

The 
Nation's
Report Card

Mathematics 2007
Performance of Public School
Students in Puerto Rico

FOCUS ON THE CONTENT AREAS

National Assessment of Educational Progress at Grades 4 and 8

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Photo by Norma Curet, Communications Office, Puerto Rico Department of Education

What is The Nation's Report Card™?

The Nation's Report Card™ informs the public about the academic achievement of elementary and secondary students in the United States. Report cards present the findings of the National Assessment of Educational Progress (NAEP), a continuing and nationally representative measure of achievement in various subjects over time.

For over three decades, NAEP assessments have been conducted periodically in reading, mathematics, science, writing, U.S. history, civics, geography, and other subjects. By collecting and reporting information on student performance at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education. Only information related to academic achievement and relevant variables is collected. The privacy of

individual students and their families is protected, and the identities of participating schools are not released.

NAEP is a congressionally authorized project of the National Center for Education Statistics (NCES) within the Institute of Education Sciences of the U.S. Department of Education. The Commissioner of Education Statistics is responsible for carrying out the NAEP project. The National Assessment Governing Board oversees and sets policy for NAEP.

Executive Summary

In 2007, public school students in Puerto Rico at grades 4 and 8 participated in a Spanish-language version of the National Assessment of Educational Progress (NAEP) in mathematics. A representative sample of approximately 2,800 students from 100 public schools was assessed at each grade.

This report contains performance results on NAEP mathematics questions for public school students in Puerto Rico and the nation. Results are presented as the average scores for the correct answers (see box below)—expressed as decimals ranging from 0.00 to 1.00—for all the questions in the NAEP mathematics assessment and for questions in each of the five mathematics content areas (as shown in figures A and B).

At grade 4

- The average of the question scores for students in Puerto Rico was lower than the score for students in the nation overall and within each content area.
- There was no statistically significant difference in performance between male and female students in Puerto Rico overall and in each content area.

At grade 8

- The overall average of the question scores for students in Puerto Rico was lower than the score for students in the nation. Results were similar for each content area.
- While there was no significant difference between the performance of male and female students in Puerto Rico overall, male students had a higher score than female students in the measurement content area, and female students had a higher score than their male peers in the data analysis and probability content area.

Figure A. Average of the question scores in NAEP mathematics at grade 4, by content area: 2007

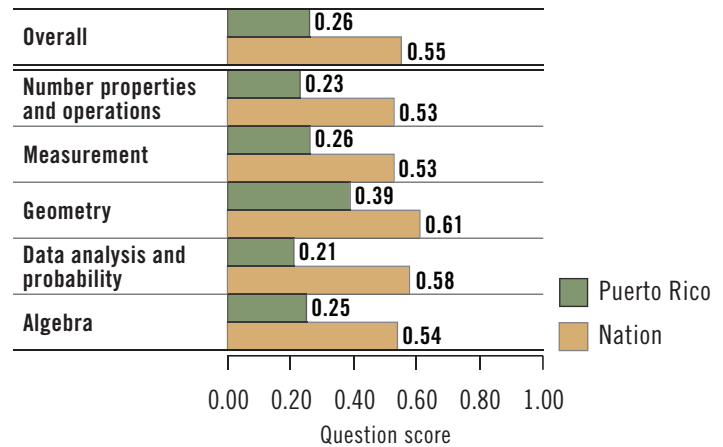
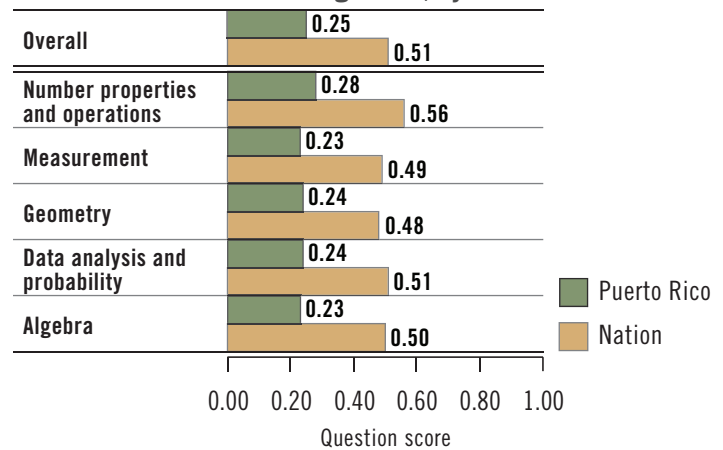


Figure B. Average of the question scores in NAEP mathematics at grade 8, by content area: 2007



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Interpreting Results for Puerto Rico

Question scores are calculated as the percentages of correct responses for multiple-choice questions and for constructed-response questions that are scored either correct or incorrect. For constructed-response questions that allow for partial credit, the question score is the sum of the percentage of students receiving full credit and a fraction of the percentage receiving partial credit. Individual question scores are then averaged together to report an average question score for the entire mathematics assessment or for each of the five content areas.

Because of technical concerns regarding the placement of the 2007 results for Puerto Rico on the NAEP mathematics scale, performance results could not be reported as average scale scores for Puerto Rico in this report, and students' performance in 2007 could not be compared to performance in previous assessments.

When comparing the results for students in Puerto Rico to students in the nation, it is important to consider some of the differences in demographics. For example, between 76 and 78 percent of fourth- and eighth-graders in Puerto Rico attended public schools compared to 91 percent in the nation. All of the public school students in Puerto Rico were eligible for the National School Lunch Program compared to between 41 and 46 percent of fourth- and eighth-graders in the nation.

The Mathematics Assessment

The Framework

The content of all NAEP assessments is determined by subject-area frameworks developed by the National Assessment Governing Board in a comprehensive process involving a broad spectrum of stakeholders, including teachers, curriculum specialists, subject-matter specialists, school administrators, parents, and members of the general public. Frameworks in NAEP are developed to survey student understanding over a broad range of content. The current frameworks are available at <http://www.nagb.org/>.

The NAEP mathematics framework serves as the blueprint for the assessment, describing the specific mathematical skills that should be assessed at grades 4 and 8. Two dimensions of mathematics, *content areas* and *mathematical complexity*, are used to guide the assessment. Each question is designed to measure one of five content areas. However, certain aspects of mathematics, such as computation, occur in all content areas.

The levels of complexity of a mathematics question are defined in the shaded box on the following page. This differs from the difficulty of a question, which is defined by the percentage of correct student responses; a lower percentage of students answering correctly indicates a more difficult question. For example, a question asking grade 8 students to interpret a number given in scientific notation has a low level of complexity but may be difficult, that is, few students answer it correctly. A question with a high level of complexity might ask students to explain or justify their solutions to a problem.



Mathematics Content Areas

Number properties and operations measures students' understanding of ways to represent, calculate, and estimate with numbers.

Measurement measures students' knowledge of measurement attributes, such as capacity and temperature, and geometric attributes, such as length, area, and volume.

Geometry measures students' knowledge and understanding of shapes in a plane and in space.

Data analysis and probability measures students' understanding of data representation, characteristics of data sets, experiments and samples, and probability.

Algebra measures students' understanding of patterns, using variables, algebraic representation, and functions.

Levels of Mathematical Complexity

Low complexity questions typically specify what a student is to do, which is often to carry out a routine mathematical procedure.

Moderate complexity questions involve more flexibility of thinking and often require a response with multiple steps.

High complexity questions make heavier demands and often require abstract reasoning or analysis in a novel situation.

Assessment Design

The NAEP mathematics framework specifies the percentage of questions to be assessed in each content area. The distribution of questions across content areas for grades 4 and 8 is shown in table 1. Topics in the Puerto Rico mathematics standards, *Estándares de Excelencia Programa de Matemáticas* (Departamento de Educación 2000) are organized around the same five content areas as the *Mathematics Framework for the 2007 National Assessment of Educational Progress* (National Assessment Governing Board 2006).

The entire NAEP mathematics assessment consists of 10 sections of mathematics questions at each grade. Each section includes 14 to 20 questions covering all five content areas. Because the assessment covers a breadth of content and includes more questions than any one student could reasonably answer, each student takes just a portion of the assessment, answering two sections of mathematics questions.

Students were asked to respond to multiple-choice questions, as well as constructed-response questions that required them to produce their own answers. Scorers evaluated student responses written in Spanish. Some questions at each grade level incorporated the use of calculators, rulers (grade 4), ruler/protractors (grade 8), or manipulatives such as geometric shapes or spinners that were provided

for students. On approximately one-third of the assessment, a four-function calculator was provided for students at grade 4, and a scientific calculator was provided for students at grade 8.

The 2007 NAEP mathematics assessments at grades 4 and 8 were translated into Spanish for administration in Puerto Rico (see the Technical Notes for more information). The content was the same as that for all students in the nation. Students in Puerto Rico were given a total of 70 minutes to complete the two sections of mathematics questions (20 minutes more than the time allotted for students assessed nationally).

Table 1. Target percentage distribution of NAEP mathematics questions, by grade and content area: 2007

Content area	Grade 4	Grade 8
Number properties and operations	40%	20%
Measurement	20%	15%
Geometry	15%	20%
Data analysis and probability	10%	15%
Algebra	15%	30%

SOURCE: U.S. Department of Education, National Assessment Governing Board, *Mathematics Framework for the 2007 National Assessment of Educational Progress*, 2006.



Reporting Student Results

Approximately 76 percent of fourth-graders and 78 percent of eighth-graders in Puerto Rico were enrolled in public schools in 2007 (table 2). Representative samples of public schools and their students at grades 4 and 8 in Puerto Rico participated in the NAEP mathematics assessment during the same time period in which students were assessed nationally. Approximately 2,800 students from 100 public schools at each grade were assessed in Puerto Rico.

Table 2. Percentage of students in Puerto Rico and the nation in NAEP mathematics, by grade and selected characteristics: 2007

Characteristic	Grade 4		Grade 8	
	Puerto Rico	Nation	Puerto Rico	Nation
Private school	24	9	22	9
Public school	76	91	78	91
Eligible for free/reduced-price school lunch	100	46	100	41
Identified as students with disabilities	20	14	14	13

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment. Percentages of Puerto Rico students in private and public schools provided by the Puerto Rico General Council of Education and the Puerto Rico Department of Education.

Performance Results

Student results are presented in this report as the average of the question scores for all the questions in the assessment and for questions in each content area. In the following pages, sample questions are shown as examples of what students in Puerto Rico know and are able to do within each of the five mathematics content areas. The percentages of students in each response category are presented in a table for each question. For a multiple-choice question, the response categories are the answer choices for the question. For a constructed-response question, the response categories are the ratings defined in the scoring guide for the question. The row with the percentages for the correct or most complete response is highlighted in the tables. Results for other released sample questions from the 2007 mathematics assessment are available at <http://nces.ed.gov/nationsreportcard/itmrsl/>.

Interpreting Results for Puerto Rico

Because of technical concerns regarding the placement of 2007 results for Puerto Rico on the NAEP mathematics scale, students' performance results could not be reported as average scale scores for Puerto Rico in this report, and students' performance in 2007 could not be compared to performance in previous assessments. NCES is continuing to investigate ways to make meaningful comparisons between the performance of students in Puerto Rico and students in the nation (see the Technical Notes for more information). For comparison purposes, the average of the question scores and percentages of responses for sample questions are shown for public school students in the nation (excluding Puerto Rico).

NAEP results are based on samples of student responses, and there is a margin of error associated with every result. Any differences that are mentioned in the text as "higher" or "lower" are statistically significant at the .05 level. Statistical significance is not marked in the figures and tables in the body of this report, but in almost all cases, the average of the question scores and percentages of correct or most complete responses for Puerto Rico were lower than those for the nation.



Overall Results

At both grades 4 and 8, the overall average of the question scores on the NAEP mathematics assessment was lower for students in Puerto Rico than for public school students nationally (figures 1 and 2). In Puerto Rico, the averages of the question scores were 0.26 for grade 4 and 0.25 for grade 8, compared to national averages of 0.55 and 0.51, respectively.

The pattern of results in Puerto Rico by gender differed slightly from that in the nation. While the national average of the question scores for male students was higher than for female students at both grades 4 and 8, there was no significant difference in the average of the question scores for male and female students in Puerto Rico at either grade.

Figure 1. Average of the question scores in NAEP mathematics at grade 4, by gender: 2007

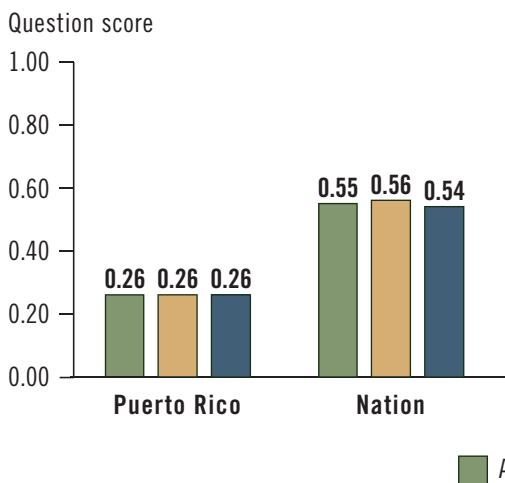
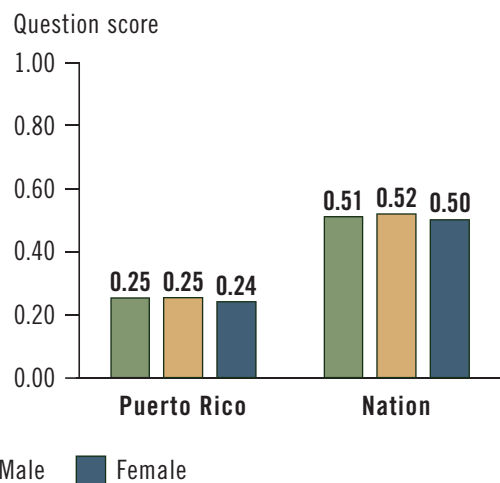


Figure 2. Average of the question scores in NAEP mathematics at grade 8, by gender: 2007



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

The Question Score

For a multiple-choice or a constructed-response question that is scored either “Correct” or “Incorrect,” the question score is the percentage of correct responses expressed as a decimal. For a constructed-response question in which students could earn partial credit if they did not have a completely correct response, the question score is computed by adding the percent of students receiving full credit to a fraction of the percent of students receiving partial credit. The average of the question scores for a set of assessment questions ranges from 0.00 to 1.00. See the Technical Notes for more details about the question score, including an example of how it is calculated. Question scores for released questions from the 2007 NAEP mathematics assessment are also presented in appendix tables A-3 and A-4.



Photo by Norma Curet, Communications Office, Puerto Rico Department of Education

Number Properties and Operations

Numbers are our main tools for describing the world quantitatively, so the ability to use number properties and operations is an important expectation of the 2007 NAEP mathematics framework. This content area focuses on students' understanding of ways to represent, calculate, and estimate numbers. In addition to basic computation skills, NAEP assesses students' ability to order and compare numbers and to solve problems in real-world settings using arithmetic operations.

At grade 4, students are expected to have a solid grasp of whole numbers and a beginning understanding of fractions. At grade 8, students are expected to be able to work with rational numbers (both fractions and decimals), ratios and proportional reasoning, scientific notation, and naturally occurring irrational numbers such as square roots and π ().

Subtopics of the number properties and operations content area covered in the assessment are

- Number sense
- Estimation
- Number operations
- Ratios and proportional reasoning
- Properties of number and operations

Student Results

In Puerto Rico, the average of the question scores in the number properties and operations content area was 0.23 for fourth-graders (figure 3). The score for eighth-graders was 0.28 (figure 4). These scores were lower on average for students in Puerto Rico than for public school students in the nation.

At both grades 4 and 8, the average of the question scores in the number properties and operations content area did not differ significantly between male and female students in Puerto Rico. For students in both grades nationally, the scores in this content area for male students were higher than those for female students.

The next few pages contain sample questions from the number properties and operations content area in the 2007 NAEP mathematics assessment. These sample questions do not represent the entire range of content assessed in this area and are for illustrative purposes only. For each sample question, the percentages of students for each response category are reported for both Puerto Rico and the nation. The row corresponding to the correct or most complete response is highlighted.

Figure 3. Average of the question scores in NAEP mathematics for number properties and operations at grade 4, by gender: 2007

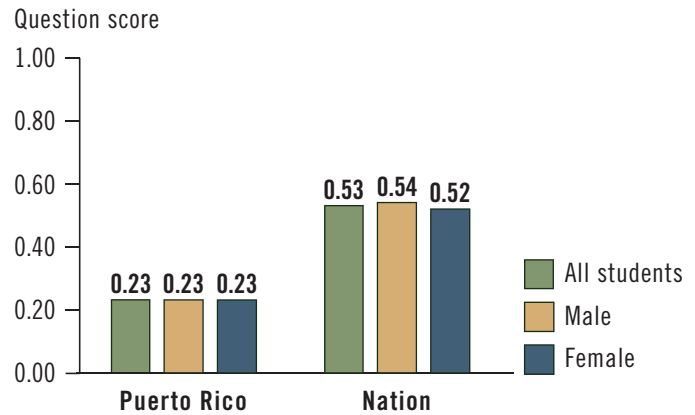
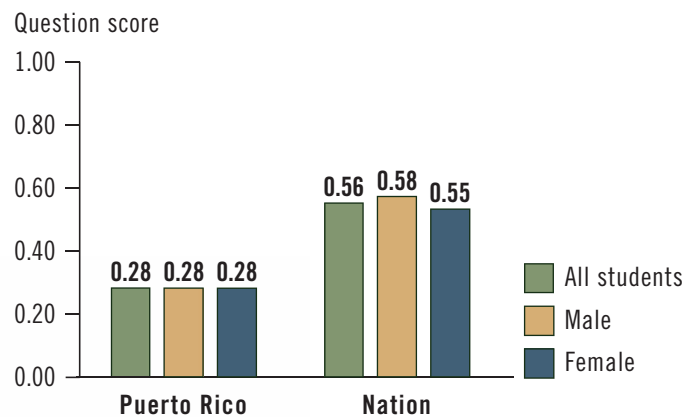


Figure 4. Average of the question scores in NAEP mathematics for number properties and operations at grade 8, by gender: 2007



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample question 1 addresses the number operations subtopic. This subtopic includes questions about computation, the effects of operations on numbers, relationships between operations, and application problems involving numbers and operations.

Sample question 2 addresses the properties of number and operations subtopic. This subtopic includes questions about odd and even numbers, factors of whole numbers, basic properties of operations, and explaining mathematical relationships.



Sample Multiple-Choice Question

Sample question 1 is a computation problem in a real-world setting. To solve the problem of finding how many more people went to the zoo requires subtraction with regrouping to find the difference between the number of people at the zoo on Saturday and Sunday: $983 - 789 = 194$. This question was included in a section that allowed the use of a calculator. The framework objective measured in this question is about solving application problems involving numbers and operations.

In Puerto Rico, 17 percent of grade 4 students answered this question correctly (choice A). Some misconceptions and errors represented by the incorrect answer choices for this question are given below:

- Subtracting incorrectly (choice B and choice C)
- Adding the two numbers given in the problem instead of subtracting (choice D)

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	17	73
Choice B	3	2
Choice C	4	5
Choice D	74	19
Omitted	2	1

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 1

On Saturday 789 people went to the zoo. On Sunday 983 people went to the zoo. How many more people went to the zoo on Sunday than on Saturday?

- 194
- (B) 204
- (C) 206
- (D) 1,772

Sample Constructed-Response Question

Sample question 2 is a conceptual question about even and odd numbers. To answer the question, students must recognize that while an even number of objects can be paired, an odd number of objects will have one remaining when paired. This question is about pairing the students in three classes, and Mr. West’s class, with 25 students, will have one child without a partner when the students line up with partners. The framework objective for this question asks students to identify odd and even numbers.

Student responses for this question were rated using the following three-level scoring guide:

- Correct** A response that answered “Mr. West’s Class” (or 25) with an explanation or picture indicating that there are an odd number of students in Mr. West’s class
- Partial** A response that answered “Mr. West’s Class” (or 25) without an acceptable explanation for this choice
OR
A response with an explanation or a picture indicating an odd number of students without answering “Mr. West’s Class” (or 25)
- Incorrect** All incorrect responses

In Puerto Rico, 12 percent of the grade 4 student responses were rated “Correct,” and 24 percent of the students gave responses that were rated “Partial.”

Sample Question 2

Number of Students	Ms. King’s Class	Mr. West’s Class	Ms. Chang’s Class
	20	25	28

In each class listed above, the students are lining up with a partner to walk to lunch. Which class will have one child with no other child for a partner?

Answer: _____

Explain your choice.

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Correct	12	56
Partial	24	19
Incorrect	52	23
Omitted	10	2

NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample question 3 addresses the ratios and proportional reasoning subtopic. This subtopic includes questions about ratios, proportions, and percentages.

Sample question 4 addresses the number sense subtopic. This subtopic includes questions about place value, ordering and comparing numbers, and translating between different representations of numbers. The focus of this subtopic is on rational numbers and includes scientific notation and absolute value.



Sample Multiple-Choice Question

Sample question 3 asks students to compute a percentage in a real-world context. This question was included in a section that allowed the use of a calculator. The framework objective measured in this question is to solve problems involving percentages (including percent increase and decrease, interest rates, tax, discount, tips, or part/whole relationships).

In Puerto Rico, 28 percent of grade 8 students answered this question correctly (choice D). Some common errors that may lead to the incorrect answer choices for this question are given below:

- Interpreting the decimal representation of 91% (i.e., 0.91) as the percentage (choice A)
- Dividing the whole (57) by the part (52) and not converting to a percentage (choice B)
- Interpreting the score as the percentage (choice C)
- Finding the difference between the values ($57 - 52 = 5$) and subtracting this as a percentage from 100% (choice E)

Sample Question 3

Tammy scored 52 out of 57 possible points on a quiz. Which of the following is closest to the percent of the total number of points that Tammy scored?

- (A) 0.91%
- (B) 1.10%
- (C) 52%
- (D) 91%
- (E) 95%

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Choice A	27	12
Choice B	9	4
Choice C	19	5
Choice D	28	62
Choice E	15	16
Omitted	1	1

NOTE: Detail may not sum to totals because of rounding.
 SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Constructed-Response Question

Sample question 4 is a short constructed-response question that asks students to express the sum of three fractions as a decimal, where each fraction has a denominator that is a power of 10. This question was included in a section that allowed the use of a calculator. This is a conceptual question about place value, measuring the framework objective of using place value to model and describe integers and decimals.

Student responses for this question were rated using the following two-level scoring guide:

Correct A response of 0.777

Incorrect All incorrect responses

In Puerto Rico, 7 percent of the grade 8 student responses were rated “Correct.”

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Correct	7	45
Incorrect	80	52
Omitted	13	3

NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 4

Add the numbers

$$\frac{7}{10}, \frac{7}{100}, \text{ and } \frac{7}{1,000}.$$

Write this sum as a decimal.

