

# *WTAQ*—A Computer Program for Calculating Drawdowns and Estimating Hydraulic Properties for Confined and Water-Table Aquifers

By Paul M. Barlow *and* Allen F. Moench

## **Abstract**

The computer program *WTAQ* calculates hydraulic-head drawdowns in a confined or water-table aquifer that result from pumping at a well of finite or infinitesimal diameter. The program is based on an analytical model of axial-symmetric ground-water flow in a homogeneous and anisotropic aquifer. The program allows for well-bore storage and well-bore skin at the pumped well and for delayed drawdown response at an observation well; by including these factors, it is possible to accurately evaluate the specific storage of a water-table aquifer from early-time drawdown data in observation wells and piezometers. For water-table aquifers, the program allows for either delayed or instantaneous drainage from the unsaturated zone. *WTAQ* calculates dimensionless or dimensional theoretical drawdowns that can be used with measured drawdowns at observation points to estimate the hydraulic properties of confined and water-table aquifers. Three sample problems illustrate use of *WTAQ* for estimating horizontal and vertical hydraulic conductivity, specific storage, and specific yield of a water-table aquifer by type-curve methods and by an automatic parameter-estimation method.

## **INTRODUCTION**

Determination of aquifer hydraulic properties is a basic component of most ground-water supply and contaminant-transport investigations. A frequently used method for estimating hydraulic properties is graphical type-curve analysis of aquifer tests, in which dimensionless type curves derived from an assumed analytical model of ground-water flow to a pumped well are used to analyze time-drawdown measurements of hydraulic head in observation wells and piezometers. These analyses are done to estimate the transmissivity and storativity of confined aquifers or the hydraulic conductivity and specific yield of water-table (unconfined) aquifers. An alternative approach to dimensionless type-curve analysis is to generate dimensional time-drawdown curves from the analytical model that are compared directly to the measured values. In this approach, the hydraulic properties of the model are adjusted in a series of model simulations until the model-calculated drawdowns closely match the measured values. This procedure is called model calibration and can be done graphically, as in the dimensionless type-curve approach, or automatically by use of a parameter-estimation technique.

Many analytical models have been developed for evaluation of axial-symmetric flow to a well that pumps from a confined or water-table aquifer. The ability of these models to represent realistic field conditions such as well-bore storage and skin, partial penetration of wells, and, in the case of a water-table aquifer, drainage from the unsaturated zone, has steadily improved since the pioneering work of Theis (1935), who presented a transient analytical model of flow to a fully penetrating well in a confined aquifer. Recently, Moench (1997) developed an analytical model of flow to a partially penetrating, finite-diameter well in a homogeneous, anisotropic water-table aquifer. The model accounts not only for well-bore storage and skin at the pumped well, but also for delayed drawdown response of an observation well. By including these factors, it is possible to accurately evaluate the specific storage of a water-table aquifer from early-time drawdown data in observation wells and piezometers. It is also theoretically possible to use the model to interpret pumped-well data. For confined aquifers, the model expands upon the work of Dougherty and Babu (1984) and allows for anisotropic hydraulic conductivity. For unconfined aquifers, the model expands upon the early work of Boulton (1954, 1963) and of Neuman (1972, 1974) and allows for well-bore storage and skin, delayed piezometer response, and delayed drainage from the unsaturated zone.

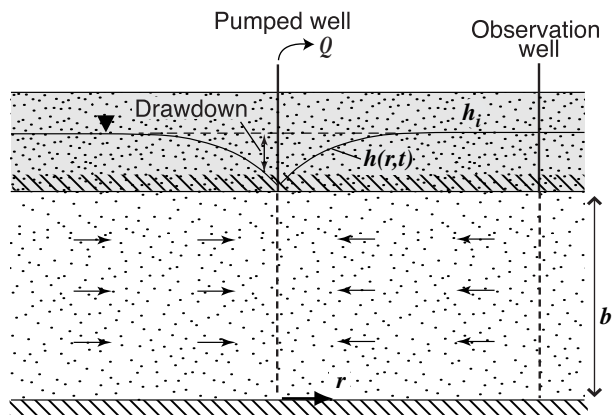
This report describes and provides documentation for the computer program *WTAQ*, which is based on Moench's (1997) analytical model for axial-symmetric flow in a confined or water-table aquifer. *WTAQ* calculates dimensionless or dimensional drawdowns that can be used with measured drawdowns at observation points to estimate hydraulic properties of confined and water-table aquifers. Three sample problems illustrate use of *WTAQ* for estimating horizontal and vertical hydraulic conductivity, specific storage, and specific yield of a water-table aquifer by graphical, type-curve methods and by an automatic parameter-estimation method. The report also includes instructions for constructing the necessary data-input files for program execution.

The authors thank Thomas Reilly (U.S. Geological Survey) for providing the results of a numerical simulation of the hypothetical aquifer described in the sample problems.

## AXIAL-SYMMETRIC FLOW IN CONFINED AND WATER-TABLE AQUIFERS

When a well is pumped at a constant rate  $Q$ , ground-water levels around the well decline, and a hydraulic gradient is established that causes ground water to flow toward the well. These water-level declines are referred to as drawdowns, and are largest in the immediate vicinity of the well (fig. 1). When the aquifer from which the well pumps is homogeneous, of infinite lateral extent, horizontal, and of uniform thickness, drawdowns will be symmetric about the axis of the well, and the resulting flow toward the well will have axial symmetry. Conceptually and mathematically, the simplest type of axial-symmetric flow occurs in an aquifer that is overlain and underlain by impermeable geologic materials that prevent vertical flow into or out of the aquifer (confined aquifer). If the pumped well fully penetrates the thickness ( $b$ ) of a horizontal confined aquifer, then the flow caused by pumping will be horizontal (that is, one-dimensional, or radial), and drawdown in the aquifer will be a function of the radial distance from the axis of the pumped well ( $r$ ) and time ( $t$ ) (fig. 1). If, however, the pumped well only partially penetrates the thickness of the aquifer, flow will consist of both horizontal and vertical components (that is, two-dimensional flow), and drawdown will be a function of the vertical position in the aquifer ( $z$ ) as well as the radial distance from the pumped well and time (fig. 2).

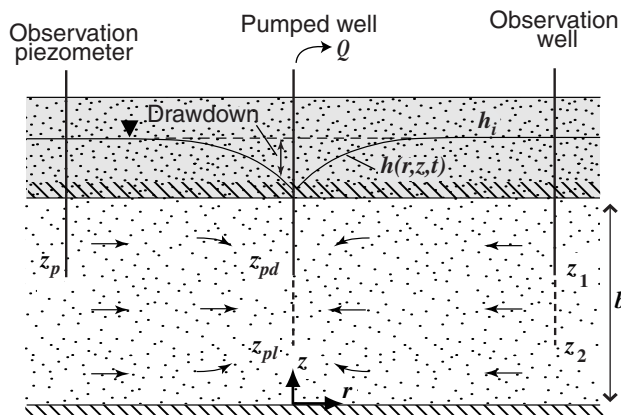
Theis (1935) developed an analytical model for one-dimensional, radial flow in a confined aquifer in which the hydraulic conductivity of the aquifer is assumed to be isotropic (that is,  $K_r = K_z$ , where  $K_r$  and  $K_z$  are the horizontal and vertical hydraulic conductivity of the aquifer, respectively). His solution provides a means to determine the transmissivity ( $T = K_r b$ ) and storativity (or storage coefficient) ( $S = S_s b$ ) of the aquifer, where  $S_s$  is specific storage. Later, Hantush (1961) developed an analytical model for two-dimensional, axial-symmetric flow in confined aquifers that accounts for the effects of partial penetration of the pumped well and anisotropy of the aquifer material ( $K_r \neq K_z$ ).



**EXPLANATION**

- AQUIFER
- CONFINING LAYER
- IMPERMEABLE (NO-FLOW) BOUNDARY
- DIRECTION OF GROUND-WATER FLOW
- POTENTIOMETRIC SURFACE IN AQUIFER—Dashed portion indicates initial potentiometric surface
- SCREENED INTERVAL OF WELL

**Figure 1.** One-dimensional, axial-symmetric flow in a confined aquifer with a fully penetrating pumped well. [ $b$ , aquifer thickness;  $h(r,t)$ , head in aquifer at radial distance  $r$  from axis of pumped well at time  $t$ ;  $h_i$ , initial head in aquifer;  $Q$ , pumping rate of well.]



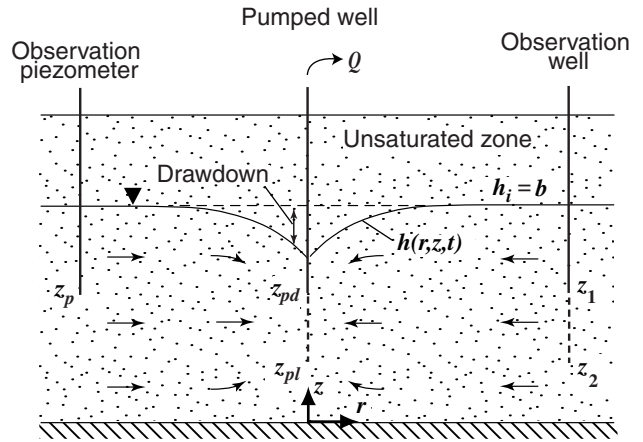
**EXPLANATION**

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




**Figure 2.** Two-dimensional, axial-symmetric flow in a confined aquifer with a partially penetrating pumped well. [ $b$ , aquifer thickness;  $h(r,z,t)$ , head in aquifer at radial distance  $r$  from axis of pumped well and depth  $z$  below top of aquifer at time  $t$ ;  $h_i$ , initial head in aquifer;  $Q$ , pumping rate of well;  $z_{pd}$ ,  $z_{pl}$ , depth below top of aquifer to the top and bottom, respectively, of the screened interval of the pumped well;  $z_1$ ,  $z_2$ , depth below top of aquifer to the top and bottom, respectively, of the screened interval of the observation well;  $z_p$ , depth below top of aquifer to center of piezometer.]

Two-dimensional, axial-symmetric flow in a water-table aquifer is more complex than that in a confined aquifer because the water table, which is the upper boundary of the aquifer, is free to move in response to pumping (fig. 3). When the water table is lowered, the saturated thickness of the aquifer is reduced; this reduction in thickness contrasts with the response of a confined aquifer in which the thickness of the aquifer remains constant during pumping. Furthermore, as the water table is lowered by pumping, water drains from pores at the water table and from the overlying unsaturated zone and flows vertically into the aquifer. The specific yield ( $S_y$ ) of a water-table aquifer is defined as the volume of pore water that is released from storage per unit surface area of aquifer per unit decline in the water table (Freeze and Cherry, 1979). Specific yields of water-table aquifers are much larger than the storativities of confined aquifers because specific yields reflect an actual drainage of the pores in the zone above the water table. In a confined aquifer, on the other hand, water is released from storage only by compression of the aquifer matrix and expansion of the water.

A comparison between drawdowns at observation wells in hypothetical confined and water-table aquifers is illustrated in figure 4 for the condition of a fully penetrating pumped well. The aquifers are equivalent in all respects with the exception of the upper boundary condition of each aquifer. Because most of the water pumped from the water-table aquifer is supplied by dewatering of pores, drawdown at the observation well in the water-



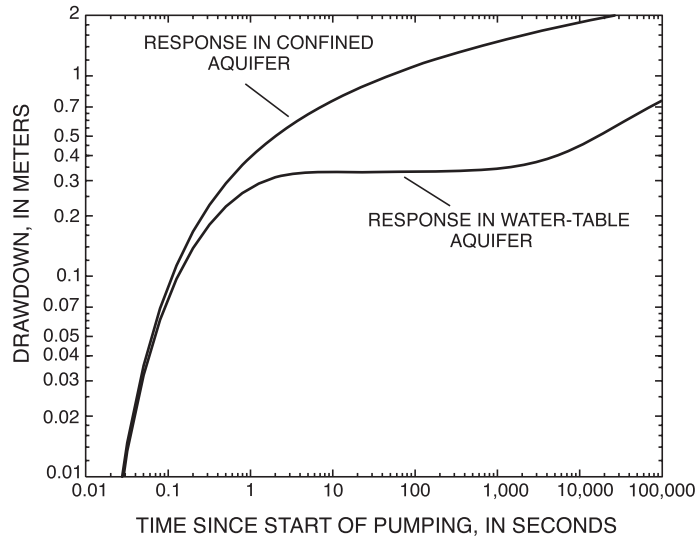
### EXPLANATION

- |   |                                |   |  |
|---|--------------------------------|---|--|
|  | AQUIFER                        |  | WATER TABLE—Dashed portion indicates initial water table |
|  | IMPERMEABLE (NO-FLOW) BOUNDARY |  | SCREENED INTERVAL OF WELL                                |
|  | DIRECTION OF GROUND-WATER FLOW |   |  |

**Figure 3.** Two-dimensional, axial-symmetric flow in a water-table aquifer with a partially penetrating pumped well. [ $b$ , initial saturated thickness of aquifer;  $h(r, z, t)$ , head in aquifer at radial distance  $r$  from axis of pumped well and depth  $z$  below initial water table at time  $t$ ;  $h_i$ , initial head in aquifer;  $Q$ , pumping rate of well;  $z_{pd}$ ,  $z_{pl}$ , depth below initial water table to the top and bottom, respectively, of the screened interval of the pumped well;  $z_1$ ,  $z_2$ , depth below initial water table to the top and bottom, respectively, of the screened interval of the observation well;  $z_p$ , depth below initial water table to center of piezometer.]

table aquifer is always less and is slower than drawdown in the confined aquifer. A characteristic feature of the drawdown curve that is frequently seen in log-log plots for the water-table aquifer is the inflection in the curve during the intermediate time (between about 1 to 3,000 seconds after the start of pumping in fig. 4). This inflection is caused by vertical drainage from the zone above the water table, which slows the drawdown response in the aquifer. Following the inflection there is an apparent (due to log-scale compression of time) increase in the rate of drawdown as the drainage from the zone above the water table keeps pace with the decline in the free surface and "...the time-drawdown curve merges with the Theis non-equilibrium curve associated with the coefficient of storage for water-table conditions" (that is, specific yield) (Boulton, 1963, p. 470).

Two conceptual approaches have been used to analytically model the release of water from the zone above the water table. In the first approach, developed primarily by Boulton (1954, 1963), drainage from the zone above the water table is assumed to occur gradually in a manner that varies exponentially with time in response to a unit decline in the elevation of the water table. As part of his mathematical treatment of the problem, Boulton introduced an empirical drainage constant. In the second approach, developed by Neuman (1972, 1974), water is assumed to be released instantaneously from the zone above the water table in response to lowering of the water table. Neuman justified this assumption, in part, based upon numerical models that appeared to show that effects of flow from the unsaturated zone upon drawdown in the saturated zone were negligible.

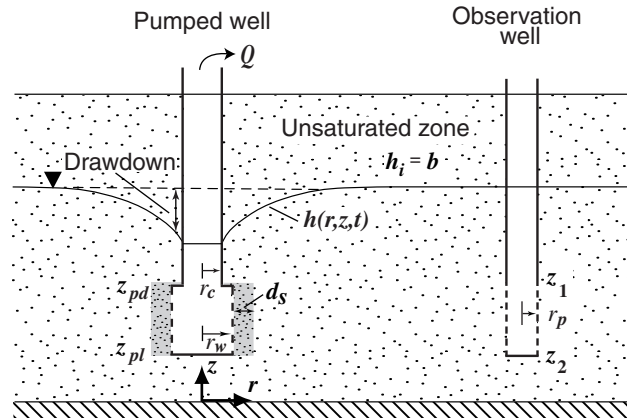


**Figure 4.** Drawdown at an observation well 3.2 meters from a fully penetrating pumped well in hypothetical confined and water-table aquifers.






Moench (1995) developed an analytical model of axial-symmetric flow in water-table aquifers that combines the Boulton and Neuman approaches. His model allows for either gradual (delayed) or instantaneous release of water from the zone above the water table, and also accounts for vertical flow in the aquifer. Moench showed that inclusion of the delayed drainage process in the analytical model improved the match between theoretically derived drawdowns and those measured in shallow, water-table observation piezometers. He also infers that the use of multiple empirical drainage constants, as opposed to a single constant, may better represent the delayed drainage process, but did not explicitly incorporate such constants into his model.

Most of the early analytical models of axial-symmetric flow in confined and water-table aquifers were developed with the assumption that the pumped well is infinitesimal in diameter, such as shown schematically in figures 1–3. This assumption is made to simplify the mathematics, but it is unrealistic because it does not allow for water stored in the well bore (fig. 5). Also unrealistic is the lack of accounting for the resistance to flow, or “skin effect,” that can occur at the interface between the aquifer and well bore. This resistance is caused by restrictions at the well screen, bridging by sand particles across screen openings, and damage to the aquifer caused by drilling. As a result of these factors, a skin of material of lower hydraulic conductivity than that of the aquifer can form around the well-bore casing (fig. 5). Well-bore storage and well-bore skin at the pumped well can affect drawdowns in the aquifer and at the pumped well itself. In particular, it has been shown (Narasimhan and Zhu, 1993; Moench, 1997) that such effects can mask early-time drawdown data that are required for accurate determination of the specific storage of a water-table aquifer. Well-bore storage effects were included in analytical models developed by Papadopoulos and Cooper (1967) and Papadopoulos (1967); well-bore storage and skin effects combined have been considered by Agarwal and others (1970), Dougherty and Babu (1984), and Moench (1997).

Additionally, most analytical models have been developed under the assumption that observation wells and piezometers are infinitesimal in diameter, are in good hydraulic connection with the aquifer material, and respond instantaneously to aquifer head changes. Because of stored water in the observation well or piezometer or because of a poor hydraulic connection with the aquifer material, however, a time delay often is observed between the drawdown in the aquifer and that in the observation installation (Hvorslev, 1951). The magnitude of this delay is a function of the geometry of the observation well or piezometer, the permeability and specific storage of the aquifer, and the integrity of the hydraulic connection between the observation installation and aquifer. Delayed drawdown response in an observation well or piezometer can be important during the early period of an aquifer test in which



**EXPLANATION**

- |   |                                |   |  |
|---|--------------------------------|---|--|
|  | AQUIFER                        |  | WATER TABLE—Dashed portion indicates initial water table |
|  | WELL-BORE SKIN                 |   |  |
|  | IMPERMEABLE (NO-FLOW) BOUNDARY |  | SCREENED INTERVAL OF WELL                                |

**Figure 5.** Finite-diameter, partially penetrating pumped well in a water-table aquifer. [ $b$ , initial saturated thickness of aquifer;  $d_s$ , thickness of the well-bore skin;  $h(r,z,t)$ , head in aquifer at radial distance  $r$  from axis of pumped well and depth  $z$  below initial water table at time  $t$ ;  $h_i$ , initial head in aquifer;  $Q$ , pumping rate of well;  $r_c$ ,  $r_p$ , inside radius of the pumped well and observation well, respectively, in the interval where water levels are changing during pumping;  $r_w$ , radius of the screened interval of the pumped well;  $z_{pd}$ ,  $z_{pl}$ , depth below initial water table to the top and bottom, respectively, of the screened interval of the pumped well;  $z_1$ ,  $z_2$ , depth below initial water table to the top and bottom, respectively, of the screened interval of the observation well.]

heads are changing rapidly, and, in addition to well-bore storage and skin at the pumped well, is an important factor to consider in the accurate evaluation of specific storage. Moench (1997) has shown that the effects of this delayed response are most important in observation wells closest to the pumped well where drawdown is most rapid.

**PROGRAM WTAQ**

Program *WTAQ* implements the Laplace-transform solutions for drawdown at a pumped well, observation well, or observation piezometer that are presented in equations 18–27 and 30–31 in Moench (1997). The Laplace-transform solutions are numerically inverted to the time domain by means of the Stehfest (1970) algorithm in the same manner as was done in Moench (1996, 1997). Some improvements were made to the numerical inversion algorithm as a result of the work of Gieske (1999). The program calculates dimensionless or dimensional drawdowns for a given set of input conditions that are specified by the user in a data-input file. Instructions for

preparing the data-input files are provided in this section, as are a summary of the simplifying assumptions used in the program, a description of selected program options and variables, and a description of the result and plot files generated by the program. A narrative description of the computer code is provided in Attachment 1.

## Simplifying Assumptions

Several simplifying assumptions were required in the development of the analytical model on which program *WTAQ* is based. Discussions pertaining to the adequacy and limitations of these assumptions can be found in Boulton (1954, 1963), Neuman (1972, 1974), Black and Kipp (1977), Dougherty and Babu (1984), and Moench (1995, 1997). The assumptions are as follows:

Assumptions for both confined and water-table aquifers (no well-bore storage or skin effects)—

1. The aquifer is homogeneous, of infinite lateral extent, horizontal, and of uniform thickness.
2. The aquifer can be anisotropic provided that the principal directions of the hydraulic-conductivity tensor are parallel to the  $r, z$  coordinate axes.
3. Vertical flow across the lower boundary of the aquifer is negligible (that is, the lower boundary is impermeable).
4. A well discharges at a constant rate from a specified zone below an initially horizontal potentiometric surface (for a confined aquifer) or water table.
5. The porous medium and fluid are slightly compressible and have constant physical properties.
6. The pumped well and observation wells or piezometers are infinitesimal in diameter.

Additional assumption for confined aquifers—

7. Vertical flow across the upper boundary of the aquifer is negligible (that is, the upper boundary is impermeable).

Additional assumptions for water-table aquifers—

8. The upper boundary of the aquifer is the water table. Vertical flow of water into the aquifer from the unsaturated zone can be instantaneous in response to a decline in the elevation of the water table (Neuman, 1972, 1974), or can vary exponentially with time in response to a step decline in the elevation of the water table (Boulton, 1954, 1963; Moench, 1995).
9. The change in saturated thickness of the aquifer due to pumping is small compared with the initial saturated thickness.

Additional assumptions for the condition of a finite-diameter pumped well (Dougherty and Babu, 1984)—

10. The head within the pumped well does not vary spatially.
11. The radial flow from the aquifer to the pumped well does not vary in magnitude along the length of the screened section.
12. Vertical flow from the aquifer through the base of the pumped well is negligible.
13. A thin skin of low-permeability material having no storage capacity may be present at the interface between the pumped well screen and the aquifer.

Additional assumptions for the condition of delayed drawdown response in a finite-diameter observation well (Black and Kipp, 1977)—

14. The hydraulic head in the observation well changes at a rate that is proportional to the head difference between the observation well and the adjacent aquifer.

## Description of Selected Program Options and Variables

Several of the program options and variables that are used in *WTAQ* are described below. Program variables are shown in upper-case text.

**Analysis formats.**--Two analysis formats are provided in the program: type curve and dimensional. Separate input files are required for each of the analysis formats. The character variable *FORMAT* is used to specify either the *TYPE CURVE* or *DIMENSIONAL* format. Type curves are the theoretical drawdown responses that result from pumping at a constant rate. For the type-curve format, *WTAQ* generates dimensionless drawdowns at observation wells and/or piezometers (but not at the pumped well) that can be used for graphical matching with measured time-drawdown data. Procedures for estimating aquifer hydraulic properties by type-curve matching can be found in many journal articles and texts on ground-water hydrology and aquifer-test analysis, such as those by Freeze and Cherry (1979), Marsily (1986), Driscoll (1986), Walton (1970, 1987), and Kruseman and de Ridder (1991).

For the dimensional format, *WTAQ* generates dimensional drawdowns for the pumped well or observation wells and/or piezometers. Using the dimensional format, the user typically adjusts the hydraulic parameters of the analytical model in a series of simulations until drawdowns calculated by *WTAQ* closely match those measured during the aquifer test. This approach for estimating aquifer hydraulic properties by calibration of the analytical model often is referred to as the trial-and-error approach. The dimensional format also can be used with automated parameter-estimation techniques for model calibration.

Example applications of these two formats are described in the sample problems.

**Aquifer type and hydraulic properties.**--*WTAQ* simulates either confined or water-table aquifer types. The characteristics that differentiate the two aquifer types are the condition of the upper boundary and the thickness of the aquifer. For confined aquifers, the upper boundary is impermeable, and there is no vertical flow into the aquifer; the thickness of the aquifer remains constant during pumping. For water-table aquifers, the upper boundary is the water table, water flows vertically into the aquifer from the overlying unsaturated zone in response to a decline in the elevation of the water table caused by pumping, and the saturated thickness of the aquifer decreases as the elevation of the water table falls.

Aquifer type is specified by use of character variable *AQTYPE*, which is either *CONFINED* or *WATER TABLE*. The thickness of the aquifer, or, for water-table aquifers, the saturated thickness of the aquifer at the beginning of the simulation, is specified by use of variable *BB*. For the type-curve format, the user specifies the ratio of the vertical to horizontal hydraulic conductivities using variable *XKD*. For the dimensional format, both the horizontal (variable *HKR*) and vertical (variable *HKZ*) hydraulic conductivity must be specified. Transmissivity of the aquifer is calculated by the program for the dimensional format and written to the result file.

Aquifer storage properties are specified by use of variables *SIGMA* (the ratio of aquifer storativity to specific yield,  $\sigma$ ), *SS* (specific storage), and/or *SY* (specific yield). *SIGMA* is used for the type-curve format. It is set equal to zero for a confined aquifer and to  $S_s b / S_y$  for a water-table aquifer. Variables *SS* and *SY* are used for the dimensional format. *SS* is specified for both confined and water-table aquifers; *SY* is specified for water-table aquifers only. Aquifer storativity is calculated by the program (variable *SS* multiplied by variable *BB*) for the dimensional format and written to the result file.



**Drainage at water table.**--For water-table aquifers, *WTAQ* simulates either instantaneous or delayed drainage into the aquifer from the unsaturated zone. The type of drainage is specified by variable *IDRA*, which is set to 0 for instantaneous drainage and to 1 for delayed drainage. (For a confined aquifer, *IDRA* = 0). For delayed drainage, up to five empirical drainage constants can be included. The number of drainage constants is specified by variable *NGAMMA* for the type-curve format and by variable *NALPHA* for the dimensional format. In Moench (1995, 1996, 1997), only a single value of *NGAMMA* or *NALPHA* is indicated. Multiple empirical drainage constants have been used in an analysis of aquifer-test data in a sand and gravel water-table aquifer, Cape Cod, Massachusetts (A.F. Moench, U.S. Geological Survey, written commun., 1999). The analysis shows that three such constants are adequate to account for deviations between measured drawdown and drawdown simulated by *WTAQ* under the assumption of instantaneous drainage.

Dimensionless drainage constants are specified for the type-curve format using the array *GAMMA*. These constants are calculated by the user independently of the program from

$$GAMMA(I) = \frac{\alpha_i b S_y}{K_z},$$

where

- GAMMA(I)* is the *i*th dimensionless empirical drainage constant;
- $\alpha_i$  is the *i*th empirical drainage constant (units of inverse time);
- b* is the initial saturated thickness of the aquifer (units of length);
- S<sub>y</sub>* is the specific yield of the aquifer (dimensionless); and
- K<sub>z</sub>* is the vertical hydraulic conductivity of the aquifer (units of length per time).

For the dimensional format, the empirical drainage constants are specified using the array *ALPHA*; *WTAQ* calculates the dimensionless values of *GAMMA* internally. For instantaneous drainage at the water table (*IDRA* = 0), the user enters a single value of 1.0D09 (an essentially infinite value) for either *GAMMA(1)* or *ALPHA(1)*, depending on the analysis format that is being used. (It is recommended that instantaneous drainage be assumed for initial estimates of aquifer parameters.)

**Time information.**--For the type-curve format, dimensionless drawdowns are calculated for a series of equally-spaced, dimensionless, and logarithmically-scaled times. These times are calculated from values specified for variables *TDLAST*, *NLC*, and *NOX*. Variable *TDLAST* is the largest value of dimensionless time that is desired; variable *NLC* is the total number of logarithmic cycles on the time scale for which drawdown will be calculated; and variable *NOX* is the number of equally spaced times per logarithmic cycle for which drawdown will be calculated. The total number of time steps for which drawdown will be calculated is equal to  $(NLC)(NOX) + 1$ , and the length of each time step equals  $10^{1.0/(NOX)}$ .

Two options are provided for specifying the times of drawdown calculations in the dimensional format. In the first option (*ITS* = 0), dimensional drawdown is calculated for a series of equally-spaced, dimensional, and logarithmically-scaled times. These times are calculated from values specified for variables *TLAST*, *NLC*, and *NOX*. Variable *TLAST* is the largest value of dimensional time that is desired; variables *NLC* and *NOX* are defined above. In the second option (*ITS* = 1), the user enters a series of specific times at which drawdown will be calculated for the pumped well or each of the observation wells and piezometers. The number of specific times at

which drawdown will be calculated is set using variable NTSPW for the pumped well and variable NTSOB for each of the observation wells and piezometers. The actual times are read using arrays TIMEPW for the pumped well and TIMEOB for observation wells and piezometers.

**Program solution variables.**--Five program-solution variables (RERRNR, RERRSUM, NMAX, NTMS, and NS) are used in the numerical-inversion algorithms of *WTAQ*. Suggested values are provided for each of these variables and the user should not have to make substantial changes to these values. Variables RERRSUM, NMAX, and NS are used for confined aquifers and variables RERRNR, NTMS, and NS are used for water-table aquifers. Variable RERRSUM is the relative error in the finite summations of the analytical solutions for drawdown in a confined aquifer. Decreasing the value of RERRSUM will increase solution precision and time. A value between 1.0D-07 to 1.0D-08 is suggested. Variable NMAX is the maximum number of terms permitted in the finite summations of the analytical solutions for drawdown in a confined aquifer. The suggested value is 200. If the relative error criterion (RERRSUM) has not been met after NMAX terms have been summed, an error message is printed to the result file and the program continues to the next time. The user should ensure that a sufficient number of terms is being used in the summations by making multiple runs in which NMAX is increased from one simulation to the next, and should continue these runs until simulation results do not vary substantially when NMAX is increased.

Variable NS is the number of terms used in the Stehfest algorithm. NS must be an even number, the value of which depends upon computer precision. If the computer holds 16 significant figures in double precision, let  $NS = 8$  to 12. A value of 8 is often sufficient. If calculated drawdowns are unstable, NS can be reduced to 6 (or even 4). Precision will be reduced, however, and results should be checked for accuracy. The user can compare simulation results using alternative values of NS (such as 6, 8, and 12) to determine if numerically stable results have been attained.

Variable RERRNR is the relative error for Newton-Raphson iteration used in the finite summations of the analytical solutions for drawdown in a water-table aquifer. Smaller values of RERRNR will increase solution precision and time. A value of 1.0D-10 is suggested. If the value of RERRNR is exceeded after 100 Newton-Raphson iterations, a message is printed to the result file and the program is stopped. Should this happen, the user can increase the value of RERRNR and re-run the simulation. Variable NTMS is a factor that is used to determine the number of terms in the finite summations of the analytical solutions for drawdown in a water-table aquifer. Suggested values are 20 or 30. The user should ensure that a sufficient number of terms is being used in the summations by making multiple runs in which NTMS is increased from one simulation to the next (for example, doubled), and should continue until simulation results do not vary substantially when NTMS is increased.

**Characteristics of the pumped well.**--The simulated pumped well can either partially or fully penetrate the aquifer and can be either finite or infinitesimal in diameter. These options are provided for both confined and water-table aquifer types. For all options, a nonzero value of the radius of the pumped well screen (fig. 5) must be specified for variable RW. The screened interval of the pumped well is specified using variables ZPD and ZPL, which are the depths below the top of the aquifer or initial water table to the top (ZPD) and bottom (ZPL) of the screened interval of the well (fig. 5). For the dimensional format, the pumping rate of the well must be specified by use of variable QQ.

Well-bore storage effects are simulated when the finite-diameter pumped-well option is used (IPWD = 1). Simulation of well-bore storage requires specification of additional information on the pumped well. For the type-curve format, this information is the dimensionless well-bore storage parameter (variable WD), which is calculated independently of the program from

$$WD = \frac{\pi r_c^2}{[2\pi r_w^2 S_s(z_{pl} - z_{pd})]}$$

where

- $r_c$  is the inside radius of the pumped well in the interval where water levels are changing during pumping (units of length);
- $r_w$  is the radius of the pumped well screen (units of length);
- $S_s$  is the specific storage of the aquifer (units of inverse length);
- $z_{pl}$  is the depth below top of aquifer or initial water table to the bottom of the screened interval of the pumped well (units of length); and
- $z_{pd}$  is the depth below top of aquifer or initial water table to the top of the screened interval of the pumped well (units of length). [Note that  $z_{pl} - z_{pd}$  is the length of the screened interval of the pumped well.]

For the dimensional format, the user must specify values of  $r_c$ ,  $r_w$ ,  $S_s$ ,  $z_{pl}$  and  $z_{pd}$ .

Well-bore skin can be specified for either a finite-diameter or infinitesimal-diameter pumped well. The value of the well-bore skin parameter (variable SW) is calculated independently of the program from

$$SW = \frac{K_r d_s}{K_s r_w}$$

where

- $K_s$  is the hydraulic conductivity of the well-bore skin (units of length per time); and
- $d_s$  is the thickness of the well-bore skin (units of length);

and other parameters are defined previously. In the absence of well-bore skin, variable SW is set to 0.0D0.

Well-bore skin can be simulated with or without well-bore storage at the pumped well. When a well-bore skin of low permeability relative to the aquifer material is simulated but well-bore storage is not, drawdowns in the aquifer (and, thus, at observation wells and piezometers) will not be affected by the skin effect at the pumped well. Drawdowns at the pumped well, however, will be larger than they would be in the absence of the skin. When both well-bore skin and well-bore storage are simulated (the usual situation), early-time drawdowns at observation wells and piezometers will be delayed relative to those that occur in the absence of the skin, and drawdown at the pumped well will be larger than it would be in the absence of well-bore storage (Moench, 1985, fig. 7).

**Characteristics of observation wells and piezometers.--**WTAQ can simulate drawdowns at up to 25 locations in an aquifer; the number of observation locations is specified with variable NOBWC. The three types of observation wells or piezometers that can be simulated are a partially penetrating observation well (IOWS = 0), a fully penetrating observation well (IOWS = 1), or an observation piezometer (IOWS = 2). For both partially and fully penetrating observation wells, the screened interval of the well must be specified using variables Z1 and Z2, which are the depths below the top of the aquifer or initial water table to the top (Z1) and bottom (Z2) of the screened interval of the well, respectively (fig. 3). For an observation piezometer, the depth below the top of the aquifer or initial water table to the center of the piezometer is specified with variable ZP (fig. 3). In all cases, the radial distance from the axis of the pumped well to the observation well or piezometer is specified with variable R.

Delayed drawdown response in an observation well can be simulated with or without well-bore storage at the pumped well. The option is activated by setting variable IDPR = 1. For the type-curve format, the dimensionless delayed response factor,  $W_D'$ , is specified for each observation well. This factor (variable WDP in the program) must be specified by the user, and is determined from

$$WDP = \frac{\pi r_p^2}{[2\pi r_w^2 S_s F']},$$

where  $r_p$  is the inside radius of the observation well in the interval where water levels are changing during pumping (units of length) and  $F'$  is a shape factor (units of length). For a well in a uniform aquifer medium,  $F'$  is defined by (Hvorslev, 1951, case 8)

$$F' = \frac{L}{\ln(x + \sqrt{1 + x^2})},$$

where

$$x = \frac{mL}{2r_p}, \quad m = \sqrt{\frac{K_r}{K_z}},$$

and  $L$  is the length of the screened interval of the observation well ( $z_1 - z_2$ ). For the dimensional format, the user need only specify the inside radius of the observation well over the range of changing water levels (variable RP) and the length of the screened interval of the observation well (variable XLL); the program will calculate WDP internally. Note that delayed drawdown response in an observation piezometer as defined here cannot be simulated using *WTAQ* because of the dependence of the delayed response on the screened length of the observation well. To do so requires treating the zero length piezometer as a finite-length observation well and specifying  $z_1$  and  $z_2$ .

**Measured drawdown data.**--Measurements of drawdown made at specific times at either the pumped well or observation wells and piezometers may be specified using the dimensional format. Measured drawdown data are used for model calibration and the user may want to include these data in the input file for comparison to model calculated drawdowns (see section on "Result and Plot Files"). When measured data are specified, variable IMEAS is set to 1 (IMEAS is set to 0 when no data are specified). The measured data are read using arrays XMEASPW for the pumped well and XMEASOB for the observation wells and piezometers. (The times associated with these data are read using arrays TIMEPW and TIMEOB.) If IMEAS=1, drawdown measurements must be specified at each time. An option also is provided to suppress drawdown calculations during a simulation for the pumped well or any of the observation wells and piezometers. This option allows the user to specify time-drawdown data in the input file without having that data used in all simulations. The option is activated by setting IRUN = 0; hence, IRUN will probably most often be set to 1.

## Data-Input Files

Each of the two analysis formats requires a separate data-input file for execution of *WTAQ*. Most of the data and other information that is required for the data-input files is described in detail in the preceding section. All input data are read using free-format style, which means that data values in each line of input do not have to be in specific columns; however, data values must be separated by one or more blank spaces. A consistent set of length and time units must be used throughout the input file, for example, meters and seconds or feet and days. All real-valued variables are double-precision format in *WTAQ*; consequently, double-precision format should be used for real-valued variables in the input file.

## Type-Curve Format

Line-by-line instructions for creating a data-input file for the type-curve format follow. Variable names that are used in the input file and computer program are shown in upper-case text.

### Part I: Aquifer Characteristics and General Information

#### Line 1:

TITLE--Title of simulation, which can be up to 70 characters in length. Leave this line blank if no title is specified.

#### Line 2:

FORMAT--Analysis format. Enter TYPE CURVE.

#### Line 3:

AQTYPE--Type of aquifer being simulated. Two options are provided:

AQTYPE = CONFINED or

AQTYPE = WATER TABLE.

#### Line 4:

BB--Thickness or saturated thickness of aquifer at beginning of simulation, in units of length.

XKD--Ratio of vertical to horizontal hydraulic conductivity of aquifer, dimensionless.

SIGMA--Ratio of storativity to specific yield of aquifer, dimensionless. Enter 0.0D0 if AQTYPE = CONFINED.

#### Line 5:

IDRA--Type of drainage at water table. Enter 0 if AQTYPE = CONFINED. Two options are provided:

IDRA = 0: Instantaneous drainage.

IDRA = 1: Delayed drainage.

NGAMMA--Number of dimensionless drainage constants. Enter 0 if IDRA = 0. NGAMMA must be less than or equal to 5.

#### Line 6:

GAMMA(I)--A single row of NGAMMA dimensionless drainage constants. Successive values must be separated by at least 1 space. Maximum of 5 values is allowed. Enter 1.0D09 if IDRA = 0.

#### Line 7:

TDLAST--Largest value of dimensionless time.

NLC--Number of logarithmic cycles on the time scale for which drawdown will be calculated.

NOX--Number of equally spaced times per logarithmic cycle for which drawdown will be calculated.

**Line 8:** Program-solution variables. See "Description of Selected Program Options and Variables" for information on these variables.

RERRNR--Relative error for Newton-Raphson iteration and finite summations of drawdown for water-table aquifers. A value of 1.0D-10 is suggested. Enter 0.0D0 for AQTYPE = CONFINED.

RERRSUM--Relative error for finite summations of drawdown for confined aquifers. Suggested value is 1.0D-07 to 1.0D-08. Enter 0.0D0 if AQTYPE = WATER TABLE.

NMAX--Maximum number of terms permitted in the finite summations of drawdown for confined aquifers. Suggested value is 200. Enter 0 if AQTYPE = WATER TABLE.

NTMS--Factor used to determine number of terms in the finite summations for drawdown for water-table aquifers. Suggested values are 20 or 30. Enter 0 if AQTYPE = CONFINED.

NS--Number of terms used in the Stehfest algorithm. This must be an even integer, the value of which depends upon computer precision. If the computer holds 16 significant figures in double precision, let NS = 8 to 12. A value of 8 is often sufficient.

## Part II: Pumped-Well Information

### Line 9:

IPWS--Type of pumped well:

IPWS = 0: Partially penetrating pumped well.

IPWS = 1: Fully penetrating pumped well.

IPWD--Type of diameter of pumped well:

IPWD = 0: Infinitesimal diameter (line-source theory).

IPWD = 1: Finite diameter.

### Line 10:

RW--Radius of pumped well screen, in units of length.

ZPD--Depth below top of aquifer or initial water table to the top of the screened interval of the pumped well, in units of length.

ZPL--Depth below top of aquifer or initial water table to the bottom of the screened interval of the pumped well, in units of length.

WD--Well-bore storage parameter, dimensionless. Enter 0.0D0 if IPWD = 0.

SW--Well-bore skin parameter, dimensionless.

## Part III: Observation-Well Information

### Line 11:

NOBWC--Number of observation wells or piezometers for which type curves will be calculated. Must be less than or equal to 25.

**Lines 12 and 13:** Repeat lines 12 and 13 for each of the NOBWC observation wells or piezometers.

### Line 12:

IOWS--Type of observation well or piezometer:

IOWS = 0: Partially penetrating observation well.

IOWS = 1: Fully penetrating observation well.

IOWS = 2: Observation piezometer.

IDPR--Option for delayed response of observation well. Valid only for IOWS = 0 or 1:

IDPR = 0: No delayed response.

IDPR = 1: Delayed response.

### Line 13:

R--Radial distance from axis of pumped well to observation well or piezometer, in units of length.

Z1--Depth below top of aquifer or initial water table to the top of screened interval of observation well, in units of length. Use for IOWS = 0 or 1. Enter 0.0D0 if IOWS = 2.

Z2--Depth below top of aquifer or initial water table to the bottom of screened interval of observation well, in units of length. Use for IOWS = 0 or 1. Enter 0.0D0 if IOWS = 2.

ZP--Depth below top of aquifer or initial water table to center of piezometer, in units of length. Use for IOWS = 2. Enter 0.0D0 if IOWS = 0 or 1.

WDP--Delayed response factor, dimensionless. Enter 0.0D0 if IDPR = 0.

A summary of the data-input requirements for the type-curve format is shown in table 1. An example data-input file that is based on the hypothetical water-table aquifer described in the sample problems is shown in figure 6. Variable names are shown for convenience on the right side of each line of the example data-input file (fig. 6).

**Table 1.** Input data for type-curve format

[Free-format input style—each variable in a line must be separated by at least one space]

Line	Variable name	Type	Explanation
<b>PART I: AQUIFER CHARACTERISTICS AND GENERAL INFORMATION</b>			
1	TITLE	Character	Title of simulation (up to 70 characters)
2	FORMAT	Character	Analysis format. Enter TYPE CURVE
3	AQTYPE	Character	Aquifer type: AQTYPE = CONFINED or AQTYPE = WATER TABLE
4	BB	Real	Thickness or saturated thickness of aquifer
	XKD	Real	Ratio of vertical to horizontal hydraulic conductivity of aquifer
	SIGMA	Real	Ratio of storativity to specific yield of aquifer. Enter 0.0D0 if AQTYPE = CONFINED
5	IDRA	Integer	Type of drainage at water table. Enter 0 if AQTYPE = CONFINED. IDRA = 0 : instantaneous drainage IDRA = 1 : delayed drainage
	NGAMMA	Integer	Number of dimensionless drainage constants. Enter 0 if IDRA = 0. NGAMMA must be less than or equal to 5
6	GAMMA(I)	Real	A single row of NGAMMA dimensionless drainage constants, successive values separated by at least 1 space. Enter 1.0D09 if IDRA = 0
7	TDLAST	Real	Largest value of dimensionless time
	NLC	Integer	Number of logarithmic cycles on time scale
	NOX	Integer	Number of equally spaced times per logarithmic cycle for which drawdown will be calculated
8	RERRNR	Real	Relative error for Newton-Raphson iteration and finite summations for water-table aquifers. Suggested value is 1.D-10. Use for AQTYPE = WATER TABLE. Enter 0.0D0 if AQTYPE = CONFINED
	RERRSUM	Real	Relative error for finite summations for confined aquifers. Suggested value is 1.D-07 to 1.D-08. Enter 0.0D0 if AQTYPE = WATER TABLE
	NMAX	Integer	Maximum number of terms permitted in the finite summations for confined aquifers. Suggested value is 200. Enter 0 if AQTYPE = WATER TABLE
	NTMS	Integer	Factor used to determine number of terms in the finite summations for water-table aquifers. Suggested values are 20 or 30. Enter 0 if AQTYPE = CONFINED
	NS	Integer	Number of Stehfest terms. Must be an even integer. Eight terms are usually sufficient
	<b>PART II: PUMPED-WELL INFORMATION</b>		
9	IPWS	Integer	Type of pumped well: IPWS = 0: Partially penetrating pumped well IPWS = 1: Fully penetrating pumped well
	IPWD	Integer	Type of diameter of pumped well: IPWD = 0 : Infinitesimal diameter (line-source theory) IPWD = 1 : Finite diameter
10	RW	Real	Radius of pumped well screen
	ZPD	Real	Depth below top of aquifer or initial water table to the top of the screened interval of the pumped well
	ZPL	Real	Depth below top of aquifer or initial water table to the bottom of the screened interval of the pumped well
	WD	Real	Well-bore storage parameter. Enter 0.0D0 if IPWD = 0
	SW	Real	Well-bore skin parameter

**Table 1.** Input data for type-curve format—*Continued*

Line	Variable name	Type	Explanation
<b>PART III: OBSERVATION-WELL INFORMATION</b>			
11	NOBWC	Integer	Number of observation wells or piezometers for which type curves will be calculated. Must be less than or equal to 25
12	IOWS	Integer	Repeat lines 12 and 13 for each observation well or piezometer Type of observation well or piezometer: IOWS = 0: Partially penetrating observation well IOWS = 1: Fully penetrating observation well IOWS = 2: Observation piezometer
	IDPR	Integer	Option for delayed response of observation well. Valid only for IOWS = 0 or 1 IDPR = 0 : No delayed response IDPR = 1 : Delayed response
13	R	Real	Radial distance from axis of pumped well to observation well or piezometer
	Z1	Real	Depth below top of aquifer or initial water table to top of screened interval of observation well. Use for IOWS = 0 or 1. Enter 0.0D0 if IOWS = 2
	Z2	Real	Depth below top of aquifer or initial water table to bottom of screened interval of observation well. Use for IOWS = 0 or 1. Enter 0.0D0 if IOWS = 2
	ZP	Real	Depth below top of aquifer or initial water table to center of piezometer. Use for IOWS = 2. Enter 0.0D0 for IOWS = 0 or 1
	WDP	Real	Delayed response factor. Enter 0.0D0 if IDPR = 0

Example input file using type-curve format.

```

TYPE CURVE                                FORMAT
WATER TABLE                              AQTYPE
10.0D0 0.5D0 1.0D-3                        BB XKD SIGMA
0 0                                         IDRA NGAMMA
1.0D09                                      GAMMA (I)
1.0D12 11 5                                TDLAST NLC NOX
1.0D-10 0.0D0 0 30 8                      RERRNR RERRSUM NMAX NTMS NS
0 1                                         IPWS IPWD--PUMPED WELL
0.10D0 5.0D0 10.0D0 5.0D03 0.0D0         RW ZPD ZPL WD SW
4                                           NOBWC
2 0                                         IOWS IDPR--OBPIEZ PD1
3.16D0 0.0D0 0.0D0 7.5D0 0.0D0           R Z1 Z2 ZP WDP
2 0                                         IOWS IDPR--OBPIEZ PD2
31.6D0 0.0D0 0.0D0 7.5D0 0.0D0          R Z1 Z2 ZP WDP
2 0                                         IOWS IDPR--OBPIEZ PS1
3.16D0 0.0D0 0.0D0 1.0D0 0.0D0          R Z1 Z2 ZP WDP
2 0                                         IOWS IDPR--OBPIEZ PS2
31.6D0 0.0D0 0.0D0 1.0D0 0.0D0          R Z1 Z2 ZP WDP

```

**Figure 6.** Example data-input file using type-curve format.



## Dimensional Format

Line-by-line instructions for creating a data-input file for the dimensional format follow. Variable names that are used in the input file and computer program are shown in upper-case text.

### Part I: Aquifer Characteristics and General Information

#### Line 1:

TITLE--Title of simulation, which can be up to 70 characters in length. Leave this line blank if no title is specified.

#### Line 2:

FORMAT--Analysis format. Enter DIMENSIONAL.

#### Line 3:

AQTYPE--Type of aquifer being simulated. Two options are provided:

AQTYPE = CONFINED or

AQTYPE = WATER TABLE.

#### Line 4:

BB--Thickness or saturated thickness of aquifer at beginning of simulation, in units of length.

HKR--Horizontal hydraulic conductivity of aquifer, in units of length per time.

HKZ--Vertical hydraulic conductivity of aquifer, in units of length per time.

SS--Specific storage of aquifer, in units of inverse length.

SY--Specific yield of aquifer, dimensionless. Enter 0.0D0 if AQTYPE = CONFINED.

#### Line 5:

IDRA--Type of drainage at water table. Enter 0 if AQTYPE = CONFINED. Two options are provided:

IDRA = 0: Instantaneous drainage.

IDRA = 1: Delayed drainage.

NALPHA--Number of drainage constants. Enter 0 if IDRA = 0. NALPHA must be less than or equal to 5.

#### Line 6:

ALPHA(I)--A single row of NALPHA drainage constants, in units of inverse time. Successive values must be separated by at least 1 space. Maximum of 5 values is allowed. Enter 1.0D09 if IDRA = 0.

#### Line 7:

ITS--Time specification:

ITS = 0: Log-cycle time steps (use when dimensional theoretical curves are desired).

ITS = 1: User-specified times.

IMEAS--Specification of measured drawdown data. Enter 0 if ITS = 0. Options for ITS = 1:

IMEAS = 0: Measured drawdown data not specified for each time.

IMEAS = 1: Measured drawdown data specified for each time.

#### Line 8:

TLAST--Largest value of time. Enter 0.0D0 if ITS = 1.

NLC--Number of logarithmic cycles on the time scale for which drawdown will be calculated. Enter 0 if ITS = 1.

NOX--Number of equally spaced times per logarithmic cycle for which drawdown will be calculated. Enter 0 if ITS = 1.

**Line 9:** Program-solution variables. See “Description of Selected Program Options and Variables” for information on these variables.

RERRNR--Relative error for Newton-Raphson iteration and finite summations of drawdown for water-table aquifers. A value of 1.0D-10 is suggested. Enter 0.0D0 for AQTYPE = CONFINED.

RERRSUM--Relative error for finite summations of drawdown for confined aquifers. Suggested value is 1.0D-07 to 1.0D-08. Enter 0.0D0 if AQTYPE = WATER TABLE.

NMAX--Maximum number of terms permitted in the finite summations of drawdown for confined aquifers. Suggested value is 200. Enter 0 if AQTYPE = WATER TABLE.

NTMS--Factor used to determine number of terms in the finite summations for drawdown for water-table aquifers. Suggested values are 20 or 30. Enter 0 if AQTYPE = CONFINED.

NS--Number of terms used in the Stehfest algorithm. This must be an even integer, the value of which depends upon computer precision. If the computer holds 16 significant figures in double precision, let NS = 8 to 12. A value of 8 is often sufficient.

## **Part II: Pumped-Well Information**

### **Line 10:**

IPWS--Type of pumped well:

IPWS = 0: Partially penetrating pumped well.

IPWS = 1: Fully penetrating pumped well.

IPWD--Type of diameter of pumped well:

IPWD = 0: Infinitesimal diameter (line-source theory).

IPWD = 1: Finite diameter.

### **Line 11:**

QQ--Pumping rate of well, in units of cubic length per time.

RW--Radius of pumped well screen, in units of length.

RC--Inside radius of pumped well in the interval where water levels are changing during pumping, in units of length. Enter 0.0D0 if IPWD = 0.

ZPD--Depth below top of aquifer or initial water table to the top of the screened interval of the pumped well, in units of length.

ZPL--Depth below top of aquifer or initial water table to the bottom of the screened interval of the pumped well, in units of length.

SW--Well-bore skin parameter, dimensionless.

**Lines 12 and 13:** Information on user-specified times for which drawdowns will be calculated for the pumped well. Skip to line 14 if ITS = 0.

### **Line 12:**

NTSPW--Number of user-specified times for which drawdown at the pumped well will be calculated. If NTSPW = 0, no drawdowns are calculated for the pumped well.

IRUN--Option to suppress drawdown calculations for the pumped well. Allows user to specify time-drawdown data (Line 13), but those data are ignored during the simulation. Options are:

IRUN = 0: Drawdowns not calculated.

IRUN = 1: Drawdowns calculated.

**Line 13:** Time and measured drawdown data for pumped well. Repeat this line of input data NTSPW times (if NTSPW = 0, skip to line 14):

TIMEPW(I)--Time I.

XMEASPW(I)--Measured drawdown at pumped well for time I, in units of length. Leave blank if IMEAS = 0.

### Part III: Observation-Well Information

#### Line 14:

NOBWC--Number of observation wells or piezometers for which drawdown curves will be calculated. Must be less than or equal to 25.

**Lines 15 through 18:** Repeat lines 15 through 18 for each of the NOBWC observation wells or piezometers.

#### Line 15:

IOWS--Type of observation well or piezometer:

IOWS = 0: Partially penetrating observation well.

IOWS = 1: Fully penetrating observation well.

IOWS = 2: Observation piezometer.

IDPR--Option for delayed response of observation well. Valid only for IOWS = 0 or 1:

IDPR = 0: No delayed response.

IDPR = 1: Delayed response.

#### Line 16:

R--Radial distance from axis of pumped well to observation well or piezometer, in units of length.

Z1--Depth below top of aquifer or initial water table to the top of screened interval of observation well, in units of length. Use for IOWS = 0 or 1. Enter 0.0D0 if IOWS = 2.

Z2--Depth below top of aquifer or initial water table to the bottom of screened interval of observation well, in units of length. Use for IOWS = 0 or 1. Enter 0.0D0 if IOWS = 2.

ZP--Depth below top of aquifer or initial water table to center of piezometer, in units of length. Use for IOWS = 2. Enter 0.0D0 if IOWS = 0 or 1.

RP--Inside radius of the observation well standpipe in the interval over which water levels are changing during pumping, in units of length. Enter 0.0D0 if IDPR = 0.

XLL--Length of screened interval of observation well, in units of length. Enter 0.0D0 if IDPR = 0.

**Lines 17 and 18:** Information on user-specified times for which drawdowns will be calculated for the observation well or piezometer. Skip these lines if ITS = 0.

#### Line 17:

NTSOB--Number of user-specified times for which drawdown at the observation well or piezometer will be calculated. If NTSOB = 0, no drawdowns are calculated for the observation well or piezometer.

IRUN--Option to suppress drawdown calculations for the observation well or piezometer. Allows user to specify time-drawdown data (Line 18), but those data are ignored during the simulation. Options are:

IRUN = 0: Drawdowns not calculated.

IRUN = 1: Drawdowns calculated.

**Line 18:** Time and measured drawdown data for observation well or piezometer. Repeat this line of input data NTSOB times (if NTSOB = 0, skip to next observation point):

TIMEOB(I)--Time I.

XMEASOB(I)--Measured drawdown at observation well or piezometer for time I, in units of length. Leave blank if IMEAS = 0.

A summary of the data-input requirements for the dimensional format is shown in table 2. An example data-input file that is based on the hypothetical water-table aquifer described in the sample problems is shown in figure 7. Variable names are shown for convenience on the right side of each line of the example data-input file (fig. 7).

**Table 2.** Input data for dimensional format

[Free-format input style—each variable in a line must be separated by at least one space]

Line	Variable name	Type	Explanation
<b>PART I: AQUIFER CHARACTERISTICS AND GENERAL INFORMATION</b>			
1	TITLE	Character	Title of simulation (up to 70 characters)
2	FORMAT	Character	Analysis format. Enter DIMENSIONAL
3	AQTYPE	Character	Aquifer type: AQTYPE = CONFINED or AQTYPE = WATER TABLE
4	BB	Real	Thickness or saturated thickness of aquifer
	HKR	Real	Horizontal hydraulic conductivity of aquifer
	HKZ	Real	Vertical hydraulic conductivity of aquifer
	SS	Real	Specific storage of aquifer
	SY	Real	Specific yield of aquifer. Enter 0.0D0 if AQTYPE = CONFINED
5	IDRA	Integer	Type of drainage at water table. Enter 0 if AQTYPE = CONFINED. IDRA = 0 : instantaneous drainage IDRA = 1 : delayed drainage
	NALPHA	Integer	Number of drainage constants. Enter 0 if IDRA = 0. NALPHA must be less than or equal to 5
6	ALPHA(I)	Real	A single row of NALPHA drainage constants, successive values separated by at least 1 space. Enter 1.0D09 if IDRA = 0
7	ITS	Integer	Time specification: ITS = 0: Log-cycle times ITS = 1: User-specified times
	IMEAS	Integer	Measured drawdown data. Enter 0 if ITS = 0. Options for ITS = 1: IMEAS = 0: Measured data not specified for each time IMEAS = 1: Measured data specified for each time
8	TLAST	Real	Largest value of time. Enter 0.0D0 if ITS = 1
	NLC	Integer	Number of logarithmic cycles on time scale. Enter 0 if ITS = 1
	NOX	Integer	Number of equally spaced times per logarithmic cycle for which drawdown will be calculated. Enter 0 if ITS = 1
9	RERRNR	Real	Relative error for Newton-Raphson iteration and finite summations for water-table aquifers. Suggested value is 1.D-10. Use for AQTYPE = WATER TABLE. Enter 0.0D0 if AQTYPE = CONFINED
	RERRSUM	Real	Relative error for finite summations for confined aquifers. Suggested value is 1.D-07 to 1.D-08. Enter 0.0D0 if AQTYPE = WATER TABLE
	NMAX	Integer	Maximum number of terms permitted in the finite summations for confined aquifers. Suggested value is 200. Enter 0 if AQTYPE = WATER TABLE
	NTMS	Integer	Factor used to determine number of terms in the finite summations for water-table aquifers. Suggested values are 20 or 30. Enter 0 if AQTYPE = CONFINED
	NS	Integer	Number of Stehfest terms. Must be an even integer. Eight terms are usually sufficient

**Table 2.** Input data for dimensional format—*Continued*

Line	Variable name	Type	Explanation
<b>PART II: PUMPED-WELL INFORMATION</b>			
10	IPWS	Integer	Type of pumped well: IPWS = 0: Partially penetrating pumped well IPWS = 1: Fully penetrating pumped well
	IPWD	Integer	Type of diameter of pumped well: IPWD = 0 : Infinitesimal diameter (line-source theory) IPWD = 1 : Finite diameter
11	QQ	Real	Pumping rate of well
	RW	Real	Radius of pumped well screen
	RC	Real	Inside radius of pumped well in the interval where water levels are changing during pumping. Enter 0.0D0 if IPWD = 0
	ZPD	Real	Depth below top of aquifer or initial water table to the top of the screened interval of the pumped well
	ZPL	Real	Depth below top of aquifer or initial water table to the bottom of the screened interval of the pumped well
	SW	Real	Well-bore skin parameter
User-specified times and drawdowns. Skip lines 12 and 13 if ITS = 0			
12	NTSPW	Integer	Number of user-specified times at which drawdown will be calculated (if NTSPW = 0, no drawdowns are calculated for the pumped well)
	IRUN	Integer	Option to suppress drawdown calculations for the pumped well: IRUN = 0: Drawdowns not calculated IRUN = 1: Drawdowns calculated
13			Time and measured drawdown data for pumped well. Repeat this line NTSPW times (if NTSPW = 0, skip to line 14):
	TIMEPW(I)	Real	Time I
	XMEASPW(I)	Real	Measured drawdown for time I. Leave blank if IMEAS = 0
<b>PART III: OBSERVATION-WELL INFORMATION</b>			
14	NOBWC	Integer	Number of observation wells or piezometers for which drawdown curves will be calculated. Must be less than or equal to 25
Repeat lines 15 - 18 for each observation well or piezometer			
15	IOWS	Integer	Type of observation well or piezometer: IOWS = 0: Partially penetrating observation well IOWS = 1: Fully penetrating observation well IOWS = 2: Observation piezometer
	IDPR	Integer	Option for delayed response of observation well. Valid only for IOWS = 0 or 1 IDPR = 0 : No delayed response IDPR = 1 : Delayed response
16	R	Real	Radial distance from axis of pumped well to observation well or piezometer
	Z1	Real	Depth below top of aquifer or initial water table to top of screened interval of observation well. Use for IOWS = 0 or 1. Enter 0.0D0 if IOWS = 2
	Z2	Real	Depth below top of aquifer or initial water table to bottom of screened interval of observation well. Use for IOWS = 0 or 1. Enter 0.0D0 if IOWS = 2
	ZP	Real	Depth below top of aquifer or initial water table to center of piezometer. Use for IOWS = 2. Enter 0.0D0 for IOWS = 0 or 1

**Table 2.** Input data for dimensional format—*Continued*

Line	Variable name	Type	Explanation
<b>PART III: OBSERVATION-WELL INFORMATION—<i>Continued</i></b>			
	RP	Real	Inside radius of the observation well standpipe in the interval over which water levels are changing during pumping. Enter 0.0D0 if IDPR = 0
	XLL	Real	Length of screened interval of observation well. Enter 0.0D0 if IDPR= 0
			User-specified times and drawdowns. Skip lines 17 and 18 if ITS = 0
17	NTSOB	Integer	Number of user-specified times at which drawdown will be calculated (if NTSOB = 0, no drawdowns are calculated for the observation well or piezometer)
	IRUN	Integer	Option to suppress drawdown calculations for the observation well or piezometer: IRUN = 0: Drawdowns not calculated IRUN = 1: Drawdowns calculated
18			Time and measured drawdown data for observation well or piezometer. Repeat this line NTSOB times (if NTSOB = 0, skip to next observation point):
	TIMEOB(I)	Real	Time I
	XMEASOB(I)	Real	Measured drawdown for time I. Leave blank if IMEAS = 0

Sample problem 2, using dimensional format.

```

DIMENSIONAL                                FORMAT
WATER TABLE                               AQTYPE
10.0D0 1.0D-4 0.5D-4 2.0D-5 2.0D-01      BB HKR HKZ SS SY
0      0                                     IDRA NALPHA
1.0D09                                     ALPHA (I)
1 1                                         ITS IMEAS
0.0D0 0 0                                   TLAST NLC NOX
1.D-10 0.0D0 0 30 8                       RERRNR RERRSUM NMAX NTMS NS
0 1                                         IPWS IPWD--P.WELL
2.0D-3 0.10D0 0.10D0 5.0D0 10.0D0 0.0D0  QQ RW RC ZPD ZPL SW
14 1                                       NTSPW IRUN
9.28D00 0.51                             TIMEPW(ID) , XMEASPW(ID)
2.00D01 0.97
4.31D01 1.64
9.28D01 2.31
2.00D02 2.65
4.31D02 2.71
9.28D02 2.71
2.00D03 2.72
4.31D03 2.74
9.28D03 2.76
2.00D04 2.81
4.31D04 2.89
9.28D04 2.98
2.00D05 3.09

```

**Figure 7.** Example data-input file using dimensional format.

```

4          NOBWC
2  0      IOWS IDPR--OBPIEZ PD1
3.16D0  0.0D0  0.0D0  7.5D0  0.0D0  0.0D0  R Z1 Z2 ZP RP XLL
14  1      NTSOB IRUN
9.28D00  0.090      TIMEOB (ID) XMEASOB (ID)
2.00D01  0.204
4.31D01  0.384
9.28D01  0.568
2.00D02  0.662
4.31D02  0.680
9.28D02  0.683
2.00D03  0.690
4.31D03  0.703
9.28D03  0.729
2.00D04  0.775
4.31D04  0.847
9.28D04  0.942
2.00D05  1.052
2  0      IOWS IDPR--OBPIEZ PD2
31.6D0  0.0D0  0.0D0  7.5D0  0.0D0  0.0D0  R Z1 Z2 ZP RP XLL
14  1      NTSOB IRUN
9.28D00  0.0000      TIMEOB (ID) XMEASOB (ID)
2.00D01  0.0005
4.31D01  0.0028
9.28D01  0.0071
2.00D02  0.0099
4.31D02  0.0105
9.28D02  0.0110
2.00D03  0.0119
4.31D03  0.0140
9.28D03  0.0186
2.00D04  0.0292
4.31D04  0.0524
9.28D04  0.0983
2.00D05  0.1722
2  0      IOWS IDPR--OBPIEZ PS1
3.16D0  0.0D0  0.0D0  1.0D0  0.0D0  0.0D0  R Z1 Z2 ZP RP XLL
14  1      NTSOB IRUN
9.28D00  0.007      TIMEOB (ID) XMEASOB (ID)
2.00D01  0.019
4.31D01  0.039
9.28D01  0.060
2.00D02  0.072
4.31D02  0.077
9.28D02  0.084
2.00D03  0.098
4.31D03  0.126
9.28D03  0.177
2.00D04  0.258

```

**Figure 7.** Example data-input file using dimensional format—*Continued.*

```

4.31D04  0.365
9.28D04  0.487
2.00D05  0.612
2  0
31.6D0  0.0D0  0.0D0  1.0D0  0.0D0  0.0D0  IOWS IDPR--OBPIEZ PS2
14  1 R Z1 Z2 ZP RP XLL
9.28D00  0.0000 NTSOB IRUN
2.00D01  0.0001 TIMEOB (ID) XMEASOB (ID)
4.31D01  0.0005
9.28D01  0.0012
2.00D02  0.0017
4.31D02  0.0019
9.28D02  0.0022
2.00D03  0.0028
4.31D03  0.0042
9.28D03  0.0075
2.00D04  0.0160
4.31D04  0.038
9.28D04  0.084
2.00D05  0.162

```

**Figure 7.** Example data-input file using dimensional format—*Continued.*

## Result and Plot Files

*WTAQ* generates a result file and, optionally, a plot file for each simulation. The contents of these files differ between the type-curve and dimensional formats. Example result and plot files generated using the type-curve format are shown in figures 13 and 14 (at back of report), respectively, and are based on the data-input file shown for the water-table aquifer example in figure 6. The result file consists of three main sections. The first includes the program banner, analysis format, aquifer type and hydraulic properties, and program time specification and solution variables. The second section lists the characteristics and calculated drawdowns for the pumped well, and the third section lists the characteristics and calculated drawdowns for observation wells and piezometers. Two dimensionless variables, BETAW and BETA, which are calculated by *WTAQ* from information provided in the data-input file, are listed in the second and third sections. These variables are based on the dimensionless parameters  $\beta_w$  and  $\beta$ , and are defined as (Moench, 1997)

$$\beta_w = \frac{K_z}{K_r} \left( \frac{r_w}{b} \right)^2,$$

and

$$\beta = \frac{K_z}{K_r} \left( \frac{r}{b} \right)^2.$$



In the type-curve format, *WTAQ* calculates dimensionless time-drawdown values for the pumped well and observation wells and piezometers. Dimensionless time ( $t_D$ ) is divided by the dimensionless radial distance of each well or piezometer from the axis of the pumped well ( $r_D = r/r_w$ ) for use in composite time-drawdown plots (see sample problem 1). The resulting dimensionless time variable TDRDSQ is written to the result file, and is defined as

$$t_D/r_D^2 = \frac{K_r t}{r^2 S_s} \quad (1)$$

(For the pumped well,  $r_D = 1.0$ .) In the remainder of this report, the dimensionless time divided by the dimensionless radial distance of each well or piezometer will be referred to simply as dimensionless time.

Dimensionless drawdown ( $h_D$ ) is defined as

$$h_D = \frac{4\pi K_r b}{Q} (h_i - h) \quad (2)$$

where

$h_i$  is the initial hydraulic head at the pumped well, observation well, or observation piezometer (units of length), and

$h$  is the hydraulic head at the pumped well, observation well, or observation piezometer at time  $t$  since the start of pumping (units of length). [Note that  $(h_i - h)$  is drawdown at the pumped well, observation well, or observation piezometer (units of length).]

Other parameters in equation 2 were defined previously. Calculated values of dimensionless drawdown less than 0.01 are set to zero.

The plot file for the type-curve format provides a tabulation of all dimensionless time-drawdown calculations for a particular simulation. Several variables shown in the plot file are equivalent in definition to those in the result file. These include dimensionless time (TDRDSQ), which is defined by equation 1; dimensionless drawdowns at the pumped well (HDPW), which are defined by equation 2; and dimensionless drawdowns at the observation wells and piezometers (HDOB1, HDOB2, and so forth), which also are defined by equation 2. Three additional variables are listed in the plot file that are not written to the result file. The first of these is HDT, which is the dimensionless drawdown at dimensionless time TDRDSQ that results from strictly one-dimensional, radial flow in a confined aquifer with storativity equal to  $S_s b$  (that is, Theis-type flow). The second and third variables, TDYRDSQ and HDTY, are the dimensionless time and dimensionless drawdown, respectively, that result from strictly one-dimensional, radial flow in a confined aquifer with storativity equal to  $S_y$  (that is, Theis-type flow with respect to specific yield). This second dimensionless time— $t_{Dy}/r_D^2$ —is defined as

$$t_{Dy}/r_D^2 = \frac{K_r b t}{r^2 S_y} \quad (3)$$

and is related to  $t_D/r_D^2$  by  $\sigma$

$$(t_{Dy}/r_D^2) = \sigma (t_D/r_D^2) \quad (4)$$

or

$$t_{Dy} = \sigma t_D \quad (5)$$

The alternative dimensionless times and drawdowns are provided in columns 1-4 of the plot file for the convenience of the user. The choice of dimensionless time (equation 1 or 3) is at the discretion of the user.

Example result and plot files generated using the dimensional format are shown in figures 15 and 16 (at back of report), respectively, and are based on the data-input file shown in figure 7. The result file consists of three main sections that are similar in scope to those generated by the type-curve format. The primary difference between results generated for the dimensional format and those generated for the type-curve format is that time-drawdown values for the dimensional format are expressed in units of time and length, respectively. Also, when measured drawdowns are specified in the input file, *WTAQ* calculates a relative error between the measured and calculated drawdowns at each time. This error is defined as

$$\frac{(h_m - h_c)}{h_m} \times 100 \quad , \quad (6)$$

where

$h_m$  is the measured drawdown at time  $t$  at the pumped well, observation well, or observation piezometer (units of length); and

$h_c$  is the calculated drawdown at time  $t$  at the pumped well, observation well, or observation piezometer (units of length).

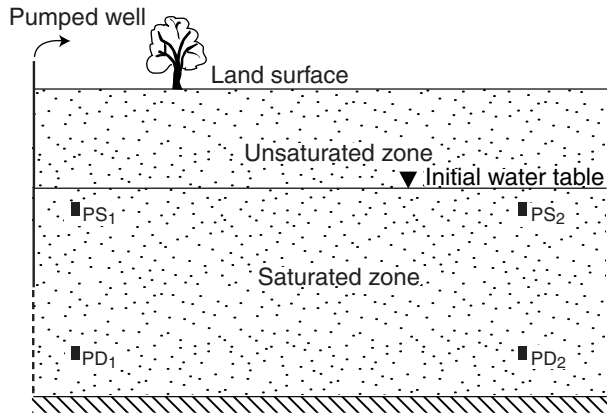
To avoid division by zero in equation 6, *WTAQ* automatically sets the relative error equal to zero when the measured drawdown equals zero, and writes asterisks in the relative error column of the result and plot files.

There are two formats for writing results to the plot file for the dimensional format. In the first, which is used when log-cycle time steps are selected ( $ITS = 0$ ), time-drawdown calculations are written for all wells and piezometers in tabular form, as is done for the type-curve format. In the second, which is used when user-specified times are selected ( $ITS = 1$ ), time-drawdown calculations are written separately for each well or observation piezometer, as shown in figure 16. This is done because the user-specified times may not be the same for the different wells and piezometers.





## SAMPLE PROBLEMS

Three sample problems demonstrate application of *WTAQ* to the estimation of hydraulic properties of a hypothetical, homogeneous water-table aquifer. A cross section illustrating the aquifer and the locations of four observation piezometers is shown in figure 8. It is assumed that the effects of delayed drawdown response at the observation piezometers have been effectively eliminated with hydraulic packers. Aquifer parameters and characteristics of the pumped well are given in table 3, and observation-piezometer locations are given in table 4. Water is assumed to drain instantaneously from the unsaturated zone [variables  $GAMMA(1)$  or  $ALPHA(1)$  equal to  $1.0 \times 10^9$  and variable  $IDRA = 0$ ] and well-bore skin is assumed to be negligible (variable  $SW = 0$ ). As seen in figure 8, the pumped well partially penetrates the aquifer. These partial-penetration effects are accounted for in each of the sample problems.

A finite-difference numerical model also was used to simulate the hypothetical water-table aquifer. The purpose of using a numerical model to simulate the aquifer was to generate a set of synthetic drawdowns at the pumped well and observation piezometers that could be used to demonstrate application of *WTAQ*. The USGS *MODFLOW* model (McDonald and Harbaugh, 1988) was used in conjunction with a radial-flow preprocessor (Reilly and Harbaugh, 1993a,b) for the simulations. The numerical model was developed by T.E. Reilly (U.S. Geological Survey) and is described in detail by Moench (1997). Drawdowns calculated by the numerical model differ from those reported by Moench who used a value of  $S_s$  equal to  $2 \times 10^{-6} \text{ m}^{-1}$ . The value used here ( $2 \times 10^{-5} \text{ m}^{-1}$ ) is an order of magnitude larger and is considered to be a more realistic value of the specific storage of a shallow, water-table aquifer.



#### EXPLANATION

 AQUIFER	 SCREENED INTERVAL OF PUMPED WELL
 IMPERMEABLE (NO-FLOW) BOUNDARY	 PS <sub>1</sub> PIEZOMETER AND IDENTIFIER

**Figure 8.** Hypothetical, homogeneous water-table aquifer with a partially penetrating pumped well of finite diameter and the locations of the open interval of the four observation piezometers. (Piezometers extend to land surface, as shown in fig. 3.)

**Table 3.** Parameters of the hypothetical water-table aquifer and pumped well

[m, meter; m/s, meter per second, m<sup>3</sup>/s, cubic meter per second]

Parameter	Value
Horizontal hydraulic conductivity ( $K_r$ )	$1.0 \times 10^{-4}$ m/s
Vertical hydraulic conductivity ( $K_z$ )	$0.5 \times 10^{-4}$ m/s
Specific storage ( $S_s$ )	$2.0 \times 10^{-5}$ m <sup>-1</sup>
Specific yield ( $S_y$ )	0.2
Saturated thickness ( $b$ )	10 m
Radius of the screened interval of the pumped well ( $r_w$ )	0.1 m
Inside radius of the pumped well ( $r_c$ )	0.1 m
Pumping rate of well ( $Q$ )	$2 \times 10^{-3}$ m <sup>3</sup> /s
Depth below initial water table to top of screened interval of pumped well ( $z_{pd}$ )	5 m
Depth below initial water table to bottom of screened interval of pumped well ( $z_{pl}$ )	10 m
Well-bore skin ( $S_w$ )	0
Empirical drainage constant ( $\alpha_1$ )	$1.0 \times 10^9$

**Table 4.** Locations of observation piezometers in hypothetical water-table aquifer

[distances in meters]

Piezometer	Radial distance from axis of pumped well	Depth below initial water table to center of piezometer
PD <sub>1</sub>	3.16	7.5
PD <sub>2</sub>	31.6	7.5
PS <sub>1</sub>	3.16	1.0
PS <sub>2</sub>	31.6	1.0

Drawdowns calculated for the hypothetical aquifer using the data-input file shown in figure 7 and table 4 by *WTAQ* and those calculated by *MODFLOW* are summarized in table 5. Drawdowns calculated by *MODFLOW* differ slightly from those calculated for the same conditions by *WTAQ*. Possible explanations for the discrepancies between the analytical (*WTAQ*) and numerical results are given by Moench (1997). Drawdowns calculated by *MODFLOW* will be referred to as measured drawdowns for the remainder of this section.

### Sample Problem 1—Estimation of Horizontal and Vertical Hydraulic Conductivity and Specific Yield Using Type-Curve Format

In this first sample problem, *WTAQ* is used to generate dimensionless type curves for the purpose of estimating horizontal hydraulic conductivity ( $K_r$ ), vertical hydraulic conductivity ( $K_z$ ), and specific yield ( $S_y$ ) of the hypothetical aquifer. Moench (1994) provides a detailed description of the methods that are followed here. The data-input file for a type-curve analysis requires specification of three dimensionless parameters ( $K_D$ ,  $\sigma$ , and  $W_D$ ). These dimensionless parameters are initially unknown, because they depend upon the as-yet-undetermined values of  $K_r$ ,  $K_z$ ,  $S_s$ , and  $S_y$ . For the determination of  $K_r$ ,  $K_z$ , and  $S_y$ , the effects of  $S_s$ , which occur at early time, can be ignored. ( $S_s$  will be reconsidered in Sample Problem 2.)

**Table 5.** Drawdown versus time for the pumped well and four observation piezometers in the hypothetical water-table aquifer

[Time, in seconds; Drawdown, in meters;  $t/r^2$ , time/distance<sup>2</sup>, in seconds per square meter, where distance is radial distance from axis of pumped well; A, drawdown calculated by WTAQ; N, drawdown calculated by MODFLOW]

Time	Pumped well						PD <sub>2</sub>						PS <sub>1</sub>						PS <sub>2</sub>					
	Drawdown		Drawdown		Drawdown		Drawdown		Drawdown		Drawdown		Drawdown		Drawdown		Drawdown		Drawdown		Drawdown		Drawdown	
	A	N	$t/r^2$	A	N	$t/r^2$	A	N	$t/r^2$	A	N	$t/r^2$	A	N	$t/r^2$	A	N	$t/r^2$	A	N	$t/r^2$	A	N	
$9.28 \times 10^0$	0.521	0.508	$9.28 \times 10^{-1}$	0.091	0.090	$9.28 \times 10^{-3}$	0.0000	0.0000	$9.28 \times 10^{-1}$	0.006	0.007	$9.28 \times 10^{-3}$	0.0000	0.0000	$9.28 \times 10^{-3}$	0.0000	0.0000	$9.28 \times 10^{-3}$	0.0000	0.0000	$9.28 \times 10^{-3}$	0.0000	0.0000	
$2.00 \times 10^1$	.996	.971	$2.00 \times 10^0$	.211	.204	$2.00 \times 10^{-2}$	.0000	.0005	$2.00 \times 10^0$	.018	.019	$2.00 \times 10^{-2}$	.0000	.0000	$2.00 \times 10^{-2}$	.0000	.0000	$2.00 \times 10^{-2}$	.0000	.0000	$2.00 \times 10^{-2}$	.0000	.0001	
$4.31 \times 10^1$	1.70	1.64	$4.31 \times 10^0$	.402	.384	$4.31 \times 10^{-2}$	.0029	.0028	$4.31 \times 10^0$	.039	.039	$4.31 \times 10^{-2}$	.0029	.0028	$4.31 \times 10^{-2}$	.0000	.0005	$4.31 \times 10^{-2}$	.0000	.0005	$4.31 \times 10^{-2}$	.0000	.0005	
$9.28 \times 10^1$	2.39	2.31	$9.28 \times 10^0$	.594	.568	$9.28 \times 10^{-2}$	.0076	.0071	$9.28 \times 10^0$	.060	.060	$9.28 \times 10^{-2}$	.0076	.0071	$9.28 \times 10^{-2}$	.0000	.0012	$9.28 \times 10^{-2}$	.0000	.0012	$9.28 \times 10^{-2}$	.0000	.0012	
$2.00 \times 10^2$	2.70	2.65	$2.00 \times 10^1$	.682	.662	$2.00 \times 10^{-1}$	.0101	.0099	$2.00 \times 10^1$	.071	.072	$2.00 \times 10^{-1}$	.0101	.0099	$2.00 \times 10^{-1}$	.0018	.0017	$2.00 \times 10^{-1}$	.0018	.0017	$2.00 \times 10^{-1}$	.0018	.0017	
$4.31 \times 10^2$	2.74	2.71	$4.31 \times 10^1$	.693	.680	$4.31 \times 10^{-1}$	.0106	.0105	$4.31 \times 10^1$	.075	.077	$4.31 \times 10^{-1}$	.0106	.0105	$4.31 \times 10^{-1}$	.0019	.0019	$4.31 \times 10^{-1}$	.0019	.0019	$4.31 \times 10^{-1}$	.0019	.0019	
$9.28 \times 10^2$	2.74	2.71	$9.28 \times 10^1$	.695	.683	$9.28 \times 10^{-1}$	.0110	.0110	$9.28 \times 10^1$	.082	.084	$9.28 \times 10^{-1}$	.0110	.0110	$9.28 \times 10^{-1}$	.0022	.0022	$9.28 \times 10^{-1}$	.0022	.0022	$9.28 \times 10^{-1}$	.0022	.0022	
$2.00 \times 10^3$	2.74	2.72	$2.00 \times 10^2$	.702	.690	$2.00 \times 10^0$	.0119	.0119	$2.00 \times 10^2$	.095	.098	$2.00 \times 10^0$	.0119	.0119	$2.00 \times 10^0$	.0028	.0028	$2.00 \times 10^0$	.0028	.0028	$2.00 \times 10^0$	.0028	.0028	
$4.31 \times 10^3$	2.76	2.74	$4.31 \times 10^2$	.715	.703	$4.31 \times 10^0$	.0140	.0140	$4.31 \times 10^2$	.123	.126	$4.31 \times 10^0$	.0140	.0140	$4.31 \times 10^0$	.0042	.0042	$4.31 \times 10^0$	.0042	.0042	$4.31 \times 10^0$	.0042	.0042	
$9.28 \times 10^3$	2.79	2.76	$9.28 \times 10^2$	.741	.729	$9.28 \times 10^0$	.019	.019	$9.28 \times 10^2$	.174	.177	$9.28 \times 10^0$	.019	.019	$9.28 \times 10^0$	.0074	.0075	$9.28 \times 10^0$	.0074	.0075	$9.28 \times 10^0$	.0074	.0075	
$2.00 \times 10^4$	2.83	2.81	$2.00 \times 10^3$	.788	.775	$2.00 \times 10^1$	.029	.029	$2.00 \times 10^3$	.256	.258	$2.00 \times 10^1$	.029	.029	$2.00 \times 10^1$	.016	.016	$2.00 \times 10^1$	.016	.016	$2.00 \times 10^1$	.016	.016	
$4.31 \times 10^4$	2.91	2.88	$4.31 \times 10^3$	.861	.847	$4.31 \times 10^1$	.053	.052	$4.31 \times 10^3$	.365	.365	$4.31 \times 10^1$	.053	.052	$4.31 \times 10^1$	.037	.038	$4.31 \times 10^1$	.037	.038	$4.31 \times 10^1$	.037	.038	
$9.28 \times 10^4$	3.01	2.98	$9.28 \times 10^3$	.957	.942	$9.28 \times 10^1$	.099	.098	$9.28 \times 10^3$	.488	.487	$9.28 \times 10^1$	.099	.098	$9.28 \times 10^1$	.086	.084	$9.28 \times 10^1$	.086	.084	$9.28 \times 10^1$	.086	.084	
$2.00 \times 10^5$	3.12	3.09	$2.00 \times 10^4$	1.068	1.052	$2.00 \times 10^2$	.175	.172	$2.00 \times 10^4$	.613	.612	$2.00 \times 10^2$	.175	.172	$2.00 \times 10^2$	.166	.162	$2.00 \times 10^2$	.166	.162	$2.00 \times 10^2$	.166	.162	

This is done by setting  $\sigma$  to some small value (such as  $10^{-9}$ ). Also, well-bore storage should be ignored (that is, set equal to zero) because it does not substantially affect the intermediate- and late-time drawdown data that are used to estimate  $K_r$ ,  $K_z$ , and  $S_y$ . Well-bore storage effects are removed from the analysis by setting variables IPWD = 0 and WD = 0.0D0 in the data-input file. The only remaining dimensionless parameter that must be specified, therefore, is  $K_D$ . For this analysis, an initial estimate of  $K_D$  equal to 0.1 was assigned. The data-input file used to generate dimensionless type curves for this initial simulation is shown in figure 9.

Several authors have discussed the importance of including drawdown data from all available observation points simultaneously in the analysis of aquifer-test results (Stallman, 1971; Weeks, 1977; van der Kamp, 1985; Moench, 1994). This is accomplished by creating a single composite plot of all measured drawdown data plotted against  $t/r^2$ , where  $t$  is the time of each drawdown measurement and  $r^2$  is the square of the radial distance of the observation point from the axis of the pumped well. This approach was taken for this analysis, and the resulting values of  $t/r^2$  for the four hypothetical observation piezometers are shown with the drawdown data in table 5. A double-logarithmic composite plot using data from all four observation points was made, and is shown in both figure 10A and 10B.

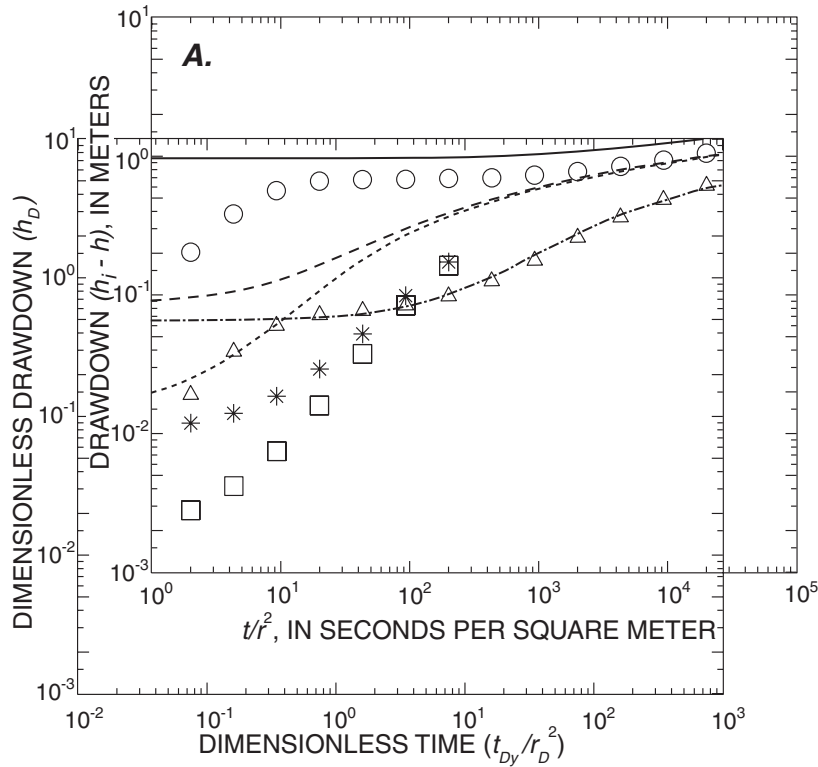
WTAQ calculates two values of dimensionless time for the type-curve format ( $t_D/r_D^2$  and  $t_{Dy}/r_D^2$ ). These dimensionless times are defined in equations 1 and 3, and are identified in the plot file by variables TDRDSQ and TDYRDSQ, respectively. A double-logarithmic graph is made of either TDRDSQ or TDYRDSQ plotted against each of the dimensionless drawdowns (shown in figure 14 as HDOB1, HDOB2, and so forth). For matching purposes this graph must be at the same size and have the same number of log cycles that were used for the composite plot of the measured data. For this analysis, dimensionless-time values TDYRDSQ were used (fig. 10) because of the emphasis on late-time data.

```

Sample problem 1, using infinitesimal diameter pumped well.
TYPE CURVE                                FORMAT
WATER TABLE                              AQTYPE
10.0D0  0.1D0  1.0D-9                     BB  XKD  SIGMA
0        0                                     IDRA  NGAMMA
1.0D09                                     GAMMA (I)
1.0D12  11        5                         TDLAST  NLC  NOX
1.0D-10 0.0D0  0  30  8                   RERRNR  RERRSUM  NMAX  NTMS  NS
0  0                                         IPWS  IPWD--PUMPED WELL
0.10D0  5.0D0  10.0D0  0.0D00  0.0D0     RW  ZPD  ZPL  WD  SW
4                                             NOBWC
2  0                                         IOWS  IDPR--OBPIEZ  PD1
3.16D0  0.0D0  0.0D0  7.5D0  0.0D0       R  Z1  Z2  ZP  WDP
2  0                                         IOWS  IDPR--OBPIEZ  PD2
31.6D0  0.0D0  0.0D0  7.5D0  0.0D0       R  Z1  Z2  ZP  WDP
2  0                                         IOWS  IDPR--OBPIEZ  PS1
3.16D0  0.0D0  0.0D0  1.0D0  0.0D0       R  Z1  Z2  ZP  WDP
2  0                                         IOWS  IDPR--OBPIEZ  PS2
31.6D0  0.0D0  0.0D0  1.0D0  0.0D0       R  Z1  Z2  ZP  WDP

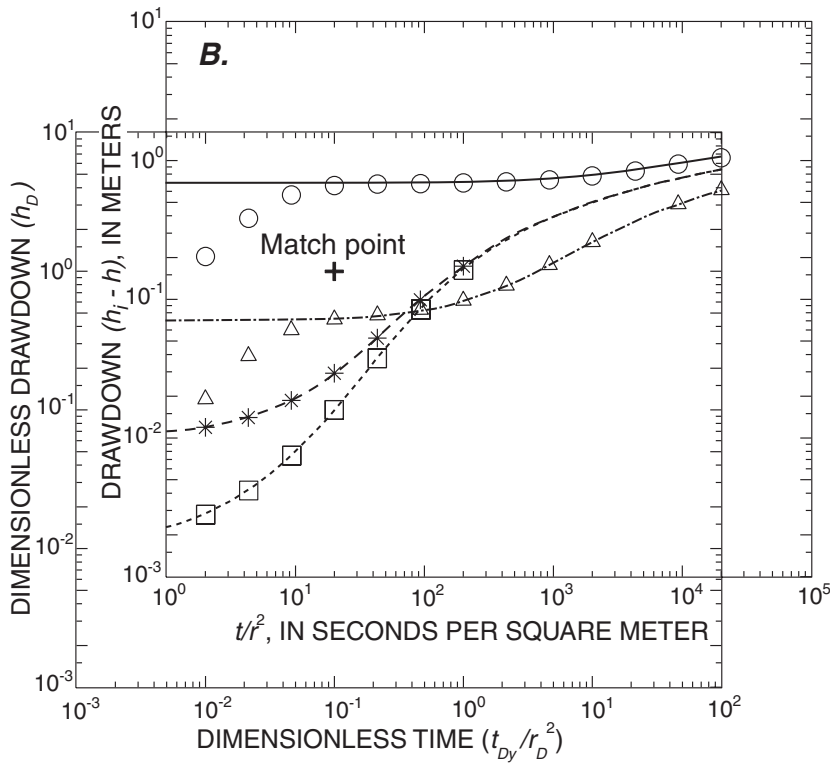
```

Figure 9. Data-input file using type-curve format for sample problem 1.



**EXPLANATION**

Observation piezometer	Measured data	Calculated type curve
PD <sub>1</sub>	○	—
PD <sub>2</sub>	*	- - - -
PS <sub>1</sub>	△	- · - · - ·
PS <sub>2</sub>	□	- · - · - ·



**Figure 10.** Composite plots of measured drawdown ( $h_i - h$ ) and calculated dimensionless drawdown (type curves) for (A)  $K_D = 0.1$  and (B)  $K_D = 0.5$  for the hypothetical water-table aquifer. ( $K_D$ , ratio of horizontal to vertical hydraulic conductivity;  $t/r^2$ , time divided by the square of the radial distance of the piezometer from the axis of pumped well.)

The composite plot of the measured data is superimposed onto the theoretical type curves, while the coordinate axes of each graph are kept parallel, such that the late-time drawdown data for a given piezometer overlies as much of the late-time type curve for the same piezometer as possible (fig. 10A). The large deviation between measured and theoretical data in figure 10A is due to the erroneous first guess of 0.1 for  $K_D$ . Because the measured data cannot be overlain onto the type curves in figure 10A, the value of  $K_D$  must be adjusted in the data-input file and new type curves generated. In this analysis,  $K_D$  was increased from 0.1 to 0.5 because, with the exception of the type curve for piezometer PS<sub>1</sub>, type-curve drawdowns were larger than the measured drawdown data for  $K_D = 0.1$  (see, for example, the curve for piezometer PD<sub>1</sub> in fig. 10A). The increase in  $K_D$  effectively decreases the calculated drawdowns by increasing vertical flow in the hypothetical aquifer, and, as seen in fig. 10B, resulted in an improvement in the match between the intermediate- and late-time portions of the measured drawdown data with the intermediate- and late-time portions of the type curves. The deviation at early time for piezometers PS<sub>1</sub> and PD<sub>1</sub> in figure 10B is due to the assumption that  $S_s = 0$ . Therefore, the  $K_D$  of the aquifer is taken to be 0.5, which is equal to the ratio of the values of vertical and horizontal hydraulic conductivity given in table 3.

A single, global match point is now selected on the superimposed graphs (fig. 10B) from which simultaneous values of  $t_{Dy}/r_D^2$  and  $h_D$ , and of  $t/r^2$  and  $(h_i - h)$  can be determined (the values of these four variables at the match point are identified by asterisks). This global match point is chosen arbitrarily, and for this problem was chosen to be  $(t_{Dy}/r_D^2)^* = 0.1$  and  $h_D^* = 1.0$ , at which  $(t/r^2)^* = 20.0 \text{ s/m}^2$  and  $(h_i - h)^* = 0.16 \text{ m}$  (fig. 10B).

Transmissivity of the aquifer is calculated from

$$T = \frac{Q}{4\pi} \left( \frac{h_D^*}{(h_i - h)^*} \right). \quad (7)$$

Using the values of  $h_D^*$  and  $(h_i - h)^*$  determined in figure 10B and the pumping rate given in table 3,

$$T = \frac{(2.0 \times 10^{-3} \text{ m}^3/\text{s})}{4\pi} \left( \frac{1.0}{0.16 \text{ m}} \right) = 1.0 \times 10^{-3} \text{ m}^2/\text{s}.$$

Horizontal hydraulic conductivity of the aquifer is calculated from the relation  $T = K_r b$ , where  $b = 10 \text{ m}$  (table 3):

$$K_r = T/b = \frac{1.0 \times 10^{-3} \text{ m}^2/\text{s}}{10 \text{ m}} = 1.0 \times 10^{-4} \text{ m/s}.$$

This calculated value equals that for the aquifer given in table 3. Vertical hydraulic conductivity is then calculated from the definition of  $K_D$  and the calculated value of  $K_r$ ; that is,

$$K_z = K_D K_r = (0.5)(1.0 \times 10^{-4} \text{ m/s}) = 0.5 \times 10^{-4} \text{ m/s}.$$

Specific yield is calculated using either

$$S_y = \frac{T (t/r^2)^*}{\sigma (t_{Dy}/r_D^2)^*} \quad (8)$$

if values of  $t_{Dy}/r_D^2$  are used in the analysis, or

$$S_y = T \frac{(t/r^2)^*}{(t_{Dy}/r_D^2)^*} \quad (9)$$

if values of  $t_{Dy}/r_D^2$  are used. Because values of  $t_{Dy}/r_D^2$  were used in this problem, equation 9 gives

$$S_y = 1.0 \times 10^{-3} \text{ m}^2/\text{s} \frac{(20.0 \text{ s}/\text{m}^2)}{0.1} = 0.2,$$

which equals the value of specific yield given in table 3.

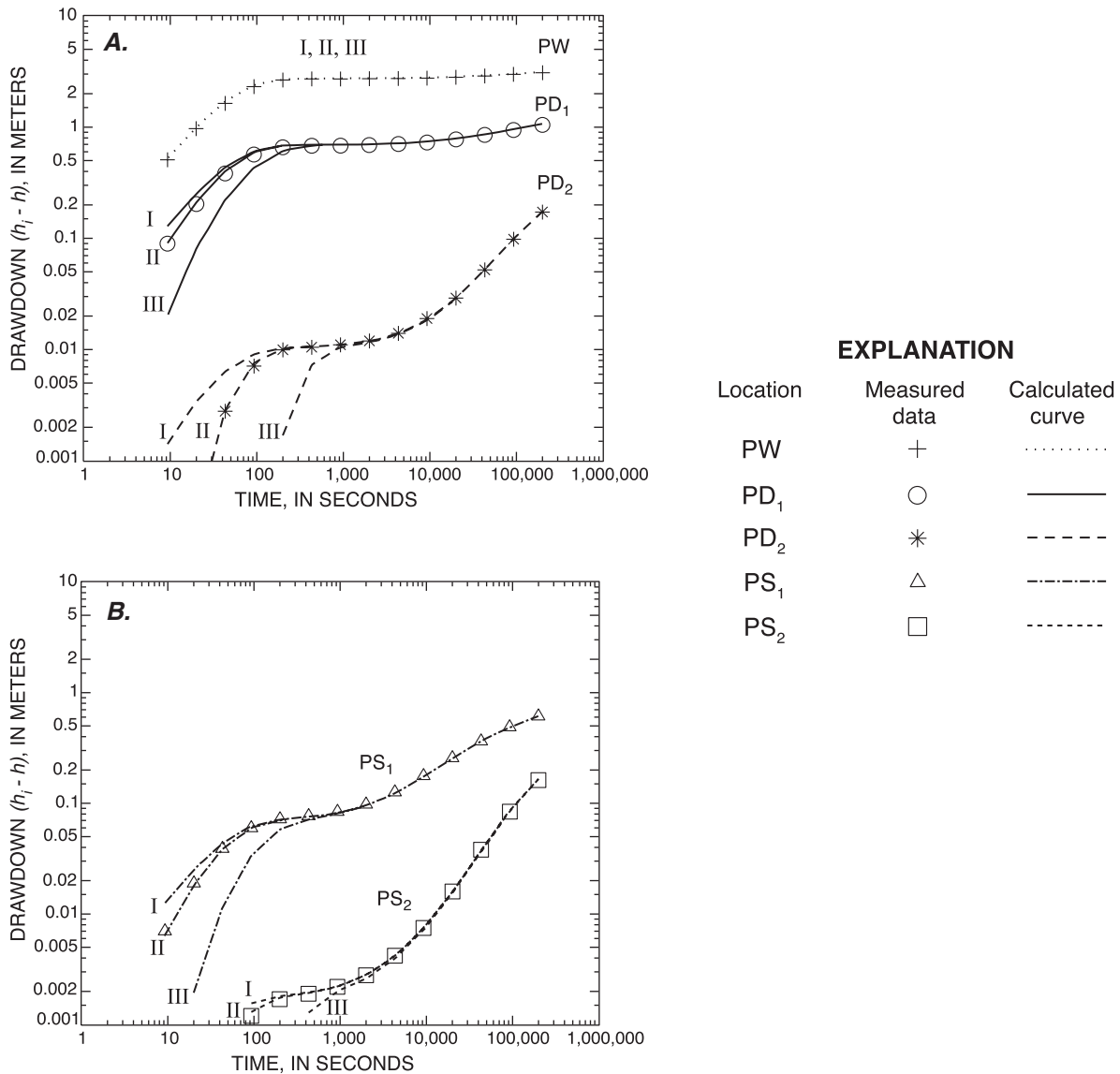
## Sample Problem 2—Estimation of Specific Storage Using Dimensional Format

A type-curve analysis was done in sample problem 1 to estimate the horizontal and vertical hydraulic conductivity and specific yield of the hypothetical water-table aquifer. The advantage of the type-curve approach to analysis of aquifer tests conducted in water-table aquifers is that only one parameter,  $K_D$ , needs adjusting to evaluate  $K_r$ ,  $K_z$ , and  $S_y$ . Of the four hydraulic properties of the aquifer that must be estimated ( $K_r$ ,  $K_z$ ,  $S_s$ , and  $S_y$ ), only  $S_s$  remains unknown. To estimate  $S_s$ , the dimensional format of *WTAQ* is now used to generate theoretical, dimensional time-drawdown responses that are compared graphically to the measured time-drawdown data. The value of  $S_s$  is varied in a series of simulations until the measured data overlies the theoretical responses. Because well-bore storage at the pumped well affects the estimation of  $S_s$ , the finite diameter of the pumped well must now be taken into consideration. Well-bore storage is accounted for using the dimensional format by setting variable  $IPWD = 1$ , specifying a value for the inside radius of the pumped well (variable  $RC$ , which equals 0.1 m in this example; table 3), and specifying a value of  $S_s$  (variable  $SS$ ). An example data-input file for this problem is shown in figure 7.

Dimensional time-drawdown graphs that show the measured data overlain onto theoretical time-drawdown curves calculated by *WTAQ* are given in figure 11. Three separate simulations were done with *WTAQ* using values of  $S_s$  equal to  $2.0 \times 10^{-6} \text{ m}^{-1}$  (simulation I),  $2.0 \times 10^{-5} \text{ m}^{-1}$  (simulation II), and  $2.0 \times 10^{-4} \text{ m}^{-1}$  (simulation III). The best match between the measured and theoretical responses corresponded to a value of  $S_s$  equal to  $2.0 \times 10^{-5} \text{ m}^{-1}$ , which is the value given for the hypothetical aquifer in table 3.

The theoretical time-drawdown curves demonstrate several aspects of the drawdown response of a water-table aquifer to variations in specific storage. First, calculated drawdowns for the pumped well are not noticeably changed by variations in the value of  $S_s$ . Second, as noted by Moench (1997), reductions in the value of specific storage below  $2.0 \times 10^{-6} \text{ m}^{-1}$  bring about no substantial change in the calculated time-drawdown responses. This is because hydraulic head at the observation piezometers in the hypothetical aquifer respond almost instantaneously to changing drawdown in the pumped well when  $S_s$  is less than about  $10^{-6} \text{ m}^{-1}$ . Under this condition, the response of the water-table aquifer approaches that of a rigid aquifer in which specific storage equals zero. Finally, figure 11 also shows that as the value of specific storage is increased from  $2.0 \times 10^{-6} \text{ m}^{-1}$  to  $2.0 \times 10^{-4} \text{ m}^{-1}$ , there is an increased delay in the early-time drawdowns at the observation piezometers.





**Figure 11.** Drawdown versus time for the hypothetical water-table aquifer. Calculated curves use values of specific storage equal to  $2.0 \times 10^{-6} \text{ m}^{-1}$  (I),  $2.0 \times 10^{-5} \text{ m}^{-1}$  (II), and  $2.0 \times 10^{-4} \text{ m}^{-1}$  (III). (A) Wells PW, PD<sub>1</sub>, and PD<sub>2</sub>. (B) Wells PS<sub>1</sub> and PS<sub>2</sub>.

### Sample Problem 3—Automatic Parameter Estimation Using Dimensional Format

In the previous two problems, trial-and-error approaches were used to estimate  $K_r$ ,  $K_z$ ,  $S_s$ , and  $S_y$  of the hypothetical water-table aquifer. In both approaches, the user adjusts the hydraulic parameters of the analytical model in a series of simulations until the measured time-drawdown data can be overlain graphically onto either the dimensionless or dimensional theoretical time-drawdown curves. The process of finding a set of hydraulic parameters that causes the theoretical curves to match the measured data closely is called model calibration. An alternative to this graphical approach is the use of automatic parameter estimation based on nonlinear-regression techniques. In that approach, a computer program is used to find a set of hydraulic parameters of the model that minimizes the sum of squared errors between measured and model-calculated drawdowns at each observation

location and time. The set of model parameters that minimizes these errors is said to be the optimal set of parameter values. The use of automatic parameter-estimation techniques for estimating aquifer hydraulic properties from aquifer-test data is described by Sayed (1990), Johns and others (1992), and Heidari and Moench (1997), as well as in several references listed in those citations.

The purpose of this sample problem is to demonstrate how aquifer hydraulic properties can be estimated by linking *WTAQ* with an automatic parameter-estimation program. The computer program *UCODE* (Poeter and Hill, 1998) was chosen for the parameter estimation, although other parameter-estimation programs may have worked equally well. The objective of the analysis was to estimate all four hydraulic properties of the hypothetical water-table aquifer ( $K_r$ ,  $K_z$ ,  $S_s$ , and  $S_y$ ) simultaneously. It is necessary to use the dimensional format in the *WTAQ* simulations so that calculated drawdowns can be compared directly with the measured data.

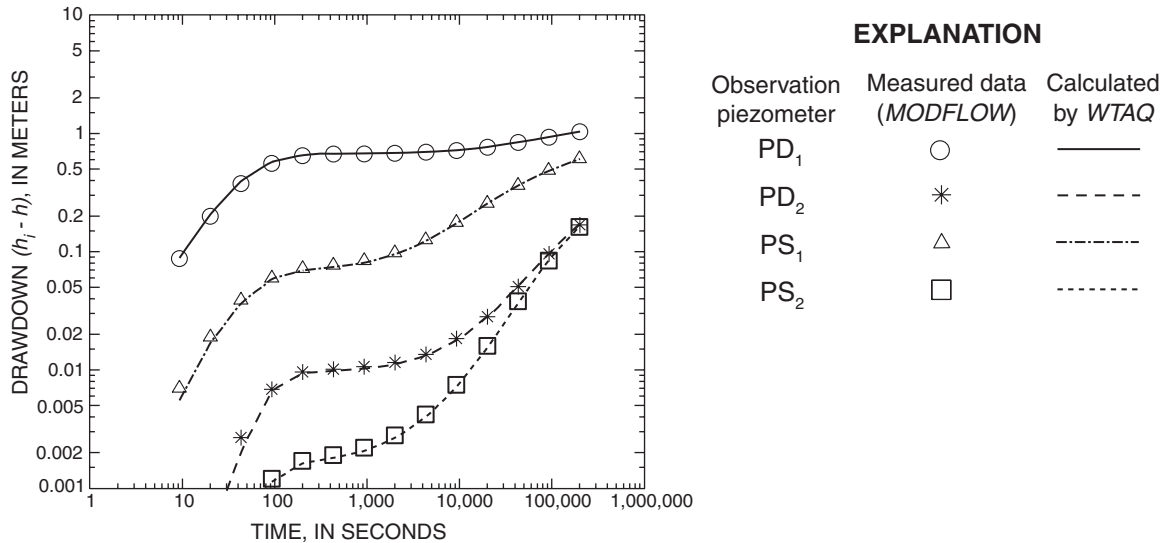
The parameter-estimation program requires that measured drawdowns at observation locations be specified. In this analysis, measured data from all four of the observation piezometers were used. The parameter-estimation program also requires specification of initial estimates of each of the parameters that are to be determined; those used in this analysis are shown in table 6. The values of these initial estimates should be varied in a series of parameter-estimation runs to ensure that the same set of optimal parameter values is determined regardless of the particular values selected for the initial estimates. Data-input files used by *UCODE* for this problem are given in Attachment 2.

As shown in table 6, the estimated values determined by *UCODE* for each of the hydraulic properties of the water-table aquifer are close to the true values. The relatively large discrepancy between the true value ( $2.00 \times 10^{-5} \text{ m}^{-1}$ ) and the estimated value ( $2.53 \times 10^{-5} \text{ m}^{-1}$ ) of  $S_s$  is most likely because of differences in drawdowns calculated at the four piezometer locations by *WTAQ* (the theoretical drawdowns) and by *MODFLOW* (the ‘measured’ drawdowns) at early times. Also shown in table 6 are the lower and upper 95-percent confidence intervals for each of the estimated values. These optimal estimates were obtained after only eight iterations of the parameter-estimation process, in which *WTAQ* was called by *UCODE* to simulate eight separate combinations of hydraulic properties. As shown by the time-drawdown curves on figure 12, drawdowns calculated at each of the piezometer locations using the estimated values of the hydraulic properties are nearly identical to those calculated by *MODFLOW*.

**Table 6.** Hydraulic properties of the hypothetical water-table aquifer determined using parameter estimation

[m, meter; m/s, meter per second]

Parameter (units)	True value	Initial estimate	Estimated value	95-percent confidence intervals of estimated parameter	
				Lower	Upper
$K_r$ (m/s)	$1.00 \times 10^{-4}$	$1.00 \times 10^{-5}$	$1.01 \times 10^{-4}$	$1.01 \times 10^{-4}$	$1.02 \times 10^{-4}$
$K_z$ (m/s)	$0.50 \times 10^{-4}$	$1.00 \times 10^{-5}$	$0.52 \times 10^{-4}$	$0.52 \times 10^{-4}$	$0.53 \times 10^{-4}$
$S_s$ ( $\text{m}^{-1}$ )	$2.00 \times 10^{-5}$	$1.00 \times 10^{-2}$	$2.53 \times 10^{-5}$	$2.35 \times 10^{-5}$	$2.71 \times 10^{-5}$
$S_y$	$2.00 \times 10^{-1}$	$1.00 \times 10^{-1}$	$2.02 \times 10^{-1}$	$1.97 \times 10^{-1}$	$2.06 \times 10^{-1}$



**Figure 12.** Drawdown versus time at the four observation piezometers for the hypothetical water-table aquifer. Drawdowns calculated by *WTAQ* using estimated values of hydraulic properties (lines) and by *MODFLOW* using true values of hydraulic properties (symbols).

## SUMMARY

This report describes the computer program *WTAQ*, which was developed to implement a previously derived analytical solution for axial-symmetric flow to a partially penetrating, finite-diameter well in a homogeneous and anisotropic water-table aquifer. The solution accounts for well-bore storage and well-bore skin at the pumped well, delayed response at an observation piezometer, and delayed or instantaneous drainage from the unsaturated zone. For the particular case of zero drainage from the unsaturated zone, the solution simplifies to that of axial-symmetric flow in a confined aquifer.

*WTAQ* calculates dimensionless or dimensional drawdowns that can be used in conjunction with measured drawdowns at observation points to estimate hydraulic properties of confined and water-table aquifers. Three sample problems illustrate use of *WTAQ* for estimating horizontal and vertical hydraulic conductivity, specific storage, and specific yield of a hypothetical water-table aquifer by type-curve methods and by use of an automatic parameter-estimation method. In the first sample problem, dimensionless type-curve analysis is used to estimate the horizontal and vertical hydraulic conductivity and specific yield of the aquifer. This is done by ignoring specific storage of the aquifer and well-bore storage, which do not affect the intermediate- and late-time drawdown data that are used to estimate  $K_r$ ,  $K_z$ , and  $S_y$ . In the second sample problem, specific storage of the aquifer is estimated by using *WTAQ* to generate dimensional time-drawdown curves that include well-bore storage effects caused by the finite diameter of the pumped well. These theoretical, model-derived time-drawdown curves are then compared directly to the measured time-drawdown data to estimate  $S_s$ . A third sample problem illustrates use of *WTAQ* in conjunction with an automatic parameter-estimation program that is based on nonlinear regression to estimate all four hydraulic properties of the water-table aquifer simultaneously. In all three sample problems, the estimation techniques yield parameter values that are equal or nearly equal to those specified for the hypothetical aquifer.

The report includes a summary of the simplifying assumptions used in the development of the analytical model, descriptions of the program variables, and instructions for constructing the necessary data-input files for program execution.

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FIGURES 13–16

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*****
*
*          ****  U.S. GEOLOGICAL SURVEY  ****
*
*          ****  WTAQ: PROGRAM OUTPUT  ****
*
*  COMPUTER PROGRAM FOR CALCULATING DRAWDOWN
*
*  IN A CONFINED OR WATER-TABLE AQUIFER WITH
*
*  AXIAL-SYMMETRIC FLOW TO A FINITE- OR
*
*  INFINITESIMAL-DIAMETER PUMPED WELL
*
*  VERSION CURRENT AS OF 10/01/99
*
*****

```

Example input file using type-curve format.

```

TYPE-CURVE FORMAT          WATER-TABLE AQUIFER

```

```

*** AQUIFER HYDRAULIC PROPERTIES ***

```

```

SATURATED THICKNESS (BB):          0.100D+02 (units of length)
RATIO OF VERTICAL TO HORIZONTAL
HYDRAULIC CONDUCTIVITY (XKD):      0.500D+00 (dimensionless)
RATIO OF STORATIVITY TO
SPECIFIC YIELD (SIGMA):            0.100D-02 (dimensionless)
DRAINAGE AT WATER TABLE (IDRA):   0 (instantaneous)

```

```

*** PROGRAM SOLUTION VARIABLES ***

```

LARGEST VALUE OF TIME (TDLAST)	NUMBER OF LOG CYCLES (NLC)	DRAWDOWN CALCULATIONS PER LOG CYCLE (NOX)		
0.100D+13	11	5		
RERRNR	RERRSUM	NMAX	NTMS	NS
0.100D-09	0.000D+00	0	30	8

Figure 13. Example result file using type-curve format.

\*\*\* PUMPED-WELL CHARACTERISTICS AND CALCULATED DRAWDOWN \*\*\*

WELL-DIAMETER TYPE (IPWD): 1 (finite diameter)  
 SCREENED INTERVAL (IPWS): 0 (partially penetrating)

WELL RADIUS	SCREENED INTERVAL		WELL BORE	WELL BORE
	ZPD	ZPL	STORAGE	SKIN
0.10D+00	0.50D+01	0.10D+02	0.50D+04	0.00D+00

BETAW = 0.500D-04

DIMENSIONLESS TIME (TDRDSQ)	DIMENSIONLESS DRAWDOWN (HD)
0.1000D+02	0.0000D+00
0.1585D+02	0.1267D-01
0.2512D+02	0.2006D-01
0.3981D+02	0.3178D-01
0.6310D+02	0.5031D-01
0.1000D+03	0.7962D-01
0.1585D+03	0.1259D+00
0.2512D+03	0.1989D+00
0.3981D+03	0.3138D+00
0.6310D+03	0.4937D+00
0.1000D+04	0.7742D+00
0.1585D+04	0.1208D+01
0.2512D+04	0.1868D+01
0.3981D+04	0.2856D+01
0.6310D+04	0.4286D+01
0.1000D+05	0.6260D+01
0.1585D+05	0.8784D+01
0.2512D+05	0.1164D+02
0.3981D+05	0.1428D+02
0.6310D+05	0.1612D+02
0.1000D+06	0.1697D+02
0.1585D+06	0.1718D+02
0.2512D+06	0.1720D+02
0.3981D+06	0.1720D+02
0.6310D+06	0.1721D+02
0.1000D+07	0.1724D+02
0.1585D+07	0.1729D+02
0.2512D+07	0.1736D+02
0.3981D+07	0.1746D+02
0.6310D+07	0.1761D+02
0.1000D+08	0.1781D+02
0.1585D+08	0.1807D+02

Figure 13. Example result file using type-curve format—Continued.

0.2512D+08	0.1840D+02
0.3981D+08	0.1877D+02
0.6310D+08	0.1917D+02
0.1000D+09	0.1960D+02
0.1585D+09	0.2004D+02
0.2512D+09	0.2049D+02
0.3981D+09	0.2094D+02
0.6310D+09	0.2140D+02
0.1000D+10	0.2186D+02
0.1585D+10	0.2231D+02
0.2512D+10	0.2277D+02
0.3981D+10	0.2323D+02
0.6310D+10	0.2369D+02
0.1000D+11	0.2415D+02
0.1585D+11	0.2461D+02
0.2512D+11	0.2507D+02
0.3981D+11	0.2553D+02
0.6310D+11	0.2600D+02
0.1000D+12	0.2646D+02
0.1585D+12	0.2692D+02
0.2512D+12	0.2738D+02
0.3981D+12	0.2784D+02
0.6310D+12	0.2830D+02
0.1000D+13	0.2876D+02

\*\*\* OBSERVATION-WELL CHARACTERISTICS AND CALCULATED DRAWDOWN \*\*\*

\*\*\*\* OBSERVATION WELL OR PIEZOMETER 1 \*\*\*\*

OBSERVATION PIEZOMETER

DISTANCE FROM CENTER OF PUMPED WELL	ZP	DELAYED RESPONSE FACTOR
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0.316D+01	0.750D+01	0.000D+00

DRAWDOWN CALCULATED FOR BETA = 0.499D-01

Figure 13. Example result file using type-curve format—Continued.

DIMENSIONLESS TIME (TDRDSQ)	DIMENSIONLESS DRAWDOWN (HD)
0.1000D+02	0.1321D+01
0.1585D+02	0.2001D+01
0.2512D+02	0.2788D+01
0.3981D+02	0.3527D+01
0.6310D+02	0.4043D+01
0.1000D+03	0.4282D+01
0.1585D+03	0.4346D+01
0.2512D+03	0.4355D+01
0.3981D+03	0.4363D+01
0.6310D+03	0.4380D+01
0.1000D+04	0.4408D+01
0.1585D+04	0.4451D+01
0.2512D+04	0.4516D+01
0.3981D+04	0.4612D+01
0.6310D+04	0.4753D+01
0.1000D+05	0.4947D+01
0.1585D+05	0.5202D+01
0.2512D+05	0.5518D+01
0.3981D+05	0.5883D+01
0.6310D+05	0.6284D+01
0.1000D+06	0.6709D+01
0.1585D+06	0.7148D+01
0.2512D+06	0.7596D+01
0.3981D+06	0.8048D+01
0.6310D+06	0.8503D+01
0.1000D+07	0.8961D+01
0.1585D+07	0.9419D+01
0.2512D+07	0.9878D+01
0.3981D+07	0.1034D+02
0.6310D+07	0.1080D+02
0.1000D+08	0.1126D+02
0.1585D+08	0.1172D+02
0.2512D+08	0.1218D+02
0.3981D+08	0.1264D+02
0.6310D+08	0.1310D+02
0.1000D+09	0.1356D+02
0.1585D+09	0.1402D+02
0.2512D+09	0.1448D+02
0.3981D+09	0.1494D+02
0.6310D+09	0.1540D+02
0.1000D+10	0.1586D+02
0.1585D+10	0.1632D+02
0.2512D+10	0.1678D+02
0.3981D+10	0.1724D+02
0.6310D+10	0.1770D+02

Figure 13. Example result file using type-curve format—Continued.

0.1000D+11	0.1817D+02
0.1585D+11	0.1863D+02
0.2512D+11	0.1909D+02
0.3981D+11	0.1955D+02
0.6310D+11	0.2001D+02
0.1000D+12	0.2047D+02
0.1585D+12	0.2093D+02
0.2512D+12	0.2139D+02
0.3981D+12	0.2185D+02
0.6310D+12	0.2231D+02
0.1000D+13	0.2277D+02

\*\*\*\* OBSERVATION WELL OR PIEZOMETER 2 \*\*\*\*

OBSERVATION PIEZOMETER

DISTANCE FROM CENTER OF PUMPED WELL	ZP	DELAYED RESPONSE FACTOR
0.316D+02	0.750D+01	0.000D+00

DRAWDOWN CALCULATED FOR BETA = 0.499D+01

DIMENSIONLESS TIME (TDRDSQ)	DIMENSIONLESS DRAWDOWN (HD)
0.1000D+02	0.7498D-01
0.1585D+02	0.8151D-01
0.2512D+02	0.9203D-01
0.3981D+02	0.1091D+00
0.6310D+02	0.1369D+00
0.1000D+03	0.1824D+00
0.1585D+03	0.2565D+00
0.2512D+03	0.3741D+00
0.3981D+03	0.5502D+00
0.6310D+03	0.7928D+00
0.1000D+04	0.1099D+01
0.1585D+04	0.1455D+01
0.2512D+04	0.1848D+01
0.3981D+04	0.2265D+01
0.6310D+04	0.2698D+01
0.1000D+05	0.3141D+01
0.1585D+05	0.3591D+01

Figure 13. Example result file using type-curve format—Continued.

0.2512D+05	0.4044D+01
0.3981D+05	0.4500D+01
0.6310D+05	0.4958D+01
0.1000D+06	0.5417D+01
0.1585D+06	0.5876D+01
0.2512D+06	0.6336D+01
0.3981D+06	0.6796D+01
0.6310D+06	0.7256D+01
0.1000D+07	0.7717D+01
0.1585D+07	0.8177D+01
0.2512D+07	0.8638D+01
0.3981D+07	0.9098D+01
0.6310D+07	0.9559D+01
0.1000D+08	0.1002D+02
0.1585D+08	0.1048D+02
0.2512D+08	0.1094D+02
0.3981D+08	0.1140D+02
0.6310D+08	0.1186D+02
0.1000D+09	0.1232D+02
0.1585D+09	0.1278D+02
0.2512D+09	0.1324D+02
0.3981D+09	0.1370D+02
0.6310D+09	0.1416D+02
0.1000D+10	0.1462D+02
0.1585D+10	0.1508D+02
0.2512D+10	0.1555D+02
0.3981D+10	0.1601D+02
0.6310D+10	0.1647D+02
0.1000D+11	0.1693D+02
0.1585D+11	0.1739D+02
0.2512D+11	0.1785D+02
0.3981D+11	0.1831D+02
0.6310D+11	0.1877D+02
0.1000D+12	0.1923D+02
0.1585D+12	0.1969D+02
0.2512D+12	0.2015D+02
0.3981D+12	0.2061D+02
0.6310D+12	0.2107D+02
0.1000D+13	0.2153D+02

**Figure 13.** Example result file using type-curve format—*Continued.*

OBSERVATION PIEZOMETER

DISTANCE FROM CENTER OF PUMPED WELL	ZP	DELAYED RESPONSE FACTOR
0.316D+01	0.100D+01	0.000D+00

DRAWDOWN CALCULATED FOR BETA = 0.499D-01

DIMENSIONLESS TIME (TDRDSQ)	DIMENSIONLESS DRAWDOWN (HD)
0.1000D+02	0.1144D+00
0.1585D+02	0.1860D+00
0.2512D+02	0.2711D+00
0.3981D+02	0.3529D+00
0.6310D+02	0.4122D+00
0.1000D+03	0.4441D+00
0.1585D+03	0.4605D+00
0.2512D+03	0.4766D+00
0.3981D+03	0.5011D+00
0.6310D+03	0.5397D+00
0.1000D+04	0.5993D+00
0.1585D+04	0.6895D+00
0.2512D+04	0.8230D+00
0.3981D+04	0.1014D+01
0.6310D+04	0.1274D+01
0.1000D+05	0.1606D+01
0.1585D+05	0.2003D+01
0.2512D+05	0.2444D+01
0.3981D+05	0.2907D+01
0.6310D+05	0.3378D+01
0.1000D+06	0.3847D+01
0.1585D+06	0.4314D+01
0.2512D+06	0.4779D+01
0.3981D+06	0.5243D+01
0.6310D+06	0.5705D+01
0.1000D+07	0.6167D+01
0.1585D+07	0.6628D+01
0.2512D+07	0.7089D+01
0.3981D+07	0.7550D+01
0.6310D+07	0.8011D+01
0.1000D+08	0.8471D+01

Figure 13. Example result file using type-curve format—Continued.

0.1585D+08	0.8932D+01
0.2512D+08	0.9392D+01
0.3981D+08	0.9853D+01
0.6310D+08	0.1031D+02
0.1000D+09	0.1077D+02
0.1585D+09	0.1123D+02
0.2512D+09	0.1170D+02
0.3981D+09	0.1216D+02
0.6310D+09	0.1262D+02
0.1000D+10	0.1308D+02
0.1585D+10	0.1354D+02
0.2512D+10	0.1400D+02
0.3981D+10	0.1446D+02
0.6310D+10	0.1492D+02
0.1000D+11	0.1538D+02
0.1585D+11	0.1584D+02
0.2512D+11	0.1630D+02
0.3981D+11	0.1676D+02
0.6310D+11	0.1722D+02
0.1000D+12	0.1768D+02
0.1585D+12	0.1814D+02
0.2512D+12	0.1860D+02
0.3981D+12	0.1906D+02
0.6310D+12	0.1952D+02
0.1000D+13	0.1998D+02

\*\*\*\* OBSERVATION WELL OR PIEZOMETER 4 \*\*\*\*

OBSERVATION PIEZOMETER

DISTANCE FROM CENTER OF PUMPED WELL	ZP	DELAYED RESPONSE FACTOR
0.316D+02	0.100D+01	0.000D+00

DRAWDOWN CALCULATED FOR BETA = 0.499D+01

DIMENSIONLESS TIME (TDRDSQ)	DIMENSIONLESS DRAWDOWN (HD)
0.1000D+02	0.1751D-01
0.1585D+02	0.2176D-01
0.2512D+02	0.2884D-01
0.3981D+02	0.4088D-01
0.6310D+02	0.6179D-01
0.1000D+03	0.9879D-01

Figure 13. Example result file using type-curve format—Continued.



0.1585D+03	0.1646D+00
0.2512D+03	0.2783D+00
0.3981D+03	0.4592D+00
0.6310D+03	0.7156D+00
0.1000D+04	0.1040D+01
0.1585D+04	0.1413D+01
0.2512D+04	0.1819D+01
0.3981D+04	0.2246D+01
0.6310D+04	0.2685D+01
0.1000D+05	0.3132D+01
0.1585D+05	0.3584D+01
0.2512D+05	0.4040D+01
0.3981D+05	0.4497D+01
0.6310D+05	0.4955D+01
0.1000D+06	0.5414D+01
0.1585D+06	0.5874D+01
0.2512D+06	0.6334D+01
0.3981D+06	0.6794D+01
0.6310D+06	0.7254D+01
0.1000D+07	0.7715D+01
0.1585D+07	0.8175D+01
0.2512D+07	0.8636D+01
0.3981D+07	0.9096D+01
0.6310D+07	0.9557D+01
0.1000D+08	0.1002D+02
0.1585D+08	0.1048D+02
0.2512D+08	0.1094D+02
0.3981D+08	0.1140D+02
0.6310D+08	0.1186D+02
0.1000D+09	0.1232D+02
0.1585D+09	0.1278D+02
0.2512D+09	0.1324D+02
0.3981D+09	0.1370D+02
0.6310D+09	0.1416D+02
0.1000D+10	0.1462D+02
0.1585D+10	0.1508D+02
0.2512D+10	0.1554D+02
0.3981D+10	0.1600D+02
0.6310D+10	0.1646D+02
0.1000D+11	0.1692D+02
0.1585D+11	0.1739D+02
0.2512D+11	0.1785D+02
0.3981D+11	0.1831D+02
0.6310D+11	0.1877D+02
0.1000D+12	0.1923D+02
0.1585D+12	0.1969D+02
0.2512D+12	0.2015D+02
0.3981D+12	0.2061D+02
0.6310D+12	0.2107D+02
0.1000D+13	0.2153D+02

**Figure 13.** Example result file using type-curve format—*Continued.*

TDRDSQ	HDT	TDYRDSQ	HDITY	HDPW	HDOB1	HDOB2	HDOB3	HDOB4
0.100E+02	0.314E+01	0.100E-01	0.000E+00	0.000E+00	0.132E+01	0.750E-01	0.114E+00	0.175E-01
0.158E+02	0.359E+01	0.158E-01	0.000E+00	0.127E-01	0.200E+01	0.815E-01	0.186E+00	0.218E-01
0.251E+02	0.404E+01	0.251E-01	0.000E+00	0.201E-01	0.279E+01	0.920E-01	0.271E+00	0.288E-01
0.398E+02	0.450E+01	0.398E-01	0.000E+00	0.318E-01	0.353E+01	0.109E+00	0.353E+00	0.409E-01
0.631E+02	0.496E+01	0.631E-01	0.000E+00	0.503E-01	0.404E+01	0.137E+00	0.412E+00	0.618E-01
0.100E+03	0.542E+01	0.100E+00	0.248E-01	0.796E-01	0.428E+01	0.182E+00	0.444E+00	0.988E-01
0.158E+03	0.588E+01	0.158E+00	0.899E-01	0.126E+00	0.435E+01	0.257E+00	0.461E+00	0.165E+00
0.251E+03	0.634E+01	0.251E+00	0.222E+00	0.199E+00	0.435E+01	0.374E+00	0.477E+00	0.278E+00
0.398E+03	0.680E+01	0.398E+00	0.430E+00	0.314E+00	0.436E+01	0.550E+00	0.501E+00	0.459E+00
0.631E+03	0.726E+01	0.631E+00	0.709E+00	0.494E+00	0.438E+01	0.793E+00	0.540E+00	0.716E+00
0.100E+04	0.772E+01	0.100E+01	0.104E+01	0.774E+00	0.441E+01	0.110E+01	0.599E+00	0.104E+01
0.158E+04	0.818E+01	0.158E+01	0.142E+01	0.121E+01	0.445E+01	0.146E+01	0.690E+00	0.141E+01
0.251E+04	0.864E+01	0.251E+01	0.183E+01	0.187E+01	0.452E+01	0.185E+01	0.823E+00	0.182E+01
0.398E+04	0.910E+01	0.398E+01	0.225E+01	0.286E+01	0.461E+01	0.227E+01	0.101E+01	0.225E+01
0.631E+04	0.956E+01	0.631E+01	0.269E+01	0.429E+01	0.475E+01	0.270E+01	0.127E+01	0.269E+01
0.100E+05	0.100E+02	0.100E+02	0.314E+01	0.626E+01	0.495E+01	0.314E+01	0.161E+01	0.313E+01
0.158E+05	0.105E+02	0.158E+02	0.359E+01	0.878E+01	0.520E+01	0.359E+01	0.200E+01	0.358E+01
0.251E+05	0.109E+02	0.251E+02	0.404E+01	0.116E+02	0.552E+01	0.404E+01	0.244E+01	0.404E+01
0.398E+05	0.114E+02	0.398E+02	0.450E+01	0.143E+02	0.588E+01	0.450E+01	0.291E+01	0.450E+01
0.631E+05	0.119E+02	0.631E+02	0.496E+01	0.161E+02	0.628E+01	0.496E+01	0.338E+01	0.496E+01
0.100E+06	0.123E+02	0.100E+03	0.542E+01	0.170E+02	0.671E+01	0.542E+01	0.385E+01	0.541E+01
0.158E+06	0.128E+02	0.158E+03	0.588E+01	0.172E+02	0.715E+01	0.588E+01	0.431E+01	0.587E+01
0.251E+06	0.132E+02	0.251E+03	0.634E+01	0.172E+02	0.760E+01	0.634E+01	0.478E+01	0.633E+01
0.398E+06	0.137E+02	0.398E+03	0.680E+01	0.172E+02	0.805E+01	0.680E+01	0.524E+01	0.679E+01
0.631E+06	0.142E+02	0.631E+03	0.726E+01	0.172E+02	0.850E+01	0.726E+01	0.571E+01	0.725E+01
0.100E+07	0.146E+02	0.100E+04	0.772E+01	0.172E+02	0.896E+01	0.772E+01	0.617E+01	0.771E+01
0.158E+07	0.151E+02	0.158E+04	0.818E+01	0.173E+02	0.942E+01	0.818E+01	0.663E+01	0.818E+01
0.251E+07	0.155E+02	0.251E+04	0.864E+01	0.174E+02	0.988E+01	0.864E+01	0.709E+01	0.864E+01
0.398E+07	0.160E+02	0.398E+04	0.910E+01	0.175E+02	0.103E+02	0.910E+01	0.755E+01	0.910E+01
0.631E+07	0.165E+02	0.631E+04	0.956E+01	0.176E+02	0.108E+02	0.956E+01	0.801E+01	0.956E+01
0.100E+08	0.169E+02	0.100E+05	0.100E+02	0.178E+02	0.113E+02	0.100E+02	0.847E+01	0.100E+02
0.158E+08	0.174E+02	0.158E+05	0.105E+02	0.181E+02	0.117E+02	0.105E+02	0.893E+01	0.105E+02
0.251E+08	0.178E+02	0.251E+05	0.109E+02	0.184E+02	0.122E+02	0.109E+02	0.939E+01	0.109E+02

Figure 14. Example plot file using type-curve format.

0.398E+08 0.183E+02 0.398E+05 0.114E+02 0.188E+02 0.126E+02 0.114E+02 0.985E+01 0.114E+02  
0.631E+08 0.188E+02 0.631E+05 0.119E+02 0.192E+02 0.131E+02 0.119E+02 0.103E+02 0.119E+02  
0.100E+09 0.192E+02 0.100E+06 0.123E+02 0.196E+02 0.136E+02 0.123E+02 0.108E+02 0.123E+02  
0.158E+09 0.197E+02 0.158E+06 0.128E+02 0.200E+02 0.140E+02 0.128E+02 0.112E+02 0.128E+02  
0.251E+09 0.202E+02 0.251E+06 0.132E+02 0.205E+02 0.145E+02 0.132E+02 0.117E+02 0.132E+02  
0.398E+09 0.206E+02 0.398E+06 0.137E+02 0.209E+02 0.149E+02 0.137E+02 0.122E+02 0.137E+02  
0.631E+09 0.211E+02 0.631E+06 0.142E+02 0.214E+02 0.154E+02 0.142E+02 0.126E+02 0.142E+02  
0.100E+10 0.215E+02 0.100E+07 0.146E+02 0.219E+02 0.159E+02 0.146E+02 0.131E+02 0.146E+02  
0.158E+10 0.220E+02 0.158E+07 0.151E+02 0.223E+02 0.163E+02 0.151E+02 0.135E+02 0.151E+02  
0.251E+10 0.225E+02 0.251E+07 0.155E+02 0.228E+02 0.168E+02 0.155E+02 0.140E+02 0.155E+02  
0.398E+10 0.229E+02 0.398E+07 0.160E+02 0.232E+02 0.172E+02 0.160E+02 0.145E+02 0.160E+02  
0.631E+10 0.234E+02 0.631E+07 0.165E+02 0.237E+02 0.177E+02 0.165E+02 0.149E+02 0.165E+02  
0.100E+11 0.238E+02 0.100E+08 0.169E+02 0.242E+02 0.182E+02 0.169E+02 0.154E+02 0.169E+02  
0.158E+11 0.243E+02 0.158E+08 0.174E+02 0.246E+02 0.186E+02 0.174E+02 0.158E+02 0.174E+02  
0.251E+11 0.248E+02 0.251E+08 0.178E+02 0.251E+02 0.191E+02 0.178E+02 0.163E+02 0.178E+02  
0.398E+11 0.252E+02 0.398E+08 0.183E+02 0.255E+02 0.195E+02 0.183E+02 0.168E+02 0.183E+02  
0.631E+11 0.257E+02 0.631E+08 0.188E+02 0.260E+02 0.200E+02 0.188E+02 0.172E+02 0.188E+02  
0.100E+12 0.261E+02 0.100E+09 0.192E+02 0.265E+02 0.205E+02 0.192E+02 0.177E+02 0.192E+02  
0.158E+12 0.266E+02 0.158E+09 0.197E+02 0.269E+02 0.209E+02 0.197E+02 0.181E+02 0.197E+02  
0.251E+12 0.271E+02 0.251E+09 0.202E+02 0.274E+02 0.214E+02 0.202E+02 0.186E+02 0.202E+02  
0.398E+12 0.275E+02 0.398E+09 0.206E+02 0.278E+02 0.218E+02 0.206E+02 0.191E+02 0.206E+02  
0.631E+12 0.280E+02 0.631E+09 0.211E+02 0.283E+02 0.223E+02 0.211E+02 0.195E+02 0.211E+02  
0.100E+13 0.284E+02 0.100E+10 0.215E+02 0.288E+02 0.228E+02 0.215E+02 0.200E+02 0.215E+02

Figure 14. Example plot file using type-curve format—Continued.

```

*****
*
*          **** U.S. GEOLOGICAL SURVEY ****
*
*          **** WTAQ: PROGRAM OUTPUT ****
*
*    COMPUTER PROGRAM FOR CALCULATING DRAWDOWN
*
*    IN A CONFINED OR WATER-TABLE AQUIFER WITH
*
*    AXIAL-SYMMETRIC FLOW TO A FINITE- OR
*
*    INFINITESIMAL-DIAMETER PUMPED WELL
*
*          VERSION CURRENT AS OF 10/01/99
*
*****

```

Sample problem 2, using dimensional format.

DIMENSIONAL FORMAT

WATER-TABLE AQUIFER

\*\*\* AQUIFER HYDRAULIC PROPERTIES \*\*\*

SATURATED THICKNESS (BB):	0.100D+02 (units of length)
HORIZONTAL HYDRAULIC CONDUCTIVITY (HKR):	0.100D-03 (units of length per time)
VERTICAL HYDRAULIC CONDUCTIVITY (HKZ):	0.500D-04 (units of length per time)
RATIO OF VERTICAL TO HORIZONTAL HYDRAULIC CONDUCTIVITY (XKD):	0.500D+00 (dimensionless)
CALCULATED TRANSMISSIVITY:	0.100D-02 (units of length squared per time)
SPECIFIC STORAGE (SS):	0.200D-04 (units of inverse length)
SPECIFIC YIELD (SY):	0.200D+00 (dimensionless)
CALCULATED STORATIVITY:	0.200D-03 (dimensionless)
RATIO OF STORATIVITY TO SPECIFIC YIELD (SIGMA):	0.100D-02 (dimensionless)
DRAINAGE AT WATER TABLE (IDRA):	0 (instantaneous)

Figure 15. Example result file using dimensional format.

\*\*\* PROGRAM SOLUTION VARIABLES \*\*\*

USER-SPECIFIED TIMES; MEASURED DRAWDOWN DATA SPECIFIED

RERRNR	RERRSUM	NMAX	NTMS	NS
0.100D-09	0.000D+00	0	30	8

\*\*\* PUMPED-WELL CHARACTERISTICS AND CALCULATED DRAWDOWN \*\*\*

WELL-DIAMETER TYPE (IPWD): 1 (finite diameter)  
 SCREENED INTERVAL (IPWS): 0 (partially penetrating)  
 PUMPING RATE OF WELL (QQ): 0.200D-02 (cubic length per time)

WELL RADIUS	SCREENED INTERVAL ZPD	INTERVAL ZPL	WELL BORE STORAGE	WELL BORE SKIN
0.10D+00	0.50D+01	0.10D+02	0.50D+04	0.00D+00

BETAW = 0.500D-04

TIME	MEASURED DRAWDOWN	CALCULATED DRAWDOWN	RELATIVE ERROR (PERCENT)
0.9280D+01	0.5100D+00	0.5214D+00	-0.2244D+01
0.2000D+02	0.9700D+00	0.9963D+00	-0.2713D+01
0.4310D+02	0.1640D+01	0.1699D+01	-0.3626D+01
0.9280D+02	0.2310D+01	0.2388D+01	-0.3394D+01
0.2000D+03	0.2650D+01	0.2700D+01	-0.1895D+01
0.4310D+03	0.2710D+01	0.2737D+01	-0.9962D+00
0.9280D+03	0.2710D+01	0.2738D+01	-0.1016D+01
0.2000D+04	0.2720D+01	0.2744D+01	-0.8975D+00
0.4310D+04	0.2740D+01	0.2759D+01	-0.6761D+00
0.9280D+04	0.2760D+01	0.2786D+01	-0.9340D+00
0.2000D+05	0.2810D+01	0.2834D+01	-0.8686D+00
0.4310D+05	0.2890D+01	0.2910D+01	-0.6791D+00
0.9280D+05	0.2980D+01	0.3008D+01	-0.9310D+00
0.2000D+06	0.3090D+01	0.3119D+01	-0.9524D+00

Figure 15. Example result file using dimensional format—Continued.

\*\*\* OBSERVATION-WELL CHARACTERISTICS AND CALCULATED DRAWDOWN \*\*\*

\*\*\*\* OBSERVATION WELL OR PIEZOMETER 1 \*\*\*\*

OBSERVATION PIEZOMETER

DISTANCE FROM CENTER OF PUMPED WELL	ZP	DELAYED RESPONSE FACTOR
0.316D+01	0.750D+01	0.000D+00

DRAWDOWN CALCULATED FOR BETA = 0.499D-01

TIME	MEASURED DRAWDOWN	CALCULATED DRAWDOWN	RELATIVE ERROR (PERCENT)
0.9280D+01	0.9000D-01	0.9071D-01	-0.7844D+00
0.2000D+02	0.2040D+00	0.2106D+00	-0.3236D+01
0.4310D+02	0.3840D+00	0.4018D+00	-0.4629D+01
0.9280D+02	0.5680D+00	0.5939D+00	-0.4567D+01
0.2000D+03	0.6620D+00	0.6816D+00	-0.2954D+01
0.4310D+03	0.6800D+00	0.6928D+00	-0.1881D+01
0.9280D+03	0.6830D+00	0.6951D+00	-0.1769D+01
0.2000D+04	0.6900D+00	0.7015D+00	-0.1669D+01
0.4310D+04	0.7030D+00	0.7148D+00	-0.1676D+01
0.9280D+04	0.7290D+00	0.7407D+00	-0.1609D+01
0.2000D+05	0.7750D+00	0.7875D+00	-0.1608D+01
0.4310D+05	0.8470D+00	0.8606D+00	-0.1610D+01
0.9280D+05	0.9420D+00	0.9572D+00	-0.1615D+01
0.2000D+06	0.1052D+01	0.1068D+01	-0.1526D+01

Figure 15. Example result file using dimensional format—Continued.

\*\*\*\* OBSERVATION WELL OR PIEZOMETER 2 \*\*\*\*

OBSERVATION PIEZOMETER

DISTANCE FROM CENTER OF PUMPED WELL	ZP	DELAYED RESPONSE FACTOR
0.316D+02	0.750D+01	0.000D+00

DRAWDOWN CALCULATED FOR BETA = 0.499D+01

TIME	MEASURED DRAWDOWN	CALCULATED DRAWDOWN	RELATIVE ERROR (PERCENT)
0.9280D+01	0.0000D+00	0.0000D+00	****
0.2000D+02	0.5000D-03	0.0000D+00	0.1000D+03
0.4310D+02	0.2800D-02	0.2890D-02	-0.3217D+01
0.9280D+02	0.7100D-02	0.7606D-02	-0.7127D+01
0.2000D+03	0.9900D-02	0.1012D-01	-0.2258D+01
0.4310D+03	0.1050D-01	0.1059D-01	-0.8321D+00
0.9280D+03	0.1100D-01	0.1100D-01	0.3762D-01
0.2000D+04	0.1190D-01	0.1194D-01	-0.3075D+00
0.4310D+04	0.1400D-01	0.1400D-01	-0.2910D-01
0.9280D+04	0.1860D-01	0.1861D-01	-0.4377D-01
0.2000D+05	0.2920D-01	0.2905D-01	0.4981D+00
0.4310D+05	0.5240D-01	0.5245D-01	-0.8710D-01
0.9280D+05	0.9830D-01	0.9934D-01	-0.1057D+01
0.2000D+06	0.1722D+00	0.1750D+00	-0.1641D+01

Figure 15. Example result file using dimensional format—Continued.

\*\*\*\* OBSERVATION WELL OR PIEZOMETER 3 \*\*\*\*

OBSERVATION PIEZOMETER

DISTANCE FROM CENTER OF PUMPED WELL	ZP	DELAYED RESPONSE FACTOR
0.316D+01	0.100D+01	0.000D+00

DRAWDOWN CALCULATED FOR BETA = 0.499D-01

TIME	MEASURED DRAWDOWN	CALCULATED DRAWDOWN	RELATIVE ERROR (PERCENT)
0.9280D+01	0.7000D-02	0.6376D-02	0.8912D+01
0.2000D+02	0.1900D-01	0.1824D-01	0.4023D+01
0.4310D+02	0.3900D-01	0.3858D-01	0.1066D+01
0.9280D+02	0.6000D-01	0.5984D-01	0.2670D+00
0.2000D+03	0.7200D-01	0.7069D-01	0.1825D+01
0.4310D+03	0.7700D-01	0.7491D-01	0.2714D+01
0.9280D+03	0.8400D-01	0.8152D-01	0.2949D+01
0.2000D+04	0.9800D-01	0.9542D-01	0.2633D+01
0.4310D+04	0.1260D+00	0.1231D+00	0.2311D+01
0.9280D+04	0.1770D+00	0.1739D+00	0.1729D+01
0.2000D+05	0.2580D+00	0.2558D+00	0.8476D+00
0.4310D+05	0.3650D+00	0.3652D+00	-0.6745D-01
0.9280D+05	0.4870D+00	0.4878D+00	-0.1672D+00
0.2000D+06	0.6120D+00	0.6126D+00	-0.9086D-01

Figure 15. Example result file using dimensional format—Continued.



\*\*\*\* OBSERVATION WELL OR PIEZOMETER 4 \*\*\*\*

OBSERVATION PIEZOMETER

DISTANCE FROM CENTER OF PUMPED WELL	ZP	DELAYED RESPONSE FACTOR
0.316D+02	0.100D+01	0.000D+00

DRAWDOWN CALCULATED FOR BETA = 0.499D+01

TIME	MEASURED DRAWDOWN	CALCULATED DRAWDOWN	RELATIVE ERROR (PERCENT)
0.9280D+01	0.0000D+00	0.0000D+00	****
0.2000D+02	0.1000D-03	0.0000D+00	0.1000D+03
0.4310D+02	0.5000D-03	0.0000D+00	0.1000D+03
0.9280D+02	0.1200D-02	0.0000D+00	0.1000D+03
0.2000D+03	0.1700D-02	0.1762D-02	-0.3626D+01
0.4310D+03	0.1900D-02	0.1929D-02	-0.1507D+01
0.9280D+03	0.2200D-02	0.2193D-02	0.3342D+00
0.2000D+04	0.2800D-02	0.2788D-02	0.4134D+00
0.4310D+04	0.4200D-02	0.4152D-02	0.1136D+01
0.9280D+04	0.7500D-02	0.7423D-02	0.1024D+01
0.2000D+05	0.1600D-01	0.1575D-01	0.1575D+01
0.4310D+05	0.3800D-01	0.3728D-01	0.1898D+01
0.9280D+05	0.8400D-01	0.8546D-01	-0.1738D+01
0.2000D+06	0.1620D+00	0.1656D+00	-0.2233D+01

Figure 15. Example result file using dimensional format—Continued.

```

**** PUMPED WELL ****

TIME          MEASDD          CALCDD          RELERR
0.9280E+01    0.5100E+00      0.5214E+00     -0.2244E+01
0.2000E+02    0.9700E+00      0.9963E+00     -0.2713E+01
0.4310E+02    0.1640E+01      0.1699E+01     -0.3626E+01
0.9280E+02    0.2310E+01      0.2388E+01     -0.3394E+01
0.2000E+03    0.2650E+01      0.2700E+01     -0.1895E+01
0.4310E+03    0.2710E+01      0.2737E+01     -0.9962E+00
0.9280E+03    0.2710E+01      0.2738E+01     -0.1016E+01
0.2000E+04    0.2720E+01      0.2744E+01     -0.8975E+00
0.4310E+04    0.2740E+01      0.2759E+01     -0.6761E+00
0.9280E+04    0.2760E+01      0.2786E+01     -0.9340E+00
0.2000E+05    0.2810E+01      0.2834E+01     -0.8686E+00
0.4310E+05    0.2890E+01      0.2910E+01     -0.6791E+00
0.9280E+05    0.2980E+01      0.3008E+01     -0.9310E+00
0.2000E+06    0.3090E+01      0.3119E+01     -0.9524E+00

**** OBSERVATION WELL OR PIEZOMETER 1 ****

TIME          MEASDD          CALCDD          RELERR
0.9280E+01    0.9000E-01      0.9071E-01     -0.7844E+00
0.2000E+02    0.2040E+00      0.2106E+00     -0.3236E+01
0.4310E+02    0.3840E+00      0.4018E+00     -0.4629E+01
0.9280E+02    0.5680E+00      0.5939E+00     -0.4567E+01
0.2000E+03    0.6620E+00      0.6816E+00     -0.2954E+01
0.4310E+03    0.6800E+00      0.6928E+00     -0.1881E+01
0.9280E+03    0.6830E+00      0.6951E+00     -0.1769E+01
0.2000E+04    0.6900E+00      0.7015E+00     -0.1669E+01
0.4310E+04    0.7030E+00      0.7148E+00     -0.1676E+01
0.9280E+04    0.7290E+00      0.7407E+00     -0.1609E+01
0.2000E+05    0.7750E+00      0.7875E+00     -0.1608E+01
0.4310E+05    0.8470E+00      0.8606E+00     -0.1610E+01
0.9280E+05    0.9420E+00      0.9572E+00     -0.1615E+01
0.2000E+06    0.1052E+01      0.1068E+01     -0.1526E+01

**** OBSERVATION WELL OR PIEZOMETER 2 ****

TIME          MEASDD          CALCDD          RELERR
0.9280E+01    0.0000E+00      0.0000E+00      ****
0.2000E+02    0.5000E-03      0.0000E+00      0.1000E+03
0.4310E+02    0.2800E-02      0.2890E-02     -0.3217E+01
0.9280E+02    0.7100E-02      0.7606E-02     -0.7127E+01
0.2000E+03    0.9900E-02      0.1012E-01     -0.2258E+01

```

Figure 16. Example plot file using dimensional format.

0.4310E+03	0.1050E-01	0.1059E-01	-0.8321E+00
0.9280E+03	0.1100E-01	0.1100E-01	0.3762E-01
0.2000E+04	0.1190E-01	0.1194E-01	-0.3075E+00
0.4310E+04	0.1400E-01	0.1400E-01	-0.2910E-01
0.9280E+04	0.1860E-01	0.1861E-01	-0.4377E-01
0.2000E+05	0.2920E-01	0.2905E-01	0.4981E+00
0.4310E+05	0.5240E-01	0.5245E-01	-0.8710E-01
0.9280E+05	0.9830E-01	0.9934E-01	-0.1057E+01
0.2000E+06	0.1722E+00	0.1750E+00	-0.1641E+01

\*\*\*\* OBSERVATION WELL OR PIEZOMETER 3 \*\*\*\*

TIME	MEASDD	CALCDD	RELERR
0.9280E+01	0.7000E-02	0.6376E-02	0.8912E+01
0.2000E+02	0.1900E-01	0.1824E-01	0.4023E+01
0.4310E+02	0.3900E-01	0.3858E-01	0.1066E+01
0.9280E+02	0.6000E-01	0.5984E-01	0.2670E+00
0.2000E+03	0.7200E-01	0.7069E-01	0.1825E+01
0.4310E+03	0.7700E-01	0.7491E-01	0.2714E+01
0.9280E+03	0.8400E-01	0.8152E-01	0.2949E+01
0.2000E+04	0.9800E-01	0.9542E-01	0.2633E+01
0.4310E+04	0.1260E+00	0.1231E+00	0.2311E+01
0.9280E+04	0.1770E+00	0.1739E+00	0.1729E+01
0.2000E+05	0.2580E+00	0.2558E+00	0.8476E+00
0.4310E+05	0.3650E+00	0.3652E+00	-0.6745E-01
0.9280E+05	0.4870E+00	0.4878E+00	-0.1672E+00
0.2000E+06	0.6120E+00	0.6126E+00	-0.9086E-01

\*\*\*\* OBSERVATION WELL OR PIEZOMETER 4 \*\*\*\*

TIME	MEASDD	CALCDD	RELERR
0.9280E+01	0.0000E+00	0.0000E+00	****
0.2000E+02	0.1000E-03	0.0000E+00	0.1000E+03
0.4310E+02	0.5000E-03	0.0000E+00	0.1000E+03
0.9280E+02	0.1200E-02	0.0000E+00	0.1000E+03
0.2000E+03	0.1700E-02	0.1762E-02	-0.3626E+01
0.4310E+03	0.1900E-02	0.1929E-02	-0.1507E+01
0.9280E+03	0.2200E-02	0.2193E-02	0.3342E+00
0.2000E+04	0.2800E-02	0.2788E-02	0.4134E+00
0.4310E+04	0.4200E-02	0.4152E-02	0.1136E+01
0.9280E+04	0.7500E-02	0.7423E-02	0.1024E+01
0.2000E+05	0.1600E-01	0.1575E-01	0.1575E+01
0.4310E+05	0.3800E-01	0.3728E-01	0.1898E+01
0.9280E+05	0.8400E-01	0.8546E-01	-0.1738E+01
0.2000E+06	0.1620E+00	0.1656E+00	-0.2233E+01

Figure 16. Example plot file using dimensional format.



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ATTACHMENT 1.—NARRATIVE FOR  
PROGRAM *WTAQ*

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Program *WTAQ* calculates ground-water-level drawdowns in confined and water-table aquifers in response to withdrawals at a pumped well. *WTAQ* is written in the FORTRAN-77 computer language. A description of the numbered steps in the code follows:

Main program:

1. Call subroutine *OFFILE* to open input (variable *IN*), result (variable *IO*), and plot (variable *IP*) files.
2. Call subroutine *DATAPREP* to read most input data from input file, write input data to result file, and prepare input data for use in main program.
3. Define selected program parameters.
4. Call subroutine *LINVST* to calculate coefficients used for the Stehfest (1970) algorithm.
5. Calculate time-step parameters for log-cycle formats.
6. Calculate required number of time-drawdown curves.
7. Calculate time-drawdown curves for each well:
  - a. Calculate dimensionless variables to pass to Laplace transform solution subroutines.
  - b. Define dimensionless time (*TD*) and iteration variables.
  - c. Calculate dimensionless drawdown (*HD*) for each dimensionless time (*TD*). If aquifer is confined, call subroutine *LTST2*; if water-table aquifer, call subroutine *LTST3*.
  - d. Write time-drawdown calculations to result file and, for dimensional data, to plot file.
8. Calculate Theis drawdowns and write all time-drawdown calculations to plot file for type-curve formats (including dimensional):
  - a. Calculate Theis drawdowns using subroutine *LTST1*.
  - b. Write dimensionless time-drawdown calculations to plot file.
  - c. Write dimensional time-drawdown calculations to plot file.
9. Close input, result, and plot files; stop program execution; end.

Subroutine *OFFILE*:

10. Subroutine *OFFILE* opens input, result, and plot files.

Function *LENCHR*:

11. Function *LENCHR* calculates the length of a character string (code from R. S. Regan, USGS, written commun., 1997).

Subroutine *DATAPREP*:

12. Subroutine *DATAPREP* reads most input data from the input file, writes this data to the result file, and prepares input data for use in main program.

Subroutine *BANNER*:

13. Subroutine *BANNER* writes a program banner to the result file.

Subroutine *RDERR*:

14. If an error is found in the input data, subroutine *RDERR* writes a message to the result file indicating the input line on which the error was found and then stops program execution.

Subroutine *LINVST*:

15. Subroutine *LINVST* calculates coefficients used for the Stehfest (1970) algorithm.

Subroutine LTST1:

16. Subroutine LTST1 calculates the Laplace-transform solution for drawdown for flow to a fully penetrating well of infinitesimal diameter in a confined aquifer (Theis solution). A value of TD (dimensionless time) is passed to LTST1 from the Main program and a value of HDT (dimensionless drawdown) is returned to the Main program from LTST1.

Subroutine LTST2:

17. Subroutine LTST2 calculates the Laplace-transform solution for drawdown for flow to a finite diameter, partially penetrating well in a confined aquifer (modified solution of Dougherty and Babu, 1984). A value of TD (dimensionless time) is passed to LTST2 from the Main program and a value of HD (dimensionless drawdown) is returned to the Main program from LTST2.

Subroutine LTST3:

18. Subroutine LTST3 calculates the Laplace-transform solution for drawdown for flow to a finite diameter, partially penetrating well in a water-table aquifer with delayed drawdown response at observation wells. A value of TD (dimensionless time) is passed to LTST3 from the Main program and a value of HD (dimensionless drawdown) is returned to the Main program from LTST3.

Function BESSK0:

19. Function BESSK0 calculates the zero-order modified Bessel function of the second kind. Mathematical basis of the function is given in Press and others (1992).

Function BESSI0:

20. Function BESSI0 calculates the zero-order modified Bessel function of the first kind. Mathematical basis of the function is given in Press and others (1992).

Function BESSK1:

21. Function BESSK1 calculates the first-order modified Bessel function of the second kind. Mathematical basis of the function is given in Press and others (1992).

Function BESSI1:

22. Function BESSI1 calculates the first-order modified Bessel function of the first kind. Mathematical basis of the function is given in Press and others (1992).



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ATTACHMENT 2.—SAMPLE INPUT  
FILES FOR USE OF *WTAQ* WITH *UCODE*

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Several input files that were used by *WTAQ* and the parameter-estimation program *UCODE* for sample problem 3 are listed in this Attachment. The files are provided for reference. Detailed information on preparing input files for *UCODE* are not given here; that information is provided by Poeter and Hill (1998). Four *UCODE* input files are shown, the universal (file *prob3.uni*), prepare (file *prob3.pre*), template (file *prob3.tpl*), and extract (file *prob3.ext*) files. In addition, two short files that are used to run *WTAQ* in batch mode (which is required by *UCODE*) on a UNIX-language computer also are given. These files (*prob3.bat* and *prob3.in*) are at the end of the Attachment.

**Universal file (*prob3.uni*):**

```

3          #Phase: 1=test files; 3=perform regression
###Sensitivity and regression control information
1          #Forward differencing
0.01      #TOL
0.0       #SOSC
0         #NOPT
10        #MAX-ITER
2.0       #MAX-CHANGE
###Inversion algorithm
/export/home/pbarlow/ucode1.08/bin/mrdrive
###Application model
1          #Number of application models
prob3.bat  #Name of batch file to run WTAQ executable
###Output control information
3          #SCALE-SENSITIVITIES
1          #PRINT-INTERMEDIATE
1          #GRAPH
4          #NUMBER-RESIDUAL-SETS
#OBS-NAME  OBS-VALUE  STATISTIC  STAT-FLAG  PLOT-SYMBOL
PD1-1      0.090      1.0        0          1
PD1-2      0.204      1.0        0          1
PD1-3      0.384      1.0        0          1
PD1-4      0.568      1.0        0          1
PD1-5      0.662      1.0        0          1
PD1-6      0.680      1.0        0          1
PD1-7      0.683      1.0        0          1
PD1-8      0.690      1.0        0          1
PD1-9      0.703      1.0        0          1
PD1-10     0.729      1.0        0          1
PD1-11     0.775      1.0        0          1
PD1-12     0.847      1.0        0          1
PD1-13     0.942      1.0        0          1
PD1-14     1.052      1.0        0          1
PD2-1      0.0000     1.0        0          1
PD2-2      0.0005     1.0        0          1
PD2-3      0.0028     1.0        0          1
PD2-4      0.0071     1.0        0          1
PD2-5      0.0099     1.0        0          1
PD2-6      0.0105     1.0        0          1
PD2-7      0.0110     1.0        0          1
PD2-8      0.0119     1.0        0          1
PD2-9      0.0140     1.0        0          1

```

PD2-10	0.0186	1.0	0	1
PD2-11	0.0292	1.0	0	1
PD2-12	0.0524	1.0	0	1
PD2-13	0.0983	1.0	0	1
PD2-14	0.1722	1.0	0	1
PS1-1	0.007	1.0	0	1
PS1-2	0.019	1.0	0	1
PS1-3	0.039	1.0	0	1
PS1-4	0.060	1.0	0	1
PS1-5	0.072	1.0	0	1
PS1-6	0.077	1.0	0	1
PS1-7	0.084	1.0	0	1
PS1-8	0.098	1.0	0	1
PS1-9	0.126	1.0	0	1
PS1-10	0.177	1.0	0	1
PS1-11	0.258	1.0	0	1
PS1-12	0.365	1.0	0	1
PS1-13	0.487	1.0	0	1
PS1-14	0.612	1.0	0	1
PS2-1	0.0000	1.0	0	1
PS2-2	0.0001	1.0	0	1
PS2-3	0.0005	1.0	0	1
PS2-4	0.0012	1.0	0	1
PS2-5	0.0017	1.0	0	1
PS2-6	0.0019	1.0	0	1
PS2-7	0.0022	1.0	0	1
PS2-8	0.0028	1.0	0	1
PS2-9	0.0042	1.0	0	1
PS2-10	0.0075	1.0	0	1
PS2-11	0.0160	1.0	0	1
PS2-12	0.0375	1.0	0	1
PS2-13	0.0844	1.0	0	1
PS2-14	0.1624	1.0	0	1

END

**Prepare file (*prob3.pre*):**

#Do not use a function file

F no

#Names of template and substitution files

<prob3.tpl

>prob3.sub

#Parameters to estimate

#NAME	START	MIN	MAX	PERT	FORM	LOG	EST?
/!Kr,,,,!	1.00e-5	1.00e-6	1.00e-2	0.01	%7.2e	1	1
/!Kz,,,,!	1.00e-5	1.00e-6	1.00e-2	0.01	%7.2e	1	1
/!Ss,,,,!	1.00e-2	1.00e-7	1.00e-2	0.01	%7.2e	1	1
/!Sy,,,,!	1.00e-1	1.00e-2	1.00e+0	0.01	%7.2e	1	1

END

**Template file (prob3.tpl):**

Sample problem 3, parameter estimation using dimensional format.

```

DIMENSIONAL                                FORMAT
WATER TABLE                               AQTYPE
 10.0D0 !Kr,,,,! !Kz,,,,! !Ss,,,,! !Sy,,,,! BB HKR HKZ SS SY
 0      0                                  IDRA NALPHA
 1.0D09                                    ALPHA(I)
 1 1                                         ITS IMEAS
 0.0D0 0 0                                  TLAST NLC NOX
 1.D-10 0.0D0 0 30 8                       RERRNR RERRSUM NMAX NTMS NS
 0 1                                         IPWS IPWD--P.WELL
 2.0D-3 0.10D0 0.10D0 5.0D0 10.0D0 0.0D0 QQ RW RC ZPD ZPL SW
 14 0                                        NTSPW IRUN
 9.28D00 0.51                               TIMEPW(ID), XMEASPW(ID)
 2.00D01 0.97
 4.31D01 1.64
 9.28D01 2.31
 2.00D02 2.65
 4.31D02 2.71
 9.28D02 2.71
 2.00D03 2.72
 4.31D03 2.74
 9.28D03 2.76
 2.00D04 2.81
 4.31D04 2.89
 9.28D04 2.98
 2.00D05 3.09

4                                             NOBWC
2 0                                           IOWS IDPR--OBPIEZ PD1
3.16D0 0.0D0 0.0D0 7.5D0 0.0D0 0.0D0 R Z1 Z2 ZP RP XLL
14 1                                         NTSOB IRUN
9.28D00 0.090                               TIMEOB(ID) XMEASOB(ID)
2.00D01 0.204
4.31D01 0.384
9.28D01 0.568
2.00D02 0.662
4.31D02 0.680
9.28D02 0.683
2.00D03 0.690
4.31D03 0.703
9.28D03 0.729
2.00D04 0.775
4.31D04 0.847
9.28D04 0.942
2.00D05 1.052

2 0                                           IOWS IDPR--OBPIEZ PD2
31.6D0 0.0D0 0.0D0 7.5D0 0.0D0 0.0D0 R Z1 Z2 ZP RP XLL
14 1                                         NTSOB IRUN
9.28D00 0.0000                             TIMEOB(ID) XMEASOB(ID)
2.00D01 0.0005
4.31D01 0.0028
9.28D01 0.0071

```



**Extract file (*prob3.ext*):**

```
# Open file wtaq.plt and read simulated drawdowns
<wtaq.plt
# Read simulated value for observation PD1-1
o PD1-1
+6
c33_42
# Read next simulated value (PD1-2)
o PD1-2
+1
c33_42
# And so forth....
o PD1-3
+1
c33_42
o PD1-4
+1
c33_42
o PD1-5
+1
c33_42
o PD1-6
+1
c33_42
o PD1-7
+1
c33_42
o PD1-8
+1
c33_42
o PD1-9
+1
c33_42
o PD1-10
+1
c33_42
o PD1-11
+1
c33_42
o PD1-12
+1
c33_42
o PD1-13
+1
c33_42
o PD1-14
+1
c33_42
# Now, read simulated values for PD2
o PD2-1
+6
c33_42
```

```
o PD2-2
+1
c33_42
o PD2-3
+1
c33_42
o PD2-4
+1
c33_42
o PD2-5
+1
c33_42
o PD2-6
+1
c33_42
o PD2-7
+1
c33_42
o PD2-8
+1
c33_42
o PD2-9
+1
c33_42
o PD2-10
+1
c33_42
o PD2-11
+1
c33_42
o PD2-12
+1
c33_42
o PD2-13
+1
c33_42
o PD2-14
+1
c33_42
# Now, read simulated values for PS1
o PS1-1
+6
c33_42
o PS1-2
+1
c33_42
o PS1-3
+1
c33_42
o PS1-4
+1
c33_42
o PS1-5
+1
```



c33\_42  
o PS1-6  
+1  
c33\_42  
o PS1-7  
+1  
c33\_42  
o PS1-8  
+1  
c33\_42  
o PS1-9  
+1  
c33\_42  
o PS1-10  
+1  
c33\_42  
o PS1-11  
+1  
c33\_42  
o PS1-12  
+1  
c33\_42  
o PS1-13  
+1  
c33\_42  
o PS1-14  
+1  
c33\_42  
# Now, read simulated values for PS2  
o PS2-1  
+6  
c33\_42  
o PS2-2  
+1  
c33\_42  
o PS2-3  
+1  
c33\_42  
o PS2-4  
+1  
c33\_42  
o PS2-5  
+1  
c33\_42  
o PS2-6  
+1  
c33\_42  
o PS2-7  
+1  
c33\_42  
o PS2-8  
+1  
c33\_42  
o PS2-9

```
+1
c33_42
o PS2-10
+1
c33_42
o PS2-11
+1
c33_42
o PS2-12
+1
c33_42
o PS2-13
+1
c33_42
o PS2-14
+1
c33_42
END
```

**Batch file to run WTAQ (*prob3.bat*):**

```
rm wtaq.scratch
rm wtaq.out
rm wtaq.plt
wtaq.exe <prob3.in >wtaq.scratch
```

**File containing names of input, result, and plot files used or generated by WTAQ (*prob3.in*):**

```
prob3.sub
wtaq.out
wtaq.plt
```



