On the Maintenance of the Polar Front Jet Stream

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ABSTRACT

A calculation of the mean transverse circulation about the polar front jet stream is performed by using a diagnostic balance, ω-equation method. The results show a thermally-direct mean transverse circulation about the jet stream system for this case study.

An examination of the kinetic energy balance of this jet stream reveals that the direct transverse circulation is probably strong enough to maintain the jet against frictional dissipation but not enough to provide large lateral export of energy. However, significant amounts of energy are transferred upward across the tropopause.

Further considerations are employed to argue that the mean transverse circulation obtained here is compatible with the observed distributions of temperature and potential vorticity about the jet core.

1. Introduction

Since its quantitative documentation over 25 years ago, the polar front or mid-latitude jet stream has been the subject of numerous intensive investigations. Much of the early interest in this jet stream system was due to its obvious connection with the mid-latitude cyclone waves and their associated weather processes. Because these seemed to form an interacting system, a number of investigators became interested in the dynamical processes responsible for producing and maintaining this jet stream.

In this study the description "polar front jet stream" refers to the meridionally meandering core of high wind speed associated with mid-latitude cyclonic disturbances. It is located at about 250–300 mb and is normally characterized by a strong baroclinic zone below the core of highest wind speeds. In contrast, the "subtropical jet stream" is found at about 200 mb in the 25°–30° latitude range. It is a much steadier, lower amplitude system than the polar front jet stream and generally possesses a much weaker baroclinic zone below the jet core. In fact, the baroclinic zone cannot normally be found below this jet at 500 mb, while it is clearly visible at 500 mb below the polar front jet stream.

To avoid possible confusion in later discussions, the phrase "mean transverse circulation" refers to the flow normal to the jet stream axis averaged along its length. On the other hand, the "mean meridional circulation" is the zonally-averaged motion normal to latitude circles. Such mean circulations are classified as "thermally direct" or "thermally indirect," depending upon whether they are kinetic energy releasing or kinetic energy consuming, respectively.

According to a theory of Rossby (1947) and Staff Members of the University of Chicago (1947), the westerly jet stream in mid-latitudes is formed by lateral mixing processes with constant absolute angular momentum to the south of the jet core and constant absolute vorticity to the north. This assumption yields a lateral velocity profile which is qualitatively reasonable. It was pointed out, however, that such a mechanism implies a thermally-indirect transverse circulation about the jet core itself. Although such a mechanism is attractive because the systematic ascent of cold air and descent of warm air helps to maintain the strong lateral temperature contrast across the jet core, it leads to the possibly more difficult question as to how the kinetic energy of the jet stream is maintained. Making use of this scheme, Palmén (1951) included the assumed indirect transverse circulation about the polar front jet stream in his model of the mean meridional circulation. However, he assumed a direct cell about the subtropical jet stream.

In a study by Endlich (1953), fields of adiabatic vertical velocities were computed in the vicinity of a polar front jet stream. Although fields of rising and sinking motion were found on both sides of the jet core, the statistics indicated a slight preference for the indirect transverse circulation. However, no systematic averaging was performed relative to the jet core itself. In a separate study, Riehl and Teweles (1953) also indicated results which were not incompatible with the original concept of such an indirect transverse circulation. Also, Newton and Carson (1953) and Newton (1958) indicated that, in the vicinity of a developing jet stream and an associated frontal zone, the vertical motion branch of the transverse circulation about the jet core must be in an *indirect* sense. However, they contended that the *total* transverse circulation in the region of the polar front jet stream should remain thermally direct.

An alternative hypothesis was advanced by Namias and Clapp (1949), who proposed that jet streams form as a result of a "confluence" of a preexisting horizontal temperature gradient. In this case, the compensating vertical transverse circulation is thermally direct. A similar concept was also proposed by Nyberg (1949, 1950, 1953).

The thermally-direct transverse circulation also resulted from theoretical studies by Van Mieghem (1950) and Kuo (1954). This is suggested as well in theoretical studies of atmospheric frontal structure by Sawyer (1956) and by Eliassen (1959, 1962). Similar results were indicated in numerical simulations of frontogenesis by Williams (1967) and Hoskins (1971).

The only previous systematic attempt to establish the mean transverse circulation about the polar front jet stream was that of Clapp and Winston (1951). They found tentative evidence in favor of a direct circulation by using adiabatic vertical velocities, averaged from the surface to 10,000 ft. This, however, is considerably below the jet core itself. Similar contentions were also presented as a result of qualitative studies by Hubert (1953), Raethjen (1953), Vuorela (1953, 1957), Reiter (1961, 1963a), Briggs and Roach (1963), and by Endlich and McLean (1965).

A number of authors have investigated the vertical motions in the vicinity of specific portions of polar front jet streams. Murray and Daniels (1953), for example, showed that the transverse circulation is direct in the entrance zone and indirect in the exit zone of a high-speed center along the jet core. In a study of upper-tropospheric frontogenesis, Reed and Sanders (1953) showed intense sinking motions in the "jet stream front" on the cyclonic side of the jet core. This was corroborated in subsequent studies by Reed (1955), Reed and Danielsen (1959), and Danielsen (1959a). These authors argued further that this frontal zone may contain subsiding stratospheric air rather than being a zone of mixing between two tropospheric air masses. This contention was corroborated in case studies by Danielsen (1959b, 1964a, b, 1968), Danielsen *et al.* (1962), Staley (1960, 1962), Reiter (1963a, b, 1964), Reiter and Mahlman (1964, 1965a, b), Reiter *et al.* (1969), and by Mahlman (1964, 1965). Moreover, this intense sinking of stratospheric air north of the jet core was shown by Danielsen (1964a) and by Mahlman (1964, 1965) to be strongly associated with cyclogenesis in the upper troposphere. Furthermore, the relationship of this exchange mechanism to the cyclogenetic process was verified statistically by Mahlman (1966, 1969a).

Even though the above mechanism is important in the exchange of mass between the stratosphere and troposphere and in the dynamics of cyclone development, it occurs only during certain periods along limited sections of the jet core. As a result, such studies may not give any real insight into the nature of the *net* transverse circulation about the polar front jet stream.

A valuable contribution to the problem of the maintenance of jet stream systems was that presented by Riehl and Fultz (1957, 1958). From an analysis of a steady three-wave case in a rotating dishpan experiment, the authors were able to demonstrate that the mean transverse circulation about the dishpan jet stream is thermally direct. This was the case even though the mean meridional circulation in the same latitudes was found to operate in a thermally-indirect sense. Thus, the analysis showed that the mechanisms responsible for the maintenance of the jet stream may be fundamentally different from those acting to maintain the zonal-mean wind at the same latitudes. Drawing from this work, Palmén (1959) predicted that a systematic averaging along the polar front jet stream would also yield a direct transverse circulation. Riehl and Fultz's results on the energetics of the dishpan jet stream will be compared later against the conclusions of the analysis to be performed here.

A study by Elsberry (1968) of a dishpan five-wave vacillation case also indicated the existence of a thermally-direct transverse circulation around the dishpan jet stream. In this case the direct circulation was a significant residual between thermally-direct and indirect transverse circulations operating at the poleward and equatorward branches of the jet stream maximum, respectively. However, in the cases studied by Riehl and Fultz (1957, 1958) and by Elsberry (1968), the lateral wind shears in the jet stream regions were significantly *less* relative to the Coriolis parameter than those observed in the atmosphere. Thus, it is not clear whether or not such results can be immediately applied to a system such as the polar front jet stream.

The success of Riehl and Fultz (1957, 1958) encouraged efforts to obtain analogous measurements for atmospheric jet streams. Krishnamurti (1961a, b), in a detailed study of the subtropical jet stream, showed the existence of a thermally-direct transverse circulation about the jet core. Thus, the transverse circulation about the subtropical jet stream operates in the same sense as the mean meridional (Hadley) circulation in the same region.

In a study of the stratospheric polar night vortex, Mahlman (1966, 1969b) found a direct transverse circulation operating below the polar night jet stream just before a major breakdown period. This was the case even though the mean meridional circulation in the same region was shown to operate in the indirect sense.

The above studies have been useful in developing our understanding of the maintenance of jet stream systems. The polar front jet stream, however, has not yet been studied in quantitative detail as to its actual mean transverse circulation and corresponding energetics. This is to a great extent due to the difficulty in defining an adequate curvilinear coordinate system for such a strongly time-dependent phenomenon. Investigators have been hampered by the lack of data and diagnostic techniques capable of resolving the vertical motion field with sufficient accuracy to provide reasonably consistent answers. The intent of this study is to obtain a systematic calculation of the net transverse circulation about the polar front jet stream. Further, it is desired to investigate the implications of such a circulation on the energetics of this jet and its surroundings. This will be accomplished through use of a diagnostic balance, ω-equation method developed originally by Krishnamurti (1966, 1968).

2. Experimental approach

When attempting to provide a meaningful calculation of the net transverse circulation occurring in the vicinity of a polar front jet stream, it is highly desirable that the case selected for computation be one in which the time variations of the system are not particularly large. This is not only advantageous from the point of view of establishing a realistic coordinate system, but also because the result is likely to be more representative of the actual processes usually acting to maintain such a system.

Because of the above considerations, the case selected for analysis was an area centered over the contiguous United States for the time period 1200 GMT 15 November to 1200 GMT 17 November, 1966. This period was dominated by a polar front jet stream of moderate intensity. The flow at jet stream level was somewhat anticyclonic over the continent bounded by relatively weak cyclones off each coast (Fig. 1). Although the Pacific Coast cyclone did move slowly eastward, bringing some precipitation to the California region, no significant cyclogenesis or large-scale system movements were observed during the five upper-air observation times contained in this selected time period. Thus, this particular case appears satisfactory for attempting to determine the basic dynamical processes acting to maintain the polar front jet stream.

As mentioned in the Introduction, the approach used here is to employ the diagnostic balance, ω-equation method developed originally by Krishnamurti (1966, 1968) in which the equations are scaled for motions of characteristic Rossby number less than unity (Ro<1). The initial data input is obtained from the National Meteorological Center objectively analyzed geopotential fields at 1000, 850, 700, 500, 300 and 200 mb. The streamfunction (ψ) is assumed to be related to the geopotential field through the nonlinear balance equation[1]

$$\nabla\cdot f\nabla\psi+2J\left(\frac{\partial\psi}{\partial x},\frac{\partial\psi}{\partial y}\right)=\nabla^2\Phi. \tag{1}$$

In this study ψ is specified at the lateral boundaries by setting $\psi=\Phi/f_0$. In the event that the ellipticity condition for Eq. (1), i.e.,

$$\nabla^2\Phi+\tfrac{1}{2}f^2-\nabla f\cdot\nabla\psi>0, \tag{2}$$

is not satisfied, Eq. (1) is replaced by

$$\nabla\cdot f\nabla\psi=\nabla^2\Phi-2J(u_g,v_g). \tag{3}$$

Such an adjustment is frequently required on the anticyclonic side of the jet stream. Use of Eq. (2) in such cases in place of Eq. (1) permits retention of strong anticyclonic vorticities from the diagnosed height fields.

By using the balance equation (1) combined with the continuity, vorticity and thermodynamic equations, one obtains the "complete" (Ro<1) ω-equation system:

$$\begin{aligned}\nabla^2\sigma\omega+f^2\frac{\partial^2\omega}{\partial p^2}
&=f\frac{\partial}{\partial p}J(\psi,\zeta_a)+\frac{\alpha}{\theta}\nabla^2J(\psi,\theta)-2\frac{\partial}{\partial t}\frac{\partial}{\partial p}J\left(\frac{\partial\psi}{\partial x},\frac{\partial\psi}{\partial y}\right)\\
&\quad-f\frac{\partial}{\partial p}(\zeta\nabla^2\chi)+f\frac{\partial}{\partial p}\left(\omega\frac{\partial}{\partial p}\nabla^2\psi\right)\\
&\quad+fg\frac{\partial^2}{\partial p^2}\left(\frac{\partial\tau_y}{\partial x}-\frac{\partial\tau_x}{\partial y}\right)-\frac{\chi}{p}\nabla^2H_L-f\frac{\partial}{\partial p}(\nabla\chi\cdot\nabla\zeta_a)\\
&\quad+f\frac{\partial}{\partial p}\left(\nabla\omega\cdot\nabla\frac{\partial\psi}{\partial p}\right)-\frac{\alpha}{\theta}\nabla^2(\nabla\chi\cdot\nabla\theta)-\beta\frac{\partial}{\partial p}\frac{\partial}{\partial y}\frac{\partial\psi}{\partial t},\end{aligned} \tag{4}$$

$$\nabla^2\chi=\frac{\partial\omega}{\partial p}, \tag{5}$$

$$\begin{aligned}\nabla^2\frac{\partial\psi}{\partial t}&=-J(\psi,\zeta_a)+\nabla\chi\cdot\nabla\zeta_a+\zeta_a\nabla^2\chi\\
&\quad-\nabla\omega\cdot\nabla\frac{\partial\psi}{\partial p}-\omega\frac{\partial}{\partial p}\nabla^2\psi-g\frac{\partial}{\partial p}\left(\frac{\partial\tau_y}{\partial x}-\frac{\partial\tau_x}{\partial y}\right).\end{aligned} \tag{6}$$

Given suitable boundary conditions for ω, χ and $\partial\psi/\partial t$, this set of equations is solved for ω by the method of sequential relaxation, with the initial solution including only the first two terms on the right-hand side of Eq. (4). Note that this initial step is similar in form to the quasi-geostrophic ω equation except f and σ vary in the horizontal and ψ is the complete balanced streamfunction rather than the simple geostrophic form. Once the initial approximation for ω is determined,

[1] See the Appendix for a list of symbols.

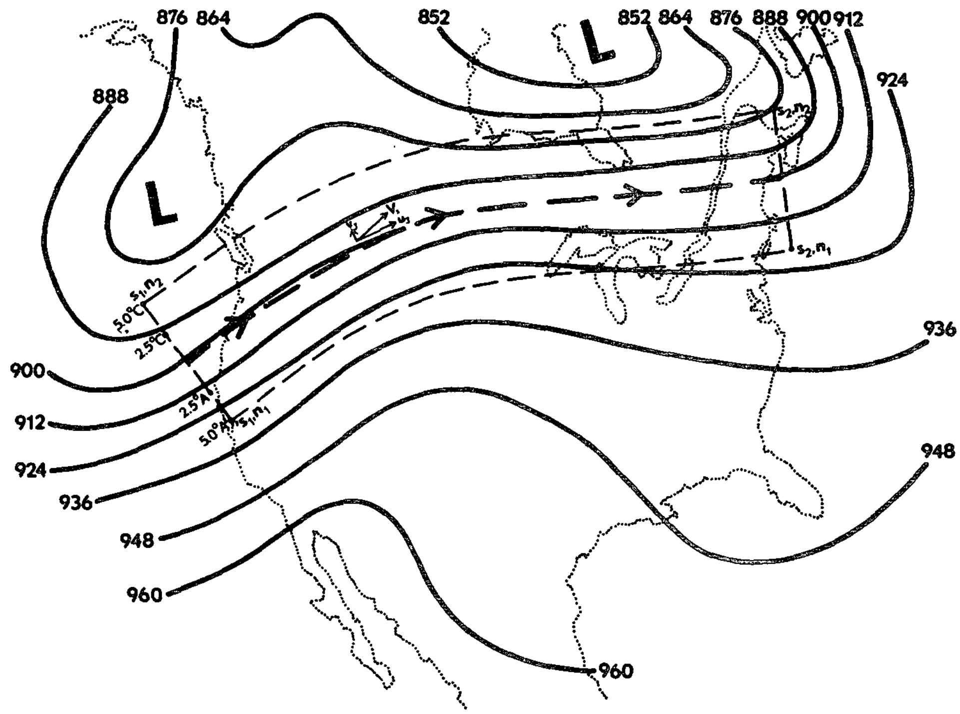

Fig. 1. Geopotential height (dkm) at 300 mb at 0000 GMT on 16 November 1966. Heavy dashed arrow shows position of the polar front jet stream and dashed box shows lateral boundary of computational grid. °C and °A represent normal distance in degrees latitude on the cyclonic and anticyclonic sides of the jet axis, respectively.

Eq. (5) is solved for χ, the velocity potential. These initial fields of ω and χ, along with Eq. (6), are then used to evaluate the remaining terms on the right-hand side of (4). Then Eq. (4) is solved by the method of sequential relaxation as before, only with all forcing functions included. This procedure is repeated until the difference between successive approximations of the ω field becomes acceptably small. In this study, five passes were found to be sufficient. For further details in the computational procedure, see Krishnamurti (1968).

The computations are made from a five-level grid in the vertical. Fields of ψ, χ, Φ, $\partial\psi/\partial t$ appear at the 1000-, 800-, 600-, 400- and 200-mb surfaces, while ω, T, θ, q, q_s are given at the 900-, 700-, 500- and 300-mb levels. The horizontal grid points are spaced 2.5° latitude and longitude apart. The grid extends from 55 to 135W and from 25 to 60N. There are 15 grid points in the meridional direction and 33 in the zonal direction. The last six grid points do not contain initial data, but are used to provide cyclic continuity so that grid points 1 and 33 have the same value for any given dependent variable. Grid point values for points 28 through 32 are provided by interpolation from values at grid points 1, 2, 26 and 27. At the north and south boundaries ω is set equal to zero.

Once the calculations for each observation time are completed, a coordinate line is defined along the polar front jet stream axis at 300 mb (as defined from a careful isotach analysis). Then parallel coordinate lines are defined at distances 2.5° and 5.0° latitude perpendicular to the jet axis in the cyclonic and anticyclonic directions, respectively (see Fig. 1). It is then a comparatively straightforward procedure to calculate "mean" distributions of variables relative to the jet core. For an example of the mean wind and temperature structure relative to this jet, see Fig. 2. In this system, an "eddy" quantity is defined as the value of a variable at a given point minus its "mean" along the same line parallel the jet axis. At times it will be useful to discuss "mean" and "eddy" processes relative to the jet core. These are to be carefully distinguished from the mean and eddy concepts arising from the zonal averaging often employed in general circulation studies.

3. Mean transverse circulation about the jet core

The calculations of ω, averaged along one complete wavelength of the polar front jet stream, were prepared for all levels at the specified normal distances to the 300-mb jet core. These were prepared individually for each observation time from 1200 GMT 15 November to 1200 GMT 17 November, 1966.

The individual calculations, derived from the five observation times, all showed for the troposphere a distinct *thermally-direct pattern of mean vertical motion about the jet core* (i.e., systematic ascent of the warm air

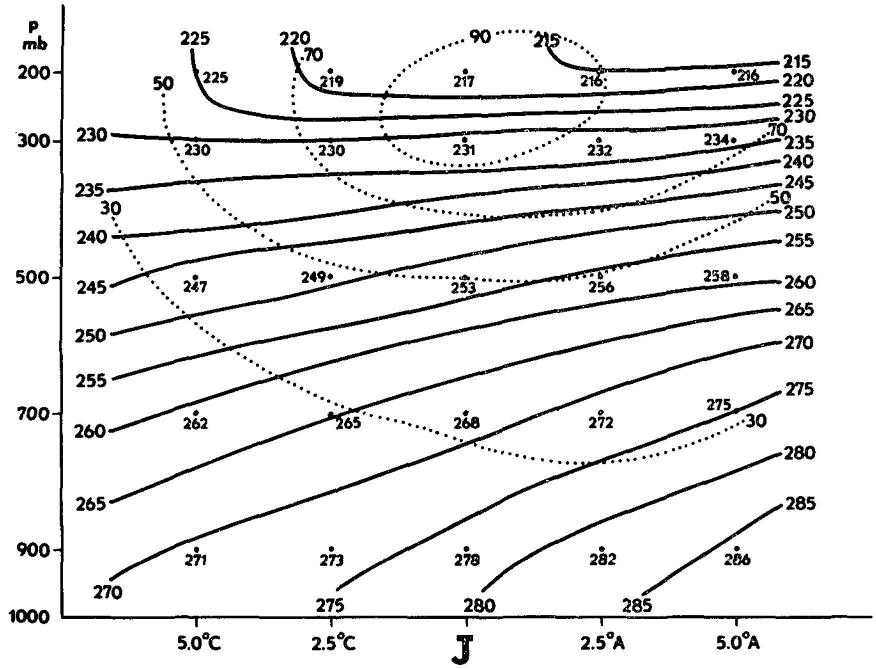

FIG. 2. Mean wind (kt, dashed lines) and temperature (°K, solid lines) along the polar front jet stream averaged from 1200 GMT 15 November to 1200 GMT 17 November, 1966.

on the anticyclonic side of the jet core relative to the colder air on the cyclonic side). Although each independent calculation revealed this, in the interest of increasing the reliability of subsequent inferences, the individual calculations were combined to form a time-averaged mean ω relative to the jet core.

The quantitative results are given in Fig. 3 along with the 95% confidence sampling interval, determined from the time variability of the individual $[\omega]_s$ calculations (assuming all variability to be the result of sampling error). The figure shows a systematic mean ascent on the anticyclonic side of the jet core with maximum

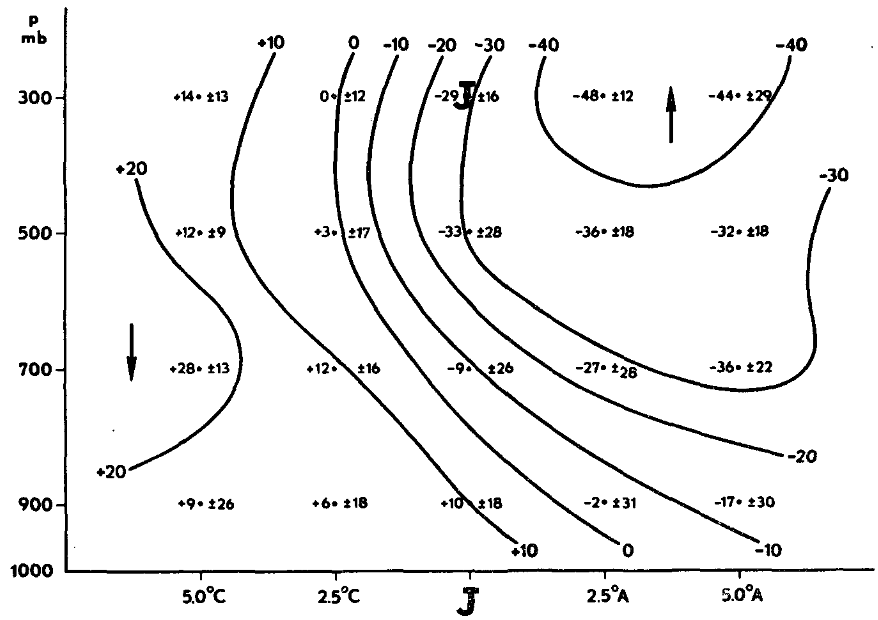

FIG. 3. Mean vertical motion ($[\omega]_s$, 10^{-5} mb sec^{-1}) relative to the jet axis averaged over the period 1200 GMT 15 November to 1200 GMT 17 November, 1966. Numbers to the right of the points give the 95% confidence interval for the value determined from the calculations.

values at 300 mb. This characteristic agrees well with observations of cirrus cloud shields on the anticyclonic side of the jet core (McLean, 1957; Conover, 1960; Kadlec, 1963; Oliver *et al.*, 1964; Bittner, 1967; Anderson *et al.*, 1969). On the other hand, Fig. 3 shows ascending motion through the jet core. This result may disagree with satellite studies which contend that the cirrus shield cuts off at the jet core or just to the anticyclonic side (Oliver *et al.*, 1964; Bittner, 1967; Anderson *et al.*, 1969). Nevertheless, this calculated ascent in the jet core is not inconsistent with the result of Starrett (1949), which shows that precipitation tends to be a maximum directly underneath the jet core. It is also compatible with the distributions of ozone and water vapor mixing ratio in the vicinity of the jet core given by Briggs and Roach (1963). This result is further supported by the observation that the tropopause "gap" (apparently dynamically produced) is generally located just to the cyclonic side of the jet core.

In a study such as this, where a number of types of averaging procedures must necessarily appear, it is convenient to employ the averaging notation proposed by Reiter (1969a, b). In this system the mean of an arbitrary quantity, say Q, is given by $[Q]_{(\ )}$, where the quantity enclosed by small parentheses indicates the coordinate (or coordinates) over which the averaging was performed. The deviation of Q from that average is given by $(Q)_{(\ )}$. Thus, for example, one might write $Q=[Q]_{(s)}+(Q)_{(s)}$ for averaging processes along the jet axis.

A useful way to view the mean motion relative to the jet core is through use of a mass transport streamfunction. Because the computation volume of Fig. 1 was chosen so that the mean speed in the left inflow region was very nearly equal to that of the right outflow region, and also due to the large ratio of the length to the width of the jet stream in this study, one may write the averaged mass continuity equation as

$$\frac{\partial[v_J]_{(s)}}{\partial n}+\frac{\partial[\omega]_{(s)}}{\partial p}=0. \qquad (7)$$

One can write a solution to this equation in the form of a streamfunction such that

$$[v_J]_{(s)}=-\frac{\partial[\Psi]_{(s)}}{\partial p}, \quad [\omega]_{(s)}=+\frac{\partial[\Psi]_{(s)}}{\partial n}. \qquad (8)$$

The diagnostic balance, ω-equation set is solved with a boundary condition of $\omega=0$ at 100 mb. Because of this, $[\Psi]_{(s)}$ is arbitrarily set equal to zero on that boundary. The implied net inflow into the region is obtained by integrating Eq. (7) from 5° latitude on the anticyclonic side to 5° latitude on the cyclonic side of the jet core. It is then assumed that the net computed inflow or outflow can be partitioned equally on the anticyclonic and cyclonic boundaries. Once the lateral boundary values for $[v_J]_{(s)}$ are set, then the entire field of $[\Psi]_{(s)}$ is determined by numerical integration of Eq. (8). The results of this determination along with the mean wind structure of the region are given in Fig. 4. This figure shows a number of interesting features. Rather strong confluence of warm and cold air appears to be taking place at about the 700-mb level under the jet core, in good agreement with the concept presented by Namias and Clapp (1949). Note the well-defined flow across the jet axis at 200 mb and the horizontal splitting of streamlines on the anticyclonic side of the jet axis. This implies rather strong energy generation through $-[v_J]_{(s)}\partial[\Phi]_{(s)}/\partial n$ at the jet core. Also, there is an implied horizontal divergence on the anticyclonic side of the jet core. This is compatible with the observed lateral spreading of cloud elements in the cirrus shield on the anticyclonic side of the subtropical jet stream as seen in ATS-I satellite time-lapse movies. Further, a study by Fujita *et al.* (1968) of these ATS-I time-lapse movies shows significant lateral convergence of plumes from the tops of lower to mid-tropospheric convective cells to the south of the cirrus shield. This is also in agreement with the results given in Fig. 4.

It is of interest to note that the structure given in Fig. 4 is also quite similar in overall character to the qualitative model of transverse circulation about the polar front jet core given by Danielsen (1968) for cases of strong import of stratospheric air into the "jet stream front" in association with cyclogenesis. This appears to be of fundamental interest, since the case selected for analysis here had *no* occurrences of large-scale cyclogenesis during the period. Therefore, even though the sinking of stratospheric air on the cyclonic side of the jet core in such cases is about an order of magnitude more intense than the mean values given here (see numerous references on this phenomenon cited in the Introduction), the qualitative forms of the transverse circulations are very similar. Because of this correspondence, it is then reasonable to hypothesize that some of the fundamental dynamical mechanisms responsible for maintenance of the polar front jet stream and for producing rapid upper tropospheric cyclogenesis (with associated import of stratospheric air into the troposphere) may be essentially the same. This may be the case even though the former exhibits a quasi-steady-state character while the latter is essentially discrete in nature. This hypothesis, of course, needs much more thorough examination before its validity can be strongly claimed. Such comparisons and examinations are difficult, however, not only because these are energetically open systems, but also because they interact very strongly with each other.

4. Jet stream "mean" kinetic energy maintenance

Given the mean transverse circulations about the polar front jet stream from the previous section, it is possible to provide quantitative estimates of the various contributions which this circulation produces toward maintaining the "mean" kinetic energy of this jet stream. The conventional approach in evaluating the energetics of open systems for volumes which do not encircle the globe is to derive expressions for the time rate of change of the *total* kinetic energy of the given volume. Since the major interest here is in the maintenance of the "*mean*" structure of the polar front jet stream, only the balance equation for the mean kinetic energy (averaged along the jet axis) will be evaluated in any detail.

The derivation of the mean kinetic energy balance in an arbitrary curvilinear (s,n,p) coordinate system is obtained by first considering the equations of motion in this system:

$$\frac{\partial u_J}{\partial t}+u_J\frac{\partial u_J}{\partial s}+v_J\frac{\partial u_J}{\partial n}+\omega\frac{\partial u_J}{\partial p}-fv_J+\frac{\partial \Phi}{\partial s}+F_s=0, \tag{9}$$

$$\frac{\partial v_J}{\partial t}+u_J\frac{\partial v_J}{\partial s}+v_J\frac{\partial v_s}{\partial n}+\omega\frac{\partial v_J}{\partial p}+fu_J+\frac{\partial \Phi}{\partial n}+F_n=0. \tag{10}$$

It should be pointed out here that the s axis is along the jet axis and not necessarily along the instantaneous streamlines. Also, u_J is the wind component along this jet axis and v_J the component along the normal n axis.

Now, by multiplying Eq. (9) by $[u_J]_{(s)}$ and Eq. (10) by $[v_J]_{(s)}$, adding, averaging along s and then along n, manipulating and integrating over the mass contained between two pressure surfaces, one obtains the equation for the time rate of change of "mean" kinetic energy of the jet stream:

$$\begin{aligned}
&\frac{\partial}{\partial t}\int_{p_u}^{p_l}\left\{\frac{[[u_J]^2_{(s)}+[v_J]^2_{(s)}]}{2}{}_{(n)}\right\}\frac{dp}{g}\\
&=\frac{\partial K_M}{\partial t}=+\int_{p_u}^{p_l}\frac{1}{A}\oint_B c_n\left\{\overbrace{\frac{[u_J]^2_{(s)}+[v_J]^2_{(s)}}{2}}^{(a_1)}+\overbrace{[u_J]_{(s)}(u_J)_{(s)}+[v_J]_{(s)}(v_J)_{(s)}}^{(a_2)}\right\}dl\frac{dp}{g}\\
&\quad-\left[\frac{\omega}{g}\left\{\overbrace{\frac{[u_J]^2_{(s)}+[v_J]^2_{(s)}}{2}}^{(b_1)}+\overbrace{[u_J]_{(s)}(u_J)_{(s)}+[v_J]_{(s)}(v_J)_{(s)}}^{(b_2)}\right\}\right]_{(s,n)}\Bigg|_{p_l}\\
&\quad+\left[\frac{\omega}{g}\left\{\overbrace{\frac{[u_J]^2_{(s)}+[v_J]^2_{(s)}}{2}}^{(c_1)}+\overbrace{[u_J]_{(s)}(u_J)_{(s)}+[v_J]_{(s)}(v_J)_{(s)}}^{(c_2)}\right\}\right]_{(s,n)}\Bigg|_{p_u}\\
&\quad+\int_{p_u}^{p_l}\overbrace{\left\{\left[(u_J)_{(s)}(v_J)_{(s)}\frac{\partial}{\partial n}[u_J]_{(s)}\right]_{(s,n)}+\left[(u_J)_{(s)}(\omega)_{(s)}\frac{\partial}{\partial p}[u_J]_{(s)}\right]_{(s,n)}+\left[(v_J)^2_{(s)}\frac{\partial}{\partial n}[v_J]_{(s)}\right]_{(s,n)}+\left[(v_J)_{(s)}(\omega)_{(s)}\frac{\partial}{\partial p}[v_J]_{(s)}\right]_{(s,n)}\right\}}^{(d)}\frac{dp}{g}\\
&\quad+\int_{p_u}^{p_l}\overbrace{\{[[u_J]_{(s)}(f)_{(s)}(v_J)_{(s)}]_{(s,n)}-[[v_J]_{(s)}(f)_{(s)}(u_J)_{(s)}]_{(s,n)}\}}^{(e)}\frac{dp}{g}+\int_{p_u}^{p_l}\overbrace{\frac{\{[\Phi]_{(B)}-[\Phi]_{(s,n)}\}}{A}}^{(f)}\oint_B c_n dl\frac{dp}{g}\\
&\quad\overbrace{-\frac{1}{g}[([\omega]_{(s)})_{(n)}([\Phi]_{(s)})_{(n)}]_{(n)}\Big|_{p_l}}^{(g)}\;\overbrace{+\frac{1}{g}[([\omega]_{(s)})_{(n)}([\Phi]_{(s)})_{(n)}]_{(n)}\Big|_{p_u}}^{(h)}\\
&\quad-\int_{p_u}^{p_l}\overbrace{[([\omega]_{(s)})_{(n)}([\alpha]_{(s)})_{(n)}]_{(n)}}^{(i)}\frac{dp}{g}-\int_{p_u}^{p_l}\overbrace{[[u_J]_{(s)}[F_s]_{(s)}+[v_J]_{(s)}[F_n]_{(s)}]_{(n)}}^{(j)}\frac{dp}{g}.
\end{aligned}\tag{11}$$

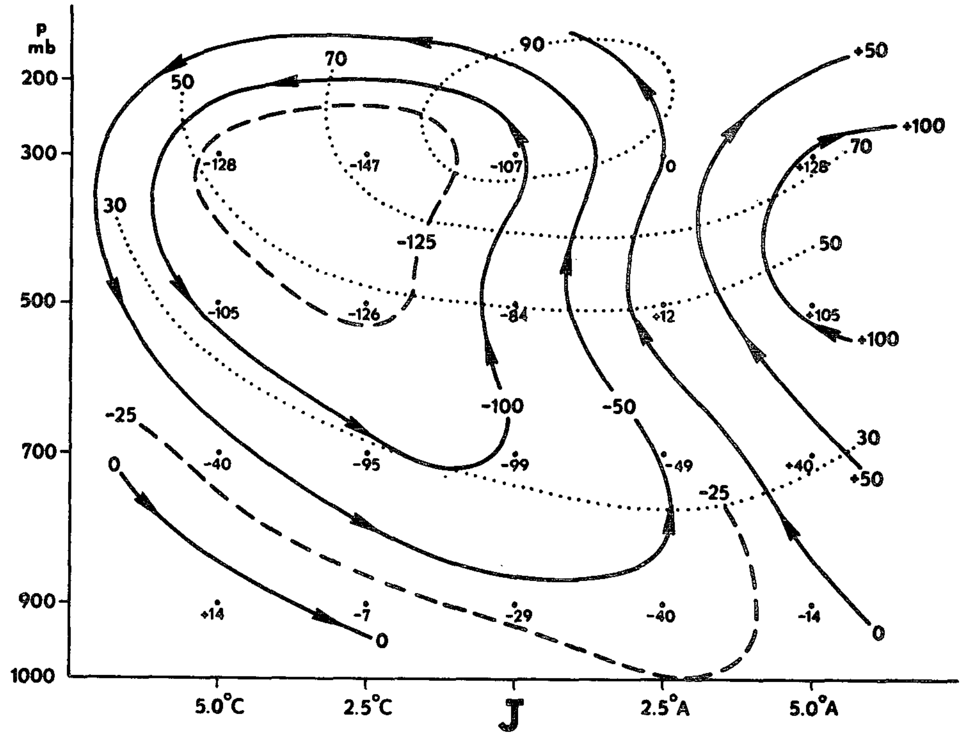

FIG. 4. Mean mass transport streamfunction ($[\Psi]_s$, mb m sec^{-1}) computed from the values of $[\omega]_{s,t}$ given in Fig. 3.

In the above expression the separate terms, labeled (a)–(j), have the following interpretation:

(a) flux of "mean" kinetic energy through the side boundaries;
(b), (c) flux of "mean" kinetic energy through the lower and upper boundaries, respectively;
(d) conversion of "eddy" kinetic energy to "mean" kinetic energy;
(e) an additional term in the conversion from "eddy" to "mean" kinetic energy which arises because the Coriolis parameter varies along the axis over which the initial average is performed. It is interpreted as a conversion term because it appears with opposite algebraic sign in the "eddy" kinetic energy equation;
(f) mean pressure interaction term at the side boundaries arising from energy conversion in an open system;
(g), (h) mean pressure interaction terms at the lower and upper boundaries, respectively;
(i) conversion of potential energy into "mean" kinetic energy due to mean transverse circulation about the jet core;
(j) dissipation of "mean" kinetic energy due to friction and subgrid-scale mixing.

Because the coordinate system tends to be aligned nearly parallel to the flow, not only are the terms involving "eddy" products difficult to calculate, but many times they may prove to be negligibly small. Since the intent of this work is to assess the effect of the "mean" transverse circulation about the jet stream, only quantities (a_1), (b_1), (c_1), (f), (g), (h) and (i) are computed here. A program to provide explicit calculation of "eddy" effects for this case study has been developed by Riordan (1971), but the energetics part of his calculation gave inconclusive results. As will be pointed out later, there may be some aspects of jet stream dynamics which are strongly dependent upon "eddy" processes relative to the jet stream. For the present analysis, the "mean" kinetic energy balance is put in the symbolism implied in Eq. (11), i.e.,

$$\frac{\partial K_M}{\partial t}=a_1+b_1+c_1+f+g+h+i-\text{"dissipation."} \quad (12)$$

Here, "dissipation" is actually a combination of terms a_2, b_2, c_2, d, e and j. As a result, its algebraic sign, as well as its magnitude, is presently uncertain. The approach here is to assess the values for the remaining terms as well as for $\partial K_M/\partial t$ so that some knowledge can be gained about the combined "eddy" and dissipation effects. The calculations are performed from 900 to 100 mb over an area $\pm 5°$ latitude perpendicular to the jet core. In order to gain further clarification of the processes involved, separate calculations are performed for the volumes 100–300 mb and 300–900 mb.

TABLE 1. Contribution of various terms in Eq. (11). All units are ergs cm^{-2} mb^{-1} sec^{-1}.

	900–100 mb layer	900–300 mb layer (below jet)	300–100 mb layer (above jet)
$\frac{\partial K_M}{\partial t}$	-5.1	-10.6	$+11.2$
$\int_{p_u}^{p_l} \frac{1}{A} \oint_B [c_n]_B \left\{ \frac{[u_J]^2{}_{(s)} + [v_J]^2{}_{(s)}}{2} \right\} dl \frac{dp}{g} = (\mathrm{a}_1)$	-0.8	$+\ 0.3$	$-\ 3.9$
$-\left[[\omega]_{(s)} \left\{ \frac{[u_J]^2{}_{(s)} + [v_J]^2{}_{(s)}}{2} \right\} \right]_{(n)} \Big\|_{p_l} = (\mathrm{b}_1)$	$+0.0$	$+\ 0.0$	$+\ 9.6$
$+\left[[\omega]_{(s)} \left\{ \frac{[u_J]^2{}_{(s)} + [v_J]^2{}_{(s)}}{2} \right\} \right]_{(n)} \Big\|_{p_u} = (\mathrm{c}_1)$	$+0.0$ ($\omega=0$ at 100 mb)	$-\ 3.2$	0.0 ($\omega=0$ at 100 mb)
$+\int_{p_u}^{p_l} \frac{\{[\Phi]_B - [\Phi]_{(s,n)}\}}{A} \oint_B c_n\, dl \frac{dp}{g} = (\mathrm{f})$	-1.8	$+\ 0.2$	$-\ 8.0$
$-\frac{1}{g}[([\omega]_{(s)})_{(n)}([\Phi]_{(s)})_{(n)}]_{(n)} \Big\|_{p_l} = (\mathrm{g})$	$+0.2$	$+\ 0.2$	$+14.3$
$+\frac{1}{g}[([\omega]_{(s)})_{(n)}([\Phi]_{(s)})_{(n)}]_{(n)} \Big\|_{p_u} = (\mathrm{h})$	$+0.0$ ($\omega=0$ at 100 mb)	$-\ 4.8$	0.0 ($\omega=0$ at 100 mb)
$-\int_{p_u}^{p_l} [([\omega]_{(s)})_{(n)}([\alpha]_{(s)})_{(n)}]_{(n)} \frac{dp}{g} = (\mathrm{i})$	$+1.9$	$+\ 3.0$	$-\ 1.3$
Total of right-hand side terms (a_1, b_1, c_1, f, g, h, i)	-0.5	$-\ 4.3$	$+10.7$
Implied "dissipation"	$+4.6$	$+\ 6.3$	-0.5

This roughly approximates the regions above and below the jet core.

The results of these calculations are given in Table 1. First, noting the results for the entire 100–900 mb volume, it may be seen that terms a_1, b_1 and c_1 in Eqs. (11) and (12) produce a small net loss of energy (-0.8 erg cm^{-2} mb^{-1} sec^{-1}) to the volume, due essentially to mean lateral outflow (see Fig. 5) occurring at larger mean kinetic energies than the mean inflow. It should be pointed out, however, that term a_1 is moderately sensitive to the previously stated assumption that the net calculated inflow can be partitioned equally on the anticyclonic and cyclonic side boundaries. The mean pressure interaction terms (f, g and h) also show a net loss for the 100–900 mb volume during this time period (-1.6 erg cm^{-2} mb^{-1} sec^{-1}). The conversion from potential to mean kinetic energy of the jet (term i) shows a net gain of $+1.9$ units for the entire volume. This conversion is especially significant since the local change term ($\partial K_M/\partial t$) represents a net loss of 5.1 units for the period. The net transverse circulation about the polar front jet is thus shown to be thermally direct even when the jet is rapidly becoming less intense. As a result, the mean transverse circulation calculated here may be less intense than in cases where the jet may be in approximate steady state or accelerating. The implied "dissipation" required for balance in this case is 4.6 ergs cm^{-2} mb^{-1} sec^{-1}.

A number of studies have been undertaken recently by Kung (1966a, b, 1967, 1968, 1969) on the generation and dissipation of kinetic energy in the atmosphere. It is interesting to note that his most recent estimate of the annual average energy dissipation over North America is 4.7 ergs cm^{-2} mb^{-1} sec^{-1} for the 100–900 mb layer. Although this value is virtually identical with the implied "dissipation" obtained here, there are a number of important differences. The transfer of kinetic energy, due to the unevaluated eddy terms in Eq. (11), is potentially important and could be of either algebraic sign. Also, Kung's dissipation values are for the total kinetic energy and not for the mean jet stream component, as calculated here. Furthermore, the computational uncertainty in the leading terms of Eq. (11) is too large to allow much confidence to be placed in the value of the "dissipation" obtained here.

If one partitions the calculations into balances for the regions below (300–900 mb) and above (100–300 mb) the jet, considerable additional information is gained. For the region below the jet, Table 1 shows a very pronounced decrease of mean kinetic energy with time ($\partial K_M/\partial t = -10.6$ ergs cm^{-2} mb^{-1} sec^{-1}). It is especially interesting that the conversion to mean kinetic energy (term i) is $+3.0$ units. There are significant losses through the boundary terms (c_1 and h) across the 300-mb surface. For this volume the implied "dissipation" is 6.3 units. As before, it is not presently possible to separate reliably the eddy terms from the true energy dissipation.

The region above 300 mb appears to possess entirely different characteristics. Table 1 shows that this layer is increasing its kinetic energy ($\partial K_M/\partial t=+11.2$ ergs cm^{-2} mb^{-1} sec^{-1}). A large portion of this increase is due to flux of mean kinetic energy (term b) into the region across the 300-mb surface (+9.6 units). Also, a very large flux of energy into the region is produced by the mean pressure interaction term at 300 mb (+14.3 units). Some of this energy is lost through the side boundaries (term $a_1=-3.9$ units and term $f=-8.0$ units). It is interesting to note that the conversion term (i) is -1.3 units, thus acting to build up potential energy at the expense of mean kinetic energy. This arises because of the reversal of the mean lateral temperature contrast above the jet core. This is also compatible with the contention of Riehl (1962) that this temperature reversal above the jet core is produced dynamically by ascent on the anticyclonic side and descent on the cyclonic side of the jet core. The implied "dissipation" for this layer is -0.5 unit. This value is too small to be considered reliable, even with respect to algebraic sign.

A truly steady-state condition would have been more favorable for interpretative purposes. It does seem clear, however, that the mean polar front jet stream is maintained to a large degree by a thermally-direct transverse circulation below the core, and that appreciable energy is transported upward across the tropopause indirectly as a result of this process.

These results carry some interesting implications. First, it is clear that the mechanisms for maintaining the jet stream at these altitudes are qualitatively different from the processes acting to maintain the zonal mean westerlies, as viewed in the traditional approach to general circulation problems. Therefore, it appears to be inconsistent to equate these two phenomena directly when discussing the mechanisms involved in their maintenance. The second implication is that the polar front jet stream provides an efficient mechanism for producing upward flux of energy across the tropopause due to the pressure interaction effect. In fact, the flux across the 300-mb surface due to term (g) in Eq. (11) and Table 1 is 2860 ergs cm^{-2} sec^{-1}. In the usual general circulation formulation of the energy equations, this would show up as a strong eddy effect at about wavenumber 4 or 5. The above is quite compatible with the vertical eddy pressure interaction term for the same general region obtained by Manabe and Hunt (1968) in an 18-level general circulation model. It is interesting to note that this large energy flux into the stratosphere occurs at the approximate location of the winter mid-latitude warm belt in the lower stratosphere. This region is known to be a strong sink of zonal and eddy available potential energy. Thus, the above process may be of some importance in determining the structure of this somewhat unusual region of the stratosphere.

5. Some additional considerations

In the previous section considerable attention was given to the balance of the "mean" kinetic energy along the polar front jet stream. One can also write a companion expression to Eq. (11) for the "eddy" kinetic energy balance along the jet axis.

In the present experimental setup, it has been impossible to calculate all terms in such an "eddy" kinetic energy balance because of the difficulty in obtaining "eddy" products. However, as a means of obtaining a preliminary estimate of the energetic contribution of "eddies" relative to the polar front jet stream, a calculation of the

$$-\int_{p_u}^{p_l} [(\omega)_{(s)}(\alpha)_{(s)}]_{(s,n)}\frac{dp}{g}$$

term in the "eddy" kinetic energy balance was performed. Physically, this represents a conversion from potential energy into "eddy" kinetic energy measured relative to the jet axis. This particular term is interesting because it helps to determine whether the "eddies," relative to the jet axis, have an internal energy source or have been produced by either boundary fluxes or conversions from "mean" kinetic energy. Fig. 5 shows the average of the above expression over the five observation times at each grid axis (in terms of energy units). The volume average of this conversion is +5.1 ergs cm^{-2} mb^{-1} sec^{-1}. This represents a rather strong energy source in the jet region and is probably large enough to balance the subgrid-scale dissipation of "eddy" kinetic energy without necessitating a large input from the various other possible mechanisms.

Fig. 5. shows that this energy conversion has an interesting distribution relative to the jet axis. A very strong source of "eddy" kinetic energy ($\sim$20 ergs cm^{-2} mb^{-1} sec^{-1}) exists in the middle troposphere on the cyclonic side of the jet axis. This source is apparently due to the asymmetric distribution of the cyclone-scale waves relative to the jet axis. On the anticyclonic side of the jet axis in the lower troposphere a significant (but weak) sink of energy appears. This strong variation across the jet axis is somewhat surprising. Its dynamical significance is not presently obvious, although it seemingly must play an important role in the redistribution of kinetic energy in the jet stream region. The effect seems to be significant because it appears in each of the five calculations at the individual observation times. As in any case study, however, companion calculations for other cases would yield valuable insight into the generality of these results.

As pointed out by Riehl (1962) and others, one of the principal difficulties with a thermally-direct transverse circulation about the jet axis is that such a circulation acts to rotate the isentropic surfaces into horizontal planes. Thus, the solenoidal field necessary to support a geostropic jet stream is weakened by the transverse circulation. Yet, in the first approximation, geostrophic balance holds for the jet stream. This implies a restriction on the intensity of the transverse circulation depending upon the intensity of the processes capable of counteracting the adiabatic cooling on the anticyclonic side and the adiabatic heating on the cyclonic side implied in Fig. 3. Some insight can be gained into the possible processes responsible by considering the formulation for the mean heat balance along the jet axis. By averaging the first law of thermodynamics along the jet axis and separating "mean" and "eddy" processes one, obtains

$$\frac{\partial[T]_{(s)}}{\partial t}=\overbrace{-[v_J]_{(s)}\frac{\partial[T]_{(s)}}{\partial n}}^{(a)}\overbrace{-\frac{\partial}{\partial n}[(v_J)_{(s)}(T)_s]_{(s)}}^{(b)}$$

$$\overbrace{-\frac{1}{s_2-s_1}\{u_J(T)_{(s)}|_{s2}-u_J(T)_{(s)}|_{s1}\}}^{(c)}$$

$$\overbrace{+\frac{[H]_{(s)}}{c_p}}^{(d)}\overbrace{+\left\{\frac{\kappa}{p}-\frac{\partial}{\partial p}\right\}[(\omega)_{(s)}(T)_{(s)}]_{(s)}}^{(e)}$$

$$\overbrace{+[\omega]_{(s)}\left\{\frac{[\alpha]_{(s)}}{c_p}-\frac{\partial[T]_{(s)}}{\partial p}\right\}}^{(f)}. \quad (13)$$

For this case study, Riordan (1971) has investigated processes which might counteract the anticyclonic side cooling and the cyclonic side heating that result from term (f) in Eq. (13). His results indicate that no single term in Eq. (13) is fully responsible for providing a balance to (f). Rather, it is a combination of terms (a) and (c), the perpendicular and parallel horizontal temperature advection. Terms (b), the convergence of "eddy" heat flux relative to the jet axis, and (d), the mean diabatic heating, also play a role.

Another very important feature of the structure of an atmospheric jet stream is that the potential vorticity $[P=-(\partial\theta/\partial p)(f+\zeta_\theta)]$ is almost discontinuous across the jet axis. As pointed out by Riehl (1962), such a potential vorticity distribution in the vicinity of the jet axis places a strong restriction on the type of dynamical process that can be responsible for maintenance of the jet stream. Realizing this, it may be instructive to consider whether or not the transverse circulation, alleged here to be responsible for maintenance of the polar front jet stream, is compatible with the observed strong gradients of potential vorticity across the jet axis. In this case, a potential vorticity equation considering non-conservative effects is appropriate, since the time required for a parcel to make a complete circuit about the jet axis (as obtained from Fig. 4) is the order of 40–50 days. As shown by Staley (1960), potential vorticity can only be modified by a curl of frictional force and gradients of diabatic heating. (It should be pointed out, however, that such a parcel would not, in general, make such a complete circuit about the axis because of "eddy" and subgrid-scale turbulence effects.)

A preliminary analysis was performed to estimate possible magnitudes of the various terms in the "mean" potential vorticity balance equation. The transverse circulation of Fig. 4 and reasonable assumptions on the friction and diabatic heating were employed to provide these estimates. The analysis showed that the non-conservative terms are important on the longer time scales, but are not directly responsible for producing the observed potential vorticity gradient across the jet axis. The results show that the probable source for this is that the jet core occurs at the tropopause level, where the static stability tends to be discontinuous in the vertical. As a result the transverse circulation, as seen in Figs. 3 and 4, leads naturally to such a potential vorticity distribution. On the cyclonic side of the jet axis, stable stratospheric air is being slowly brought downward while tropospheric air is being slowly lifted on the anticyclonic side. Because of the relatively small net cross-axis velocity component required to maintain the jet energetically, the mean advection normal to the jet core tends to be considerably smaller. This is particularly true at the jet core itself. In more quantitative terms, the $-[\omega]_{(s)}\partial[P]_{(s)}/\partial p$ term in a potential vorticity balance relative to the jet axis clearly dominates the $-[v_J]_{(s)}\partial[P]_{(s)}/\partial n$ term. Although the horizontal convergence of "eddy" potential vorticity flux cannot yet be evaluated directly, it is probable that this effect is not strong across the jet core itself, because streamlines at this level tend to be quite parallel to the axis of maximum wind.

Furthermore, from the case studies of discrete intrusions of stratospheric air into the troposphere mentioned in the Introduction, considerable downward vertical "eddy" flux of potential vorticity is to be expected on the cyclonic side of the polar front jet stream. In this case the resultant vertical convergence of "eddy" potential vorticity flux will also act to reinforce the observed near-discontinuity across the jet axis. The essential similarity of this process (which occurs only at particular regions and times along the jet axis) to the net transverse circulation has already been pointed out in Section 3. As a result, the conclusion that these are reinforcing effects is not surprising.

Thus, one may argue that the deduced net transverse circulation about the jet core is not contradicted by

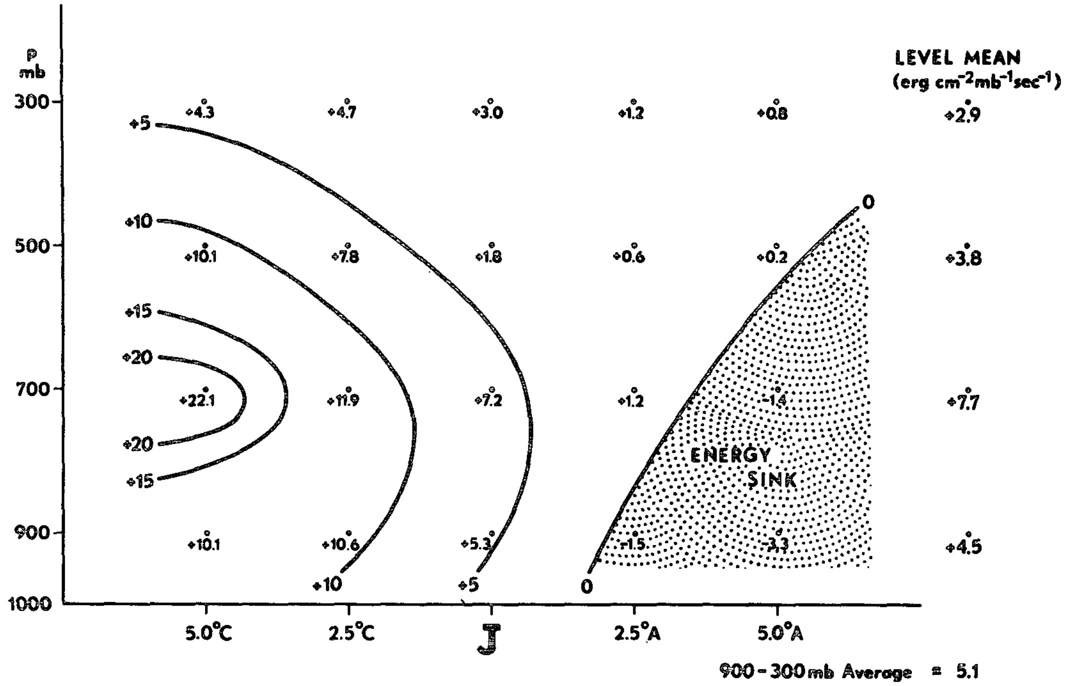

FIG. 5. Distribution of potential into "eddy" kinetic energy conversion relative to the jet axis averaged over the period 1200 GMT 15 November to 1200 GMT 17 November, 1966. Units are expressed in ergs cm^{-2} mb^{-1} sec^{-1}.

either heat balance requirements or potential vorticity considerations.

6. Summary and conclusions

Calculations have been made of the mean transverse circulation about the polar front jet stream, using a diagnostic balance, ω-equation method. The results show that such a circulation is thermally direct in the troposphere, with ascent on the anticyclonic side and through the jet core and descent on the cyclonic side.

A calculation of the "mean" kinetic energy balance for this jet stream shows that it is probably maintained by release of potential energy resulting from this direct circulation, and also that no appreciable amounts of energy are either imported into or exported horizontally out of the jet stream region. The results do show, however, the existence of a comparatively large upward flux of energy at the tropopause, resulting from the transverse circulation.

Preliminary analyses of the heat and potential vorticity budgets indicate that the mean transverse circulation obtained here is compatible with the observed temperature and potential vorticity distributions in the vicinity of the polar front jet stream.

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APPENDIX

List of Symbols

A	area enclosed by lateral boundaries
B	index denoting averaging or integration to be performed along lateral boundaries
c_n	velocity component normal to the lateral boundary (+ inward)
c_p	specific heat of air at constant pressure
f	Coriolis parameter
f_0	average f over region
F_s, F_n	frictional force in the s and n directions, respectively
g	acceleration of gravity
H	diabatic heating rate
H_L	diabatic heating rate due to release of latent heat
J	Jacobian operator

K_M mean kinetic energy measured with respect to the jet stream axis
l length along the boundary B
n coordinate direction oriented perpendicular to the jet stream axis
P potential vorticity
p_l, p_u pressure at lower and upper boundaries of integration volume, respectively
q, q_s specific humidity and saturation specific humidity, respectively
R gas constant for dry air
Ro Rossby number
s coordinate direction oriented along the jet stream axis
T temperature
u_g, v_g zonal and meridional components of geostrophic wind, respectively
u_J, v_J wind components along and normal to the jet stream axis, respectively
V_s wind speed
α specific volume
β Rossby parameter
ζ, ζ_θ relative vorticity on p and θ surfaces, respectively
ζ_a $f+\zeta$
θ potential temperature
κ R/c_p
σ a static stability parameter $[=-(\alpha/\theta)(\partial\theta/\partial p)]$
τ_x, τ_y frictional stress in the x and y directions, respectively
Φ geopotential
χ velocity potential
ψ streamfunction for horizontal wind
Ψ streamfunction for mean transverse circulation about the jet stream axis
ω dp/dt

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