## **VALIDATION OF PROBABILISTIC FORECASTS**

## Zoltan Toth<sup>1</sup>

## Environmental Modeling Center, NCEP

## ASSESSING THE VALUE OF PROBABILISTIC FORECASTS FROM A SCIENTIFIC PERSPECTIVE

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http://sgi62.wwb.noaa.gov:8080/ens/enshome.html

- 1) TYPES OF WEATHER FORECASTS WRT UNCERTAINTY
- 2) TYPES OF WEATHER FORECASTS WRT GENERATION

3) ATTRIBUTES OF (PROBABILISTIC) FORECASTS

4) **PROBABILISTIC VERIFICATION MEASURES** 

5) VALUE OF ENSEMBLE VS. CONTROL FCST

6) HOW MUCH DETAIL CAN ENSEMBLES FITHFULLY DEFINE IN PDF?

## TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS

1) Types of weather forecasts wrt uncertainty

DICHOTOMOUS (CATEGORICAL) VS. PROBABILISTIC

2) Types of weather forecasts wrt generation

STATSTICAL; DYNAMICAL – SINGLE VS. ENSMEBLE IN ANY CASE, PDF IS DESIRED, GENERAL FORMAT ONLY LIOUVILLE EQS PROVIDE THAT – NEED FOR POSTPROC.

3) Attributes of (probabilistic) forecasts

RELIABILITY (NO BIAS) & RESOLUTION (SMALL RANDOM ERROR)

4) **Probabilistic verification measures** 

BRIER, RANKED PROB., ROC, INFO CONTENT, ETC.

5) Value of ensemble vs. control fcst

1ST MOMENT BETTER; TEMOPRAL VARIATIONS IN 2ND MOMENT BETTER DEFINED PDF

6) How much detail can ensembles faithfully define in pdf?

BIMODALITY CAPTURED – JUMPS IN CONSECUTIVE CONTROL FCSTS ASSOCIATED WITH IT Predictability Seminar, ECMWF, Sept. 9–13 2002

#### TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS FORMAT OF FORECASTS RELATED TO UNCERTAINTY

		FORECAST FORMAT			
	PROBABILISTIC		DICHOTOMOUS		
			(CATEGORICAL)		
UNCERTAINTY	Substantial		Little or no		
FORMAT	0–100%		Yes or No		
EXAMPLE	Precip above 5 mm				
	80%		Yes		
TYPE	General		Special		

Resolution in probability space can be set at different levels: Very high: Continuous values

Intermediate: Every 10% (0, 10, 20, etc)

Very low: 0% (No) and 100% (Yes)

Dichotomous and probabilistic forecasts are fundamentally not different => Quasi–continuous transition

EXAMPLE: User wants to know if min temp will be below 5 C *IF* expected value is below –5 C *OR* above +15C *AND*expected error less than +/–5 C *THEN*use of dichotomous format justified (ie, no fcst uncertainty) *ELSE* use of dichotomous format TRUNCATES fcst info

**PROBABILITY DENSITY FUNCTION** (PDF) is the complete format, allowing all queries to be answered

We must **CONDENSE ALL KNOWLEDGE** on future weather into **PDF**s

#### TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS TYPES OF WEATHER FORECASTS WRT GENERATION

Fcsts in probabilistic & dichotomous format

can be generated by same methods:

- 1) **STATISTICAL** (based on observations)
- DYNAMICAL, based on NWP model integration:
   Single (combined with past verification statistics)
   Ensemble (sample of multiple realizations)
- 3) LIOUVILLE EQS (in prob space) not practical

## **FCST UNCERTAINTY**

 The atmosphere is a deterministic system AND has at least one direction in which perturbations grow
 Initial state (and model) has error in it ==> Chaotic system + Initial error =(Loss of) Predictability 90% Fcst probability



Ensemble approach potentially offers more fcst info – how do we tell? Predictability Seminar, ĘCMWF, Sept. 9–13 2002

# TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS PDF ESTIMATION BASED ON FINITE SAMPLE

Dynamical method (single or multiple realizations) – not proper format Must *INTER–/EXTRAPOLATE* probabilities to generate PDF format *Single integration* (combined with past verification statistics)



WHAT TO MEASURE? Predictability Seminar, ECMWF, Sept. 9–13 2002



Reliability can be statistically corrected (assuming stationary processes



Resolution CANNOT be statistically corrected –

**INTRINSIC VALUE** of fcst systems

If fcsts perfectly reliable, resolution = spread in ens. = spread in obs. => Perfect forecast system uses 0 & 100% probs & always correct TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS VERIFICATION MEASURES FOR PROBABILISTIC FCSTS

## 1) **RELIABILITY**

Reliability diagram (graphical) Reliability component of Brier Score

> RELATED ENSEMBLE FCST MEASURE: Analysis Rank Histogram (Talagrand diagram)

## 2) **RESOLUTION**

Reliability/Attributes diagram (graphical) Resolution component of the Brier Score Relative Operating Characteristics Economic Value (D. Richardson's presentation)

## FOR PERFECTLY RELIABLE FCSTS: Brier Skill Score

Ranked Probability Skill Score Information content *RELATED ENSEMBLE FCST MEASURE:* RMS error of ensemble mean ( = ensemble spread) Verifying analysis indistinguishable from ens members Smaller spread = fcst problem better resolved





#### TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS RELIABILITY / ATTRIBUTES DIAGRAM



**FCST OUTCOMES** 

#### TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS ANALYSIS RANK HISTOGRAM (TALAGRAND DIAGRAM)



Percentage of cases when the verifying analysis falls in any of the bins defined by the ordered series of the 17 ensemble members, at 120-hour lead time for April–June 1999. The expected value next to one particular forecast in the 17–member ensemble at each grid point at 120 hours lead time for March–May 1997, 500 hPa height over the NH extratropics. The expected value (5.55 %) is marked as a dotted blue line.



Percentage of cases when the 17–member ensemble does not encompass the 500 hPa height verifying analysis over the NH extratropics (in excess of the 11.1% that is expected due to the limited size of the ensemble.) April–June 1999. Predictability Seminar, ECMWF, Sept. 9–13 2002



- 2) a) **Correct for conditional bias** (shift) in distribution
  - b) Correct for error in spread

### Balance must be found between

- a) Details sought in calibration (time, space, meteor. & prob values)
- b) Available fcst-obs archive



#### TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS **RELATIVE OPERATING CHARACTERISTICS (ROC)** Application of signal detection theory for measuring discrimination between two alternative outcomes Worded, categorical and probab. forecasts can be compared Missed events not considered directly FORECAST Stratification YES NO DBSERVAT according to observations -M(isses) H(its) reliability NOT C(orrect rejections) F(alse alarms) measured Hit Rate (HR) = $\frac{H}{H + M}$ False Alarm Rate (FAR) = $\frac{\Gamma}{F+C}$

Use 10 climatologically equally likely bins to define events

<u>Categorical forecast:</u> If control falls in a given climate bin, forecast is YES and NO otherwise

**Ensemble forecast:** 

Probabilities converted to a categorical fcst given the probability exceeds a certain threshold. Eg., all 30% or higher probabilities count as YES. Using different threshold probabilities yield an HR/FA diagram.

<u>Measures:</u> 1) Area between HR–FAR curve and diagonal

2) How different forecast probabilities are given different observations



ROC (Relative Operating Characteristics) curve for a 5–day lead time 14–member T62 ensemble of forecasts and for the T126 and T62 control forecasts predicting events defined in terms of 10 climatologically equally likely bins for the 500 hPa height, NH extratropics, April–June 1999.

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## **INFORMATION CONTENT**

Use 10 climatologically equally likely bins to define events

 $Entropy = Plog_2 P_1$ 

Information in one forecast =  $I = 1 - \sum_{i=1}^{10} P_i \log_{10} P_i$ Average info in n independent fcsts =  $I_{ave} = \frac{1}{n} \sum_{i=1}^{n} I_i$ 



Information content of probabilistic forecasts based on the full ensemble distribution (red continuous semble distribution (rea continuous line), the mode (most frequent val-ue) of a 10-member ensemble (pur-ple dotted), and the T62 (greed short dash) and T126 (blue long dash) control forecasts for the **NH extratropics**, for March–May 1997. Expression and for 10 climato. Forecasts are made for 10 climatologically equally likely bins. The bin where the control or ensemble mode falls is assigned a probability corresponding to the observed frequency of the verifying analysis falling into the same bin (P), while the remaining 9 bins are assigned (1–P)/9 (assuming perfect reliability that is close to be satisfied when using calibrated forecasts). Probabilities for the full ensemble are based on the number of ensemble members falling into the various bins. Note that the ensemble-based forecast probabilities can vary widely from case to case, depending on how the ensemble members spread while they are fixed for the control forecasts. The advance knowledge of the case dependent reliability of the forecasts transletes into substantial gains in terms of the information content the forecasts carrv.

ON AVERAGE A 7.5–DAY FULLY PROBABILISTIC FORECAST OR A 6–DAY CATEGORICAL FORECAST ASSOCIATED WITH CASE DE-PENDENT RELIABILITY ESTIMATES HAS AS MUCH INFORMATION CONTENT AS A 5–DAY CATEGORICAL FORECAST

A 7.5–DAY FULLY PROBABILISTIC FORECAST HAS MORE THAN TWICE AS MUCH INFORMATION CONTENT THAN A 5–DAY CATE-GORICAL FORECAST

## IF ENSEMBLE IS MORE USEFUL THAN CONTROL FCST, WHAT EXPLAINS THE DIFFERENCE?

### 1) Expected value: Ensemble mean better than control? YES

- 2) Case dependent variations in spread: Ensemble has skill?
- 3) More detailed pdf from ensemble (m vs. 1 members)?
- 4) Is it only 2nd moment (spread), or further details in ensemble?

1) **Ensemble mean** has lower RMS error than control fcsts => Smaller random error/ Better estimate of 1st moment / Higher resolution



1000 hPa height RMS error for the T170/T126 (for the first 60 hrs, then T62) control forecast (short dash blue/long dash green) and the T126 (for first 60 hrs, T62 afterwards) ensemble mean (solid red), for June 28 – July 22 2000, over the NH extratropics.

- 1) Expected value: Ensemble mean better than control?
- 2) More detailed pdf from ensemble (m vs. 1 members)? YES
- 3) Case dependent variations in spread: Ensemble has skill?
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## RELATIVE OPERATING CHARACTERISTICS



ROC (Relative Operating Characteristics) area skill score for the T126 (dashed) and T62 (dotted) control, and the 14–member T62 ensemble forecasts (solid) predicting events defined in terms of 10 climatologically equally likely bins for the 500 hPa height, NH extratropics, for April–June 1999. Scale on vertical axis is logarithmic.





ROC (Relative Operating Characteristics) curve for a 5–day lead time 14–member T62 ensemble of forecasts and for the T126 and T62 control forecasts predicting events defined in terms of 10 climatologically equally likely bins for the 500 hPa height, NH extratropics, April–June 1999.

Same as figure above except for ROC–distance, defined (on linear vertical axis) as the distance between a control point and the closest point on the ensemble polygon. Positive (negative) values indicate the control point is above (below) the ensemble curve.

- 1) Expected value: Ensemble mean better than control?
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## Ensemble mode not much better than control

Overall reliability of T62 and T126 controls and 10–member ensemble mode (most frequent value) forecasts for the NH extratropics, for March–May 1997.



Brier Skill Score for the NH extratropics, for March-May 1997. Forecasts are made for 10 climatologically equally likely bins; results shown here are the average for the two extreme bins. The bin where the control or ensemble mode falls is assigned a probability corresponding to the observed frequency of the verifying analysis falling into the same bin (P), while the remaining 9 bins are assigned (1–P)/9 (assuming perfect reliabil-ity). Note that depending on the value of the mode (1 < M < 10),the corresponding observed frequency for the ensemble (but not for the control) varies widely.

#### TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS RESOLUTION OF ENSEMBLE BASED PROB. FCSTS

## **QUESTION:**

What are the typical variations in foreseeable forecast uncertainty? What variations in predictability can the ensemble resolve?

## **METHOD**:

Ensemble mode value to distinguish high/low predictability cases Stratify cases according to ensemble mode value – Use 10–15% of cases when ensemble mode is highest/lowest

### DATA:

NCEP **500 hPa NH extratropical ensemble fcsts** for March–May 1997 14 perturbed fcsts and high resolution control

## **VERIFICATION:**

Hit rate for ensemble mode and hires control fcst





THE UNCERTAINTY OF FCSTS CAN BE QUANTIFIED IN ADVANCE HIT RATES FOR 1–DAY FCSTS

CAN BE AS LOW AS 36%, OR AS HIGH AS 92% 10-15% OF THE TIME A 12-DAY FCST CAN BE AS GOOD, OR A 1-DAY FCST CAN BE AS POOR AS AN AVERAGE 4-DAY FCAST 1–2% OF ALL DAYS THE 12–DAY FCST CAN BE MADE WITH MORE CONFIDENCE THAN THE 1-DAY FCST AVERAGE HIT RATE FOR EXTENDED-RANGE FCSTS IS LOW -

VALUE IS IN KNOWING WHEN FCST IS RELIABLE

100 90 (%) **DBSERVED FREQUENCY** 80 600 400 70 200 828855 0 60 50 40 30 NCEP 20 10 0 0 10203040506070809000 FORECAST PROBABILITY (%) Predictability Seminar, ECMWF, Sept. 9–13 2002

Reliability diagram for 240-hour lead time 500 hPa height NH extratropics forecasts between March and May 1997. Forecast probabilities are based on how many ensemble members fell in any of 10 climatologically equally likely bins at each gridpoint, and are calibrated using verification statistics from the winter of 1995–96. Insert in upper left corner shows in how many events a particular forecast probability was used for the most likely bin (ensemble mode).





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#### TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS

![](_page_22_Figure_1.jpeg)

(25.4 mm/day)

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- - -

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## CAN ENSEMBLES SKILLFULLY PREDICT BIMODALITY? WORK IN PROGRESS

## Difficult to verify, **NEEDS LOTS OF DATA** (too much)

![](_page_23_Figure_7.jpeg)

Each fcst pdf pattern needs large number of realizations to establish associated distribution of observations

## **APPROACH:**

**Use climate pdf as reference** (10 climatologically equally likely bins) **Drastically reduce dof** by compositing pdf according to location of max

- 1) Identify bimodal distributions wrt climate pdf
- 2) Locate local maxima & minima in terms of 10 climate bins
- 3) Establish frequency of verifying analysis falling in max/min bins

![](_page_23_Figure_14.jpeg)

#### TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS CAN ENSEMBLES SKILLFULLY PREDICT BIMODALITY?

1) Given overall ensemble fcst distribution -

Does bimodality occur more frequent	tly than	expecte	d by ch	ance?
Ratio between multi/unimodal fcst pdfs	12	168	288	360h
NH	0.12	1.1	12	23
SH	0.93	5.3	14	17

Many bimodal pdfs must be due to sampling; have not tested stat. signif

# 2) In bimodal fcst cases, do obs confirm bimodality?

![](_page_24_Figure_5.jpeg)

**EXPECTATION:** Verification of bias-reduced fcsts will show stronger multimodal behavior

#### TOTH, Z.: VALIDATION OF PROBABILISTIC FORECASTS CAN ENSEMBLES SKILLFULLY PREDICT BIMODALITY?

4) Does multimodality as described here have fcst implications?

## CASE STUDY OF LARGE VARIATIONS IN CONSECUTIVE CONTROL FCSTS

![](_page_25_Figure_3.jpeg)

## **USE 50-MEMBER TIME-LAGGED ENSEMBLE**

initialized 0909 & 0910 00 &12Z, 0911 00Z

![](_page_25_Figure_6.jpeg)

a) # bimodal gridpoints vsaverage # for Sept 2001(Ratio)

## NUMBER OF MULTIMODAL GRIDPOINTS MUCH HIGHER THAN USUAL

Difference in ratio significant? Probably yes (have not checked) Predictability Seminar, ECMWF, Sept. 9–13 2002

## CASE STUDY OF LARGE VARIATIONS IN CONSECUTIVE CONTROL FORECASTS

![](_page_26_Figure_1.jpeg)

### Distribution of High–Low MSLP difference

STRONGLY BIMODAL

Statistically significant? Have not tested

## **CLUSTER ANALYSIS** – Two dominant patterns

![](_page_26_Figure_6.jpeg)

**GOOD CONTROL FCST** 

**BAD CONTROL FCST** 

![](_page_26_Figure_9.jpeg)

CAN CASES LIKE THIS BE IDENTIFIED BY STAT METHODS AS LIKELY REAL? Predictability Seminar, F,CMWF, Sept. 9–13 2002

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