Chapter 3

Study of Mathematics-in-Context

Overview

The Study of Mathematics-in-Context (also referred to as the Theme Study) was implemented to address specific reform recommendations regarding mathematics instruction and assessment. These recommendations assert that students benefit from mathematics instruction and assessments that enable them to make connections across mathematics content areas (e.g., Number Sense, Properties, and Operations with Geometry and Spatial Sense, or Measurement with both Geometry and Spatial Sense and Algebra and Functions) and to real-world contexts.[1] These instructional and assessment strategies also reflect reform efforts to include more integrated curriculum and project-based instruction in our mathematics classrooms.

NAEP Administration

Students in grades 4, 8, and 12 who participated in the Theme Study were selected through the same procedures used in selecting students for the national NAEP mathematics assessment (also referred to as the main NAEP assessment).[2] Two 30-minute Theme blocks based on sustained real-life scenarios were constructed for each grade level. The use of a single real-world problem contexts for each Theme block was expected to engage students' interest and sustain motivation throughout the assessment. Additionally, the 30-minute Theme blocks provided an opportunity to explore student responses to questions that were more detailed and complex than questions in the main NAEP assessment, in which all blocks were 15 minutes in length.

Students participating in the Theme Study were each given an assessment booklet containing one of the Theme blocks for their grade level and a 15-minute, grade-appropriate block from the main NAEP assessment. In addition, the Theme Study booklets included blocks of background questions that also were administered to students in the main NAEP assessment.

[1] National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author; Brutlag, D. & Maples, C. (1992). Making connections: Beyond the surface. *Arithmetic Teacher*, *85*(3), 230–235.

[2] For information on sampling for the national NAEP mathematics assessment, see Appendix A and Allen, N. J., Jenkins, F., Kulick, E., & Zelenak, C. A. (1997). *Technical report of the NAEP 1996 state assessment program in mathematics*. Washington, DC: National Center for Education Statistics.

As with the main assessment, teachers of participating fourth- and eighth-grade students and principals from schools in which the fourth-, eighth-, and twelfth-grade students were enrolled also were surveyed. In addition, students were provided with calculators that they could use without restriction, and they were asked to indicate whether or not they had used the calculator when answering each question.

Although all of the questions in the Theme blocks were centered around a single theme, a correct response to one question did not depend on correct responses to other questions in the block, and each question was scored independently. However, some questions did include two or more interdependent parts that were scored as a single unit.

Unlike the Estimation Study or the main NAEP assessment, no achievement scale was developed for the Theme Study. Because the Theme Study was not intended to measure a unique aspect of mathematics achievement, a separate achievement scale was not appropriate. Moreover, the Theme Study booklets did not have enough questions per content strand (subscale) in common with the main assessment to allow for valid linking to the scale used in the main assessment.

Organization of Chapter

The performance data presented in this chapter include block-level average percentage correct scores as well as average percentage correct scores for individual questions: for all students, and by gender and race/ethnicity subgroups where sufficient sample sizes allow. One Theme block from each grade was released for public use, and these released questions are discussed in detail, although student performance on the unreleased blocks also is referenced.

Theme-block questions included mathematics content from more than one content strand and assessed a variety of mathematical abilities as defined by the 1996 NAEP Mathematics Framework.[3] Although assessment developers attempted to provide students with questions from an appropriate range of grade-level difficulty, there was no explicit attempt to make the two Theme blocks at each grade level (or across grade levels) equivalent in terms of overall difficulty level. Therefore, the reader should not make any formal comparisons of performance at different grade levels or across the two Theme blocks at any of the grade levels.

The remainder of this chapter is divided into four sections, one for each grade level and one that summarizes the findings across grade levels. Each grade-level section begins with background information on student demographics and the prevalence of two specific instructional practices thought to be related to skills required for answering the Theme block questions. The instructional practices data are based on teacher responses to the following questions: (1) How often are students asked to write a few sentences about how to solve a mathematics problem? and (2) How often are students asked to write reports or do mathematics projects? It was hypothesized that answering a series of thematically-related mathematics questions might be a more familiar task for students who had carried out sustained mathematics projects, and that students who had experience writing about mathematics might perform better

[3] For information about the NAEP 1996 Mathematics Framework, see National Assessment Governing Board. (1996). *Mathematics framework for the 1996 National Assessment of Educational Progress*. Washington, DC: Author.

on questions asking students to explain how they arrived at their answers. The background and instructional information is followed by a presentation of overall student performance on each Theme block administered to that grade and, finally, by a discussion of each of the questions students encountered in the released Theme block.

Grade 4

Student characteristics

The data in Tables 3.1 and 3.2 show that the fourth-grade students who participated in the Theme Study were similar to the students who participated in the main NAEP assessment on a variety of demographic variables and on the two classroom practice variables. Nearly equal numbers of male and female students took each of the Theme blocks and the main NAEP assessment. The majority of students in each of the samples were White; very small percentages were Asian/Pacific Islander or American Indian. When asked about the highest level of education attained by their parents, students most often indicated that their mothers and fathers had "graduated from college." However, more than a third of the students were unable to provide any information about their parents' education. Nearly 90 percent of students attended public as opposed to nonpublic schools. About one-fourth of the students participated in the Title I program, and about a third were identified as being eligible for the federal Free/Reduced-Price Lunch program.

The data in Table 3.2 show that fewer than 10 percent of the students in each of the samples had teachers who reported that students in their mathematics classes wrote a few sentences about solving a mathematics problem "nearly every day." Teacher responses for the remainder of the students were distributed fairly evenly across the other response categories: "once or twice a week," "once or twice a month," and "never or hardly ever." Teachers of these fourth-grade students reported asking students to write reports and do projects even less frequently than asking them to write a few sentences about solving a mathematics problem. For all samples, fewer than five percent had teachers who reported asking their students to write reports or do projects "once or twice a week" or more, and two-thirds or more had teachers who reported "never or hardly ever" asking their students to engage in such practices. However, the reader should keep in mind that reports and projects are typically long-term activities, and this factor may limit the frequency with which teachers could ask students to do them.

Given these similarities between students in the Theme Study samples and those participating in the main NAEP assessment, it is reasonable to expect that fourth-grade students across the nation would have performed similarly on the Theme blocks to the students who actually participated in the Theme Study.

Table 3.1 ***Student Demographic Distributions by Assessment, Grade 4, 1996***

THE NATION'S REPORT CARD naep

	Percentage of Students		
	Main Assessment	Theme Block 1 The Butterfly Booth	Theme Block 2 Recycling
Grade 4			
Gender			
Males	51	51	53
Females	49	49	47
Students who Indicated Their Race/Ethnicity as...			
White	68	68	69
Black	15	15	14
Hispanic	13	12	13
Asian/Pacific Islander	3	3	3
American Indian	2	2	1
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	4	4	6
Graduated From High School	13	12	13
Some Education After High School	7	7	8
Graduated From College	40	42	38
I Don't Know	36	36	35
Students who Attend...			
Public Schools	89	87	88
Nonpublic Schools	11	13	12
Title I Participation...			
Participated	22	24	23
Did Not Participate	78	76	77
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	31	33	35
Not Eligible	53	55	54
Information Not Available	16	12	11

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 3.2 ***Percentage of Students by Teachers' Reports on Classroom Practices, Grade 4, 1996***

THE NATION'S REPORT CARD

	Percentage of Students		
	Main Assessment	Theme Block 1 The Butterfly Booth	Theme Block 2 Recycling
Grade 4			
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem…			
Nearly Every Day	9	7	6
Once or Twice a Week	26	28	28
Once or Twice a Month	36	35	33
Never or Hardly Ever	29	30	33
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project…			
Nearly Every Day	1	0	0
Once or Twice a Week	4	2	2
Once or Twice a Month	29	27	25
Never or Hardly Ever	66	70	72

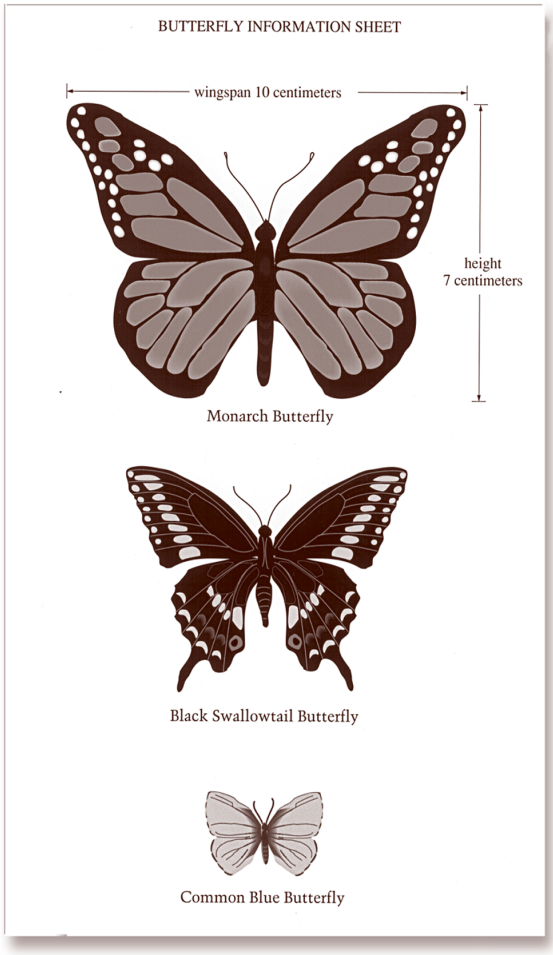
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

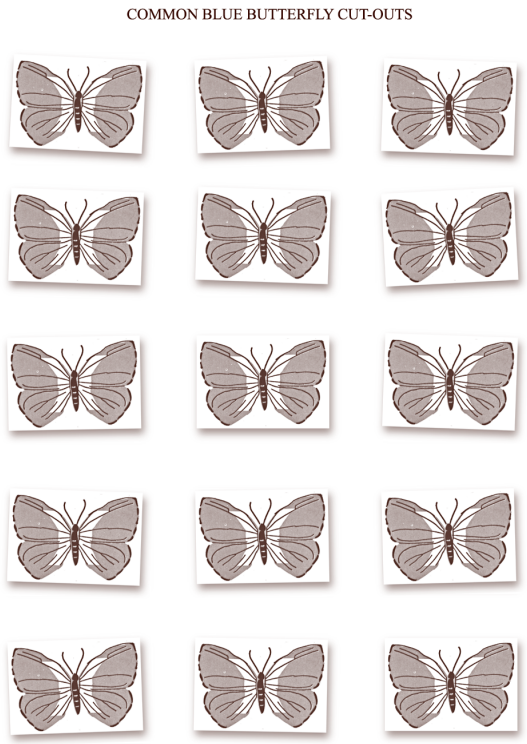
Content of the Theme blocks

The developers of the assessment questions for the two fourth-grade Theme blocks selected real-life contexts that should have been familiar to most, if not all, fourth-grade students in the United States. The released block involved setting up a Butterfly Booth at a school science fair; the unreleased block was about recycling. With each of these blocks, students were provided with a packet of materials to be used in answering some of the questions. The packet that accompanied the Butterfly Booth block contained a Butterfly Information Sheet, which featured beautifully colored illustrations of a Monarch butterfly, a Black Swallowtail butterfly, and a Common Blue butterfly; a 6-inch ruler marked with both inches and centimeters; and 15 cut-outs of the Common Blue butterfly. The Butterfly Information Sheet also included information about the wingspan and height of the Monarch butterfly. Providing students with pictures of butterflies in such vivid, realistic colors may have made the task more interesting for many of the students.

Packet Contents for "The Butterfly Booth"

Not shown to size





RULER

The questions in both blocks were similar in format. The six questions in the Butterfly Booth block were either short or extended constructed-response questions. Of the eight questions in the Recycling block, one was a multiple-choice question and the remainder were short or extended constructed-response questions. In both blocks, there were three or four questions in which students were asked to provide an explanation for how they arrived at their answer.

Overall student performance

Students' overall performance, as measured by average percentage correct scores, is presented in Table 3.3. For both blocks of questions, students' average percentage correct score was 30 percent. Male and female students performed similarly; White, Hispanic, and Asian/Pacific

Islander students outperformed Black students, and White and Asian/Pacific Islander students also outperformed Hispanic students. The sample of American Indian students for both Theme blocks was too small to permit reliable estimates of their performance on the block as a whole or on individual questions. Therefore, the remainder of the discussion will not include information about the performance of American Indian students.

Classroom practices, as measured by the frequency of writing a few sentences about how to solve a mathematics problem, or writing reports or doing mathematics projects, were not found to be related to student performance on either of the Theme blocks.

Table 3.3 ***Average Percentage Correct Scores by Theme Block, Grade 4, 1996***

THE NATION'S REPORT CARD

	The Butterfly Booth	Recycling
Grade 4		
All Students	30	30
Gender		
Males	29	30
Females	31	29
Race/Ethnicity		
White	34	33
Black	17	20
Hispanic	21	22
Asian/Pacific Islander	35	35
American Indian	***	***
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem…		
Nearly Every Day	35	34
Once or Twice a Week	30	32
Once or Twice a Month	29	27
Never or Hardly Ever	32	30
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project…		
Nearly Every Day	***	***
Once or Twice a Week	***	***
Once or Twice a Month	30	30
Never or Hardly Ever	30	30

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Planning a Butterfly Booth

The introduction to the block of questions about planning a Butterfly Booth for a school science fair is shown in Figure 3.1. The instructions included the following information about the expectations for acceptable responses: "In those questions where you must write an answer, it is important that your answer be clear and complete and that you show all of your work since partial credit may be awarded." The instructions also informed students that "[s]ome questions may each require 5 minutes or more to think about and answer." Thus, students were expected to take time to think about how to solve some of these problems. Finally, the manner in which the task of planning the booth was presented provided students the opportunity to take ownership of the task. For example, the introduction says, "Your class is planning to have a Butterfly Booth," and "You need to make decorations for the booth... " The hope was that students would feel that they were no longer just answering questions on another test but solving problems that they might encounter in their own lives at school, making the task of answering the questions more interesting and meaningful.

Figure 3.1 ***Introduction to "Planning a Butterfly Booth" Theme Block, Grade 4, 1996***

THE NATION'S REPORT CARD

This part has 6 questions. Mark your answers in your booklet. You will have to fill in an oval or write your answer as directed. In those questions where you must write an answer, it is important that your answer be clear and complete and that you show all of your work since partial credit may be awarded. Some questions may each require 5 minutes or more to think about and answer. After each question, fill in the oval to indicate whether you used the calculator.

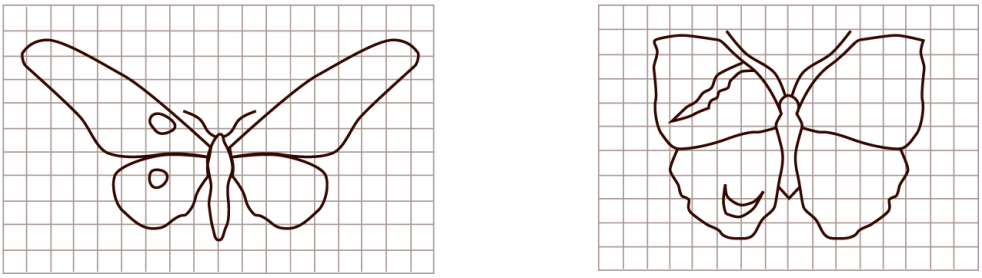
Use the packet you have been given to help you answer the questions in this section.

Each class in Oakville School will have a booth at the Science Fair. Your class is planning to have a Butterfly Booth.

Your class has a lot to do to get ready for the Science Fair. You need to make decorations for the booth, plan activities, and order materials.

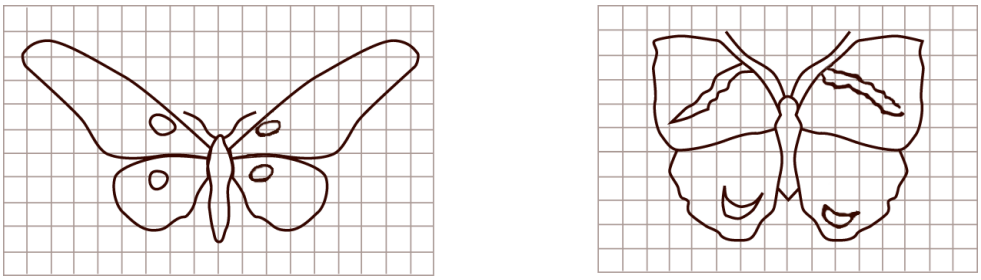
Question 1. Draw symmetrical figure. The first question that students encountered in the block was classified as a Geometry and Spatial Sense question and was designed to assess Problem Solving ability. The question provided students with a context for the task they were asked to complete. That is, students could imagine rendering butterfly drawings that would be used to decorate the booth.

1. The butterfly booth will be decorated with butterfly drawings. Draw only the missing markings on each picture to make each butterfly symmetrical.



To respond successfully to this question, students needed to know what "symmetrical" meant, be familiar with using grids to complete drawings, and understand that the grids were important for getting a correct response. This question was scored with a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect."[4] To be scored "satisfactory," responses had to have all four markings correctly drawn as shown in the sample student response. Students were given full credit if their drawings appeared to show that they understood the concept of symmetry, even if the symmetrical markings were not perfectly drawn or not placed exactly on the grid.

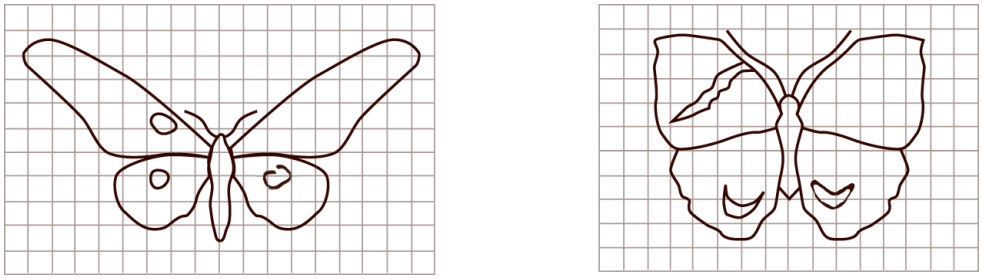
Sample "satisfactory" response



[4] Student responses for this and all other constructed-response questions also were scored as "off task" if the student provided a response that was deemed not related in content to the question asked. There are many examples of this type of response, but a simple one would be "I don't like this test." In contrast, responses scored as "incorrect" were valid attempts to answer the question that simply were wrong.

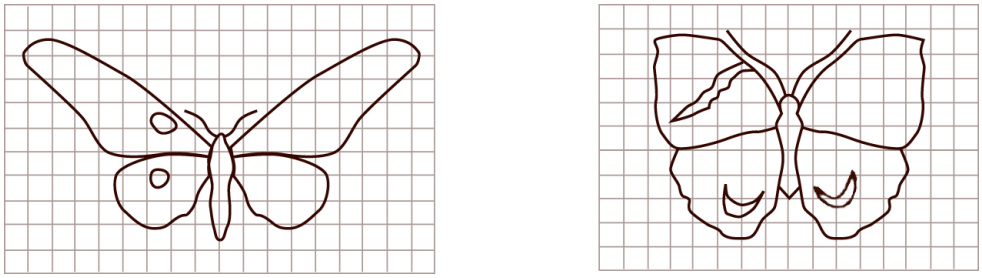
A "partial" response was one in which three of the four markings were correctly drawn; the fourth marking was either omitted or incorrectly drawn. In the "partial" response that follows, the student appears to have understood the concept of symmetry but did not duplicate the fourth marking. The student may have decided that, because this marking had a less uniform shape, it was too difficult to attempt to draw. Another possibility is that the student did not have enough time to finish the task.

Sample "partial" response

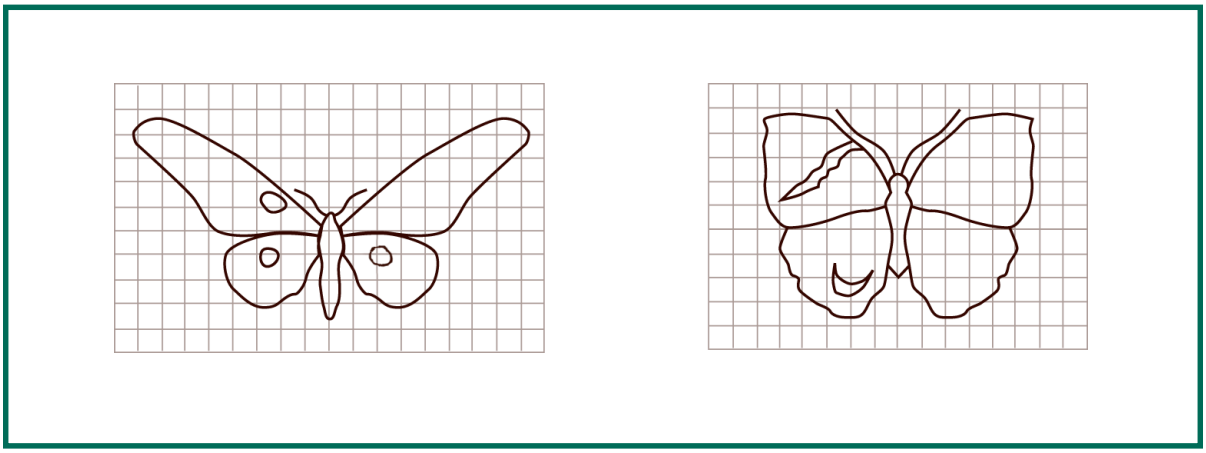


"Minimal" responses included those in which one or two of the markings were drawn correctly, and the remaining markings were either omitted or incorrectly drawn. The two "minimal" responses shown each have one marking correctly drawn. It is not clear why the students chose to draw only one marking. It is possible they did not fully understand the meaning of symmetrical. "Incorrect" responses had none of the parts correctly drawn.

Sample "minimal" response 1



Sample "minimal" response 2



Information on students' performance on this question is presented in Table 3.4. The majority of students were able to draw at least some of the parts correctly; however, only 28 percent of the students drew all four parts correctly. Furthermore, a relatively large proportion of students (33%) did not attempt to draw symmetrical butterflies at all; that is, they skipped over this problem entirely. It is not clear why so many students omitted this question; possibly they did not know the word "symmetrical" or did not know how to use the grid paper.

Table 3.4 ***Score Percentages for "Draw Symmetrical Figure," Grade 4***

THE NATION'S REPORT CARD

	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 4					
All Students	28	17	11	11	33
Males	30	14	8	11	36
Females	27	20	13	10	29
White	34	19	10	10	26
Black	8	14	13	15	50
Hispanic	18	12	11	10	48
Asian/Pacific Islander	38	21	8	6	27
American Indian	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 2. Measure length using ruler. In the second question, students were asked to use the ruler that was provided in their packets to measure the wingspans of the Black Swallowtail and the Common Blue butterflies and report their measurements to the nearest centimeter. This question was classified as a Measurement question and was designed to assess Procedural Knowledge. Students needed to know how to measure using the ruler provided, what centimeters are, and what it means to measure to the nearest centimeter. Students also needed to know what the term "wingspan" means or be able to discern its meaning from the indication of the Monarch butterfly's wingspan on the Butterfly Information Sheet. Furthermore, although centimeters are clearly indicated on the ruler provided, knowledge of and experience in using centimeter measurements may have been helpful.

2. Take the Butterfly Information Sheet from your packet.

On the Butterfly Information Sheet the wingspan of the Monarch butterfly is shown.

Use your ruler to measure the wingspans of the other two butterflies on the sheet, the Black Swallowtail butterfly and the Common Blue butterfly, to the nearest centimeter.

Black Swallowtail	Wingspan: _____ centimeters
Common Blue	Wingspan: _____ centimeters

The correct answer to the question is seven centimeters for the wingspan of the Black Swallowtail butterfly and three centimeters for the wingspan of the Common Blue butterfly. Students' responses were scored on a 5-point rubric: "extended," "satisfactory," "partial," "minimal," and "incorrect." An "extended" response was one in which both measurements were correct. Responses were scored "satisfactory" if one of the measurements was correct, if the two measurements were correct but reversed on the answer sheet, or if students did not round to the nearest centimeter but provided answers that would round to seven (6.5 to 7.5) and three (2.5 to 3.5). "Partial" responses were those in which students appeared to have measured the heights of the butterflies rather than their wingspans; that is, "partial" responses included answers for both measurements, with the wingspan ranging between 5 and 5.5 centimeters for the Black Swallowtail and between 2 and 2.5 centimeters for the Common Blue butterfly. Responses were scored "minimal" if they were in the correct range for inches, not centimeters. That is, if the measurements were 2.5 to 3 inches for the Black Swallowtail and 1 to 1.5 inches for the Common Blue. Responses that met none of the criteria already mentioned were scored as "incorrect."

Information on response scores for this question is presented in Table 3.5. Forty percent of fourth-grade students were able to provide the correct measurements for the wingspans of both butterflies. From student responses, it appears that many fourth-graders are familiar with metric measurements such as centimeters or are at least able to translate what they know about

using a ruler with inch markings into using a ruler with centimeter markings. Twenty-six percent of the responses were scored as "satisfactory" or "partial," suggesting that these students may not have been careful in their measurements or in their reading of the question, may not have understood the term "wingspan," or may not have understood what it means to measure to "the nearest centimeter."

Perhaps as evidence that fourth-grade students were more familiar with the task involved in this question compared to the first question, a much higher percentage of students chose to respond to this question (96%) than chose to respond to the first question (67%). Furthermore, the percentage of "extended" responses (40%) for this question was higher than the percentage of "satisfactory" responses (28%) for Question 1.

Table 3.5

Score Percentages for "Measure Length Using Ruler," Grade 4

THE NATION'S REPORT CARD

Grade 4	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	40	14	12	3	27	4
Males	41	12	12	2	28	5
Females	40	16	12	4	26	3
White	47	14	12	3	23	2
Black	20	18	11	3	41	8
Hispanic	26	13	15	3	31	12
Asian/Pacific Islander	54	7	5	2	27	5
American Indian	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 3. Solve packing problem. Students were asked to use the 15 rectangular cut-outs of the Common Blue butterflies that were provided in their packets to answer this two-part question. The question included a two-dimensional representation of a storage case, drawn to size. Part 1 of the question asked students to determine the greatest number of Common Blue butterflies (of the size of the cut-outs) that could be stored in the case and to show how those butterflies would be arranged in the case; butterflies could not overlap each other or be stacked on top of each other. In the second part of the question, students were asked to determine how many storage cases of the same size they would need to store 28 Common Blue butterflies. Students also were asked to explain with drawings, words, or numbers how they arrived at their answers. Question 3 was classified as a Measurement question and was designed to measure Problem-Solving ability.

In responding to this question, students were not required to do any actual measuring with a ruler. To determine the number of butterflies that would fit, they simply needed to place the Common Blue butterfly cut-outs onto the picture of the storage case. To indicate how the butterflies would lie in the case, students could trace the outline of the cut-outs or they could draw butterflies of the approximate size of the Common Blue cut-outs. Having answered Part 1, there were numerous strategies students could use to solve Part 2. Whatever strategy students used, they were required to describe it in order to answer the question completely.

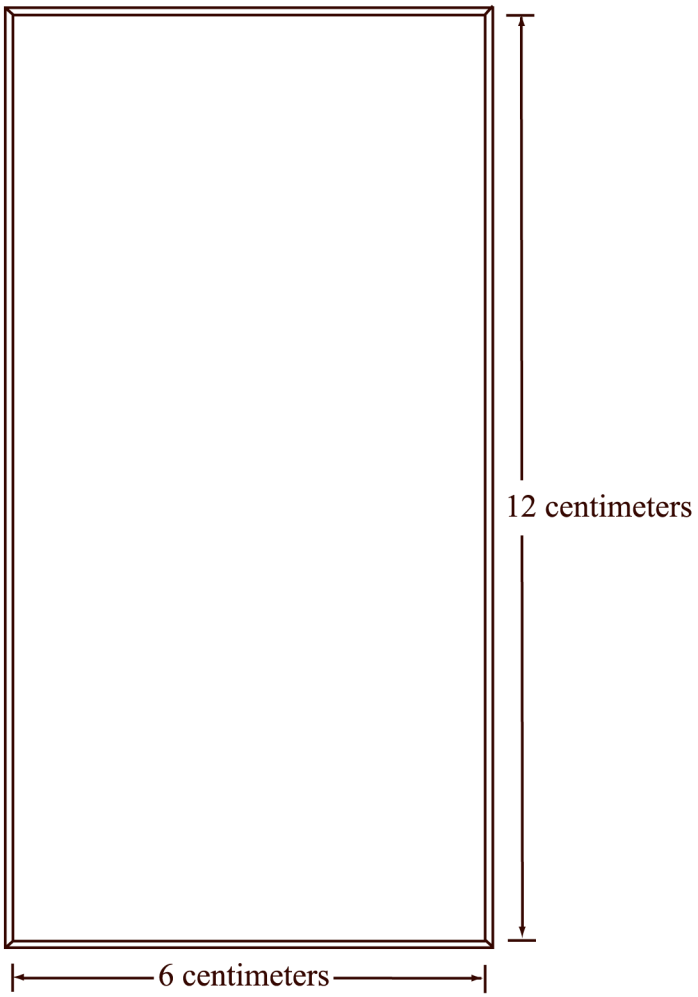
3. Take the butterfly cutouts from your packet.

What is the greatest number of Common Blue butterflies that can be stored in the case below? (When you put butterflies in the case, you can't stack them. The butterflies can touch, but they can't overlap at all.)

Answer: _____

Show how the butterflies fit in the case.

Storage Case



How many storage cases would you need to store 28 Common Blue butterflies?

Answer: _____

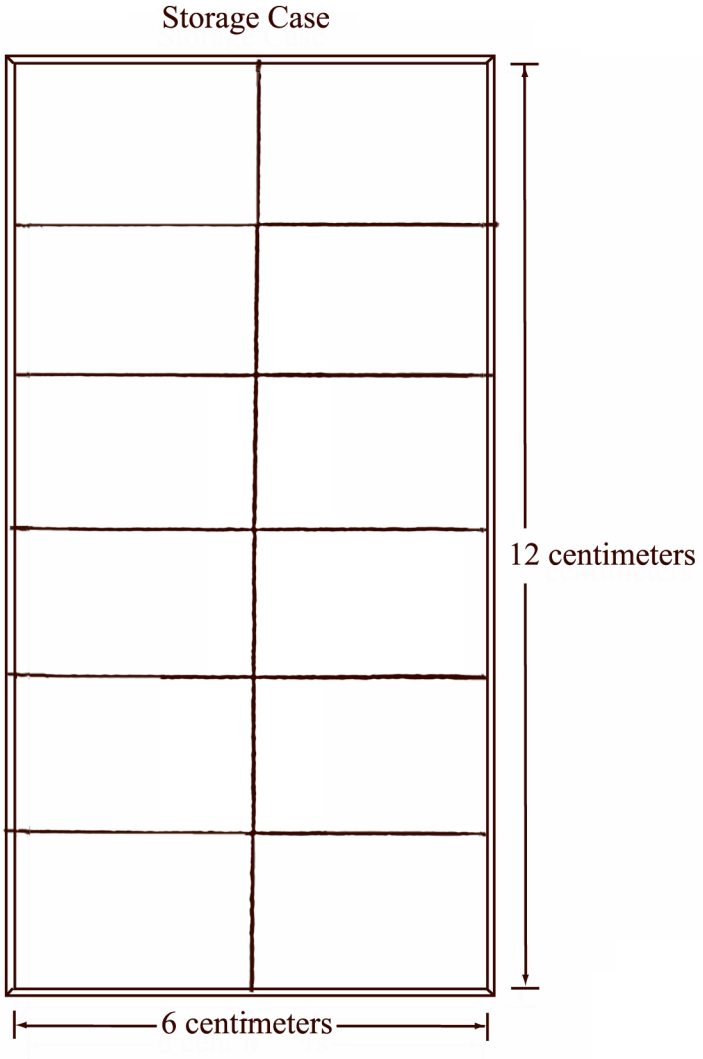
Use drawings, words, or numbers to explain how you got your answer.

This question was scored on a 5-point rubric: "extended," "satisfactory," "partial," "minimal," and "incorrect." Responses scored as "extended" provided the answer 12 for the number of butterflies that would fit in the case, with a drawing of how the 12 would lie in the case, and the answer 3 for the number of cases needed to accommodate 28 butterflies, with an appropriate explanation for arriving at 3 cases. In the sample "extended" response that follows, the student subdivided the picture of the storage case into 12 appropriately sized rectangles to show the placement of the 12 non-overlapping butterflies. For the second part of the question, the student showed, through addition, how she or he arrived at the answer of 3 cases needed to accommodate the 28 butterflies.

Sample "extended" response

Answer: 12 butterflies

Show how the butterflies fit in the case.

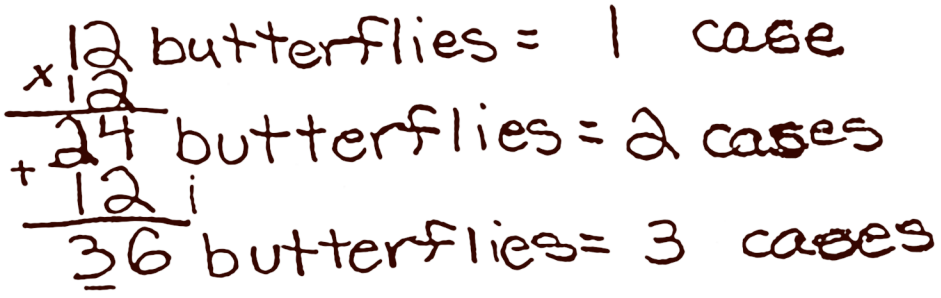


Sample "extended" response (continued)

How many storage cases would you need to store 28 Common Blue butterflies?

Answer: 3

Use drawings, words, or numbers to explain how you got your answer.



A response could be scored as "satisfactory" if it met any of the following criteria:

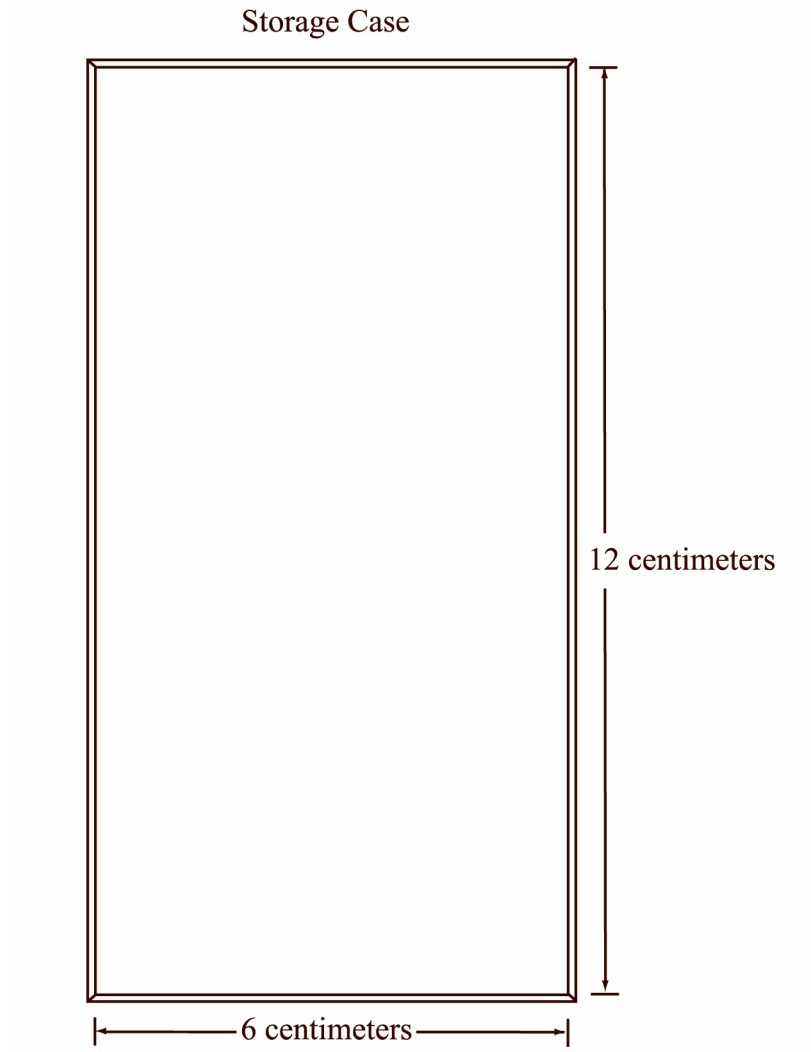
- Part 1, 12 butterflies with a correct drawing; and Part 2, an incorrect answer but a correct explanation;
- Part 1, 12 butterflies with a correct drawing; and Part 2, a correct answer but an inadequate or missing explanation;
- Part 1, 12 butterflies with drawing missing; and Part 2, a correct answer with a correct explanation; or
- Part 1, 10 or 11 butterflies with a correct drawing; and Part 2, a correct answer with a correct explanation.

The following sample "satisfactory" response includes the correct answers for Parts 1 and 2 and an acceptable explanation for Part 2, but the drawing is missing for Part 1. It is not evident why the student did not complete the drawing. It is possible that she or he simply did not read the directions carefully and therefore did not realize that it was necessary to show how the 12 butterflies would fit in the case.

Sample "satisfactory" response

Answer: 12 butterflies

Show how the butterflies fit in the case.



Sample "satisfactory" response (continued)

How many storage cases would you need to store 28 Common Blue butterflies?

Answer: 3

Use drawings, words, or numbers to explain how you got your answer.

I new 12 could fit in a case. Then I multiplied 12 times and got 24. Then I new I would have to get another case because I couldnt any more in that case

A "partial" response would meet one of the following criteria:

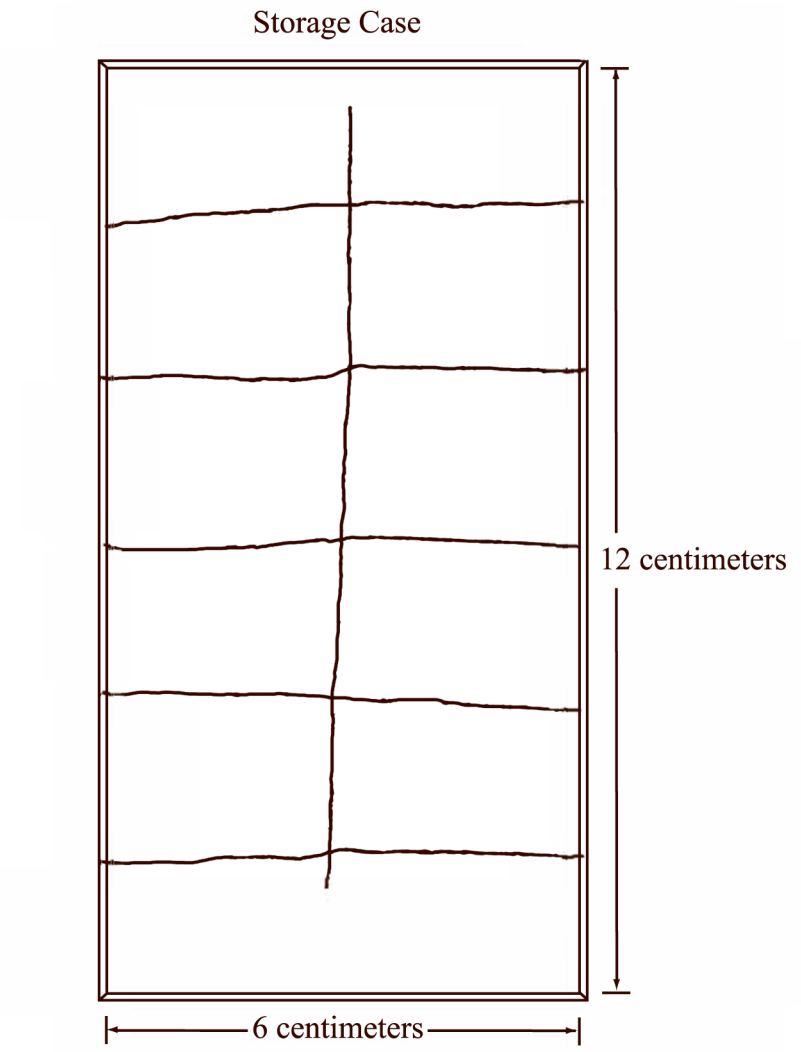
- Part 1, 10 or 11 butterflies with or without a drawing; and Part 2, an incorrect answer with a correct explanation of the strategy used to determine that number;
- Part 1, 10 or 11 butterflies without a drawing; and Part 2, a correct answer with an acceptable explanation;
- Part 1, 10 or 11 butterflies with a correct drawing; and Part 2, a correct answer but a missing or inadequate explanation;
- Part 1, 12 butterflies without a drawing; and Part 2, an incorrect answer but an acceptable explanation; or
- Part 1, 12 butterflies without a drawing; and Part 2, a correct answer but an inadequate or missing explanation.

The sample "partial" response that follows indicated that 10 butterflies would fit in the case and provided a correct drawing of the 10 in the case. For Part 2, the student offered the correct answer of 3 cases but an inappropriate explanation of the solution process. The explanation in Part 2 is actually an explanation of deriving the answer in Part 1.

Sample "partial" response

Answer: 10

Show how the butterflies fit in the case.



How many storage cases would you need to store 28 Common Blue butterflies?

Answer: 3

Use drawings, words, or numbers to explain how you got your answer.

I mesured the Blue Butterfly's wing span and hight then I took my ruler Put it on the box and mesured 5 dow and 2 across and I multiplied and got the answer 10

The different responses that could be scored as "minimal" include the following:

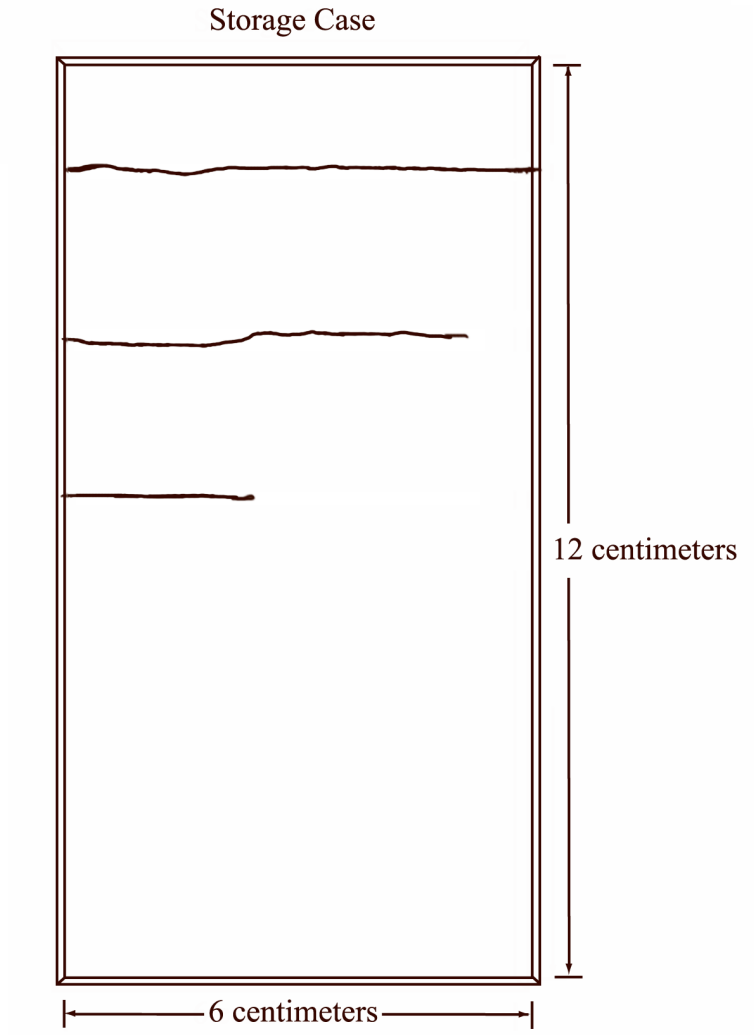
- Part 1, 10 or 11 butterflies with or without a correct drawing; and Part 2, an incorrect number of cases and explanation;
- Part 1, 10 or 11 butterflies with an incorrect drawing; and Part 2, a correct answer but an explanation that is missing or inadequate; or
- Part 1, 12 butterflies with or without a correct drawing; and Part 2, an incorrect answer and explanation.

The sample "minimal" response that follows has an acceptable number of 10 butterflies but an incomplete drawing for Part 1 and the correct number of 3 cases but an inadequate explanation for Part 2.

Sample "minimal" response

Answer: 10

Show how the butterflies fit in the case.



How many storage cases would you need to store 28 Common Blue butterflies?

Answer: 3

Use drawings, words, or numbers to explain how you got your answer.

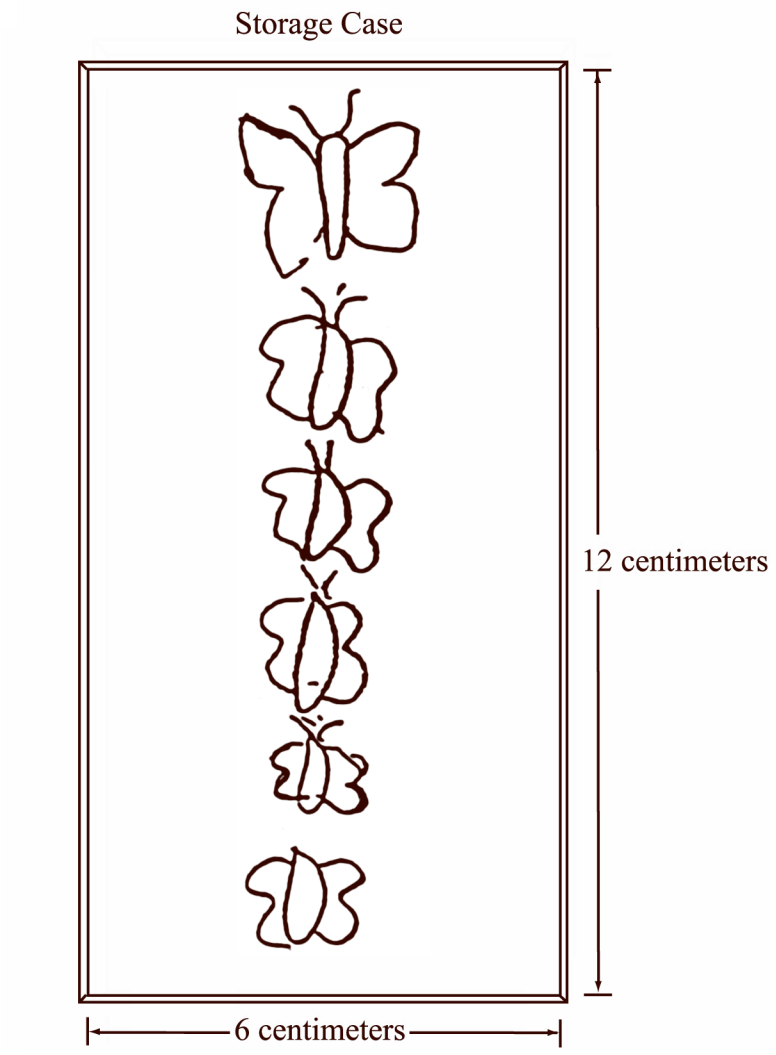
I put the Butterflys in the storage case.

An "incorrect" response is one that meets none of the criteria previously mentioned. In particular, it provides no answer for the number of butterflies or an answer other than 10, 11, or 12. The following "incorrect" response indicates that the greatest number of butterflies that would fit in the case is 6, and that 34 storage cases would be necessary to fit 28 butterflies. Although the answer 6 is incorrect, the drawing for Part 1 shows some understanding of the task of explaining how the butterflies would lie in the case. The answers to the second part of the question appear to indicate that the student did not understand at all what the question was asking.

Sample "incorrect" response

Answer: 6

Show how the butterflies fit in the case.



Sample "incorrect" response (continued)

How many storage cases would you need to store 28 Common Blue butterflies?

Answer: 34

Use drawings, words, or numbers to explain how you got your answer.

IF There are 6 butterflies in one storage case add ~~6~~ and 28 to get 34

Student performance information for this question is presented in Table 3.6. Students appear to have found this question difficult compared to the previous measurement question. Only a third of the responses were scored "partial" or higher, about a third were scored "minimal," and just under a third of the responses were "incorrect." Despite the apparent difficulty students encountered with this question, only one percent of the students chose not to attempt to answer it. It is perhaps both the multistep feature and the need to provide an explanation of the process of getting to an answer that accounted for much of the difficulty students had with this question.

Table 3.6 ***Score Percentages for "Solve Packing Problems," Grade 4***

THE NATION'S REPORT CARD

	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 4						
All Students	4	13	16	35	30	1
Males	4	12	14	36	34	1
Females	5	14	18	36	27	0
White	6	16	18	34	25	1
Black	2	4	8	34	50	2
Hispanic	1	6	12	42	39	0!
Asian/Pacific Islander	2	18	20	36	22	2!
American Indian	***	***	***	***	***	***

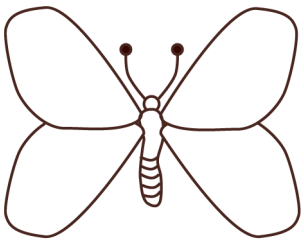
NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 4. Determine number of models. The fourth question explained that students who visit the booth would be building models of butterflies. Students were told how many of each part of the butterfly — 4 wings, 1 body, and 2 antennae — were needed to build one butterfly. They were asked to determine how many complete butterflies could be built with the supply of 29 wings, 8 bodies, and 13 antennae that the class had. Students also were asked to explain how they arrived at their answer using drawings, words, or numbers. This question was designed to assess Problem-Solving ability and content from the area of Number Sense, Properties, and Operations.

To answer this question correctly, students needed to understand that, after building all possible complete butterflies, they might have remaining wings, bodies, and/or antennae. Their task, actually, was to determine for each part how many models of butterflies they could accommodate. That is, they had enough wings for 7 butterflies, enough bodies for 8 butterflies, and enough antennae for 6 butterflies. So, students had to know that the answer they were seeking was the smallest of those three numbers, namely, 6.

4. The children who visit your booth are going to build models of butterflies. For each model, they will need the following:

When the model is put together it looks like this:



If the class has a supply of 29 wings, 8 bodies, and 13 antennae, how many complete butterfly models can be made?

Answer: _____

Use drawings, words, or numbers to explain how you got your answer.

This question was scored using a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect." "Satisfactory" responses had the correct number of butterflies, 6, and a correct explanation for how that number was determined. The sample "satisfactory" response shown had a correct answer and a complete explanation.

Sample "satisfactory" response

Answer: 6

Use drawings, words, or numbers to explain how you got your answer.

4 wings for each model.
4)29 with one leftover, There
is eight bodies so you'll have
1 left over but there is only
13 antennae, so you can only make
6

Responses scored as "partial" either had the correct answer but an incomplete or erroneous explanation or had a correct explanation with the six-and-a-half pairs of antennae rounded to 7 butterfly models. The "partial" answer that follows had the correct answer 6 but is lacking an explanation.

Sample "partial" response

Answer: 6

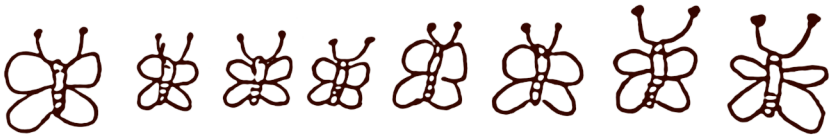
Use drawings, words, or numbers to explain how you got your answer.

Responses scored as "minimal" had an incorrect number of butterfly models but provided some evidence that the student understood that the number of parts (wings, bodies, and antennae) available determined the number of complete butterfly models that could be built. The following sample "minimal" response has the wrong answer, 8, but the drawing of the butterflies shows an understanding of how the different parts are needed to build a complete butterfly model.

Sample "minimal" response

Answer: 8

Use drawings, words, or numbers to explain how you got your answer.

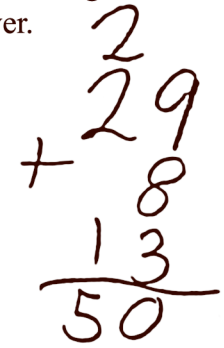


An "incorrect" response showed no understanding of how to solve the problem. For example, the following "incorrect" response shows that the student simply added the various available butterfly parts and came up with the sum, 50, which is the wrong number of complete models possible.

Sample "incorrect" response

Answer: 50

Use drawings, words, or numbers to explain how you got your answer.



Student performance information on this question is presented in Table 3.7. Eighteen percent of the students were able to provide a response that received at least a "partial" score. Such responses appear to indicate that the student had some idea of how to solve the problem correctly. However, the majority of responses were scored as "incorrect," indicating that many students found this question very difficult.

Table 3.7 ***Score Percentages for "Determine Number of Models," Grade 4***

THE NATION'S REPORT CARD

	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 4					
All Students	3	15	18	61	2
Males	2	15	15	64	3
Females	3	15	21	59	2
White	3	19	21	55	2
Black	1	5	9	81	3
Hispanic	1	8	11	75	3
Asian/Pacific Islander	6	22	17	51	4
American Indian	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 5. Determine number of leaves. The fifth question used the context of the Butterfly Booth indirectly; that is, the question is about feeding caterpillars. Students could have answered this question correctly even if they did not know that butterflies are transformed caterpillars, but having that knowledge might have made the problem more interesting. In the question, students were told that a class has two caterpillars and needs five leaves a day to feed them. Students were asked to determine how many leaves the class would need each day if it had 12 caterpillars to feed. This question was classified as a Number Sense, Properties, and Operations question and was designed to assess Problem-Solving ability. As with other questions, in addition to specifying the number of leaves needed, students were asked to explain how they determined their answer.

There were a number of strategies that students might have used to answer this question. For example, students may have reasoned that 12 is 6 groups of 2, so 6 times 5 leaves, or 30 leaves are needed.

5. A fourth-grade class needs 5 leaves each day to feed its 2 caterpillars. How many leaves would they need each day for 12 caterpillars?

Answer:_____

Use drawings, words, or numbers to show how you got your answer.

This question was scored on a 3-point rubric: "complete," "partial," and "incorrect." Responses were scored "complete" if they had the right number of leaves, 30, and a correct explanation. The "complete" response that follows has the correct answer and an adequate computational explanation. Through the computations, the student appears to show an understanding that the number of caterpillars and the number of leaves needed to be multiplied by the same number, 6.

Sample "complete" response

Answer: 30 leaves

Use drawings, words, or numbers to show how you got your answer.

$\begin{array}{r} 2 \\ \times 6 \\ \hline 12 \end{array}$ $\begin{array}{r} 6 \\ \times 5 \\ \hline 30 \end{array}$

"Partial" responses either had the correct number of leaves with an incomplete, erroneous, or no explanation or a correct explanation with a wrong number of leaves because of a computational error. In the sample response below, the student has the correct number of leaves, 30, but the explanation is incomplete. The student started with an acceptable process of determining the number of leaves one caterpillar needs a day, that is, two-and-a-half, but then jumped to the conclusion of needing 30 leaves without explaining that 30 leaves is the answer to two-and-a-half leaves per caterpillar multiplied by 12 caterpillars.

Sample "partial" response

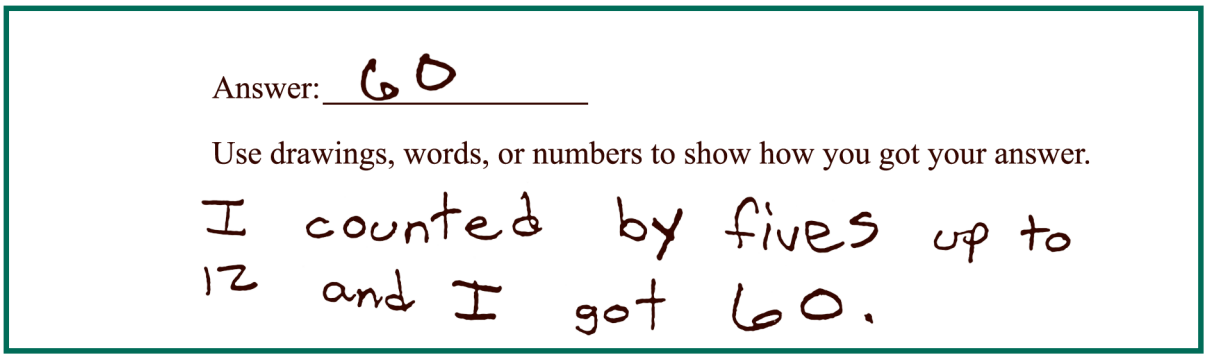
Answer: 30

Use drawings, words, or numbers to show how you got your answer.

If each caterpillar gets 2½ leaves you need 30 leaves

"Incorrect" responses had the wrong number of leaves and either an erroneous or no explanation. In the sample "incorrect" response shown below, the student forgot that the five leaves were for two caterpillars instead of one. The process explained was an acceptable process. It would have resulted in the correct answer had the student remembered that the five leaves were for a pair of caterpillars and therefore only counted by fives 6 times rather than 12.

Sample "incorrect" response



Student performance information on this question is presented in Table 3.8. It proved to be a very difficult question for students; 86 percent of their responses were scored as "incorrect." Despite the fact that students had difficulty determining the correct answer, most students attempted to answer the question.

Table 3.8 ***Score Percentages for "Determine Number of Leaves," Grade 4***

THE NATION'S REPORT CARD

	Complete	Partial	Incorrect	Omit
Grade 4				
All Students	6	7	86	2
Males	6	7	85	2
Females	5	6	87	1
White	7	8	84	1
Black	2	3	92	3
Hispanic	3	2	93	2
Asian/Pacific Islander	11	8	75	6
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

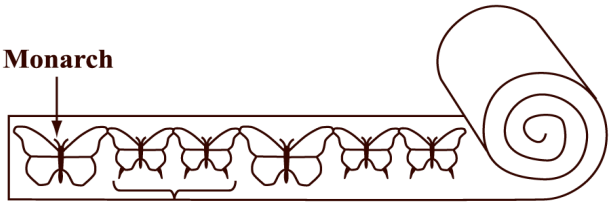
Question 6. Interpret pattern of figures. The last question in the Butterfly Booth block was about making a banner for the booth. Students were told that they should use the Butterfly Information Sheet to help them solve this problem. They were told the length of the banner, 130 centimeters, and the design on the banner; that is, a repeating pattern of one Monarch butterfly followed by two Black Swallowtail butterflies with wings touching but not overlapping. Students were asked to determine how many Monarch and Black Swallowtail butterflies would be needed to fill the banner. This question was classified as an Algebra and Functions question and was designed to assess Problem-Solving ability. As with other questions, students were asked to explain how they got their answer.

6. Use the Butterfly Information Sheet and your answer from question 2 to solve this question.

Your class has decided to have a banner that will be 130 centimeters long. This banner will have a repeating pattern of one Monarch butterfly followed by two Black Swallowtail butterflies, as shown here.

This part keeps repeating across the banner.

The butterflies will just touch but will not overlap.



How many of each type of butterfly are needed for the banner?

Monarch _____

Black Swallowtail _____

Show how you got your answers.

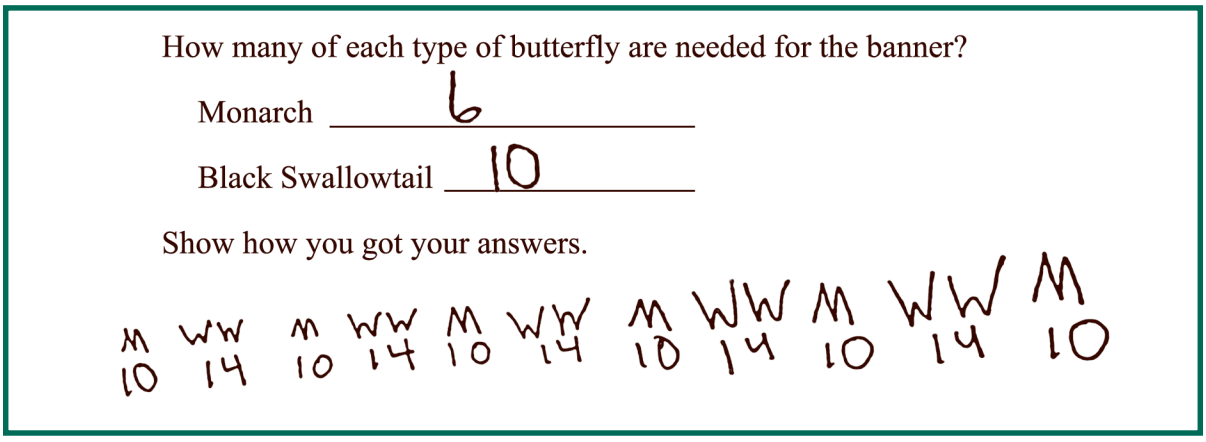
This was a relatively complex question. Students first had to determine which of all the information available was needed to solve the problem and, second, to determine a multistep process for arriving at the number of Monarchs and Black Swallowtails they needed to complete the banner. As with Question 2, students who previously had conducted measurements using centimeters may have had an advantage in solving this problem. The correct response was 6 Monarch and 10 Black Swallowtail butterflies and an adequate explanation of the process for obtaining that response.

Although there are other strategies that students could have used to solve the problem, the following is one possible strategy:

1. Measure in centimeters the wingspan of the Monarch butterfly and the Black Swallowtail butterfly — 10 centimeters and 7 centimeters.
2. Add the wingspan measurements of one Monarch and two Black Swallowtails — 24 centimeters.
3. Divide 24 centimeters into 130 centimeters, the length of the banner, to get the number of patterns needed to cover the banner — five patterns.
4. Realize that there was 10 centimeters of banner remaining — enough to fit Monarch butterfly, but no additional Black Swallowtails.

The responses to this question were scored on a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect." A "satisfactory" response had the correct number of Monarchs and Black Swallowtails and an adequate explanation of how the student arrived at those answers. The sample "satisfactory" response provides the correct answer and provides, as the explanation, a drawing of the how the butterflies (indicated with an "M" for Monarchs and a "W" for Black Swallowtails) would be positioned on the banner with the correct number of centimeters (for one Monarch and two Black Swallowtails) indicated below these letters.

Sample "satisfactory" response



"Partial" responses either had the correct number of Monarchs and Black Swallowtails but no explanation or an incomplete explanation, or had the correct strategy explained with the correct number of patterns but the wrong number of Monarchs and Black Swallowtails. The "partial" response shown had the wrong number of butterflies, but provided an explanation that showed an appropriate strategy for solving the problem. In the explanation, the student showed knowledge of the number of centimeters needed for the pattern, 24; had a counting strategy to get to the number of patterns; and understood that the banner could have an incomplete pattern at the end. Although it is not completely clear, it appears that the student made a mathematical calculation error while counting by 24 and came up with four repeating patterns with centimeters left over for the Monarch but no other Black Swallowtails.

Sample "partial" response

How many of each type of butterfly are needed for the banner?

Monarch 5

Black Swallowtail 8

Show how you got your answers.

Both of the wingspans on one set was 24 and I just kept adding and adding it together and counting as I go to get the answer of 5 Monarchs and 8 Black Swallowtail.

Responses that had any of the following pairs of numbers for Monarchs and Black Swallowtails with no or an inadequate explanation were scored as "minimal": 4 and 8, 5 and 8, 5 and 10, 6 and 12, or 7 and 12. These responses, which were classified as minimally acceptable, indicated measurement or computational error but showed that the student had some understanding of how to solve the problem. A sample "minimal" response follows. The student had a minimally acceptable number of repeating patterns, but did not provide any explanation and did not appear to take into consideration the remaining centimeters that could accommodate an additional Monarch butterfly.

Sample "minimal" response

How many of each type of butterfly are needed for the banner?

Monarch 4

Black Swallowtail 8

Show how you got your answers.

"Incorrect" responses involved pairs of numbers other than those mentioned above or had missing numbers. The following "incorrect" response shows an understanding of some aspects of the problem, but the student clearly did not understand that it was necessary to take into account the wingspans of all three butterflies in the pattern. Although it is not possible to know with certainty from the response how the student went about solving the problem, it appears the student took the wingspan measurement of the Monarch butterfly, 10 centimeters, and determined that 13 such butterflies could fit on a 130-centimeter banner by dividing 130 by 10. Then, while not attending to the fact that the Black Swallowtails also have wingspans that take up space, the student laid out the repeating patterns with the 13 Monarchs. It is interesting, however, that the student did not add the two Black Swallowtails after the thirteenth Monarch butterfly. Therefore, the total number of Black Swallowtails summed to 24.

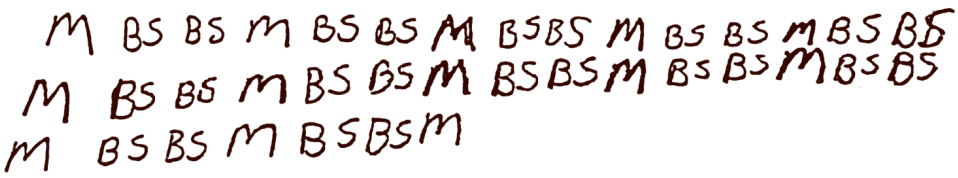
Sample "incorrect" response

How many of each type of butterfly are needed for the banner?

Monarch 13

Black Swallowtail 24

Show how you got your answers.



Student performance data on this last question in the Butterfly Booth block are presented in Table 3.9. Students apparently found this question very difficult to solve: 90 percent of the responses were scored as "incorrect."

Table 3.9 ***Score Percentages for "Interpret Pattern of Figures," Grade 4***

THE NATION'S REPORT CARD

	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 4					
All Students	1	3	4	90	1
Males	1	2	5	90	1
Females	1	4	4	89	1
White	2	4	4	89	1
Black	0!	1	5	93	1
Hispanic	0!	1	4	91	1
Asian/Pacific Islander	0!	1!	9	87	3
American Indian	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Grade 8

Student characteristics

Eighth-grade students who participated in the Theme Study were similar to eighth-grade students in the main NAEP assessment in terms of a variety of demographic characteristics. (See Table 3.10.) Students in grade 8 also were similar to students in grade 4. For example, there were similar percentages of male and female students; White students were in the majority; the modal response regarding parents' highest level of education was "graduated from college." In addition, about 90 percent of the students were from public schools; the large majority were not Title I students; and just over a quarter of the students were eligible for the federal Free/Reduced-Price Lunch program.

Table 3.10 ***Student Demographic Distributions by Assessment, Grade 8, 1996***

THE NATION'S REPORT CARD

	Percentage of Students		
	Main Assessment	Theme Block 1 Building a Doghouse	Theme Block 2 Flooding
Grade 8			
Gender			
Males	52	53	52
Females	48	47	48
Race/Ethnicity			
White	69	70	70
Black	14	14	14
Hispanic	12	12	12
Asian/Pacific Islander	3	2	2
American Indian	1	2	1
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	7	8	8
Graduated From High School	22	23	25
Some Education After High School	19	18	18
Graduated From College	42	42	39
I Don't Know	11	10	9
Students who Attend...			
Public Schools	89	90	91
Nonpublic Schools	11	10	9
Title I Participation...			
Participated	12	11	10
Did Not Participate	88	89	90
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	27	25	26
Not Eligible	55	54	54
Information Not Available	17	21	21

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The data in Table 3.11 show similar levels of exposure across the three samples of eighth-grade students to the classroom practice of writing a few sentences about how to solve a mathematics problem. Similarly, exposure to writing reports or doing mathematics projects did not vary significantly across the three samples of eighth-grade students.

Table 3.11 ***Percentage of Students by Teachers' Reports on Classroom Practices, Grade 8, 1996***

THE NATION'S REPORT CARD NAEP

	Percentage of Students		
	Main Assessment	Theme Block 1 Building a Doghouse	Theme Block 2 Flooding
Grade 8			
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem…			
Nearly Every Day	5	5	4
Once or Twice a Week	25	19	21
Once or Twice a Month	37	40	36
Never or Hardly Ever	33	37	38
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project…			
Nearly Every Day	0!	0!	0!
Once or Twice a Week	3	4	4
Once or Twice a Month	33	34	31
Never or Hardly Ever	64	62	65

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

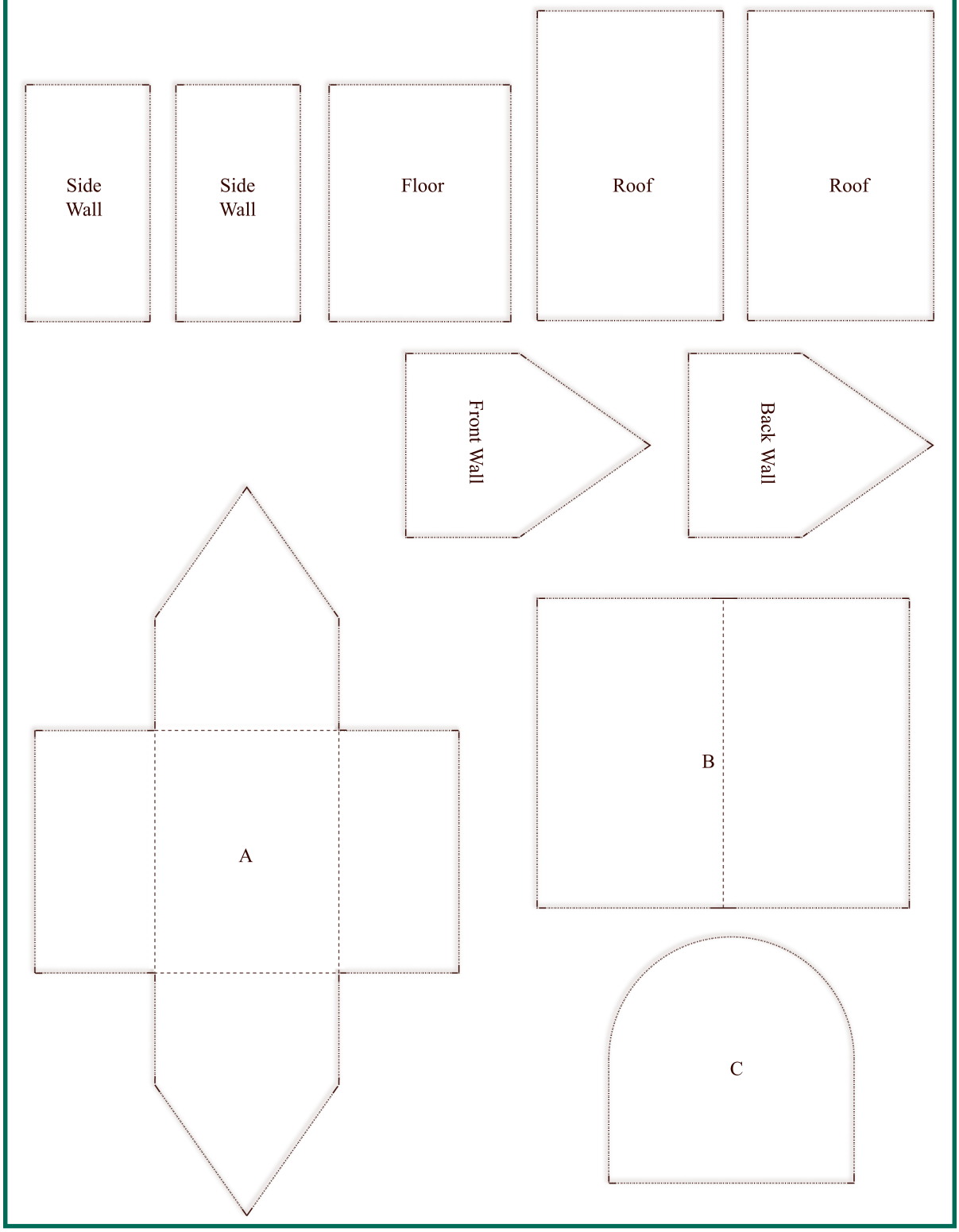
Content of the Theme blocks

The two blocks of questions for the eighth-grade Theme Study also had interesting and relevant contexts. The released block involved building a doghouse; the unreleased block was based on the flooding of the Mississippi River in the summer of 1993. Some eighth-grade students may have pets for which they have considered building a house. With regard to the Theme block on the flooding in the summer of 1993, most eighth-grade students had just completed their fifth-grade year at that time and may have been exposed to media attention about the floods. The flooding of the Mississippi River also was the context of the unreleased Theme block for grade 12. Seven of the 11 questions in that block were given to both eighth- and twelfth-grade students.

Students taking the Doghouse block of questions were given a sheet of push-outs representing parts of a doghouse. A picture of the sheet follows. The sheet included a set of seven push-outs that were parts of the doghouse — two roof pieces, two side walls, a front wall, a back wall, and a floor; a set of two push-outs (indicated with an "A" and a "B") that when folded together formed a model of the doghouse; and a separate push-out (indicated with a "C") that represented the door opening of the doghouse. Students were also provided with a ruler/protractor and a calculator.

Sheet of Push-Outs

Not shown to size



Both Theme blocks included multiple-choice and constructed-response questions. The Doghouse block included four multiple-choice and six constructed-response questions, whereas the Flooding-of-the-Mississippi block included four multiple-choice and seven constructed-response questions. In two of the questions in the Doghouse block, students were asked to show work that supported their answers. In the Flooding block, students were asked to explain their answers in five of the questions.

Overall student performance

Students' overall performance on the two blocks is presented in Table 3.12. The average percentage correct score was 41 percent for the Doghouse block and 30 percent for the Flooding block. In the Doghouse block, female students had a significantly higher percentage correct score than male students. White, Hispanic, and Asian/Pacific Islander students outperformed Black students, and White and Asian/Pacific Islander students also outperformed Hispanic students. For the Flooding block, male and female students performed similarly, while the pattern of performance by racial/ethnic groups was the same as noted on the Doghouse block. The frequency with which students engaged in the two classroom practices highlighted in this chapter was not found to be related to student performance. The sample of American Indian students for both Theme blocks was too small to permit reliable estimates of their performance on either the blocks as a whole or on individual questions. Therefore, the performance of American Indian students is not discussed.

Table 3.12 ***Average Percentage Correct Scores by Theme Block, Grade 8, 1996***

THE NATION'S REPORT CARD

Grade 8	Build a Doghouse	Flooding
All Students	41	30
Gender		
Males	39	31
Females	43	30
Race/Ethnicity		
White	45	34
Black	27	18
Hispanic	33	22
Asian/Pacific Islander	43	35
American Indian	***	***
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...		
Nearly Every Day	42	29
Once or Twice a Week	44	32
Once or Twice a Month	40	30
Never or Hardly Ever	42	31
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...		
Nearly Every Day	***	***
Once or Twice a Week	39	30
Once or Twice a Month	42	32
Never or Hardly Ever	42	30

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

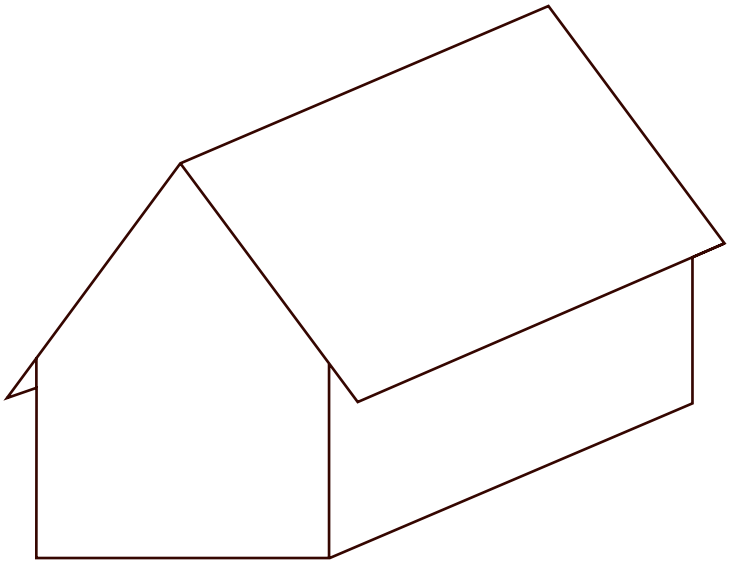
Building a doghouse

The introduction to the Doghouse block is shown in Figure 3.2. The instructions clarified what was expected of students with regard to responses. The description of the context for the block of questions brings the students directly into the task by telling them that Julie would like their help in building a doghouse. Students were asked first to put together a model of the doghouse using pieces "A" and "B" and following the directions given. The results of these efforts were not, however, collected or scored.

Figure 3.2 ***Introduction to the "Building a Doghouse" Theme Block, Grade 8, 1996***

THE NATION'S REPORT CARD

This part has 10 questions. Mark your answers in your booklet. You will have to fill in an oval or write your answer as directed. In those questions where you must write an answer, it is important that your answer be clear and complete and that you show all of your work since partial credit may be awarded. The last two questions may each require 5 minutes or more to think about and answer. After each question, fill in the oval to indicate whether you used the calculator. If you are asked to round your answer, do not round any numbers except your final answer.



Julie wants to build a doghouse like the one shown in the picture above. She has asked you to help her build the doghouse.

The kit you have been given contains a model of a doghouse like the one Julie wants to build. Please put the model together now by following these instructions.

1. Separate pieces A and B from the paper. **Do not separate any other pieces from this paper until you are told to do so.**

2. Fold up the four walls on piece A so that they form right angles with the rectangular floor.

3. Fold the roof (piece B) in half, and set it on top of the house. The edges of the roof will extend slightly beyond the walls.

   Note: When Julie builds the house, the roof will be made up of two identical pieces of wood, since wood cannot be folded the way you folded the piece of paper just now to make the roof.

You may also use your calculator and ruler/protractor to help answer the questions in this part.

Question 1. Identifying needed information. The first actual question the students encountered asked them to consider whether each of five different measurements would help Julie determine whether the finished doghouse will be large enough to accommodate her dog. The question was designed to assess content from the Geometry and Spatial Sense content strand and the mathematical ability Conceptual Understanding. Students were asked to reply "yes" or "no" to the utility of each of the measurements, and each response was scored "correct" or "incorrect." Only measurement "d" contained specific mathematical language (i.e., "rectangular").

1. Consider each of the following measurements. Will knowing the measurement help Julie to determine whether the doghouse she plans to build will be large enough for her dog to sleep in and to go in and out of comfortably? (Answer "Yes" or "No" for each part.)

a. The length of the floor	● Yes	○ No
b. The height of the house	● Yes	○ No
c. The weight of the house	○ Yes	● No
d. The width of the rectangular floor	● Yes	○ No
e. The width and height of the door's opening	● Yes	○ No

Student performance information for this question is presented in Tables 3.13 and 3.14. The data in Table 3.13 summarizes the percentage of students by the number of measurements correctly identified as useful in helping Julie determine whether the doghouse would accommodate her dog; Table 3.14 details the percentage correct scores for individual measurements. Students appear to have done relatively well on this question. Fifty-five percent of the students correctly evaluated the utility of each of the five measurements, and 23 percent evaluated four of the five measurements correctly.

Table 3.13

Score Percentages for "Identifying Needed Information," Grade 8

THE NATION'S REPORT CARD

	5 Correct	4 Correct	3 Correct	2 Correct	1 Correct	0 Correct	Omit
Grade 8							
All Students	55	23	9	6	6	1	0
Males	51	24	10	6	7	1	0
Females	60	22	7	6	4	0	0
White	62	20	7	5	5	1	0!
Black	34	30	14	10	10	1	1
Hispanic	35	37	13	7	6	1!	0
Asian/Pacific Islander	73	16	5!	4	1!	1!	0!
American Indian	***	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 3.14

Percentages Correct for "Identifying Needed Information," Grade 8

THE NATION'S REPORT CARD

	1a — Yes	1b — Yes	1c — No	1d — Yes	1e — Yes
Grade 8					
All Students	90	86	72	82	83
Males	87	85	69	79	81
Females	92	87	76	86	85
White	91	88	77	85	86
Black	82	77	58	70	74
Hispanic	86	83	61	79	74
Asian/Pacific Islander	98!	94	84	86	90
American Indian	***	***	***	***	***

***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The data in Table 3.14 show that, in general, students appear quite knowledgeable about the use of each of these measurements: that is, for each measurement more than 70 percent of students were able to assess correctly whether the measurement would help Julie. Students had more difficulty correctly assessing what appeared to be more complicated or more formal measurements. For example, they were better at assessing correctly whether the "length of the floor" or the "height of the house" would be helpful than they were at assessing the usefulness of the "width of the rectangular floor" or the "width and height of the door's opening." Interestingly, students had most difficulty determining whether the "weight of the house" would be a helpful measurement, or they may have been inclined to answer "yes" to measurement "c" because "yes" was the correct answer to each of the other measurements presented.

Question 2. Determine minimum measuring needed. In the second question, students were first given information about the pieces that made up the doghouse — four walls, two roof pieces, and the floor. They were then told that some of the pieces were exactly the same size and shape, and that Julie did not want to measure all of the pieces, if not necessary. Students were presented with the problem of determining the smallest number of individual pieces Julie would need to actually measure in order to have the information she needed to cut out all of the pieces for the doghouse. The question was classified as a Geometry and Spatial Sense question and was designed to assess Conceptual Understanding.

In order to answer this question correctly, students could use either the model of the doghouse or the push-out pieces to determine that there were three matched pairs among the seven pieces that Julie needed to cut; therefore, she only needed to make four unique measurements. The problem was simplified if students realized that they did not need to actually measure any of the pieces themselves.

2. Seven pieces — four walls, two roof pieces, and the floor — make up the doghouse. Since some of the pieces are exactly the same in size and shape, Julie does not need to measure every piece. She can measure and cut a piece and then make identical pieces without measuring by tracing an outline of the cut piece onto the wood and then cutting out the traced shape.

How many of the seven pieces does Julie need to measure before she cuts?

(A) Two

(B) Three

(C) Four

(D) Five

(E) Seven

Information on student performance on this multiple-choice question is presented in Table 3.15. Forty-two percent of the students were able to correctly identify four pieces as the correct response. The second highest percentage, 24 percent, chose Option B, three pieces. There are a number of reasons that students could have chosen three pieces. This is the answer students would arrive at if they assumed that all the walls were the same size, the two roof pieces were of the same size, and the floor was of a size different from the walls or roof pieces. This would also be the correct response if students assumed that they could use the two adjacent walls to trace out the floor and therefore did not need to measure the floor piece itself.

Table 3.15 ***Percentage Correct for "Determine Minimum Measuring Needed," Grade 8***

THE NATION'S REPORT CARD

Grade 8	Percentage Correct
All Students	42
Males	40
Females	43
White	46
Black	28
Hispanic	32
Asian/Pacific Islander	37
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 3. Measure lengths using ruler. The third question asked students to actually measure, in inches, the pieces of the model doghouse that they put together initially. For example, students were told to measure "the longer side of the rectangular floor." This meant that the student had to know what a "rectangular floor" was, determine which was the "longer side," and use the ruler correctly to measure. This question was designed to assess Measurement content and Procedural Knowledge.

3. The model of the doghouse that you put together is a smaller version of the actual house. Measure the following lengths, in inches, on your model and record your results in the spaces below.

Longer side of rectangular floor _____ inches

Shorter side of rectangular floor _____ inches

Height from floor to highest point of roof _____ inches

The responses to this question were scored on a 3-point rubric: "complete," "partial," and "incorrect." As shown below, a "complete" response gave three correct measurements: two inches for the longer side of the rectangular floor, one-and-one-half inches for the shorter side of the rectangular floor, and two inches for the height from the floor to the highest point of the roof.

Sample "complete" response

3. The model of the doghouse that you put together is a smaller version of the actual house. Measure the following lengths, in inches, on your model and record your results in the spaces below.

Longer side of rectangular floor 2 inches

Shorter side of rectangular floor 1.5 inches

Height from floor to highest point of roof 2 inches

A "partial" response provided only one or two correct measurements in inches. In the sample "partial" response shown, it appears that the student may have been careless in conducting the measurements and used the centimeter side of the ruler to measure the longer side of the rectangular floor, arriving at 5 (centimeters rather than inches). The remaining dimensions were measured correctly in inches.

Sample "partial" response

3. The model of the doghouse that you put together is a smaller version of the actual house. Measure the following lengths, in inches, on your model and record your results in the spaces below.

Longer side of rectangular floor 5 inches

Shorter side of rectangular floor 1 ½ inches

Height from floor to highest point of roof 2 inches

Finally, an "incorrect" response gave no correct measurements in inches. The measurements in the sample "incorrect" response appear to have been measured in centimeters rather than inches. It is not clear if the student did not understand the difference between centimeters and inches or if the student simply was careless and used the wrong side of the ruler to conduct the measurements.

Sample "incorrect" response

3. The model of the doghouse that you put together is a smaller version of the actual house. Measure the following lengths, in inches, on your model and record your results in the spaces below.

Longer side of rectangular floor 5 inches

Shorter side of rectangular floor 4 inches

Height from floor to highest point of roof 5 inches

Student performance information on this question is presented in Table 3.16. The modal score for the responses was "complete" (46%) with similar percentages of responses scored "partial" and "incorrect" (23% and 22%, respectively).

Table 3.16 ***Score Percentages for "Measure Lengths Using Ruler," Grade 8***

THE NATION'S REPORT CARD NAEP

Grade 8	Complete	Partial	Incorrect	Omit
All Students	46	23	22	8
Males	46	22	23	8
Females	47	24	21	8
White	54	21	18	6
Black	17	31	34	17
Hispanic	36	25	31	8
Asian/Pacific Islander	48	24	25	2!
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 4. Apply concept of ratio. The fourth question was prefaced by a scale that showed an inch representing 1½ feet (18 inches). Students were asked to explain how they would use the scale to determine in feet the measurements they obtained in inches in Question 3. Students were also told that they were not required to find the actual measurements; that is, students were simply being asked to provide an explanation of the procedure they would follow to convert measurements in inches to measurements in feet when given the conversion scale. This question was classified as a Number Sense, Properties, and Operations question and was designed to assess Conceptual Understanding, specifically, of the use of ratios.

1 inch represents $1\frac{1}{2}$ feet (18 inches)

4. Explain what you would need to do to each of your measurements in question 3 to find the measurements in feet of the actual house.

(You do not have to find each of the actual measurements.)

Responses to the question were scored using a 3-point rubric: "complete," "partial," and "incorrect." A "complete" response provided an explanation of the correct procedure for translating the inches into feet using the scale provided. The sample response shown below describes that procedure in one sentence. Although not necessary, it also provides the measurements in feet.

Sample "complete" response

You must multiply the measurments (previous question) by 1.5 to get the # of feet.

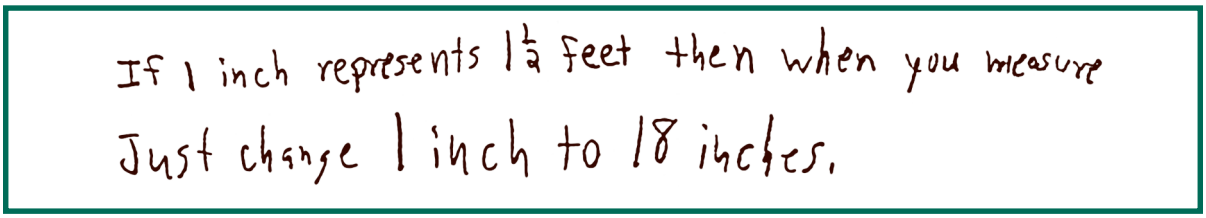
1. 3 ft.
2. 2.25 ft.
3. 3 ft.

Responses were scored as "partial" if they met any of the following criteria:

- the correct procedure for finding the measurements in inches rather than feet;
- the correct procedure using one of the measurements from Question 3, but not generalizing to the other measurements;
- the three correct measurements in feet, but no explanation of the procedure; or
- an example of the correct procedure using one of the measurements in Question 3 multiplied by 18, 1.5, or 18/12.

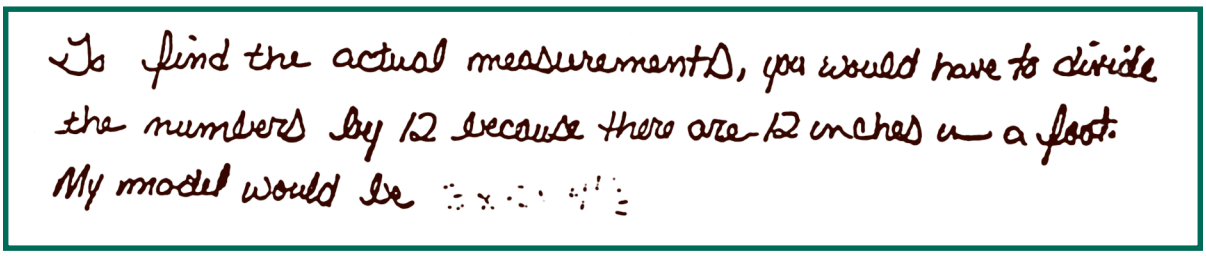
The "partial" response that follows provides a procedure for finding the measurement in inches rather than feet.

Sample "partial" response



An "incorrect" response met none of the previous criteria. The following "incorrect" response begins with an attempt to explain how to solve the problem, "To find the actual measurements…," and ends the sentence with the general procedure to convert inches into feet with no reference to use of the scale provided.

Sample "incorrect" response



Student performance data for this question are presented in Table 3.17. Fourteen percent of the students were able to provide "complete" responses; however, 45 percent of students were not able to provide even a partially correct answer. It is not clear whether eighth-grade students do not understand how to use a scale, or whether they cannot explain in words a general mathematical procedure, or both. Nevertheless, there is certainly hesitancy on the part of many students to attempt to respond to a question like this because a fifth of the students chose not to attempt to answer this question at all.

Table 3.17 *Score Percentages for "Apply Concept of Ratio," Grade 8*

THE NATION'S REPORT CARD

	Complete	Partial	Incorrect	Omit
Grade 8				
All Students	14	17	45	21
Males	14	16	44	24
Females	15	19	46	18
White	17	21	44	16
Black	3	6	53	36
Hispanic	8	10	49	31
Asian/Pacific Islander	22	21	40	16
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 5. Understand concept of ratio (I). The fifth question students encountered was a multiple-choice question classified in the Number Sense, Properties, and Operations content strand that was designed to assess Conceptual Understanding, specifically of the use of scales and ratios. The question asked students to select, from among five options, the scale that would produce the largest doghouse. There are a number of different strategies students could use to solve the problem and thereby select the correct option. One strategy, for example, would involve referencing all of the scales to the same number of inches; that is, convert all of the scales so that each indicates the number of feet represented by one inch. The scale with the largest number of feet is then the correct response. Some students also could have been able immediately to see ways to eliminate certain options before doing any computations. For example, some students may have seen that Option D would produce a larger doghouse than Option E, and that Option B would produce a larger doghouse than Option C, and therefore eliminated Options E and C at the outset.

5. Of the following scales, which one would produce the largest doghouse?

(A) 2 inches on model represents 5 feet on actual house.

(B) 1 inch on model represents 3 feet on actual house.

(C) 1 inch on model represents $1\frac{1}{2}$ feet on actual house.

(D) $\frac{1}{2}$ inch on model represents 1 foot on actual house.

(E) $\frac{1}{2}$ inch on model represents $\frac{3}{4}$ foot on actual house.

Student performance data on this question are presented in Table 3.18. Just over a third of the students were able to select the correct scale that would produce the largest doghouse, Option B. Forty-seven percent of students selected Option A. From this high percentage, it appears that many students did not know how to use scales, and consequently, used other reasoning in selecting their answer. Option A has both the largest number of inches and the largest number of feet, so it appears that many students may have simply selected that option because it had the largest numbers. It is not clear, however, from students' choices, whether they have little or no understanding of scales or simply do not know how to compare differing ratios.

Table 3.18 ***Percentage Correct for "Understand Concept of Ratio (I)," Grade 8***

THE NATION'S REPORT CARD

	Percentage Correct
Grade 8	
All Students	35
Males	39
Females	31
White	38
Black	21
Hispanic	27
Asian/Pacific Islander	42
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 6. Understand concept of ratio (II). In the sixth question, students were provided with a scale they were to use in answering the question and were asked to indicate, based on the scale provided, how much taller the actual doghouse would be compared to the model. This question also was designed to assess Conceptual Understanding, specifically the use of scales and ratios, and content from the area of Number Sense, Properties, and Operations.

1 inch represents $1\frac{1}{2}$ feet (18 inches)

6. The height of the actual house will be how many times as tall as the height of the model?

- (A) $1\frac{1}{2}$
- (B) 9
- (C) 18
- (D) 24
- (E) 27

If, in answering the question, students understood the use of scales and also realized that they should be comparing inches to inches rather than converting the height of the actual house to feet, the correct response, Option C, would be straightforward. That is, they would see that an inch on the model represents 18 inches of the actual doghouse, and that therefore the actual doghouse would be 18 times as tall as the height of the model. Student performance data on this question are presented in Table 3.19. Thirty-five percent of the students were able to get the correct answer. The next highest percentage of students, 26 percent, selected Option A, which specified that the actual doghouse would be one-and-a-half times as tall as the model. It may be that this option attracted students who did not think about the metric in which the height was being measured, or they may have been drawn to it because of the repeated number (1½).

Table 3.19 ***Percentage Correct for "Understand Concept of Ratio (II)," Grade 8***

THE NATION'S REPORT CARD

	Percentage Correct
Grade 8	
All Students	35
Males	34
Females	35
White	38
Black	24
Hispanic	30
Asian/Pacific Islander	39
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

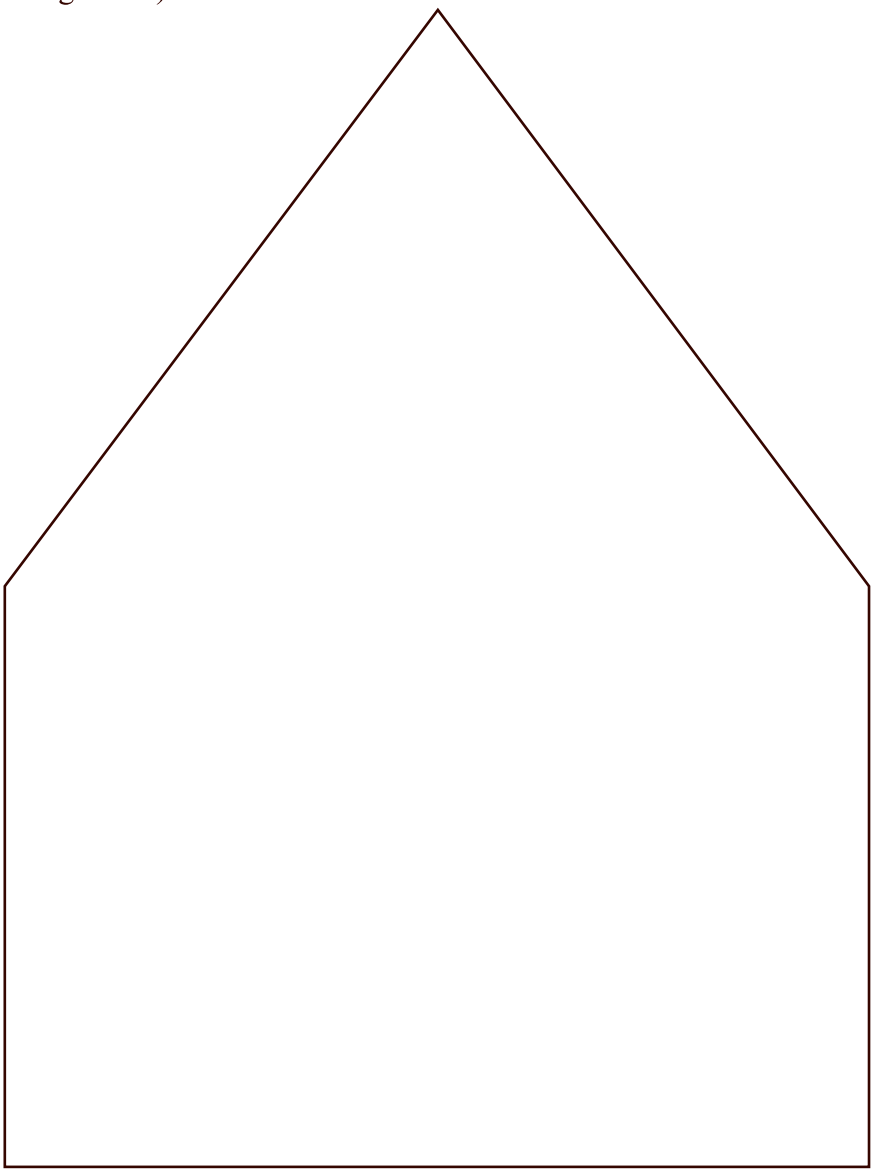
Question 7. Correctly position door. The seventh question required students to trace the correct position of the door opening onto a drawing of the front wall of the doghouse, using push-out piece "C" as a template. Students were told how to position the door, using the measurements of the actual doghouse, and given a conversion scale for the model pieces. In addition, they were cautioned that this conversion scale was not the same as the one used in the previous question. This question was classified as a Geometry and Spatial Sense question and was designed to assess Problem-Solving ability.

7. You will now need piece C to answer this question. Separate piece C from the paper.

2 inches represents 1 foot (12 inches)

Piece C represents a scale model of the door for the doghouse. The front wall of the doghouse, shown below, as well as piece C has been drawn to a different scale than the one used in the previous question. The scale is 2 inches represents 1 foot.

On the drawing below, use piece C to locate the door on the wall so that it will be $\frac{1}{2}$ foot above the floor level of the doghouse (to keep the water out) and centered exactly between the vertical edges of the wall. When you have correctly positioned the door, trace its location on the drawing. (Disregard the thickness of the wood that will be used to build the doghouse.)

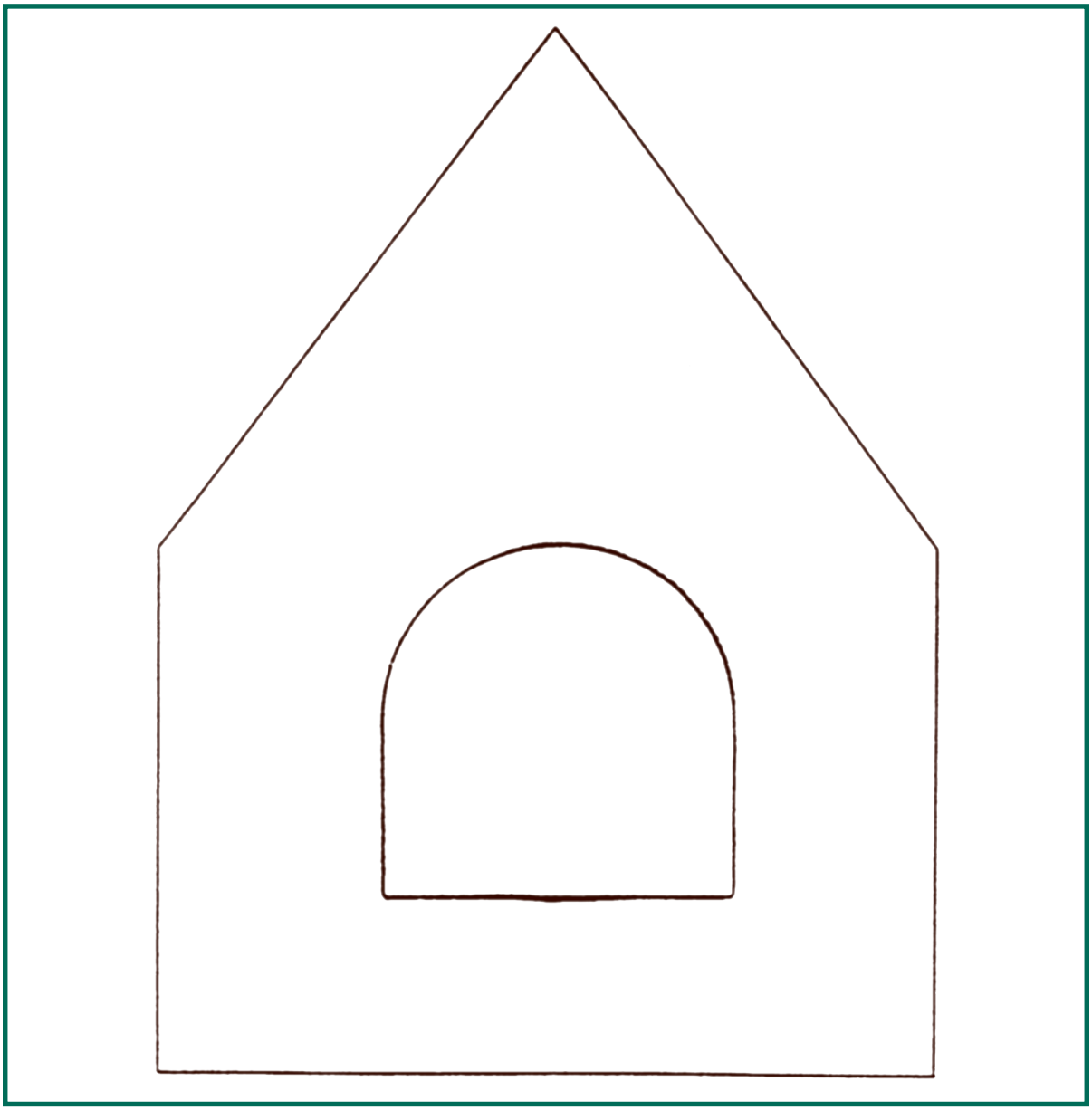


Floor

In order to solve this problem correctly, students had to convert half-a-foot above the floor level to one inch above the floor line on the drawing of the front wall. There were several strategies students could use to center the door opening. One of the simplest is to measure both the width of the front wall and the width of the door opening, find the centers of both, and align the centers. Finally, the student had to be able to trace the door opening in the correct position.

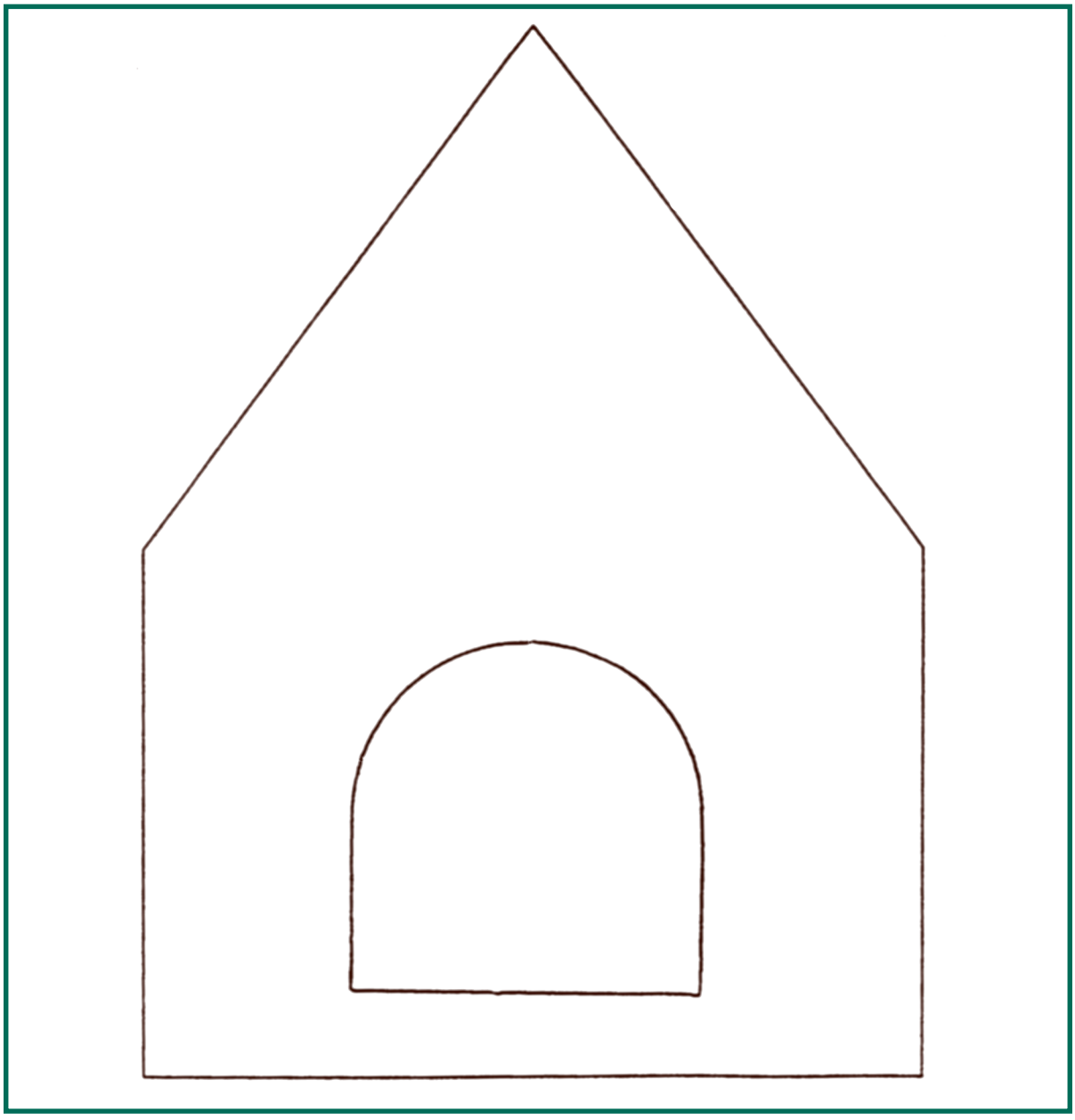
The responses to this question were scored on a 3-point rubric: "complete," "partial," and "incorrect." "Complete" responses had the bottom of the door opening positioned between and including 0.906 to 1.064 inches above the floor and the sides of the door opening positioned between and including 1.142 to 1.314 inches from the side edges of the front wall. A "complete" response follows.

Sample "complete" response



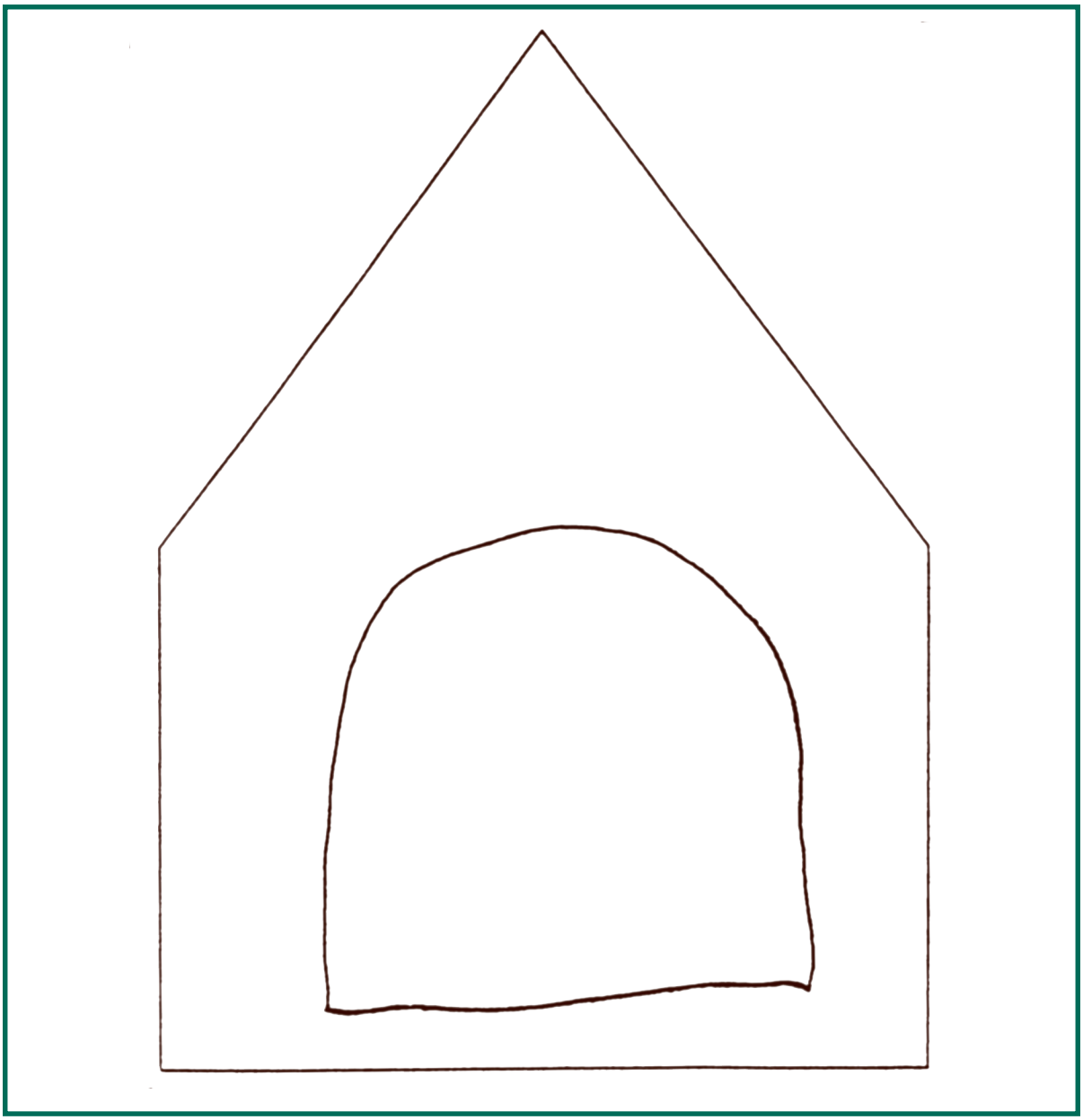
A "partial" response either had the door centered correctly but not located an inch above floor level, or located an inch above the floor level but not centered correctly. The door opening in the "partial" response shown below is correctly centered but only half an inch above the floor of the doghouse. Although the exact reason for this mistake is not clear, the student appears to have forgotten about the scale and instead erroneously translated the half-a-foot instruction into half an inch on the drawing.

Sample "partial" response



Responses scored as "incorrect" met none of the criteria mentioned above. In the following "incorrect" response, the door opening appears to be a free-hand drawing that is larger than the push-out, less than an inch above the floor, and not centered properly. If the drawing shown was actually traced, as required by the question, the tracing was rather imprecise.

Sample "incorrect" response



Information on how students performed on this question is presented in Table 3.20. Almost a fifth of the responses were scored as "complete," and 44 percent were scored as "partial."

Table 3.20 ***Score Percentages for "Correctly Position Door," Grade 8***

THE NATION'S REPORT CARD

	Complete	Partial	Incorrect	Omit
Grade 8				
All Students	19	44	26	10
Males	16	42	29	13
Females	21	46	24	8
White	23	45	25	7
Black	5	40	31	24
Hispanic	11	43	30	16
Asian/Pacific Islander	14	40	32	15
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 8. Visualize cut-outs on grid. The eighth question that students encountered was classified as a Problem-Solving question in the Measurement content strand. Students were instructed to use the seven pieces of the doghouse — the four walls, two roof pieces, and the floor — to help answer the question. They were presented with a scale, told that the plywood sheets from which doghouse pieces would be cut were four feet wide by eight feet long, and provided with drawings of three plywood sheets. (Only one piece is shown here.) The students were further told that the pieces of the doghouse and the representations of the plywood sheet were all drawn to the same scale. Students were asked to trace the doghouse pieces onto the drawings of the plywood sheet in order to demonstrate the fewest number of plywood sheets needed to cut out all seven pieces of the doghouse.

Not shown to size

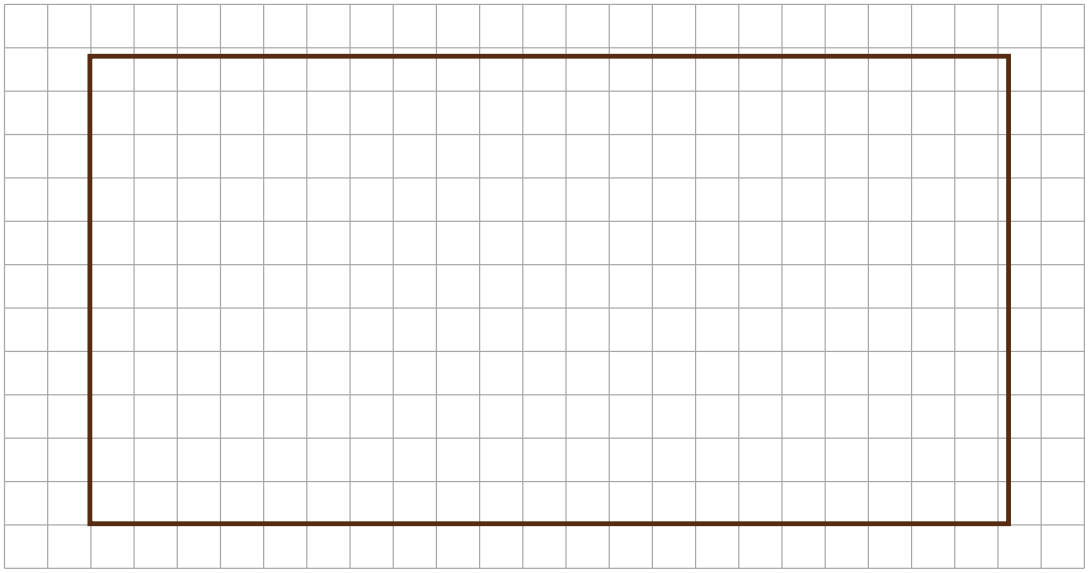
8. Separate the remaining seven pieces from the paper, and use only those seven pieces to help you answer the following question.

1 inch represents $1\frac{1}{2}$ feet (18 inches)

Julie plans to use plywood to build her doghouse, using the scale above. The plywood is sold in rectangular sheets that are each 4 feet wide and 8 feet long. She wants to determine the fewest number of sheets that she will need.

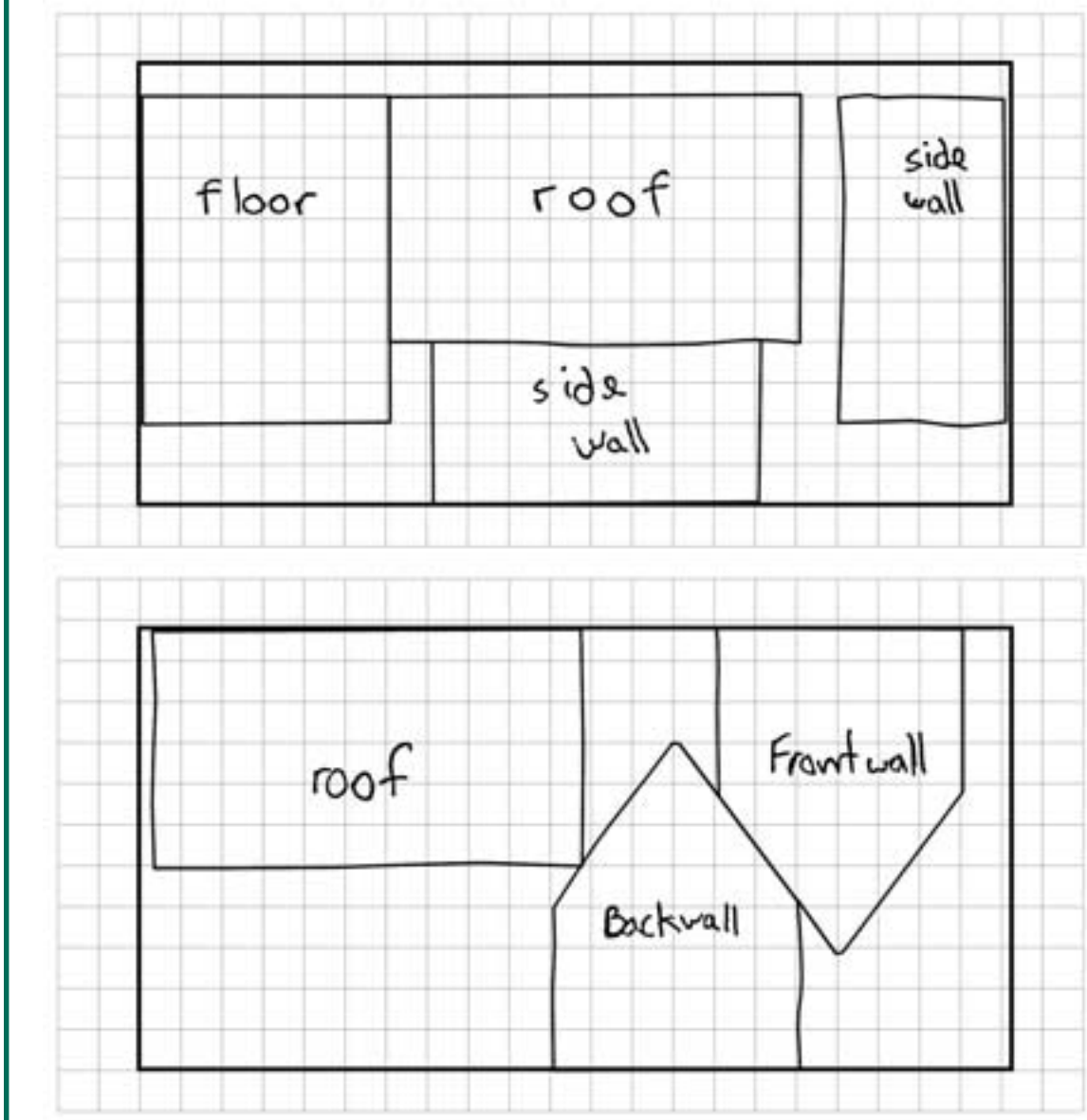
On the grids below and on the next page, the plywood sheets have been drawn to the same scale as the seven pieces. Show how the seven pieces (four walls, two roof pieces, and the floor) could be cut from the plywood sheets so that the fewest number of sheets are used. This should be done by tracing the pieces on the sheets.

(Note: There may be more sheets shown than you will need to use.)



The responses to this question were scored with a 3-point rubric: "complete," "partial," and "incorrect." The fewest number of plywood sheets needed for the seven pieces was two. A "complete" response was one in which the seven pieces were correctly drawn onto two plywood sheets. Responses that showed two side pieces, two front or back wall pieces, or two roof pieces drawn as one piece but correctly labeled were considered correctly drawn. Although students were not required to label the pieces, if they did so incorrectly, their responses were not scored as "complete." The following "complete" response is just one of several configurations that would accommodate the seven pieces on two plywood sheets.

Sample "complete" response

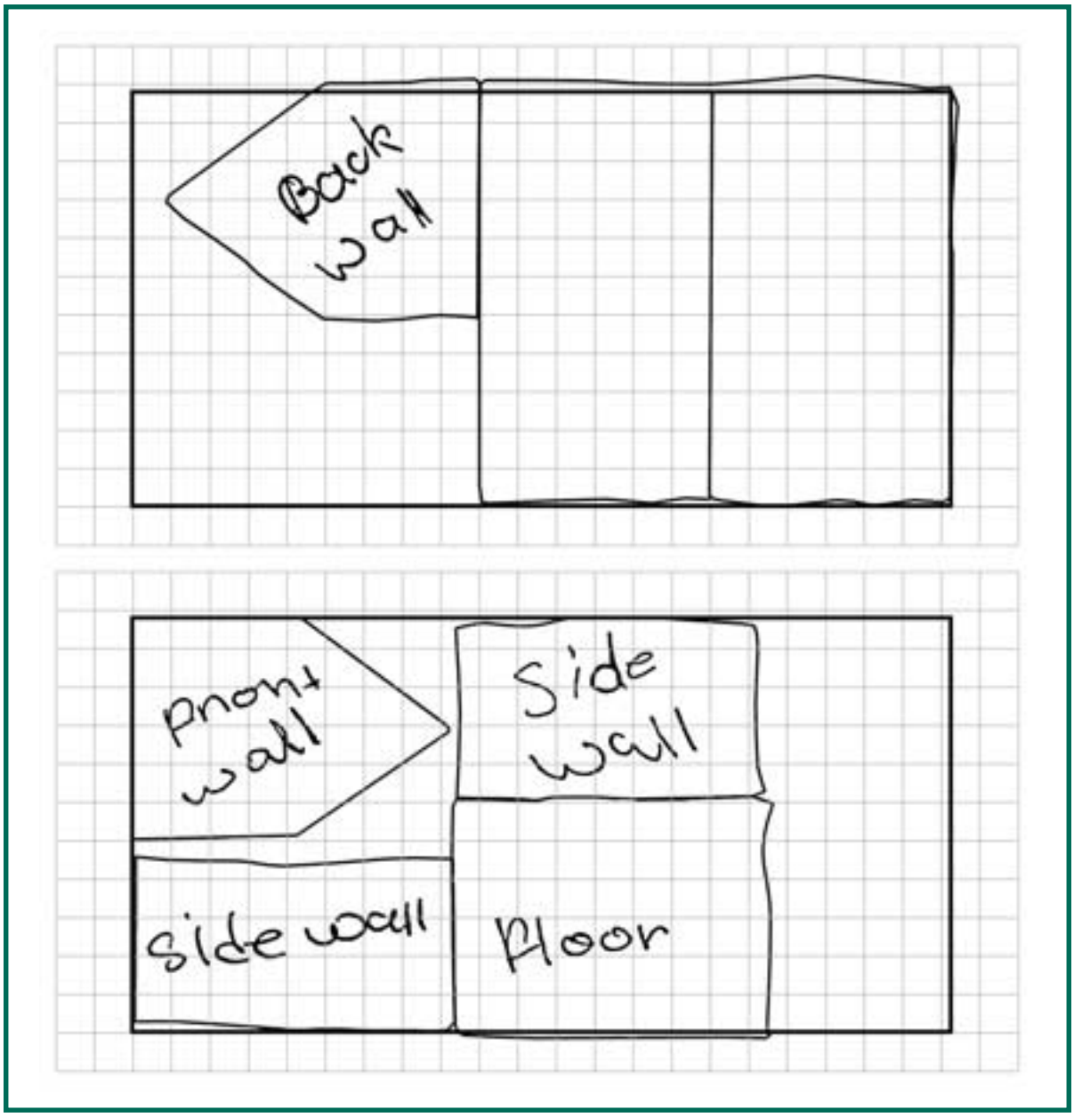


Responses meeting one of the following criteria were scored as "partial":

- six of the seven pieces correctly traced on two sheets with the seventh piece missing;
- six of the seven pieces correctly traced on two sheets with the seventh piece drawn incorrectly, but the seventh piece is not the door opening (piece "C") or a duplicate piece already accounted for; or
- seven pieces correctly traced on to two sheets, but also piece "C" drawn on one of the two sheets or on a third sheet.

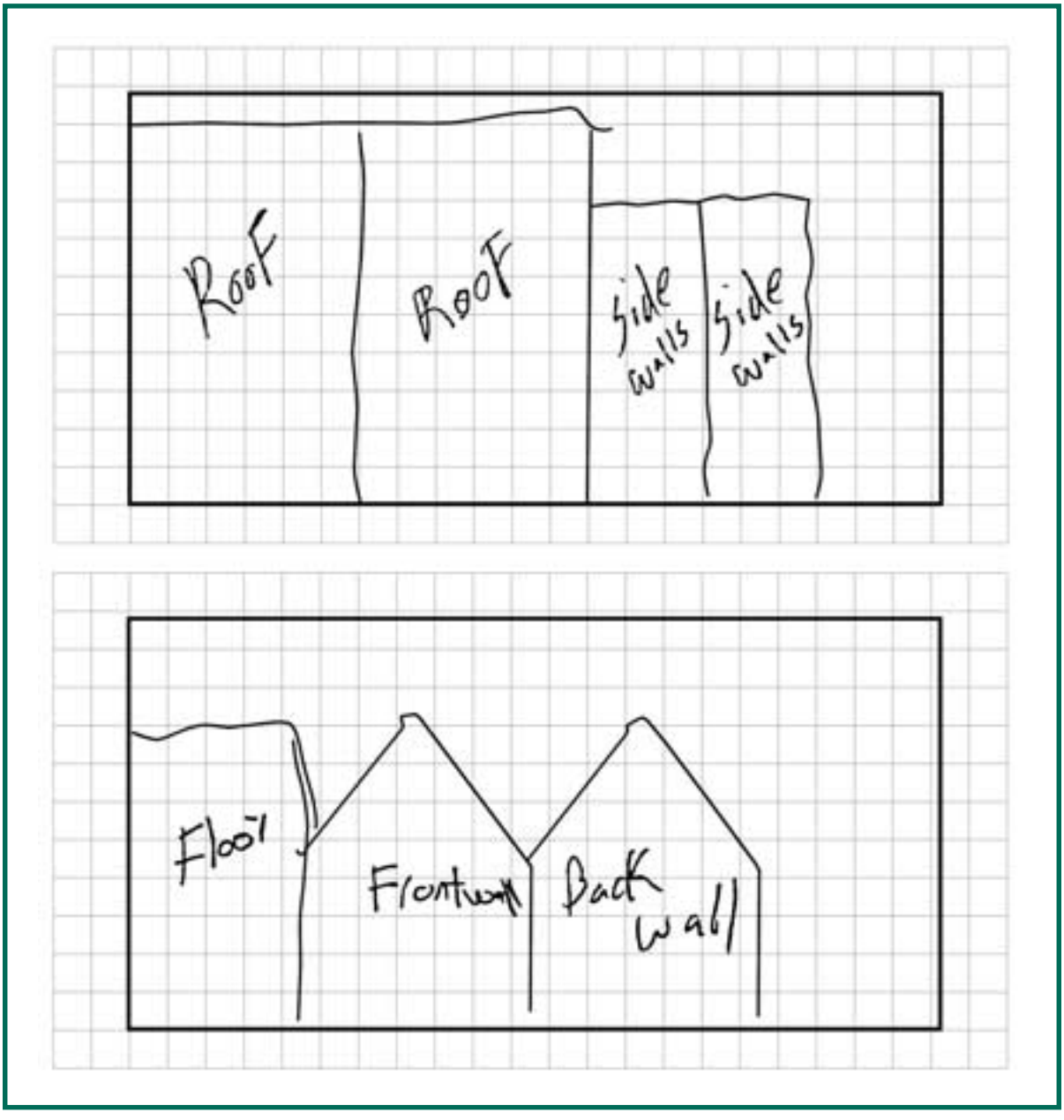
If pieces were labeled incorrectly, responses were still scored as "partial." In the "partial" response shown below, the student appears to have drawn one of the side walls larger than the push-out piece provided.

Sample "partial" response



An "incorrect" response is one that used three sheets, had drawings of pieces that appeared to be freehand or not drawn to scale, or included more than one of any of the seven requisite pieces. In the "incorrect" response shown below, it is apparent that although the roof pieces, front wall, and back wall were traced, the side walls and floor were drawn freehand and were not of the correct size.

Sample "incorrect" response



As the data in Table 3.21 show, 48 percent of the students understood and correctly solved the problem. This was a problem where few students provided responses that were scored as "partial." That is, if students were able to correctly trace and position six of the seven pieces, it was uncommon for them not to succeed with the seventh piece as well. A substantial percentage of the responses, 36 percent, were scored as "incorrect." In addition, 10 percent of the students chose not to attempt this question.

Table 3.21 ***Score Percentages for "Visualize Cut-Outs on Grid, Grade 8"***

THE NATION'S REPORT CARD

	Complete	Partial	Incorrect	Omit
Grade 8				
All Students	48	7	36	10
Males	43	8	37	12
Females	54	6	34	7
White	56	7	30	7
Black	20	2	58	20
Hispanic	36	6	45	13
Asian/Pacific Islander	45	11	34	10
American Indian	***	***	***	***

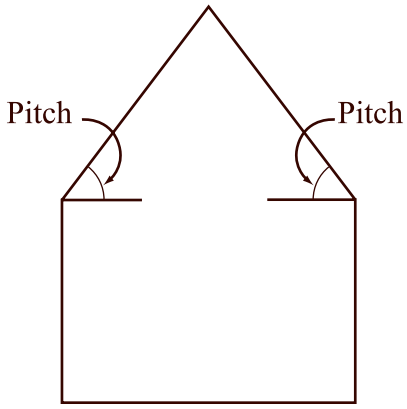
NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 9. Apply geometry in model. The ninth question was a multistep question classified as Problem-Solving and designed to assess content from both the Algebra and Functions and the Geometry and Spatial Sense content strands. In the introduction to the problem, students were told that they were required to show how they solved the problem and to explain their reasoning process. A scale that students had seen in previous problems was provided. The question began with a definition of the pitch of a roof. Students were told that Julie had read that the optimal pitch for best air flow was 30°, and that the pitch of the model doghouse was 53°. They then were asked to determine the number of feet by which the height of the actual doghouse would decrease if Julie decreased the pitch from 53° to 30°.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.

1 inch represents $1\frac{1}{2}$ feet (18 inches)

9. The drawing below shows a wall of Julie's doghouse. The **pitch** is defined as the slope of the roof; it can also be described as the angle formed between the roof and a horizontal line, as shown in the drawing.



The pitch of the roof in the drawing is slightly more than 53 degrees, which is the same as the roof pitch on your model.

Julie read in a book that the best air flow inside a doghouse occurs when the roof pitch is 30°. If the height of the doghouse is measured from the floor to the highest point on the roof, by about how many feet is the actual height of the doghouse decreased when the pitch is decreased from 53° to 30°?

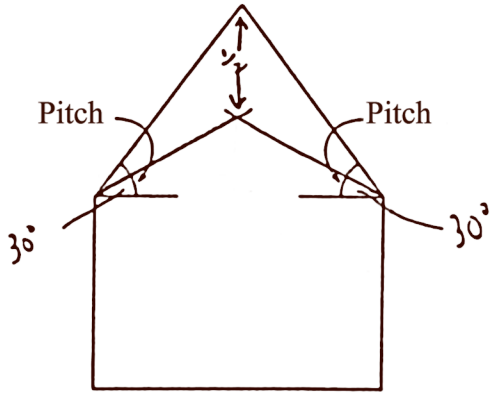
Show how you got your answer. (You may find it helpful to mark on the drawing.)

To solve the problem, students needed to understand the definition of pitch and be able to draw a new roof slope based on a pitch of a different degree. That is, they needed to know what an angle is, what a horizontal line is, and how to use a protractor to measure an angle. Finally, students needed to know how to use the scale provided to convert measures on the drawing of the model to measures of the actual doghouse. Responses to this question were scored using a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect." The responses were scored on the presence of three attributes:

1. correct indication of the 30° angle on the drawing;
2. correct measurement of the height of the old doghouse, the height of the new doghouse, or the sides opposite the 30° angle in the right triangle; or correct subtraction or measurement to obtain the difference between the heights of the old and new doghouses; and
3. correct conversion of the decrease in height from inches to feet using the scale provided.

A "satisfactory" response included all three of the attributes detailed above. The sample "satisfactory" response shown has the correct indication of the 30° angle on the drawing, the correct measurement (with explanation) of the decrease in the height of the roof from the old to the new, and proper conversion of inches to feet using the scale provided, resulting in the correct answer of ¾ of a foot.

Sample "satisfactory" response

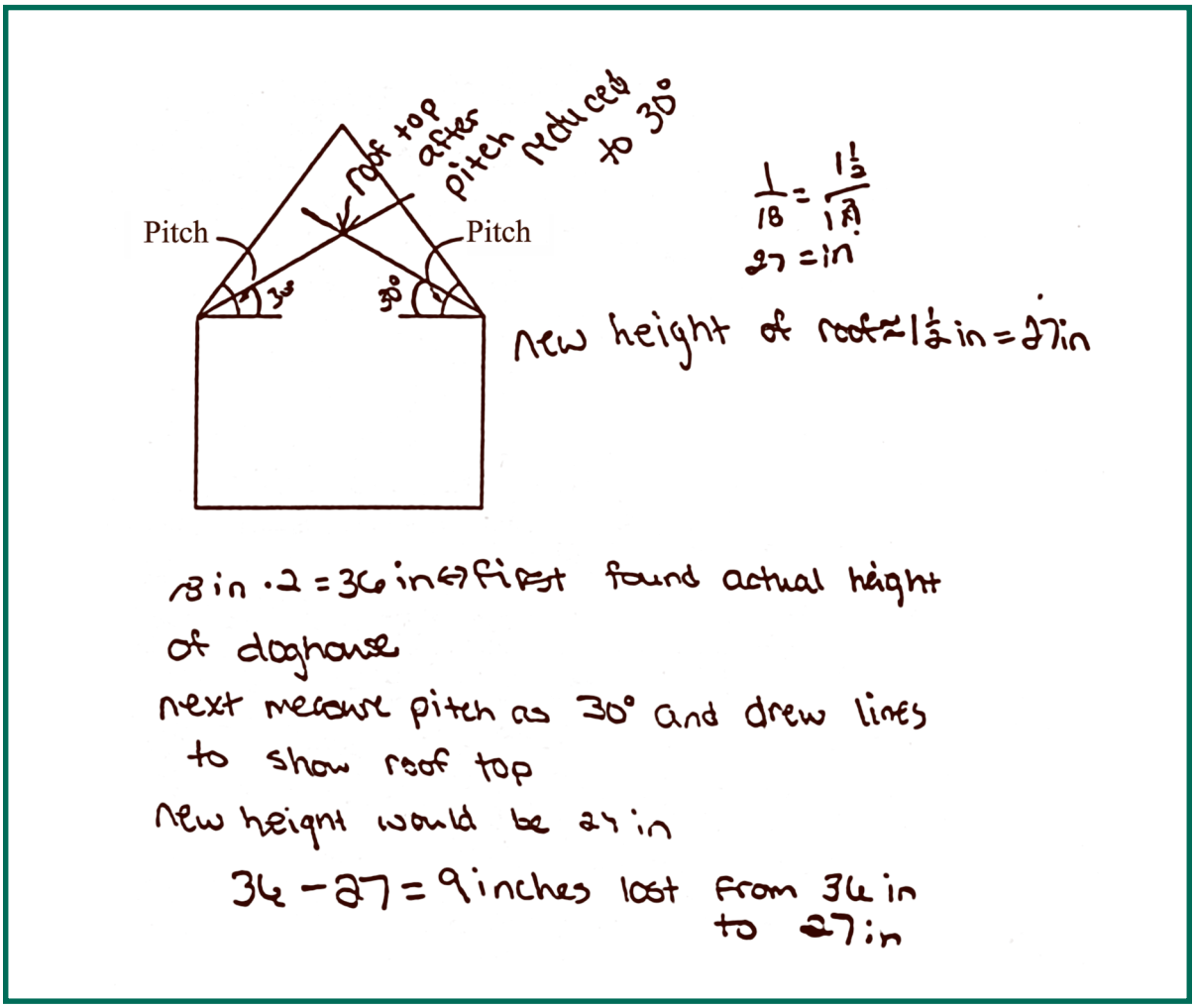


First you mark a 30° angle on the drawing. Then make the roof to fit that. Measure the length of the top of the original roof to the new one. The length is half of an inch.

$\left(\frac{1}{2}'\right)\left(\frac{1\frac{1}{2}\text{ feet}}{1\text{ in.}}\right) = \frac{3}{4}$ of a foot

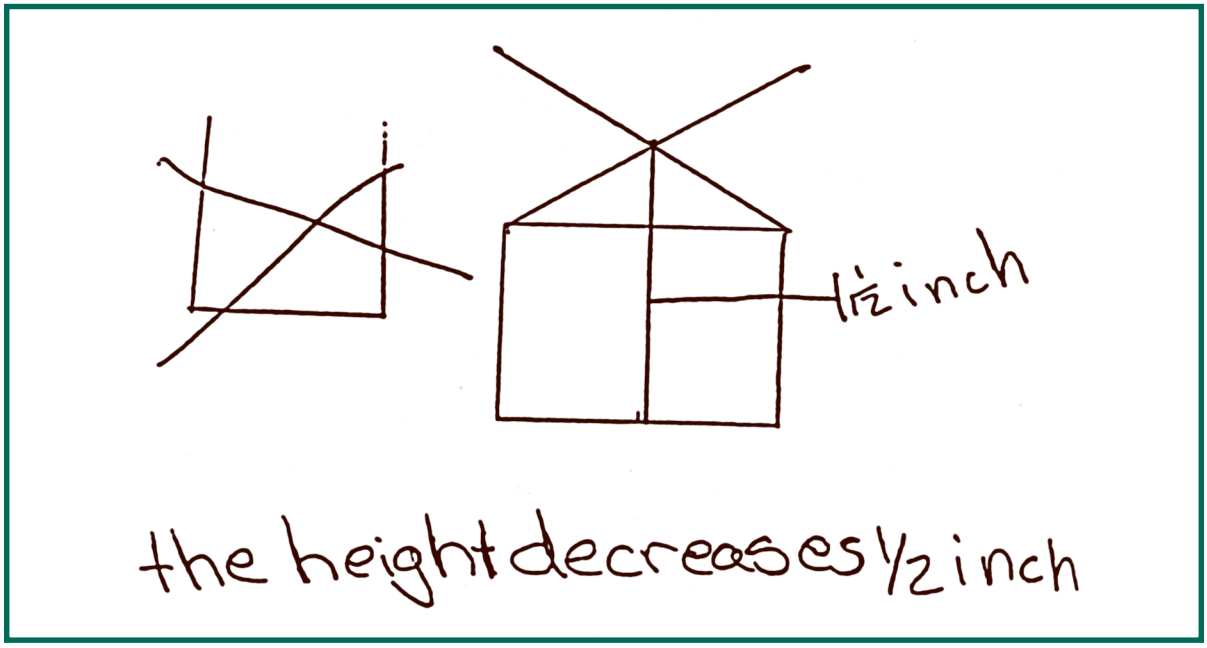
A "partial" response had two of the attributes detailed above. The following "partial" response has the 30° angle correctly measured and drawn and has the correct measurements of the old and new heights for the full-scale doghouse shown in inches. The response also includes the decrease in the height but leaves it in inches and does not carry out the conversion to feet.

Sample "partial" response



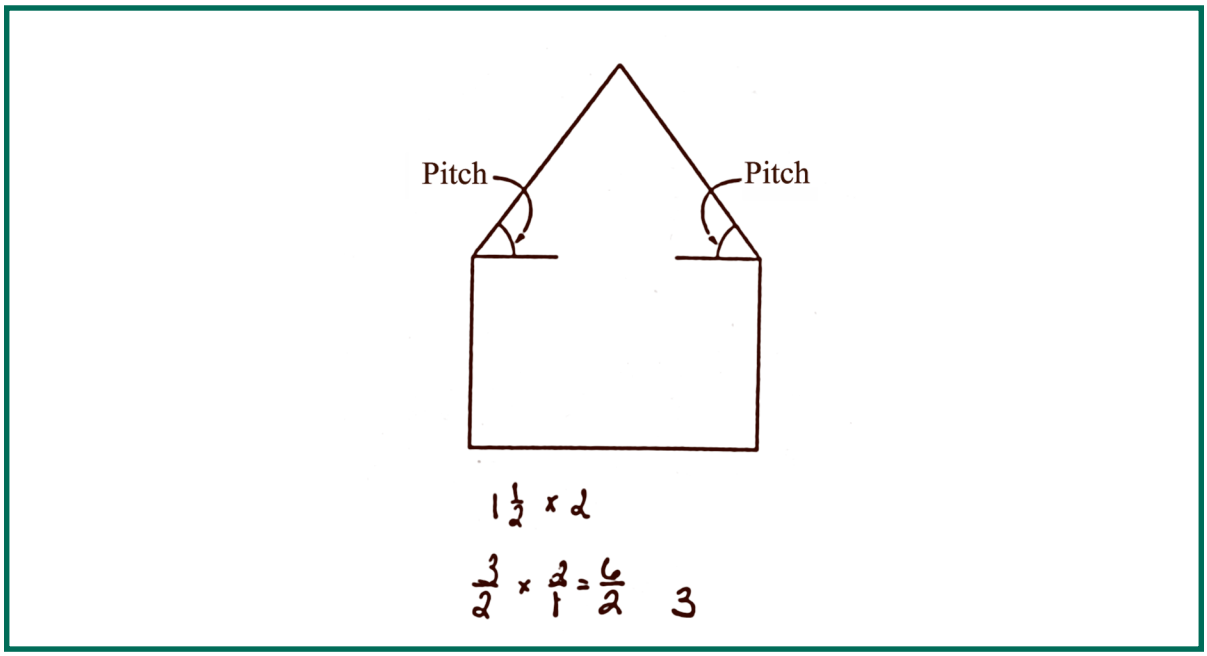
A "minimal" response has one of the correct attributes. The sample "minimal" response shows a correct measure of the new height of the model doghouse. It also includes the decrease in heights of the old and new models but does not show or mention the use of the 30° angle and does not convert inches into feet using the scale provided.

Sample "minimal" response



An "incorrect" response had none of the correct attributes. The sample "incorrect" response shows little understanding of how to approach solving the problem.

Sample "incorrect" response



The data in Table 3.22 show that nearly half of the responses from students were scored as "incorrect." Furthermore, only 22 percent of the responses received any credit, and 24 percent of the students chose not to attempt to solve this problem at all.

Table 3.22 ***Score Percentages for "Apply Geometry in Model," Grade 8***

THE NATION'S REPORT CARD

Grade 8	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	2	6	14	49	24
Males	1	6	13	47	28
Females	2	7	14	52	19
White	2	8	16	47	21
Black	0!	0!	6	60	32
Hispanic	0!	4	8	53	32
Asian/Pacific Islander	***	***	***	***	***
American Indian	***	***	***	***	***

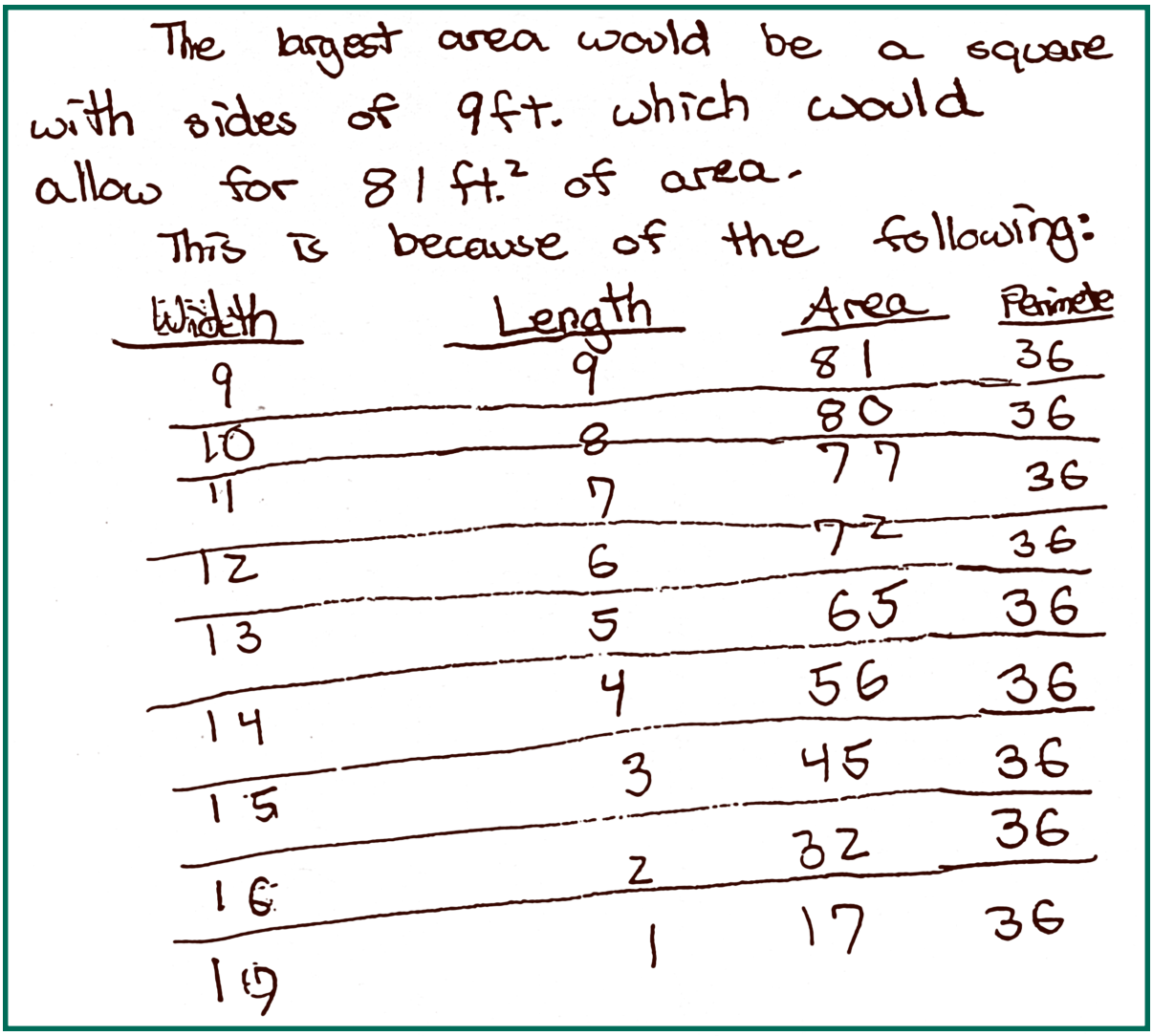
NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 10. Find maximum area when perimeter is fixed. The final question was classified as a Measurement question and was designed to assess Problem-Solving ability. Students were introduced to this question with information about what was required of their responses. Then they were presented with the problem. They were told that Julie wanted to build a fence around a section of the yard for her dog and that she was able to purchase 36 feet of fencing. They also were told what the enclosed area should look like (i.e., a rectangle with whole number lengths). Students were asked to determine the largest area that Julie could enclose with her fencing and to show work that would convince Julie that their solution had indeed yielded the largest area.

> This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.
>
> **10.** Julie wants to fence in an area in her yard for her dog. After paying for the materials to build her doghouse, she can afford to buy only 36 feet of fencing.
>
> She is considering various different shapes for the enclosed area. However, she wants all of her shapes to have 4 sides that are whole number lengths and contain 4 right angles. All 4 sides are to have fencing.
>
> What is the largest area that Julie can enclose with 36 feet of fencing?
>
> Support your answer by showing work that would convince Julie that your area is the largest.

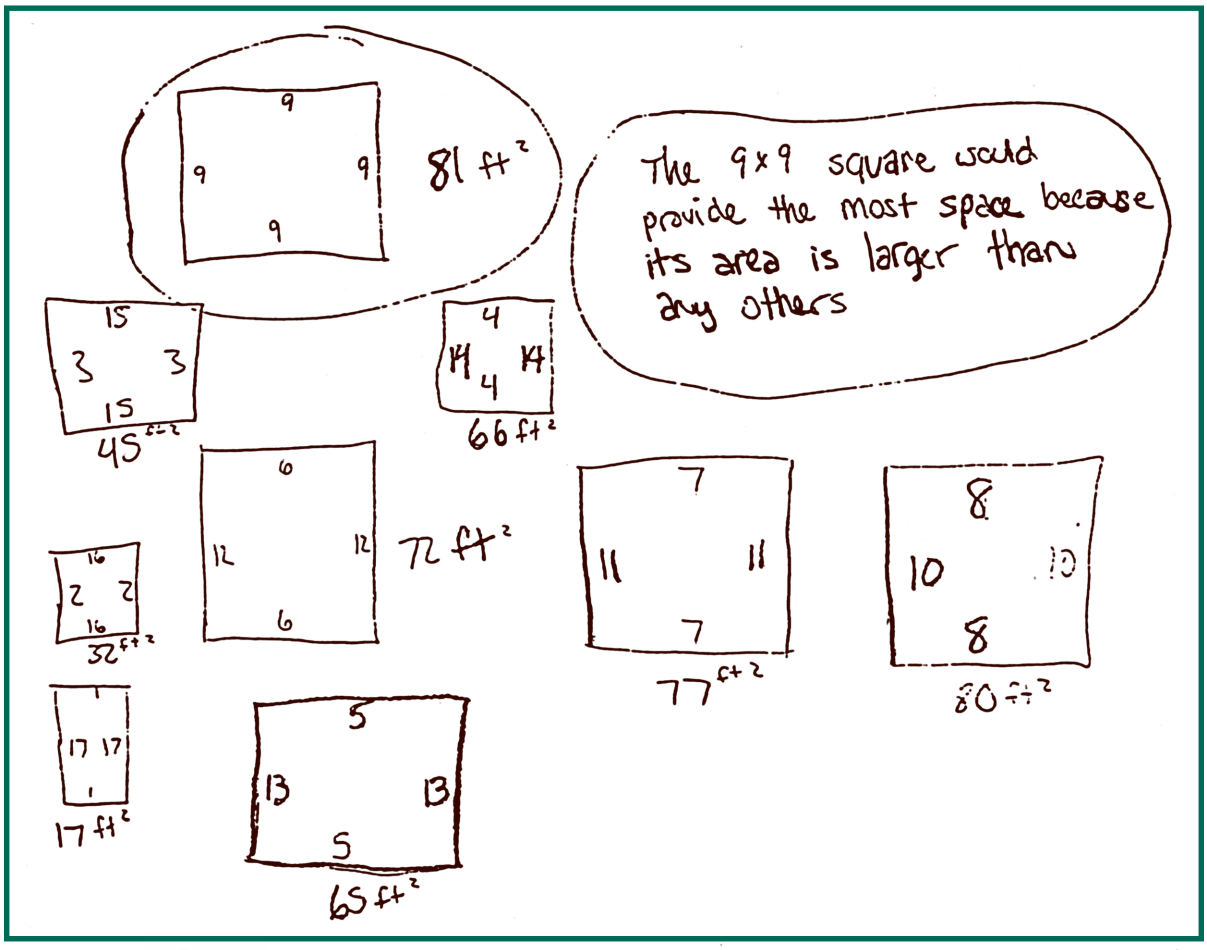
To answer this question students needed to know what a rectangle is; have a conceptual understanding of the fact that rectangles with the same perimeter could have different areas; and be able to calculate the area from the length and width of the sides of the rectangle. This question was scored on a 5-point scale: "extended," "satisfactory," "partial," "minimal," and "incorrect." An "extended" response needed to provide the correct answer of 81 square feet along with evidence that all length/width combinations had been considered in determining the largest area enclosed. The sample "extended" response clearly explained that the largest rectangular area of perimeter 36 feet would be a 9-foot square, which provides an area of 81 square feet. It also included a table that listed all the possible combinations of widths and lengths that would result in a perimeter of 36 feet, thus providing evidence that could "convince Julie" and also showing that the student clearly understood the relationships between the lengths of the sides of the rectangle, perimeter, and area.

Sample "extended" response



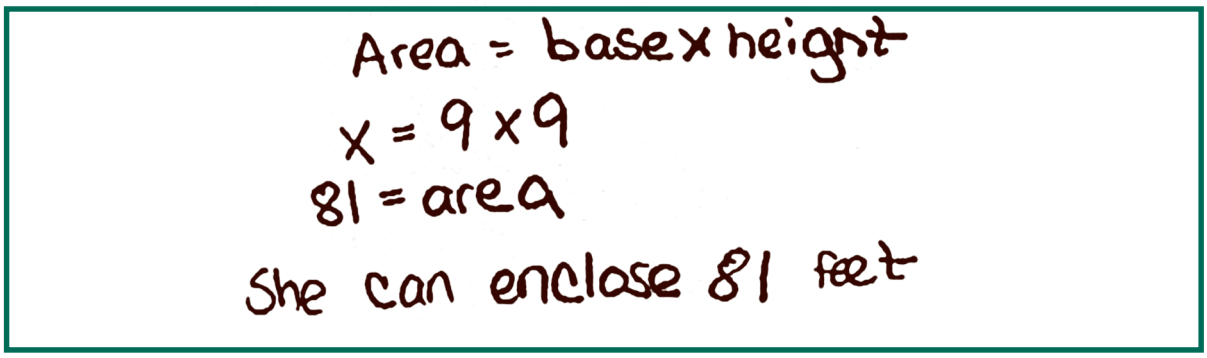
"Satisfactory" responses indicated that the 9-by-9 square has the maximum area of 81 square feet; indicated that another rectangle has the maximum area, due to a mathematical error in the accompanying work; or contained calculations for all nine rectangles (i.e., widths of 1 though 9) but did not specify the maximum area. The sample "satisfactory" response shown correctly designates the rectangle with largest area; however there is a computational error in the supporting work.

Sample "satisfactory" response



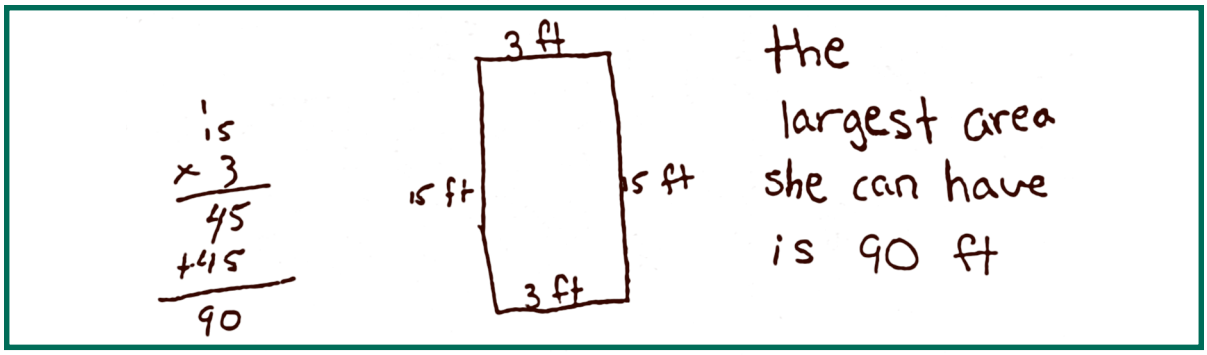
A "partial" response was one that a) showed at least three different rectangles with their dimensions and areas and might have included an indication that one of those rectangles yielded the maximum area, or b) indicated that the 9-by-9 square had the maximum area of 81 square feet, but showed no accompanying work. The "partial" response shown below specifies the correct rectangle and area (although the area units are incorrectly labeled as "feet" rather than "square feet"), but the work shows no consideration of other possible rectangles.

Sample "partial" response



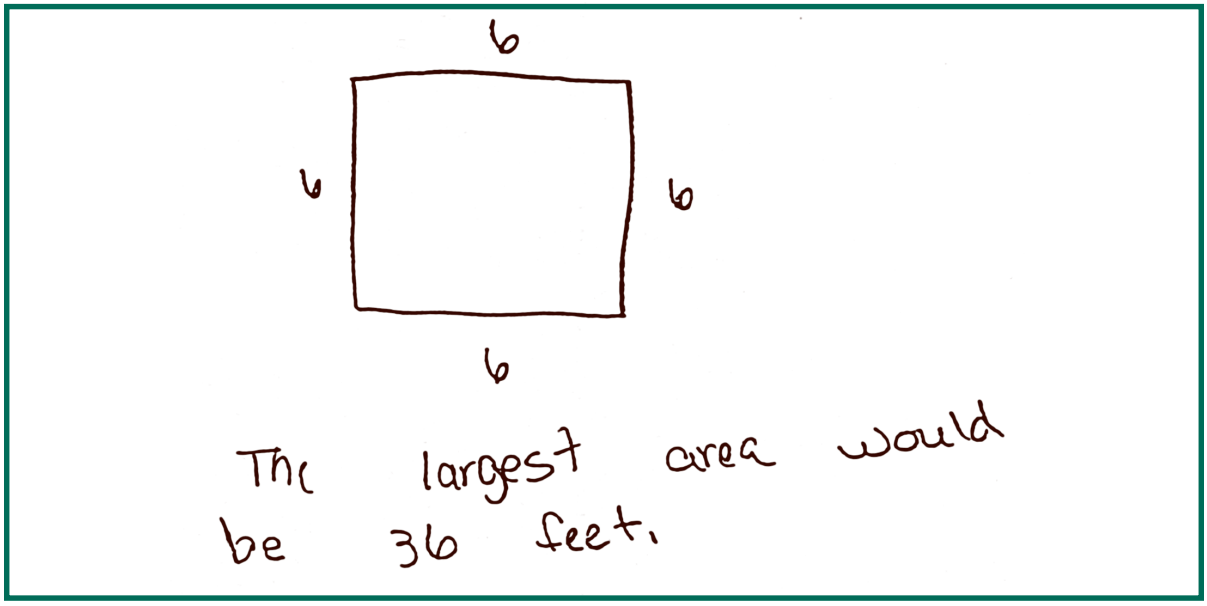
A "minimal" response must have included some evidence of understanding that area and perimeter formulas for rectangles were needed to solve the problem and might have included an attempt to organize the data. The sample "minimal" response shows that the student had an understanding of using the 36-foot-long fencing to form a rectangle, but was unclear as to the exact formula for calculating the area of that rectangle.

Sample "minimal" response



An "incorrect" response had none of the preceding criteria. Although it is not definitive from the "incorrect" response shown, it appears that the student knew how to calculate the area of a rectangle, but did not fully understand the task explained in the problem, since the perimeter of the student's rectangle does not equal 36 feet.

Sample "incorrect" response



Student performance data are presented in Table 3.23. Less than one-half of one percent of the responses were scored "extended" or "satisfactory." A third of the responses received at least minimal credit, while 38 percent were scored as "incorrect." A large percentage of students, 27 percent, chose not to attempt this question.

Table 3.23 ***Score Percentages for "Find Maximum Area When Perimeter is Fixed," Grade 8***

THE NATION'S REPORT CARD

	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 8						
All Students	0	0	29	4	38	27
Males	0	0!	28	4	35	30
Females	0	0	29	3	40	23
White	0	0!	34	4	38	21
Black	0!	0!	10	2	40	45
Hispanic	0!	0!	13	2	37	44
Asian/Pacific Islander	***	***	***	***	***	***
American Indian	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Grade 12

Student characteristics

Twelfth-grade students who participated in the Theme Study were similar demographically to students who participated in the national NAEP assessment in mathematics. The data in Table 3.24 show that similar percentages of male and female students participated in each of the samples; the majority of students were White students; and the modal response from students regarding their parents' highest level of education was "graduated from college." In addition, the large majority of twelfth-grade students were from public schools, very few students were part of the Title I program, and about 10 percent of students were eligible for the federal Free/Reduced-Price Lunch program.

Teachers of twelfth-grade students were not surveyed either in the main NAEP assessment or in the Theme Study. Therefore, the information presented on classroom practices in Table 3.25 is based on responses from the twelfth-grade students themselves. Students in each of the different samples indicated similar levels of frequency of writing about solving a mathematics problem and of writing reports or doing a mathematics project. Notice that only small percentages of students reported engaging in either of these practices with any frequency, while 60–70 percent of students reported "never or hardly ever" doing so. In interpreting these data, however, one must keep in mind that the "never or hardly ever" category also includes responses from students not currently enrolled in mathematics. In the main NAEP assessment, only 62 percent of grade 12 students indicated being enrolled in mathematics.

Table 3.24 ***Student Demographic Distributions by Assessment, Grade 12, 1996***

THE NATION'S REPORT CARD

	Percentage of Students		
	Main Assessment	Theme Block 1 Buying a Car	Theme Block 2 Flooding
Grade 12			
Gender			
Males	48	50	49
Females	52	50	51
Race/Ethnicity			
White	70	69	69
Black	14	14	14
Hispanic	11	12	12
Asian/Pacific Islander	4	4	4
American Indian	1	0	0
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	6	7	7
Graduated From High School	19	18	20
Some Education After High School	25	26	27
Graduated From College	47	46	43
I Don't Know	3	3	3
Students who Attend...			
Public Schools	88	89	88
Nonpublic Schools	12	11	12
Title I Participation...			
Participated	2	2	2
Did Not Participate	98	98	98
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	13	10	10
Not Eligible	60	64	65
Information Not Availabile	27	26	25

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 3.25 ***Percentage of Students by Reports on Classroom Practices, Grade 12, 1996****

THE NATION'S REPORT CARD

Grade 12	Percentage of Students		
	Main Assessment	Buying a Car	Flooding
Students who Report Writing a Few Sentences About How to Solve a Mathematics Problem…			
Nearly Every Day	6	6	7
Once or Twice a Week	12	13	13
Once or Twice a Month	18	19	18
Never or Hardly Ever	64	63	63
Students who Report Writing Reports or Doing a Mathematics Project…			
Nearly Every Day	2	1	2
Once or Twice a Week	4	4	5
Once or Twice a Month	24	22	25
Never or Hardly Ever	70	72	69

* Teachers of twelfth-grade students were not surveyed in these assessments.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Content of the Theme blocks

As with grades 4 and 8, the two blocks of Theme Study questions at the twelfth-grade level also were structured around engaging contexts. The released block involved buying or leasing a car, a topic that is likely to be of interest to twelfth-grade students. The unreleased block had the same context and many of the same questions as the unreleased block at the eighth-grade level: It concerned the flooding of the Mississippi River during the summer of 1993.

Both of the Theme blocks at grade 12 included multiple-choice and constructed-response questions. In the Buying-a-Car block, 2 of the 7 questions were multiple-choice. Four of the remaining five constructed-response questions required the students to show how they solved the problem. In the Flooding block, 4 of the 11 questions were multiple-choice, and in 4 of the 7 constructed-response questions, students were asked to show how they arrived at their answers.

Overall student performance

Information on students' performance on the two Theme blocks is presented in Table 3.26. The average percentage correct score is 41 percent for the Buying-a-Car block and 38 percent for the Flooding block. On both of the Theme blocks, male and female students performed similarly. However, on the Buying-a-Car block, White, Hispanic, and Asian/Pacific Islander students outperformed Black students, and White students outperformed Hispanic students, whereas on the Flooding block, White students outperformed Black, Hispanic, and Asian/Pacific Islander students, and Asian/Pacific Islander students outperformed Black students. The sample

of American Indian students for both Theme blocks was too small to permit reliable estimates if their performance on either the blocks as a whole or on individual questions. Therefore, the performance of American Indian students is not discussed.

In terms of the highlighted classroom practices, the levels of frequency of writing a few sentences and of writing reports or doing a mathematics project were not found to be related to student performance on the questions in the Buying-a-Car block. Performance on the Flooding block, however, was found to have some relationship to the frequency of those classroom practices, although not necessarily in the direction that one might predict. The data show that students who "nearly every day" are asked to write a few sentences about how to solve a mathematics problem were outperformed by students who indicated a lower frequency, and students who indicated writing reports or doing mathematics projects "once or twice a week" were outperformed by students who indicated a lower frequency. One can only speculate about the reasons for these unexpected relationships. It is possible that they reflect ambiguities in the meaning of phrases such as "mathematics projects" or other sources of inaccuracies in the self-reported answers to the survey questions. Another possibility is that activities such as mathematics projects and writing about how to solve mathematics problems are being used less frequently in classes that enroll more high achieving students. Finally, it is possible, that contrary to expectations, introduction of these types of teaching practices actually decrease mathematics learning, perhaps by taking time away from more effective instructional activities.

Table 3.26 ***Average Percentage Correct Scores by Theme Block, Grade 12, 1996***

THE NATION'S REPORT CARD

Grade 12	Buying a Car	Flooding
All Students	41	38
Gender		
Males	40	39
Females	42	37
Race/Ethnicity		
White	46	44
Black	24	23
Hispanic	30	26
Asian/Pacific Islander	45	34
American Indian	***	***
Students Whose Teachers Ask Them to Write a Few Sentences About How to Solve a Mathematics Problem…		
Nearly Every Day	33	31
Once or Twice a Week	43	38
Once or Twice a Month	43	40
Never or Hardly Ever	41	39
Students Whose Teachers Ask Them to Write Reports or Do a Mathematics Project…		
Nearly Every Day	***	***
Once or Twice a Week	37	31
Once or Twice a Month	42	40
Never or Hardly Ever	42	39

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Buying a car

The released Theme block at the twelfth-grade level is about buying versus leasing a car. Unlike the released Theme blocks in grades 4 and 8, this block was not accompanied by any supplemental materials other than a calculator. All of the materials or information needed to solve the problems were provided in the questions themselves. The introduction to the block of items is shown in Figure 3.3. In this introduction, students were told about the expectation for completeness of responses, given information that Question 5 in the block might require more time for thinking and answering, told that they should indicate whether they used a calculator on each question, and instructed to round only at the final answer when calculating. Then, students were introduced to the context for the block of questions: A family is trying to decide whether to buy a new car with a loan or to lease the car. Although familiarity with buying or leasing a car might have some influence on students' comfort level with the questions in this block, prior knowledge about buying and leasing cars was not essential to solving the problems.

Figure 3.3 ***Introduction to "Buying a Car" Theme Block, Grade 12, 1996***

THE NATION'S REPORT CARD

This part has 7 questions. Mark your answers in your booklet. You will have to fill in an oval or write your answer as directed. In those questions where you must write an answer, it is important that your answer be clear and complete and that you show all of your work since partial credit may be awarded. Question 5 may require 5 minutes or more to think about and answer. After each question, fill in the oval to indicate whether you used the calculator. If you are asked to round your answer, do not round any numbers except your final answer.

A family is trying to decide how to pay for the new car they want. Should they buy the car with a loan or should they lease it? If they lease the car, then they must return the car to the dealer at the end of the lease period. If they buy the car, then at the end of the loan period the car belongs to them. The monthly payments for the car, whether they buy it or lease it, will depend upon the size of their down payment. If they make a big down payment now, they will have smaller monthly payments, but they might like to use the down payment money to buy something else. How will the interest rate and the period of time for the loan or lease affect the total amount they will pay?

All the problems on this test are related to financial questions similar to those asked above that involve buying or leasing a car.

Question 1. Find amount of down payment. In the first question, students were introduced to Donna, who wants to buy a car, and were provided with the selling price of the car she wants to buy, $16,500. They were asked to determine the required down payment in dollars, given a down payment requirement of 20 percent of the selling price. This is a relatively straightforward computational question that was designed to assess content from the Number Sense, Properties, and Operations content strand and Procedural Knowledge. One way to answer the question is to translate the 20 percent into the decimal 0.20 and then to multiply the selling price by the decimal fraction. However, students could have used a number of other strategies. For example, some students may have known that 20 percent is equivalent to 1/5 and multiplied the selling price by 1/5 or divided it by 5. Others may simply have used the percent key on their calculators.

1. Donna decides to buy a new car that is selling for $16,500. If she is required to pay 20 percent of the selling price as a down payment, what is the number of dollars required for the down payment?

(A) $330

(B) $1,650

(C) $3,300

(D) $4,125

(E) $13,200

The correct response to this multiple-choice question is $3,300, Option C. Student performance information is presented in Table 3.27. A large majority of the students, 82 percent, were able to answer this question correctly.

Table 3.27 — ***Percentage Correct for "Find Amount of Down Payment," Grade 12***

THE NATION'S REPORT CARD

Grade 12	Percentage Correct
All Students	82
Males	82
Females	83
White	86
Black	73
Hispanic	70
Asian/Pacific Islander	83
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 2. Find total amount paid for car. In the introduction to the second question, students were told that the information given was to be used in Questions 2 and 3. The information included the $19,200 selling price of Bill's car; his down payment of $4,800; and the duration of his loan, 36 months. In Question 2, students were asked to determine how much Bill paid in total for his car, given the down payment and monthly payments of $451. This question was classified as a Number Sense, Properties, and Operations question and was designed to assess Procedural Knowledge. Like the first question, this was a relatively straightforward computational question. The one difficulty that students might have had was in realizing that the selling price of the car, $19,200, was extraneous to answering the question.

Questions 2-3 refer to the following information.

Bill purchased a new car that was selling for a price of $19,200. He paid $4,800 as a down payment and obtained a 36-month car loan to finance the remainder of the selling price.

2. If his monthly payments were $451, what was the total amount, including the down payment, that Bill paid for the car?

Ⓐ $11,436

Ⓑ $16,236

Ⓒ $19,200

Ⓓ $21,036

Ⓔ $24,000

The data on students' performance presented in Table 3.28 show that a large percentage (83%) of students were able to select the correct option, D. The second highest percentage of students, seven percent, selected Option B, which is the number of dollars for the 36 monthly payments of $451 without the down payment. Five percent of the students selected Option E, which is the sum of the selling price of the car and the down payment.

Table 3.28 ***Percentage Correct for "Find Total Amount Paid For Car," Grade 12***

THE NATION'S REPORT CARD

	Percentage Correct
Grade 12	
All Students	83
Males	83
Females	82
White	89
Black	62
Hispanic	74
Asian/Pacific Islander	77
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 3. Find difference between total amount paid and price. The third question builds on the results of the second question: students were asked to indicate how much more Bill paid for his car, that is, with the down payment and the loan, than the selling price of the car. This question, like Questions 1 and 2, was designed to assess content from the Number Sense, Properties, and Operations strand and Procedural Knowledge.

> **3.** By how much did the total amount Bill paid for the car exceed the selling price for the car?

In order to answer this short constructed-response question correctly, students needed to determine which numbers they needed for solving the problem and perform the subtraction accurately. The question was scored simply "correct" or "incorrect"; that is, students were not given partial credit for their work. To be scored "correct," the response must have been $1,836 (that is, $21,036 − $19,200). Any other response was scored as "incorrect." Information on student performance on this dichotomously-scored question is presented in Table 3.29; 80 percent of the students were able to determine the correct answer.

Table 3.29 ***Percentage Correct for "Find Difference Between Total Amount Paid and Price," Grade 12***

THE NATION'S REPORT CARD

	Percentage Correct
Grade 12	
All Students	80
Males	79
Females	80
White	86
Black	60
Hispanic	69
Asian/Pacific Islander	75
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

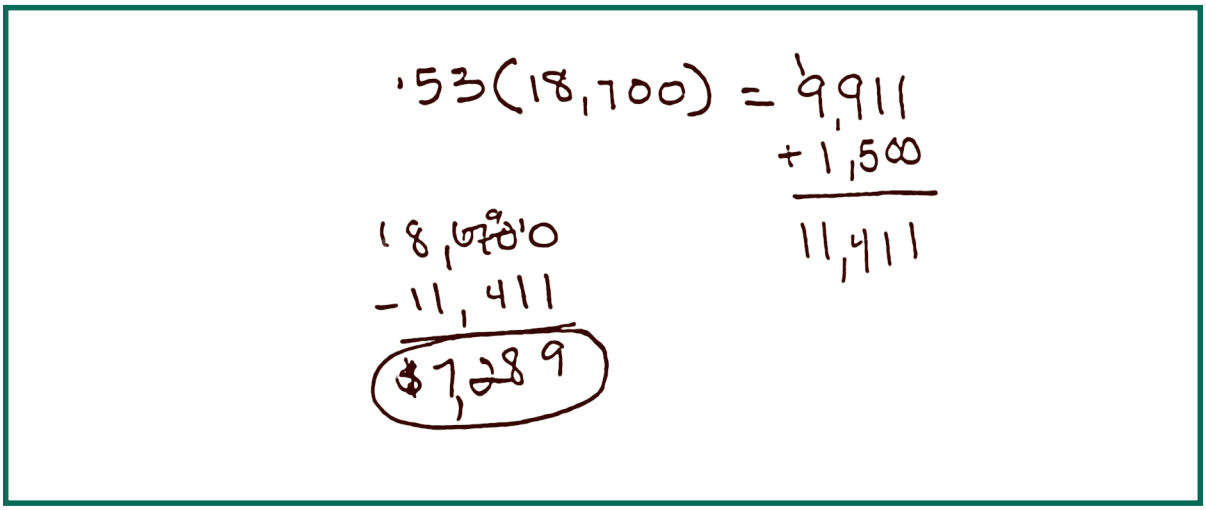
Question 4. Find amount to be financed. The fourth question introduced the term "residual value." Students were asked to determine the number of dollars that would be needed to finance a two-year lease on a car, if the amount to be financed is the selling price minus the sum of the down payment and the residual value of that car at the end of the lease period. They were given the down payment, $1,500; the selling price of the car, $18,700; and were told that the residual value of that car after a two-year lease was 53 percent of the selling price. This question also was classified as a Number Sense, Properties, and Operations question and was designed to assess Procedural Knowledge. Students were asked to show their work.

> **4.** When leasing a new car, the amount to be financed is the amount left when the down payment and residual value are subtracted from the selling price. (The residual value is the value of the car at the end of the lease period.) The residual value for a two-year-old car, whose original selling price was $18,700, is estimated to be 53 percent of the selling price. If the amount of the down payment is $1,500, what is the number of dollars to be financed for a two-year lease? Show the work that led to your answer.

In essence, students were asked to use knowledge and skills similar to those that they needed for the first three questions, only this time in combination, to solve a multistep problem. The computational skills required to solve the problem were relatively simple; the difficult part of the problem was in setting up the problem and determining the steps involved. One way to solve for the amount to be financed is first to calculate the residual value, 53 percent of $18,700 or $9,911; add that amount to the down payment to get $11,411; and, finally, subtract that sum from the selling price to get $7,289.

The responses to this question were scored on a 3-point rubric: "complete," "partial," and "incorrect." A "complete" response had the correct answer and showed an acceptable process for getting to that answer. The sample "complete" response shows the process through mathematical calculations.

Sample "complete" response

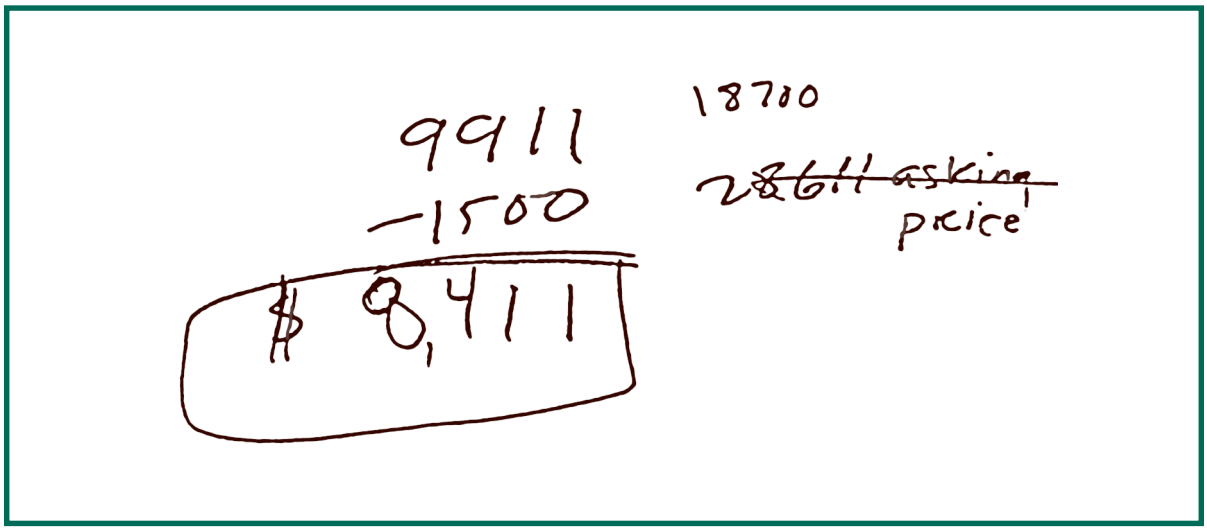


The following types of responses were scored as "partial" responses:

- the correct residual value — $9,911;
- the correct difference between selling price and residual value — $8,789;
- an incorrect residual value, but with all calculations performed correctly — for example, finds 53 percent of $17,200, which is the selling price minus the down payment, instead of 53 percent of the selling price;
- a correct procedure but work shown contains arithmetic errors;
- the correct sum of the residual value and the down payment — $11,411; and
- the correct difference between the residual value and the down payment — $8,411.

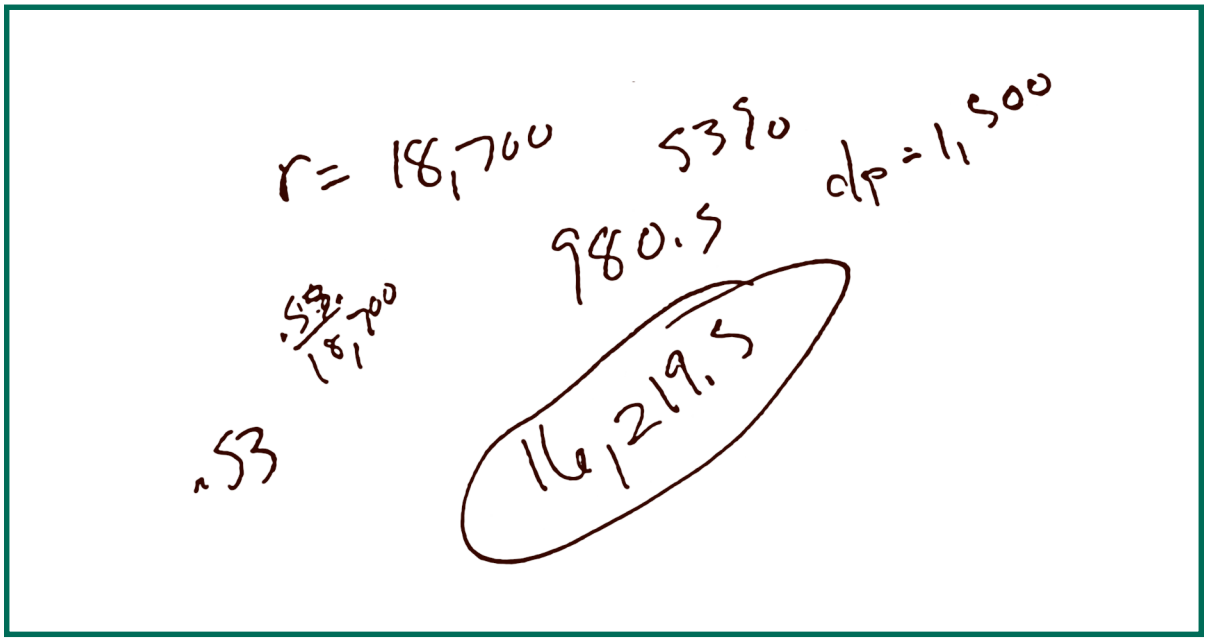
The sample "partial" response is an example of the correct difference between the residual value and the down payment.

Sample "partial" response



An "incorrect" response met none of the preceding criteria. The "incorrect" response shown below contains evidence of some understanding of the process for solving the problem. That is, it is not clear how the student arrived at 980.5, but if that were the residual value, when added to the down payment of 1,500 and subtracted from the selling price of 18,700, the answer obtained would be the 16,219.5 shown. The response nevertheless is scored as "incorrect."

Sample "incorrect" response



Information on students' performance on this question is presented in Table 3.30. Thirty-four percent of student responses were scored as "complete," and 31 percent were scored as "partial."

Table 3.30 ***Score Percentages for "Find Amount To Be Financed," Grade 12***

THE NATION'S REPORT CARD

	Complete	Partial	Incorrect	Omit
Grade 12				
All Students	34	31	24	8
Males	33	28	26	10
Females	36	34	22	6
White	39	30	22	6
Black	17	30	34	16
Hispanic	27	36	26	7
Asian/Pacific Islander	42	30	18	7
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 5. Use formula to find total cost. The fifth question is prefaced with requirements for the students about showing their work and explaining their reasoning. In the question, students were asked to calculate the total cost of a car given the following information: a formula for calculating monthly car payments, the selling price of the car, the down payment, the amount to be financed, the annual percent rate of the amount to be financed, and the length of the loan. Students also were given an approximate value to use for one of the quantities used in the formula — $\left(1+\frac{.08}{12}\right)^{48}$. This question was classified as an Algebra and Functions question and was designed to assess Problem-Solving ability.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.

5. A formula used to calculate a monthly car payment is

$$MP = A\left[\frac{\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}\right]$$

where MP is the monthly payment,
A is the number of dollars to be financed,
r is the annual percent rate (e.g., if the annual percent rate is 7%, then r is 0.07), and
n is the length of the loan in months.

Use the formula shown above to help calculate the total cost of purchasing an $18,000 car if there is a down payment of $4,000 and the remaining $14,000 is to be financed at an annual percent rate of 8 percent for 48 months.

Show the work that led to your answer.

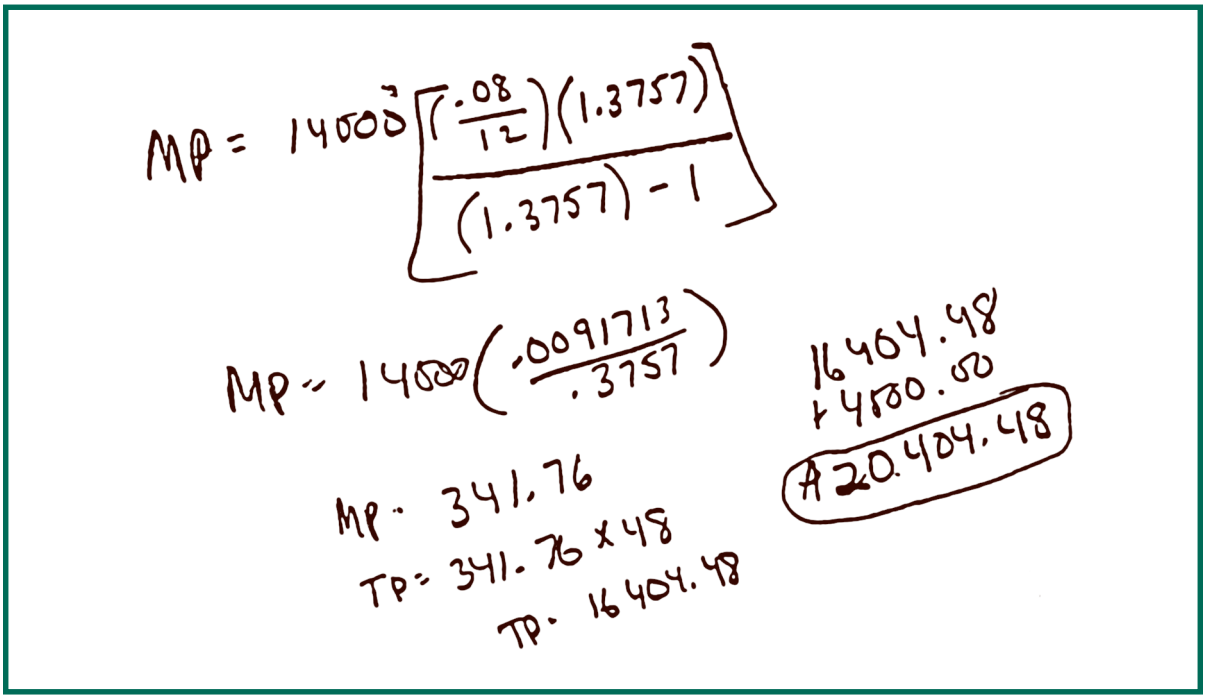
Note: In your calculations, use 1.3757 as the approximate value of

$$\left(1+\frac{0.08}{12}\right)^{48}$$

This problem was difficult because students needed to determine how to organize and use the information provided in order to solve the problem. In other words, students needed to know how to use a formula with variables to compute the monthly payments and how to correctly substitute the approximate value provided in the "Note" into the formula. They also had to recognize that the formula is for a monthly payment; because the question asks for the total cost of purchasing the car, the monthly payment needs to be multiplied by 48 months and added to the down payment in order to arrive at the total cost.

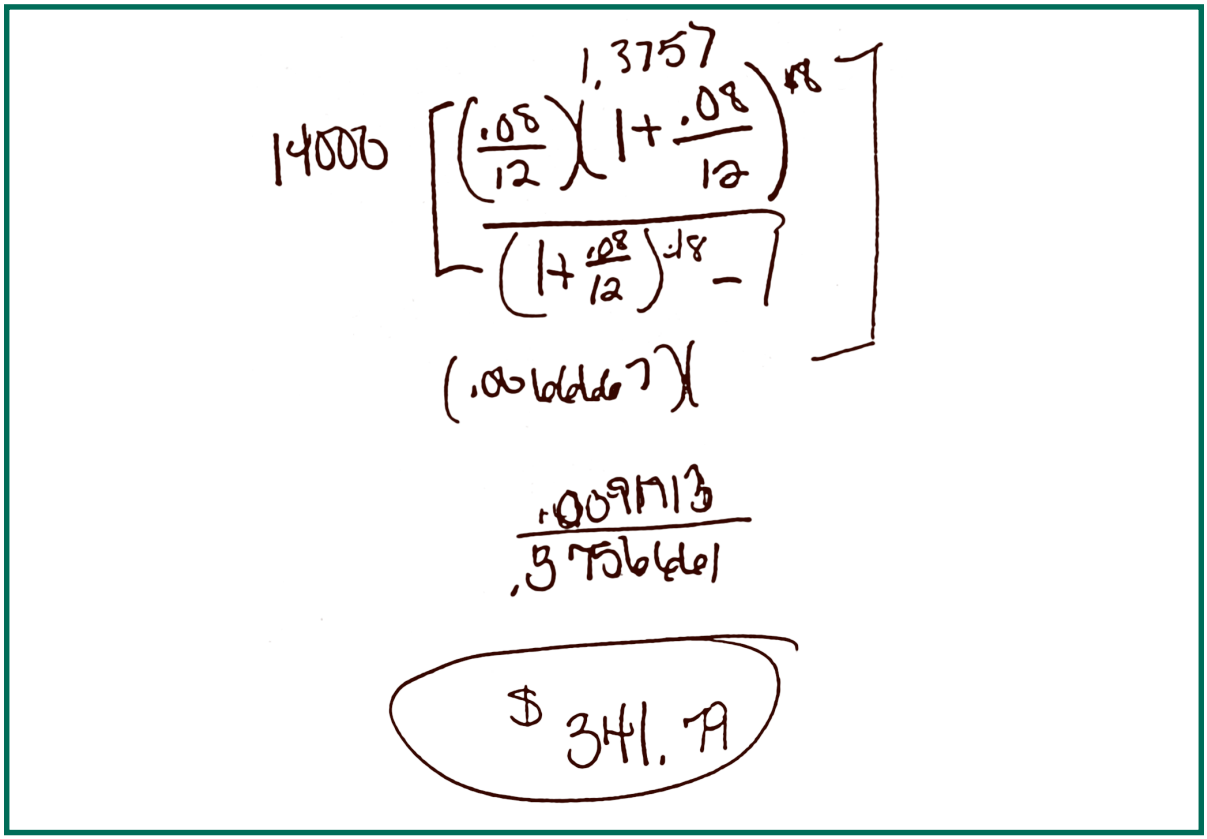
The responses to this question were scored with a 5-point rubric: "extended," "satisfactory," "minimal," "partial," and "incorrect." The scoring rubrics took into consideration that the answers students gave may have differed because of rounding, although students were told at the beginning of the block that they should not round in their calculations until the final answer. Therefore, the following answers, if accompanied by evidence of correct work, were all scored as "extended" — $20,404.48; $20,404.41; $20,404.35; or $20,405.48. The sample "extended" response has the correct answer, $20,404.48 and shows the mathematical calculations done to arrive at the correct answer. The student first calculated the monthly payment, using the approximate value, 1.3757, in the calculations; then calculated the total payments by multiplying the monthly payment by 48 months; and, finally, calculated the total cost by adding the total of the monthly payments and the down payment.

Sample "extended" response



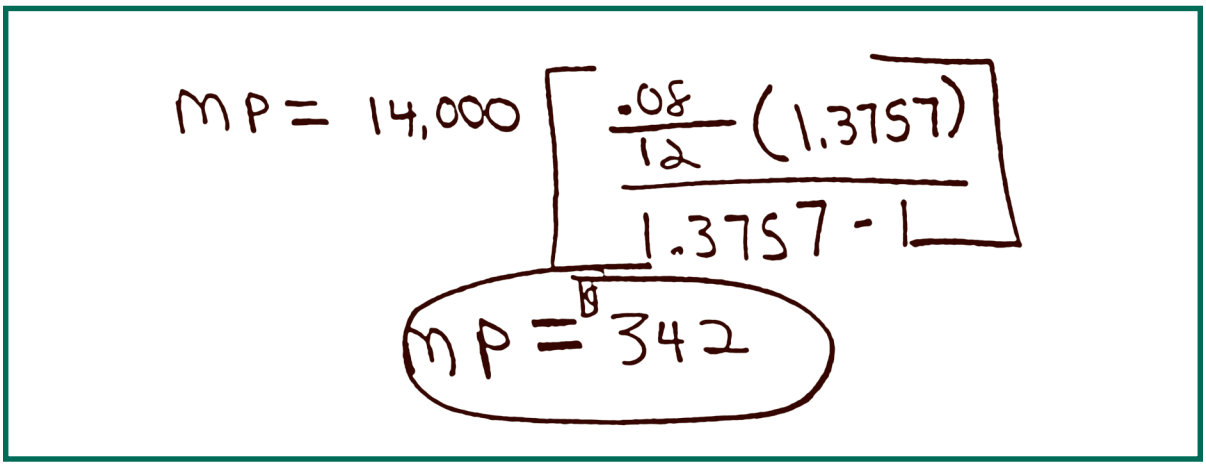
A "satisfactory" response was one that produced an acceptable monthly payment shown in dollars and cents or produced a total cost derived from rounded values (e.g., using 0.0092 for 0.0091713 in the numerator of the monthly payment formula). The sample "satisfactory" response provides an acceptable answer for the monthly payment, but fails to show the total cost of purchasing the car. If the student had completed the calculations, a correct answer would have been obtained.

Sample "satisfactory" response



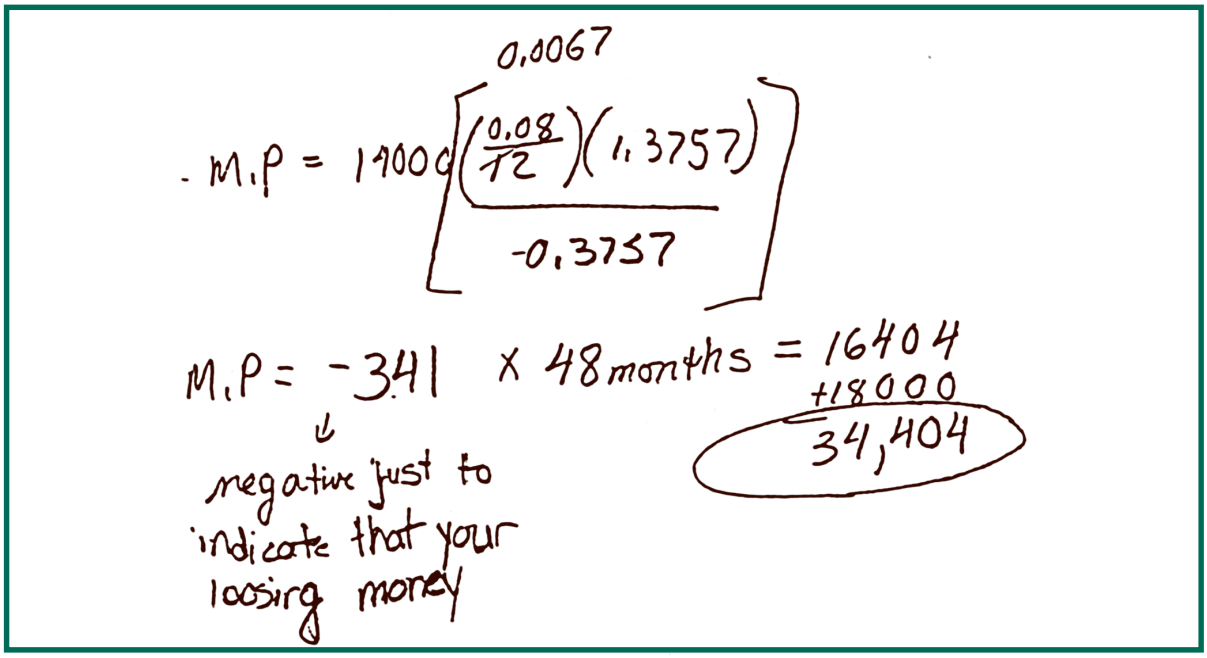
A "partial" response is one in which any of the following are correctly calculated and indicated: the monthly payment using rounded values; the numerator in the brackets, 0.0091713; and/or the amount to be financed. The following "partial" response shows a rounded monthly payment of $342. However, unlike the "satisfactory" response, this response would have led to an incorrect final answer if the student had completed the calculations, due to the intermediate rounding up to $342.

Sample "partial" response



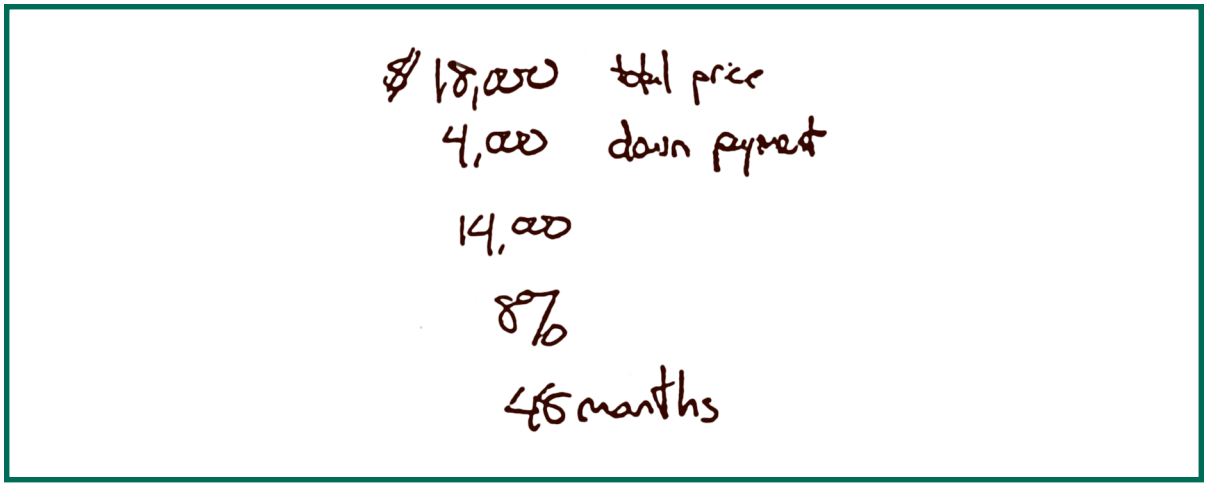
A "minimal" response is one that shows a correct substitution of the given values into the formula. In the "minimal" response shown below, the student rounded the monthly payment down incorrectly, made an error in multiplying that monthly payment by 48 months, and finally, added the selling price of the car to the monthly payments rather than the down payment, to arrive at a total cost for the car that was nearly double the $18,000 selling price.

Sample "minimal" response



An "incorrect" response met none of the criteria mentioned above. In the sample "incorrect" response shown below, the student relisted the information provided but did not attempt any calculations with that information.

Sample "incorrect" response



The data on student performance on this question are presented in Table 3.31. Twenty-three percent of the responses were scored at least "satisfactory," but 44 percent of the responses were scored as "incorrect."

Table 3.31 ***Score Percentages for "Use Formula to Find Total Cost," Grade 12***

THE NATION'S REPORT CARD

	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 12						
All Students	9	14	7	12	44	9
Males	9	12	7	10	45	12
Female	8	16	8	14	43	7
White	10	16	8	12	40	8
Black	2	4	4	12	54	19
Hispanic	4	11	4	13	55	8
Asian/Pacific Islander	19	15	8	10	38	8
American Indian	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 6. Find amount saved if leased. As students encountered Question 6, they were provided with information that they were told would be relevant for Questions 6 and 7. This information included formulas for computing the cost of leasing or buying a particular car and for determining the amount of money for which the car could be resold after 4 years.

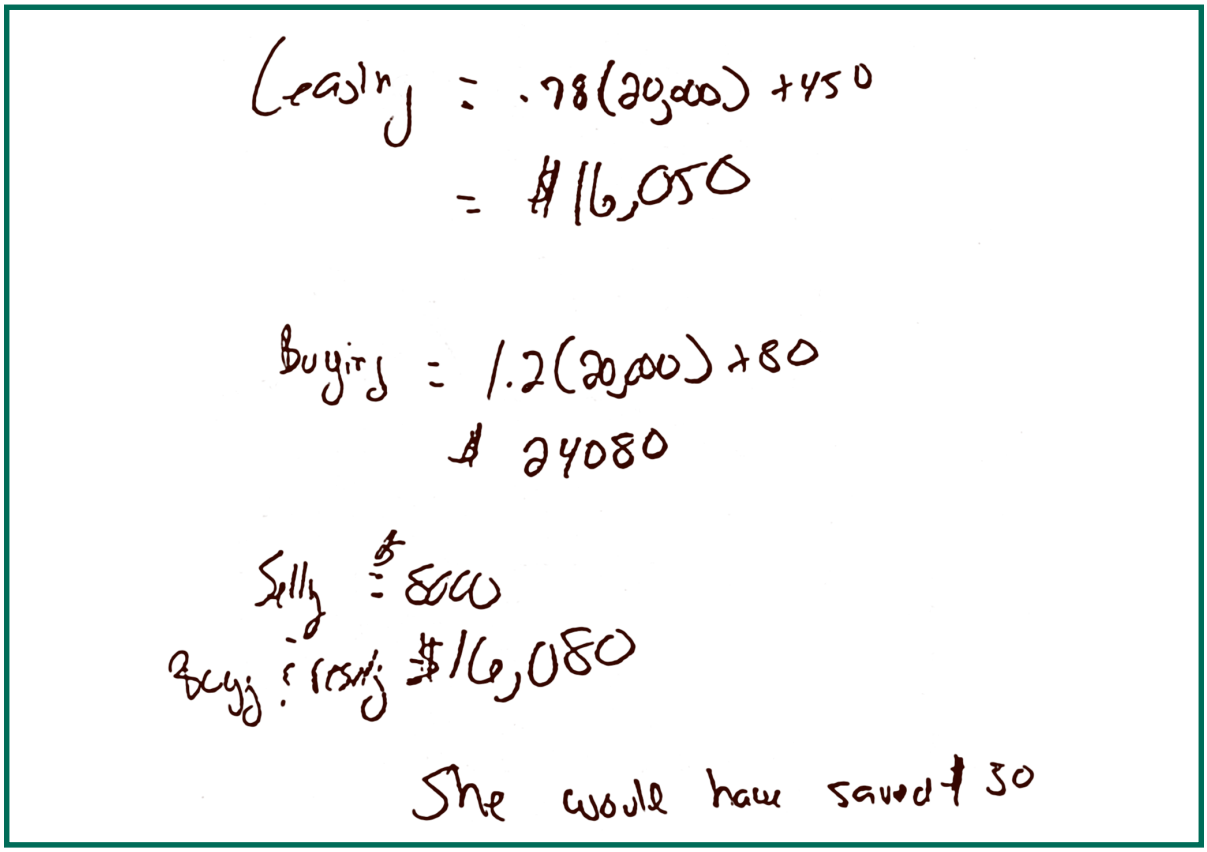
In the sixth question, students were told that Mary bought a car and sold it after 4 years. They were asked to determine the amount of money Mary could have saved if she had leased the car for 4 years instead of buying it. This question was designed to assess content from the Algebra and Functions strand and Procedural Knowledge. Students were instructed to show how they solved the problem.

> **Questions 6-7** refer to the following information.
>
> Mary is interested in leasing or buying a particular car model. She has determined that the cost of leasing this car for 4 years is $0.78p + 450$, where p is the selling price. Mary has also determined that the cost of buying this car, including finance charges, is $1.2p + 80$. At the end of 4 years, when the car is paid off, it can be sold for $0.4p$.
>
> **6.** Mary decided to buy the car at a selling price of $20,000. She kept the car for 4 years and then sold the car to recover some of the costs. If Mary had decided to lease the same car for 4 years, what amount of money would she have saved? Show the work that led to your answer.

To solve the problem, students needed to understand how to use the formulas provided, correctly substituting in the selling price of $20,000. Computing the cost of leasing is relatively straightforward, $0.78(20,000) + 450$. However, computing the cost of buying involved adding the finance charges, $1.2(20,000) + 80$, and also subtracting out the amount for which the car can be sold at the end of 4 years, $0.4(20,000)$. Finally, students had to subtract the cost of leasing from the cost of buying and reselling to get the answer of $30, the amount saved by leasing.

The responses to this question were scored on a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect." Responses scored as "satisfactory" had the correct number of dollars saved, $30, and showed an acceptable procedure for getting to that answer. The sample "satisfactory" response shows the correct procedure for calculating the cost of leasing, then the cost of buying and reselling, and, finally, the correct savings of $30.

Sample "satisfactory" response

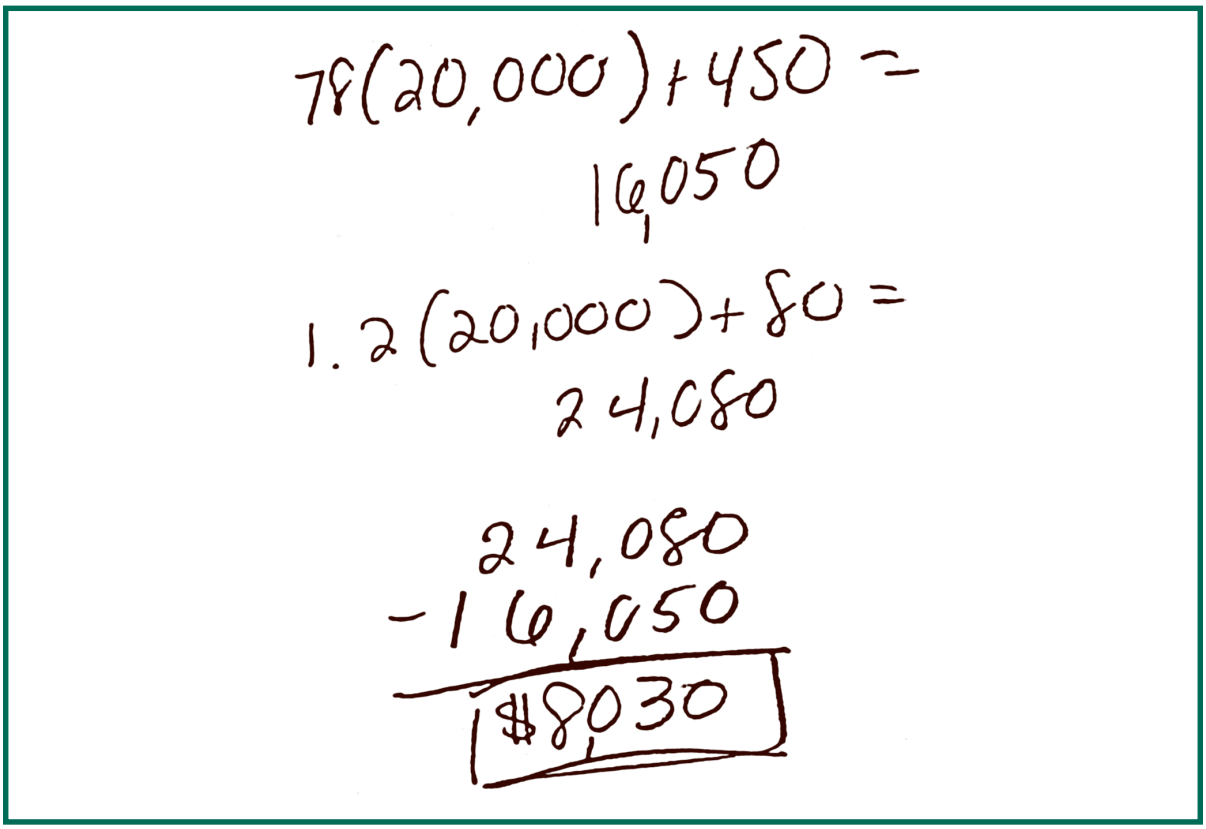


"Partial" responses satisfied one of the following criteria:

- both costs were correctly calculated, but the amount saved with leasing was not specified;
- the cost to buy the car was calculated without subtracting out the amount gained from reselling; or
- the correct amount saved, $30, was indicated, but the work involved in calculating that number was not shown.

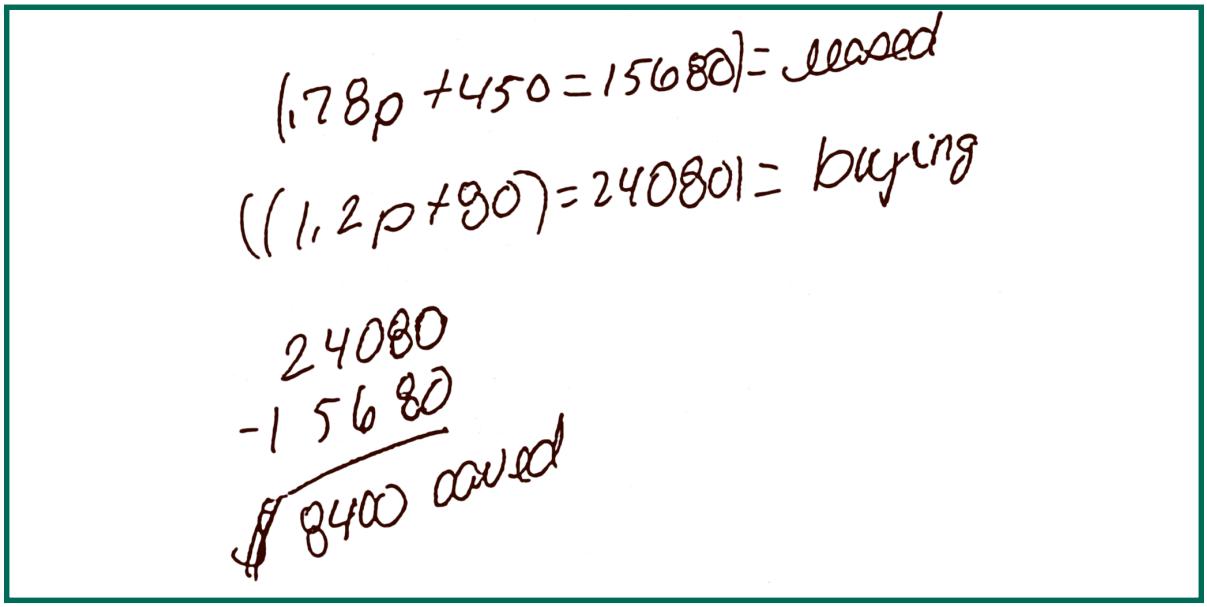
The sample "partial" response shown below includes the correctly calculated cost of leasing and buying the car but fails to take into account the amount gained from reselling the car; it therefore gives $8,030 as the amount saved by leasing.

Sample "partial" response



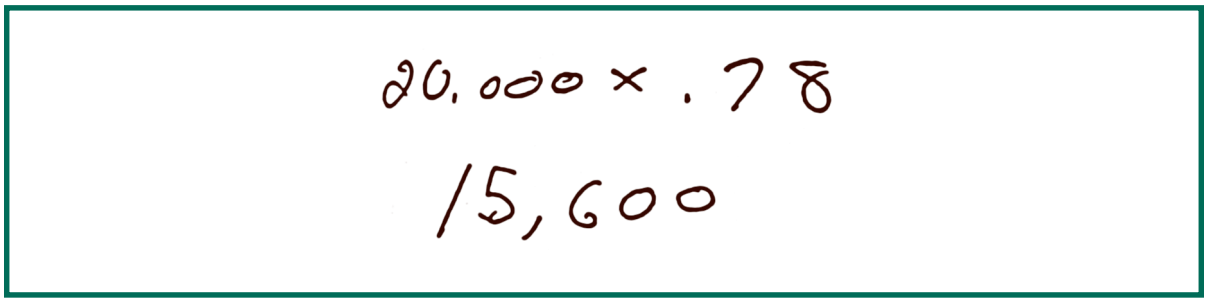
A "minimal" response is one in which the cost of either leasing or buying is correctly calculated, or the cost to buy is correctly calculated but the gain from reselling is not taken into consideration. The sample "minimal" response correctly calculates the cost of buying, and although it has the correct formula for leasing, incorrectly calculates the cost of leasing. Furthermore, in determining the dollars saved by leasing, the student failed to take into account the amount gained from reselling.

Sample "minimal" response



An "incorrect" response satisfied none of the criteria mentioned above. The following "incorrect" response correctly calculates a part of the formula for the cost of leasing, but goes no further.

Sample "incorrect" response



The information presented in Table 3.32 shows that the students did relatively well on this question. Twenty-seven percent of the responses were scored as "satisfactory," and 23 percent were scored as "partial." This means that students at least were able to calculate the cost of leasing and buying; in some cases the failure to include the gain from the resale was probably an oversight rather than inability to understand how to solve the problem.

Table 3.32 ***Score Percentages for "Find Amount Saved if Leased," Grade 12***

THE NATION'S REPORT CARD

Grade 12	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	27	23	20	21	5
Males	27	19	18	24	7
Females	27	27	21	18	3
White	33	24	20	16	4
Black	6	24	19	36	11
Hispanic	12	23	22	30	7
Asian/Pacific Islander	29	16	20	32	2
American Indian	***	***	***	***	***

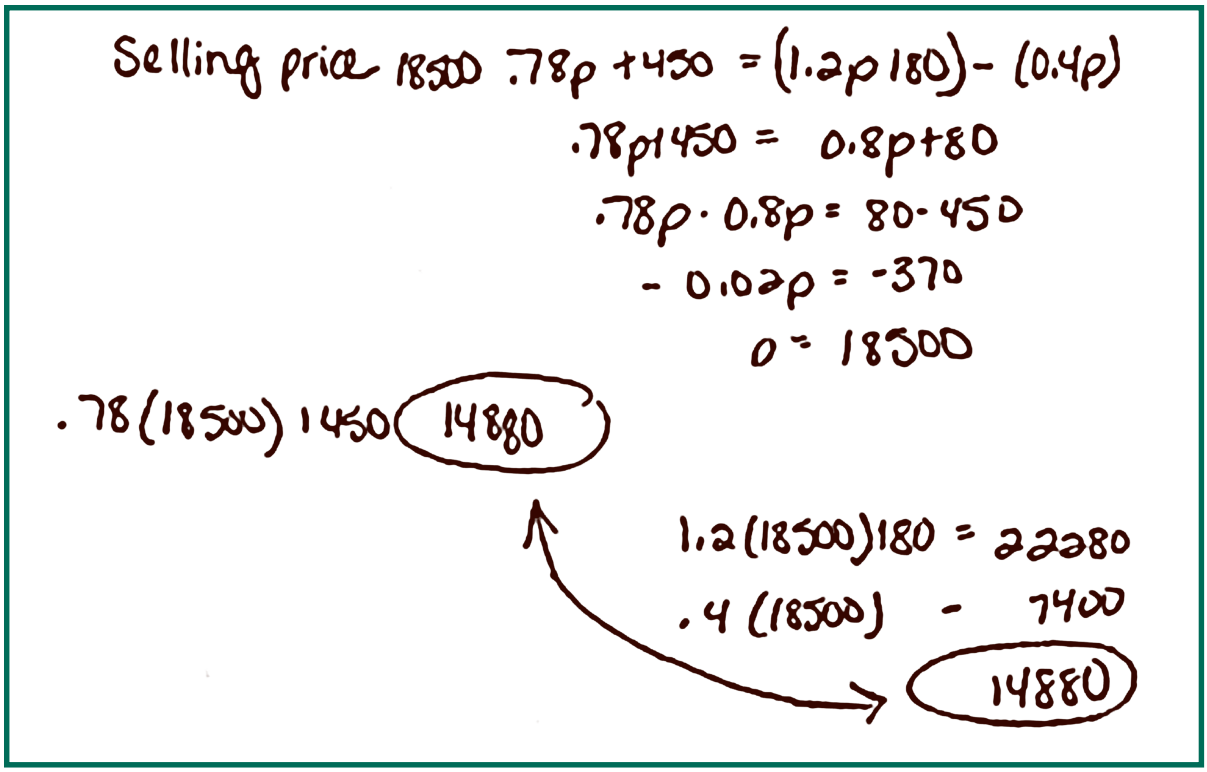
NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 7. Price lease vs. buy. In the seventh question, students were asked to find the selling price of a car for which the leasing cost for 4 years and the buying cost with a resale after 4 years are the same. As in the other questions in the question block, students also were asked to show how they got their answer. This question was classified as an Algebra and Functions question designed to assess Procedural Knowledge.

> **7.** For the particular car model that Mary is interested in, determine the selling price for which the car costs the same to buy and resell at the end of 4 years as it costs to lease the car for 4 years. Show the work that led to your answer.

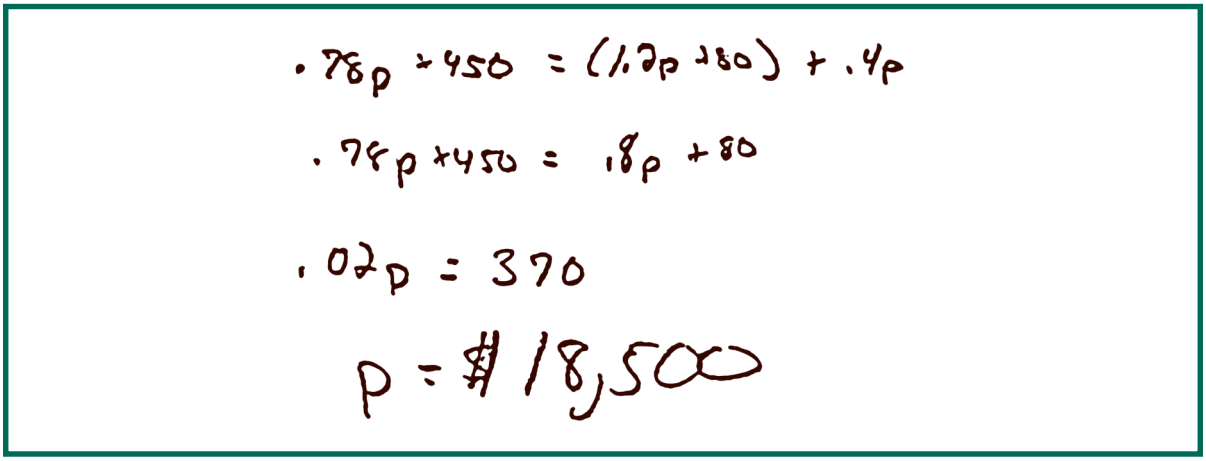
To answer this question most efficiently, students had to know how to set up an equation that equated the cost of leasing with the cost of buying and reselling, and to solve for the selling price of the car that would satisfy that equation. The responses to this question were scored using a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect." A "satisfactory" answer is one in which the students find the correct selling price of $18,500 either through setting up and solving the correct equation or through trial and error. In the sample "satisfactory" response shown below, the student wrote out the correct equation and calculated the selling price correctly. The student also included computations to show that, with the selling price of $18,500, both buying and leasing the car would cost $14,880.

Sample "satisfactory" response



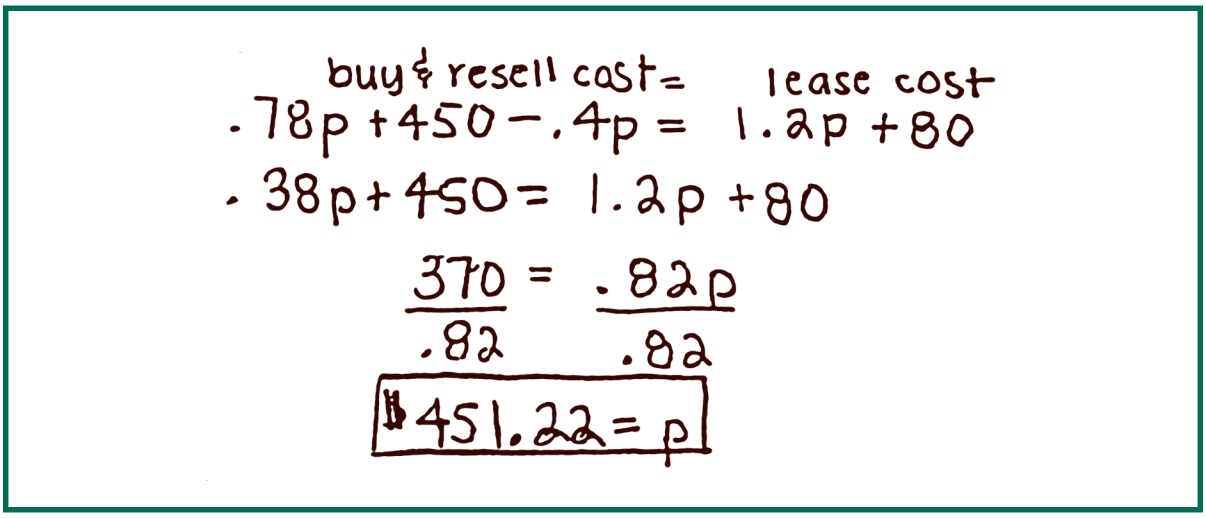
Responses scored as "partial" were those in which the correct equation was written out, but not solved, or the correct selling price was obtained, but the work contained a computational error. In the "partial" response shown, the correct answer is obtained, but there appears to be some carelessness in writing out the computations. For example, in the original equation, the student shows the resale price as being added to the selling price rather than subtracted from it. However, in the next line of the solution, the student has correctly subtracted the resale price after all.

Sample "partial" response



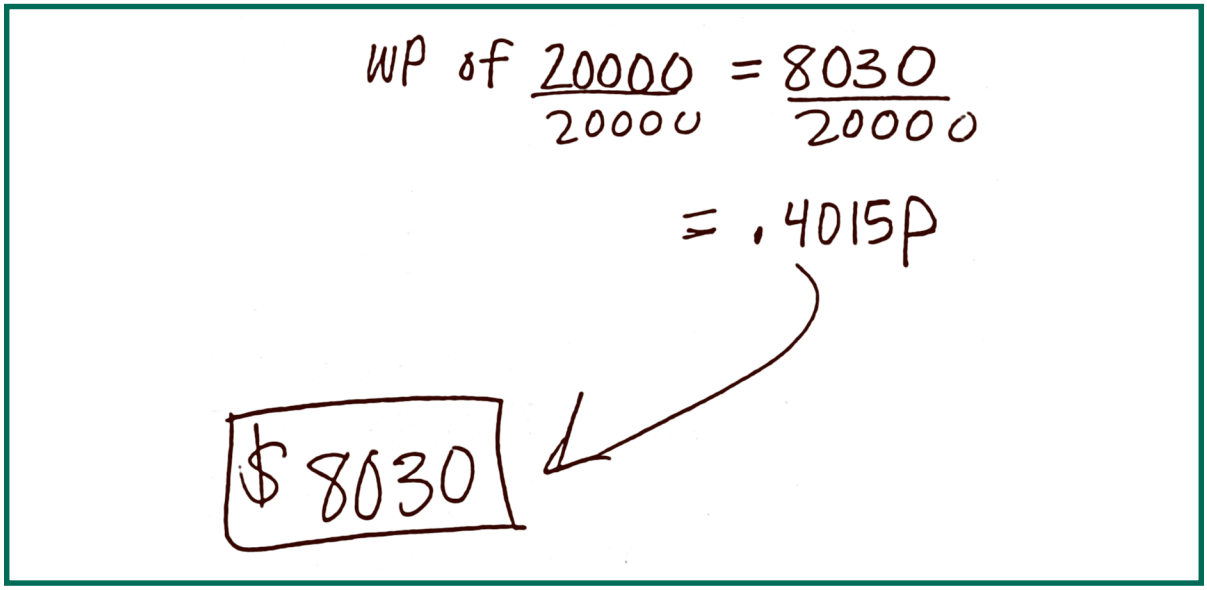
"Minimal" responses included an equation without the resale price and therefore arrival at a wrong answer; or use of the resale price, 0.4*p*, incorrectly in the algebraic relationship. In the following "minimal" response, the student confused the equations for leasing and buying the car, although the resale proceeds were correctly subtracted out from what the student labeled as the cost of buying the car. The calculations were carried out correctly but, of course, the wrong answer was obtained.

Sample "minimal" response



An "incorrect" response is one in which none of the above criteria were satisfied. The sample "incorrect" response used the selling price of the car mentioned in Question 6 and therefore appeared to show that the student did not understand how to approach solving this problem.

Sample "incorrect" response



Information on student performance on this item is presented in Table 3.33. Sixteen percent of the responses were scored as "satisfactory," while 57 percent were scored "incorrect." Student responses were more likely to be scored "satisfactory" than "partial" or "minimal," suggesting that, if they understood the question at all, they were likely to carry through to the final solution correctly.

Table 3.33 ***Score Percentages for "Price Lease vs. Buy," Grade 12***

THE NATION'S REPORT CARD

	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 12					
All Students	16	2	7	57	8
Males	17	2	7	52	11
Females	16	1	7	62	6
White	19	2	8	55	7
Black	4	1!	4	59	19
Hispanic	7	0!	3	66	9
Asian/Pacific Islander	18	3	14	57	2
American Indian	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

The fourth-grade released Theme block was about planning a Butterfly Booth for the school's science fair. Of the six questions posed, one was classified as measuring Procedural Knowledge, the remaining five were classified as measuring Problem-Solving abilities. Questions were designed to assess content in four content strands: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; and Algebra and Functions. In addition to solving the problems, students were generally asked to provide explanations of their answers, using mathematical computations, drawings, or words.

The following are findings of what students appear to know and are able to do based on their performance on the questions in the Butterfly Booth block:

- A majority of fourth-grade students appeared to be familiar with centimeter measurements and were able to measure using a ruler marked off in centimeters. A substantial proportion of students, however, appeared not to understand what it means to measure to the nearest centimeter.

- In most of the questions, students were required to solve a multistep problem. Students appeared to have difficulty with multistep problems, even those that required straightforward calculations at each step of the problem.

- Students appeared either to not understand what is meant by the word "symmetrical" or not understand how to use grid paper to show symmetry in a drawing.

- Students appeared to be familiar with using proportions in reasoning but, in many cases, they had difficulty carrying out proportional operations with acceptable precision.

- With the exception of the first question, most students attempted to answer the questions posed, even though large percentages produced responses that were scored as "incorrect."

- Although students' explanations were often incomplete, most students attempted to provide explanations for their answers.

- In many cases, students seemed to lack the mathematical knowledge needed to solve the problems. In other cases, students appeared to understand the underlying mathematics but provided incorrect or incomplete responses as a result of carelessness, inexperience in writing out solutions to problems, or confusion over the wording of the question.

The eighth-grade released Theme block was about building a doghouse. Of the 10 questions that students encountered in this block, 4 were designed to assess Conceptual Understanding, 1 to assess Procedural Knowledge, and 5 to assess Problem-Solving ability. The questions were classified in the content strands of Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; and Algebra and Functions. In three of the

questions, students were asked to write an explanation of how they solved a problem or to describe a mathematical operation. The following are findings of what students appear to know and are able to do based on their performance on the questions in the Doghouse block:

- Many eighth-grade students appeared to understand how to interpret and use linear dimensions.
- Some students showed understanding of the use of scales and ratios; however, for many students this understanding was not fully developed.
- Some students appeared to understand the relationship between perimeters and areas but had difficulty applying these concepts in a relatively complex problem.
- Many students appeared to lack the depth of mathematical understanding required to plan and execute solutions to the problems. Others appeared to have the requisite conceptual and procedural knowledge, but were not careful in reading the problems or precise in carrying out the operations necessary for solving them.
- Omit rates for the constructed-response questions were much higher than those observed at grade 4, particularly for the two questions at the end of the block, which also happened to be among the most difficult.

In the twelfth-grade released block, students were told that a family was considering whether to buy or lease a car. Most of the questions involved evaluating the costs of one of these options in relation to the other. Of the seven questions included in the block, six were designed to assess Procedural Knowledge and one to assess Problem-Solving ability. Four of the questions were classified as Number Sense, Properties, and Operations, and the other three were classified as Algebra and Functions. The following are findings based on student performance on the Buying-a-Car Theme block questions:

- Most students were able to solve problems involving simple computational skills. However, some of these students seemed to have difficulty with more complicated multistep problems, even those that required only simple computational skills at each step of the problem.
- Many, but not nearly the majority, of twelfth-grade students appeared able to solve an algebraic problem if they were given the equation and values for variables in the equation. However, many students appeared to have trouble reading through a relatively complicated word problem in order to isolate the information needed to solve the problem completely.

At *all* grade levels, the frequency with which students engaged in writing a few sentences about how to solve a mathematics problem, or engaged in writing reports or doing mathematics projects, was examined in relationship to student performance on the Theme blocks. Given the many recommendations to increase attention to extended-response problems in the classroom, it is perhaps significant that no positive relationship was seen between these two instructional practice variables and performance on the Theme blocks.