MATHEMATICAL SCIENCES

Today's discoveries in science, engineering, and technology are intertwined with advances across the mathematical sciences. New mathematical tools disentangle the complex processes that drive the climate system; mathematics illuminates the interaction of magnetic fields and fluid flows in the hot plasmas within stars; and mathematical modeling plays a key role in research on microscale, nanoscale, and optical devices. Innovative optimization methods form the core of computational algorithms that provide decision-making tools for Internet-based business information systems.

The fundamental mathematical sciences – embracing mathematics and statistics – are essential not only for the progress of research across disciplines, they are also critical to training a mathematically literate workforce for the future. Technology-based industries that help fuel the growth of the U.S. economy and increasing dependence on computer control systems, electronic data management, and business forecasting models, demand a workforce with effective mathematical and statistical skills, well-versed in science and engineering.

It is vital for mathematicians and statisticians to collaborate with engineers and scientists to extend the frontiers of discovery where science and mathematics meet, both in research and in educating a new generation for careers in academia, industry, and government. For the United States to remain competitive among other Nations with strong traditions in mathematical sciences education, we must attract more young Americans to careers in the mathematical sciences. These efforts are essential for the continued health of the Nation's science and engineering enterprise.

The role of mathematics has expanded in science and society, but the resources devoted to three key areas – fundamental mathematical and statistical research, interdisciplinary collaboration between the mathematical sciences and other disciplines, and mathematics education – have not kept pace with the needs, thus limiting the Nation's scientific, technical, and commercial enterprises. To strengthen the mathematical foundations of science and society, NSF has supported the Mathematical Sciences Priority Area since FY 2002. This investment focuses on the mathematical sciences, encompassing interdisciplinary efforts in all areas of science, engineering, and education supported by the Foundation.

Mathematical Sciences Funding
(Dollars in Millions)

	FY 2005 Actual	FY 2006 Current Plan	FY 2007 Request	Change over FY 2006 Amount	Percent
Biological Sciences	2.21	2.21	1.11	-1.10	-49.8%
Computer and Information Science and Engineering	2.29	2.29	1.15	-1.14	-49.8%
Engineering	2.91	2.88	1.46	-1.42	-49.3%
Geosciences	7.07	7.00	3.53	-3.47	-49.6%
Mathematical and Physical Sciences	70.21	69.69	69.26	-0.43	-0.6%
Social, Behavioral and Economic Sciences	1.50	1.50	0.75	-0.75	-50.0%
Office of International Science and Engineering	0.32	-	-	-	N/A
Office of Polar Programs	0.20	0.20	0.10	-0.10	-50.0%
Subtotal, Research and Related Activities	86.71	85.77	77.36	-8.41	-9.8%
Education and Human Resources	2.85	2.20	1.09	-1.11	-50.5%
Total, Mathematical Sciences	$89.56	$87.97	$78.45	-$9.52	-10.8%

Totals may not add due to rounding.

Long-term Goals: The goal of this priority area is to advance frontiers in three interlinked areas: (1) fundamental mathematical and statistical sciences; (2) interdisciplinary research involving the mathematical sciences with science and engineering and focused on selected themes; and (3) critical investments in mathematical sciences education. The investment plan (FY 2002 – FY 2007) will allow efforts in research and education to take root and begin a long-term transformation in the way mathematics, science, and education interact. The long-term goals of the investments in the priority area that were articulated during its initial stages and continue as important goals are to:

- Foster significant advances in fundamental mathematics and statistics together with important benefits for the mathematical and other sciences and engineering;
- Foster interdisciplinary research partnerships that integrate the mathematical sciences with other science and engineering disciplines and recognize mathematicians and statisticians as full partners;
- Integrate the most appropriate, state-of-the-art, statistical principles and mathematical tools and concepts into all NSF sponsored research;
- Train a new generation of researchers in interdisciplinary approaches to future science and engineering challenges;
- Increase the numbers and diversity of U.S. students trained in the mathematical and statistical sciences to meet the increasing demands of scientific research, engineering, and technology in academic institutions, industry, and government laboratories; and
- Develop a framework to significantly advance the image and understanding of mathematics in the general population.

Long-term Funding for Mathematical Sciences
(Dollars in Millions)

FY 2002 Actual	FY 2003 Actual	FY 2004 Actual	FY 2005 Actual	FY 2006 Current Plan	FY 2007 Request
$30.00	$60.42	$91.56	$89.56	$87.97	$78.45

FY 2007 Areas of Emphasis: NSF plans to invest $78.45 million in the Mathematical Sciences activities described below, while starting to mainstream interdisciplinary research partnerships. FY 2007 is the last year of funding of the Mathematical Sciences Priority Area. In future years, these activities will be part of ongoing programs in the participating areas. There is strong commitment to continuing partnerships.

- **Fundamental Mathematical and Statistical Sciences** – Fundamental research areas include themes such as dynamical systems and partial differential equations, geometry and topology, stochasticity, number theory, algebraic and quantum structures, the mathematics of computation, statistics, and multi-scale and multi-resolution analysis. To enhance research in these areas, the NSF will provide improved support for mathematical sciences through research groups and individual investigator grants, as well as through institute and undergraduate, graduate, and postdoctoral training activities.

- **Advancing Interdisciplinary Science and Engineering** – The concepts and structures developed by fundamental mathematics often provide just the right framework for the formulation and study of applications in other disciplines. Mathematics and statistics have yielded new analytical, statistical, computational, and experimental tools to tackle a broad range of scientific and technological challenges long considered intractable. This success has fueled a demand for increased support for collaborative research in which teams containing both mathematical scientists and researchers from other science and engineering disciplines work together: (a) to develop new mathematical approaches to concrete scientific or engineering problems for which adequate mathematical tools do not yet exist as well as (b) to apply these sophisticated techniques to significant problems in science and engineering. Such interdisciplinary collaborations will also nurture a new breed of researchers, broadly trained in both mathematics and science or engineering disciplines, needed to tackle the increasingly complex multidisciplinary research topics that confront society. Three broad, interdisciplinary research themes are being emphasized in the mathematical sciences priority area:

- **Mathematical and statistical challenges posed by large data sets** – Much of modern science and engineering involves working with enormous data sets. Major challenges include: the identification and recovery of meaningful relationships between data; the identification and validation of the structure of large data sets, which require novel mathematical and statistical methods; and improvement of theories of control and decision-making based on large, complex data streams. These challenges arise in such diverse arenas as: large genetic databases; the explosion of data gathered from earth monitoring systems (satellite observation systems, seismic networks, and global observation systems); situations in which privacy and missing data are major concerns; the massive data streams generated by automated physical science instruments, which must be compressed, stored and accessed for analysis; and data produced by modern engineering systems that place networked sensors and actuators on scalable networks to support dynamic interactions.

- **Managing and modeling uncertainty** – Predictions and forecasts of phenomena – bracketed by measures of uncertainty – are critical for making better decisions, whether in public policy or in research. Improved methods for assessing uncertainty will increase the utility of models across the sciences and engineering and result in better predictions of phenomena. Improving the ability to forecast extreme or singular events will improve safety and reliability in such systems as power grids, the Internet, and air traffic control. Advancing techniques to assess uncertainty has applications ranging from forecasting the spread of an invasive species, to predicting genetic change and evaluating the likelihood of complex climate change scenarios. In the social sciences, methods for assessing uncertainty will improve the utility of forecasts of phenomena such as market behavior.

- **Modeling complex nonlinear systems** – Advances in mathematics are necessary for a fundamental understanding of the mechanisms underlying interacting complex systems and systems far from equilibrium. They are essential to the further development of modern physical theories of the structure of the universe at the smallest and largest scales. Across the sciences, there is a great need to analyze and predict emergent complex properties and understand multi-scale phenomena, from social behaviors to brain function, and from communication networks to multi-scale business information systems to complex engineered systems.

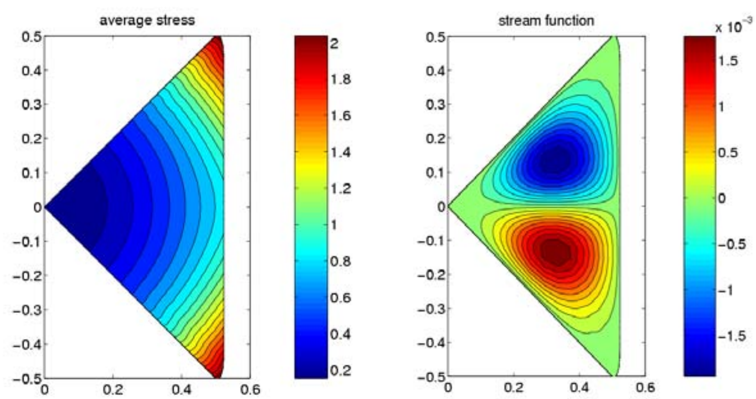
To enhance research in these areas of science and engineering, which depend on cross-cutting themes in the mathematical sciences, NSF will support opportunities encompassing interdisciplinary research groups, interdisciplinary centers, interdisciplinary cross-training programs, and partnership activities with other federal agencies. Training activities will cover interdisciplinary professional development at many levels and those that link highly innovative training activities with research.

- **Advancing Mathematical Sciences Education –** This effort will support innovative educational activities, centered on the research priorities highlighted above. Activities that foster closer connections between research and education include: curriculum development both in the mathematical sciences and in incorporating sophisticated mathematics into other disciplines, introducing new ideas across the K-16 spectrum; and research on how mathematics is learned, particularly in light of new learning technologies and emerging mathematical fields. Investments include support for undergraduate and graduate education and postdoctoral training coupled with curriculum reform, and for mentoring at key transition points in the careers of mathematical scientists. An area of focus that will continue in FY 2007 is to enhance undergraduate research experiences at the interface between the mathematical and biological sciences.

Recent Research Highlights

► **Computational Methods for Bulk Solid Handling Problems:** The handling of granular materials such as ores, building materials, chemical and pharmaceutical powders poses serious industrial problems. Silos routinely malfunction or even collapse. Some of those problems can be traced to the use of oversimplified models from the 1950s. NCSU Professor Pierre Gremaud in collaboration with Professor John Matthews (U. of Tennessee, Chattanooga) has recently made significant progress in the calculation of slow granular flows in industrial hoppers. Their approach allows the computational study of realistic industrial cases. This work is done in consultation with engineers at Jenike & Johanson, Inc. Once coupled with existing shell mechanics codes, those results will lead to the first comprehensive predictive tool for this type of phenomena and ultimately to more stable silos that are far less likely to collapse or malfunction. This research is representative of the type of contribution that mathematics can make for very real problems and the potential for positive economic impact.

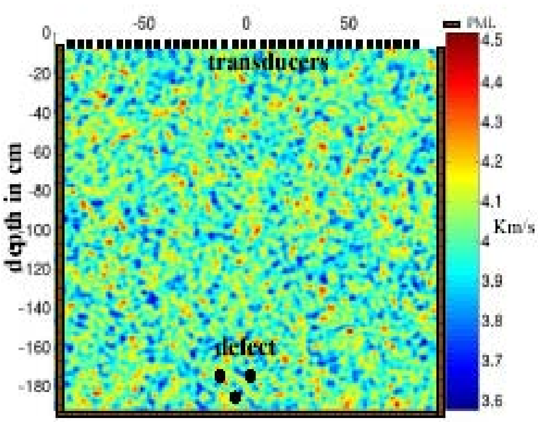
Collapse of Silo



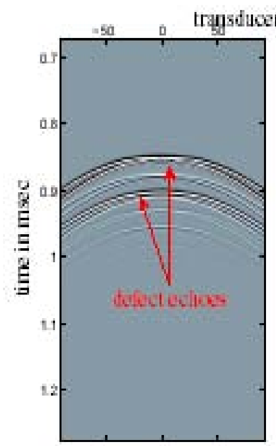
Calculations of the flow in a conical hopper with square cross section. Secondary circulation is observed. Only a quarter of the cross section is shown.

Credits: P. Gremaud, North Carolina University, J. Matthews, University of Tennessee, Chattanooga

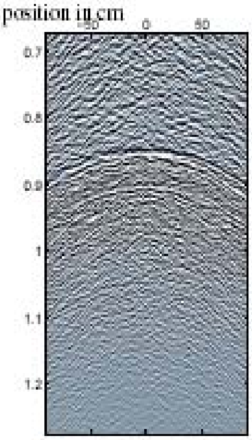
► **Robust Imaging Algorithms for Nondestructive Testing of Materials:** Accurately inferring the internal structure of a body is a great challenge in all imaging applications, such as medical imaging, remote sensing, nondestructive testing of materials, object detection, and monitoring of underground flows, etc., because of the inherent inhomogeneity of the media. A group of mathematicians consisting of Professor Liliana Borcea of Rice University, Professor George Papanicolaou of Stanford University and Professor Chrysoula Tsogka of the University of Chicago, has been working to advance the mathematical techniques for imaging. Through collaborations with engineers and physicists, including A. Paulraj and J. Claerbout of Stanford University and W. Scott of the Georgia Institute of Technology, they made significant progress in developing a statistically stable imaging algorithm and applied it to nondestructive detection of defects in aging concrete. Their method involves the use of an array of transducers that sends ultrasonic waves and records that scattered echoes at the surface of the concrete structure and an Adaptive Interferometric Imaging algorithm that exploits the coherence in the data by calculating cross-correlations of the recorded echoes at the array over carefully chosen space-time windows. This approach is statistically stable in that it is insensitive to changes of the detailed structure of the material and gives a reliable identification of the defects.

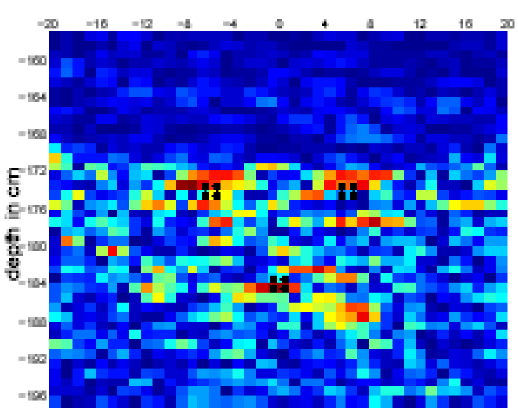


A schematic of sensing scenario

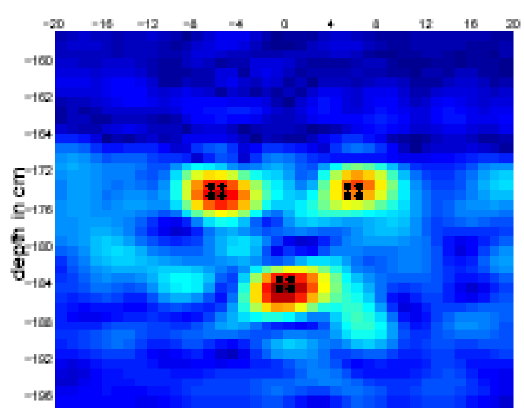


Resulting image assuming homogeneous medium


Resulting image in realistic inhomogeneous medium



Detection using conventional imaging algorithms



Detection using Adaptive Interferometric Imaging

Credit: Liliana Borcea, Rice University; George Papanicolaou, Stanford University; Chrysoula Tsogka, University of Chicago