activity, a teacher pointed out how helpful it had been for her to work individually on the task first. Others corroborated this observation, noting the value of getting personally engaged in a task before interacting with others. In contrast, one participant expressed his relief at knowing that this individual stage would last only a few minutes, since he initially believed he would never be able to compute the area of the "fish" alone. This discordant opinion invited some considerations about differences in individuals' preferences and learning styles. Other people then commented on the power of the whole group discussion and how it had helped them go well beyond what they had achieved working with just one partner. The group agreed on the value of being able to explain one's strategies and solutions to another person first, and all the participants felt that this stage had been beneficial not only for gathering courage to report their ideas to the whole group but also for clarifying and expanding ideas by talking with a partner.

This reflective session also enabled the participants to recognize and discuss the role of less evident yet equally important pedagogical decisions, such as starting the unit with the complex, open-ended task of finding the area of the "fish." Participants noted the marked contrast between this decision and the traditional practice of assigning complex problems only *after* students have learned specific procedures that are presumed to be prerequisites for solving problems efficiently. This insight led to discussing the different assumptions about *learning* that distinguish constructivist/inquiry-based mathematics from traditional practices grounded in behaviorist learning theories.

Illustration 2: Working alongside mathematicians in a real-life setting

We adapted the illustration in this section from the *Growth in Education through a Mathematical Mentorship Alliance Project* (*GEMMA*) (ENC, 2000; Farrell, 1994).

As part of the *GEMMA* project, teachers participated in an eight-week summer internship in local businesses heavily involved in the use of mathematics and science, such as consumer marketing companies, scientific consulting firms, and automobile and other manufacturing companies. Each teacher was assigned a mentor in a company, and they worked together solving authentic problems that confronted the business. These projects included analyzing market surveys, testing fan blades for engines, researching the operation of a microwave that was being installed on a factory production line, determining and graphically displaying the relationship among molecules in a new material, and creating a computerized model of transportation systems. The companies expected teachers to be fully contributing members of the problem-solving team. In doing so, teachers had to learn about current industry practices for solving problems and to identify where and how mathematics was used.

During the internship, teachers attended a series of seminars where they discussed what they were doing, what mathematical applications they were learning, and what new instructional practices they were generating from their experiences with industry. By the end of the summer internship, teachers were expected to have designed some applied mathematical problems that they would pilot in their own classrooms. The project goal was to create a booklet of such "applications problems" to share with the other mathematics teachers.

The outcomes far surpassed the *GEMMA* project directors' expectations. They hoped the teachers would discover applications for the kind of mathematics they taught, which they did. However, the directors found that the internship experiences also introduced and/or reinforced many of the current reforms in pedagogy. In their final papers, for example, teachers wrote that they teach with a greater purpose and that they feel a need to integrate mathematics and science. They also wished to create collaborative learning environments in their classrooms and to give students much more responsibility for their learning.

Main elements and variations

The previous illustrations highlight several of the elements we believe need to be a part of any high-quality experience-as-learners.

Some of these elements have to do with the nature of the *mathematical learning experience* for the teachers. In order to be effective, we believe that these mathematical experiences need to accomplish the following:

- ***Challenge the participants intellectually,*** regardless of their mathematical backgrounds or the grade levels they teach. Only under these conditions can teachers be genuine learners and benefit fully from participating in these instructional experiences.
- ***Be mathematically sound and address key concepts.*** In order to strengthen teachers' knowledge of mathematics and invite them to rethink the goal of school mathematics, these experiences must offer opportunities to learn worthwhile and significant mathematics.
- ***Allow for mathematical reflection and discussion in addition to mathematical problem-solving.*** Doing so is essential to ensure that teachers revise and enhance their current understanding of key mathematical concepts and procedures, and do not just engage in "activities for activity sake."
- ***Model non-traditional ways of learning and/or teaching mathematics.*** Participants must experience alternatives to traditional school mathematics in order to appreciate their potential for student learning.

Another set of characterizing elements involves the *reflections* that follow the mathematical learning experience itself. As both illustrations show, these reflections are critical to the success of any experience-as-learners in initiating teachers' rethinking of their views of mathematics, teaching and learning. The following list captures the characteristics of optimal reflective activities:

- ***Reflective activities should occur after the learning experience is over, not during it.*** In this way, participants may find it easier to abandon their teacher roles as they engage in the mathematical learning experience and be genuine learners in it.

- ***There should be opportunities for individual reflections as well as group discussion.*** Participants need to make personal sense of the experience as well as hear other people's insights and perspectives.

Despite these common characteristics, successful experiences-as-learners can also differ in substantial ways, as reflected by our two illustrations. Important variations can occur along any of the following dimensions:

- ***Duration and complexity of the mathematical experience.*** Both of our illustrations included intense mathematical experiences – a 7-hour inquiry on area in Illustration 1, and a summer-long project in Illustration 2. In contrast, there are examples in the literature of shorter mathematical experiences, involving the solution of a problem or other isolated mathematical tasks.
- ***Diversity of participants.*** Participants may be a rather homogeneous groups of mathematics teachers teaching at the same level of schooling or they may include mathematics specialists and non-mathematics specialists at different grade levels (as it was the case in Illustration 1).
- ***Facilitator's role.*** The facilitator may purposefully model some innovative teaching practices (as in the inquiry on area reported in Illustration 1) or simply work alongside teachers in a joint task (as expert mathematicians did in Illustration 2).
- ***Scope and structure of follow-up reflections***. Reflective activities may be open-ended or focused explicitly on specific aspects of the learning experience. For example, facilitators may ask teachers to reflect on the teaching practices modeled, the reactions of different learners to the experience, or their views of mathematics. Leaders may also elicit individual reflections in different ways, such as asking teachers to respond in writing to written prompts, to write in journals or to brainstorm ideas with a partner before having teachers share and discuss them.

Experiences-as-learners can also take place in a variety of professional development formats. They can be part of an after-school workshop, a summer institute, a university course, an on-site study group, or even an immersion situation in which teachers become mathematics-learners and problem-solvers alongside mathematicians in real-world settings. In many cases, part of the participants' mathematical experience may require projects or other assignments that are undertaken by each teacher independently.

Experiences-as-learners can be conducted by facilitators with a variety of backgrounds. Although mathematicians might seem to be ideal facilitators for this type of professional development, they may need to work collaboratively with experienced teachers or mathematics educators who can complement their subject matter expertise with experience in instructional innovation. Conversely, experienced teachers playing the facilitator's role may benefit from coaching on the differences between teaching adults and K-12 students and from readings about the "big mathematical ideas" that form the core of any experience as learners. Regardless of their affiliation, facilitators leading experiences-as-learners need both a strong mathematical background and the ability to model innovative teaching practices.

Teacher learning needs addressed

Experiences-as-learners have the potential to address many of the teacher learning needs we identified in Chapter 1, yet the extent to which they do so depends on how the activity is implemented. In this section, we discuss what specific variations of experiences-as-learners can best help meet the needs of teachers who are interested in pursuing school mathematics reform and how.

- ***Developing a vision and commitment to school mathematics reform.*** Mathematical experiences-as-learners can be powerful to help teachers understand what school mathematics reform really mean and why it should be promoted. When a skilled mathematics teacher educator designs the activities to demonstrate the kind of mathematics instruction promoted by the reform movement, teachers can appreciate the vast difference between traditional and constructivist-based practices. For example, the inquiry on area reported in Illustration 1 allowed the teachers themselves to learn about a traditional mathematical topic by focusing on big mathematical ideas, solving problems through inquiry and constructing knowledge with others. It also illustrated concretely the new roles that teachers and students must play when a constructivist view of learning informs mathematics instruction.

The personal success and enjoyment that participants experience in novel mathematical activities are powerful motivators toward instructional innovation. Committed teachers want their students to experience the same positive emotions about mathematics. We have observed this happen, especially with teachers who have bad memories of being students in traditional mathematics classes. Even teachers who were successful students in traditional settings, however, can experience vicariously their colleagues' delight when they share such thoughts as "I never knew I could do mathematics! If only I had been taught this way!" This kind of response is especially common when the group includes non-mathematics specialists.

The personal success and enjoyment that participants experience in novel mathematical activities are powerful motivators toward instructional innovation.

- ***Strengthening one's knowledge of mathematics.*** Experiences-as-learners are ideal for strengthening teachers' knowledge of mathematics. However, the nature and extent of this learning depends on the duration and design of the learning experience. For example, immersion experiences (as shown in Illustration 2) expose teachers to mathematical tools and applications used in business, not in the traditional school curriculum. By seeing what mathematical knowledge and skills are really needed to solve real-life problems, teachers may begin to question what their students should learn. Consequently, they may rethink the goals of the mathematics courses they teach.

Teachers can also learn something new about topics that are currently in the K-12 curriculum, as shown in the area inquiry in Illustration 1. There are many benefits to doing so, since teachers – even those who have taken several college-level mathematics courses – often lack the deep conceptual understanding of mathematical topics in the K-12 curriculum that are necessary to implement reform lessons. As reported earlier, several teachers in the inquiry on area had never questioned the significance of using squares as units when measuring area, nor had they really understood what area formulas are or where they come from. However, the mathematical insights these teachers gained might not have been achieved at the same level without the reflection and discussions that followed the learning experience itself. Follow-up reflective discussions, such as the "What I have learned" analysis that followed the inquiry on area, are critical to challenge participants' views of mathematics as a discipline and their perceptions of themselves (and their students) as learners of mathematics.

- ***Understanding the pedagogical theories that underlie school mathematics reform.*** Experiencing mathematics as learners has also the potential to help teachers understand better the pedagogical theories that inform current reform efforts. As Simon's (1994) model of learning cycles suggests, this kind of professional development activity not only provides an experiential basis for new learning approaches but also stimulates teachers to reflect on, and inquire further about, the theories of learning and teaching on which these approaches are based. To ensure a thorough understanding of learning theories, however, personal reflections need to be augmented by specially designed follow-up readings and/or presentations, something that was missing in our illustrations.

- ***Understanding students' mathematical thinking.*** Because experiences-as-learners focus on the *teachers*' learning, they are not an ideal vehicle to pursue an understanding of *students*' learning and thinking processes. However, these experiences do help teachers become aware of their own – and other adults' – mathematical thinking and problem-solving strategies. This awareness can be eye-opening for many teachers, and it can inspire them to examine their students' thinking in the future.

- ***Learning to use effective teaching and assessment strategies.*** Experiences-as-learners are especially appropriate for modeling effective teaching practices, at least when the facilitator has the expertise to do so. As we argued earlier, modeling is a critical part of learning complex tasks (Collins, Brown, & Newman, 1989). To be most effective, modeling should not stop with the expert performing the novel task in front of the novice. Rather, it should be accompanied by *explicit reflection* on the teaching practice that was demonstrated so that participants can recognize and internalize its key elements. We believe, therefore, that a focused follow-up reflective session is necessary to help teachers identify the teaching practices modeled and to analyze the implications for mathematics instruction (as shown in Illustration 1).

- ***Becoming familiar with exemplary instructional materials and resources.*** Depending on the content of the mathematical learning experience, experiences-as-learners may or may not help participants become familiar with exemplary instructional materials and resources. Teacher educators who want to introduce participants to an exemplary curriculum series or to a replacement unit that teachers will be expected to implement later in their classes need to select mathematical tasks from these materials and adapt them for an adult audience. This is what happened in the inquiry on area we featured in Illustration 1, and it is a practice used in many projects designed to support the implementation of NSF-funded curricula.

- ***Understanding equity issues and their implications for the classroom.*** By doing mathematics in a group, teachers are inescapably confronted with the diversity in learning styles and approaches that exist. This is especially the case, though, when the mathematical task is open-ended and there are opportunities to share different solution processes. The experience can be especially powerful when the group is highly diverse and the implications of the differences are addressed explicitly. However, it is our experience that given an appropriate mathematical task, any group of learners will produce enough diversity in responses to begin a conversation. Facilitated experiences-as-learners are also ideal for modeling strategies for differentiated instruction based on diverse learning needs and, then, discussing participants' reactions to these strategies.

- ***Coping with the emotional aspects of engaging in reform.*** Coping with the emotional aspects of engaging in reform is not a central goal of engaging teachers in experiences as learners of mathematics. Nevertheless, using this kind of professional development experience early in a program can be instrumental in creating a bond among participants and engendering a "community of learners" that can offer emotional support as the participants undertake instructional innovation in their classrooms later on. It is also important to recognize that for some elementary and special education teachers just engaging as learners in a mathematical task may evoke painful memories of failure and raise anxiety levels. Acknowledging and addressing these feelings within the context of an experience as learners may help these teachers overcome their fears, thus mitigating emotional obstacles to their individual efforts at instructional innovation later on.

As teachers critically analyze the experience they participated in as learners, they begin to appreciate the power of reflecting on instructional practice.

- ***Developing an attitude of inquiry toward one's instructional practice.*** As teachers critically analyze the experience they participated in as learners, they begin to appreciate the power of reflecting on instructional practice. These reflective sessions can also model ways for teachers to structure their own reflections to make the process more productive. Therefore, experiences-as-learners can be valuable in addressing this teacher learning need, provided that the follow-up reflective sessions are designed to achieve that goal.

Summary

Our analysis shows that activities in which teachers become learners of mathematics can be a powerful way to accomplish multiple professional development goals, especially when they are thoughtfully designed and led by a capable facilitator. Any variation within this type of professional development experience can promote the learning of new mathematics and challenge teachers' beliefs about what students should learn and how. These experiences can also help teachers develop a vision for school mathematics reform, examine pedagogical theories and effective teaching practices and become aware of diversity in approaches to problem-solving and learning styles. However, we caution that these benefits depend on whether a facilitator carefully models novel teaching strategies and orchestrates focused reflections on these experiences. The length of the activity, the complexity of the tasks, the design of the format, and the structure of the follow-up reflection may also determine the extent to which this kind of professional development experience can meet various kinds of teacher learning needs.

Suggested follow-up resources

If you are interested in learning more about exemplary professional development materials that can help teacher educators plan and facilitate mathematical experiences-as-learners, we recommend the following resources:

Corwin, R.B., Price, S.L., and Storeygard, J. (1996). *Talking mathematics: Resources for developing professionals*. Portsmouth, NH: Heinemann.

This multi-media package is intended to support teacher educators in planning professional development for elementary teachers to help them promote and facilitate in their classes the kind of mathematical discourse recommended by the NCTM Standards. A main component of the proposed professional development program are experiences-as-learners where the teachers engage in a number of mathematical problems, chosen because they are mathematically rich and "engaging" yet accessible to elementary students. The materials include a facilitator guide, videotapes providing images of elementary classrooms engaged in mathematical discourse and a book for the participants. The facilitator guide provides considerable support for setting-up and facilitating the suggested experiences-as-learners.

Friel, S.N., and Joyner, J.M. (Eds.). (1997). *Teach-Stat for teachers: Professional development manual*. Palo Alto, CA: Seymour.

This manual is intended to support teacher educators interested in replicating the 3-week summer institute developed and field-tested by the NSF-funded Teach-Stat project. This program was designed to prepare elementary teachers to teach statistics and at its core has a carefully-designed series of experiences where the teachers themselves learn statistics in the inquiry-oriented way they are expected to encourage in their students. The manual provides valuable directions and support about how to plan and implement the summer institute.

Fonzi, J., and Borasi, R. (2000). *Orchestrating math experiences for teachers.* (videotape + facilitator's guide) (available from the authors).

This 50-minute videotape features the mathematical inquiry on area described in Illustration 1. The accompanying guide provides additional information about and a commentary on this mathematical learning experience and a rich set of questions to help teacher educators use the illustration to design similar mathematical learning experiences for teachers.

Fonzi, J., and Borasi, R. (2000). *Promoting focused reflections on learning experiences.* (videotape + facilitator's guide) (available from the authors).

This 40-minute videotape features excerpts from three reflective sessions that followed the inquiry on area featured in *Orchestrating math experiences for teachers* and another inquiry on the topic of tessellations. Taken together, the three sessions illustrate complementary ways to focus and structure follow-up reflections, a critical component of effective experiences as learners. The accompanying guide offers additional information about and a commentary on the illustrations and questions to help teacher educators analyze what it takes to successfully design and facilitate this kind of reflective session.

Borasi, R., and Fonzi, J. (in preparation). *Introducing math teachers to inquiry: A framework and supporting materials for teacher educators.* (multi-media package) (available from the authors).

This multimedia package supports mathematics teacher educators who want to implement a professional development program to begin the process of school reform. It shows teacher educators how to design experiences as learners that introduce teachers to an inquiry approach to teaching mathematics. The package contains two 2-hour-long videos, each featuring an experience-as-learners. The CD-ROM included in the package contains a detailed set of artifacts from these experiences and suggestions for implementing similar ones.

CHAPTER 5

Analyzing Students' Thinking

In this chapter, we examine the type of professional development experience in which teachers analyze student thinking as revealed in students' written assignments, think-aloud problem-solving tasks, class discussions and clinical interviews. Within this kind of professional development sessions, teachers learn to observe various types of student mathematical activity and to interpret what they observe, with the ultimate goal of enhancing their students' learning opportunities.

Theoretical rationale and empirical support

In Chapter 1, we discussed the research evidence that supports teachers learning about students' mathematical thinking. We argued that doing so can help teachers develop not only a knowledge base about students' conceptions and problem-solving strategies that they can use in planning instruction but also skills for listening to students and interpreting their thinking.

Professional development that helps teachers analyze students' mathematical work is a logical vehicle to achieve these goals. First, it is consistent with the professional development principle that teachers should engage actively in concrete activities close to their own practice, not just abstract discussions. Second, according to Simon's (1994) Learning Cycles model, analyzing student artifacts creates the context necessary to start a learning cycle focusing on students' thinking. As groups of teachers examine artifacts together, they can engage in active learning, experience cognitive dissonance as different interpretations are proposed and construct new meanings. Third, examining students' work and thinking is precisely what we want teachers to do as part of their everyday teaching practice. Therefore, engaging in these tasks with the guidance of an expert is a valuable way to learn to do the same tasks independently (Collins, Brown, & Newman, 1989).

Research shows that analyzing student thinking can promote instructional practices that result in higher student achievement. Evidence supporting this claim comes from several research studies on outcomes of professional development programs for elementary teachers based on a Cognitive Guided Instruction (CGI) model (e.g., Carpenter & Fennema, 1992; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996), as well as research conducted by the *Integrating Mathematical Assessment* (*IMA*) project involving middle school students (Gearhart, Saxe & Stipek, 1995).

Moreover, comparison studies between teachers who had participated in CGI training and those who had not showed that CGI teachers were using some highly effective practices in their classroom teaching:

> [T]eachers who had been in CGI workshops spent more time having children solve problems, expected multiple solution strategies from their children, and listened to their children more than did control teachers. (Fennema, Carpenter & Franke, 1997, p.194)

Case studies of teachers who participated in CGI programs (Fennema Carpenter, Franke, & Carey, 1992; Fennema, Franke, Carpenter, & Carey, 1993) also show that these teachers gained a better understanding of student thinking and expressed views about the learning and teaching of mathematics consistent with the goals of school mathematics reform. We attribute these results mainly to the teachers' analysis of student thinking, as this was the key professional development activity used in the CGI programs.

Illustration 3: Building a classification of addition/subtraction problems from the analysis of a videotaped problem-solving session.

This illustration depicts a typical 2-hour-long session in a CGI program. We adapted this vignette from the description provided in Fennema, Carpenter, Levi, Franke & Empson (1999). In this session, teachers viewed a videotape of a first-grade child solving four word problems. The goal was for them to identify different types of problems involving addition and subtraction. From this activity, the teachers were able to reconstruct the "Classification of Word Problem Chart" (shown later in Figure 9) that is part of the research model informing the CGI program.

The session opened with teachers discussing three mathematical word problems:

1. *Lucy has 8 fish. She wants to buy 5 more fish. How many fish would Lucy have then?*
2. *TJ has 13 chocolate chip cookies. At lunch she ate 5 of these cookies. How many cookies did TJ have left?*
3. *Janelle has 7 trolls in her collection. How many more does she have to buy to have 11 trolls?*

These problems represent different types of addition and subtraction problems. At first glance, problem 1 seems to involve addition, and problems 2 and 3 seem to require subtraction. However, problems 1 and 3 can also be characterized as having to do with "joining" two sets, while problem 2 is about "separating" an original set into two subsets. Characterizing problems in this way suggests that subtraction may not be the only approach to solving problem 3, for example.

After the participants had a chance to solve the three problems on their own, the facilitator initiated the discussion by asking, "Which of these two problems are most alike and why?" Besides noticing that problems 2 and 3 involved subtraction, a teacher also commented that problem 3 would be harder for his/her students. After a brief discussion of this point, the facilitator introduced the videotape, in which a first-grade child, Rachel, solves the same three problems. (The videotape is available in the CGI professional development support materials available from the authors.)

The facilitator invited the participants to watch how the child solved these problems and to think about how *the child* perceived these problems in terms of similarity, difference and level of difficulty. Rachel's approach surprised the teachers, as Rachel solved the third problem by "joining," while most teachers had solved the same problem by subtraction. In the ensuing discussion, problems involving a joining action were distinguished from ones involving a separating action. To clarify the difference, the facilitator asked teachers to write a problem of each kind and then to share and discuss these problems with the group. In the course of the discussion, participants also agreed that problem 3 must have been more difficult for Rachel because "the child just can't go step by step through the problem and do what it says."

The facilitator then introduced the next segment of videotape, in which Rachel solves yet another addition/subtraction problem:

4. *Max had some money. He spent $9.00 on a video game. Now he has $7.00 left. How much money did Max have to start with?*

The follow-up discussion on the child's solution of this problem led the group to realize that this problem, too, could not be easily solved "step by step." In addition, this problem could be even harder to approach because the child would not know where to start.

Building on these observations, the facilitator pointed out that addition and subtraction problems may vary not only according to the type of action involved in solving them (i.e., "joining" or "separating") but also according to where the unknown appears in the story. After some discussion, the leader suggested that the following variables could be used to organize the four problems:

A. Involve joining
B. Involve separating

and,

i) The unknown is introduced at the end of the word problem
ii) The unknown is introduced in the middle of the problem
iii) The unknown is introduced at the beginning of the problem

The group then used these variables to create the 2x3 matrix reproduced in Figure 9. When the matrix was completed, the leader also introduced the "official names" used in the CGI project to refer to each of these six types of addition and subtraction problems (highlighted in boldface in Figure 9).

Figure 9
CGI classification of word problems chart

1. Lucy has 8 fish. She wants to buy 5 more fish. How many fish would Lucy have then? **(join-result unknown)**	*3. Janelle has 7 trolls in her collection. How many more does she have to buy to have 11 trolls?* **(join-change unknown)**	**join-start unknown**
2. TJ has 13 chocolate chip cookies. At lunch she ate 5 of these cookies. How many cookies did TJ have left? **(separate-result unknown)**	**(separate-change unknown)**	*5. Max had some money. He spent $9.00 on a video game. Now he has $7.00 left. How much money did Max have to start with?* **(separate-start unknown)**

The session concluded with further discussion about each type of problem.

Illustration 4: Supporting teachers in analyzing the results of a test on area

The episode we report in this section occurred in the Making Mathematics Reform a Reality (MMRR) project described in Chapter 2. It was part of the field experiences that took place in the first year of the professional development program. In the MMRR project a mathematics teacher educator was assigned to each school as school facilitator to support participating teachers as they implemented innovative instructional experiences in their classes. The professional development activity described below took place while one of the school facilitators worked with two 7^{th} grade teachers implementing their first inquiry unit, an adaptation of the inquiry on area described in Chapter 1.

The two teachers had designed a comprehensive paper-and-pencil test to assess what their students had learned about area at the end of the unit. This test included items to assess whether students could compute the area of different figures, describe the strategies they used to solve these problems and show understanding of some basic concepts about area. The teachers had already graded these tests, but when the school facilitator asked them to say what they thought their students actually learned about area and what aspects of area might still be a problem, neither teacher felt able to respond.

The facilitator then suggested that each teacher select three or four student papers that presented interesting differences in students responses and re-examine these tests to determine what each student knew or did not know about area. In the after-school meeting scheduled to discuss their findings, both teachers expressed surprise at the challenge this analysis presented, especially since grading the test had been rather straightforward. In several cases, they came to the meeting with just a guess about why a student might have answered a question in a certain way. The discussion that developed as everyone tried to make sense of such puzzling responses was very informative. It often clarified some mathematical points about area, uncovered the student's thinking process and helped teachers further articulate their instructional goals for the unit. Since some student work revealed particular misconceptions, the facilitator also asked both teachers to brainstorm ideas about how to help each student gain a better understanding, either in individual after-school sessions or in future classroom instruction.

Although not planned as part of the professional development program, this experience was an eye-opener for the both the teachers and the school facilitator. Among other things, it engendered a greater appreciation for the importance of analyzing students' work, and it also called into question the grading process that the teachers had so far taken for granted as a viable way to measure student learning.

Main elements and variations

As stated at the beginning of the chapter, analyzing students' thinking involves primarily the in-depth examination and discussion of selected artifacts of students' mathematical activity. Effective implementations of this type of professional development also require the following:

- ***Worthwhile student artifacts for analysis.*** Discussions around the selected artifact will be rich only when the mathematical task(s) assigned to the students admit more than one solution and/or methods of solution, and result in partial or incorrect solutions by some students.
- ***Alternative interpretations to be examined.*** As teachers first analyze the artifacts, they should be requested to generate a variety of hypotheses about possible interpretations. The group can then examine each *hypothesis* for its likelihood of being correct.

Although analyzing students' thinking may at first appear straightforward, our illustrations show that there is not just one way to implement this kind of professional development. Considerable variations can occur depending on the kind of student artifacts available, who provides them, and how teachers analyze them.

For example, teachers can analyze productively the following kinds of student artifacts:

- ***Written work*** students produce in response to homework assignments or assessments.
- ***Videotaped "clinical interviews***," where the interviewer presents a student with a mathematical task and asks probing questions about what the child is doing and why.
- Videotaped excerpts and/or written transcripts of ***actual lessons*** in which students actively discuss a mathematical topic, solve problems in a group or report on the results of individual and/or small-group work.
- "***Cases***" or narratives of classroom experiences created to highlight the mathematical thinking and activities of selected students.

The suitability of each type of artifact depends on the goals of the professional development experience. For example, among the artifacts listed above, written work may reveal the least because it is only a product of student thinking, and even the student's written explanation of his/her solution may not always be enlightening. On the other hand, this kind of artifact presents some unique advantages, as teachers can quickly skim through the work of several different students, noting similarities and differences that can generate interesting questions and speculations. Clinical interviews are more likely to reveal the thinking processes of an individual student working to solve a problem alone. Video excerpts from a mathematics lesson may instead allow teachers to analyze the interaction among several learners working on a mathematical task. Finally, while videos and/or transcripts of a problem-solving session can capture the actual dialogue of students working on mathematical tasks, they do not provide background information on the individual learners or the instructional context to support interpretations of the learning event. Cases, or classroom narratives, on the other hand, usually do offer such information, but they are necessarily based on the writer's interpretation of the event, which may unduly influence the teachers' analysis of the students' thinking and reasoning.

Who provided the artifacts to be examined can also affect the implementation of this type of professional development. The main options in this case are as follows:

- The ***facilitator*** provides the artifacts, or
- The ***teachers*** themselves collect the artifacts from their own students.

Once again, each option has its strengths and weaknesses. Only when the facilitator provides the artifacts can these be carefully selected beforehand to illustrate specific kinds of student strategies or misconceptions. Also, some teachers may feel somewhat uncomfortable and defensive when using their own students' work. On the other hand, teachers may be more interested and motivated in analyzing their own students' work. Moreover, collecting and making sense of their own students' work apprentices teachers immediately to the daily process of analyzing student thinking. Several programs, cognizant of the benefits and limitations of each option, do both. That is, teachers experience a guided analysis of pre-selected artifacts first, and then they collect and analyze student work from their own classroom.

How the artifacts are analyzed also varies, depending on the main goals of the professional development experience. The most interesting variations occur along the following dimensions:

- The extent to which the facilitator structures and focuses the analysis.
- The role the facilitator plays in the analysis and/or discussion of the artifacts.
- The role that research-based knowledge of student thinking about the mathematical topic plays in the analysis. It is worth noting that, while using research is always highly desirable, to date there are only a few mathematical topics for which substantial research on student thinking is available.

- The extent to which instructional implications of the analysis are explicitly addressed.
- The nature and extent of follow-up experiences that could extend what teachers learn from analyzing the artifacts.

Analyzing students' thinking can occur in any of the formats we identified in Chapter 3: summer institutes, university courses, workshops, study groups, one-on-one interactions with a teacher educator, and independent work.

Facilitators for this type of professional development experience are most effective if they understand clearly the mathematics principles underlying the tasks being analyzed and know well the research on students' thinking in the particular mathematical topic.

Teacher learning needs addressed

At first, the activity of analyzing student thinking might seem to relate only to the teacher learning need we have called "understanding student thinking." While this is indeed a main goal of this kind of professional development experience, our two illustrations show that analyzing student mathematical activity can achieve much more than that. In this section, we discuss how this type of professional development experience can contribute to most of the teacher learning needs we identified in Chapter 1:

- ***Developing a vision and commitment to school mathematics reform.*** Although teachers focus on what students do and think in this type of experience, the act of examining students' mathematical activity in innovative learning situations can also contribute to teachers developing a vision and commitment to school mathematics reform. In this case, teachers can develop images of school mathematics reform in action from the instructional context that generated the student samples. The samples themselves can also show evidence of what students can accomplish when offered the kind of learning opportunities promoted by reform. This may then lead teachers to challenge traditional learning goals and practices and to experience a felt need for instructional change. The potential for this type of experience to engender a vision of reform, however, depends on the artifacts chosen and the structure and facilitation of the experience. If participants are to draw larger implications for the teaching and learning of mathematics, facilitators must help them move beyond the specifics of the learning situation they are analyzing and encourage the discussion to develop in that direction.

- ***Strengthening one's knowledge of mathematics.*** As our examples illustrate, analyzing student thinking can lead teachers to a better understanding of mathematical ideas. This is especially true when the facilitator carefully selects and sequences artifacts around a "big mathematical idea" and then focuses part of the conversation on uncovering and examining that idea. Teachers' learning of new mathematics can further be enhanced through presentations or follow-up reading assignments on the mathematical idea examined.

- ***Understanding the pedagogical theories that underlie school mathematics reform.*** Analyzing student thinking can also introduce teachers to the constructivist theories of learning that inform the current recommendations for school mathematics reform. However, in order to truly meet this teacher learning need, the analysis of students' artifacts should be supplemented by readings and/or presentations about the theoretical foundations and empirical research supporting a constructivist perspective. This component is missing in both our illustrations.

- ***Understanding students' mathematical thinking.*** Understanding students' mathematical thinking is obviously at the core of this kind of professional development experience. As both examples illustrate, examining specific examples of students' mathematical activity in depth gives teachers valuable insights about the many different ways in which students at different grade levels approach problems or develop specific concepts or skills. Even more importantly, it can help teachers learn to conduct a similar analysis of their own students' work, to both understand where students might be in their development of key mathematical ideas and to devise learning experiences to best help them progress. This second goal, however, calls for teachers to collect and analyze artifacts from their own classes.

- ***Learning to use effective teaching and assessment strategies.*** While learning new teaching practices is not an explicit goal of this kind of professional development experience, there are two notable exceptions. First, teachers can learn strategies for encouraging students to share their thinking and approaches to solutions. Second, teachers can learn to interpret students' work. We argue that both these strategies are at the core of school mathematics reform.

Supporters of this kind of professional development experience would also argue that these practices are likely to result in better instruction. Knowing how their students' think can empower teachers to make informed instructional decisions and to devise effective assessments. As the vignette on examining the results of a test on area (Illustration 4) shows, even well-designed assessment tools can prove ineffective unless teachers learn to interpret the results and use them to inform instruction.

Finally, we should not forget that teachers, whenever they examine student thinking that takes place in reform mathematics classrooms, are exposed to other teachers' worthwhile teaching practices.

- ***Becoming familiar with exemplary instructional materials and resources.*** Becoming familiar with exemplary instructional materials and resources is not typically a goal of analyzing student thinking. One exception occurs when teachers examine student work in lessons adapted from exemplary instructional materials. In this case, the analysis of the students' work can become an effective vehicle to examine the potential outcomes and goals of the materials.

- ***Understanding equity issues and their implications for the classroom.*** Analyzing student thinking can be powerful for exploring issues of equity in learning mathematics in schools. Teachers have reported being surprised by the reasoning skills that students from disadvantaged backgrounds and students with disabilities reveal when given the opportunity to explain their solutions. These experiences can challenge teachers' biases against students with different learning styles or cultural backgrounds. At the same time, knowing how differently students may approach a task alerts teachers to the influence that race, class, gender and disability may have on students' mathematical performance. We need to keep in mind, however, that to capitalize on this potential, the selected artifacts must represent a wide-range of abilities and socio-cultural backgrounds.

- ***Coping with the emotional aspects of engaging in reform.*** While coping with the emotional aspects of engaging in reform is not an explicit goal of experiences that analyze students' thinking, some teachers may need help dealing with the discomfort and frustration this kind of professional development activity may generate. It is not uncommon for teachers to feel overwhelmed as they realize how powerful, yet time consuming, it is to examine the thinking process of each of their students in-depth. Therefore, facilitators should watch for and be ready to address these feelings. Although there is no easy way to resolve the time constraints teachers must live with, facilitators can discuss realistic expectations for analyzing students' thinking as part of everyday practice and suggest some concrete strategies to make it a possibility.

> ***One of the most desirable outcomes of examining student thinking is that teachers develop the habit of paying careful attention to students' work.***

- ***Developing an attitude of inquiry towards one's practice.*** As we mentioned earlier, one of the most desirable outcomes of examining student thinking is that teachers develop the habit of paying careful attention to students' work. Teachers can then determine what students already know and do not know and make better instructional decisions. In other words, developing an attitude of inquiry toward students' work is a central goal of this type of professional development experience, although it may not necessarily invite teachers' inquiry on other aspects of their practice.

Summary

Although analyzing students' thinking might seem at first to be a rather narrowly focused strategy, our analysis reveals that this type of professional development experience is complex and powerful. The analysis of students' thinking can take a number of different forms, depending on what kind of artifacts are examined and who provides them. The implementation of this activity also depends on how the facilitator focuses the process of analysis, the specific tasks that enable the analysis, and the role the facilitator plays in both the design and the implementation of the professional development experience. The choices that the facilitator makes on each of these dimensions determines which different teacher learning needs can be met.

Suggested follow-up resources

If you are interested in learning more about exemplary professional development materials that can help teacher educators plan and facilitate the analysis of student thinking, we recommend the following resources:

Fennema, E., Carpenter, T., Levi, L., Franke, M.L., and Empson, S.B. (1999). *Children's mathematics: Cognitively guided instruction. Professional development materials.* Portsmouth, NH: Heinemann. (videotapes available from the University of Wisconsin at Madison).

The creators of CGI offer a detailed and varied set of materials to support teacher educators in implementing a professional development program based on this approach. These materials provide first of all a description of the research model for studying students' thinking about numbers and operations that informs the program. They also include suggestions for planning a comprehensive professional development program designed to introduce this research model, invite teachers to examine their own students' thinking, and help them make instructional decisions accordingly. Facilitators of such program can also find examples of lesson plans for specific sessions, problems sets and students' work to use with participants, and tips about various implementation issues. Videotapes of students' problem solving are not included in the published materials, but they are available directly from the authors.

Schifter, D., Bastable, V., and Russell, S. J. (1999). *Developing mathematical ideas (DMI)* (casebooks + facilitator's guides + videos) Parsippany, NJ: Dale Seymour.

This set of materials for teacher educators supports the implementation of an entire professional development program for elementary teachers who want to focus on numbers and operations. The sixteen 3-hour sessions that comprise this program have the analysis of students' thinking at their very core – whether the analysis is conducted through a written "case," video images of students engaged in mathematical activities, or student work the participants collect from their own classes. In each session, the Facilitator's Guide provide concrete suggestions about how to analyze the student artifacts and develop productive discussions about them.

CHAPTER 6

Discussing Cases

In this chapter, we consider professional development experiences based on the "case study method." Here teachers analyze and discuss "cases" that are written narratives or video excerpts of events that are used as catalysts for raising and discussing important issues regarding school mathematics reform.

Because several cases currently being used in professional development programs show students working on mathematical tasks, there is some overlap between this category of professional development experiences and analyzing students' thinking, the category discussed in the previous chapter. However, cases can be used to focus on other educational issues besides students' mathematical thinking. Furthermore, case discussions more generally have a long tradition in a number of professions besides education. These combined reasons led us to the decision of examining the use of cases in professional development as a separate category.

Theoretical rationale and empirical support

While using cases to develop professional knowledge in education has not been widespread, there is a strong tradition of using cases in other fields, such as law and business. Engaging mathematics teachers in the analysis of practice is certainly consistent with the principle of focusing professional development on the concrete activities of teaching and learning rather than abstractions and generalities. Appropriately selected cases can also be the starting point for all the six teacher learning cycles identified by Simon (1994), as reported in Chapter 3. The guided discussion of examples of practice can indeed provide the stimulus for new constructions of meaning by evoking cognitive dissonance, especially when the cases show a problematic situation. Furthermore, discussing such concrete examples offers teachers an ideal context for reflection and for hearing alternative viewpoints.

Indeed, Barnett (1998) has argued that the public scrutiny of ideas that takes place during a case discussion often leads teachers to new knowledge about mathematics, pedagogy and student thinking. Such knowledge is co-generated by the group in a way that significantly enhances what individuals could have come up with on their own.

Proponents of using cases in professional development have also pointed out that this kind of experience can potentially develop teachers' habits of inquiry into practice (Barnett, 1998; Schifter, Bastable & Russell, 1997). Empirical evidence in support of using cases comes from research studies evaluating the effects of the Mathematics Case Methods project, a program based entirely on case discussions (Barnett, 1991; Barnett & Ramirez, 1996; Barnett & Tyson, 1993 a&b; Gordon & Heller, 1995; Gordon & Tyson, 1995; Tyson, Barnett & Gordon, 1995). Barnett & Friedman (1997) write that these studies show the following:

> Teachers involved in case discussions move towards a more student-centered approach, learn to adapt and choose materials and methods that reveal student thinking, and anticipate and assume rationality in students' misunderstandings. Moreover, it appears that without being exposed to these ideas in research literature, teachers naturally move towards constructivist views of learning and develop a complex knowledge of students' thinking processes and underlying mathematical concepts. (p. 389)

While these findings could be attributed to the particular focus for the case discussions that the Mathematics Case Methods project employed (where all cases show classroom vignettes of students grappling with ideas about rational numbers), similar outcomes were found in field-testing the *Developing Mathematical Ideas* (*DMI*) program, which also uses cases (personal communication with Keith Cochran, 2001).

Schifter, Bastable and Russell (1997) have also pointed out the value of teachers creating their own cases, not just discussing ready-made ones. In their project, Teaching for the Big Ideas, a number of teachers successfully created cases.

Illustration 5: A case discussion about rational numbers

The vignette we present in this section illustrates a typical case discussion in the Mathematics Case Methods project (Barnett, Goldenstein & Jackson, 1994 a&b). The 2-hour session featured here occurred in a training session for experienced teachers who, although they had not had previous experience with case discussions, expressed interest in this approach and in the possibility of eventually becoming case discussion facilitators. This case discussion was the first for these teachers. The case, called “Beans, Rulers and Algorithms,” is the first in a series of cases about rational numbers that the Mathematics Case Methods project (Barnett, Goldenstein & Jackson, 1994a) developed.

The session began with a brief ice-breaker activity in which participants introduced themselves by saying their name and giving an adjective to describe their personality. Then, participants worked independently on the following problem designed to engage them personally with the key mathematical ideas in the case:

> Think about what might be difficult or confusing for a child. Use beans to solve this problem: 1/3 + 3/12.

Teachers then read the case silently. It is a two-page narrative reporting a teacher’s experience in a combined fifth/sixth-grade class working on fractions (Barnett, Goldenstein, & Jackson, 1994a). The students in this class had already worked with equivalent fractions, addition and subtraction of fractions with the same denominator, improper fractions and mixed numbers. They had done so with success, using both manipulatives and pencil-and-paper tasks. The class had then moved to adding fractions with different denominators. The teacher introduced this new situation by providing the students with 12 beans, asking them what part of this whole would correspond to 1/2, 1/3, 1/4, 1/6 and 1/12. She also showed them a ruler, pointing out how each inch is divided in 16 parts and asked students to locate various fractions on the ruler. Using this information and the two tools (i.e., the beans and the ruler), the teacher asked the students to add several fractions, including such problems as 1/3 + 3/12 and 1/2 + 5/16. Once again, the students seemed to understand and had no difficulty with these problems, at least as long as they worked with the manipulatives. However, when the teacher moved to adding fractions on paper a few weeks later, the students seemed suddenly to “switch from understanding the concepts to memorizing a formula,” and mistakes such as 1/6 + 2/7 = 3/13, or 1/6 + 2/7 = 7/42 + 6/42 = 13/42 surfaced. These outcomes puzzled the teacher, and she questioned what the students really understood about adding fractions. She wondered what she should do in the next lessons to help them.

When almost everyone had finished reading this case, the facilitator asked the participants what they thought were the important *facts* about this case. As participants offered suggestions, the facilitator recorded them on newsprint without comment. A list of about a dozen items was quickly generated, including such information as “it was a fifth/sixth-grade class,” “students already knew how to add fractions with common denominators,” “they had been working on this for some time (but not clear how long),” “they were using manipulatives,” and “they did not understand the process.”

The facilitator then asked participants to work in pairs to generate issues for discussion about the case, requesting that each issue be expressed in the form of a question. She pointed out that issues could be about the mathematics involved in the case, the children’s thinking, aspects of the instructional practice, the materials used or even the language used. She noted that based on past case discussions, some kinds of questions generated more interesting discussions than others. Therefore, she suggested teachers avoid yes/no answer questions, such as “Did the …?” and try instead to express their questions in a more open-ended way, such as the following:

“Why might a student . . . ?”
“What might happen if . . .?”
“What does . . . mean?”
“What if the problem/manipulatives were . . .?”
“What are the benefits/limitations of . . . ?”

She elicited a few examples of each kind of question from participants to serve as models before the group broke into pairs to work on the task.

After about 10 minutes, the group reconvened and each pair shared some of the questions it had generated. Once again, the facilitator recorded all these questions on newsprint with minimal comment, making sure that every pair had an equal chance to contribute and that every voice was heard. The list contained about 15 items that addressed a variety of elements in the case, all using the format for questions suggested by the facilitator. They included very specific questions, such as “What might have happened if they had used fraction bars or paper folding instead of beans and rulers?” to more general ones, such as “How do you make the connection from the manipulatives to the paper-and-pencil process?” and “What does ’basically understand’ mean?” While the majority of questions were about the teacher’s instructional choices and alternative possibilities, some questions looked more at the children's thinking, such as: "Why might students not understand the concept using beans?" and "Why would they add numerators and denominators?" Other questions focused on the mathematics, for example: "What do the beans represent?"

The group then picked one question for further discussion: "What does 'basically understand' mean?" In the remainder of the session, teachers discussed just this one question, although several other questions on the list were also addressed in the process.

The discussion began with several teachers trying to articulate what "understanding addition of fraction," or even "understanding fractions," meant for them. To help clarify their position, the facilitator occasionally invited them to come to the board and illustrate the point they were trying to make with an example. These examples usually made the discussion more concrete and raised some interesting mathematical questions about fractions and their representations. For example, participants generated new insights about the complexity of using beans to represent fractions. They noted that, depending on the number of beans chosen as the "unit," one single bean might represent a different fraction. For example, if the unit is 12 beans, 1 bean represents 1/12, but if the unit is 8 beans, one bean (the *same* bean!) represents 1/8. This suggested to another participant a possible explanation for why students might have added numerators and denominators in the problem 1/4+1/3 when using the beans, as shown in Figure 10.

Figure 10
A participant's graphical explanation of the mistake 1/4+1/3=2/7

1/4	+ 1/3	= 2/7
0000	000	0000000
one out of four	one out of three	two out of seven

This discussion led several teachers to appreciate the importance of clearly specifying what the *unit* is whenever using discrete representations for fractions. It also revealed that students might reasonably be puzzled by the fact that the teacher chose different sets of beans as the unit depending on the problem. It also suggested the value of making the reasons behind that choice explicit for students.

Throughout the discussion, the facilitator tried not to drive the conversation in a specific direction although she was not neutral either. Rather, she tried to deepen the participants' analysis and challenge their thinking through a combination of "pulling probes:"

> "Can you show us what you mean?"
> "What do other people think about that?"
> "What are benefits/drawbacks of this position/idea? Why do you think so?"

She also used "pushing probes:"

> "What about [counterexample]?"
> "Is that always true?"
> "What might be the impact on students?"
> "What new ideas can you envision for this situation?"

The facilitator also had to interrupt the discussion before the group could reach closure on the original question. She explained that while it is always hard to interrupt a good discussion, it is almost impossible to reach closure on this or any case. However frustrating this may feel at first, it also has the advantage that participants can continue to think on their own about the issues raised in the true spirit of inquiry.

The session concluded with a brief round-robin closure activity in which each teacher identified something he/she was thinking about differently as a result of the experience. Participants also gave feedback on the process by filling a process check form; the facilitator quickly reviewed the results of this feedback before the end of the session so that the group could think about how the process could be improved the next time around.

Illustration 6: Examining an example of teaching mathematics through inquiry

We took the next illustration from the Leadership Seminar in the Making Mathematics Reform a Reality (MMRR) project that we described in Chapter 2. At the beginning of this project, one of the main goals of the Leadership Seminar was to develop a common understanding among lead teachers of what it means to teach mathematics through inquiry and what it takes to put such an approach into practice.

To these ends, the facilitators devoted a 1 1/2-hour session to discussing a vignette of an inquiry lesson. The participants first read a four-page account of a lesson on constructing a congruent triangle given a side and two angles, where the students used creatively what they already knew about triangles and constructions to accomplish this novel task (Borasi, 1995, pp. 44-48).

The facilitators then carefully framed the discussion of this teaching episode. They asked the teachers to refrain from commenting on the quality of the lesson or the suitability of the example for teaching mathematics through inquiry. Instead, they should identify the elements of teaching mathematics through inquiry that were illustrated in the vignette.

As individual teachers shared the elements they had identified, facilitators asked them to explain why they had reached their conclusions and encouraged other participants to challenge these conclusions and ask for further explanation if it seemed necessary. A facilitator then recorded on newsprint the elements of inquiry-based instruction that the group agreed upon.

This exercise produced an extensive list of elements that characterize teaching mathematics through inquiry. It represented the group's shared understanding of this instructional approach at this point in time. This list was later reproduced for all participants, and they referred to it frequently in later sessions as the group continued to refine its understanding of inquiry-based mathematics as a vehicle for mathematics reform.

Main elements and variations

The two illustrations we offer in this chapter only begin to illustrate the variety of interpretations about what constitutes a case and how cases can be used in mathematics teacher education. However, a number of elements are common to all these interpretations and are thus worth highlighting as characteristic of this kind of professional experience, despite its many variations:

- ***Teachers engage in the in-depth analysis of a shared example of practice.*** The concreteness of the case enables participants to ground their reflection and discussion of more abstract ideas about school mathematics reform.

- ***Each case is carefully selected to stimulate debate on specific issues.*** A case is not simply a story, but rather a story with a "point" – although case discussions may sometimes surprise the facilitator by developing in unexpected directions!
- ***Facilitators elicit and explore multiple perspectives and opinions about the cases.*** One of the main benefits of case discussions is that teachers can benefit from the group interaction to construct meaning and knowledge that goes beyond what they, as individual participants, could have achieved. However, this requires careful facilitation of the discussion.

Within these guidelines, case discussions may differ considerably with respect to both the *nature of the case* used as a starting point and the *nature of the discussion* that is orchestrated around the case. Cases may differ along the following important dimensions:

- ***The content of the case.*** While most cases used in teacher education deal directly with classroom instruction, some feature other aspects of teachers' and/or students' practice. For example, there are cases that portray teachers' interactions with colleagues, teachers' experiences in professional development settings or even students' learning as it occurs outside of the classroom.
- ***The format in which the case is presented.*** The vignette may be presented as a story, in narrative form, or conveyed through a video. Each of these media has unique advantages and disadvantages. Most notably, while videos can allow the direct observation of non-verbal as well as verbal behaviors, they are less flexible than a narrative format and less able to convey background information about the event.
- ***Whether the case is a "stand-alone" or part of a collection.*** While almost any case can be used in isolation, programs that rely on case discussions as their primary vehicle tend to use carefully sequenced collections of cases, designed to provide teachers with multiple opportunities to examine a complex concept in different contexts. Multiple cases examined in a sequence make it possible to highlight different aspects of a topic each time, allowing for meanings to be constructed and revised over time.

- ***The extent to which the case illustrates exemplary practice.*** While most cases make no assumptions about the quality of the practice they portray (as, for example, the case on rational numbers used in Illustration 5), some are created specifically to illustrate exemplary – although never perfect! – practice (as exemplified by the case used in Illustration 6).

- ***How "real" the case is.*** The cases currently available in the literature cover the entire spectrum from faithful representations of real-life events to fictitious situations. Most cases, however, are composites of several real-life events that have been created for the purpose of illustrating specific issues.

- ***How "pointed" the case is.*** A case is usually selected or constructed to illustrate specific points. This is especially true in collections of cases designed to help teachers grapple with different topics, such as elementary students' developing conceptions of numbers and operations. However, Illustration 6 shows that almost any account of practice can become a case if it is appropriately framed for participants.

The other major area of variation depends on how the facilitator organizes the discussion about the case. Important variations can occur along any of the following dimensions:

- ***How the case discussion is framed.*** Facilitators may determine the specific goals and foci for the discussion in advance and communicate this to the participants upfront, or be more open-ended and willing to set goals together with the participants.

- ***How the case discussion is facilitated.*** As mentioned earlier, all facilitators should ensure that participants feel free to express their opinions and show respect for others' ideas. Facilitators should also try to elicit multiple opinions, encourage debate, and invite further articulation of ideas among the participants. However, there are various ways to achieve these goals. Some programs, such as the Mathematics Case Method featured in Illustration 5, expect facilitators to follow a carefully articulated set of practices, while others are less prescriptive about what the facilitator should do.

- ***What activities may accompany the case discussion.*** While case discussions may occur in isolation, most often they are accompanied by other activities intended to strengthen or extend the outcomes of the discussion. For example, teachers in the rational numbers case discussion (Illustration 5) engaged first as learners in the same mathematical tasks discussed in the case. In this way, they gained a personal understanding of the mathematics involved and began to think about alternative ways to approach these tasks. In other implementations, teachers have been invited to further pursue issues raised in the discussion through follow-up readings, or even mini action research projects in their own classrooms.

Cases can be used in a great variety of professional development formats – including summer institutes, courses, workshops and study groups.

Case discussion facilitators may require different kinds of expertise depending on the content and focus of the case. Whenever the case involves mathematics, a good understanding of the mathematical topic involved is critical to be able to direct the discussion in productive ways. However, cases focusing on leadership and school reform issues more generally may not require any mathematical expertise in the facilitator. Regardless of the content of the case, facilitators can greatly benefit from specific training in conducting case discussions, to learn strategies to set a conducive learning environment and to ask questions that can move the conversation in productive directions without dominating it.

Teacher learning needs addressed

Cases are indeed a flexible professional development tool that can address most of the teacher learning needs we identified in Chapter 1. The extent to which this potential can be met for each specific need, however, depends on both the content of the case and the nature of the discussion about it.

- ***Developing a vision and commitment to school mathematics reform.*** Barnett (1998) argues that cases are a non-threatening way to expose teachers to innovative pedagogical practices and to help them develop pedagogical content knowledge even before they have made any commitment to reform. This exposure may in turn engender an interest in teachers toward changing their instructional practices and in becoming a part of reform efforts.

Cases that portray learning experiences and/or teaching practices consistent with school mathematics reform, such as the congruent triangle case reported in Illustration 6, can contribute to teachers' images of what reform looks like. When developing a vision for school mathematics reform is one of the main goals, however, cases should be chosen to represent exemplary practice.

Cases that capture the conflicts and challenges that reform teachers may encounter contribute an additional dimension to understanding the demands of school mathematics reform. Thus, they help teachers develop realistic expectations before committing to reform.

- ***Strengthening one's knowledge of mathematics.*** Although it may seem surprising at first, developing teachers' mathematical knowledge is a stated goal of some professional development programs that use cases extensively, such as the Developing Mathematical Ideas project and the Mathematics Cases Method. To achieve this goal, a sequence of cases is carefully constructed around a key mathematical concept. Before they read the case, teachers work the same mathematical tasks featured in it. In this way, they engage personally with the mathematical concept before they examine other learners' approaches to the same task and speculate on their thinking processes, as shown in Illustration 5.

  Misconceptions and errors often play an important role in these cases because teachers may uncover some important mathematical ideas while trying to explain the origin of the errors. When developing mathematical understanding is a focus, the facilitator needs to pay special attention to eliciting alternative mathematical ideas from the participants and helping them see the significance and connections between ideas.

- ***Understanding the pedagogical theories that underlie school mathematics reform.*** Cases that focus on classroom instruction are likely to stimulate observations and analyses that challenge teachers' taken-for-granted beliefs about teaching and learning. These situations, in turn, may be used to motivate additional inquiry into the learning theories that distinguish school mathematics reform from traditional mathematics instruction, through readings, presentations and further discussions. Interestingly, Barnett and Friedman (1997) report gains in teachers' understanding of the basic tenets of constructivist learning theories even from just experiencing the workshop, without additional readings or presentations about the research that supports those theories.

- ***Understanding students' mathematical thinking.*** Many cases currently available in the literature have the analysis of student thinking at their very core. These cases all include students' mathematical activities as a central part of the vignette. To facilitate teachers' understanding of students' mathematical thinking most effectively, the discussion of these cases should focus, at least in part, on making sense of the thinking behind the activities. Barnett (1998) also suggests that, prior to reading the case, participants should engage as learners in the same mathematical tasks featured in the vignette and speculate about how their students would see and approach the same task – in other words, try to see the task through their students' eyes.

  The goal of understanding student mathematical thinking can be furthered if teachers test the insights generated in the case discussion with students in their own classes.

- ***Learning to use effective teaching and assessment strategies.*** Many cases showing instructional episodes can, at the very least, expose teachers to effective instructional practices. Whether the case features traditional mathematics instruction or shows practices promoted by school mathematics reform, teachers may benefit from critically examining these practices from various perspectives. For example, participants can explore the assumptions about student and teacher roles, examine the students' responses these roles elicit, and discuss their effectiveness in promoting student learning. Results from the Mathematics Case Method project suggest that these experiences often make teachers more willing to experiment with new practices and then reflect on these experiences. In addition, cases that portray exemplary practices have the added benefit of providing teachers with an image and a model of reform practices that they can refer to as they begin to experiment with instructional changes.

- ***Becoming familiar with exemplary instructional materials and resources.*** Cases that portray instructional episodes in which exemplary materials are featured help teachers get to know such materials and use them effectively. Although only a few such cases are currently available in the literature, teachers who are using the materials could choose to create such cases themselves and learn even more about the materials by doing so! Cases of this kind certainly help teachers anticipate students' responses to the non-traditional tasks at the core of the exemplary materials, examine the nature of the mathematical learning that results and grapple themselves with the mathematics involved. They can also provide an image of the kind of teaching practices that such materials support and help teachers begin to identify what it takes to implement such practices effectively. Even "non-exemplary" vignettes featuring exemplary materials can be helpful because they can make teachers aware of potential pitfalls in using the materials.

- ***Understanding equity issues and their implications for the classroom***. Cases featuring inclusive classrooms, or even just classrooms with diverse students, can generate worthwhile discussions about equity issues and their implications for teaching mathematics. Especially when the case is presented as a video excerpt with little interpretation of the events, participants can observe teacher-student interactions and draw their own conclusions about possible biases at work. An in-depth analysis of the interactions observed may indeed bring prejudices to the surface that participants may not know they have. Because discussions about issues of equity are often accompanied by strong feelings, they must be facilitated sensitively.

- ***Coping with the emotional aspects of engaging in reform.*** Cases can also feature the struggles and emotional challenges experienced by teachers engaged in reform. This element is often present in cases designed to support participants' inquiry about school reform (Miller & Kantrov, 1998). However, participation in any case discussion, regardless of its focus, is likely to address some of the teachers' emotional needs because it breaks their isolation and offers them opportunities to share and discuss their concerns with other colleagues.

- ***Developing an attitude of inquiry toward one's practice.*** Many proponents of the use of cases (e.g., Barnett, 1998; Shulman, 1992) state that a central goal of this kind of professional development experience is to help teachers develop an attitude of inquiry toward their practice. By definition, the discussion of a case, regardless of its specific content or focus, engages participants in a critical reflection on practice. As importantly, these reflections can benefit from the guidance of an expert and the generation of ideas with other practitioners. Barnett and Friedman (1997) also suggest that avoiding closure on case discussions may contribute to developing habits of inquiry. Leaving issues unresolved may motivate teachers to pursue them on their own.

The discussion of a case, regardless of its specific content or focus, engages participants in a critical reflection on practice.

Summary

Cases discussions have a multitude of possible uses in professional development. Depending on the content of the case and the focus of the discussion, this type of activity can address all the teacher learning needs we identified in Chapter 1. The extreme flexibility in using cases is one of its greatest strengths as a professional development tool. At the same time, because cases can vary so much, it is more difficult to evaluate their effectiveness without context-specific information.

Suggested follow-up resources

If you are interested in learning more about how to use cases for a variety of professional development goals, we recommend the following resources, in addition to the *Developing Mathematics Ideas* (*DMI*) materials already mentioned in Chapter 5:

Barnett, C., Goldenstein, D., and Jackson, B. (Eds.) (1994b). *Fractions, decimals, ratios, and percents: Hard to teach and hard to learn?* (casebook and facilitator's guide) Portsmouth, NH: Heinemann.

This set of 29 teacher-written cases illustrates recurring dilemmas and problems in teaching and learning fractions, decimals, ratios and percents. The editors primarily intend these cases for mathematics teachers in grades 4 – 8; however, we find them to be beneficial for teachers from kindergarten through grade 12. The facilitator's guide identifies the central mathematical and pedagogical issues addressed by each case, offers suggestions for facilitating the discussions, and identifies some of the common misconceptions that can emerge during the discussions.

Stein, M. K., Smith, M. S., Henningsen, M. A., and Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College Press.

This book, intended for teacher educators and teachers, is more than a set of cases. The authors introduce their *mathematical task framework* and describe the typical pedagogical patterns teachers use in implementing tasks, uncovered as the result of their research of middle school mathematics classrooms. The explicit description of the task framework and the pedagogical patterns helps teachers become aware of the cognitive demands of a mathematical task and of the issues involved in maintaining the cognitive level of a task. The cases, inspired by real classroom experiences, provide opportunities for teachers to practice identifying the cognitive demands of a particular mathematical task, to see firsthand how pedagogical practices impact the task, and to grapple with the issues raised by the example. Each case includes a section in which the featured teacher discusses her class and a section describing her implementation. In addition, each case is accompanied by a set of discussion questions and notes to support the case discussion.

Harvard Mathematics Case Development Project (in press). *Cases in secondary mathematics classrooms*. New York: Teachers College Press.

This book includes several cases at each level of high school mathematics (i.e., Pre-Algebra and Algebra, geometry, Algebra II and Trigonometry, Probability and Statistics, Pre-Calculus and Calculus). Before presenting the cases, the authors outline in detail their theoretical framework for constructing them. They examine the mathematical, pedagogical, student assessment and contextual issues they believe teachers need in order to promote learning at high levels. The book also includes a guide for case facilitators and for participants in case discussions, and it lists the major mathematical and pedagogical issues raised by each case. Each case is supplemented by notes for the facilitator that include a prediscussion activity, tips for teaching the case, suggested discussion questions, possible extensions and annotated references for further reading on either the mathematics content or the pedagogy.

Miller, B., and Kantrov, I. (1998). *Casebook on school reform*. Portsmouth, NH: Heinemann.

This book includes six ready-to-use cases that describe teachers' reform efforts in mathematics or science. The introduction of the book provides a rationale for using cases, an explanation of why and how the cases were developed and some suggestions for how to use them. The cases, developed to highlight issues raised when educators engage in school reform, are intended to stimulate thinking and discussions from multiple perspectives. Each case is accompanied by a facilitator's guide that suggests ways to elicit discussion about the "big ideas" underlying the case.

CHAPTER 7

Engaging in Scaffolded Instructional Innovation

Many successful programs working toward school mathematics reform include scaffolded field experiences. That is, opportunities for participating teachers to experiment with instructional innovation while receiving support. However, there is great variation in the kind of innovative instructional experiences that teachers undertake and in the kinds of support that can be offered. As we examine this kind of professional development experience we will often refer to it as "scaffolded instructional innovation."

Theoretical rationale and empirical support

Research conducted in several areas supports the value of scaffolded instructional innovation. First, studies of teachers' beliefs point out that the relationship between pedagogical beliefs and practices is not unidirectional (Thompson, 1992). That is, while teachers' beliefs clearly inform their practices, we might also expect experiencing "alternative practices" to challenge their existing beliefs. This change is especially apparent when teachers observe their own students demonstrating a higher level of learning and thinking in non-traditional instruction than they did in traditional instruction.

The importance of scaffolded field experiences is also emphasized in Simon's (1994) learning cycles model of teacher development introduced in Chapter 3. Simon identified the planning and implementation of innovative instruction as a possible catalyst for the fifth and sixth stages of a teacher's learning cycle.

At the same time, putting novel instructional techniques into practice presents a considerable challenge for most teachers, and many may fail in their first attempts unless they are supported appropriately. Some initial scaffolded practice is indeed recognized as a key component in the model developed by Collins, Brown, and Newman, (1989) to shed light on the process of learning complex tasks, which we introduced earlier in Chapter 1.

While it is difficult to evaluate the effect of scaffolded field experiences alone, many successful professional development programs have used this strategy extensively. The changes in teachers' beliefs and instructional practices reported by Simon and Schifter (1991), Schifter and Fosnot (1993), and Borasi, Fonzi, Smith, and Rose (1999), for example, document the success of combining experiences-as-learners with scaffolded field experiences. Furthermore, the latter two studies include case studies and anecdotal evidence that point to the specific contributions of scaffolded field experiences.

Indirect evidence in support of scaffolded field experiences is found in the positive outcomes reported by projects that implemented one of the NSF-funded comprehensive curricula (these data can be found in each project website, listed earlier in Figure 7). These projects showed long-term gains in student achievement in schools that implemented these curricula, especially when high-quality professional development helped teachers use these exemplary instructional materials appropriately (Russell, 1997).

Illustration 7: A scaffolded implementation of an illustrative inquiry unit

We derived this illustration from the Making Mathematics Reform a Reality (MMRR) project described in Chapter 2. As teachers joined the program, they agreed to participate in a week-long Introductory Summer Institute and to implement one of two illustrative inquiry units in at least one class at the beginning of the following school year.

Both illustrative units highlight fundamental features of teaching mathematics through inquiry and present "big ideas" in geometry and measurement while focusing specifically on the topics of tessellation and area. A team of researchers and teachers created and field-tested these units in a variety of middle school settings. Based on careful documentation and analysis of these experiences, instructional materials were created to support the planning and implementation of each unit at different grade levels and in different school contexts. The materials include an overview and discussion of the key components of the unit, a mathematical essay highlighting the "big ideas" addressed in the unit, a timeline, and selected artifacts (e.g., hand-outs and assessment tools) from implementations of the unit in different settings.

Scaffolding teachers' implementation of these illustrative units began as early as the Summer Institute. First, participants engaged in two experiences-as-learners lasting 5 to 7 hours each. The activities were designed to highlight key components of the two illustrative units, both of which were adapted to challenge adult learners. The inquiry on area reported as Illustration 1 in Chapter 4 was one of these experiences. These experiences, together with the reflection on the mathematical content and pedagogy that followed, gave teachers a personal understanding of the goals, rationale and overall design of the two units. Several participants reported that the positive feelings they experienced as learners in these inquiries motivated them to take the risk to try them in their own classes.

These experiences-as-learners were then supplemented by images of what the two units might look like in middle school classrooms. Excerpts of an implementation of the tessellation unit were presented in a 2-hour-long video while a 50-page narrative provided a detailed story of an implementation of the area unit. After participants watched the video and read the story, they had opportunities to share their impressions and to question teachers who had already implemented the units. Their concerns ranged from the management of materials and group work to information about student outcomes and potential pitfalls. Mostly, participants emerged from these conversations with more experienced teachers reassured that these experiences could work in middle school and encouraged by their colleagues' enthusiasm.

During the Summer Institute, the facilitators introduced participants to the instructional materials created to support the implementation of the two units. Selected readings from these materials were assigned for homework and later discussed. When facilitators asked teachers to comment on the value of these readings, they said that encountering the materials for the first time when trying to plan their unit would have been truly overwhelming because of their unusual content and structure and might have easily discouraged them from using them. Thus, assigning the readings in the Summer Institute was an important way to enable teachers to benefit from the instructional materials intended to support their first field experience.

As teachers began to plan their unit at the beginning of the school year, facilitators encouraged them to consult individually with mathematics teacher educators on the project staff or with a lead teacher at their school site. Although not everyone took advantage of these opportunities, those that did found them very helpful. In some cases, especially when the teacher felt overwhelmed by the novelty and complexity of the task, these sessions involved brainstorming and writing an overall plan together. In addition, the teacher received help writing lesson plans for the first few days of the unit. In other cases, teachers came to these meetings with drafted lesson plans that were then discussed and refined. In all cases, these consultations made it possible to address teachers' possible misconceptions and resulted in lessons that offered much better learning opportunities for the students.

As teachers began to implement their units, they could request further support from project staff or lead teachers. This support usually took the form of classroom visits followed by debriefing meetings. Whenever possible, support staff visited classrooms for a few consecutive days in order to observe how suggestions and decisions made during previous debriefing meetings played out. The teacher educator's role in the classroom visits and the nature of the follow-up meetings varied considerably, depending on the personality and needs of each teacher. In most classroom visits, the teacher educator simply observed the class, moving around to help individuals and small groups during the lesson. This strategy allowed the classroom teacher to spend more time with other students and enabled the teacher educator to report observations about students' work and thinking that the teacher might not have known otherwise. Other times, the teacher educator played a more direct role in the instruction, perhaps introducing selected activities, demonstrating the use of certain materials or recording on the board the key points of a discussion the teacher was facilitating. In either case, the debriefing meetings that followed the lesson played a key role. These meetings focused not so much on providing feedback on the teacher's performance, but rather on discussing students' work and what had been observed about their learning and thinking. This kind of knowledge helped teachers to consider in more depth the mathematical concepts they were working on and to plan for future lessons.

Occasionally, two or three teachers from the same school who were teaching the same mathematics courses worked as a team. In that case, they usually planned the units together and met regularly to discuss the outcomes of specific activities and to make revisions to the original plan. These teams were encouraged to observe each other if possible, but few managed to put this suggestion into practice.

Finally, in November, after everyone had concluded the implementation of the first inquiry unit, all Summer Institute participants were called back together for a day-long meeting. To prepare for the meeting, teachers were asked to look back at their experience and to identify at least one success and one concern that they would like to share with the rest of the group. The meeting began with each teacher briefly sharing these reflections.

Overall, the reports were quite positive, and in most cases, even enthusiastic. Most of the successes had to do with student accomplishments; several teachers reported their surprise at seeing some of their weakest students blossom during this unit and reveal abilities they had never imagined!

Most of the successes had to do with student accomplishments; several teachers reported their surprise at seeing some of their weakest students blossom during this unit and reveal abilities they had never imagined!

This sharing also revealed some common concerns and challenges. Several teachers reported feeling panicked when students came up with solutions they could not understand or questions they had no idea how to answer. Others were worried about being able to follow through, given the enormous amount of time and energy this way of teaching requires. In the second half of the meeting, these common concerns were addressed in small groups. While the small groups did not always reach satisfactory solutions, teachers generally agreed that it was helpful just to know that other people had encountered similar problems or worried about the same issues.

Illustration 8: Creating a study group to support a new curriculum

This second illustration occurred during the third and final year of the MMRR project. It involved a group of teachers who had been participating in teacher enhancement experiences for 2 to 5 years. At the end of the previous school year, the mathematics department in their school had decided to adopt the *Connected Mathematics Project* (*CMP*) series. All the teachers of the seventh-grade mathematics courses had agreed to implement several *CMP* units in their classes the following year. Since several of these units were new to the teachers, they decided to create a study group to become familiar with the units and prepare to implement them.

The study group met weekly after school. The teachers worked independently, but they followed a format that had been modeled the previous year by a mathematics teacher educator assigned to support instructional innovation at that school site.

Scaffolded field experiences can vary a great deal, depending on the nature of the innovative teaching experience and the kind of support that is provided.

To prepare for teaching each new *CMP* unit, the teachers first read the introductory information at the beginning of the teacher's guide and then worked through the mathematical investigations comprising the unit on their own, doing the same tasks they would ask their students to do. Then they met a few times to share their results and discuss the mathematics covered in the unit. They devoted the remaining sessions to planning how to introduce and pace each investigation. They read the relevant "Teaching the Investigation" sections of the materials to glean valuable tips for orchestrating classroom activities. During the group planning sessions, teachers divided up the tasks of preparing the necessary materials, such as handouts, manipulatives, assignment sheets, tests and so on, in order to accomplish them in the most efficient way.

As they implemented lessons, the teachers also sought opportunities in and even outside their regular weekly meetings to share what was happening in their classes. This sharing focused primarily on how specific activities developed. Occasionally, however, the teachers also discussed students' responses that had puzzled them.

Overall, the teachers found this experience extremely beneficial and decided to continue it the following year. They continued to add new *CMP* units to their repertoires and to refine the implementation of units they had already done.

Main elements and variations

Variations in scaffolded field experiences are many and substantial, but most successful implementations of this type of professional development experience have the following elements in common:

- ***Some scaffolding occurs at BOTH the planning and implementation stages of the innovative teaching experience.*** Both stages present unique challenges for teachers engaging in instructional innovation and call for different kinds of support.
- ***Teachers are provided opportunities to reflect on their field experience and share these reflections with others.*** Not only do teachers learn from reflecting on their experiences, but sharing is one way to address the emotional challenges of taking on instructional innovation.

Within these parameters, scaffolded field experiences can vary a great deal, depending on *the nature of the innovative teaching experiences* and *the kind of support* that is provided.

With respect to the first point, the nature of the innovative teaching experience is affected both by the *duration/extent* of the field experience requirement and by the *teacher's role in its design*. For example, teachers may be expected to do the following:

- Design and implement one or more isolated lessons consistent with a proposed innovation.
- Design an innovative unit independently and implement it.
- Implement a replacement unit (i.e., a unit that experts have designed and field-tested and for which supporting instructional materials are available) adapted appropriately to the setting.

- Gradually implement an entire "reform curriculum," that is, a comprehensive curriculum informed by the NCTM Standards, which experts have designed and field-tested to ensure appropriate student learning outcomes.

While it is certainly a valuable learning experience for any teacher to design his or her own lesson or unit, there are limitations to this practice. First, it is unlikely that the first efforts of a teacher new to reform will incorporate fully the desired mathematical content or pedagogical practices. Second, shortcomings in the design of the instructional experience are likely to produce negative outcomes, and the teacher might feel unimpressed or even discouraged by what the students gain from the experience. Finally, the time and effort required to design an innovative instructional experience may take precious resources away from other aspects of implementing that experience, such as attending to the introduction of new teaching strategies or analyzing students' responses. On the other hand, when teachers experience the complexity and challenges of designing quality instructional units, they may appreciate more fully the value of pre-made exemplary instructional materials and may develop more effective ways to use such materials.

Professional development projects that incorporate scaffolded field experiences may also differ widely according to the kind of support provided to teachers. As projects struggle to meet their participants' needs in cost-effective ways, many kinds of support strategies have been developed. We report the most commonly used ones here, organizing them according to the four different stages at which support can be offered.

Support provided prior to planning:

- Facilitators introduce teachers to the exemplary instructional materials they are going to use. The goal is to empower teachers to use these materials effectively as they start planning their experience, by becoming familiar with their overall scope, philosophy, contents and structure.

- Teachers engage as learners, independently or with a group of colleagues, in the same mathematical tasks their students are going to experience. In this way, they become familiar with the mathematics covered in the unit and personally engage with the "big ideas" they are expected to incorporate.

- Teachers participate in facilitated experiences-as-learners that mirror the kinds of learning experiences they will be offering their students. In this way, they can personally experience the impact of some new pedagogical practices, as well as gain an understanding of the goals, rationale and design of the experiences they are getting ready to teach.

- Teachers read stories or watch videos that provide a detailed account of the kinds of experiences they are going to implement in their classes. These activities give them a sense of how the experience might play out in a classroom and help them anticipate possible student responses.

- Teachers look at samples of student work for the tasks they are going to use in their classes. Looking at these artifacts can help them anticipate their own students' responses and outcomes.

- Teachers attend presentations by, and/or have conversations with, teachers who have already implemented similar experiences in their classrooms. They thus benefit from others' experiences and insights. Hearing from other teachers can also allay some of their fears before they try their first innovative experience.

- Teachers observe a colleague's implementation of the same unit on a regular basis. This can provide a concrete image of one implementation, which can serve as a model. Teachers also get a sense of the pacing, begin to anticipate students' possible responses and learn some useful tips.

Support provided during planning:

- Teachers brainstorm ideas for their unit with a small group of colleagues interested in developing a similar unit. They get feedback on their own ideas and learn from listening to the ideas of others.

- Teachers work in teams with one or two other colleagues to develop daily plans for the unit and prepare all the necessary materials to implement it. Here teachers benefit from the feedback received and from dividing up the time-consuming task of preparing instructional materials.

- Teachers (or teams) capitalize on exemplary instructional materials to create their daily plans for the unit and prepare materials for the implementation. They thereby benefit from the thinking and field-testing that went into the design of these materials. They also save time in preparing the necessary handouts, assessments and so on.
- Individual teachers (or teams) meet with a mathematics teacher educator to review and refine their plans. This enables them to benefit from an expert's feedback and provides the opportunity to brainstorm more ideas.

Support provided during classroom implementation:

- A mathematics teacher educator or more experienced colleague teaches (or co-teaches) a few demonstration lessons in the teacher's classroom at the beginning of the unit. The demonstration provides a model and helps establish a supportive classroom climate.
- A mathematics teacher educator or more experienced colleague observes a few classes and then meets with the teacher. These debriefing meetings provide the teacher with the opportunity to gather feedback, reflect on students' thinking and learning and revise their lesson plans.
- A mathematics teacher educator or more experienced colleague provides some in-class support, so that the classroom teacher can focus on selected aspects of an innovative instructional approach.
- Members of the team that planned the unit together observe each other and debrief these observations on a regular basis. All members benefit from each other's feedback and can use the discussions as a starting point to plan future implementations.

Support provided after the classroom implementation:

- The teacher records key concerns, observations and insights in a journal that is shared and discussed with a mentor or a colleague.
- The teacher collects and examines artifacts from the field experience (e.g., handouts, assignments, assessment instruments, lesson plans, student work, etc.) to create a record of the implementation that can

be used in the future. The record can also be used to evaluate the outcomes of the experience.

- The teacher participates in facilitated meetings with other peers in which they all share and discuss their field experiences. In these meetings, teachers can benefit from articulating their experiences and hearing other people's experiences and insights without having to engage in any writing.

- The teacher participates in an ongoing peer support group in which field experiences are shared and discussed informally. Again, these opportunities for reflection do not involve writing, yet teachers benefit from sharing and reflecting on their experience and from hearing other people's experiences and insights. The peer support group can also provide immediate feedback and help when facing a problem, as well as on-going emotional support.

All the options listed above can support the efforts of teachers engaging in instructional innovation. The choice of specific options, however, will depend for the most part on the available personnel and financial resources and the expressed needs of the teachers.

The variations discussed in this section show that supported field experiences do not just take place in the teacher's own classroom or in one-on-one interactions with a teacher educator. Rather, important scaffolding can occur before and after the field experience in different settings, such as Summer Institutes or other large group meetings and in small groups, too.

Scaffolded field experiences are probably one of the most challenging forms of professional development because the provider must have high levels of expertise in multiple areas. In order to evaluate and guide other teachers' efforts toward instructional innovation, teacher educators facilitating these experiences need to have a good understanding of mathematics in a wide variety of areas and considerable pedagogical expertise. Specific training in classroom observation and mentoring strategies is also advisable.

Teacher learning needs addressed

Our discussion thus far suggests that, depending on the nature of the innovative teaching experience and the support provided for it, scaffolded field experiences may effectively address several of the teacher learning needs we identified in Chapter 1:

- ***Developing a vision and commitment to school mathematics reform.*** For many teachers, seeing a non-traditional approach to teaching mathematics succeed in their classrooms and witnessing their students' enthusiastic responses may be the most powerful way to grasp what school mathematics reform is all about. Indeed, once teachers see what their students can do when given the opportunity to explore and make sense of mathematics, they are hooked!

  Nevertheless, certain conditions need to occur for this to happen. First, the innovations that teachers implement in their classes need to truly enact school mathematics reform. Second, they have to be sufficiently well-designed and implemented, so that students actually have new opportunities to learn and thus to show their teacher what they can do. Either conditions are difficult to ensure in the case of teacher-designed experiences. Therefore, having teachers begin with field-tested materials, in addition to receiving sufficient in-class support, may be advisable to ensure that teachers' first attempts at innovation are successful.

  It is also critical to offer opportunities for individual reflection and sharing so that teachers can recognize the significance of the changes they witness in their classrooms and the implications for school mathematics reform. Such cognizance is illustrated by the conversations that took place when teachers shared their first experience with inquiry in Illustration 7.

- ***Strengthening one's knowledge of mathematics.*** From years of offering scaffolded field experiences, we know that the maxim, "You learn something best when you have to teach it," is really true. After they use open-ended problems and a student-centered approach in their classrooms, teachers regularly report learning new solutions and strategies from their own students! Even more substantial opportunities to learn new mathematics occur when the scaffolded field experience entails implementing replacement units or units from one of the new Standards-based comprehensive curricula. Since these materials have been designed to address new learning standards and to highlight "big mathematical ideas," they offer new perspectives and insights on familiar – and not so familiar – mathematical topics for both teachers and students. Again, opportunities to learn new mathematics and to challenge dysfunctional mathematical beliefs are enhanced when providers build time for reflection and sharing into the field experiences that focuses on mathematical issues.

- ***Understanding the pedagogical theories that underlie school mathematics reform.*** While scaffolded field experiences by themselves are not sufficient to teach teachers the theories that underlie the teaching and learning practices of mathematics reform, they can help further this goal. First, scaffolded experiences can motivate teachers to learn more about pedagogical theories not only as a way to make sense of what they witness in their classes but also to justify their instructional choices to other teachers, parents and administrators. Consequently, teachers may be more willing to attend presentations or read articles they may have previously dismissed as "too theoretical" and, therefore, irrelevant to classroom practice. Second, these classroom experiences can provide an experiential base for teachers to interpret and critically examine competing pedagogical theories.

- ***Understanding students' mathematical thinking.*** Scaffolded field experiences can provide teachers with multiple opportunities to understand their students' thinking. This understanding occurs to some extent any time teachers listen to their students' explain how they solved complex and open-ended tasks, which is one of the key practices promoted by school mathematics reform. However, this teacher learning need is supported best when the scaffolded field experience includes opportunities to examine students' work systematically with other teachers.

- ***Learning to use effective teaching and assessment strategies.*** Addressing this teacher learning need is probably the most obvious goal of scaffolded field experiences, especially at the beginning of a professional development program. No matter how effectively a new teaching practice is modeled in an experience-as-learners or in a classroom video, it is only when teachers try it out in their own classrooms that they really understand what it takes to make it work. However, the extent to which this happens depends once again on the *design* of the innovative teaching experience and teachers' opportunities for receiving *feedback* on their implementations of the new teaching practice.

- ***Becoming familiar with exemplary instructional materials and resources.*** Scaffolded field experiences are the best way for teachers to become acquainted with exemplary instructional materials and to appreciate fully the role these materials can play in supporting instructional innovation. Many of the teachers who participated in the experiences reported in Illustrations 7 and 8 voiced the belief that they could not have come up with a unit of the same quality on their own. To a lesser extent, scaffolded field experiences based on teacher-designed units might also provide motivation and opportunities to examine exemplary instructional materials, especially when teachers are encouraged to look at these resources for ideas to adapt for their own unit.

- ***Understanding equity issues and their classroom implications.*** Scaffolded field experiences have the potential to contribute greatly to teachers' understanding issues of equality in the classroom, especially when the implementation takes place in a diverse instructional setting and strategies for differentiated instruction are explicitly introduced. Implementing a unit that has been designed to address multiple learning styles and needs can allow *all* students in the class to show what they are capable of doing. This, in turn, may surprise many teachers and invite them to critically examine their expectations and biases. Explicit reflections about equity issues and their implications in each teacher's specific context are also critical to capitalize on the potential of scaffolded field experiences to address this teacher learning need.

- ***Coping with the emotional aspects of engaging in instructional innovation.*** Teachers are likely to experience emotions ranging from elation to despair as they try innovative instructional experiences, especially the first time. Consequently, it is especially important that *any* scaffolded field experience include ongoing opportunities for teachers to share their experiences and feelings with peers. They need reassurance that their reactions are not unique. They also need to hear from more experienced peers and mentors that there is “light at the end of the tunnel.” Scaffolded field experiences should include a reflective component to meet this teacher learning need.

- ***Developing an attitude of inquiry towards one’s practice.*** Helping teachers become more reflective about their practice should indeed be one of the main goals of any scaffolded field experience. The extent to which such experiences can promote the habit of inquiry, however, depends on the structures and opportunities for reflecting and sharing provided to participants. The more teachers are invited to critically examine what they have done in their field experiences, whether in reflective journals, discussions with peer-support groups, or debriefing meetings, the more they can appreciate the value of such reflections and learn strategies to continue reflecting on their own.

Summary

Scaffolded field experiences can be extremely effective in addressing many of the teacher learning needs we identified in Chapter 1. At the same time, the potential of this type of professional development for providing teachers with opportunities to learn new mathematics, to try out new teaching practices and materials, and to understand equity is greatly increased when teachers use exemplary instructional materials rather than units of their own design. Structures for teachers to talk and share with others, both peers and experts, also ensure that teachers can not only learn from their experiences but also get emotional support. The success of scaffolded field experiences also depends on sufficient resources being available to provide the support that teachers need.

Suggested follow-up resources

Most of the new Standards-based exemplary materials now available (including all the NSF-funded comprehensive curricula listed earlier in Figure 7, along with the address of their respective websites) come together with information designed to provide support to the teachers implementing them. These may include explanations about the mathematics addressed in various units, examples of lesson plans, suggestions about how to implement certain activities, and even recommendations about how specific tasks may be modified to meet the needs of students disadvantaged by some disabilities or limited language proficiency. These supporting materials can also be extremely helpful for teacher educators who want to support the implementation of any of these curricula.

There are not, instead, many professional development materials that have been published specifically to support teacher educators in orchestrating effective field experiences. If you are interested in learning more about ways to organize and support innovative teaching experiences, we recommend the following unpublished resources:

Fonzi, J. & Borasi, R. (2000). *Providing in-class support* (videotape + facilitator's guide) (available from the authors)

This 40-minute videotape captures a classroom experience in which a teacher educator plays a number of different roles to support the classroom teacher in implementing an inquiry unit with her sixth grade class. The accompanying guide offers additional information and a commentary on this experience and a set of questions to help teacher educators use this illustration as a catalyst for an inquiry on providing effective in-class support.

Fonzi, J. & Borasi, R. (2000). *Debriefing classroom observations* (videotape + facilitator's guide) (available from the authors)

This 40-minute videotape features excerpts from a series of classroom observations and debriefing meetings about the implementation of an inquiry unit in a eighth-grade class. The accompanying guide offers additional information, a commentary on this experience and a set of questions to help teacher educators use this illustration for an inquiry on conducting classroom observations. The goal of the inquiry is to show how debriefings can be a vehicle for professional development rather than teacher evaluation.

Borasi, R. & Fonzi, J. (in preparation). *Introducing math teachers to inquiry: A framework and supporting materials for teacher educators.* (multimedia package) (available from the authors)

These materials provide descriptions and supporting materials for orchestrating a supported field experience similar to the one portrayed in Illustration 7.

CHAPTER 8

Gathering and Making Sense of Information

We described a number of creative and novel learning experiences for teachers in the previous chapters, but some traditional learning experiences still have much to contribute to teacher learning. Indeed, in several of the illustrations reported in the previous four chapters, participants read articles or listened to presentations. In this chapter, we show how teachers can benefit from these as well as other forms of data gathering and sense-making, including action research, as a main venue for learning. More specifically, we will examine ways in which teacher education informed by a constructivist paradigm can facilitate teachers' learning *from and with* texts, videos, presentations, and even data they have gathered in their own research.

Theoretical rationale and empirical support

Having teachers listen to experts' presentations and doing assigned readings has been the preferred mode of professional development so far at both pre-service and in-service levels. Interestingly, however, not much research documents the effects of these learning modes on teachers' knowledge, beliefs or practice.

Nevertheless, gathering and making sense of information continues to be a valuable tool for teachers and any other learners. This mode of learning can become an integral part of constructing a personal understanding of issues and theories that are at the core of school mathematics reform. Indeed, readings, presentations, and data collection and analysis can all contribute to teacher education although they may take on different forms and purposes when informed by a constructivist perspective.

Recent research on reading, in particular, can help us begin to reconceptualize how *making sense of information* can become an active and socially constructed process. Reading researchers have argued that reading does not need to occur as an isolated, or even individual, activity (e.g., Harste & Short, 1988). First, reading should be purposeful. In other words, teachers should read either to address questions that *they* feel the need to know more about or because their concerns could not be resolved through discussion. Reading can also be a catalyst for other experiences. Indeed, reading can fulfill many functions while teachers inquire into any topic (Siegel, Borasi & Fonzi, 1998). Readings can provide background information, raise questions for further inquiry about a topic, synthesize different points of view, and offer models for teachers' own practice. Research also teaches us that reading is not a passive or straightforward matter of decoding or extracting information from text (e.g., Pearson & Fielding, 1991; Rosenblatt, 1994). Rather, readers always construct meaning in interaction with the text, their own background and interests, and their purposes for reading the text. Furthermore, such construction of meaning can be even more productive when it is augmented by interactions with other learners, so that different interpretations can be shared and discussed.

Action research thus offers an ideal way for teachers to learn more about teaching and learning mathematics and to apply the results immediately to their own practice.

Reading researchers also argue for expanding our notion of what constitutes a text (e.g., Bloome & Egan-Robertson, 1993; Green & Meyer, 1991), noting that the principles of reading outlined above also hold true for other "texts," such as videos, presentations or electronic media. Indeed, teachers can benefit from actively constructing and negotiating meaning not only through written texts but also videos they watch together or independently, information they gather on the Internet or presentations made by an expert or a colleague.

In addition to benefiting from information others provide, teachers can gather their own data to illuminate issues of particular interest to them. Teachers can gain from participating in many forms of research, but "action research" is especially promising as a form of professional development (Holly, 1991; Eisenhower National Clearinghouse, 2000). Action research is defined as "an ongoing process of systematic study in which teachers examine their own teaching and students' learning through descriptive reporting, purposeful conversation, collegial sharing, and reflection for the purpose of improving classroom practice" (Eisenhower National Clearinghouse, 2000, p.18). Action research thus offers an ideal way for teachers to learn more about teaching and learning mathematics and to apply the results immediately to their own practice, although conducting full-blown action research studies is not the only way that teachers can benefit from gathering and analyzing classroom data.

Illustration 9: Using a variety of resources to rethink the teaching and learning of geometry in middle school

The experience captured in this illustration took place in the Leadership Seminar that was one of the components of the Making Mathematics Reform a Reality (MMRR) project we described in Chapter 2. After several teachers had participated in the first year of the program, they wanted to make more radical changes in their teaching. During the first year, they had attended a Summer Institute introducing them to an inquiry approach to mathematics instruction and then implemented an illustrative inquiry unit on either tessellations or area in their own classrooms. Their experiences with the tessellation and area units made them aware of the inadequacy of traditional approaches to teaching geometry in the middle school curriculum. Although they felt that the next logical step would be to revise their school's geometry curriculum, they were not sure how to proceed. In the usual process for rewriting curriculum, teachers sat around a table, and based on the current textbook, discussed what contents should be covered at each grade and how. The teachers suspected that this process might at best eliminate some repetition in the existing curriculum, but that it was not likely to help them reconceive the entire middle school geometry curriculum.

After some lead teachers shared these concerns in the Leadership Seminar, the facilitators decided to use this opportunity to lead the group in a systematic rethinking of the teaching and learning of geometry in middle school. Such an experience could serve as a model for lead teachers interested in replicating a similar process with colleagues in their own school. An even more important goal for this experience, however, was to familiarize the lead teachers with the resources offered by relevant research studies and exemplary instructional materials, so they could use these resources well in the future.

The group inquiry started with a few readings about geometry. As a homework assignment, participants read two mathematical essays from the book *On the Shoulders of Giants* (Steen, 1990). One essay focused on the concept of “Shape” (by Senechal) and the other on “Dimension” (by Banchoff). As part of the same assignment, participants reviewed the NCTM Standards (1989) for geometry in middle school.

In the group discussion that resulted, the lead teachers analyzed the meaning and rationale of each of the NCTM geometry standards in light of the “big ideas” of geometry presented in the two essays. This discussion enabled participants to enhance their understanding of the mathematical concepts presented in the two essays and to consider implications for instruction. For example, some teachers said they found it very helpful to think of geometry as the study of “shapes,” especially as they had come to realize the connection between the geometric properties of a shape and its possible functions. This realization helped them frame in a more meaningful way the study of geometric figures for their students. It also helped them change their instructional goals because they agreed that students should learn strategies for identifying the attributes of any geometric figure, not just memorize a pre-established set of properties for a few standard figures.

Although very helpful, this activity did not immediately result in a plan for what to teach about geometry, and how, at different grade levels in middle school. The facilitators then suggested that the group look at the choices made by two of the comprehensive middle school math curricula funded by the National Science Foundation, the *Connected Mathematics Project* and *Mathematics in Context*. In both cases, groups composed of mathematicians, mathematics educators, and teachers had grappled for years with the same question: What should students learn about geometry in middle school? The facilitators argued, then, that the group should capitalize on all the thinking that had gone into the development of these exemplary curricula.

However, it turned out to be difficult to extract from the curricula the choices that the authors had made about what geometry content to cover and how, and the rationale for these decisions. Although the background materials accompanying each of these curricula did address, to some extent, these choices and how they were made, the information was not specific enough for the group. It soon became clear that the group needed to examine the individual geometry units in each curriculum.

To make this task less daunting and time-consuming, the group divided up the responsibilities. Each participant, including the facilitators, agreed to review one or two units from each curriculum to identify what was taught and how and to present their findings to the group. To ensure consistency, the facilitators proposed some guidelines for the review and report on each unit and then modeled a presentation.

A 3-hour session was then devoted to the geometry unit presentations. To get a sense of how topics in each curriculum were sequenced, participants presented the units in the order they were intended to be taught. As each unit was presented, a facilitator recorded on newsprint the key ideas about geometry that the unit addressed. At the end of the presentations, the teachers had a detailed list of the geometry content that each curriculum covered.

The group then compared these lists to identify similarities and differences between these two Standards-based curricula and the traditional middle school geometry curriculum. Many teachers were amazed at the richness of the lists describing the new curricula when compared with the traditional middle school math curriculum. They were struck especially by the emphasis in both of the new curricula on three-dimensional geometry and spatial visualization, topics they rarely covered but that were highlighted in the geometry essays they had read. On the other hand, they were puzzled by the presence of some new topics, such as Euler's formula and graph theory in the *Mathematics in Context* curriculum.

The facilitators then suggested they seek a mathematician's help to examine further the relative importance of the topics on the lists. The facilitators met independently with Dr. Sanford Segal, a research mathematician on the faculty at the University of Rochester, to share the group's lists and ask whether he felt comfortable commenting on the mathematical significance of the topics listed. They also shared some information about the group's background and goals to help him prepare his contribution.

Dr. Segal then joined the group for a 2-hour session in which he presented his comments on the relative importance of items on the lists from a mathematical stand-point, and then he answered questions. His presentation and the follow-up discussion further confirmed the critical role of spatial visualization in mathematics, and hence the importance of developing this skill in middle school through appropriate learning experiences. On the other hand, Dr. Segal's personal position on the relative importance of graph theory and transformation geometry challenged the need to introduce these topics at the middle school level.

Overall, all participants, facilitators included, emerged from this inquiry with a much deeper understanding of what the "big ideas" in geometry are and a greater appreciation for the complexity of making good choices about mathematics content at any grade level.

Illustration 10: A teacher's action research on her own biases

We adapted the illustration in this section from a teacher's personal account of her eye-opening experience with action research (Wickett, 1997). Her experience took place in the context of the NSF-funded Equity in Mathematics Education Leadership Institute project (also known as the EMELI project).

In a workshop on equity issues, this teacher learned about the empirical evidence showing that teachers call on boys more often than girls in mathematics classrooms. She became interested in exploring whether she, too, had some unrecognized biases in the way she called upon students in her class. She feared such biases might impede her goal of providing equitable access and support to *all* her students.

While the focus of her action research was clear, she struggled with the decision of what kind of data to gather. She searched for a systematic way to examine her classroom practices that would not make her self-conscious and unduly influence her daily practice. After rejecting, for various reasons, the options of audiotaping or videotaping some of her classes, she decided to examine the charts that she routinely created to record students' contributions in a mathematical discussion. As it was her practice to create these charts by writing down each student's contribution verbatim, followed by the student's name, these existing records were indeed ideal to address her question.

Her analysis of the charts created over several weeks revealed some interesting and surprising patterns. While there was not much difference in the numbers of girls and boys she called on, she noticed that she tended to call on the boys first. She also noticed that she usually included students with limited English proficiency only toward the end of the discussions. The teacher describes these findings as "upsetting" to her because they suggested unconscious biases in her behavior.

These finding led to the teacher reflecting on the *reasons* she called on students in a mathematical discussion and the potential *implications* of these instructional choices for her students' learning opportunities. She realized that she tended to call on certain students first because she expected their contributions to be catalysts for other students' ideas; she was also hesitant to call on students until they volunteered, and some students (especially students with limited English proficiency) tended to do so only later in the lesson, if at all. Despite these reasonable justifications, she concluded that her current practices were not truly giving all students equal opportunities to participate in her mathematics classes. She decided to try to change these practices.

To make sure that she gave all students an equal opportunity to answer first, she made a conscious effort to pause before calling on students during a mathematical discussion. Whenever possible, she asked other adults in the class to write down the students' responses so that she could pay more attention to facilitating the discussion and to asking questions that could invite more students to contribute. To encourage more students to share in a large group, she also successfully experimented with the use of "dyad." In this technique, each student has the opportunity to express his or her thoughts to a partner without interruption; each partner is allotted an equal amount of time and students may choose to use their primary language.

The teacher reports feeling empowered by this process. She was able to make positive changes in her classroom practice that resulted in better learning opportunities for her students. At the same time, she had done it at her own pace, taking only the steps she felt comfortable taking at the moment. She sums up her experience in this way:

> I had enough information that I could make positive changes yet not so much information that I felt overwhelmed and defeated. … By looking at my practices honestly and without condemning myself, I began the process of recovery and change. … I was able to remain open, freeing myself to try new ideas with my students' best interests in mind. (Wickett, 1997, p. 104)

Main elements and variations

Teachers can gather and make sense of information in many different ways. In the illustrations in this chapter, we highlighted the following elements common to gathering information and making sense of it:

- ***Teachers gathered information for a purpose.*** In other words, teachers gathered data and evaluated it to address a felt need or answer a question they had posed themselves.
- ***Teachers actively made sense of the information.*** Teachers engaged in hands-on interpretation of data, readings or presentations in each activity we reported.
- ***Teachers made sense of the information in interaction with others.*** In all the activities, teachers at some point negotiated interpretations and made meaning with peers, facilitators and/or experts. Through this process, they benefited from different perspectives and others' constructions of meaning.

Despite these common elements, professional development experiences in which teachers gather and making sense of information can be quite varied. This was already evident in our two illustrations, and many more variations are reported in the literature. Indeed, the professional development experiences examined in this chapter can be seen as a "collection" related by the fact that each example explicitly engages teachers in learning from and with information of various kinds.

Variations within this collection mostly depend on the *source of the information, how the information is gathered*, and *how the information is examined and used.*

As we consider the first variable, the *source of the information*, the following possibilities should be considered, as they can all present valuable learning opportunities for teachers:

- ***Lectures or presentations.*** These can be offered by an expert, such as the mathematician in Illustration 9, a more experienced colleague, or even another member of the group. In Illustration 9, for example, each participant contributed a unit presentation.

- ***Published texts.*** These could include for example articles, books, textbooks or curriculum series. All these resources were used in the inquiry on the geometry curriculum reported in Illustration 9.

- ***Texts produced by other members of the learning community.*** These texts could be created by a facilitator, individual teachers or even the group as a whole. The list of key geometry ideas the group generated based on the unit presentations in Illustration 9 is a good example of this kind of text.

- ***Videotaped excerpts.*** These could capture examples of classroom practice as well as other events related to school mathematics reform.

- ***Materials available in electronic form.*** These could include CD-ROMs, information gathered from the Internet, and even data available in electronic databases.

- ***Various kinds of artifacts.*** These could have been generated in classroom implementations (such as student work, lesson plans or the "discussion charts" used by the teacher in her action research reported in Illustration 10), or in other reform-related experiences (such as agendas or minutes of important meetings, policy documents, etc.).

- ***Various kinds of data.*** These data could be the results of the teacher's own observations or analysis of artifacts and/or demographic information (such as the number of times and the sequence in which different categories of students were called upon in the teacher's classroom, as transpired from her analysis of the discussion charts in Illustration 10) or data available in the research literature or other sources (such as the data about boys being called on more than girls in mathematics classrooms that the teacher in Illustration 10 read about prior to her own action research).

Each source of information listed above may convey some informational content better than others. Also, different kinds of activities may be more appropriate than others for making sense of information conveyed from these different sources.

A second source of variation in this kind of professional development is *how the information examined was gathered*. This can happen mainly in two ways:

- The *facilitator* selects the information and makes it accessible to the participating teachers.
- The *teachers* themselves gather the information, following some directions or guidelines set by the facilitator.

The first option is often the preferred one because it saves teachers valuable time. Teachers also benefit from the facilitator's expertise. However, there is value in empowering teachers to gather their own information, at least some of the time. Whether they search the library, browse the Internet, or collect their own data, teachers can learn skills that will serve them in the future as they research issues independently.

Finally, this type of professional development varies according to *what is done with the information*. Since the options in this case are too many and too context-dependent to list, we will simply refer readers to the two illustrations featured in this chapter for some examples. We would like to point out, however, how reading and conducting action research seem greatly enhanced when they occur in conjunction with other activities in summer institutes, workshops or study groups, rather than in isolation.

The role played by the professional development provider in this type of experiences may appear to be less central, yet it is by no means unimportant. Professional development providers can serve as invaluable resources for participants as they gather and make sense of information. Moreover, providers can be very influential in framing and guiding these activities and in connecting them to other parts of the professional development program. Depending on the content and format of the information gathering activities, providers may require different kinds of expertise in order to be effective.

Teacher learning needs addressed

When presented as a purposeful, active and social process of meaning-making, gathering and learning from information has the potential to address many of the teacher learning needs we identified in Chapter 1. Of course, the content and source of the information, and even more importantly, how it is used, determine the extent to which specific teacher learning needs can be addressed in any implementation of this kind of professional development experience:

- ***Developing a vision and commitment to school mathematics reform.*** Developing a vision and commitment to school mathematics reform requires an understanding and appreciation of what such reform calls for and its rationale. Therefore, readings and presentations that explain each recommendation for mathematics reform and that review research supporting these recommendations can address this teacher learning need. When teachers also have concrete opportunities to draw implications from this information for their own practice, the benefit is even greater. Videos and stories of reform-oriented mathematics classrooms can also provide images of what reform is really about. Hearing the success stories of more experienced teachers may also motivate some teachers to attempt instructional innovation in their own classes.

- ***Strengthening one's knowledge of mathematics.*** While reading mathematics texts should not be the primary vehicle for teachers to learn new mathematics, this mode of learning has valuable potential if approached correctly. It should, for example, occurs in combination with, not as an alternative to, other experiences. For example, videos or multi-media materials that take advantage of computer animation can help teachers visualize and thus grasp specific mathematical concepts more clearly. Also, by reading mathematical essays on key mathematical ideas (as those used in the inquiry on the geometry curriculum reported in Illustration 9) or on the history and philosophy of mathematics, teachers can learn not only new mathematical content, but perhaps more importantly, begin to rethink their beliefs about the discipline of mathematics.

- ***Understanding the pedagogical theories that underlie school mathematics reform***. To understand the theories of learning and teaching that inform school mathematics reform, teachers need readings and presentations that explain and critically examine these theories. The effectiveness of this kind of information, however, depends to a great extent on the experiences organized to help teachers make sense of this information. For example, teachers are likely to perceive the information as more relevant if it is connected to experiences-as-learners or videos of mathematics lessons that exemplify some of the same or principles of learning and teaching.

- ***Understanding students' mathematical thinking.*** Reading research on students' thinking about specific mathematical topics can aid teachers in making sense of their own students' work. Again, however, these readings are most effective when they are explicitly connected to other professional development activities, such as analyzing student work around the same mathematical topics addressed in the readings. In addition, by conducting their own action research studies, teachers can enhance their understanding of the results in other studies, or they can even contribute new results in less-researched topics. Conducting such studies also helps teachers develop their skills in listening to students and interpreting their work.

- ***Learning to use effective teaching and assessment strategies.*** Readings and presentations alone are not likely to help teachers teach more effectively. However, watching video excerpts of other teachers modeling innovative practices can be quite powerful in helping teachers understand what they need to do. Action research in which teachers monitor and evaluate their own practice can also help teachers as they begin to try out new teaching and assessment practices in their classrooms.

- ***Becoming familiar with exemplary instructional materials and resources.*** Exemplary instructional materials have the potential to greatly support teachers in implementing high quality instructional innovation in their classes, but only if teachers know what is in them and how they can find that information. Because most of these resources provide much more information than traditional textbooks and have a non-linear structure, teachers need guidance in using the materials effectively at the beginning. Presentations about the origin and structure of the exemplary materials, followed by modeling of how to navigate them, may be very helpful for teachers as they are first introduced to these materials. Reading *from* and *about* the exemplary materials is essential for becoming acquainted with these resources. In addition, to understand what the materials require of students, teachers often have to do the mathematical tasks themselves first.

- ***Understanding equity issues and their implications for the classroom***. Readings and presentations about issues of diversity and equity can be valuable catalysts for discussing what it means to teach *all* students equally. Action research may be an even better way to meet this teaching learning need, as Illustration 10 shows. By researching their own practice, teachers can become aware of their own biases and prejudices, investigate the impact and implications of equity issues in their own classrooms and schools, and monitor their efforts toward more equitable teaching.
- ***Coping with the emotional aspects of engaging in instructional innovation.*** Stories of other teachers engaged in reform may help teachers headed in that direction to recognize in advance emotions they are also likely to experience. This kind of information can help teachers set realistic expectations and perhaps even suggest strategies to deal with the inevitable "emotional roller-coaster" that accompanies most first attempts at instructional innovation. An even more powerful variation on this type of professional development activity is hearing directly from teachers they know and being able to converse with them.
- ***Developing an attitude of inquiry toward one's practice.*** Engaging in any form of action research can contribute very effectively to addressing this teacher learning need. By definition, action research means that teachers systematically inquire about specific aspects of the teaching and learning of mathematics in their own classrooms.

Summary

Our analysis of information gathering and interpretation as a type of professional development activity confirms the value of more "traditional" professional development experiences, such as reading articles and hearing presentations, for teachers involved in school mathematics reform. As we stress in this chapter, however, these experiences need to be purposeful, engage teachers actively, and provide opportunities to share and discuss information with others. They should be combined with other activities that encourage teachers to use information to draw personal implications for their own beliefs and practices. Various forms of data collection and analysis, and action research in particular, can also enable teachers to gain valuable and relevant knowledge and skills that help them become reflective practitioners and life-long learners.

Suggested follow-up resources

With a notable exception in the case of action-research, there are few published materials to support teacher educators in designing and orchestrating professional development experiences within this category – perhaps because gathering and making sense of information is often not even considered as a professional development strategy for which materials, or even guidance, is needed. For teacher educators interested in promoting and supporting action research we recommend the following resources, which describe methods and approaches to conduct sound action research in educational settings:

Calhoun, E.F. (1993). Action research: Three approaches. *Educational Leadership 51* (2), 62-65.

Sagor, R. (1992). *How to conduct collaborative action research*. Alexandria, VA: Association for Supervision and Curriculum Development.

Loucks-Horsley, S., Harding, C.K., Arbuckle, M.A., Murray, L.B., Dubea, C., and Williams, M.K. (1987). *Continuing to learn: A guidebook for teacher development*. Oxford, OH: National Staff Development Council.

CHAPTER 9

Summary

What have we learned about professional development that works?

The many illustrations from actual professional development that we included in this monograph are an indication of the large number of successful professional development initiatives currently supporting school mathematics reform. The literature on mathematics teacher education reports positive outcomes for these initiatives, showing that high quality professional development can make a difference in the future of mathematics instruction. Yet, it is more difficult to pinpoint the role that specific professional development activities play in the effectiveness of different programs.

No single model of professional development emerges from the many successful examples reported in the literature on mathematics teacher education. Instead, we find many examples of worthwhile experiences that address the multiple needs of teachers engaged in school mathematics reform. In Chapter 1, we identified and discussed these needs, categorizing them as follows:

- Developing a vision and commitment to school mathematics reform.
- Strengthening one's knowledge of mathematics.
- Understanding pedagogical theories that underlie school mathematics reform.
- Understanding students' mathematical thinking.
- Learning to use effective teaching and assessment strategies.
- Becoming familiar with exemplary instructional materials and resources.
- Understanding equity issues and their classroom implications.
- Coping with the emotional aspects of engaging in reform.
- Developing an attitude of inquiry toward one's practice.

In Chapter 3, we argued that in order to address these teachers' learning needs effectively, professional development programs need to have the following characteristics:

- Be sustained and intensive.
- Be informed by what we know about how people learn best.
- Center around the critical activities of teaching and learning rather than focus primarily on abstractions and generalities.
- Foster collaboration.
- Offer a rich set of diverse experiences.

These characteristics can be embodied in a number of different *types of professional development experiences*. We found it convenient for our analysis to categorize the many forms of professional development activities suggested in the literature into five main categories:

- Engaging teachers in mathematical experiences-as-learners.
- Having teachers analyze in-depth exemplars of student work and thinking.
- Using "cases" as the catalyst for reflections and discussions on important issues related to school mathematics reform.
- Supporting teachers as they engage in structured and scaffolded attempts at instructional innovation.
- Empowering teachers to gather and make sense of information.

Our explanation and discussion of each type of professional development experiences in Chapters 4 through 8 make clear that these categories are not mutually exclusive. Rather, these five types sometimes overlap. For example, certain experiences-as-learners may provide a scaffold for instructional innovation, and many "cases" may involve the analysis of student thinking among other things. However, distinguishing these five major types of professional development experiences allowed us to study each in depth. Thus, we have been able to identify the characteristic elements of each type, consider the theoretical and empirical support for it and discuss the variations and conditions that may maximize its effectiveness. In our analysis, we also show how each type of professional development experiences may be used to address several of the teacher learning needs we identified in Chapter 1. We summarize the results of this analysis in Figure 11.

Figure 11
Teacher learning needs addressed by each type of professional development experience

Professional Development Experience: / **Teacher Learning Need:**	***Experiences as learners***	***Analyzing students' thinking***	***Case Study Method***	***Scaffolded instructional innovation***	***Gathering & making sense of information***
1. Developing a vision and commitment to math reform	●	•	●	●	●
2. Strengthening knowledge of mathematics	●	●	●	●	•
3. Understanding pedagogical theories that underlie reform	•	●	•	•	●
4. Understanding students' thinking	•	●	●	•	●
5. Learning to use effective teaching and assessment strategies	●	●	•	●	•
6. Becoming familiar with exemplary materials/resources	●	•	•	●	●
7. Understanding equity issues and their implications	●	●	●	●	●
8. Coping with emotional aspects of engaging in reform	•	•	●	●	•
9. Developing an attitude of inquiry towards one's practice	•	●	●	●	●

NOTE: In this chart, a large dot indicates that the teacher learning need can be effectively addressed by at least some variations of the corresponding type of professional development experience. A small dot indicates that the teacher learning need can be met somewhat, but it is not a primary goal of that type of professional development experience.

This analysis suggests that certain types of professional development experiences are more appropriate than others to further specific goals. It also shows that whether a type of professional development experience addresses any specific goal effectively depends to a great extent on the choices providers make in its implementation.

The analysis in this monograph supports the principle that professional development programs should include a variety of experiences. Furthermore, it suggests that programs should be comprised of a combination of the types of professional development experiences we have described, carefully selected to meet specified teacher learning needs.

While there are significant differences in the preparation, mathematical background, teaching experience and attitude of elementary and secondary mathematics teachers, we found nothing to suggest that any type of professional development experience is more or less appropriate for one or the other group of teachers. Indeed, illustrations showed successful implementation of a strategy with both levels of teachers. Working with elementary or secondary teachers, however, may affect some important choices within each implementation; for example, the mathematical content of experiences-as-learners or cases, or the exemplary instructional materials used in scaffolded field experiences. Despite these differences it is both possible and valuable to provide opportunities – at least occasionally – for elementary and secondary mathematics teachers to participate together in professional development experiences (as shown by the teachers' inquiry on area reported in Illustration 1, and the case discussion on rational numbers reported in Illustration 5).

Effective professional development may take a variety of formats, including intensive Summer Institutes, a series of workshops held during the school day or after school, study groups of teachers who meet on a regular basis, one-on-one interactions between a teacher and a teacher educator, and independent work done by the teacher. Most successful programs combine different formats to respond to the needs and constraints of their audience. They must also make sure that the chosen formats are appropriate for the type of professional development experiences planned.

Figure 12 summarizes the relationship between the format and the type of professional development activity that providers might consider in designing a program:

Figure 12
Acceptable formats for each type of professional development experience

Type of Professional Development Experience: **Format:**	***Experiences as learners***	***Analyzing students' thinking***	***Case study method***	***Scaffolded instructional innovation***	***Gathering & making sense of information***
Series of workshops	●	●	●		
Summer Institutes	●	●	●		
Study groups	●	●	●		●
One-on-one interactions		●		●	
Independant work		●		●	●

Our analysis in Chapters 4 through 8 also confirms that different types of professional development experiences call for somewhat different sets of skills and expertise in the facilitator. Interestingly, in each case we described, the provider could be a mathematics educator, a mathematician, an experienced teacher or a staff development administrator. What really matters is whether the provider has expertise in the discipline of mathematics, pedagogy, and/or mentoring, as required by the specific activity s/he is expected to facilitate.

However, with a few exceptions (e.g., sessions on developing leadership skills), some expertise in mathematics emerges as an important prerequisite for facilitating successful professional development on the teaching and learning of mathematics. At the same time, knowledge of mathematics alone is not sufficient to ensure a facilitator's success. While mathematicians with an interest in K-12 education are a powerful resource, they too need to become familiar with what helps or hinders adult learning and school reform in order to be effective professional development providers of specific professional development experiences.

Finally, our analysis also identified a number of exemplary materials for mathematics teacher educators. Each of these materials has been developed to support teacher educators in adapting and implementing a specific professional development program with documented effectiveness in supporting school mathematics reform. Just as we encourage mathematics teachers to take advantage of exemplary instructional materials, we also urge teacher educators to take advantage of these resources to strengthen the quality of the programs they offer.

We have provided some information about these materials at the end of Chapters 4 though 8. A more extensive list of worthwhile materials that can support mathematics teacher educators, along with in-depth reviews, can be found in the database for mathematics and science teacher educators (TE-MAT) recently developed by Horizon Research with the support of the National Science Foundation. This database is available on the World Wide Web (address: *www.te-mat.org*).

What should we look for when evaluating professional development programs?

While our analysis has validated many alternative approaches to professional development, we clearly do not support the notion that "anything goes" in mathematics teacher education. On the contrary, we believe that the *quality* of the professional development offered determines to a great extent whether any reform effort succeeds.

Providing quality professional development is the joint responsibility of the teacher educators who design it, the school administrators who decide what to offer or require for teachers, and the teachers who choose what programs in which to participate.

Professional development can be expensive, and resources allocated to it are usually limited. Therefore, it is critically important that consumers, decisionmakers and providers of professional development learn to evaluate the quality of available professional development programs. Too often, the decisions made about professional development – what to offer, fund or participate in – are based simply on the topic, for example, whether the professional development is on assessment, cooperative learning, technology or high school geometry. In this monograph, we have tried to alert readers to many other aspects of professional development that should be considered when evaluating available programs.

To begin evaluating a program, we suggest identifying one's own needs, priorities and constraints in the larger context of pursuing school mathematics reform. This list should yield a sense of the *larger goals* against which the *focus* and the *structure* of a specific professional development initiative should be evaluated.

Analyzing the main *experiences* in a professional development program will show its potential to meet one's goals and needs. Throughout this monograph, we emphasize that certain types of professional development experiences are more conducive than others in addressing certain teacher learning needs. Nevertheless, since our analysis in Chapters 4 through 8 shows how widely these approaches can vary, simply knowing that a program uses case discussions or analyses of student work may not be enough information to evaluate its appropriateness for furthering one's goals.

How can different constituencies contribute to more effective professional development?

We believe that providing quality professional development is the joint responsibility of the teacher educators who design it, the school administrators who decide what to offer or require for teachers, and the teachers who choose what programs in which to participate. Therefore, we conclude this monograph with suggestions for how each of these groups can promote quality professional development aimed at school mathematics reform.

First, we believe that professional development ***providers*** can design more effective professional development initiatives by doing the following:

- Developing a rich repertoire of effective professional development experiences and learning to use them appropriately.
- Identifying the specific reform goals, needs and constraints of their audience.
- Selecting and sequencing appropriate professional development experiences to address the goals, needs and constraints of their audience.

- Capitalizing on relevant exemplary materials for teacher educators instead of "reinventing the wheel."

Second, school and district ***administrators*** who decide which programs to offer teachers can contribute to quality professional development by doing the following:

- Identifying the main needs for professional development within the larger goal of pursuing school mathematics reform in their school or district and the constraints on providing professional development in their particular context.
- Knowing what different kinds of professional development experiences can be expected to achieve and what resources are needed to implement them appropriately.
- Maximizing the limited resources available for professional development by using them to fund programs that are most likely to effectively support school mathematics reform *and* to meet the school/district priorities.
- Ensuring that each professional development experience is offered only by providers with the required expertise and qualifications.
- Providing adequate resources for a quality implementation of the professional development program selected.

Last, but not least, professional development ***participants*** should become critical consumers by doing the following:

- Identifying their personal and professional goals and needs within the reform agenda of their school or district.
- Developing reasonable expectations about what professional development can and should achieve and about the time and effort required to benefit from it.
- Learning to evaluate the quality of a professional development initiative and to determine whether it can meet one's needs.

If we all do our part in these ways, we can expect to see an increase in high quality professional development opportunities for all mathematics educators.

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