BLS WORKING PAPERS

U.S. DEPARTMENT OF LABOR
Bureau of Labor Statistics

OFFICE OF PRICES AND LIVING CONDITIONS

A Comparison of the Substitution Effects for Input and Output Price Indexes

Keith Waehrer, U.S. Bureau of Labor Statistics

Working Paper 327
April 2000

The views expressed are those of the author and do not reflect the policies of the U.S. Bureau of Labor Statistics or the views of other staff members. The paper benefited from the helpful comments of Dennis Fixler, Jack Galvin, John Greenlees, and Ken Stewart.

A Comparison of the Substitution Effects for Input and Output Price Indexes

KEITH WAEHRER*
Division of Price and Index Number Research
U.S. Bureau of Labor Statistics

Current Version: February, 1999

ABSTRACT

The substitution effect is different for output and input indexes like the consumer price index and the producer price index respectively. The conceptual index for both output and input indexes are defined and compared. Then the conceptual indexes are compared with some common index number formulas. Some results relating to which formulas are closer to the conceptual index are reviewed.

* The paper benefited from the helpful comments of Dennis Fixler, Jack Galvin, John Greenlees, and Ken Stewart.

A Comparison of the Substitution Effects for Input and Output Price Indexes

The recent report to the Senate Finance Committee by the Advisory Committee on the Consumer Price Index highlighted a number of sources of bias in the CPI; one of these being substitution bias. While the Boskin Report only discusses the CPI, some have suggested that its recommendations may also apply to the producer price index. However, conceptual differences between CPI and PPI imply that substitution bias has different effects on the two indexes and, thus, will likely require different remedies.

In this paper I describe the conceptual target indexes for input and output price indexes.[1] Conceptually the CPI is an input index and the PPI is an output index. Once defined, it is straight forward to show that the Laspeyres is a lower bound on the conceptual output price index and an upper bound on the conceptual input price index. The geometric mean index has been proposed for use in the CPI, but a similar use in the PPI may be inappropriate. The geometric mean index is always lower than the Laspeyres. Hence, the Laspeyres must lie between the geometric mean and the conceptual output price index. Therefore, the Laspeyres is always closer to the conceptual target output price index than the geometric mean. I also give conditions that imply that the Laspeyres is a better estimate of the conceptual output price index than the Fisher Ideal index.

Output Price Indexes:

The important indexes produced as part of the PPI are net output indexes. Net output indexes are calculated for different industries. These indexes are used to deflate the value of goods and services produced by these industries in order to determine changes in real output. For instance, if the value of the output of the mining industry rises, the PPI net output index for mining is used to determine how much of that rise is due to an increase in the price of the goods produced by the mining industry and how much is due to an increase in production.

Conceptually output indexes depend on how one defines an increase or decrease in aggregate output. If each industry produced only one good, this problem has a straightforward solution. Suppose that the mining industry's only product is copper. Then clearly the mining industry's output increases if and only if the physical quantity of copper produced increases. However, an individual industry such as mining produces a

[1] The concept of an output price index has been well developed. Much of the analysis of this paper has been drawn from this literature. For example, see Robert Archibald (1975) "On the Theory of Industrial Price Measurement: Output Price Indexes," BLS Working Paper #44, W.E. Diewert (1983) "The Theory of the Output Price Index and the Measurement of Real Output Change," in *Price Level Measurement*, W.E. Diewert and C. Montmarquette (eds.), Ottawa: Statistics Canada, pp. 1049-1113, and Franklin M. Fisher and Karl Shell (1981) "Output Price Indices," BLS Working Paper #120.

whole array of products. Therefore, it is more difficult to define increases and decreases in aggregate output. While two output combinations may be different, they may be considered the same level of output in an aggregate sense. When an index is used as a deflator for aggregate output measures, whether intentionally or not, that index makes a particular assumption about which output combinations are taken as equivalent in terms of aggregate output.

Figure 1 diagramitically describes the difficulty when there are two goods produced. Suppose that the mining industry produces only copper and iron. Each point on the diagram represents a quantity of output for both iron and copper. It is clear that points *b* and *c* represent a higher level of output than point *a* and point *d* depicts a lower level of aggregate output than point *a*. More generally an output combination to the northeast of point *a* would depict more aggregate output and an output combination to the southwest of point *a* would represent less aggregate output. What is less clear is how point *e* should be compared to point *a*: more, less, or equal in aggregate output.

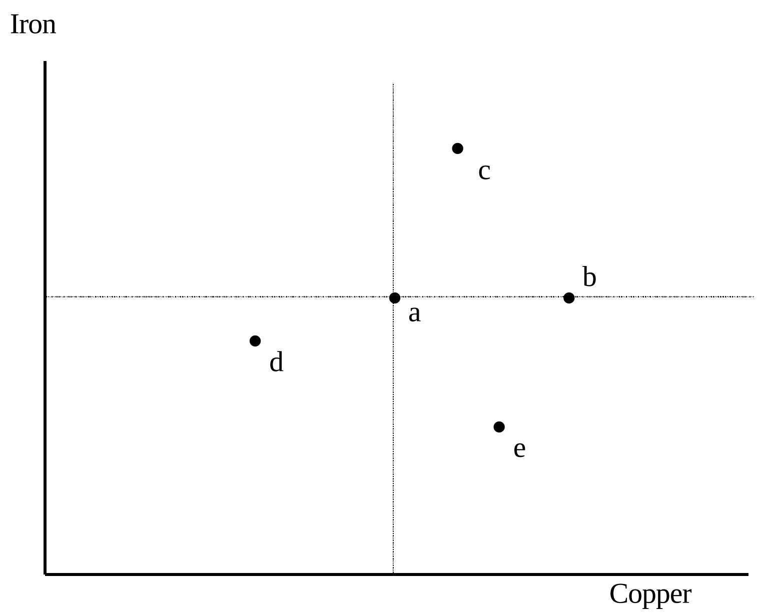

Figure 1

Economists have resolved this issue by making use of the concept of a production possibility frontier. A *production possibility set* is the set of all output combinations that can be produced from a particular set of inputs and a particular technology. A *production possibility frontier* (PPF) is the upper boundary of a production possibility set. Consider Figure 2. The area enclosed by the curved line and the axes is an example of a production possibility set. Since points *a*, *d*, and *e* are within the set, any of those combinations of copper and iron can be produced with the same set of inputs and technology. (A production possibilities set always includes its boundary.) Points *a* and *e* are on the boundary of the production possibility set and thus are on the production possibility frontier.

Once the production possibility frontier has been defined, it is easy to describe which output combinations constitute more, less, or the same level of aggregate output for the industry. Points on the same PPF as point *a* are considered the same level of aggregate output. Thus, while two output combinations may be different, they are said to represent the same level of aggregate output if they can be produced from the same initial set of inputs and technology. Output combinations above point *a*'s PPF are considered higher levels of aggregate output. And points below *a*'s PPF correspond to lower levels of aggregate output for this industry. Therefore, for the purpose of calculating an output price index, holding aggregate output constant can be thought of as staying on the same PPF.

The conceptual index underlying the PPI is the Fixed-input Output Price Index (FIOPI). Let p_{ib} and p_{ic} denote the price of good *i* in the base and current periods. Define q_{ib} and q_{ic} as the quantity produced of good *i* in the base and current periods. Then the FIOPI is defined as

$$I_O(p_b, p_c, PPF) = \frac{\max_q \left\{ \sum_i q_i \, p_{ic} \mid (q_1, \ldots, q_n) \in PPF \right\}}{\max_q \left\{ \sum_i q_i \, p_{ib} \mid (q_1, \ldots, q_n) \in PPF \right\}}$$

This notation requires some explanation. The FIOPI is the ratio of revenues: revenue in the current period divided by revenue in the base while staying on the same production possibility frontier. Recall that we defined holding aggregate output constant as staying on the same PPF or in other words holding inputs and technology constant. The numerator in the expression above is the maximum revenue that can be generated from output combinations on the production possibility frontier PPF when facing current period prices. The denominator of the expression is the maximum revenue that can be achieved from output combinations on the same production possibility frontier PPF when faced with base period prices.

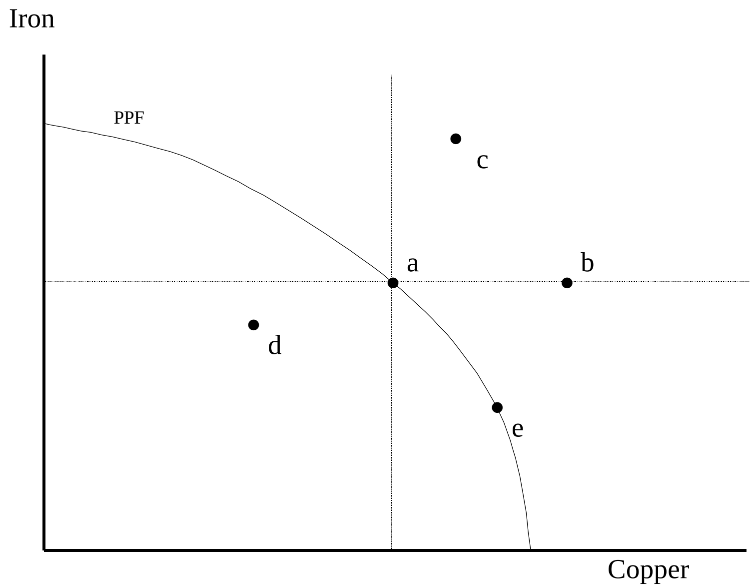

Figure 2

There are actually many possible production possibility frontiers that could be used in the calculation of output indexes. As a result of the inputs and technology in use at any given time, an industry faces a particular production possibility frontier. Over time the production possibility frontier faced by an industry changes due to changes in the inputs and technology used. In general, the choice of production possibility frontiers to use in the FIOPI formula has an impact on the resulting index. Therefore, it seems difficult to argue that the choice of production possibility frontier is completely arbitrary. Recall that the production possibility frontier is used in the FIOPI essentially defines the definition of aggregate output. Arguing that the choice of production possibility frontiers to use in a FIOPI is arbitrary implies that the definition of aggregate output and analogously the price index are to some extent arbitrary.

Two natural choices of production possibility frontier for use in the FIOPI are the production possibility frontiers in the base and current periods. The base period production possibility frontier PPF_b is the boundary of the set of output combinations that could be produced by the same technology and inputs used to produce the base period outputs $(q_{1b},..., q_{nb})$. The current period production possibility frontier PPF_c is the boundary of the set of output combinations that could be produced by the same technology and inputs used to produce the current period outputs $(q_{1c},...,q_{nc})$. There is no reason to believe that PPF_b and PPF_c are the same or equivalent for the purposes of calculating a FIOPI. I say that PPF_b and PPF_c are equivalent for the purposes of calculating a FIOPI if $I_O(p, p', PPF_b) = I_O(p, p', PPF_c)$ no matter what prices p and p' are plugged into the formula.[2]

[2]Production possibility frontiers with this relationship has also been described as parallel. This property is also related to homothetic production possibility maps. A more detailed description of this property and

There are a number of reasons for the differences between PPF_b and PPF_c. Over time advancing technology allows producers to produce the same level of output with fewer inputs. Industries may also adjust some of the inputs that are used. Between the base period and current period, the prices of the inputs used by the industry are likely to have changed. Such a change will most likely cause a change in the combination of inputs used by the industry and hence a change in the *PPF* associated with the two periods. Our use of a FIOPI does not require us to assume that the inputs and technology used in the actual economy remain fixed. However, the calculation of such an index requires us to calculate revenues as if the PPF stays the same while output prices change. (This is analogous to the calculation of a cost of living index where it is necessary to calculate a consumers cost as if they were kept on the same indifference curve.)

Having settled on either PPF_b and PPF_c as natural choices for inclusion in the FIOPI, let me define the following notation in order to simplify the expressions in this paper. First, I define what is referred to as the dot product.

$$q \cdot p = \sum_{i=1}^{n} q_i p_i$$

Define $q_{xy}^{\max}$ as the output combination that maximizes revenue when faced with period *x* prices while remaining on the PPF for period *y*. That is,

$$q_{xy}^{\max} \cdot p_x = \max_q \left\{ \sum_i q_i p_{ix} \mid (q_1, \ldots, q_n) \in PPF_y \right\}.$$

Therefore, the formula for the FIOPI can be written more compactly as

$$I_O(p_b, p_c, PPF_b) = \frac{q_{cb}^{\max} \cdot p_c}{q_{bb}^{\max} \cdot p_b}$$

$$I_O(p_b, p_c, PPF_c) = \frac{q_{cc}^{\max} \cdot p_c}{q_{bc}^{\max} \cdot p_b}$$

Under the assumption that utilized an industry's output combination is selected to maximize revenue conditional on the inputs and technology, we have

$$q_b \cdot p_b = q_{bb}^{\max} \cdot p_b .$$

Therefore, the expression for FIOPI using the base period PPF can be written as

---

its relation to homotheticity can be found in Franklin M. Fisher and Karl Shell, *Economic Analysis of Production Price Indexes* (Cambridge, Cambridge University Press, 1998).

$$I_O(p_b, p_c, PPF_b) = \frac{q_{cb}^{\max} \cdot p_c}{q_b \cdot p_b}$$

While calculating an exact FIOPI for an industry might be ideal, it is impractical since PPF_b is not generally observable or known by statistical agencies, and thus, the actual calculation of $q_{cb}^{\max} \cdot p_c$ is impossible in the absence of strong assumptions. Being unable to construct an index over fixed inputs and technology, the BLS instead constructs an index based on fixed outputs. That is, the Bureau aims to calculate a Laspeyres index as an estimate of the conceptual target. The Laspeyres index is defined as

$$I_L(p_b, p_c) = \frac{q_b \cdot p_c}{q_b \cdot p_b}$$

However, notice that $q_b \cdot p_c \leq q_{cb}^{\max} \cdot p_c$ since while q_b is on PPF_b, we would not necessarily expect q_b to continue to be the revenue maximizing output combination when faced with current period prices. We would expect the industry to adjust its output combination to capture more revenue.

To fix ideas geometrically, consider Figure 3. Suppose that point *a* corresponds to the base period output combination. Assuming that this output combination maximizes the industry's revenue conditional on its inputs, there is a line R_b that runs through point *a* and with slope equal to minus the price of copper divided by the price of iron in the base period. All of the output combinations on R_b generate the same revenue as point *a* under base period prices. All output combinations above R_b generate more revenue than R_b and all output combinations below R_b generate less revenue than R_b. It is important to see that in terms of the notation defined above $R_b = q_b \cdot p_b$.

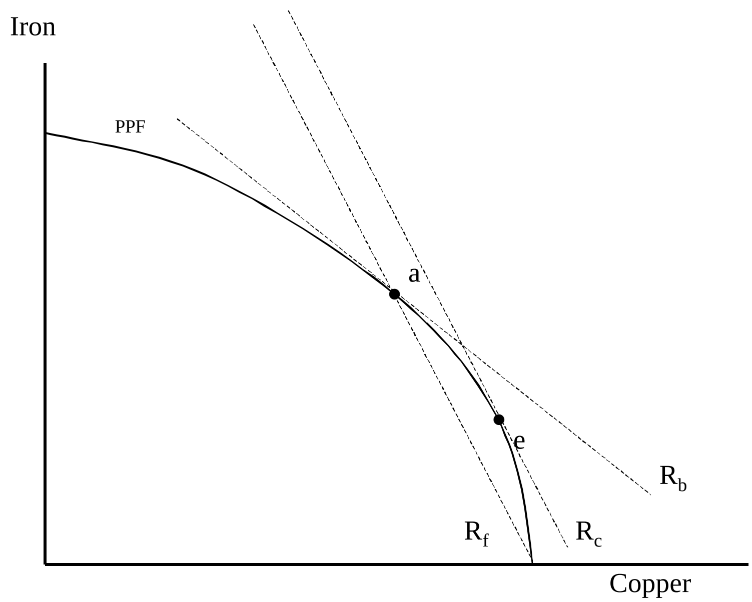

Figure 3

Now consider a price change that corresponds to a relative increase in the price of copper. If the industry does not change its output combination, then its new revenue will be $R_f = q_b \cdot p_c$. However, notice that the industry could earn higher revenue while still remaining on the base period production possibility frontier. Specifically, if the industry produced output combination *e*, it could earn higher revenue. That is, $R_c = q_{cb}^{\max} \cdot p_c \geq R_f$. Notice that when the relative price of copper rises, in order to maximize revenue the industry should change its output combination to produce more copper and less iron. That is, substitute in the direction of the good whose relative price has risen.

It is now possible to define substitution bias for output indexes. The discussion above can be summarized by the following chain of inequalities.

$$I_L(p_b, p_c) = \frac{R_f}{R_b} \leq \frac{R_c}{R_b} = I_O(p_b, p_c, PPF_b) \quad \text{(since } R_f \leq R_c\text{)}$$

Therefore, assuming that industries maximize revenue conditional on the inputs it uses, the Laspeyres index is a lower bound on the FIOPI. The difference between I_L and I_O being the so called substitution bias. Hence, one would expect that a price index based on the Laspeyres concept to be on average below the conceptual fixed-input output price index.

It is also possible to relate the relative size of a Paasche to a FIOPI in a way similar to tat just presented. The Paasche index is defined as

$$I_P(p_b, p_c) = \frac{q_c \cdot p_c}{q_c \cdot p_b}$$

Assuming that firms seek to maximize revenue conditional on their input utilization notice that $q_c \cdot p_b \leq q_{bc}^{\max} \cdot p_b$ which follows from the fact that the current period output combination is one of the feasible choices in PPF_c and $q_{cc}^{\max} \cdot p_c = q_c \cdot p_c$ which follows as long as an industry maximizes revenue subject to its inputs. Therefore,

$$I_P(p_b, p_c) = \frac{q_c \cdot p_c}{q_c \cdot p_b} \geq \frac{q_c \cdot p_c}{q_{bc}^{\max} \cdot p_b} = I_O(p_b, p_c, PPF_c)\,.$$

To summarize what has been demonstrated thus far, we have

$$I_P(p_b, p_c) \geq I_O(p_b, p_c, PPF_c)$$
$$I_O(p_b, p_c, PPF_b) \geq I_L(p_b, p_c)$$

That is, the Paasche index is greater than a FIOPI based on the *current* period PPF, and the Laspeyres index is less than a FIOPI based on the *base* period PPF. What is not known is the relationship between $I_O(p_b, p_c, PPF_c)$ and $I_O(p_b, p_c, PPF_b)$. If PPF_b and PPF_c are "equivalent", then we have $I_O(p_b, p_c, PPF_c) = I_O(p_b, p_c, PPF_b)$ and thus, $I_P \geq I_L$. However, as I have argued above there is no reason to believe that PPF_b and PPF_c are "equivalent". In fact, there is nothing in the theory that suggests that the Paasche should necessarily be greater than the Laspeyres.

Consider the following simple two product example. Suppose that each good is produced independently with different inputs and the goods' demands are independent. Let the market for good 1 remain unchanged between the base and current periods, but the price of an input used to produce good 2 rises in these periods. This change will cause the marginal cost of producing good 2 to rise and in a perfectly competitive equilibrium the output price of good 2 must also rise to equal the new marginal cost, decreasing the quantity demanded. Therefore, in this simple example we have $q_{1b} = q_{1c}$, $p_{1b} = p_{1c}$, $q_{2b} > q_{2c}$, and $p_{2b} < p_{2c}$. It is not difficult to show that in such a simple case the Laspeyres will exceed the Paasche. In fact, in a companion paper, Galvin and Stewart find empirical evidence using PPI data that the Laspeyres index is greater in general than the Paasche index.[3] A table from their analysis is reproduced below.

[3]J. Galvin and K. Stewart (1998) "Alternative Measures of Price Change for the U.S. Producer Price Index," mimeo, Bureau of Labor Statistics.

Table 1. Laspeyres and Paasche PPIs for 2-digit product groups, total percentage change, 1987-92

Producer Price Index 2-digit Product Group Category	Laspeyres	Paasche	Difference, Laspeyres minus Paasche
Farm products (01)	13.55	9.40	4.15
Processed foods and feeds (02)	15.41	14.57	0.84
Textile products and apparel (03)	11.98	11.47	0.51
Hides, skins, leather, and related products (04)	19.00	18.23	0.77
Fuels and related products and power (05)	7.40	7.36	0.04
Chemicals and allied products (06)	15.20	16.27	-1.07
Rubber and plastic products (07)	10.28	10.20	0.08
Lumber and wood products (08)	29.85	26.20	3.65
Pulp, paper, and allied products (09)	17.23	15.08	2.15
Metals and metal products (10)	12.78	10.17	2.61
Machinery and equipment (11)	12.26	9.86	2.40
Furniture and household durables (12)	10.77	9.91	0.86
Nonmetallic mineral products (13)	6.75	6.90	-0.15
Transportation equipment (14)	15.13	14.38	0.75
Miscellaneous products (15)	26.13	24.08	2.05

Reproduced from J. Galvin and K. Stewart (1998) "Alternative Measures of Price Change for the U.S. Producer Price Index," mimeo, Bureau of Labor Statistics.

While the calculation of a FIOPI requires the hypothetical output combination q_{cb}^{max} , its use does not necessarily imply that we expect supply conditions such as the quantity of inputs or technology to actually remain unchanged from period to period.

Input Price Indexes:

For input price indexes like the CPI, the substitution bias works in the opposite direction. Consumers substitute away from goods whose relative price have risen, and a Laspeyres index is an upper bound of the "true" price index concept. The concept underlying input indexes like the CPI can be referred to as Fixed-output Input Price Index (FOIPI). Fixed-output Input Price Indexes are defined as

$$I_I(p_b, p_c, BF) = \frac{\min_q \left\{\sum_i q_i p_{ic} \mid (q_1,\ldots,q_n) \in IO\right\}}{\min_q \left\{\sum_i q_i p_{ib} \mid (q_1,\ldots,q_n) \in IO\right\}}$$

where IO is an iso-output curve denoting the input combinations that produce a given output combination while utilizing a particular level of technology. The iso-output curve would be exactly analogous to an indifference curve in the theory of cost of living indexes. Therefore, the following analysis also applies to cost of living indexes.

The iso-output curve is represented graphically in Figure 4. In the diagram input combinations *a* and *e* can produce the same output combination. Combinations *b* and *c* can produce more output than combination *a*, and combination *d* cannot produce as much as input combination *a*.

It is now possible to interpret the expression for Fixed-Output Input Price Index. The numerator is the least cost way of reaching a certain output combination given the prices in the current period, and the denominator is the least cost way of producing the same output combination given the prices in the base period.

Let $q_{xy}^{\min}$ denote the input combination that minimizes the cost of producing the same output as in period *y* while facing the prices from period *x*. That is,

$$q_{xy}^{\min} \cdot p_x = \min_q \left\{\sum_i q_i p_{ix} \mid (q_1,\ldots, q_n) \in IO_y\right\}$$

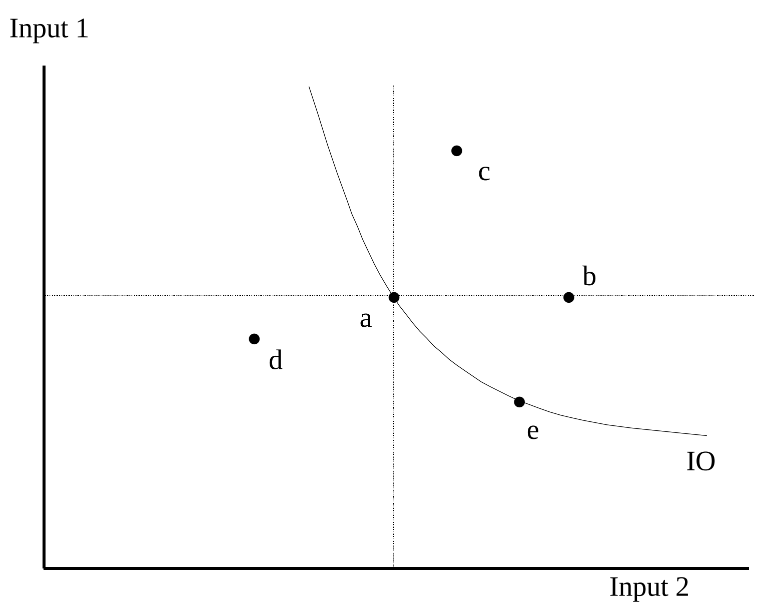

Figure 4

Let IO_b and IO_c denote the iso-output curves associated with the output combinations in the base and current periods. Assuming that producers attempt to achieve a particular output combination at minimum cost, we have

$$q_b \cdot p_b = q_{bb}^{\min} \cdot p_b$$
$$q_b \cdot p_c \geq q_{cb}^{\min} \cdot p_c$$
$$q_c \cdot p_b \geq q_{bc}^{\min} \cdot p_b$$
$$q_c \cdot p_c = q_{cc}^{\min} \cdot p_c$$

The explanations for these equalities and inequalities is similar to the arguments presented in the previous section on output price indexes. If behavior is optimal, then the input combination selected in the base period will be the least cost way of producing the base output combination, and similarly, the input combination selected in the current period will be the least cost way of producing the current period output combination. The inequalities follow from the facts that the base and current period input combinations can produce the base and current period output combinations respectively, but they do not necessarily produce those output combinations at least cost when faced with different prices.

A graphic representation of these facts is straightforward to derive using consumer theory. Consider Figure 5. Let the straight line labeled C_b represents the set of input combinations that cost the consumer the same amount of money as the base period input combination point *a* when facing base period prices. That is, $C_b = q_b \cdot p_b$.

Suppose that between the base period and the current period the relative price of input 1 rises. Let C_c' denote the set of input combinations that cost the consumer the same amount of money as the base period input combination under the current period prices. Notice that it is possible to find a different input combination that produces the same output combinations at a lower cost. That is, input combination *e* represents the input combination on IO_b that costs the least. Therefore, we have

$$C_c' = q_b \cdot p_c \geq q_{cb}^{\min} \cdot p_c = C_c$$

Notice that when the price of input 1 rises the consumer wishes to substitute some of input 1 for input 2, effectively substituting away from the more expensive good.

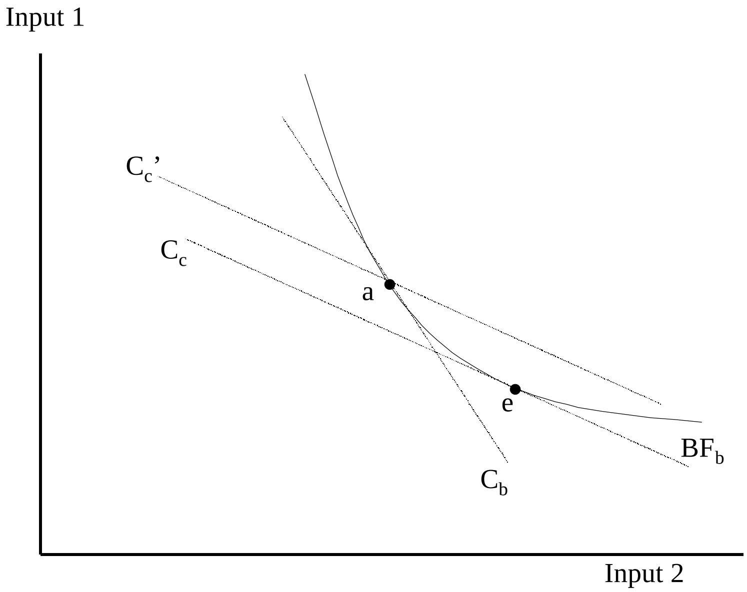

Figure 5

By making use of the expressions above we can establish the following relationships between the Laspeyres, Paasche, and FOIPI.

$$I_L(p_b, p_c, q_b) = \frac{q_b \cdot p_c}{q_b \cdot p_b} \geq \frac{q_{cb}^{\min} \cdot p_c}{q_b \cdot p_b} = I_I(p_b, p_c, IO_b)$$

$$I_I(p_b, p_c, IO_c) = \frac{q_c \cdot p_c}{q_{bc}^{\min} \cdot p_b} \geq \frac{q_c \cdot p_c}{q_c \cdot p_b} = I_L(p_b, p_c, q_c)$$

The Laspeyres is necessarily larger than the FOIPI when the base period iso-output curve is used, and similarly the Paasche is necessarily smaller than the FOIPI when the current period iso-output curve is used. However, as in the theory of output indexes, there is nothing in the arguments made thus far that allows us to draw any conclusion regarding the relative sizes of $I_I(p_b, p_c, IO_b)$ and $I_I(p_b, p_c, IO_c)$.

Taking $I_I(p_b, p_c, IO_b)$ as the conceptual target, the Laspeyres index must over estimate the target input index because it does not take into account that the consumer/producer will reoptimize the input combination in response to the change in prices. Since in the case of input price indexes, the consumer wishes to minimize its cost of achieving a certain level of utility the Laspeyres which does not account for this reaction over estimates the effect of the price increase.

Using Fisher and Geometric Means Indexes for Input and Output Price Indexes:

The actual calculation of price indexes is usually limited to formulas that only require observed price and quantity data. In general this limitation makes the calculation of FIOPI and FOIPI impossible since each would require hypothetical data. That is, the numerator of the FIOPI using a base period production possibility frontier is the maximum revenue that can be achieved under current period prices while remaining on the base period PPF. Such a revenue calculation is purely hypothetical since the actual revenue observed in the current period is likely to be associated with a PPF other than the base period PPF. A similar problem arises in the case of input price indexes.

Various strategies have been proposed to circumvent these difficulties. I consider two here: the Fisher index that requires prices and quantities from both periods and the geometric mean index that requires prices from both periods and quantities only from the base period. They are defined as follows.

$$I_F(p_b, p_c) = \left(\frac{\sum_i q_{ib} p_{ic}}{\sum_i q_{ib} p_{ib}} \frac{\sum_i q_{ic} p_{ic}}{\sum_i q_{ic} p_{ib}} \right)^{1/2}$$

$$I_G(p_b, p_c) = \prod_i (p_{ic} / p_{ib})^{q_{ib} p_{ib} / \sum_j q_{jb} p_{jb}}$$

Notice that the Fisher index, I_F, is equal to the geometric average of the Laspeyres and the Paasche indexes. Therefore, mathematically the Fisher must lie between the Laspeyres and Paasche indexes. It is straight forward prove that the geometric mean index must never be larger than the Laspeyres index. Note that,

$$\begin{aligned} \ln I_G(p_b, p_c) &= \sum_i \frac{q_{ib} p_{ib}}{\sum_j q_{jb} p_{jb}} \ln\left(\frac{p_{ic}}{p_{ib}} \right) \\ &\leq \ln \sum_i \frac{q_{ib} p_{ib}}{\sum_j q_{jb} p_{jb}} \left(\frac{p_{ic}}{p_{ib}} \right) \\ &= \ln I_L(p_b, p_c, q_c) \end{aligned}$$

The inequality in the expression above follows from Jensen's inequality.[4] Thus, $I_G(p_b, p_c) \leq I_L(p_b, p_c)$ since for any $x, y > 0$, $\ln x \leq \ln y$ implies $x \leq y$.

The Fisher index has a number of properties that make it attractive as an index number. The property most relevant for the discussion here is its status as a superlative index. The Fisher index is equal to the conceptual index when the production function takes the form of a generalized quadratic function. It has also been shown that under more general conditions the Fisher index closely tracks the conceptual index bases on a PPF that is an "average" of PPF_b and PPF_c .

[4] For a discription of Jensen's inequality see for example William Noveshek, Mathematics for Economists (San Diego, Academic Press, 1993).

Before the Fisher and geometric mean indexes can be evaluated as possible input or output price indexes it is important to settle on the conceptual target for each. I argue in the sections above that the conceptual output and input price indexes are the FIOPI and the FOIPI respectively. However, it remains to be determined what production possibility frontier and iso-output curve to use as a basis in each of the formula. As I argue above, natural choices appear to be the base or current period production possibility frontiers, PPF_b or PPF_c and the base or current period iso-output curves, IO_b or IO_c. Its seems natural to fix the PPF and IO in the base period. Therefore, for the purpose of discussion I settle on $I_O(p_b, p_c, PPF_b)$ and $I_I(p_b, p_c, IO_b)$ as the conceptual targets for output and input price indexes.

First consider the case of the geometric mean. If it can be shown that the Laspeyres always falls between the geometric mean and the target index, then it must be the case that the Laspeyres index is closer to the target than the geometric mean index. That is, the difference between the geometric mean and the target index must be larger than the difference between the Laspeyres and the target. Notice this relationship always holds for output indexes. We have shown

$$I_G(p_b, p_c) \le I_L(p_b, p_c) \le I_O(p_b, p_c, PPF_b)$$

Hence, for the case of output indexes, the geometric mean must be farther from the target index than the Laspeyres. Therefore, using closeness to the target index as the sole criteria in judging an index formula, the Laspeyres is a better formula than the geometric mean for output indexes. Intuitively, the geometric mean implies that firms would decrease their production of products whose relative price increases when its inputs stay the same. This is clearly not consistent with the hypothesis that firm maximize revenue conditional in their inputs.

The same is not true for input indexes. Recall that we have shown that

$$I_I(p_b, p_c, IO_b) \le I_L(p_b, p_c);$$

the relative quantities of the geometric mean and the target input price index cannot be definitively established without further assumptions. Therefore, it cannot be the case that the Laspeyres falls between the target input index and the geometric mean. However, with the assumptions made thus far, the geometric mean is not necessarily closer to the target index than the Laspeyres.

Now let us consider the case of the Fisher index. As stated above, the Fisher index will always fall between the Paasche and Laspeyres indexes. Therefore, just as we did in the case of the geometric mean, we can establish definitively that the Laspeyres is closer to the target index than the Fisher if the Paasche is less than the Laspeyres for output indexes and if the Paasche is greater than the Laspeyres for input indexes. That is,

$$I_P(p_b, p_c) \le I_L(p_b, p_c) \Rightarrow I_F(p_b, p_c) \le I_L(p_b, p_c) \le I_O(p_b, p_c, PPF_b)$$
and
$$I_L(p_b, p_c) \le I_P(p_b, p_c) \Rightarrow I_I(p_b, p_c, IO_b) \le I_L(p_b, p_c) \le I_F(p_b, p_c).$$

It is possible to make additional assumptions that make the Fisher index closely approximate the target index. One such assumption is that the production possibility frontiers or the iso-output curves are parallel (also know as the homothetic case). When production possibility frontiers are parallel, it must be the case that

$$I_L(p_b, p_c) \le I_O(p_b, p_c, PPF_b) = I_O(p_b, p_c, PPF_c) \le I_P(p_b, p_c).$$

Therefore, anytime $I_L(p_b, p_c) > I_P(p_b, p_c)$, the production possibility frontiers cannot be parallel.

In fact, Galvin and Stewart, in their paper (see Table 1), provide empirical evidence that for PPI data the Laspeyres is often larger than the Paasche index. In such cases the Laspeyres index will actually be closer to a FIOPI based on the base period PPF than a Fisher index. However, basing the FIOPI on an "average" of base period and current period PPFs will imply that Fisher index is a better estimate than the Laspeyres.

Summary:

The theory of input and output price indexes indicate that substitution effects in the two are fundamentally different. For input price indexes like the CPI, consumers would like to consume less of goods whose relative price increases. For output price indexes like the PPI, the theory indicates the firms would want to produce a larger quantity of those goods whose relative price increases. Hence, the two substitution effects are in fact in opposite direction.

The conceptual input and output price indexes require hypothetical calculations that are often impractical to perform. Therefore, other index formulas are used in their place. If an index formula is to be judged by its proximity to the conceptual target, then regardless of the assumptions made the geometric mean index is never a better candidate output index than the Laspeyres index. On the other hand, under certain assumptions the geometric mean index is exactly equal to the conceptual input price index. Therefore, while the geometric mean index may be appropriate for the CPI, it is not likely to be appropriate for implementation in the PPI.

The Fisher index will definitely be farther from the conceptual output price index than the Laspeyres index when the Laspeyres index is larger than the Paasche. While for input price indexes, the Fisher is a worse approximation of the conceptual input price index than the Laspeyres when the Laspeyres is less than the Paasche.

Price data reflects the interaction of supply and demand in the real world. If price fluctuations are predominantly caused by the supply curve moving up and down the demand schedule, then we should expect to see price and quantity demanded moving in opposite directions and, thus, the Laspeyres greater than the Paasche. This type of effect would result if, for instance, the price or supply of productive inputs change over time. On the other hand, if price fluctuations are predominantly caused by the demand curve moving up and down the supply curve, then we should expect to see price and quantity moving in the same direction and, thus, the Laspeyres less than the Paasche. Changes in consumer preferences or income can result in demand curve fluctuations.