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Nondirectional and Directional Wave Data Analysis Procedures

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1.0   __INTRODUCTION__

The National Data Buoy Center (NDBC), a component of the National Oceanic and Atmospheric Administration's (NOAA) National Weather Service (NWS), maintains a network of data buoys to monitor oceanographic and meteorological data off U.S. coasts including the Great Lakes. NDBC's nondirectional and directional wave measurements are of high interest to users because of the importance of waves for marine operations including boating and shipping. NDBC's wave data and climatological information derived from the data are also used for a variety of engineering and scientific applications.

NDBC began its wave measurement activities in the mid-1970's. Since then, NDBC has developed, tested, and deployed a series of buoy wave measurement systems. Types of systems for which data analysis is described in this report have been deployed since 1979. These systems are generally recognized as representing the evolving state-of-the-art in buoy wave measurement systems. As NDBC's newer systems have been developed, they have provided increasingly more comprehensive wave data. NDBC's data buoys relay wave information to shore by satellite in real time. Transmission of raw time-series data is not feasible with today's satellite message lengths. Thus, much of the data analysis is performed onboard using pre-programmed microprocessors. Key analysis results as well as Data Quality Assurance (DQA) information are relayed to shore. Additional data processing and quality control are performed onshore. Several aspects of NDBC's wave data analysis are unique to NDBC because of onboard analysis and NDBC's detailed consideration of buoy hull-mooring response functions.

This NDBC Technical Document is derived from contract deliverable CSC/ATD-91-C-002 which was completed in its original form in May 1994. While minor editorial changes have been made, the content has not been changed. Therefore, any improvements or changes in NDBC wave measurement systems and data analysis procedures since May 1994 are not included in this report. Appendix A contains definitions and acronyms used in this report.

2.0   __PURPOSE__

This report's primary purpose is to document NDBC's wave data analysis procedures in a single publication for users of NDBC's wave data and results based on these data. NDBC's methods for analysis of nondirectional and directional wave data have been described over many years in papers published in the refereed literature, papers presented at professional society meetings, reports, and internal documents and memoranda. Although these descriptions were correct at the time of their preparation, the methods have evolved over time so that no one document describes NDBC's wave data analysis procedures. In addition, concise descriptions with mathematical background are needed to facilitate technical understanding by NDBC's wave information users.

The U.S. Army Corps of Engineers Coastal Engineering Research Center (CERC) is a major NDBC sponsor and user. CERC is documenting wave data analysis procedures of organizations that provide wave data and analysis results to their Field Wave Gaging Program (FWGP) with the exception of NDBC. Partly because of onboard data analysis with satellite transmission of results and consideration of buoy hull-mooring response functions, NDBC's wave data processing is significantly different than that of the other organizations participating in the FWGP. The FWGP's wave measurements also are made with different types of sensors (e.g., fixed *in situ* sensors such as pressure sensors and wave orbital velocity sensors). This report's secondary purpose is to accompany and parallel CERC's documentation to satisfy CERC's documentation needs for the FWGP.

3.0   __DATA ANALYSIS PROCEDURES__

3.1   __DOCUMENTATION APPROACH__

Previous papers, reports, memoranda, and other pertinent documentation dating from the mid-1970's to the present were identified and obtained. This information was reviewed to develop an outline of NDBC's wave data analysis procedures. Appropriate information was then extracted and incorporated into this report.

The first part of this section provides the mathematical background and theory which serves as the scientific basis for NDBC's wave data analysis procedures. This information is described first because much of it applies to several NDBC wave measurement systems. Analysis of data from individual systems is described next. A later section

briefly describes archived results that are analysis outputs.

This report emphasizes NDBC's wave data analysis techniques, rather than wave data collection, instrumentation, or applications. Aspects of wave data collection are not described except as they directly relate to wave data analysis. The most pertinent references pertaining to analysis that are available in the open literature or through NDBC are referenced. Many of these references also provide further detail about NDBC's data collection and instrumentation. There are numerous references that describe applications (e.g., descriptions of wave conditions during particular events of meteorological or oceanographic interest) of NDBC's wave data. Unless they provide additional information about analysis procedures beyond that provided in other references, these applications oriented references are not referenced in this report.

Because this report accompanies and parallels CERC's documentation of wave data analysis procedures used by the FWGP, this report's format is similar to CERC's corresponding report. Where wave analysis theory is the same, text for the two reports is identical or similar.

3.2 MATHEMATICAL BACKGROUND AND DATA ANALYSIS THEORY

3.2.1 Overview

NDBC's wave data analysis involves application of accepted time-series analysis and spectral analysis techniques to time-series measurements of buoy motion. Hull-mooring response function corrections, methods for obtaining buoy angular motions from magnetic field measurements, corrections for use of hull-fixed accelerometers on some systems, and encoding-decoding procedures for relay of information by satellite involve techniques developed by NDBC. Following parts of this section provide the mathematical background and theory which applies to analysis of data from NDBC's wave measurement systems. Some techniques, such as calculation of confidence intervals, are not routinely used by NDBC but may be applied by users of NDBC's results. Information in this section is written to generally apply to analysis of buoy-collected wave data as much as possible so that it will have application to other buoy wave measurements. Thus, specific symbols and manners in which equations are written may be

different than in some noted references. In several cases, equations can be re-written or combined in different ways to yield the same results. Because this section is generally applicable to analysis of buoy-collected wave data, information in it does not have to be repeated in later descriptions of particular NDBC wave measurement systems. The style of presenting mathematical background and theory also parallels the corresponding CERC documentation.

3.2.2 Types of Data and Assumptions

Measured nondirectional wave data time series consist of digitized data from a single-axis accelerometer with its measurement axis perpendicular to the deck of the buoy within which it is mounted. If NDBC's buoys were perfect slope-following buoys, the accelerometer axis would be perpendicular to the wave surface.

Measured directional wave data time series consist of digitized data representing one of the following types of data sets:

(1) buoy nearly vertical acceleration, pitch, and roll measured using a Datawell Hippy 40 sensor, incorporating a nearly vertical stabilized platform, and buoy azimuth obtained from measurements of the earth's magnetic field with a hull-fixed magnetometer. On some systems, nearly vertical displacement time series are obtained by electronic double integration of nearly vertical acceleration time series.

(2) buoy acceleration from a single-axis accelerometer with its measurement axis perpendicular to the deck of the buoy within which it is mounted as well as buoy pitch, roll, and azimuth obtained from measurements of the earth's magnetic field with a hull-fixed magnetometer. Systems that provide this type of data set are sometimes called Magnetometer Only (MO) systems to emphasize that they do not use a Datawell Hippy 40 sensor.

For nondirectional wave measurement systems, spectra are calculated directly from measured acceleration time series. For directional wave measurement systems, buoy pitch and roll time series are resolved into earth-fixed east-west and north-south buoy slope time series using buoy azimuth. Spectra and cross-spectra are calculated

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from these slope time series and acceleration (or displacement) time series. Parameters derived from the spectra and cross-spectra provide nondirectional and directional wave information, including directional spectra.

Spectral analysis assumes that the measured time series represent stationary random processes. Ocean waves can be considered as a random process. For example, two wave records that are collected simultaneously a few wavelengths apart would have similar, but not identical, results due to statistical variability. Similarly, two wave records that are collected at slightly different times, even when wave conditions are stationary, would have similar, but not identical, results. For the purpose of documenting NDBC's wave data analysis, a random process is assumed simply to be one that must be analyzed and described statistically. A stationary process is one for which actual (true) values of statistical information (e.g., significant wave height or wave spectra) are time invariant. Wave conditions are not truly stationary. Thus, wave data analysis is applied to time series with record lengths over which conditions usually change relatively little with time.

In applying statistical concepts to ocean waves, the sea surface is assumed to be represented by a superposition of small amplitude linear waves with different amplitudes, frequencies, and directions. Statistically, individual sinusoidal wave components are assumed to have random phase angles (0° to 360°). To describe the frequency distribution of these wave components, wave spectra are assumed to be narrow. That is, the wave components have frequencies within a reasonably narrow range. Except for high wave conditions, the assumption of small amplitude linear wave theory is usually realistic for intermediate and deep water waves. It is not generally true for shallow water waves, but NDBC's buoys are seldom in shallow water from a wave theory viewpoint. Regardless, this assumption is well known to provide suitable results for most purposes. The narrow spectrum assumption, except in some cases of swell, is almost never strictly true but also provides suitable results for most purposes. Even though these assumptions are not always valid, they provide a theoretical basis for wave data analysis procedures and help establish a framework for a consistent approach toward analysis.

Kinsman (1965) provides useful philosophical and intuitive descriptions of these concepts. Earle and Bishop (1984) describe wave analysis procedures involving statistics in an introductory manner. Several papers by Longuet-Higgins (e.g., 1952, 1957, 1980) are among the most useful papers which describe statistical aspects of ocean waves. Longuet-Higgins, et al. (1963) provide a classic description of directional wave data analysis that is used for buoy directional wave measurements. Donelan and Pierson (1983) provide statistical results that are particularly useful for significant wave height. Dean and Dalrymple (1984) discuss theoretical and practical aspects of waves without emphasis on statistics.

Time-series analysis and spectral analysis techniques that are applied to measured buoy motion data are described in the following parts of this section.

3.2.3   Data Segmenting

A measured time series can be analyzed as a single record or as a number of data segments. Data segmenting with overlapping segments decreases statistical uncertainties (i.e., confidence intervals). Data segmenting also increases spectral leakage since, for shorter record lengths in each segment, fewer Fourier frequencies are used to represent actual wave frequencies. However, for most wave data applications, spectral leakage effects are small compared to spectral confidence interval sizes.

Several earlier NDBC wave measurement systems use data segmenting, while NDBC's newest system, the Wave Processing Module (WPM), does not. Data segmenting is based on ensemble averaging of J spectral estimates from 50 percent overlapping data segments following procedures adapted from Welch (1967). Estimates from 50 percent overlapping segments produce better statistical properties for a given frequency resolution than do estimates from the commonly used band-averaging method (Carter, et al., 1973) without data segmenting. A measured time series, $x(n\Delta t)$, with N data points digitized at a time interval, Δt, is divided into J segments of length L. Each segment is defined as $x(j,n\Delta t)$, where j represents the segment number, and x indicates that values within each segment are the same as the ones that were in the equivalent part of the original time series. Following are definitions of the segments

$$1 \quad x(1,n\Delta t) \;=\; x(n\Delta t), \; n \;=\; 0\ldots,L-1$$

$$2 \quad x(2,n\Delta t) \;=\; x(n\Delta t), \; n \;=\; \frac{L}{2},\ldots,\frac{3L}{2}\;-\;1$$

$$J \quad x(J,n\Delta t) \;=\; x(n\Delta t), \; n \;=\; (J-1)\frac{L}{2},\ldots,$$

$$(J+1)\frac{L}{2}-1$$

where

$$J \;=\; \frac{2\left(N-\dfrac{L}{2}\right)}{L}$$

The number of segments affects confidence intervals which are discussed later. Not using segmenting is equivalent to using one segment containing all data points (L = N, J = 1).

3.2.4 Mean and Trend Removal

Measured buoy motion time series (acceleration, pitch, and roll) as well as calculated east-west and north-south buoy slopes (obtained from pitch and roll with consideration of buoy azimuth) have small, often near-zero, time-averaged means. Non-zero mean values of pitch and roll may occur due to wind and/or current effects on a buoy's hull and superstructure. Means of several parameters are calculated as DQA checks. Means are also removed from buoy acceleration and slope time series before further analysis (data segmenting and spectral analysis) to obtain wave spectra.

Seiches, tides, and other long-period water elevation changes produce negligibly small trends in the buoy motion time series that are measured. Thus, trends are not calculated or removed.

3.2.5 Spectral Leakage Reduction

Spectral leakage occurs when measured wave data time series, which represent contributions from wave components with a nearly continuous distribution of frequencies, are represented by the finite number of frequencies that are used for covariance calculations or Fourier transforms. Mathematically, a data record or segment has been convolved with a boxcar window. The data begin abruptly, extend for a number of samples, and then end abruptly. The boxcar window has the

advantage that all data samples have equal weight. However, the discontinuities at the beginning and the end introduce side lobes in later Fourier transforms. The side lobes allow energy to leak from the specific frequency at which a spectral estimate is being computed to higher and lower frequencies. How to reduce spectral leakage is discussed in many time-series analysis textbooks (e.g., Bendat and Piersol, 1971, 1980, 1986; Otnes and Enochson, 1978) and leads to some disagreement in the wave measurement community.

Wave data are frequently analyzed without spectral leakage reduction. This approach is followed by the WPM. The rationale is that leakage effects are usually small for wave parameters, such as significant wave height and peak wave period, even though spectra may differ somewhat from those that would be obtained with use of leakage reduction techniques. Moreover, effects of leakage on spectra are generally far less than spectral confidence interval sizes. Not reducing leakage also eliminates the need for later variance corrections.

A measured wave data time series that has been processed by covariance methods or Fourier transforms provides estimates of wave component contributions at a finite number of frequencies. Different NDBC wave measurement systems that apply leakage reduction techniques do so in the time domain or the frequency domain. These systems use Hanning because it is the most commonly used method for reducing spectral leakage (e.g., Bendat and Piersol, 1971, 1980, 1986; Otnes and Enochson, 1978).

For time domain Hanning, a cosine bell taper over each data segment is used. The cosine bell window is given by

$$W(n\Delta t) \;=\; \frac{1}{2}\left(1-\cos\left(\frac{2\pi n}{L}\right)\right),\; 0 \le n \le L-1$$

where L is the record or data segment length.

The data are multiplied by the tapering function

$$x_w(j,n\Delta t) \;=\; x(j,n\Delta t) \;*\; W(n\Delta t)$$

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where the subscript w indicates that the data have been windowed.

Because of Hanning, the windowed data variance is less than the unwindowed data variance. Frequency domain corrections are made by multiplying spectral and cross-spectral densities by pairs of standard deviation ratios for each involved time series. A standard deviation ratio is given by

$$c = \frac{\sigma}{\sigma_w}$$

where

$$\sigma = \left[\frac{1}{L-1} \left(\sum_{n=0}^{L-1} x(j,n\Delta t)^2 - \frac{\left(\sum_{n=0}^{L-1} x(j,n\Delta t) \right)^2}{L} \right) \right]^{1/2}$$

$$\sigma_w = \left[\frac{1}{L-1} \left(\sum_{n=0}^{L-1} x_w(j,n\Delta t)^2 - \frac{\left(\sum_{n=0}^{L-1} x_w(j,n\Delta t) \right)^2}{L} \right) \right]^{1/2}$$

in which x and x_w indicate the time series before and after windowing respectively.

Leakage reduction in the frequency domain involves weighting frequency components within a window which is moved through all analysis frequencies. Because the variance of the data is reduced, a subsequent variance correction is also made.

For frequency domain Hanning, spectral estimates are smoothed using the following equations.

$$\begin{aligned} S_{w-xy}(m\Delta f) &= 0.25\, S_{xy}((m-1)\Delta f) \\ &+ 0.5\, S_{xy}(m\Delta f) \\ &+ 0.25\, S_{xy}((m+1)\Delta f) \end{aligned}$$

$$S_{w-xy}(M\Delta f) = 0.5\, S_{xy}(M-1\Delta f) + 0.5\, S_{xy}(M\Delta f)$$

where S_{xy} is a cross-spectral density (CSD) estimate for time series, x and y, M is the number of frequencies, Δf is the frequency interval, the subscript w indicates frequency domain windowing, and m = 1, 2, ... (M-1). CSD estimates are defined in the Fourier transform section. If x=y, S_{xy} becomes a power spectral density (PSD) estimate, S_{xx}.

The standard deviation ratios for frequency domain corrections can be written as

$$c = \frac{\sigma}{\sigma_w}$$

where

$$\sigma = \left[S_{xx} \right]^{1/2}$$

$$\sigma_w = \left[S_{xx-w} \right]^{1/2}.$$

As for corrections based on time domain calculations, windowed spectral densities and CSDs are multiplied by pairs of standard deviation ratios for each involved time series. For example, for buoy directional wave measurements, standard deviations may be for acceleration (or displacement), east-west slope, or north-south slope time series.

3.2.6   Covariance Calculations

The earliest system described in this report is the General Service Buoy Payload (GSBP) Wave Data Analyzer (WDA) system. This system is the only described system that determines wave spectra (nondirectional) by covariance (autocorrelation) techniques rather than by use of Fast Fourier Transforms (FFT). Spectral calculations via covariance techniques follow standard procedures (e.g., Bendat and Piersol, 1971, 1986).

For a buoy acceleration record, a covariance (autocorrelation) function for M lags is obtained from

$$R_j = \frac{1}{N-j} \sum_{n=1}^{N-j} a_n a_{n+j} - a_{mean}^2$$

where j is the lag number (1, 2, ... M), a_n indicates the nth acceleration value, N is the number of data points per record, and a_{mean} is the mean acceleration.

Acceleration spectra are computed from

$$S_{aa}(m\Delta f) = 2\Delta t \left(R_0 + 2 \sum_{j=1}^{M-1} R_{j} \cos\left(\frac{\pi j m}{M} \right) + (-1)^{kR_M} \right)$$

Spectral estimates can be obtained at frequencies, $m\Delta f$, where m ranges from 1 to M and Δf is given by

$$\Delta f = \frac{f_{Nyquist}}{M}$$

and the Nyquist frequency is given by

$$f_{Nyquist} = \frac{1}{2\Delta t}$$

where Δt is the time between data points. The Nyquist frequency is the highest resolvable frequency in a digitized time series. Spectral energy at frequencies above $f_{Nyquist}$ incorrectly appears as spectral energy at lower frequencies.

The GSBP WDA calculates spectral estimates only up to $m = (2/3)M$ so that spectra are cutoff at a frequency corresponding to the half-power frequency of the onboard low-pass antialiasing analog filter. Further GSBP WDA data analysis details are provided in a following section.

3.2.7   Fourier Transforms

Fourier transforms are calculated by FFT algorithms that were tested with known input data. Tests of system hardware also include tests with known analog inputs in place of actual sensor signals. Algorithms themselves were implemented in different computer languages, for different onboard microprocessors, and for different wave measurement systems. Thus, the algorithms themselves are not identical.

An FFT is a discrete Fourier transform that provides the following frequency domain representation, X, of a measured time series, x (or x_w with use of a window).

$$X(j,m\Delta f) = \Delta t \sum_{n=0}^{L-1} x(j,n\Delta t)e^{-i\frac{2\pi mn}{L}}$$

where

$$m = 0,1,2,...,\frac{L}{2} \quad L\ even$$
$$m = 0,1,2,...,\frac{L-1}{2} \quad L\ odd$$

The real and imaginary parts of X are given by

$$Re[X(j,m\Delta f)] = \Delta t \sum_{n=0}^{L-1} x(j,n\Delta t)\cos\left(\frac{2\pi mn}{L}\right)$$

$$Im[X(j,m\Delta f)] = -\Delta t \sum_{n=0}^{L-1} x(j,n\Delta t)\sin\left(\frac{2\pi mn}{L}\right)$$

Spectral estimates are obtained at Fourier frequencies, $m\Delta f$, where the interval between frequencies is given by

$$\Delta f = \frac{1}{L\Delta t}$$

Utilized FFT algorithms require that the number of data points be a power of two for computational efficiency so that L is even. The frequency corresponding to $m = L/2$ is the Nyquist frequency given by

$$f_{Nyquist} = \frac{1}{2\Delta t}$$

3.2.8   Spectra and Cross-Spectra

PSD estimates for the jth segment are given by

$$S_{xx}(j,m\Delta f) = \frac{X^*(j,m\Delta f)X(j,m\Delta f)}{L\Delta t}$$
$$= \frac{|X(j,m\Delta f)|^2}{L\Delta t}$$

where X^* is the complex conjugate of X.

CSD estimates for the jth segment are given by

$$S_{xy}(j,m\Delta f) = \frac{X^*(j,m\Delta f)Y(j,m\Delta f)}{L\Delta t}$$
$$= C_{xy}(j,m\Delta f) - iQ_{xy}(j,m\Delta f)$$

where C_{xy} is the co-spectral density (co-spectrum), Q_{xy} is the quadrature spectral density (quadrature spectrum), and X and Y are frequency domain representations of the time series x and y. These equations have units of spectral density which are the multiplied units of each time series divided by Hz. C_{xy} and Q_{xy} can be written as

$$C_{xy}(j,m\Delta f) = \frac{Re[X] \ Re[Y] + Im[X]Im[Y]}{L\Delta t}$$

$$Q_{xy}(j,m\Delta f) = \frac{Im[X] \ Re[Y] - Re[X]Im[Y]}{L\Delta t}$$

where the arguments, $(j,m\Delta f)$, of X and Y are not shown for brevity.

Final spectral estimates are obtained by averaging the results for all segments to obtain

$$S_{xx}(m\Delta f) = \frac{1}{J}\sum_{j=1}^{J} S_{xx}(j,m\Delta f)$$

$$C_{xy}(m\Delta f) = \frac{1}{J}\sum_{j=1}^{J} C_{xy}(j,m\Delta f)$$

$$Q_{xy}(m\Delta f) = \frac{1}{J}\sum_{j=1}^{J} Q_{xy}(j,m\Delta f)$$

With the definition of C_{xy}, the equation for S_{xx} is not needed. The co-spectra, C_{xx} and C_{yy}, are the same as the power spectra, S_{xx} and S_{yy}. In most following sections, the argument, $m\Delta f$, is dropped and f is used to indicate frequency.

If data segmenting is not used, one segment (J = 1) contains all of the data points in the original time series and spectral estimates are at individual Fourier frequencies, $m\Delta f$, with L = N (the total number of data points in the measured time series). Spectral estimates are then band-averaged over groups of consecutive Fourier frequencies to increase the degrees of freedom and the statistical confidence.

Conventional wave data analysis with band-averaging often uses frequency bandwidths within which one more or one less Fourier frequency may fall for adjacent bands. This situation occurs when bandwidths are not multiples of the Fourier frequency interval. Thus, adjacent bands may have slightly different degrees of freedom and confidence intervals. For NDBC wave measurement systems that employ band-averaging, Fourier frequencies fall precisely on boundaries between frequency bands. Spectral and cross-spectral estimates at

boundaries are spread equally into adjacent frequency bands when band-averaging is performed.

Frequency-dependent effects caused by the measurement systems (e.g., hull-mooring, sensor, electronic filtering) are corrected for at this point in the analysis. Non-zero frequency-dependent phase shifts have no effect on calculation of nondirectional wave spectra but may affect directional wave spectra. Nonunity frequency-dependent response amplitude operators affect both nondirectional and directional wave spectra. A following section describes hull-mooring response function corrections which are an important and unique aspect of NDBC's wave data analysis.

3.2.9 Spectra Confidence Intervals

Calculated spectra are estimates of the actual spectra. Degrees of freedom describe the number of independent variables which determine the statistical uncertainty of the estimates.

Following standard wave data analysis practice, confidence intervals are defined for power spectra (PSDs), but not for co-spectra and quadrature-spectra (CSDs). Confidence intervals are not provided by NDBC, but may be calculated by users of NDBC's wave information.

There is 100 α percent confidence that the actual value of a spectral estimate is within the following confidence interval

$$\left(\frac{S_{xx}(f)EDF}{\chi^2\left(EDF,\frac{(1.0-\alpha)}{2}\right)}\right), \left(\frac{S_{xx}(f)EDF}{\chi^2\left(EDF,\frac{(1.0+\alpha)}{2}\right)}\right)$$

where χ^2 are percentage points of a chi-square probability distribution and EDF are the equivalent degrees of freedom. As earlier noted, $S_{xx} = C_{xx}$. Ninety percent confidence intervals ($\alpha = 0.90$) are often used.

For data processed by Fourier transforms without data segmenting, the degrees of freedom for a frequency band are given by twice the number of Fourier frequencies within the band. For consistency, degrees of freedom are referred to as equivalent degrees of freedom to correspond to terminology used for segmented data. Without segmenting, the equivalent degrees of freedom are given by

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$$EDF = 2n_b$$

where n_b is the number of Fourier frequencies in the band. In general, n_b equals the bandwidth divided by the Fourier frequency interval.

For data processed by Fourier transforms with data segmenting, the equivalent degrees of freedom, EDF, for J segments, is given by

$$EDF = \frac{2J}{\left(1 + \frac{0.4(J-1)}{J}\right)}$$

where J is the number of overlapping segments. The value 0.4 in the above equation is an approximation, but is adequate for segment lengths ($\geq$100 data points) used by NDBC.

For data processed by covariance techniques, the equivalent degrees of freedom are given by

$$DF = \frac{2N}{M}$$

where N is the number of data points and M is the number of covariance lags. The degrees of freedom when data are processed by covariance techniques appear to be larger than the value based on the frequency separation interval because only every other spectral estimate is independent (e.g., Bendat and Piersol, 1971).

3.2.10 Directional and Nondirectional Wave Spectra

A directional wave spectrum provides the distribution of wave elevation variance as a function of both wave frequency, f, and wave direction, θ. A directional wave spectrum can be written as

$$S(f,\theta) = C_{11}(f)\ D(f,\theta)$$

where C_{11} is the nondirectional wave spectrum (which could be determined from a wave elevation time series if such a time series were available) and D is a directional spreading function. Integration of a directional wave spectrum over all directions (0 to 2π) provides the corresponding nondirectional spectrum given by

$$S(f) = \int_0^{2\pi} S(f,\theta) = C_{11}(f)$$

Following NDBC's conventions, subscripts are defined as follows:

1 = wave elevation (also called displacement) which would correspond to buoy heave after sensor and buoy hull-mooring response corrections
2 = east-west wave slope which would correspond to buoy tilt in this direction after sensor and buoy hull-mooring response corrections
3 = north-south wave slope which would correspond to buoy tilt in this direction after sensor and buoy hull-mooring response corrections

The CERC wave data analysis documentation noted earlier and some papers in the scientific literature use (x, y, and z) subscripts in place of (1, 2, and 3) subscripts. NDBC's wave measurement systems usually measure buoy acceleration rather than buoy heave. A superscript m indicates a spectral or cross-spectral estimate based on a measured time series. Estimates based on buoy acceleration are converted to estimates based on wave elevation (displacement) during subsequent analysis steps as later described.

Directional spectra are estimated using a directional Fourier series approach originally developed by Longuet-Higgins, et al. (1963). Since its development, this approach has been described and used by many others (e.g., Earle and Bishop, 1984; Steele, at al., 1985; Steele, et al., 1992). It yields directional analysis coefficients that are part of the World Meteorological Organization (WMO) FM65-IX WAVEOB code for reporting spectral wave information (World Meteorological Organization, 1988).

The directional Fourier series approach provides the directional Fourier coefficients, a_n and b_n, in the following Fourier series

$$S(f,\theta) = \frac{a_o}{2} + \sum_{n=1}^{2} [a_n\cos(n\theta) + b_n\sin(n\theta)]$$

which can also be written as

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$$S(f,\theta) \;=\; C_{11}(f)\; D(f,\theta)$$

in which

$$C_{11} \;=\; \pi\, a_o$$

and the directional spreading function is given by

$$D(f,\theta)=\frac{1}{\pi}\left(\frac{1}{2}+r_1\cos[\theta-\theta_1]+r_2\cos[2(\theta-\theta_2)]\right)$$

with

$$r_1 \;=\; \frac{1}{a_o}\left(a_1^2 \;+\; b_1^2\right)^{\frac{1}{2}}$$

$$r_2 \;=\; \frac{1}{a_o}\left(a_2^2 \;+\; b_2^2\right)^{\frac{1}{2}}$$

$$\theta_1 \;=\; \tan^{-1}\!\left(\frac{b_1}{a_1}\right)$$

$$\theta_2 \;=\; \frac{1}{2}\,\tan^{-1}\!\left(\frac{b_2}{a_2}\right)$$

The ambiguity of π for θ_2 is resolved by choosing the value that is closest to θ_1. Numerator and denominator signs in the inverse tangent functions determine the quadrants of θ_1 and θ_2. The parameter θ_1 is called mean wave direction, and the parameter θ_2 is called principal wave direction. These directional parameters (r_1, θ_1, r_2, and θ_2) are more commonly considered as analysis results than the parameters (a_1, b_1, a_2, and b_2) originally developed by Longuet-Higgins, et al. (1963). The latter parameters are often used as an intermediate calculation step.

Equations for calculation of the directional-spectrum parameters from buoy wave measurements follow. Calculated directional spectra have units of wave elevation variance/(Hz z- radians). Directional spectra are thus spectral densities in terms of both frequency, Hz, and direction, radians. Co-spectra and quadrature spectra estimates would be corrected for sensor and buoy hull-mooring responses before use of these equations. As seen

when considerations that are specific to NDBC's data processing are described, NDBC combines response corrections, conversion from spectral estimates involving acceleration to those involving displacement, and conversion to the directional parameters (r_1, θ_1, r_2, and θ_2) into a few equations that can be applied without many intermediate steps.

In the following equations, wavenumber, k, is related to frequency, f, by the dispersion relationship for linear waves given by

$$(2\pi f)^2 \;=\; (gk)\tanh(kd)$$

where g is acceleration due to gravity, and d is mean water depth during the measurements. For a given water depth, this equation can be solved for k by iterative methods, or, as later described for NDBC's wave data analysis, k can be estimated from elevation and slope spectra.

For a buoy that measures heave, pitch, roll, and buoy azimuth (heading) the Longuet-Higgins directional parameters are given by

$$a_o \;=\; \frac{C_{11}}{\pi}$$

$$a_1 \;=\; \frac{Q_{12}}{k\pi}$$

$$b_1 \;=\; \frac{Q_{13}}{k\pi}$$

$$a_2 \;=\; \frac{(C_{22} \;-\; C_{33})}{k^2\pi}$$

$$b_2 \;=\; \frac{2\,C_{23}}{k^2\pi}$$

Buoy azimuth is used to convert buoy pitch and roll values into buoy east-west and north-south slopes.

For a buoy that measures nearly vertical acceleration, pitch, roll, and azimuth, the parameters are given by

$$a_o = \frac{C_{11}}{(2\pi f)^4 \pi}$$

$$a_1 = -\frac{Q_{12}}{(2\pi f)^2 k\pi}$$

$$b_1 = -\frac{Q_{13}}{(2\pi f)^2 k\pi}$$

$$a_2 = \frac{(C_{22} - C_{33})}{k^2 \pi}$$

$$b_2 = \frac{2 C_{23}}{k^2 \pi}$$

where the subscript 1 temporarily indicates acceleration which is considered positive upward and negative downward. Signs for the parameters involving quadrature spectra are often written as either positive or negative to consider outputs of particular accelerometers used to make the measurements. For these equations, wave slopes are positive if the slopes tilt upward toward the east (for east-west slopes) or toward the north (for north-south slopes).

The direction convention for these equations is the scientific convention (e.g., Longuet-Higgins, et al., 1963) which is the direction toward which waves travel measured counterclockwise from the x axis (usually east, subscript 2, in NDBC's notation). Theoretically, this is the direction of the wavenumber vector for a particular wave component.

NDBC's direction convention is used by mariners and marine forecasters. It also corresponds to that of the WMO FM65-IX WAVEOB code (World Meteorological Organization, 1988). An NDBC wave direction is the direction from which waves come measured clockwise from true north. If the x axis is positive toward the east, an NDBC wave direction, α, is related to a scientific convention wave direction, θ, by

$$\alpha = \frac{3\pi}{2} - \theta$$

This method for estimating a directional wave spectrum is described mathematically as a directional convolution of a weighting function with the actual directional spectrum. For the utilized $D(f,\theta)$ parameters, the half-power width of the weighting function is $88°$. This width is sometimes called the directional resolution. Longuet-Higgins, et al. (1963) provide a weighting of the directional Fourier coefficients to prevent unrealistic negative values of $D(f,\theta)$ for directions far from θ_1, but this approach is not used by NDBC because it increases the half-power width to $130°$. A directional spreading function that is determined by the described procedure is a smoothed version of the true directional spreading function. Even so, estimated directional spectra are useful. Separate directions of sea and swell can usually be identified since sea and swell often occur at different frequencies.

Higher resolution techniques, most notably Maximum Entropy Methods (MEM) and Maximum Likelihood Methods (MLM), have been developed. These techniques involve the same cross-spectral values and thus could be used. NDBC has investigated versions of these methods (Earle, 1993). However, because they are not as widely used, have several nonstandardized versions, and may provide erroneous directional information unless they are carefully applied, they are not presently used by NDBC except for research purposes. Benoit (1994) summarizes and compares many higher resolution techniques.

3.2.11   Wave Parameters

NDBC calculates several widely used wave parameters. As an example of use of wave parameters, wave climatologies may be based on statistical analysis of wave parameters on a monthly, seasonal, or annual basis. As a second example, engineering calculations of extreme wave conditions may be based on statistical analysis of wave parameters near the times of highest waves during high wave events.

Significant wave height, H_{mo}, is calculated from the wave elevation variance which is also the zero moment, m_o, of a nondirectional wave spectrum using

$$H_{mo} = 4.0 \sqrt{m_o}$$

where m_o is computed from

$$m_o = \sum_{n=1}^{N_b} C_{11}(f_n)\, df_n$$

$$H_{m0}\left(\frac{TDF}{\chi^2\left(TDF, \frac{(1.0-\alpha)}{2}\right)}\right)^{\frac{1}{2}},$$

$$H_{m0}\left(\frac{TDF}{\chi^2\left(TDF, \frac{(1.0+\alpha)}{2}\right)}\right)^{\frac{1}{2}}$$

in which the summation is over all frequency bands (centered at f_n) of the nondirectional spectrum and df_n is the bandwidth of the nth band. NDBC's newest wave measurement system, the WPM, uses frequency bands that are more narrow at low frequencies than at high frequencies.

The theoretical significant wave height, $H_{1/3}$, is the average height of the highest one-third waves in a wave record. An assumption for approximating significant wave height by H_{mo} is that wave spectra are narrow-banded (e.g., Longuet-Higgins, 1952). Although this assumption is not strictly valid for actual waves, considerable wave data representative of different wave conditions show that this method for determining significant wave height is suitable for nearly all purposes. Finite spectral width is the likely explanation for differences between $H_{1/3}$ and H_{mo} with H_{mo} values typically being about 5 to 10 percent greater than $H_{1/3}$ values (Longuet-Higgins, 1980).

NDBC does not calculate significant wave height confidence intervals, but these can be estimated by users based on statistical approaches for estimating time-series variance confidence intervals (e.g., Bendat and Piersol, 1980; Donelan and Pierson, 1983). Confidence intervals depend on the total degrees of freedom, TDF, which in turn depend on spectral width. When a spectrum has been determined, the total degrees of freedom can be estimated from

$$TDF = \frac{2\left(\sum_{n=1}^{N} C_{11}(f_n)\right)^2}{\sum_{n=1}^{N} \left(C_{11}(f_n)\right)^2}$$

The 100 α percent confidence interval for H_{mo} is given by

where x^2 values are obtained from chi-square probability distribution tables. Ninety percent intervals ($\alpha = 0.90$) are generally about -10 to +15 percent below and above calculated H_{mo} values. These confidence intervals are smaller than those for individual spectral density values because the variance is based on information over all frequencies. The following equations provide accurate results for 90 percent ($\alpha = 0.90$) confidence intervals when TDF exceeds thirty which it usually does for waves.

$$\chi^2(TDF, 0.05) \approx TDF\left(1 - \frac{2}{9TDF} + 1.645\left(\frac{2}{9TDF}\right)^{\frac{1}{2}}\right)^3$$

$$\chi^2(TDF, 0.95) \approx TDF\left(1 - \frac{2}{9TDF} - 1.645\left(\frac{2}{9TDF}\right)^{\frac{1}{2}}\right)^3$$

Peak, or dominant, period is the period corresponding to the center frequency of the C_{11} (nondirectional spectrum) spectral frequency band with maximum spectral density. That is, peak period is the reciprocal of the frequency, f_p (peak frequency), for which spectral wave energy density is a maximum. It is representative of the higher waves that occurred during the wave record. Confidence intervals for peak period are not determined and there is no generally accepted method for doing so. Peak period, T_p, is given by

$$T_p = \frac{1}{f_p}$$

Average and zero-crossing periods are used less often than peak period, but provide important information for some applications. Average wave period is not calculated by NDBC, but can be calculated from nondirectional spectra by the following equation

$$T_{av} = \frac{m_o}{m_1}$$

where T_{av} is average period. NDBC calculates zero-crossing wave period from the following equation

$$T_{zero} = \left(\frac{m_o}{m_2} \right)^{\frac{1}{2}}$$

where T_{zero} is zero-crossing period. The spectral moments (m_o, m_1, and m_2) are given by

$$m_i = \sum_{n=1}^{n_b} f_n^{\;i} C_{11}(f_n)\, df_n \qquad i = 0, 1, 2$$

T_{zero} is called zero-crossing period because it closely approximates the time domain mean period which would be obtained from zero-crossing analysis of a wave elevation record. For this reason, NDBC and others sometimes refer to T_{zero} as mean wave period. Average period and zero-crossing period each represent typical wave periods rather than periods of higher waves which are better represented by peak period.

Among those involved in wave data collection and analysis, there are differences of opinion about the best spectral-width and directional-width parameters. Calculation of these parameters is left to more specialized users of NDBC's wave information. Spectral- and directional-width parameters can be calculated from nondirectional and directional spectra respectively.

Many applications of NDBC's data are based on a significant wave height, H_{mo}, peak wave period, T_p, and a representative wave direction for each data record rather than the spectrum. Mean wave direction, θ_1 using NDBC's direction convention, at the spectral peak is most often considered as the best direction to use as a representative direction. It is the mean wave direction of the nondirectional spectrum frequency band with maximum spectral density, C_{11}.

NDBC also calculates a wave direction called peak, or dominant, wave direction. This direction is the direction with the most wave energy in a directional spectrum, $S(f,\alpha)$, where NDBC's direction convention is used. NDBC typically computes $S(f,\alpha)$ at 5° intervals to estimate peak, or dominant, wave direction.

3.2.12   NDBC Buoy Considerations

3.2.12.1   Overview

This section documents aspects of NDBC's wave data analysis that are not simple applications of the provided mathematical background and theory. A key unique aspect for determination of NDBC's nondirectional spectra is use of a Power Transfer Function (PTF) to consider sensor, hull-mooring, and data collection filter responses. For directional spectra, methods for obtaining buoy azimuth, as well as pitch and roll for some wave measurement systems, from measurements of the earth's magnetic field have been developed and applied by NDBC. Directional spectra also require careful determination and application of both the PTF and phase responses associated with each measurement system. Accurate calculation of directional spectra require that frequency-dependent response information be determined for each measurement time period (i.e., set of simultaneous wave records) at each measurement location. To reduce quantities of information relayed to shore by satellite message, while preserving accuracies of relayed information, NDBC developed and uses special encoding-decoding procedures.

3.2.12.2   Nondirectional Wave Spectra

A PTF converts acceleration spectra that are transmitted to shore to displacement spectra and corrects for frequency-dependent responses of the buoy hull and its mooring, the acceleration sensor, and any filters (analog or digital) that were applied to the data. The DACT Directional Wave Analyzer (DWA) now utilizes output from a nearly vertically stabilized accelerometer, but formerly utilized an electronically double-integrated output from this accelerometer. In the latter case, the PTF considered the response of the double integration electronics and did not convert from acceleration to displacement spectra. The PTF is given by

$$PTF = \left(R^{\,hH} R^{\,sH} R^{\,fa} R^{\,fn} \right)^2$$

where

R^{hH} = hull-mooring (h) amplitude response for heave (H)

R^{sH} = sensor (s) amplitude response for heave (H)

R^{fa} = analog antialiasing filter (fa), if used, amplitude response

R^{fn} = digital (numerical) filter (fn), if used, amplitude response

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R^{hH} includes the factor, radian frequency raised to the fourth power, for conversion of acceleration spectra to displacement spectra, or the double integration electronics response. NDBC maintains tables of PTF component values for each wave measurement system and buoy. Steele, et al. (1985, 1992) also discuss obtaining these values.

The equation for calculation of nondirectional wave spectra is given by

$$C_{11} = \left(C_{11}^m - NC\right)/PTF \qquad C_{11}^m \geq NC$$

$$C_{11} = 0 \qquad\qquad\qquad C_{11}^m \leq NC$$

where NC is a frequency-dependent noise correction function. Empirical equations for values of NC were determined originally using low frequencies (e.g., 0.03 Hz) where real wave energy does not occur (Steele and Earle, 1979; Steele, et al., 1985). Use of fixed accelerometers on some wave measurement systems was demonstrated to be the main cause of noise (Earle and Bush, 1982; Earle, et al., 1984) and provided better formulations for empirical noise corrections (Lang, 1987) when such systems are used. For systems with nearly vertically stabilized accelerometers, Wang and Chaffin (1993) developed empirical NC values to correct for instrument-related low-frequency acceleration spectra noise.

3.2.12.3 Directional Wave Spectra

Directional wave spectra involve the additional measurements of buoy azimuth, pitch, and roll. Buoy azimuth (heading of bow axis) is determined onboard a buoy from measurements of the earth's magnetic field along axes aligned in the buoy-fixed bow and starboard directions.

The bow (i=1) and starboard (i=2) components of the magnetic field vector measured by a magnetometer within a buoy hull are given by

$$B_i = b_{i0} + B_{ez}\left[b_{i1}\sin(P) - b_{i2}\cos(P)\sin(R)\right] +$$

$$B_{ey}\left\{\left[-b_{i2}\cos(R)\right]\sin(A) +\right.$$

$$\left.\left[+b_{i1}\cos(P) + b_{i2}\sin(P)\sin(R)\right]\cos(A)\right\}$$

where A, P, and R are, respectively, buoy azimuth, buoy pitch, and buoy roll angles. NDBC's conventions are that azimuth of the buoy bow is positive clockwise from magnetic north, pitch is positive for upward bow motion, and roll is positive

for downward motion of the starboard side of the buoy. B_{ey} and B_{ez} are, respectively, the horizontal and vertical components of the local earth magnetic field. The other constants correct for the residual (b_{10}, b_{20}) and induced (b_{11}, b_{22}, b_{12}, b_{21}) hull magnetic fields and can be determined for a particular hull by proven methods (Steele and Lau, 1986; Remond and Teng, 1990).

The equations for B_i (i=1, 2) are two equations in two unknowns, sin(A) and cos(A). Solutions are given by

$$\sin(A) = \frac{S}{D}$$

$$\cos(A) = \frac{C}{D}$$

in which

$$S = [b_{21}\cos(P) + b_{22}\sin(P)\sin(R)][B_1 - b_{10}] -$$

$$[b_{11}\cos(P) - b_{12}\sin(P)\sin(R)][B_2 - b_{20}] -$$

$$B_{ez}\Delta\sin(R)$$

$$C = [b_{22}(B_1 - b_{10}) - b_{12}(B_2 - b_{20})]$$
$$\cos(R) - B_{ez}\Delta\sin(P)\cos(R)$$

$$D = B_{ey}\Delta\cos(P)\cos(R)$$

$$\Delta = b_{11}b_{22} - b_{12}b_{21}$$

Azimuth relative to magnetic north is obtained from

$$A = \tan^{-1}\left(\frac{\sin(A)}{\cos(A)}\right)$$

A variation (Steele, 1990) of this approach (Steele and Earle, 1991) is used by NDBC for systems first employing the MO method for obtaining buoy azimuth, pitch, and roll. Pitch and roll are obtained from high-frequency parts of measured magnetic field components and used with low-frequency parts of measured magnetic field components to provide azimuth. Using total magnetic field components in the azimuth part of the calculations in theory provides some improvement. Differences

are small because buoy azimuth motions are mainly at low frequencies.

The azimuth quadrant is determined by the signs of sin(A) and cos(A). Magnetic azimuth is converted to true azimuth by use of the magnetic variation angle at the measurement location. Azimuth components relative to true north are obtained from

$$\sin(A_{true}) = \sin(A)\cos(VAR) + \cos(A)\sin(VAR)$$

$$\cos(A_{true}) = \cos(A)\cos(VAR) - \sin(A)\sin(VAR)$$

where A_{true} is bow heading relative to true north, A is bow heading relative to magnetic north, and VAR is the magnetic variation angle.

For each data point, azimuth is used to convert pitch and roll to buoy slopes in east-west and north-south directions, z_x and z_y respectively, relative to true north using

$$z_x = \left(\frac{\sin(A_{true})\sin(P)}{\cos(P)} - \frac{\cos(A_{true})\sin(R)}{\cos(P)\cos(R)} \right)$$

$$z_y = \left(\frac{\cos(A_{true})\sin(P)}{\cos(P)} + \frac{\sin(A_{true})\sin(R)}{\cos(P)\cos(R)} \right)$$

NDBC's directional wave measurement systems obtain buoy pitch and roll in one of two ways. A Datawell Hippy 40 sensor provides direct outputs of pitch (actually sin(P)) and roll (actually sin(R)cos(P)) relative to a nearly vertically stabilized platform within the sensor. NDBC also developed a technique to obtain pitch and roll from measurements of magnetic field components made with a magnetometer (Steele, 1990; Steele and Earle, 1991; Wang, et al., 1994) that is rigidly mounted within a buoy hull. Because the second method utilizes magnetometer data and, indirectly, accelerometer data, and these sensors are on NDBC's directional wave measurement systems anyway, the second method is often referred to as an MO method.

Data show that hull inertia and a wind fin on most buoy superstructures inhibit buoy azimuth motions at wave frequencies. To obtain buoy pitch and roll, time series of B_1 and B_2 are first digitally high-pass filtered with the filter cutoff frequency just below the lowest frequency where nonnegligible wave energy appears in the nondirectional spectrum which is calculated from acceleration data as earlier described. Considering that variations in sin(A) and cos(A) are negligible above the cutoff frequency and making small angle approximations for P and R provides

$$b_{i1}\sin(P) - b_{i2}\sin(R) = \left(\frac{B_i^{/}}{B_{ez}} \right)$$

where $B_i^{/}$ represents the high-pass filtered magnetic field data. Based on pitch and roll data from Datawell Hippy sensors, the small angle approximations are generally realistic. These equations for i=1, 2 are two equations in two unknowns, sin(P) and sin(R). Solutions are given by

$$\sin(P) = \frac{b_{22}B_1^{/} - b_{12}B_2^{/}}{\Delta B_{ez}}$$

$$\sin(R) = \frac{b_{21}B_1^{/} - b_{11}B_2^{/}}{\Delta B_{ez}}$$

As noted by Steele and Earle (1991), these pitch and roll values could differ from true wave-produced pitch and roll values if forces due to winds or currents on the hull or mooring also cause nonnegligible pitch and roll.

NDBC's directional wave measurement systems determine wave direction information from cross-spectra between buoy acceleration (or displacement) and east-west and north-south buoy slopes. Depending on the system, accelerations may be measured by a nearly vertically stabilized accelerometer or a fixed accelerometer with its measurement axis perpendicular to the buoy deck. Cross-spectra are corrected for sensor and buoy hull-mooring effects as described in this section. Further mathematical detail is provided by Steele, et al. (1985, 1992).

Effects of a buoy hull, its mooring, and sensors are considered to shift wave elevation and slope cross-spectra by a frequency-dependent angle, φ, to produce hull-measured cross-spectra. The angle, φ, is the sum of two angles,

$$\varphi = \varphi^{sH} + \varphi^{h}$$

where

φ^{sH} = an acceleration (or displacement) sensor (s) phase angle for heave (H)

and

$$\varphi^h = \varphi^{hH} + \varphi^{hS}$$

in which

φ^{hH} = a hull-mooring (h) heave (H) phase angle
φ^{hS} = a hull-mooring (h) slope (S) phase angle

With a phase shift, φ, measured co-spectra and quadrature spectra are related to true quadrature spectra, Q_{12} and Q_{13}, by

$$Q_{12} = [(R^{sH}/R^h)/PTF]Q_{12}^{/}$$
$$Q_{13} = [(R^{sH}/R^h)/PTF]Q_{13}^{/}$$

where

$$R^h = (R^{hS}/R^{hH})$$

in which

R^{hS} = hull-mooring (h) amplitude response for buoy pitch and roll (same amplitude response as buoy slopes, S)

and

$$Q_{12}^{/} = Q_{12}^{m}\cos(\varphi) + C_{12}^{m}\sin(\varphi)$$
$$Q_{13}^{/} = Q_{13}^{m}\cos(\varphi) + C_{13}^{m}\sin(\varphi)$$

Steele, et al. (1985) use the following relationship from linear wave theory

$$k^2 C_{11} = (C_{22} + C_{33})$$

to show that

$$kR^h/R^{sH} = [(C_{22}^m + C_{33}^m)/C_{11}^m]^{1/2}$$

After considerable algebra, this equation combined with the quadrature spectra corrected for phase shifts and the Longuet-Higgins directional Fourier series formulation for directional spectra yield the following forms for the directional parameters used to describe directional wave spectra (Steele, et al., 1985, 1992).

$$r^1 = \left\{ \left[(Q_{12}^{/})^2 + (Q_{13}^{/})^2 \right] / \left[C_{11}^m (C_{22}^m + C_{33}^m) \right] \right\}^{1/2}$$

$$\alpha_1 = (3\pi/2) - \tan^{-1}(Q_{13}^{/}, Q_{12}^{/})$$

$$r_2 = \left[(C_{22}^m - C_{33}^m)^2 + (2C_{23}^m)^2 \right]^{1/2} / (C_{22}^m + C_{33}^m)$$

$$\alpha_2 = (3\pi/2) - \left[\frac{1}{2}\right] \tan^{-1}[2C_{23}^m, (C_{22}^m - C_{33}^m)] + either\ 0$$

$$or\ \pi,$$

whichever makes the angle between α_1 and α_2 smaller.

Directional spectra are then calculated from

$$S(f,\alpha) = C_{11}(f)D(f,\alpha)$$

where $C_{11}(f)$ is the nondirectional spectrum, $D(f,\alpha)$ is a spreading function, f is frequency, and α is direction, clockwise relative to north, from which waves come. The spreading function can be written as a Fourier series,

$$D(f,\alpha) = [1/2 + r_1\cos(\alpha - \alpha_1) + r_2\cos(\alpha - \alpha_2)]/\pi$$

The dispersion relationship is used to provide wavenumber, k, and the nondirectional spectrum is also estimated from the wave slope spectra by

$$C_{11s} = (C_{22} + C_{33})/k^2$$

This estimate, C_{11s}, is used as an ad hoc DQA tool since it should be similar to C_{11} for parts of spectra with reasonable wave energy levels.

There are other equivalent ways that these calculations can be made and that provide identical results. For example, the Longuet-Higgins directional Fourier coefficients (a_1, b_1, a_2, and b_2) described in the background and theory section could be determined explicitly with consideration of response functions as an intermediate step and used in place of the directional parameters (r_1, α_1, r_2, and α_2). The forms of the equations finally provided by Steele, et al. (1992) for the directional parameters are compact and well-suited for NDBC's operational use.

The angle, φ, is a critical parameter for obtaining proper wave directions. If φ was time-invariant, it could be determined once for each type of wave measurement system. However, as environmental conditions (e.g., winds, waves, and currents) change, NDBC's data show that phase angles change somewhat because of environment-induced

forces and tensions placed on an overall hull-mooring system. Considerably better wave directions are obtained by calculating φ as a function of frequency for each wave record using linear wave theory.

According to linear wave theory, quadrature spectra between wave elevation and wave slopes are non-zero and co-spectra are zero for directions in which waves travel. NDBC's first approach for determining φ used this criteria with weighting based on quadrature spectra involving acceleration (or displacement) and both east-west and north-south slopes to consider that quadrature spectra involving slopes perpendicular to wave directions are small. The theory involves considerable algebra and is fully described by Steele, et al. (1985). This approach generally worked well, but occasionally provided mean wave directions in the wrong compass quadrant.

During 1988-1989, an improved approach for determination of φ was implemented. This approach is based on the fact that quadrature spectra at each frequency have a maximum value for some direction. Steele, et al. (1992) perform a complicated derivation that shows how φ is calculated with this criterion. This approach's advantage is that calculations are based on a direction in which substantial wave energy travels so that φ values are minimally contaminated by low signal to noise ratios. Steele, et al. (1992) show that an ambiguity of π (180°) can be removed through use of an initial estimated φ value that is within π/2 (90°) of the correct value. For each buoy station, NDBC determines initial values as a function of frequency and adjusts the initial values if needed after the first data are acquired and processed. Procedures used since 1988-1989 for obtaining hull-mooring phase angles, φ, are described in Appendix B.

Because sensor responses (R^{sH}, φ^{sH}) are known and do not vary with time, these responses can be used to estimate hull-mooring effects (which implicitly include water depth effects on the mooring), separate from sensor effects. φ^h, which depends only on hull-mooring responses, is given by

$$\varphi^h = \varphi - \varphi^{sH}$$

which determines φ^h after φ is calculated. To separate hull-mooring and sensor effects for R^h, the parameter, q, is defined as

$$q = \tanh(kd) = \omega^2/gk$$

where k is wavenumber, d is mean water depth during a measurement time period, and g is acceleration due to gravity. From small amplitude wave theory, q = 1 in deep water and ranges from 0 to 1. Eliminating k between this equation and the equation for kR^h/R^{sH} yields

$$R^h/q = (gR^{sH}/\omega^2)[(c_{22}^m + C_{33}^m) / (c_{11}^m)]^{1/2}$$

Because q is usually 1 or nearly 1 for NDBC's measurement locations, and R^h is order of magnitude 1 for NDBC's buoys, R^h/q has a convenient magnitude.

Estimating (R^h/q) from data was reasonably successful, but division of apparent noise in the slope co-spectra by small values of ω^2 led to unsatisfactory results at low frequencies. Better estimates are obtained by replacing (C_{22}^m, C_{33}^m) in the previous equation by noise-corrected values for frequencies ≤ 0.18 Hz. These values are obtained by subtraction of an empirical frequency-dependent noise correction function based on (C_{22}^m, C_{33}^m) at 0.03 Hz where there is not real wave energy (Steele, et al., 1992).

Frequency-dependent values of φ^h and R^h/q quantify hull-mooring phase and amplitude responses. As later noted, these parameters are included in information that is relayed to shore by NDBC's newest wave measurement system, the WPM.

3.2.12.4   Spectral Encoding and Decoding

Transmitting results by satellite requires minimizing transmitted record lengths. For systems that transmit spectral estimates, a nonlinear logarithmic encoding-decoding algorithm is used to do this, rather than a linear algorithm, so that small spectral values are accurately obtained in the presence of large values at other frequencies. Logarithmic encoding-decoding is also used for some other satellite-relayed information that may have large ranges of values. Strictly speaking, this encoding-decoding negligibly affects analysis results since values can be decoded with errors of less than ±2 percent and usually less than ±1 percent . The absolute value of each spectral and cross-spectral estimate is normalized to the maximum absolute value (a different value for each type of estimate) and these ratios are encoded. For cross-spectral

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estimates that can be negative, signs are separately transmitted. The encoding equation is given by

$$I_{encoded} = 0 \quad value < value_1$$

$$I_{encoded} = \alpha + \beta \log(value) \quad value_1 \le value < value_2$$

$$I_{encoded} = 2^{nbit} - 1 \quad value \ge value_2$$

in which

$$\alpha = 1 - \beta \log(value_1)$$

$$\beta = \frac{(2^{nbit} - 2)}{\log(value_2 / value_1)}$$

where $I_{encoded}$ is an encoded integer, value is the absolute value to be encoded, nbit is the number of encoding bits, $value_1$ is the minimum value to encode, and $value_2$ is the maximum value to encode. The maximum absolute value itself is relayed either as a floating point number or is logarithmically encoded as described with many bits for high accuracy.

The decoding equation is given by

$$value_{decoded} = value_1 \exp\left(\frac{(I_{encoded} - 0.5)}{\beta} \right)$$

The separately transmitted signs (each requiring only one bit) are applied to decoded values for encoded information that can be negative.

The maximum percentage error due to encoding and decoding is given by

$$\delta_{max}(\%) = 100\left(\left(\frac{value_2}{value_1} \right)^C - 1 \right)$$

where

$$C = \left(2\left(2^{nbit} - 2\right)\right)^{-1}$$

3.3   ANALYSIS OF DATA FROM INDIVIDUAL SYSTEMS

3.3.1   Types of Systems

NDBC applies aspects of the described mathematical background and theory to analyze data from the following wave measurement systems that are used at the time of this report.

(1) GSBP WDA

(2) DACT Wave Analyzer (WA)

(3) DACT DWA

(4) DACT DWA-MO

(5) VEEP WA

(6) WPM

where

GSBP = General Service Buoy Payload
DACT = Data Acquisition and Control Telemetry
VEEP = Value Engineered Environmental Payload

Table 1 summarizes the most important data collection parameters for these systems. Segment length is not applicable to the GSBP WDA because covariance calculations are made for entire data records. Segment length also is not applicable to the WPM because entire data records are Fourier transformed without segmenting. As noted with the table, systems that do not use analog and/or digital filters rely on hull filtering of high-frequency waves to avoid aliasing. This approach works well because NDBC buoys are large compared to wave lengths of high-frequency waves that could cause aliasing. These waves also have negligible energy for most applications of NDBC wave information. A 3-m-diameter discus buoy has become NDBC's standard buoy although data covered by this report have been collected by boat-shaped hulls approximately 6 m in length (NOMAD buoys), 10-m-diameter discus buoys, and 12-m-diameter discus buoys. Different sampling rates have been used. The 1.28-Hz, 1.7066-Hz, and 2.56-Hz rates were selected to place Fourier frequencies precisely on frequency band boundaries for subsequent spectra and cross-spectra calculations. Differences in Table 1 result in negligible differences in analysis results for users of NDBC wave information.

Table 1. Wave Measurement System Data Collection Parameters

SYSTEM	GSBP WDA	DACT WA	DACT DWA	DACT DWA-MO	VEEP WA	WPM
SENSOR(S)	FIXED ACC.	FIXED ACC.	HIPPY 40 AND 3-AXIS MAG.	FIXED ACC. AND 3-AXIS MAG.	FIXED ACC.	HIPPY 40 AND 3-AXIS MAG. OR FIXED ACC. AND 3-AXIS MAG.
A-TO-D BITS	8	12	12	12	4.5 DIGIT DECIMAL	12
RECORD LENGTH	20 MIN (1200 S)	20 MIN (1200 S)	20 MIN (1200 S)	20 MIN (1200 S)	20 MIN (1200 S)	40 MIN (2400 S)
ANALOG FILTER	0.50 HZ	0.50 HZ	NONE	NONE	0.50 HZ	NONE
SAMPLING RATE	1.50 HZ	2.56 HZ	2.00 HZ	2.00 HZ	1.28 HZ	1.7066 HZ
DIGITAL FILTER	NONE	NONE	0.39 HZ, 1 HZ SUBSAMPLING	0.39 HZ, 1 HZ SUBSAMPLING	NONE	NONE
SEGMENT LENGTH	N/A	100 S	100 S	100 S	100 S	N/A

System abbreviations are defined in the text and the following tables.

A-to-D = analog-to-digital conversion

acc. = accelerometer

mag. = magnetometer. Although a 3-axis magnetometer is used, only bow and starboard components of the measured magnetic field are used during data analysis.

Half-power frequency is given for low-pass analog filters.

Hippy 40 = Datawell Hippy 40 that can provide: (1) nearly vertical acceleration, pitch, and roll, or (2) nearly vertical displacement (from electronic double integration of acceleration), pitch, and roll.

Systems that do not use analog and/or digital filters rely on hull filtering of high-frequency waves to avoid aliasing.

3.3.2 GSBP WDA

Data from these nondirectional wave measurement systems were first collected operationally in 1979. Although some systems are still used, these systems are obsolete and are being phased out as newer systems are deployed. Table 2 summarizes data analysis steps in the order that they are performed. Steele, et al. (1976), Magnavox (1979), and Steele and Earle (1979) provide further detail about GSBP WDA wave measurement systems.

For each 20-minute acceleration record consisting of 1800 data points sampled at 1.50 Hz, an acceleration covariance (autocorrelation) function for M lags (75 for normal data collection) is calculated by the onboard system. Covariance values are relayed to shore along with mean, minimum, and maximum acceleration values.

After DQA checks, acceleration spectra are computed onshore by direct summation of covariance values and cosine functions. Spectral estimates are obtained at frequencies, $m\Delta f$, where m ranges from 1 to 50 and $\Delta f = 0.01$ Hz. The time between samples, Δt, is 0.667 s and the Nyquist frequency is 0.75 Hz. Spectral estimates are calculated to 0.50 Hz (m = 50) rather than to 0.75 Hz (m = 75) so that spectra are cutoff at the half-power frequency of the onboard low-pass antialiasing analog filter. Frequency domain Hanning with variance correction is used to reduce spectral leakage. Acceleration spectra are then corrected for electronic noise and low-frequency noise due to use of a hull-fixed accelerometer. Buoy hull-mooring and onboard low-pass analog filter response functions are applied. Acceleration spectra are converted to displacement spectra by division by radian frequency to the fourth power. The number of degrees of freedom (also called equivalent degrees of freedom, EDF, to correspond

Table 2. GSBP WDA Data Analysis Steps

Calculation of mean, minimum, and maximum acceleration for entire 20-min (1200-s) record sampled at 1.5 Hz

Time domain calculation of acceleration covariance (autocorrelation) function for M lags incorporating mean removal. M = 75 (normal data collection), M = 150 (test data collection)

Transmission of acceleration covariances (normally 75), mean, minimum, and maximum values to shore

DQA
  Parity checks
  Complete message checks
  Minimum or maximum acceleration out of range

Direct calculation of acceleration spectra from covariances using cosine functions. Nyquist frequency = 0.75 Hz. Spectra cutoff at 0.50 Hz corresponding to half-power frequency of onboard low-pass analog filter.

Spectral leakage reduction (Hanning) in frequency domain

Correction of acceleration spectra for electronic noise and noise due to use of fixed accelerometer by subtraction of empirical low-frequency noise correction function. Before approximately 1984, low-frequency noise correction described by Steele and Earle (1979) used. After approximately 1984, low-frequency noise correction described by Earle, et al. (1984) used.

Conversion of acceleration spectra to displacement (elevation) spectra including use of frequency-dependent hull-mooring transfer function, correction for onboard low-pass analog filter, and division by radian frequency to the fourth power. Frequency bandwidth = 0.01 Hz, degrees of freedom = 48 for 75 covariances

Calculation of wave parameters from displacement spectra

Key References: Steele, et al. (1976)
                   Steele and Earle (1979)
                   Earle, et al. (1984)
                   Steele and Mettlach (1994)

to terminology when data segmenting is used) is 48 considering that adjacent spectral estimates are not independent as noted in the section describing spectral confidence intervals.

The main difference between GSBP WDAs and later nondirectional wave measurement systems is use of the covariance technique instead of direct Fourier transforms for onboard calculation of spectral estimates.

3.3.3   DACT WA

DACT WA data were first collected operationally in 1984. Table 3 summarizes data analysis steps in the order that they are performed. DACT WAs are nondirectional wave measurement systems. DACT directional wave measurement systems are described in the next section.

Each 20-minute buoy acceleration record consisting of 3072 data points is sampled at 2.56 Hz. Data segmenting with 50 percent overlapping segments is used to decrease spectral estimate uncertainties (i.e., confidence intervals). As acceleration data are acquired, the data are stored in two groups of 256 data points each so that data points in the first half

of the second group are the same as those in the second half of the first group. This approach permits analyzing one 256-data-point group as data to complete the next group are acquired. Twenty-three 256-point records with 50 percent overlap and lengths of 100 s are analyzed. For each segment, the pre-window variance is calculated, minimum and maximums are found, the mean is calculated and removed, Hanning is performed, and the postwindow variance is calculated. Next, an FFT provides frequency domain information from 0.00 to 1.28 Hz (Nyquist frequency) with a Fourier frequency separation of 0.01 Hz, but only spectral estimates for frequencies to 0.40 Hz are calculated. These estimates are corrected for windowing and averaged over the segments. Considering segmenting, spectral estimates have 33 equivalent degrees of freedom. Minimum and maximum accelerations for the 20-minute record are determined from segment minimums and maximums.

Acceleration spectra are normalized to the maximum spectral density (i.e., at the peak frequency), logarithmically encoded, and relayed to shore. Minimum and maximum accelerations are also relayed and used onshore for DQA. The

Table 3. DACT WA Data Analysis Steps

Segmenting of record (as data acquired with analysis after each segment is complete) into 23 50-percent overlapping segments with lengths of 100 s (256 data points sampled at 2.56 Hz)

Calculation of mean, minimum, and maximum acceleration, mean removal for each segment

Spectral leakage reduction (Hanning) in time domain

FFT

Correction for window use and spectral calculations

Averaging of spectra over segments. Nyquist frequency = 1.28 Hz. Spectra cutoff at 0.40 Hz slightly less than half-power frequency, 0.5 Hz, of onboard low-pass analog filter.

Acceleration spectra logarithmic encoding, finding overall minimum and maximum acceleration

Transmission of acceleration spectra, minimum, and maximum values to shore

DQA
        Parity checks
        Complete message checks
        Minimum or maximum acceleration out of range

Correction of acceleration spectra for electronic noise and noise due to use of fixed accelerometer by subtraction of empirical low-frequency noise correction function described by Lang (1987).

Conversion of acceleration spectra to displacement (elevation) spectra including use of frequency-dependent hull-mooring transfer function, correction for onboard low-pass analog filter, and division by radian frequency to the fourth power. Frequency bandwidth = 0.01 Hz, equivalent degrees of freedom considering segmenting = 33

Calculation of wave parameters from displacement spectra

Key References: Magnavox (1986)
                   Lang (1987)
                   Steele and Mettlach (1994)

spectra are corrected for electronic noise and low-frequency noise due to use of a hull-fixed accelerometer. Buoy hull-mooring and onboard low-pass analog filter response functions are applied. Acceleration spectra are converted to displacement spectra by division by radian frequency to the fourth power. Onshore processing is similar to that for GSBP WDA data. DACT WA systems are capable of performing onboard noise and response function corrections as well as conversion of acceleration spectra to displacement spectra if appropriate noise and power transfer functions are pre-stored in the systems, but these systems are not operated in this manner.

3.3.4 DACT DWA

Operational DACT DWA data were first collected in 1986. Table 4 summarizes data analysis steps in the order that they are performed. Steele, et al. (1992) provide detailed descriptions of most aspects of DACT DWA data collection and processing.

Each 20-minute buoy acceleration (or displacement) record consisting of 2400 data points is sampled at 2.00 Hz. East-west and north-south

buoy slopes are determined from buoy azimuth, pitch, and roll. Acceleration (or displacement) data as well as slope data are digitally low-pass filtered and subsampled at 1.00 Hz to provide 1200 data points. Mean, minimum, and maximum values of acceleration (or displacement) and the slopes are calculated and subtracted from each data point of each time series. Twenty-three 100-point records with 50 percent overlap and lengths of 100 s are analyzed. For each segment, the pre-window variance is calculated, the mean is calculated and removed, and Hanning is performed. An FFT then provides frequency domain information from 0.00 to 0.50 Hz (Nyquist frequency for subsampled data) with a Fourier frequency separation of 0.01 Hz. Postwindow variances are calculated from the spectra and corrections for windowing are made. Spectral estimates are averaged over the segments so that there are 33 equivalent degrees of freedom.

Spectra and cross-spectra are normalized to their respective maximum absolute values, logarithmically encoded, and relayed to shore along with the absolute maximum value. Sign information also is relayed for cross-spectra that may have negative values. Maximum encoding-decoding errors are less than ±1 percent. Nondirectional

Table 4. DACT DWA Data Analysis Steps

Calculation of azimuth from pitch, roll, and measured bow and starboard magnetic field components. Acceleration, pitch, and roll provided by Datawell Hippy 40 sensor.

Conversion of pitch and roll to earth-fixed east-west and north-south wave slopes

Digital low-pass filtering of acceleration (or displacement[1]) and slope data available at 2-Hz sampling rate and subsampling at 1 Hz

Calculation of mean, minimum, and maximum acceleration (or displacement[1]) and slope components for entire 20-min (1200-s) record

Segmenting of acceleration (or displacement[1]) and slope component data into 23 50-percent overlapping segments with lengths of 100 s (100 data points)

Mean removal for acceleration (or displacement[1]) and slope components for each segment

Spectral leakage reduction (Hanning) in time domain for acceleration (or displacement[1]) and slope components

FFT

Correction for window use (acceleration or displacement and slope components)

Spectral and cross-spectral calculations

Averaging of spectral estimates over segments. Nyquist frequency = 0.50 Hz. Directional spectra cutoff at 0.35 Hz slightly less than half-power frequency, 0.39 Hz, of onboard low-pass analog filter. Nondirectional spectra cutoff at 0.40 Hz.

Transmission of cross-spectra and other information to shore. Other information includes mean, minimum, and maximum values of buoy acceleration (or displacement[1]), buoy pitch, and buoy roll as well as maximum buoy combined pitch and roll tilt angle, original (first data point) buoy bow azimuth (heading), and maximum clockwise and counterclockwise deviations from original azimuth.

DQA
        Parity checks
        Complete message checks
        Parameter minimums or maximums out of range

Correction of acceleration spectra (usually transmitted rather than displacement spectra) for electronic and sensor-related noise using an empirical noise correction function (Wang and Chaffin, 1993).

Conversion of acceleration spectra to displacement (elevation) spectra (unless displacement originally used) by division by radian frequency to the fourth power, use of frequency-dependent hull-mooring transfer function, correction for onboard low-pass digital filter. Frequency bandwidth = 0.01 Hz, equivalent degrees of freedom considering segmenting = 33

Calculation of directional wave spectra including corrections for hull-mooring amplitude and phase responses that affect directional information. Techniques described by Steele, et al. (1985) followed before 1988-1989. Techniques described by Steele, et al. (1990) and Steele, et al. (1992) followed after 1988-1989.

Calculation of wave parameters from displacement and directional spectra

Key References: Steele, et al. (1985)
                                      Tullock and Lau (1986)
                                        Steele, et al. (1990)
                                        Steele, et al. (1992)
                                        Wang and Chaffin (1993)
                                        Steele and Mettlach (1994)

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[1] The Datawell Hippy 40 onboard sensor can provide either nearly vertically stabilized acceleration or displacement obtained by electronic double integration of acceleration. Either output has been used for particular buoy installations. Buoy pitch and roll are also provided.

spectra (C_{11}^m) are relayed to shore over the frequency range 0.03 Hz to 0.40 Hz. Directional spectra parameters (C_{22}^m, C_{33}^m, C_{23}^m, Q_{12}^m, C_{12}^m, Q_{13}^m, C_{13}^m) are relayed over the frequency range 0.03 Hz to 0.35 Hz. Q_{23}^m, which is theoretically zero, is not determined by DACT DWA systems. Nondirectional acceleration spectra (C_{11}^m) are corrected for electronic noise and sensor-related noise using an empirical correction developed by Wang and Chaffin (1993). Buoy hull-mooring and onboard low-pass analog filter response functions are applied to spectra and cross-spectra through use of the PTF and phase angles described in the theory and background section. Acceleration spectra are converted to displacement spectra by division by radian frequency to the fourth power.

Various information referred to as "housekeeping" information pertains to system setup and onshore DQA. This information includes mean, minimum, and maximum values of buoy acceleration (or displacement), buoy pitch, and buoy roll as well as maximum buoy combined pitch and roll tilt angle, original (first data point) buoy bow azimuth (heading), and maximum clockwise and counter-clockwise deviations from original azimuth.

3.3.5   DACT DWA-MO

Operational DACT DWA-MO data were first collected operationally in 1991. Table 5 summarizes data analysis steps in the order that they are performed.

These systems are essentially identical to DACT DWA systems except that a fixed accelerometer is used in place of a Datawell Hippy 40 sensor and that pitch, roll, and azimuth are all calculated from the magnetometer data. Low-frequency noise corrections described by Lang (1987) are used to correct for effects of using a fixed accelerometer. Steele (1990) and Steele, et al. (1992) provide detailed descriptions of most aspects of DACT DWA-MO data collection and processing.

3.3.6   VEEP WA

Data from these nondirectional wave measurement systems were first collected operationally in 1989. Table 6 summarizes data analysis steps in the order that they are performed.

From a data analysis point of view, these systems are essentially implementations of DACT WA

procedures on VEEP hardware. DACT systems were developed in the early 1980's and VEEP systems were developed in the mid- to late 1980's. The primary difference between DACT WAs and VEEP WAs is that the sampling rates are 2.56 Hz and 1.28 Hz respectively. VEEP WAs thus require less memory for processing data through use of   twenty-three 50-percent overlapping 100-s segments.

3.3.7   WPM

The WPM represents a major increase in NDBC's wave data analysis capabilities. The WPM consists of onboard software and a powerful microprocessor dedicated to processing wave data that passes wave data analysis results to other onboard systems for data relay via satellite. The WPM is being initially used with VEEP systems. However, it is a software-hardware module that can also be used with future systems that control overall buoy functions including satellite data transmission. Earlier onboard software was programmed in languages that were not widely used and that were fairly specific to buoy payload microprocessors. To provide flexible onboard software that can be modified or expanded readily, WPM software was programmed in C language (Earle and Eckard, 1989). The actual code that operates on a buoy contains some nonstandard C aspects because of the utilized C compiler (Chaffin, et al., 1994). WPMs have been successfully field tested, but are not yet operational (Chaffin, et al., 1994). With the flexibility provided by the onboard C code, changes from the following description are likely before WPMs become operational.

WPMs can analyze either nondirectional or directional wave data. Directional wave data analysis is described here since nondirectional analysis is a subset of directional analysis. Nondirectional analysis uses only buoy acceleration data. Table 7 summarizes data analysis steps in the order that they are performed. WPM data analysis aspects are further described by Earle and Eckard (1989), Chaffin, et al. (1992), and Chaffin, et al. (1994). In 1989, the prototype WPM software was called the Wave Record Analyzer (WRA) software.

Data from a magnetometer and either a Datawell Hippy 40 or a fixed accelerometer can be analyzed. If a Hippy 40 sensor is used, it provides buoy nearly vertical acceleration, pitch, and roll. Buoy azimuth is obtained from the magnetometer data.

Table 5. DACT DWA-MO Data Analysis Steps

Calculation of pitch and roll from high-frequency parts of measured bow and starboard magnetic field components

Calculation of azimuth from pitch, roll, and low-frequency parts of measured bow and starboard magnetic field components

Conversion of pitch and roll to earth-fixed east-west and north-south wave slopes

Digital low-pass filtering of acceleration and slope data available at 2-Hz sampling rate and subsampling at 1 Hz. Acceleration provided by fixed accelerometer.

Calculation of mean, minimum, and maximum acceleration and slope components for entire 20-min (1200-s) record

Segmenting of acceleration and slope component data into 23 50-percent overlapping segments with lengths of 100 s (100 data points)

Mean removal for acceleration and slope components for each segment

Spectral leakage reduction (Hanning) in time domain for acceleration and slope components

FFT

Correction for window use (acceleration and slope components)

Spectral and cross-spectral calculations

Averaging of spectral estimates over segments. Nyquist frequency = 0.50 Hz. Directional spectra cutoff at 0.35 Hz slightly less than half-power frequency, 0.39 Hz, of onboard low-pass analog filter. Nondirectional spectra cutoff at 0.40 Hz.

Transmission of cross-spectra and other information to shore. Other information includes mean, minimum, and maximum values of buoy acceleration, buoy pitch, and buoy roll as well as maximum buoy combined pitch and roll tilt angle, original (first data point) buoy bow azimuth (heading), and maximum clockwise and counterclockwise deviations from original azimuth.

DQA
        Parity checks
        Complete message checks
        Parameter minimums or maximums out of range

Correction of acceleration, spectra for electronic noise using a noise threshold following approach of Steele, et al. (1985)

Conversion of acceleration spectra to displacement (elevation) spectra by division by radian frequency to the fourth power, use of frequency-dependent hull-mooring transfer function, correction for onboard low-pass digital filter. Frequency bandwidth = 0.01 Hz, equivalent degrees of freedom considering segmenting = 33

Calculation of directional wave spectra including corrections for hull-mooring amplitude and phase responses that affect directional information. Techniques described by Steele, et al. (1985) followed before 1988-1989. Techniques described by Steele, et al. (1990) and Steele, et al. (1992) followed after 1988-1989.

Calculation of wave parameters from displacement and directional spectra

Key References: Steele (1990)
                                          Steele, et al. (1992)
                                          Steele and Mettlach (1994)

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Table 6. VEEP WA Data Analysis Steps

Segmenting of record (as data acquired with analysis after each segment is complete) into 23 50-percent overlapping segments with lengths of 100 s (128 data points sampled at 1.28 Hz)

Calculation of mean, minimum, and maximum acceleration, mean removal for each segment

Spectral leakage reduction (Hanning) in time domain

FFT

Correction for window use and spectral calculations

Averaging of spectra over segments. Nyquist frequency = 0.64 Hz. Spectra cutoff at 0.40 Hz slightly less than half-power frequency, 0.5 Hz, of onboard low-pass analog filter.

Acceleration spectra logarithmic encoding, finding overall minimum and maximum acceleration

Transmission of acceleration spectra, minimum, and maximum values to shore

DQA
        Parity checks
        Complete message checks
        Minimum or maximum acceleration out of range

Correction of acceleration spectra for electronic noise and noise due to use of fixed accelerometer by subtraction of empirical low-frequency noise correction function described by Lang (1987).

Conversion of acceleration spectra to displacement (elevation) spectra including use of frequency-dependent hull-mooring transfer function, correction for onboard low-pass analog filter, and division by radian frequency to the fourth power. Frequency bandwidth = 0.01 Hz, equivalent degrees of freedom considering segmenting = 33

Calculation of wave parameters from displacement spectra

Key References: Lang (1987)
                                    Chaffin, et al. (1992)
                                      Steele and Mettlach (1994)

If a fixed accelerometer is used, the MO technique is used to obtain buoy azimuth, pitch, and roll. In the applied MO technique (Steele and Earle, 1991), pitch and roll are obtained from high-frequency parts of measured magnetic field components and used with the total measured components to provide azimuth. As for other directional wave measurement systems, east-west and north-south buoy slopes are determined from buoy azimuth, pitch, and roll.

Forty minutes of data (4096 data points) are acquired at a sampling rate of 1.7066 Hz. Acceleration and slope data are Fourier transformed using FFTs for the full data record (4096 data points), the last 20 minutes (2048 data points) of the data record, and the last 10 minutes (1024 data points) of the data record. Spectra and cross-spectra are calculated without use of a leakage reduction window. Results based on longer data records are used for lower frequencies to improve frequency resolution while maintaining a constant 24 equivalent degrees of freedom for each frequency band. Spectral estimates are obtained at resolutions of 0.005 Hz for frequency bands centered between 0.0325 Hz and 0.0925 Hz (based on 4096 data points), 0.01 Hz for frequency bands centered between 0.1000 Hz and 0.3500 Hz (based on 2048 data points), and 0.02 Hz for frequency bands centered between 0.3650 Hz and 0.4850 Hz (based on 2048 data points). Spectra and cross-spectra involving acceleration are converted to spectra and cross-spectra involving displacement at individual Fourier frequencies before band-averaging rather than at the more widely spaced band center frequencies. To reduce the dynamic range of spectral estimates, band-averaged spectra and cross-spectra are temporarily converted (later reverse conversion performed onshore) to acceleration spectra using band center frequencies.

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Table 7. WPM Data Analysis Steps[1]

In MO mode
Calculation of pitch and roll from high-frequency parts of measured bow and starboard magnetic field components

In Hippy 40 mode
Pitch and roll provided by Hippy 40 onboard sensor

Calculation of azimuth from pitch, roll, and measured bow and starboard magnetic field components

Calculation of mean, minimum, and maximum acceleration, pitch, and roll for entire 40-min (2400-s) record sampled at 1.7066 Hz

Conversion of pitch and roll to earth-fixed east-west and north-south wave slopes

FFT of data for 4096, 2048, and 1024 data point sections each ending with the last data point of the original 4096 data point record.

Cross-spectral calculations using longer record lengths for lower frequencies to obtain finer frequency resolution. Nyquist frequency = 0.8533 Hz.

Transmission to shore of frequency-dependent parameters for calculation of nondirectional and directional spectra and other information. Other information includes mean, minimum, and maximum values as well as standard deviations of buoy pitch, buoy roll, buoy combined pitch and roll tilt angle, bow and starboard magnetic field components, buoy bow azimuth (heading), and buoy east-west and north-south slopes. Also relayed are original (first data point) buoy bow azimuth (heading), maximum clockwise and counterclockwise deviations from original azimuth, and mean buoy bow azimuth.

DQA is performed using the same procedures that are used for other NDBC directional wave measurement systems.

In MO mode
Corrections of acceleration spectra for electronic noise and noise due to use of fixed accelerometers by subtraction of empirical low-frequency noise correction function are not made but may be implemented later.

In Hippy 40 mode
Corrections of acceleration spectra for electronic noise and sensor-related noise are not made but may be implemented later.

Conversion of acceleration spectra to displacement (elevation) spectra by division by radian frequency to the fourth power and use of frequency-dependent hull-mooring transfer function. Frequency bandwidth = 0.005 Hz between 0.03250 and 0.0925 Hz, bandwidth = 0.01 Hz between 0.0100 and 0.3500 Hz, bandwidth = 0.02 Hz between 0.3650 and 0.4850 Hz, degrees of freedom = 24 for all frequency bands.

Calculation of directional wave spectra from transmitted parameters including corrections for hull-mooring amplitude and phase responses that affect directional information. Techniques described by Steele, et al. (1992).

Calculation of wave parameters from displacement and directional spectra

Key References:
Earle and Eckard (1989)
Steele, and Earle (1991)
Steele, et al. (1992)
Chaffin, et al. (1992)
Chaffin, et al. (1994)
Steele and Mettlach (1994)

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[1] System is under development (initial field tests completed) and analysis details may change. System can also operate in nondirectional mode if the only sensor is a fixed accelerometer.

Acceleration spectra are normalized to the maximum value, logarithmically encoded, and relayed to shore along with the maximum value. The maximum encoding-decoding error is less than two percent. As presently configured, a WPM calculates directional-spectra parameters (r_1, α_1, r_2, and α_2) onboard. Values of r_1 and r_2 are logarithmically encoded without normalization and relayed. Values of α_1 and α_2 are linearly encoded and relayed. Maximum encoding-decoding errors are less than 1 percent and $1°$ for r values and α values respectively. The response functions φ^h and R^h/q are also calculated onboard for each frequency band. Values of φ^h are linearly encoded and relayed. Values of R^h/q are logarithmically encoded and relayed. Maximum encoding-decoding errors are less than 0.1 percent and $1°$ for R^h/q values and φ^h values values respectively.

Nondirectional spectra (C_{11}^m) and directional spectra parameters (r_1, α_1, r_2, and α_2) relayed by WPMs are part of the WMO WAVEOB code for reporting spectral wave information (World Meteorological Organization, 1988). Following derivations by Steele, it has been shown that the standard cross-spectral parameters between wave displacement and slopes can be derived from the relayed information (Earle, 1993). With this capability and the relayed information (including response information), it is possible to re-perform response calculations and/or recalculate directional spectra onshore by other methods.

WPMs transmit considerable "housekeeping" information that pertains to system setup and onshore DQA. Information includes mean, minimum, and maximum values as well as standard deviations of buoy pitch, buoy roll, buoy combined pitch and roll tilt angle, bow and starboard magnetic field components, buoy bow azimuth (heading), and buoy east-west and north-south slopes. Also relayed are original (first data point) buoy bow azimuth (heading), maximum clockwise and counterclockwise deviations from original azimuth, and mean buoy bow azimuth.

3.3.8   Additional Information

Additional, more detailed, information for NDBC's wave measurement systems than that provided here is not generally needed by NDBC's wave information users. Considerable additional information is in the noted key references, but information in a given reference may not always

apply for times after the referenced document was written. Very detailed information, such as values of specific parameters (e.g., for response corrections, encoding-decoding, etc.) would almost never be needed by users, and would have to be obtained by NDBC inspection of databases and data analysis computer codes that are retained.

4.0   ARCHIVED RESULTS

Results of NDBC's wave data analysis are archived at and made available to users through the National Oceanographic Data Center (NODC), a component of the NOAA. Significant wave height, peak (dominant) wave period, and peak (dominant) wave direction parameters are also archived at NOAA's National Climatic Data Center (NCDC). Information is available in hardcopy, on magnetic tape, and on CD-ROM. NDBC also maintains an archive for its own use.

The most comprehensive information is available from NODC. Two similar formats, type "191" and type "291", have been used. Data that are now being acquired are provided in type "291" format. This section summarizes the wave information that can be provided by NODC, but does not describe details of these formats nor the "housekeeping", meteorological, and other information that is also provided in these formats. Format specification sheets can be obtained from NDBC or NODC.

NDBC wave data are acquired and analyzed hourly. At each measurement location for each hour that data were collected and passed DQA checks, the following information is retained and available through NODC. This information is also provided on a regular basis to other major users such as CERC and the Minerals Management Service (Department of Interior).

- Significant wave height, H_{mo}
- Mean (average) wave period, T_{zero}
- Peak (dominant) wave direction (mean direction, α_1, at spectral peak)
- Peak (dominant) wave period, T_{peak}
- Center frequency of each spectral band
- Bandwidth of each spectral band
- Nondirectional spectral density, C_{11}, for each band
- The following cross-spectra for each band C_{22}, C_{33}, C_{12}, Q_{12}, C_{13}, Q_{13}, C_{23}, Q_{23} (some systems), C_{22}-C_{33}
- The following directional Fourier coefficients for each band — a_1, b_1, a_2, b_2

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- The following directional parameters for each band — r_1, α_1, r_2, α_2
- Nondirectional spectral density, C_{11s}, estimated from wave slope spectra for each band

Although directional wave spectra are not directly available through NODC, directional spectra may be calculated from available frequency-dependent parameters.

Less comprehensive results are provided in almost real time every 3 hours to NWS marine forecasters, NDBC, and others, via NWS communication circuits after DQA and onshore processing by the NWS Telecommunications Gateway (NWSTG). Because of the almost real-time requirement to meet forecasting needs, NWSTG's DQA is not as extensive as that performed by NDBC for information that is permanently archived. Information is distributed by NWSTG in the following two message formats: Automation of Field Operations and Services (AFOS) spectral wave code format and the WMO FM65-IX WAVEOB code for reporting higher resolution spectral wave information (World Meteorological Organization, 1988). An NDBC user's guide for real-time directional wave information (Earle, 1990) describes the real-time information and the AFOS message in detail. NDBC is updating this user's guide to also cover WAVEOB messages.

5.0 SUMMARY

Wave data analysis procedures used by NDBC are documented. Wave data analysis assumptions are noted and appropriate mathematical background and theory is provided. Procedures that can be applied by users of NDBC's wave information to obtain additional useful information (e.g., confidence intervals) are described. This report summarizes NDBC's wave data analysis procedures in one document. Earlier documentation is contained in numerous documents that were prepared as NDBC's wave measurement systems and procedures evolved since the mid-1970's. This report was developed for users of NDBC's wave information, including CERC, but should also be a valuable resource as a general guide for using buoys to measure waves.

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APPENDIX A — DEFINITIONS

Azimuth: Buoys are considered to have two axes which form a plane parallel to the buoy deck. One axis points toward the "bow" and the other axis points toward the starboard (righthand) "side". Azimuth is the horizontal angular deviation of the bow axis measured clockwise from magnetic north (subsequently calculated relative to true north). Azimuth describes the horizontal orientation (rotation) of a buoy.

Confidence intervals: Spectra and wave parameters (e.g., significant wave height) have statistical uncertainties due to the random nature of waves. For a calculated value of a given spectral density or wave parameter, confidence intervals are values less than and greater than the calculated value within which there is a specific estimated probability for the actual value to occur.

Cross Spectral-Density (CSD): The CSD represents the variance of the in-phase components of two time series (co-spectrum) and the variance of the out-of-phase components of two time series (quadrature spectrum). Phase information contained in CSD estimates helps determine wave directional information.

DACT: Data Acquisition and Control Telemetry NDBC onboard system.

Deep-water waves: Waves have different characteristics in different water depths. Water depth classifications are based on the ratio of d/L where d is water depth and L is wavelength. Waves are considered to be deep-water waves when d/L > 1/2.

Directional spreading function: At a given frequency, the directional spreading function provides the distribution of wave elevation variance with direction.

Directional wave spectrum: A directional wave spectrum describes the distribution of wave elevation variance as a function of both wave frequency and wave direction. The distribution is for wave variance even though spectra are often called energy spectra. For a single sinusoidal wave, the variance is 1/2 multiplied by wave amplitude squared and is proportional to wave height squared. Units are those of energy density = amplitude^2 per unit frequency per unit direction.

Units are usually m^2/(Hz-degree) but may be m^2/(Hz-radian). Integration of a directional wave spectrum over all directions provides the corresponding nondirectional wave spectrum.

DWA: Directional Wave Analyzer NDBC onboard system.

GSBP: General Service Buoy Payload NDBC onboard system.

Intermediate (transitional) water depth waves: Waves have different characteristics in different water depths. Water depth classifications are based on the ratio of d/L where d is water depth and L is wavelength. Waves are considered to be intermediate, or transitional, waves when 1/25 < d/L < 1/2.

Leakage: Leakage occurs during spectral analysis because a finite number of analysis frequencies are used while measured time series that are analyzed have contributions over a nearly continuous range of frequencies. Because a superposition of wave components at analysis frequencies is used to represent a time series with actual wave components at other frequencies, variance may appear at incorrect frequencies in a wave spectrum.

Mean wave direction: Mean wave direction is the average wave direction as a function of frequency. It is mathematically described by $\theta_1(f)$ or $\alpha_1(f)$.

Measured time series: A sequence of digitized measured values of a wave parameter is called a measured time series. For computational efficiency reasons, the number of data points is equal to 2 raised to an integer power (e.g., 1024, 2048, or 4096 data points). A measured time series is also called a wave record.

MO Method: An NDBC technique for obtaining buoy pitch, roll, and azimuth from measurements of the two components of the earth's magnetic field parallel to a buoy's deck without use of a pitch/roll sensor.

Nondirectional wave spectrum: A nondirectional wave spectrum provides the distribution of wave elevation variance as a function of wave frequency only. The distribution is for wave variance even

though spectra are often called energy spectra. It can be calculated by integrating a directional wave spectrum over all directions for each frequency. Units are those of spectral density which are amplitude^2/Hz (e.g., m^2/Hz). Also see directional wave spectrum.

Nyquist frequency: The Nyquist frequency is the highest resolvable frequency in a digitized measured time series. Spectral energy at frequencies above the Nyquist frequency appears as spectral energy at lower frequencies. The Nyquist frequency is also called the folding frequency.

Peak (dominant) wave period: Peak, or dominant, wave period is the wave period corresponding to the center frequency of the frequency band with the maximum nondirectional spectral density. Peak wave period is also called the period of maximum wave energy.

Pitch: Buoys are considered to have two axes which form a plane parallel to the buoy deck. One axis points toward the "bow" and the other axis points toward the starboard (righthand) "side". Pitch is the angular deviation of the bow axis from the horizontal (tilt of the bow axis).

Power Spectral Density (PSD): See spectral density.

Principal wave direction: Principal wave direction is similar to mean wave direction, but it is calculated from other directional Fourier series coefficients. It is mathematically described by $\theta_2(f)$ or $\alpha_2(f)$.

Random process: Measured wave characteristics (e.g., wave elevation, pressure, orbital velocities) represent a random process in which their values vary in a nondeterministic manner over a continuous period of time. Thus, descriptions of wave characteristics must involve statistics.

Roll: Buoys are considered to have two axes which form a plane parallel to the buoy deck. One axis points toward the "bow" and the other axis points toward the starboard (righthand) "side". Roll is the angular deviation of the starboard axis from the horizontal (tilt of the starboard axis).

Sea: Sea consists of waves which are observed or measured within the region where the waves are generated by local winds.

Shallow-water waves: Waves have different characteristics in different water depths. Water depth classifications are based on the ratio of d/L where d is water depth and L is wavelength. Waves are considered to be shallow-water waves when d/L < 1/25.

Significant wave height, H_{mo}: Significant wave height, H_{mo}, can be estimated from the variance of a wave elevation record assuming that the nondirectional spectrum is narrow. The variance can be calculated directly from the record or by integration of the spectrum as a function of frequency. Using the latter approach, H_{mo} is given by $4(m_o)^{1/2}$ where m_o is the zero moment of nondirectional spectrum. During analysis, pressure spectra are converted to equivalent sea surface (elevation) spectra so that these calculations can be made. Due to the narrow spectrum assumption, H_{mo} is usually slightly larger than significant wave height, $H_{1/3}$, calculated by zero-crossing analysis.

Significant wave height, $H_{1/3}$: Significant wave height, $H_{1/3}$, is the average crest-to-trough height of the highest one-third waves in a wave record. Historically, $H_{1/3}$ approximately corresponds to wave heights that are visually observed. If a wave record is processed by zero-crossing analysis, $H_{1/3}$ is calculated by actual averaging of the highest-one heights. H_{mo} is generally used in place of $H_{1/3}$ since spectral analysis is more often performed than zero-crossing analysis and numerical wave models provide wave spectra rather than wave records.

Small amplitude linear wave theory: Several aspects of wave data analysis assume that waves satisfy small amplitude linear wave theory. This theory applies when ak, where a is wave amplitude and k is wavenumber (2π/wavelength), is small. Wave amplitudes must also be small compared to water depth. Except for waves in very shallow water and very large waves in deep water, small amplitude linear wave theory is suitable for most practical applications.

Spectral density: For nondirectional wave spectra, spectral density is the wave elevation variance per unit frequency interval. For directional wave spectra, spectral density is the wave elevation variance per unit frequency interval and per unit direction interval. Values of a directional or nondirectional wave spectrum have units of spectral density. Integration of spectral density values over all frequencies (nondirectional spectrum) or over all frequencies and directions (directional spectrum)

provides the total variance of wave elevation. Also see directional wave spectrum and nondirectional wave spectrum.

Spectral Moments: Spectral moments are used for calculations of several wave parameters from wave spectra. The rth moment of a wave elevation spectrum is given by

$$m_r = \sum_{n=1}^{N_b} (f_n)^r C_{11}(f_n) \Delta f_n$$

where Δf_n is the spectrum frequency band width, f_n is frequency, C_{11} is nondirectional-spectral density, and N_b is the number of frequency bands in the spectrum.

Stationary: Waves are considered stationary if actual (true) statistical results describing the waves (e.g., probability distributions, spectra, mean values, etc.) do not change over the time period during which a measured time series is collected. From a statistical viewpoint, this definition is one of strongly stationary. Actual waves are usually not truly stationary even though they are assumed to be so for analysis.

Swell: Swell consists of waves that have travelled out of the region where they were generated by the wind. Swell tends to have longer periods, more narrow spectra, and a more narrow spread of wave directions than sea.

VEEP: Value Engineered Environmental Payload onboard NDBC system.

WA: Wave Analyzer onboard NDBC system.

Wave amplitude: Wave amplitude is the magnitude of the elevation of a wave from mean water level to a wave crest or trough. For a hypothetical sine wave, the crest and trough elevations are equal and wave amplitude is one-half of the crest to trough wave height. For actual waves, especially high waves and waves in shallow water, crests are further above mean water level than troughs are below mean water level.

Wave crest: A wave crest is the highest part of a wave. A wave trough occurs between each wave crest.

Wave energy: For a single wave, wave energy per unit area of the sea surface is $(1/2)\rho g a^2$ where ρ is water density, g is acceleration due to gravity, and a is wave amplitude. The constant factor, ρg, is usually not considered so that energy or wave variance is considered as $(1/2)\, a^2$. Since wave components have different frequencies and directions, wave energy can be determined as a function of frequency and direction.

Wave frequency: Wave frequency is 1/(wave period). Wave period has units of s so that wave frequency has units of Hertz (Hz) which is the same as cycles/s. Radian wave frequency (circular frequency) is $2\pi f$ where f is frequency in Hz.

Wave height: Wave height is the vertical distance between a wave crest and a wave trough. Wave height is approximately twice the wave amplitude. However, for large waves and waves in shallow water, wave crests may be considerably further above mean sea level during the time of the measurements than crests are below mean sea level. When using a spectral analysis approach, wave heights are calculated from spectra using well established wave statistical theory instead of being calculated directly from a measured wave elevation time series.

Wavelength: Wavelength is the distance between corresponding points on a wave profile. It can be the distance between crests, between troughs, or between zero-crossings (mean water level crossings during measurement time period).

Wavenumber: Wavenumber is defined as $2\pi/$wavelength. Wavenumber, k, and wave frequency, f, are related by the dispersion relationship given by

$$(2\pi f)^2 = (gk)\tanh(kd)$$

where g is acceleration due to gravity and d is mean water depth during a wave record.

Wave period: Wave period is the time between corresponding points on a wave profile passing a measurement location. It can be the time between crests, between troughs, or between zero-crossings (mean sea level crossings). The distribution of wave variance as a function of wave frequency (1/period) can be determined from spectral analysis so that tracking of individual wave periods from wave profiles is not necessary when wave spectra are calculated.

Wave trough: A wave trough is the lowest part of a wave. A wave crest occurs between each wave trough.

WDA: Wave Data Analyzer onboard NDBC system.

WPM: Wave Processing Module NDBC onboard system.

Zero-crossing wave period: Zero-crossing wave period is the average of the wave periods that occur in a wave height time-series record where a wave period is defined as the time interval between consecutive crossings in the same direction of mean sea level during a wave measurement time period. It is also statistically the same as dividing the measurement time period by the number of waves. An estimate of zero-crossing wave period can be computed from a nondirectional spectrum. This estimate is statistically the same as averaging all wave periods that occurred in the wave record.

APPENDIX B — HULL-MOORING PHASE ANGLES

Effects of a buoy hull, its mooring, and sensors are considered to shift wave elevation and slope cross-spectra by a frequency-dependent angle, φ, to produce hull-measured cross-spectra. The angle, φ, is the sum of two angles,

$$\varphi = \varphi^{sH} + \varphi^{h}$$

where

φ^{sH} = an acceleration (or displacement) sensor (s) phase angle for heave (H)

and

$$\varphi^{h} = \varphi^{hH} + \varphi^{hS}$$

in which

φ^{hH} = a hull-mooring (h) heave (H) phase angle
φ^{hS} = a hull-mooring (h) slope (S) phase angle

This appendix describes how φ is determined for each frequency from each wave measurement record following Steele, et al. (1992). Described procedures were implemented for NDBC's directional wave measurement systems during 1988-1989. The main text notes references that describe earlier procedures.

The time-varying hull slopes in an earth-fixed frame of reference that has been rotated clockwise by an angle "a" relative to the true (east($z_2(0)$)/north($z_3(0)$)) frame of reference can be written as

$$z_2(a) = z_2(0)\cos(a) - z_3(0)\sin(a)$$

$$z_3(a) = z_2(0)\cos(a) - z_3(0)\sin(a)$$

If both sides of these equations are transformed to the frequency domain, and heave/slope co-spectra and quadrature spectra in the rotated frame of reference are formed, the results are

$$C_{12}^{m}(a) = C_{12}^{m}(0)\cos(a) - C_{13}^{m}(0)\sin(a)$$

$$Q_{12}^{m}(a) = Q_{12}^{m}(0)\cos(a) - Q_{13}^{m}(0)\sin(a)$$

$$C_{13}^{m}(a) = C_{12}^{m}(0)\sin(a) + C_{13}^{m}(0)\cos(a)$$

$$Q_{13}^{m}(a) = Q_{12}^{m}(0)\sin(a) + Q_{13}^{m}(0)\cos(a)$$

in which spectra on the righthand sides are those reported by a buoy. The following definitions

$$\left[Q_{12}^{/}(a)\right]^2 = \left[Q_{12}^{m}(a)\right]^2 + \left[C_{12}^{m}(a)\right]^2$$

$$\left[Q_{13}^{/}(a)\right]^2 = \left[Q_{13}^{m}(a)\right]^2 + \left[C_{13}^{m}(a)\right]^2$$

and the previous four spectra equations provide

$$\left[Q_{12}^{/}(a)\right]^2 = [\tfrac{1}{2}][W - Z\cos(2a - b)]$$

$$\left[Q_{13}^{/}(a)\right]^2 = [\tfrac{1}{2}][W - Z\cos(2a - b)]$$

where

$$W = \left[Q_{13}^{/}(0)\right]^2 + \left[Q_{12}^{/}(0)\right]^2$$

$$z = \left[X^2 + Y^2\right]^{1/2}$$

$$b = \tan^{-1}(Y,X)$$

Here, a comma separating numerator and denominator in the argument of the arc tangent means that the signs of Y and X are considered separately to place the angle b in the correct quadrant. When a slash (/) is used to separate numerator and denominator, the angle lies in the first or fourth quadrant.

$$X = \left[Q_{13}^{/}(0)\right]^2 - \left[Q_{12}^{/}(0)\right]^2$$

and

$$Y = 2\left[C_{12}^{m}(0)C_{13}^{m}(0) + Q_{12}^{m}(0)Q_{13}^{m}(0)\right]$$

$Q_{13}^{/}(a)$ has its largest magnitude(s) at either of two angles, π apart, one of which is given by

$$a_0 = b/2$$

Assuming that the best estimate of φ each hour for each frequency band can be obtained by using the values of $C_{13}^{m}(a)$ and $Q_{13}^{m}(a)$ corresponding to the earth-fixed frame of reference yielding the largest value of $Q_{13}^{/}(a)$, these cross-spectra can be written as

$$C_{12}^{m}(a_0) = C_{12}^{m}(0)\sin(a_0) + C_{13}^{m}(0)\cos(a_0)$$

$$Q_{13}^{m}(a_0) = Q_{12}^{m}(0)\sin(a_0) + Q_{13}^{m}(0)\cos(a_0)$$

If $C_{13}^m(a_0)$ and $Q_{13}^m(a_0)$ are used with

$$C_{13}^{/}(a_0) = 0 = -Q_{13}^m(a_0)\sin(\varphi) + C_{13}^m(a_0)\cos(\varphi)$$

to determine φ, the result is given by

$$\varphi = \tan^{-1}\left[C_{13}^m(a_0) \big/ Q_{13}^m(a_0)\right] + (0 \text{ or } \pi)$$

To remove the π ambiguity, the $\overline{\varphi}$ frame of reference is first rotated by an angle $\varphi(f)$, which is the best available estimate of φ for each frequency. This estimate is determined from hull-mooring models, or from previously measured data that have been supported by hull-mooring models. From $C_{13}^m(a_0)$ and $Q_{13}^m(a_0)$, the new co-spectra and quadrature spectra are given by

$$y = -Q_{13}^m(a_0)\sin\left(\overline{\varphi}\right) + C_{13}^m(a_0)\cos\left(\overline{\varphi}\right)$$

$$x = +Q_{13}^m(a_0)\cos\left(\overline{\varphi}\right) + C_{13}^m(a_0)\sin\left(\overline{\varphi}\right)$$

With these values and if the angle $\overline{\varphi}$ is within $(\pi/2)$ of the correct angle, the correct angle is given by

$$\varphi = \overline{\varphi} + \tan^{-1}(y/x)$$

The ratio (y/x) is computed first, so that φ is forced to lie within $\pm(\pi/2)$ degrees of φ, the best estimate. For each station with a directional wave measurement system, a frequency-dependent table of φ is maintained because of station-to-station differences. The equation for φ is used to estimate φ for each wave record.