Equatorial Waves in the Presence of the Equatorial Undercurrent

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ABSTRACT

Because of the narrow region over which it has high speeds, the Equatorial Undercurrent has little effect on long waves with large phase speeds, such as long Kelvin and equatorially trapped inertia-gravity modes with equivalent depths ≳30 cm. The Rossby branch of the Rossby-gravity family, and the gravest Rossby modes, have phase velocities comparable to the maximum speed of the Undercurrent and are significantly modified by this current. Meanders of the Undercurrent that are due to superimposed neutral (non-amplifying) waves must have westward phase propagation; standing or eastward traveling meanders are possible only if the Equatorial Undercurrent is unstable.

1. Introduction

In the proposed explanations for a number of physical phenomena, the role of the equator as a waveguide is of central importance. This is true, for example, of Lighthill's (1969) explanation for the generation of the Somali Current, and of the explanation of McCreary (1976) and of Hurlburt *et al.* (1976) for the initiation of El Niño events off the Peruvian coast. In these various models the existence of mean equatorial currents is neglected. The most intense of these currents is the Equatorial Undercurrent. Since this current can attain speeds comparable to the phase speeds of equatorially trapped waves, it is of importance to know to what extent the Undercurrent can modify these waves. The observations of equatorially trapped waves in the Pacific and Atlantic Oceans (Wunsch and Gill 1975; Weisberg *et al.*, 1978a,b) provide a further motivation for a study that addresses this question. Of the measurements in the Atlantic, which were made below the core of the Undercurrent in a region of relatively weak mean currents, one can ask how the observed waves were modified when they propagated downward through the Undercurrent. In the Pacific Ocean there is less information concerning equatorially trapped waves but the available data are in reasonable agreement with linear theory. Does this imply that the effect of the Undercurrent on the observed inertia-gravity waves is small?

The differential equations that describe the effect of mean currents with latitudinal and vertical shear on equatorial waves are, in general, nonseparable and hence difficult to solve. Considerable progress has been made in the meteorologically relevant case where the scale of the vertical shear of the mean flow far exceeds the vertical wavelength of the waves. [See Holton (1975) and Boyd (1979) for reviews of the subject.] This is not the case that is of most interest to oceanographers. The Equatorial Undercurrent is confined to a shallow surface layer (~150 m deep) in an ocean with a total depth of ~4 km. The two cases of most oceanographic interest therefore correspond to waves with vertical scales comparable to that of the Undercurrent, and waves with vertical scales larger than that of the Undercurrent. For example, the first baroclinic mode has a node at a depth of ~1500 m in the tropics and therefore has a vertical scale greater than that of the Undercurrent. The second baroclinic mode, on the other hand, has a large amplitude primarily in the upper 150 m because of internal reflection in the strong shallow tropical thermocline. Its scale, therefore, is comparable to that of the Undercurrent.

The simplest problem that can be posed concerns the effect of the Equatorial Undercurrent on a wave with the same vertical structure as the current. If we assume that the Undercurrent has no vertical shear and that it is in geostrophic balance in a homogeneous layer of depth H (below which there is a motionless, infinitely deep layer of slightly higher density), then the equations for linearized wave perturbations which are also confined to the layer of depth H are

$$-i(\sigma - Uk)u + vU_y - \beta yv = -ikg'\zeta, \quad (1a)$$

$$-i(\sigma - Uk)v + \beta yu = -g'\zeta_y, \quad (1b)$$

$$-i(\sigma - Uk)\zeta + ikHu + (Hv)_y = 0. \quad (1c)$$

The mean current U and mean layer-depth H depend on latitude y only and are related by the geo-

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strophic equation. Zonal (u) and meridional (v) velocity perturbations and layer-depth perturbations (ζ) are assumed to have time (t) and longitude (x) dependence of the form $e^{i(kx-\sigma t)}$. Reduced gravity is denoted by g'. For a given mean flow $U(y)$ there are two external parameters that determine the character of the perturbed flow, *viz*., β and $g'H_0$ where H_0 is the mean depth of the layer. The second parameter is usually written gh where h is the equivalent depth. The idealized model described by Eqs. (1) could be considered reasonable for the study of modifications to second baroclinic mode waves by the Equatorial Undercurrent, provided h has a value appropriate for a second baroclinic mode (25 cm). The solutions to (1) will also give insight into the effect of the Undercurrent on waves with longer vertical scales, such as the first baroclinic mode for which $h \approx 60$ cm. This can be explained by first pointing out that Eqs. (1) are satisfied by waves in the presence of a mean current $U(y)$ that depends on latitude only, in a fluid with mean stratification $N(z)$ that depends on depth z only. In such a situation the velocity components are pressure fields are separable and can be written as $\mu(z)u(x,y,t)$, $\nu(z)v(x,y,t)$ and $\pi(z)\zeta(x,y,t)$, respectively. The vertical structure of the flow is described by

$$\left(\frac{1}{N^2}\pi_z\right)_z + \frac{1}{gh}\pi = 0, \tag{2}$$

where h is the constant of separation. The horizontal structure is described by (1) provided $g'H$ is replaced by the constant gh. It is evident from (2) that we are dealing with vertically propagating waves and that h is a measure of the vertical wavelength of the perturbations. Hence the slightly modified Eqs. (1) describe the horizontal structure of waves that propagate vertically through a mean current with no vertical shear, in a fluid whose stratification depends on depth only.

This case at first appears uninteresting because we are primarily interested in waves with vertical scales that exceed that of the Undercurrent. However, we consider a two-layer system in which the shallow upper layer has stratification $N_1(z)$ and the lower deep layer has stratification $N_2(z)$. (Both N_1 and N_2 depend on depth only.) In the upper layer there is a mean current $U(y)$ that depends on latitude only. In the lower layer we could permit a different mean zonal flow but we shall assume that there is no mean motion in the deep layer. The interface between the two layers slopes in accordance with geostrophic balance and since U is discontinuous there, the density field must be discontinuous at the interface. In each of the two layers the dependent variables are separable and the flow satisfies Eqs. (1) and (2) as discussed above. The major difficulty now is in matching the solutions at the interface where all the vertical shear is concentrated. Before discussing this problem, we briefly review perturbations when there are no mean currents.

In the absence of a mean flow ($U \equiv 0$), Eqs. (1) and (2) describe linear waves on an equatorial β plane. For given values of σ and k, Eqs. (1) constitute an eigenvalue problem; h is the eigenvalue. The solution to this problem is well known (and has most recently been discussed by Philander, 1978b). If the solutions are required to be bounded at large distances from the equator then the eigenvalues can be determined from the dispersion relation

$$\frac{\sigma^2}{gh_l} - k^2 - \frac{\beta k}{\sigma} - \frac{\beta}{(gh_l)^{1/2}}(2l+1) = 0,$$
$$l = 0, 1, 2. \tag{3}$$

The associated eigenfunctions can be expressed in terms of Hermite functions. Their structure is such that the integer l can be identified with a discrete meridional wavenumber. These latitudinal modes are functions of the variable η, where

$$\eta = \left(\frac{\beta^2}{gh_l}\right)^{1/4} y. \tag{4}$$

It follows that h determines the latitudinal scale of the modes. As pointed out earlier h also determines the vertical scale of the oscillations. For large values of h the waves are of considerable latitudinal and vertical extent; for small values of h the waves are strongly trapped about the equator and have short vertical wavelengths.

Next, we include a mean flow in the upper layer. It will cause the dispersion relation in that layer to differ from (3). For given values of σ and k, h_l will have values different from those implied by (3). The structure of the associated modes will also be different. Consider one of these latitudinal modes propagating downward toward the interface between the two layers. Because the interface has a latitudinal slope, the boundary conditions there are complicated. However, we do know that the values for σ and k must be the same in the two layers. The pressure must also be continuous across the interface. The structures of the eigenfunctions that describe the pressure in the two layers are different. Hence a single mode in the upper layer will map onto a number of modes, each associated with a different value of h and l, in the lower layer. It follows that a single mode that propagates vertically toward the interface will excite a large number of latitudinal modes (each with a different vertical wavelength and a different speed of vertical propagation) in the other layer. A measure of these effects can be obtained by determining the extent to which the dispersion relation and the structure

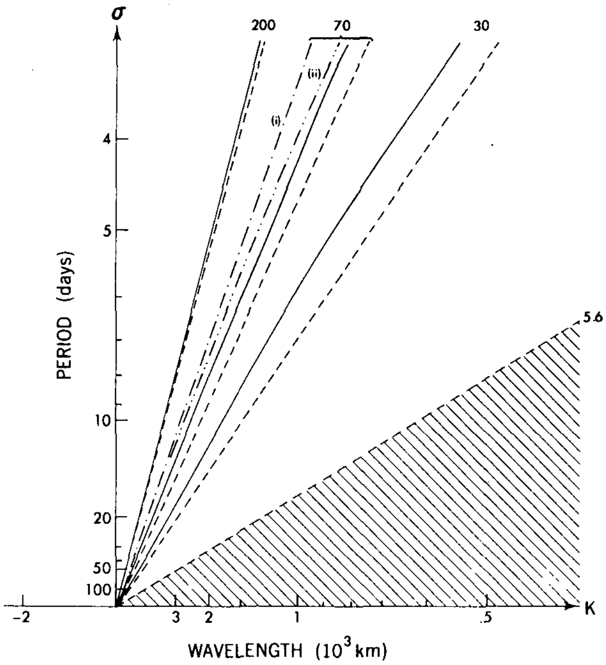

FIG. 1. The dispersion curves for Kelvin waves in the absence of mean currents (dashed lines) and in the presence of an Equatorial Undercurrent with maximum speed 75 cm s^{-1} (solid lines) for different values of h (cm). The curve marked (ii) shows the effect of an Undercurrent with a maximum speed of 125 cm s^{-1}; the curve marked (i) shows the effect of a current without shear and with a speed of 75 cm s^{-1}.

of the latitudinal modes are modified by the mean current in the upper layer.

In this paper we describe how an Equatorial Undercurrent of the form

$$U = U_0 \operatorname{sech}^2(y/120 \text{ km}), \tag{5}$$

(where y measures distance from the equator) modifies the dispersion relation and modal structure of the gravest equatorially trapped waves. Solutions to Eqs. (1) are found numerically by using a method described in a paper by Philander (1976). {The equatorial β plane is bounded by walls along circles of latitude four units of η [see Eq. (4)] from the equator. For numerical purposes this canal is divided into 300 equal intervals.} The solutions that correspond to an equivalent depth h of about 25 cm will give reasonably accurate information about the modifications to the second baroclinic mode. This is so because this mode has a large amplitude in the upper 150 m only; its vertical structure approximately coincides with that of the Undercurrent. We shall also describe solutions for values of $h \neq 25$ cm. These solutions will be used to make qualitative inferences about the effect of the vertical shear of the mean flow on the waves; we shall not attempt to solve Eq. (2) and to match solutions across the interface.

In this study we only consider mean currents described by Eq. (5). For a discussion of the effect of different mean currents on equatorial waves the reader is referred to the valuable studies by Hallock (1977) and McPhaden and Knox (1979).

2. Kelvin waves

In the absence of mean currents Kelvin waves are eastward propagating and nondispersive

$$\sigma = k(gh)^{1/2}. \tag{6}$$

There are no meridional velocity fluctuations associated with this wave and the latitudinal structure of the pressure and zonal velocity component are Gaussians, i.e.,

$$u = \exp[-\beta y^2/2(gh)^{1/2}]. \tag{7}$$

The dashed lines in Fig. 1 correspond to Eq. (6) for the indicated values of h (cm). The solid lines show how the dispersion relation is altered by the presence of an Equatorial Undercurrent with a maximum velocity of 75 cm s^{-1} [= U_0 in Eq. (5)]. It is evident that for large equivalent depths the effect of the Undercurrent is negligible. There are two reasons for this: for large values of h the Undercurrent speed is small compared to that of Kelvin waves which propagate with speed $(gh)^{1/2}$; the latitudinal scale of the Undercurrent is negligible compared to that of the Kelvin waves when h is large as is evident from (7). For equivalent depths ≤ 5.6, the Kelvin waves have phase speeds less than the maximum speed of the Undercurrent. The latitudinal mode associated with a Kelvin wave will therefore not be established because a critical layer exists. Hence, in the presence of an Undercurrent with a maximum speed of 75 cm s^{-1}, no Kelvin waves exist in the shaded part of Fig. 1.

For equivalent depths between about 200 and 6 cm the presence of the Undercurrent causes higher (Doppler-shifted) values for the frequency if the values of k and h are fixed. (Alternately, for fixed σ and k the equivalent depth is decreased by the Undercurrent.) The effect is largest for high frequencies and short zonal wavelengths. An increase in the maximum speed of the Undercurrent increases the shaded area of Fig. 1 but is shown to have little effect on the h = 70 cm line. [The line marked (ii) in Fig. 1 corresponds to U_0 = 125 cm s^{-1}]. An increase in the width of the Undercurrent has a much larger effect on Kelvin waves. [The dotted line marked (i) corresponds to a mean flow of 75 cm s^{-1} with no latitudinal shear and hence infinite width.]

Fig. 2 shows two examples of the eigenfunctions and, superimposed, the structure of the pressure field, in the absence of a mean flow. It is evident that for an Undercurrent with a maximum speed of 75 cm s^{-1}, waves with a wavelength of 2700 km

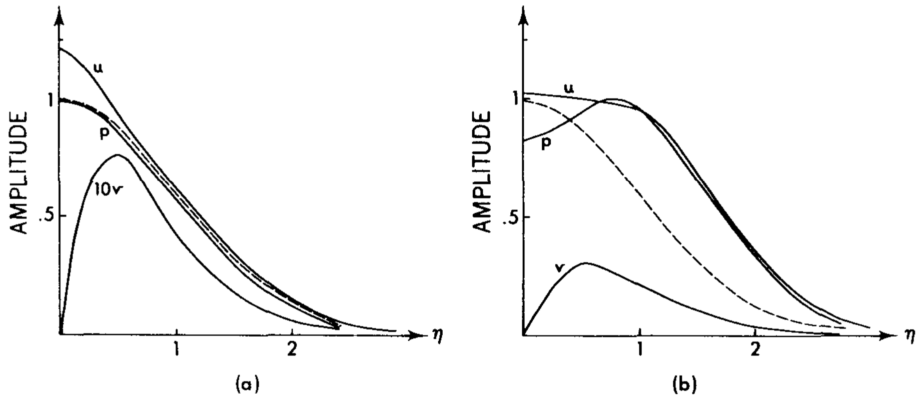

FIG. 2. Eigenfunctions corresponding to modified Kelvin waves for the cases: (a) $\alpha = 2\pi/(11 \text{ days})^{-1}$, $k = 2\pi/(2700 \text{ km})^{-1}$, $h = 70$ cm; (b) $\sigma = 2\pi/(3.3 \text{ days})^{-1}$, $k = 2\pi/(800 \text{ km})^{-1}$ $h = 70$ cm. The dashed lines show the pressure fields for the same frequencies and wavenumbers, in the absence of mean currents. The corresponding values for h are 85.3 and 83.2 cm, respectively. One unit of η corresponds to 360 km [see eq. (4)].

and a period of 11 days—the equivalent depth is 70 cm—are only slightly affected by the mean flow. Shorter waves are strongly modified, as is shown in Fig. 2 for the case of a 800 km wave. Note the significant meridional velocity component induced by the Undercurrent in the case of the short 800 km wave with a period of 3.3 days.

We now consider a Kelvin wave excited in the upper layers of the ocean by an eastward moving atmospheric disturbance. For given values of the frequency σ and wavenumber k, the equivalent depth can be determined from a dispersion diagram such as Fig. 1. In the deep ocean, a downward propagating Kelvin wave with the same value of σ and k but a larger value of h (and hence a longer vertical wavelength) will be excited. Since the structure of the Kelvin wave in the surface layers does not map exactly onto that of the Kelvin wave in the deep ocean, additional latitudinal modes are involved. At high frequencies these additional modes, which play an important role because of the large difference in the structures of the Kelvin waves, are primarily inertia-gravity modes. They all have the same values for σ and k but they have increasingly smaller values of h (see Philander 1978b). Small values of h imply large vertical and latitudinal shears and hence vulnerability to dissipation. It follows that a single, high-frequency Kelvin wave in the surface layers of the ocean will give rise to a Kelvin wave plus a set of inertia-gravity modes in the deep ocean. A considerable energy loss may be associated with dissipation of the higher order inertia-gravity waves.

At low frequencies the structures of the Kelvin waves in the two layers do not differ much. What little difference there is will not map onto inertia-gravity waves because the latitudinal scale of the low-frequency inertia-gravity waves is much smaller than that of the Kelvin waves. (The inertial waves decay exponentially poleward of their inertial latitude.) The difference between the structures of the Kelvin waves must therefore map onto modes for which h is negative (Philander, 1978b). In the vertical such modes decay exponentially. Hence the residual energy, which does not go into the excitation of a Kelvin wave in the deep ocean, remains trapped in the surface layers. This could result in a varicose mode of oscillation for the Undercurrent.

In the studies mentioned in the Introduction, the Kelvin wave is a nondispersive event that propagates along the equator in the first baroclinic mode. Since a vertically standing mode is a superposition of upward and downward propagating waves, the discussion above gives some indication as to how this mode is affected by the Equatorial Undercurrent. The equivalent depth for the first baroclinic mode is ~70 cm. The high-frequency part of a Kelvin-wavelike disturbance (with periods less than a week) will be seriously modified by the Undercurrent, and could be dissipated rapidly when energy is scattered into short inertia-gravity waves. For periods equal to or longer than a week the Undercurrent has little effect on Kelvin waves in the first baroclinic mode. (This is true for the first baroclinic mode only, not for higher baroclinic modes with smaller equivalent depths.) There are two principal reasons why the Undercurrent has little effect on long-period Kelvin waves: the waves have a high speed relative to the Undercurrent and the width of the region over which the Undercurrent has high speeds is small compared to the width of the Kelvin waves.

3. Rossby-gravity waves

The gravest antisymmetric equatorially trapped mode is the Rossby-gravity wave. We discuss separately those disturbances which have westward phase propagation (and which are similar to Rossby waves) and those disturbances which have eastward phase propagation (and which are similar to inertia-gravity waves).

a. *Westward propagating waves*

In the absence of mean currents these waves satisfy the dispersion relation

$$2\sigma/k = -(gh)^{1/2} - [gh + 4\beta(gh)^{1/2}/k^2]^{1/2}$$

which is the curve marked (iv) in Fig. 3 (when $h = 70$ cm). A mean flow without horizontal shear changes curve (iv) into (i) which, at large values of σ and k asymtotes to the line $\sigma = U_0k$ ($U_0 = 75$ cm s^{-1} for the case shown). Sufficiently short waves are seen to have eastward phase propagation, and standing waves with a wavelength of ~1500 km are also possible. Since the waves are antisymmetrical about the equator their superposition on the mean flow will result in a meandering current. If these meanders have an eastward phase speed and a low frequency, then they could transport energy eastward very efficiently. The reason for this is that in the deep ocean there are no low-frequency, eastward propagating waves that are antisymmetric about the equator (except inertia-gravity waves which are unimportant at low frequencies). Hence the meandering current in the surface layers cannot lose energy by radiating waves into the deep ocean. Consider a disturbance that causes the Undercurrent to meander in the neighbourhood of its origin in the western equatorial Pacific Ocean say. Could the Undercurrent be an efficient wave-guide for transferring energy associated with this disturbance to the eastern side of the ocean basin?

The remarks concerning meanders thus far pertain to a mean flow without latitudinal shear. The possible eastward traveling meanders described above have phase speeds less than the speed of the mean flow. Hence, if the mean flow has shear, so that its speed varies from zero to a maximum value, then the eastward traveling waves will encounter critical layers where $c = U$. (The phase speed is c.) The presence of shear therefore appears to eliminate eastward traveling meanders. Curve (iii) in Fig. 3, which shows the manner in which (i) is modified when the shear of the Undercurrent is taken into account, demonstrates that this is indeed the case. [The Undercurrent is again described by the expression in (5) with $U_0 = 75$ cm s^{-1}.] Neither eastward traveling waves nor standing waves are now possible. The principal reason for this is the narrowness of the Undercurrent. If we consider waves with an extremely small equivalent depth then they will be so strongly equatorially trapped that, to them, the Undercurrent will appear to have no shear. For this to happen the value of h must be much less than 1 mm. The vertical and latitu-

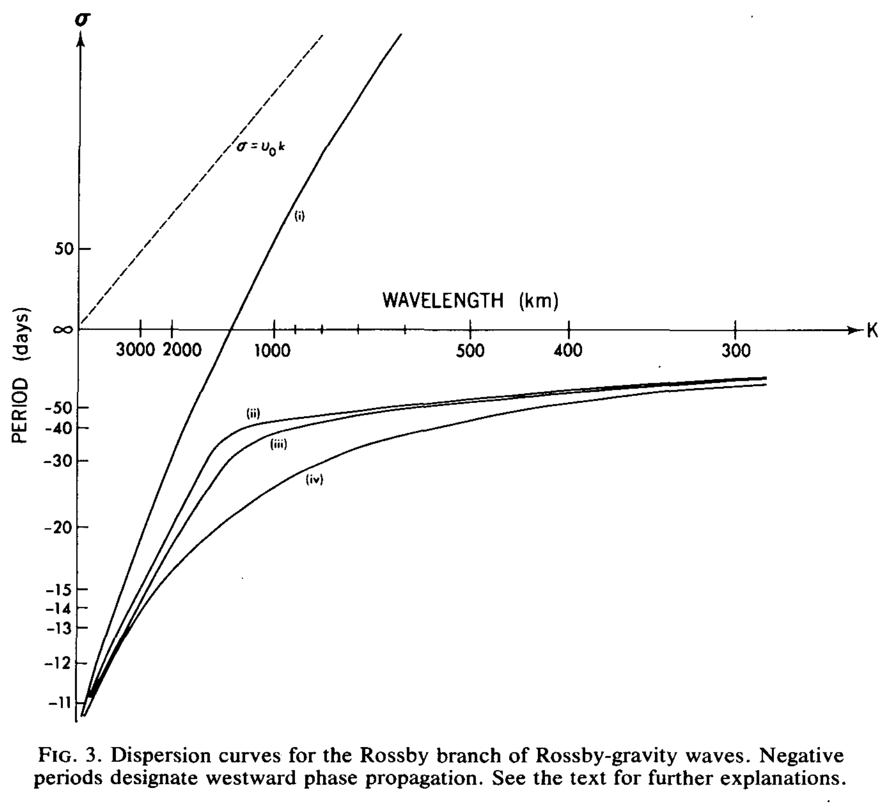

FIG. 3. Dispersion curves for the Rossby branch of Rossby-gravity waves. Negative periods designate westward phase propagation. See the text for further explanations.

dinal shear of waves with such a small value of h is so large that the waves are almost certainly unstable. For waves with reasonable values of h (a few centimeters or more) the Undercurrent appears narrow; the waves are "aware" that the Undercurrent speed varies from 0 to 75 cm s^{-1}. Hence eastward propagating meanders are eliminated (because of critical layers) and only meanders with westward phase propagation are possible. Such meanders can excite Rossby waves in the deep ocean so that the energy of the meanders does not remain trapped in the surface layers. The Equatorial Undercurrent is not an efficient waveguide for transferring energy zonally.

The presence of the Undercurrent modifies the Rossby-gravity waves considerably. It is evident from Fig. 4 that for a frequency and wavenumber for which h = 200 cm in the surface layers, h could be as small as 10 cm in the deep ocean. Since h is a measure of the latitudinal scale of the eigenfunctions [see Eq. (4)], there is a considerable mismatch between the eigenfunctions in the surface layers and those in the deep ocean. This implies that a downward propagating Rossby-gravity wave will give rise to a Rossby-gravity wave plus several Rossby modes in the deep ocean. As an example consider a Rossby-gravity wave with a wavelength of 1000 km and a period of 30 days. Satellite photographs of the sea surface temperature in the eastern equatorial Pacific show such disturbances (Legeckis, 1977) which Philander (1978a) has attributed to an instability of the surface currents. Below the surface currents, in the region of the Undercurrent, these disturbances will excite a downward propagating modified Rossby-gravity wave with an equivalent depth of ~200 cm (see Fig. 4). (Other modes will also be excited but this is probably the most important one.) Below the Undercurrent, where we assume that there is no mean flow, this modified Rossby-gravity wave will excite a Rossby-gravity wave (for which h is ~3 cm) plus the gravest equatorially trapped Rossby modes that are antisymmetrical about the equator. The equivalent depth for the gravest of these Rossby modes can be calculated from eq. (3) and is greater than 1000 cm. (The higher order modes have even larger values of h.) This value is so large and the implied turning latitude is so high, that the latitudinal mode is unlikely to be established. Hence the only equatorially trapped wave to be excited in the deep ocean is the Rossby-gravity mode. Such a wave probably accounts for the greater part of the 30 day, 1000 km signal observed near the ocean floor just west of the Galapagos Islands by Harvey and Patzert (1976). There appears to be a similar phenomenon in the Atlantic Ocean: in the surface layers there is a westward propagating undulation with a wavelength of ~1000 km and a period of a month (Brown, 1978); in the deep ocean Weisberg *et al.* (1978) observed a Rossby-gravity wave with the same scales.

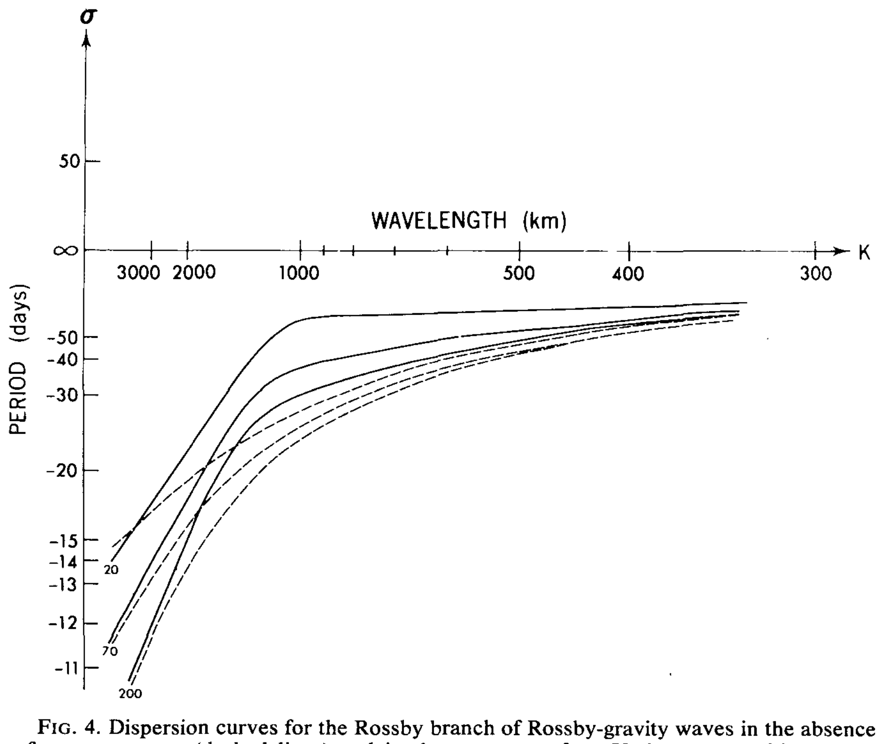

FIG. 4. Dispersion curves for the Rossby branch of Rossby-gravity waves in the absence of mean currents (dashed lines) and in the presence of an Undercurrent with a maximum speed of 75 cm s^{-1} (solid lines) for indicated values of h (cm).

b. Eastward propagating waves

The eastward propagating Rossby-gravity waves are basically inertia-gravity waves. Fig. 5 shows that these waves, particularly those with large values of h and long wavelengths, are affected relatively little by the presence of the Undercurrent. (If h is too small, the waves of course have critical layers. This happens to waves in the shaded area of Fig. 5.) The reason why inertia-gravity waves are not much affected by the mean current is their high phase speed relative to the maximum speed of the Undercurrent.

4. The gravest Rossby mode

Fig. 6 shows the dispersion curve for the gravest Rossby mode in the absence of mean currents [curve (iv)], in the presence of a mean current without shear [curve (i)], and in the presence of an Undercurrent with a maximum speed of 75 cm s^{-1} [curve (iii)] and 125 cm s^{-1} [curve (ii)]. For all these curves the value of h is 70 cm which is approximately the value for the first baroclinic mode. As in the case of the Rossby branch of the Rossby-gravity family, only perturbations with westward phase speeds are possible if the shear of the Undercurrent is taken into account. The modification of the Rossby waves by the Undercurrent is considerable. Fig. 7 shows examples of eigenfunctions. In the case of the long non-dispersive wave with a period of 83 days and wavelength of 4300 km (point A), the equivalent depth in the presence of the Undercurrent is 70 cm but in the absence of this current it is only 39 cm. This difference is reflected in the difference between the pressure functions in Fig. 7. In the case of the relatively short wave with a period of 41 days and a wavelength of 1022 km (point B) the differences between the eigenfunctions with and without an Undercurrent present, and the difference between the values for the equivalent depths, are again substantial. The reason for this large modification by the Undercurrent is the comparable speeds of the Rossby waves and the Undercurrent. (The first baroclinic mode Rossby waves have a speed of at most 80 cm s^{-1}. The corresponding speed is at least 250 cm s^{-1} for the Kelvin and inertia-gravity waves which were found not to be strongly influenced by the Undercurrent.)

Because the structures of the eigenfunctions in the two layers are mismatched, a vertically propagating mode in one layer will excite its counterpart in the other layer, plus higher order Rossby modes. The higher order modes, however, have increasingly larger values for the equivalent depth h. Large values of h imply turning latitudes that are distant from the equator, and hence imply an unlikelihood of the mode actually being established. In effect, the excitation of high-order Rossby modes will result in a loss of energy to non-equatorial latitudes. The projection of the gravest Rossby wave in one layer onto high-order modes in the other layer is likely to be small, however, because their struc-

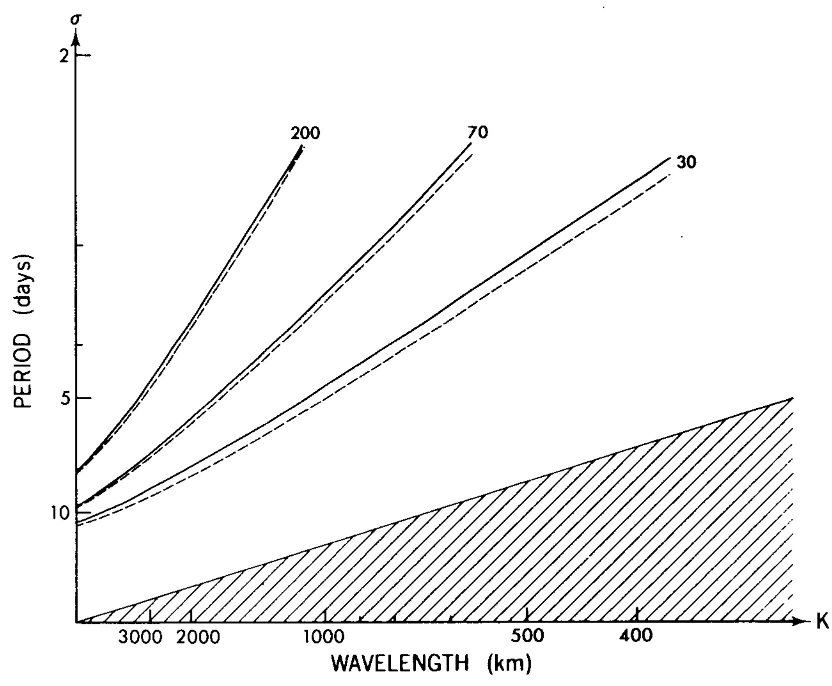

FIG. 5. Dispersion curves for the inertia-gravity branch of Rossby-gravity waves. Solid lines apply when the Undercurrent is present, dashed lines apply when there are no mean currents. Values for equivalent depths are in centimeters.

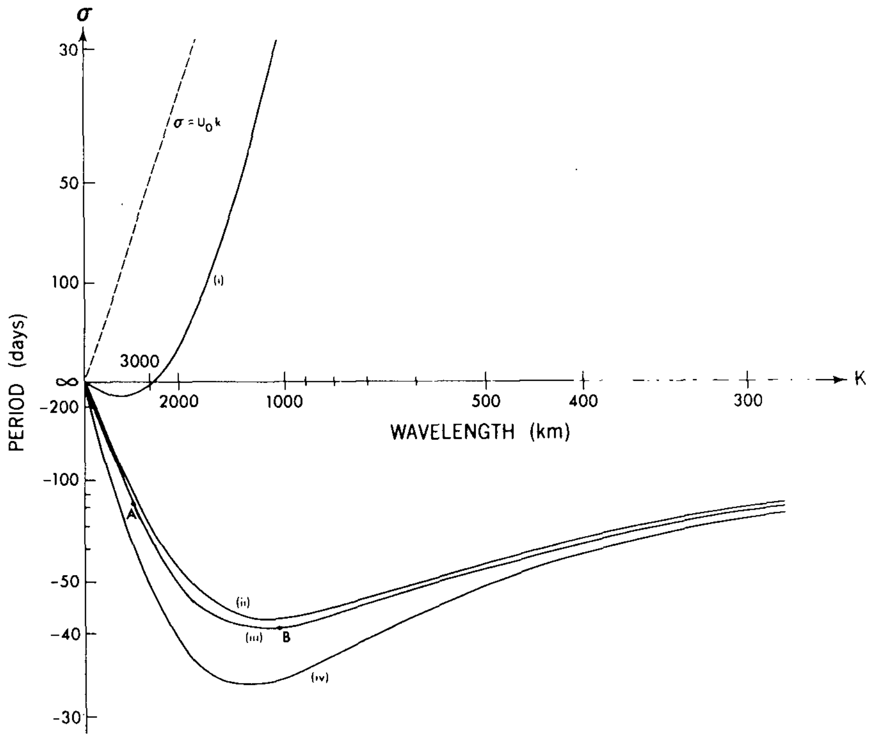

FIG. 6. Dispersion curves for the gravest Rossby mode [l = 1 in Eq. (13)] which is symmetric about the equator. Negative periods imply westward phase propagation. See the text for further information.

tures do not match. Of greater importance will be modes for which $h < 0$ so that interaction between the two layers will be limited. It is difficult to assess how the Undercurrent will modify first-baroclinic-mode equatorially trapped Rossby waves that emanate from the eastern coast of the Pacific (say). But it is evident that the waves will be slowed down, that their latitudinal width will be increased and that they will lose some energy to non-equatorial latitudes.

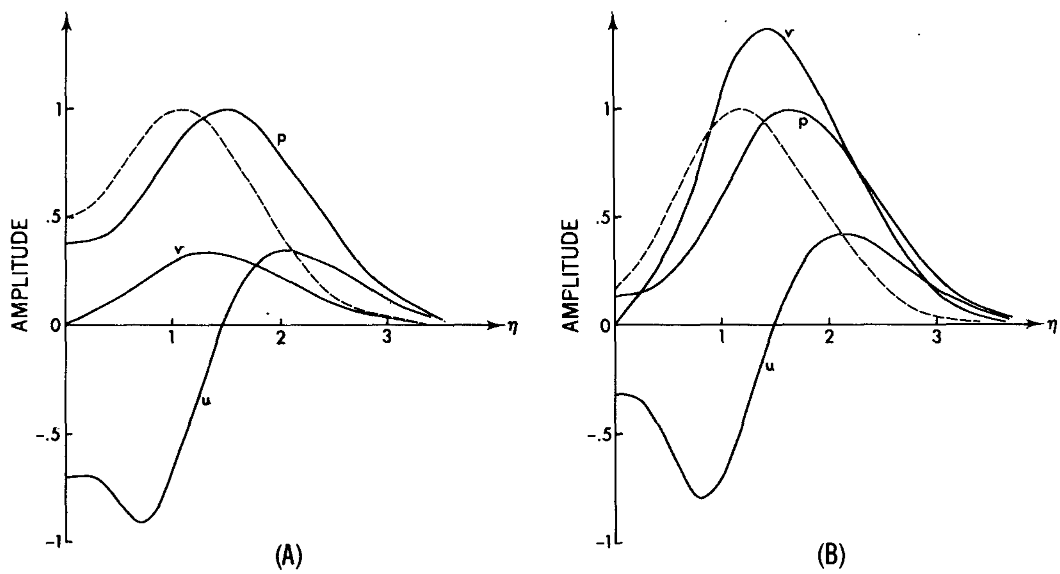

FIG. 7. Eigenfunctions corresponding to points A and B in Fig. 6. The dashed lines show the structure of the pressure functions for the same frequencies and wavenumbers but in the absence of the Undercurrent in which case h = 39 cm (case A) and h = 42 cm (case B). One unit of η is equal to 360 km [See Eq. (4)].

5. Summary

The Equatorial Undercurrent has little effect on long Kelvin and inertia-gravity waves with phase speeds considerably in excess of the maximum speed of the Undercurrent (or with equivalent depths $\gtrsim$30 cm). The inertia-gravity waves observed by Wunsch and Gill (1975) satisfy these conditions. Short Kelvin waves with periods less than a week are significantly modified by the Undercurrent: for given values of the frequency and zonal wave-number the equivalent depth in the presence of the Undercurrent is appreciably smaller than in the absence of this current. (The equivalent depth is a measure of the vertical and latitudinal scale of the waves.) This implies, as explained in Section 2, that first-baroclinic-mode Kelvin wave events (such as those said to play a role in El Niño phenomena) are likely to lose the energy associated with the high-frequency part of their spectrum, but will otherwise be unmodified.

The Rossby branch of the Rossby-gravity modes, and the gravest equatorially trapped Rossby waves, are more strongly influenced by the presence of the Undercurrent because their phase speeds are comparable to the maximum speed of the Undercurrent. It can be inferred that one of these waves propagating downward through the Undercurrent could, in the region below the Undercurrent, excite a number of Rossby modes (each with a different latitudinal structure). The Undercurrent will also decrease the speed, and alter the vertical and latitudinal structure, of the first baroclinic mode appreciably, in addition to causing it to lose a small amount of energy to non-equatorial latitudes.

Meanders of the Undercurrent that can be viewed as a superposition of propagating neutral (non-amplifying) waves on the current, can have westward phase propagation only. Because of the latitudinal shear of the current, waves that could cause eastward propagating meanders encounter critical layers. The meanders described by White (1973) and Monin (1972) are impossible; their analyses disregard the shear of the Undercurrent.

This paper describes in detail the manner in which the horizontal shear of the Equatorial Undercurrent modifies the dispersion relation and latitudinal structure of the eigenfunctions associated with the gravest equatorially trapped waves. Some inferences are made concerning the effect of the vertical shear on vertically propagating waves and vertically standing modes but these results are very qualitative. Quantitative results will require a much more ambitious study since the equations are nonseparable. In the meantime, it is hoped that these results will facilitate the interpretation of data from multilevel numerical models of the tropical oceanic circulation.

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