Instabilities of Zonal Equatorial Currents, 2

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A stability analysis of a realistic meridional profile of the surface currents in the equatorial Atlantic and Pacific oceans reveals that the most unstable waves are westward propagating and have a period of about 1 month, a wavelength of approximately 1100 km, and an *e* folding time near 2 weeks. This is proposed as an explanation for waves with these scales that have recently been observed in the Atlantic and Pacific oceans.

1. INTRODUCTION

An earlier study of the stability of equatorial currents (*Philander* [1976], hereafter referred to as I) revealed that the surface currents in the equatorial Pacific and Atlantic oceans may be inertially (barotropically) unstable. The latitudinal shear of the currents analyzed in I only crudely resembles the shear of the observed currents. Figure 1 shows a more realistic profile based on measurements by *Bubnov et al.* [1978] and on as yet unpublished measurements made during the Garp Atlantic Tropical Experiment (Gate). It is significantly different from the profile of the currents analyzed in I: the current structure is not symmetrical about the equator but is characterized by a single region of large latitudinal shear between the core of the South Equatorial Current (SEC) and the maximum of the North Equatorial Countercurrent (NECC). The maximum speed of the SEC had a mean value of about 60 cm/s during Gate. The mean currents, however, are not appropriate for a stability analysis because these currents have already been stabilized by the unstable waves. This note will therefore present the results of a stability analysis of the profile shown in Figure 1 for a range of values for the velocity amplitude U_0 and a range of values for the width of the shear zone. The range of parameters to be studied will cover the cases relevant to the currents in the Atlantic and Pacific oceans.

The results to be described here differ significantly from the results presented in I. In I it was stated that instabilities of the surface currents will give rise to waves with periods of between 2 and 3 weeks and wavelengths slightly in excess of 2000 km. An analysis of the more realistic profile shows that waves with these scales are essentially stable. Unstable waves with the shortest *e* folding time have a period of about 1 month and a wavelength of approximately 1100 km.

2. RESULTS

The model used for the stability analysis of the profile in Figure 1 is described in detail in I. The equations of motion are linearized about the mean flow which is confined to the upper layer of a two-layer system which has an infinitely deep, motionless lower layer. The flow is in geostrophic balance on an equatorial β plane so that the depth of the interface varies with latitude. Perturbations superimposed on the mean flow are assumed to have time t and zonal x dependence of the form $\exp i(kx + \sigma t)$. For given zonal wave numbers k the complex frequency σ is then an eigenvalue of a second-order differential equation in y (latitude). This equation is solved numerically; the profile shown in Figure 1 is described by 300 discrete values. The flow is bounded by walls at 15°N and 10°S.

The two nondimensional parameters that determine the nature of the solution are the Rossby and Richardson numbers:

$$Ro = U_0 a/2\Omega L^2 \qquad Ri = g'H/U_0^2$$

Here U_0 is the amplitude of the mean flow and has a value of 60 cm/s in Figure 1. L is a measure of the latitudinal scale of the shear and has a value of 550 km (equal to 5° latitude) in Figure 1. The boundaries are at $y = 3L$ and $y = -2L$. The radius and the rate of rotation of the earth are denoted by a and Ω, respectively. H is the mean depth of the interface, and g' is reduced gravity based on the density difference between the two layers. The Richardson number is a measure of the divergence. (For nondivergent flow, $Ri = \infty$ or, for practical purposes, $Ri \gtrsim 100$.)

The profile shown in Figure 1 has two unstable modes, on a β plane, in the nondivergent limit. (At two points, β-U_{yy} vanishes.) In this paper we discuss only the more unstable mode. Our principal interest is the variations in the stability properties when the shear of the flow is changed. We therefore describe results for different values of U_0 and L (and hence Ro) but for a fixed value of 10 for Ri. According to the general results presented in I, westward jets are destabilized by divergence (i.e., a decrease in the value of Ri). This result is also valid for the profile studied here. A value for Ri lower than 10 would be more appropriate for the equatorial oceans, but it is then possible for the interface between the two layers of the model to intersect the surface for some values of Ro. We therefore keep the value of Ri fixed at 10, but it should be kept in mind that this has the effect of increasing the *e* folding time.

Figure 2 shows how the *e* folding times and periods of the unstable waves are affected by changes in the values of U_0 and L. The most unstable wave typically has a period of 1 month and a wavelength of 1100 km. The *e* folding time of the wave is about 2 weeks. This time scale is sensitive to a change in the value of U_0, but the wavelength of the most unstable wave is not. An increase in the value of U_0 results in a decrease in the period of the most unstable wave. The period decreases from 36 to 26 days when U_0 increases from 50 to 80 cm/s. Changes in the width of the shear region have a large effect on the wavelength of the most unstable waves, but such changes leave their period almost unchanged (see Figure 2).

The structure of the eigenfunctions shows that the unstable waves have large amplitudes only in the region between the equator and 7°N. (This region is centered on the zone of large latitudinal shear of the mean flow.) The phase of the meridional velocity fluctuations changes by π across this shear region.

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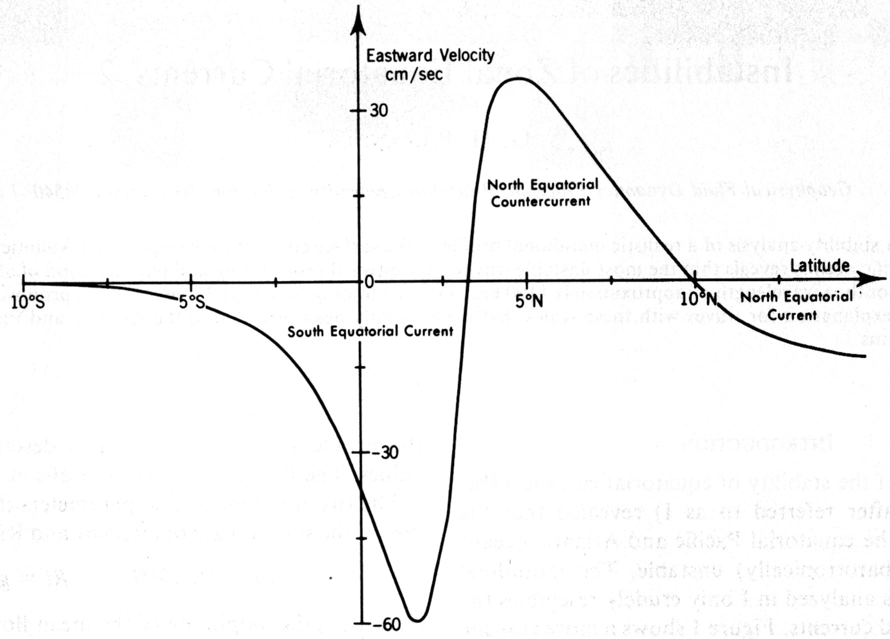

Fig. 1. Latitudinal variation of the mean zonal flow in the surface layers of the ocean.

3. Discussion

Satellite photographs of the sea surface temperature of the central equatorial Pacific, taken during the latter half of 1975, clearly show westward traveling undulations with a period of about 25 days and a wavelength of 1000 km along the temperature front that separates the NECC and SEC [*Legeckis*, 1977]. It is proposed that these waves are due to an instability of the surface currents. The unstable waves in the surface layers will radiate equatorially trapped waves which could travel into regions where the mean currents are stable. Thus the 25-day 1000-km waves observed by *Harvey and Patzert* [1976] near the ocean floor near the Galápagos Islands could be Rossby gravity waves (for example) that were generated in the surface layers further to the west. (Rossby gravity waves have an eastward group velocity but westward phase speed.) Twenty-five-day oscillations in the sea surface height and the sea surface temperature at the Line Islands (K. Wyrtki, private communication, 1977) may be indirectly related to unstable waves in a similar manner.

The various measurements of 25-day oscillations in the equatorial Pacific indicate that they occur intermittently. Satellite photographs taken during 1976, for example, show no evidence of these waves. The Line Island measurements suggest that these fluctuations occur preferentially in the fall and winter. This intermittency is probably related to the consid-

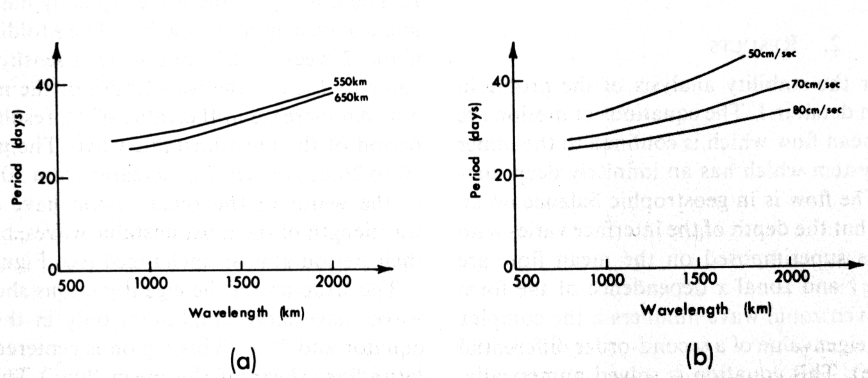

Fig. 2. The *e* folding time and period, as a function of wavelength, of unstable waves for (*a*) L = 550 and 650 km when U_0 = 70 cm/s and (*b*) U_0 = 50, 70, and 80 cm/s when L = 550 km.

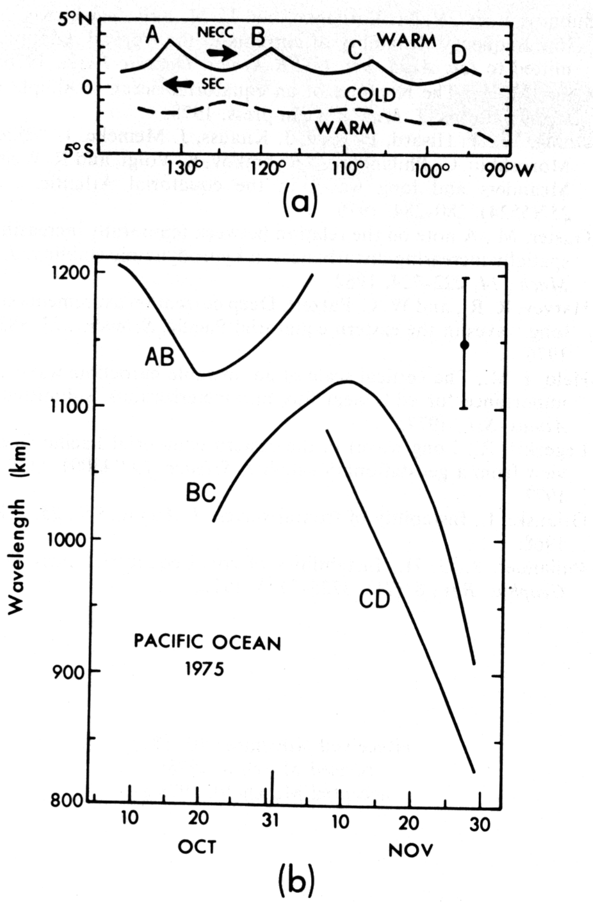

Fig. 3. (*a*) A schematic diagram of the sea surface temperature as seen on satellite photographs. The solid line indicates the temperature front. The dashed line indicates the approximate position of the southern boundary of equatorial upwelling. (*b*) Separation of successive wave crests as a function of time. (The error bar indicates the probable error.) After *Legeckis* [1977].

erable seasonal and interannual variability of the surface currents in the Pacific (and Atlantic). Typically, the westward flow at the equator is at its most intense in the autumn (October and November), at which time the NECC also has a maximum. The other extreme occurs in March and April, when the NECC is at its weakest and when the westward flow at the equator sometimes disappears. When the maximum speed of the SEC is less than 25 cm/s, then according to the analysis of section 2, the *e* folding time of unstable waves is so long that instabilities are unlikely.

The satellite photographs mentioned earlier show the 25-day fluctuations west of 95°W only. It is unclear whether or not there are waves east of this longitude because clouds make it difficult to see the sea surface in that region. Daily sea surface temperature measurements at the Galápagos Islands (90°W) in 1974, 1975, and 1976, however, show no evidence of 25-day oscillations. Presumably, the currents across the meridian of the Galápagos and the currents further east are too weak to be unstable. It is also possible that the deflection of the zonal currents by the Galápagos Islands provides the initial perturbation that grows into unstable waves of finite amplitude further downstream. The approximate equation $\lambda = c_g T$ relates the *e* folding distance λ of the spatially growing waves to the *e* folding time T as calculated above for waves that are sinusoidal in x [*Gaster*, 1962]. (The zonal group velocity of the waves is c_g.) For the case $U_0 = 70$ cm/s, $L = 550$ km, the group velocity of the most unstable wave is about 15 cm/s, and the *e* folding distance is approximately 180 km. The spatial growth of the waves is very rapid because the group velocity is small. The spatially growing waves are dispersive because the group velocity decreases as the wavelength decreases: $c_g = 18$ cm/s when $\lambda = 1300$ km and $c_g = 12$ cm/s when $\lambda = 850$ km for the case $U_0 = 70$ cm/s, $L = 550$ km. This dispersiveness could explain why the wavelengths observed in the central equatorial Pacific during October and November 1975 decreased gradually as shown in Figure 3. In September 1975 the trade winds in the vicinity of the Galápagos Islands were unusually intense, but the winds relaxed suddenly in October [*Legeckis*, 1977]. One would expect this event to excite a spectrum of waves that grow and disperse as they propagate westward from the Galápagos Islands. Hence in some downstream region the longer waves will arrive first, to be followed later by the shorter waves.

A recent analysis of daily sea surface temperature maps for the tropical Atlantic, based on measurements obtained during Gate, reveals the existence of westward drifting waves with a period of about 30 days and a wavelength of 1000 km [*Brown*, 1978]. The fluctuations have their largest amplitude in the vicinity of 3°N, the boundary between the NECC and SEC. Simultaneous current measurements along 23.5°W [*Bubnov et al.*, 1978], on the equator at 10°W (A. Rybnikov, private communication, 1977), and on the equator at 28°W (R. Weisberg, private communication, 1977) also indicate fluctuations with these scales. Along the equator the waves propagate downward into the deep ocean. The downward propagating waves are presumably equatorially trapped waves excited by the instability in the surface layers. The measurements in the Atlantic may be sufficient to determine the structure of the unstable waves.

It is noteworthy that in *Cane*'s [1978] numerical study of the generation of equatorial currents there are westward traveling waves with a wavelength of 1000 km and a period of 30 days after the model had reached an equilibrium state. This is probably due to an instability of the realistic surface currents.

During Gate the Equatorial Undercurrent was observed to meander about the equator with a period of about 16 days and a wavelength in excess of 2000 km [*Duing et al.*, 1975]. In I it was suggested that these meanders could be due to an instability of the surface currents. This suggestion was based on the results of a stability analysis of a velocity profile that only crudely resembles the structure of the observed currents. The stability analysis of the much more realistic profile shown in Figure 1 indicates that waves with a 16-day period are effectively stable. An instability excited by random perturbations is unlikely to have been the cause of the 16-day meanders observed during Gate.

4. CONCLUSION

The velocity profile shown in Figure 1, which describes the structure of the surface currents in the Atlantic and Pacific oceans realistically, is unstable and can support amplifying waves with a period of about 30 days and a wavelength of approximately 1100 km. These scales are insensitive to changes in U_0, the velocity scale of the mean currents. The results are in good agreement with measurements in the Pacific and Atlantic oceans, but further measurements are necessary, especially in the Pacific, for a rigorous test of this hypothesis. The model that has been used to study the stability properties of the currents also needs to be improved.

In the Pacific Ocean the unstable waves appear strikingly as undulations of the temperature front that is the boundary between the NECC and the SEC. In mid-latitudes a frontal instability is a hybrid between a barotropic and a baroclinic instability [*Orlanski,* 1968]. In I it was argued that equatorial currents are unlikely to be baroclinically unstable, but *Held* [1977] has pointed out that that result is an artifact of a two-layer model. In a continuously stratified fluid, baroclinic instability may be possible in very low latitudes. A multilevel model is necessary to determine the extent to which baroclinicity will modify the results obtained here.

We have suggested that the dispersion of unstable waves excited near the Galápagos Islands could explain the gradual change in the wavelengths of the waves observed west of 95°W. Further investigation of this hypothesis calls for a model that does not assume an e^{ikx} structure for the waves. Finally, we note that the waves seen in *Legeckis*'s [1977] satellite photographs have a cusplike structure. Presumably, nonlinearities have to be taken into account to simulate this feature.

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