Measurement

Measuring is the process by which numbers are assigned to describe the world quantitatively. The 2007 NAEP mathematics framework includes measurement attributes such as capacity, weight or mass, time, and temperature, as well as the geometric attributes of length, area, and volume.

Assessment questions on measurement at grade 4 focus on customary units such as inch, quart, pound, and hour, and common metric units such as centimeter, liter, and gram, as well as the geometric attribute of length. At grade 8, the emphasis is on the use of square units for measuring area and surface area, cubic units for measuring volume, degrees for measuring angles, and rates. More emphasis is placed on area and angle measurements than on linear measurements.

Subtopics in the measurement content area are

- **Measuring physical attributes**
- **Systems of measurement**

Student Results

In Puerto Rico, the averages of the question scores in measurement were 0.26 at grade 4 and 0.23 at grade 8 (figures 5 and 6). These scores were lower on average than the scores for public school students in the nation.

Results by gender for this content area varied by grade in Puerto Rico. While the average of the question scores in measurement did not differ significantly between male and female fourth-graders in Puerto Rico, the score for male eighth-graders was higher than that for their female peers. In the nation, the score was higher for male students than for female students at both grades.

The next few pages contain sample questions from the measurement content area in the 2007 NAEP mathematics assessment.

Figure 5. Average of the question scores in NAEP mathematics for measurement at grade 4, by gender: 2007

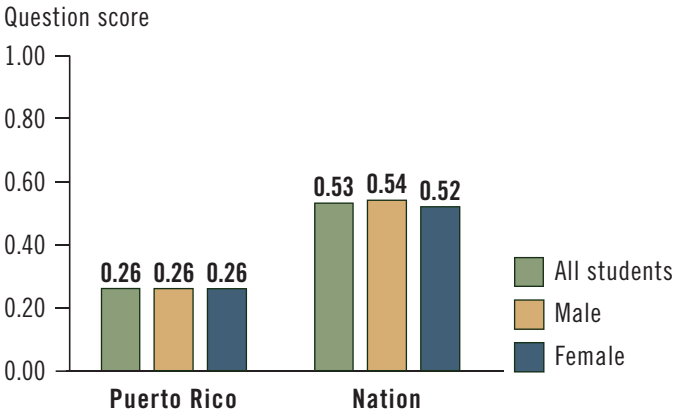
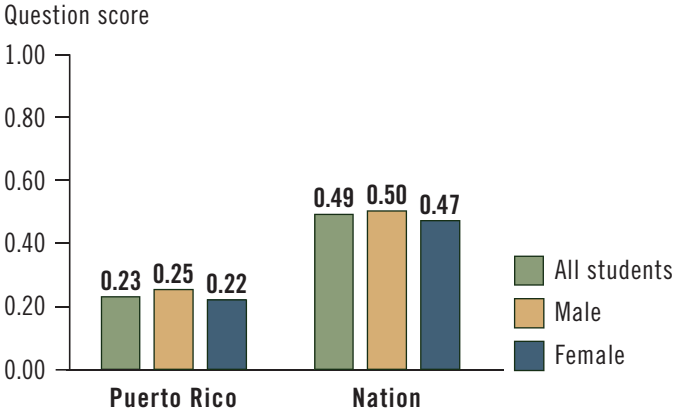


Figure 6. Average of the question scores in NAEP mathematics for measurement at grade 8, by gender: 2007



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample question 5 addresses the systems of measurement subtopic. This subtopic includes questions about appropriate units of measurement, appropriate sizes of measurements in problem situations, conversions within the same measurement system, and determining when highly accurate measurements are important.

Sample question 6 addresses the measuring physical attributes subtopic. This subtopic includes questions about identifying attributes that can be measured; comparing objects or estimating the size of an object with respect to a given attribute such as length, time, or temperature; using appropriate measurement instruments; and solving problems involving the perimeter of plane figures or the area of squares and rectangles.

Sample Multiple-Choice Question

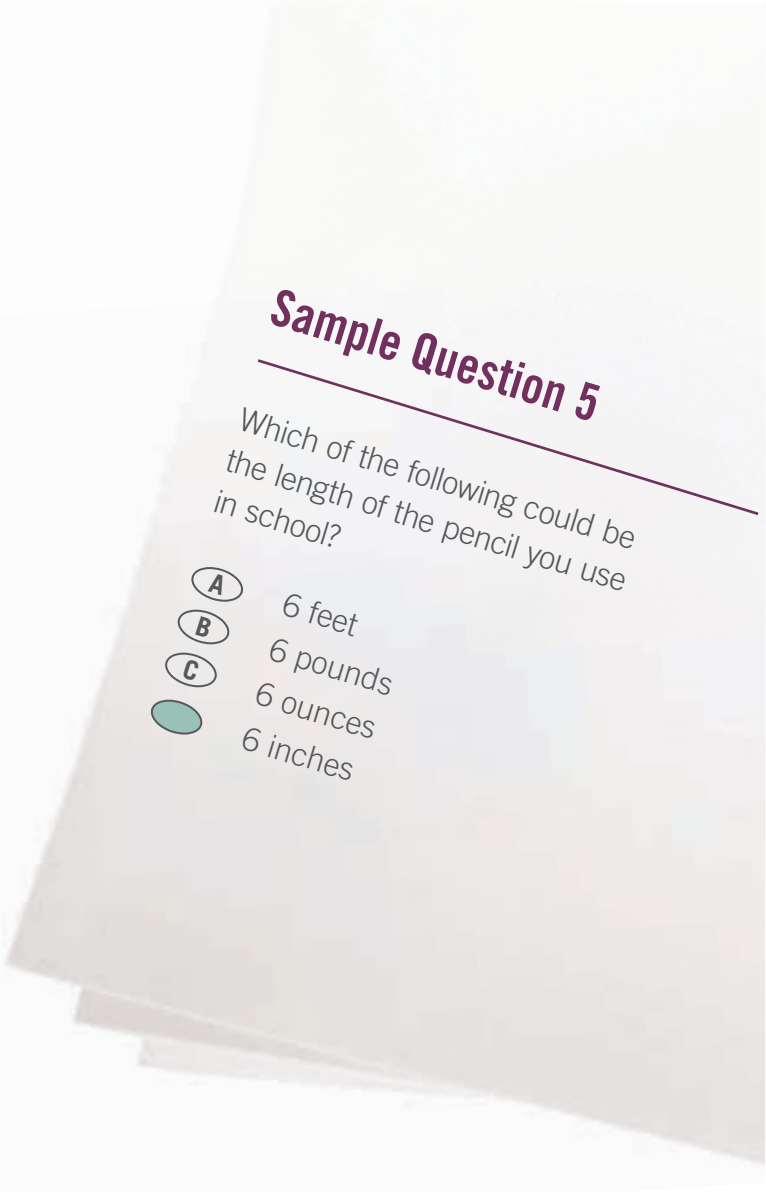
Sample question 5 asks students to identify the measurement that could be the length of a pencil. The framework objective for this question asks students to select or use the appropriate type of unit for the attribute being measured such as length, time, or temperature.

In Puerto Rico, 55 percent of grade 4 students answered this question correctly (choice D). The incorrect answer choices for the question are inappropriate measurements or units of measurement.

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	22	5
Choice B	9	1
Choice C	12	5
Choice D	55	89
Omitted	2	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.



Sample Constructed-Response Question

Sample question 6 is a two-step problem about carpeting a room. The first step in solving the problem is to determine the amount of carpet needed to cover the floor of a room (180 square feet), and the second step is to compute the cost of the carpet for this room (180 × $2.60 = $468). This question was included in a section that allowed the use of a calculator. The framework objective of solving problems involving the area of squares and rectangles is measured in this question.

Student responses for this question were rated using the following three-level scoring guide:

Correct A response that had the correct area of the floor (180 square feet) and the correct cost of the carpet ($468)

Partial A response that had the correct area of the floor but did not have the correct cost of the carpet

OR

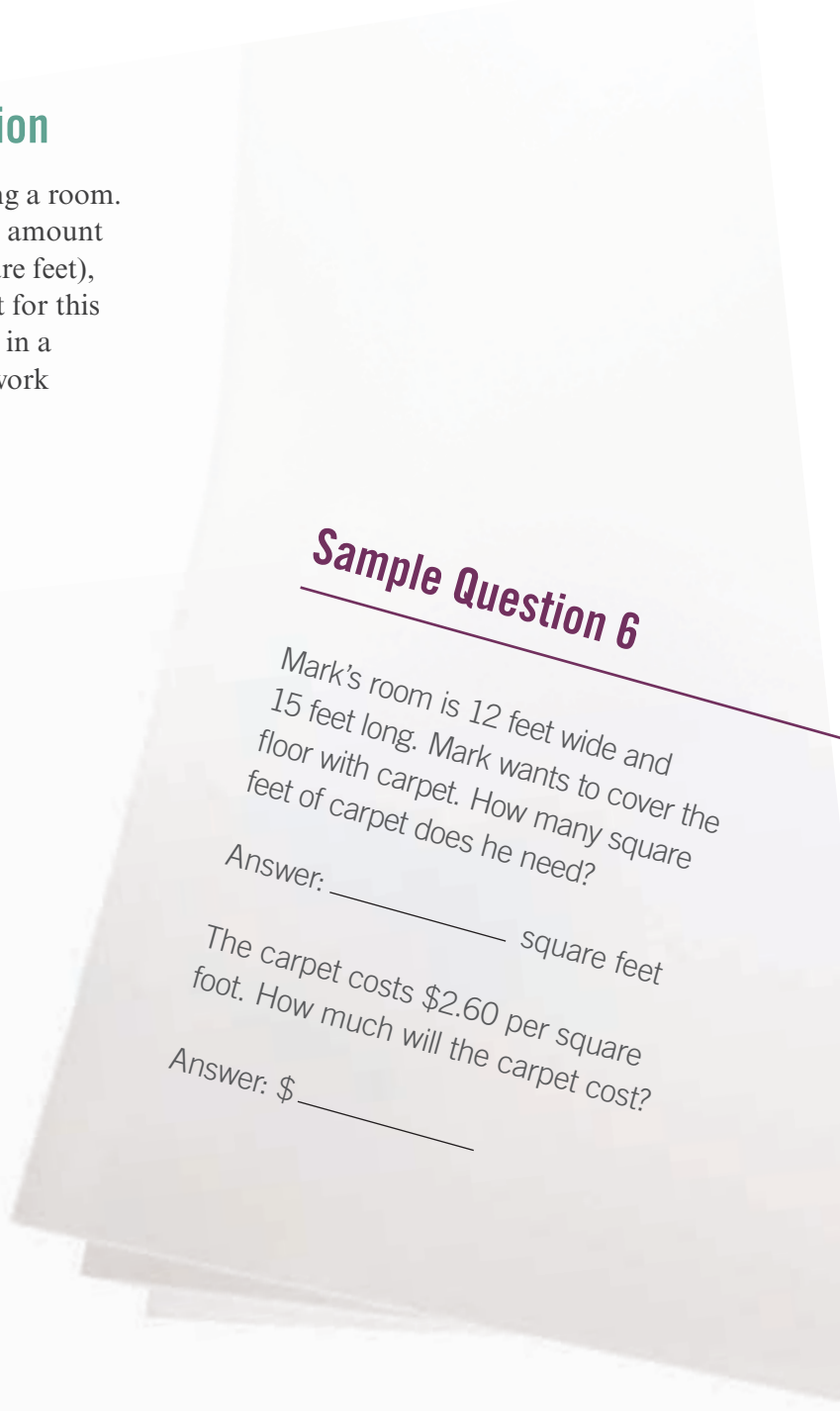
A response that did not have the correct area of the floor but had the correct cost of the carpet

OR

A response that did not have the correct area of the floor but correctly computed the cost of the carpet based on this incorrect area

Incorrect All incorrect responses

In Puerto Rico, 2 percent of the grade 4 student responses were rated "Correct," and 5 percent of the responses were rated "Partial."



Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Correct	2	9
Partial	5	30
Incorrect	87	58
Omitted	6	3

NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample questions 7 and 8 address the measuring physical attributes subtopic. This subtopic includes questions about comparing objects or estimating the size of an object with respect to a measurement attribute such as length, angle, weight, or mass; using appropriate measurement instruments; solving problems involving the perimeter or area of plane figures; and solving problems involving the volume or surface area of solids.

Sample Multiple-Choice Question

Sample question 7 asks students to identify the measurement that could be the area of a typical classroom floor. This question was included in a section that allowed the use of a calculator. The framework objective measured in this question is about estimating the size of an object with respect to a given measurement attribute (e.g., area).

In Puerto Rico, 47 percent of grade 8 students answered this question correctly (choice B). The incorrect answer choices for the question are inappropriate measurements or inappropriate units of measurement.

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Choice A	22	13
Choice B	47	68
Choice C	10	13
Choice D	7	2
Choice E	12	3
Omitted	2	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 7

Of the following, which is the best estimate for the area of a typical classroom floor?

- (A) 700 feet
- (B) 700 square feet
- (C) 700 cubic feet
- (D) 700 yards
- (E) 700 square yards

Sample Constructed-Response Question

Sample question 8 involves reasoning about the units on a scale. The figure on the left shows that two tick marks on the scale represent $\frac{1}{2}$ pound. Therefore, each tick mark represents $\frac{1}{4}$ pound, so the total weight of the two apples on the right is $\frac{7}{4}$ pounds or $1\frac{3}{4}$ pounds. The framework objective measured in this question is to compare objects with respect to length, area, volume, angle measurement, weight, or mass.

Student responses for this question were rated using the following two-level scoring guide:

Correct A response of $\frac{7}{4}$ or $1\frac{3}{4}$

Incorrect All incorrect responses

In Puerto Rico, 6 percent of the grade 8 student responses were rated "Correct."

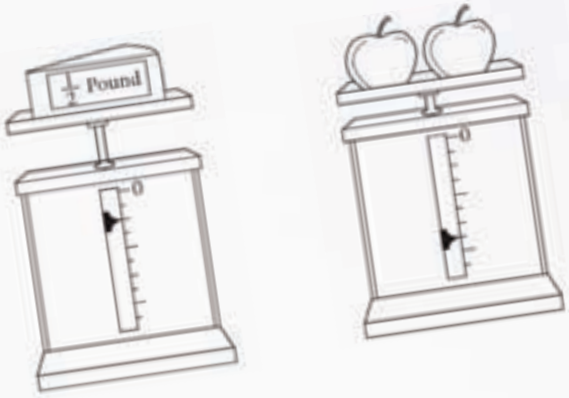
Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Correct	6	48
Incorrect	90	51
Omitted	4	1

NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 8

Both figures below show the same scale. The marks on the scale have no labels except the zero point.



The weight of the cheese is $\frac{1}{2}$ pound. What is the total weight of the two apples?

Total weight of two apples = _____ pounds.

Geometry

School geometry roughly mirrors the historical development of geometry, which began as a practical collection of rules for calculating lengths, areas, and volumes of common shapes. This expanded over time to include the study of structures in space and ideas of symmetry and transformation.

Students at grade 4 are expected to be familiar with a library of simple figures and their attributes, both in the plane and in space. At grade 8, students are expected to be familiar with the properties of plane figures, especially parallel and perpendicular lines, angle relations in polygons, cross sections of solids, and the Pythagorean theorem.

Subtopics in the geometry content area are

- **Dimension and shape**
- **Transformation of shapes and preservation of properties**
- **Relationships between geometric figures**
- **Position and direction**
- **Mathematical reasoning**

Student Results

In Puerto Rico, the averages of the question scores in the geometry content area were 0.39 at grade 4 and 0.24 at grade 8 (figures 7 and 8). Both scores were lower than those for public school students in the nation.

The patterns in results by gender for geometry were similar in Puerto Rico and the nation. Among fourth- and eighth-graders in both Puerto Rico and the nation, the average of the question scores in geometry for male students was not significantly different from that of female students.

The next few pages contain sample questions from the geometry content area in the 2007 NAEP mathematics assessment.

Figure 7. Average of the question scores in NAEP mathematics for geometry at grade 4, by gender: 2007

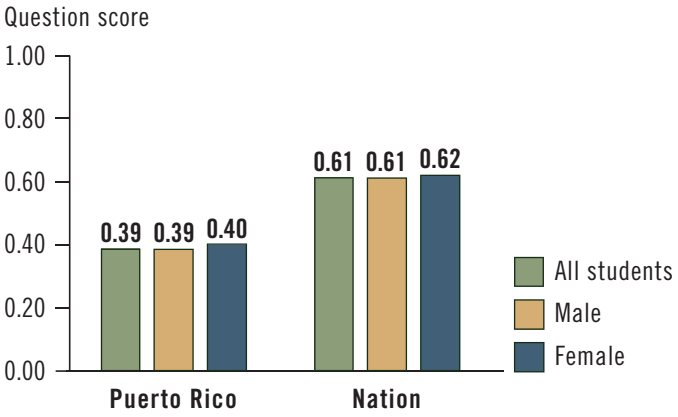
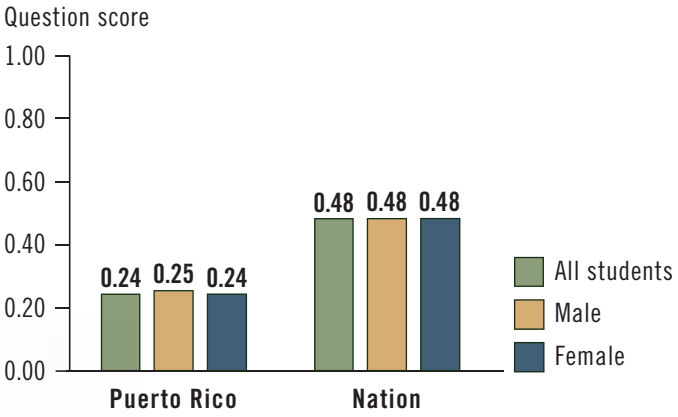


Figure 8. Average of the question scores in NAEP mathematics for geometry at grade 8, by gender: 2007



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample question 9 addresses the dimension and shape subtopic. This subtopic includes questions about identifying, describing, or drawing geometric figures in the plane; identifying or informally describing real-world objects using geometric shapes; and attributes of two- and three-dimensional shapes.

Sample question 10 addresses the mathematical reasoning subtopic. This subtopic includes questions about distinguishing objects in a collection that satisfy a given geometric definition.

Sample Multiple-Choice Question

Sample question 9 asks students to determine the number of right angles contained in a figure representing the path from a student's house to the school. The framework objective measured in this question is to identify or draw angles and other geometric figures in the plane.

In Puerto Rico, 42 percent of grade 4 students answered this question correctly (choice C).

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	22	34
Choice B	20	21
Choice C	42	41
Choice D	12	2
Omitted	3	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 9

The picture shows Rachel's path to school. How many right angle turns does Rachel make to get to school?

- (A) Two
- (B) Three
- Five
- (D) Seven

Sample Multiple-Choice Question

Sample question 10 is a reasoning question that presents four figures that have two different attributes: color and shape. The question presents information about both the color (the figure is shaded) and the shape (the figure is not a triangle) of a specific figure, requiring the student to select a rectangle that is shaded (figure D). The framework objective for this question asks students to distinguish which objects in a collection satisfy a given geometric definition and sometimes to explain their choices.

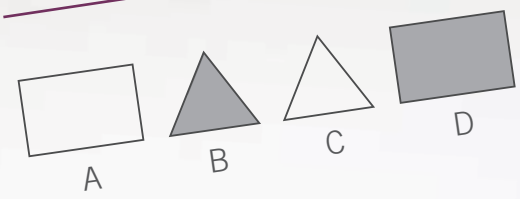
In Puerto Rico, 69 percent of grade 4 students answered this question correctly (choice D).

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	10	5
Choice B	11	3
Choice C	5	1
Choice D	69	90
Omitted	4	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 10



Melissa chose one of the figures above.

- The figure she chose was shaded.
- The figure she chose was not a triangle.

Which figure did she choose?

(A) A
(B) B
(C) C
D

Photo by Norma Curet, Communications Office, Puerto Rico Department of Education

Sample question 11 addresses the position and direction subtopic. This subtopic includes questions about relative positions of points and lines including midpoints, parallel and perpendicular lines, and points of intersection; cross sections of solids; and the representation of geometric figures in a rectangular coordinate plane.

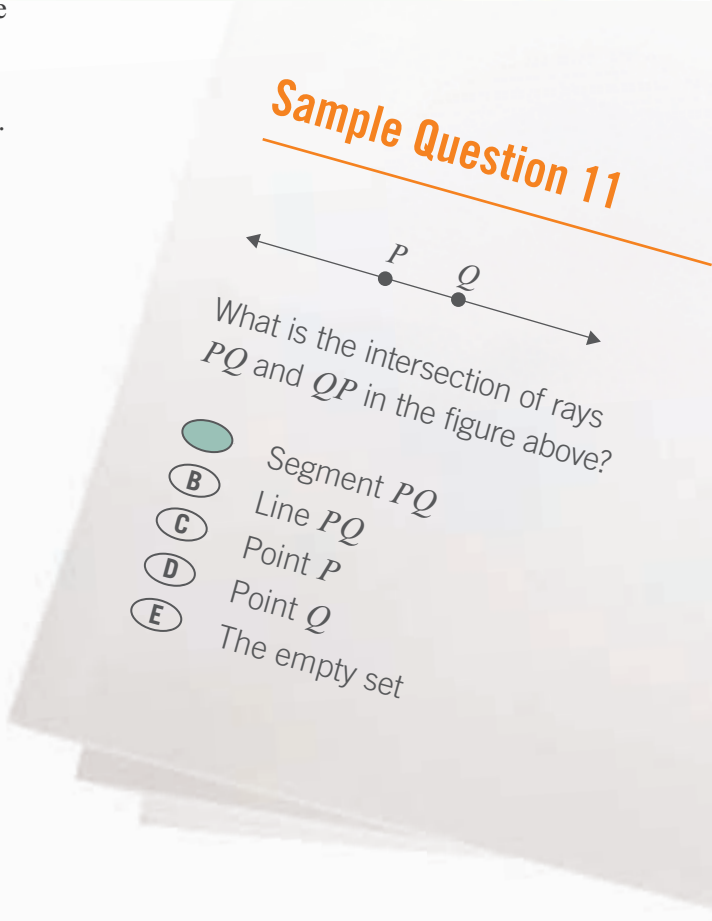
Sample question 12 addresses the subtopic of relationships between geometric figures. This subtopic includes questions about the properties of and relationships between geometric shapes in two and three dimensions; properties of and relationships between parallel or intersecting lines; and the Pythagorean theorem.

Sample Multiple-Choice Question

Sample question 11 is based on the definitions of line, ray, line segment, and the intersection of geometric figures. The framework objective measured in this question addresses how to describe the intersection of two or more geometric figures in the plane (e.g., intersection of a circle and a line).

In Puerto Rico, 29 percent of grade 8 students answered this question correctly (choice A). Some misconceptions represented by the incorrect answer choices for this question are given below:

- The union of rays PQ and QP is line PQ (choice B).
- Point P is a point on ray QP (choice C), and point Q is a point on ray PQ (choice D).
- Rays PQ and QP have no points in common, so the intersection is the empty set (choice E).



Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Choice A	29	40
Choice B	57	41
Choice C	4	3
Choice D	1	1
Choice E	8	14
Omitted	1	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Constructed-Response Question

Sample question 12 is presented in a real-world setting and requires students to analyze the relationship between the radius of the tennis balls in a can and the minimum height of the can. To answer the question, students need to recognize that the minimum height of the can is 3 times the diameter of each ball, and that the diameter of each ball is $2 \times 3 = 6$ centimeters. Therefore, the minimum height of the can is 18 centimeters. This question was included in a section that allowed the use of a calculator. The framework objective for this question is to represent problem situations with simple geometric models to solve mathematical or real-world problems.

Student responses for this question were rated using the following three-level scoring guide:

Correct A response that had the correct height of the can (18 centimeters) and a correct diagram or complete explanation

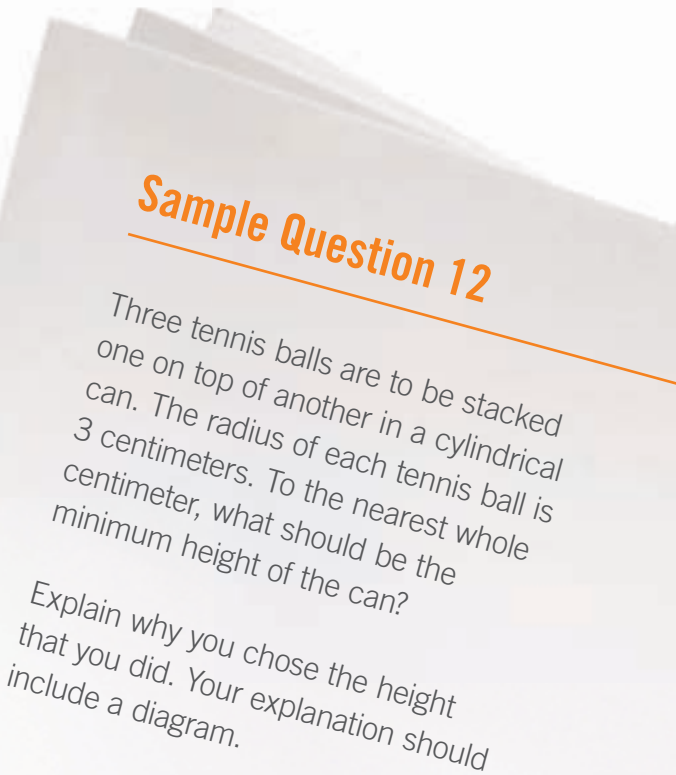
Partial A response that had the correct height of the can (18 centimeters), with an incomplete, incorrect, or missing diagram or explanation

OR

A response that had a correct diagram or complete explanation, with an incorrect height or no height for the can

Incorrect All incorrect responses

In Puerto Rico, 1 percent of the grade 8 student responses were rated "Correct," and 4 percent of the responses were rated "Partial."



Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Correct	1	17
Partial	4	20
Incorrect	53	49
Omitted	38	13

NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.