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The effects of coastal geometry on equatorial waves (Forced waves in the Gulf of Guinea)

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ABSTRACT

The response of a stratified, semi-infinite equatorial ocean, bounded by a zonal coast close to the equator, to forcing at a given frequency and zonal wavenumber, is considered. If the coast is distant from the equator, the vertically propagating waves that are excited could include an infinite set of inertia-gravity waves, a finite set of Rossby waves, a Rossby-gravity wave and a coastally or equatorially trapped Kelvin wave. If the coast is close to the equator all these waves are modified except the equatorially trapped Kelvin wave. Most severely affected by the coast are the coastally trapped Kelvin wave and the equatorially trapped mixed Rossby-gravity wave. On a dispersion diagram the lines corresponding to the latter two waves are deformed so as to give rise to a Kelvin-gravity and a Rossby-Kelvin mode, each with a point at which the zonal component of the group velocity vanishes. The modification of these waves is most severe in the neighborhood of this point, particularly for small vertical wavenumbers. Examples of waves that have been, or are likely to be observed in the Gulf of Guinea (where there is a nearly zonal coast to the equator) are discussed.

1. Introduction

The free waves of the ocean fall into two main groups: inertia-gravity and Rossby waves. The gravest latitudinal modes are equatorially trapped; their latitudinal structure is described by Hermite Functions which decay exponentially poleward of a turning latitude. It follows that the presence of a zonal coast considerably poleward of this turning latitude has little effect on the gravest equatorially trapped waves. The coast will, of course, introduce an additional mode: a westward propagating, nondispersive, coastally trapped Kelvin wave. This paper concerns the modifications to this wave, and to the gravest equatorially trapped waves, when there is a zonal coast at a latitude equal to, or less than, the turning latitude of the equatorially trapped waves. The analysis may be relevant to the Gulf of Guinea (in the eastern equatorial Atlantic Ocean) where there is a nearly zonal coast at approximately 5°N.

This problem can be approached in (at least) two ways. Hickie (1977) has determined the natural modes of oscillation of a semi-infinite equatorial ocean bounded

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by a zonal coast to the north of the equator. (He also considered the additional effect of a meridional coast.) The vertical structure of the modes corresponds to that of the barotropic and standing baroclinic modes. With each vertical mode is associated a value for the effective vertical wavenumber m. The dispersion relation

$$\sigma = \sigma\,(k,l,m) \tag{1.1}$$

which relates the frequency σ to the wavenumbers k, l and m where k and l are the zonal and meridional wavenumbers respectively, then determines the eigenfrequencies of the ocean for given values of k. This approach is particularly useful when studying resonant phenomena; it predicts frequencies at which spectral peaks will occur should the forcing be white (i.e. should the forcing have equal energy at all frequencies).

The spectrum of the atmospheric forcing in the tropics (and, in particular, in the Gulf of Guinea) is characterized by sharp peaks that correspond to tropical waves. (See §5). The forcing function for oceanic motion therefore has well-defined values of σ and k. A vertical wavenumber can then be calculated from the dispersion relation (1.1) for each latitudinal mode l. This value for m will in general not coincide with the value that corresponds to the standing vertical modes. (The atmospheric waves will in general not excite a resonant oceanic response). Since the natural modes of the ocean form a complete set, they can be used to describe this nonresonant response. It is, however, possible to have an alternative and more convenient mathematical representation for this response. This paper describes such a representation.

2. Dispersion relation

On an equatorial β-plane, with (x,y,z) the eastward, northward and upward coordinates, the linear, hydrostatic equations of motion can be reduced to the following single equation for the meridional velocity component

$$U_{yy} + (\beta^2 y^2 - \sigma^2)\frac{\partial}{\partial z}\left(\frac{1}{N^2}\frac{\partial U}{\partial z}\right) - \left(k^2 + \beta\,\frac{k}{\sigma}\right)U = F(y,z) \tag{2.1}$$

Here F is the forcing function, N is the Brunt-Vaisala frequency and it has been assumed that solutions (and the forcing) are of the form $e^{i(kx-\sigma t)}$ where t measures time and σ is the frequency. The zonal wavenumber k is positive (negative) for eastward (westward) propagating disturbances. The coordinates y and z measure distance northwards from the equator and upwards from the ocean surface respectively; $\beta = 2\Omega/a$ where Ω and a are the rate of rotation and radius of the earth, respectively.

Consider the free (unforced) modes when N is a constant. We seek solutions of the form

$$U = e^{i(kx+mz-\sigma t)}\, V_{(y)} \quad . \tag{2.2}$$

It follows from (2.1) with $F=0$ that

$$V_{yy} + \left(\frac{\sigma^2}{gh} - k^2 - \frac{\beta k}{\sigma} - \beta^2 y^2/gh\right) V = 0 \tag{2.3}$$

where we have written

$$m^2 = N^2/gh \quad . \tag{2.4}$$

Equation (2.3) must be solved subject to the boundary conditions

$V=0$ at $y=L$, a coast parallel to the equator; 2.5a

V must be bounded at large distances from the equator. 2.5b

Solutions that satisfy these conditions are possible provided

$$\frac{\sigma^2}{gh} - k^2 - \frac{\beta k}{\sigma} - \frac{\beta}{\sqrt{gh}}(2\nu_l + 1) = 0 \qquad l = 0,1,2\,.. \tag{2.6}$$

where ν_l is the lth root of the indicial equation

$$D_\nu(\eta_0) = 0 \tag{2.7}$$

Here

$$\eta = (\beta^2/gh)^{\frac{1}{4}}\, y \tag{2.8}$$

and η_0 is the value of η at the coast $y=L$. The function D_ν is a Parabolic Cylinder Function (Magnus and Oberhettinger, 1949). The eigenfunctions are

$$V_l(y) = D_{\nu_l}(\eta) \tag{2.9a}$$

and the associated zonal velocity components and pressure fields are

$$U_l = -\frac{i}{ghk^2-\sigma^2}(\sigma\beta y V - kgh\, V_y) \tag{2.9b}$$

$$P_l = -\frac{i\, gh}{ghk^2-\sigma^2}(k\beta y V - \sigma\, V_y) \tag{2.9c}$$

The solid lines in Figure 1 show the roots of (2.7) as a function of η_0. If the coast is distant from the equator ($L \to \infty$) then

$$\nu_l = l \qquad l = 0,1,2\,..$$

and the eigenfunctions are the Hermite Functions. If the coast is at the equator ($L=0$) then

$$\nu_l = 2l+1 \qquad l = 0,1,2\,..$$

and only (Hermite Function) modes that are anti-symmetrical about the equator

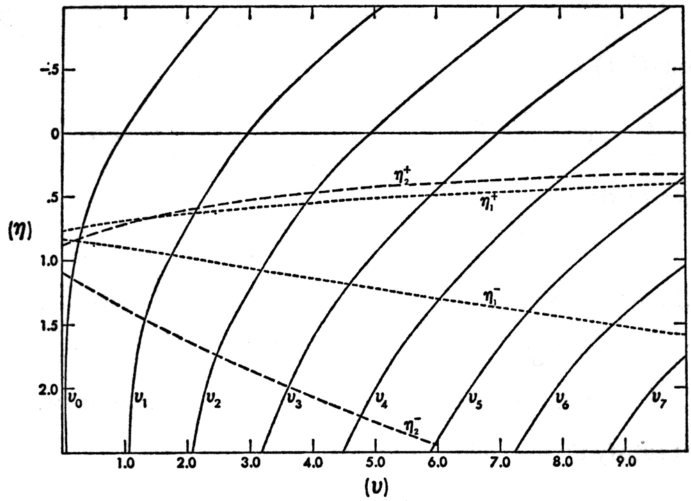

Figure 1. Solid lines: Solutions of (2.7) as a function of η. Dashed lines: $\eta(\nu)$, according to (2.6) and (2.8), for

(1) $\sigma = 2\pi/2$ days k $= -2\pi/2000$ km
(2) $\sigma = 2\pi/4$ days k $= -2\pi/3000$ km

are possible. In general the index l denotes the number of zeros of D_r and can be regarded as a discrete meridional wavenumber. Equation (2.6) is therefore a dispersion relation that relates the frequency to the wavenumbers k, l and m. This equation is quadratic in k and m ($\sim 1 \sqrt{h}$). It is cubic in σ but the solutions for σ fall into two groups. (See Matsuno, 1966). Hence, for a given integer value of l, equation (2.6) defines two surfaces. We label these l and l'. Figure 2 schematically shows some of the surface described by this dispersion relation. The surfaces can be divided into four groups.

(i) Inertia-gravity waves: $l \geqslant 1$

(ii) Rossby waves: $l' \geqslant 1$

(iii) Equatorial Kelvin wave: $l = -1$ (not shown in Figure 2). This mode satisfies (2.1) trivially since, on an equatorial β-plane, its meridional velocity component U is identically zero. It follows that its structure is unaffected by the presence of the zonal coast. For this nondispersive wave (see Philander, 1977)

$$\sigma = k\,N/m \tag{2.10a}$$

and

$$U = \exp\left(-\tfrac{1}{2}\,\frac{\beta m}{N}\,y^2\right) \tag{2.10b}$$

(iv) Coastal Kelvin-Rossby-gravity modes: $l = 0$; $l' = 0$. If the coast is very far from the equator then we expect the following two modes: (a) a Rossby-gravity, equatorially trapped wave (Matsuno, 1966) for which $\nu_0 = 0$ and

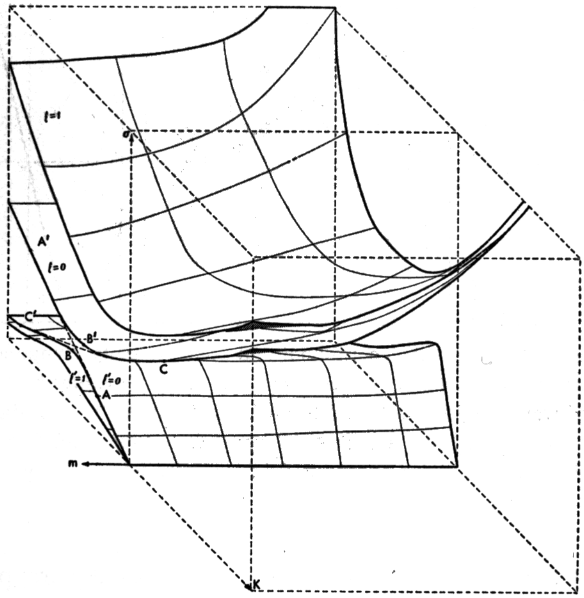

Figure 2. Schematic diagram of the surfaces defined by (2.6). Fig. 5 is a section from this figure.

$$\frac{m}{N} = \frac{\beta+\sigma k}{\sigma^2} \qquad 2.11$$

and (b) a westward propagating coastally trapped wave for which

$$\sigma = -k\,N/m\,. \qquad 2.12$$

The surfaces defined by 2.11 and 2.12 (see the dashed lines near BB′ in Figure 2) intersect. However, the true solutions to (2.6) for the gravest modes ν_0 ($\neq 0$) do not intersect. Close to the point of near intersection they behave like the two branches of a hyperbola. (See Longuet-Higgins and Pond (1970) for a discussion of the behavior of dispersion curves near a point of intersection). Hence the Rossby-gravity and coastal Kelvin wave become a Rossby-Kelvin and Kelvin-gravity wave. Furthermore, there is a range of frequencies for which neither of these mixed modes exists. (In the absence of a zonal coast there is a Rossby-gravity mode at all frequencies). This frequency gap gets increasingly smaller as the vertical wavenumber increases. (See Figure 2). This suggests that for short vertical wavelengths, waves with a hybrid coastal-equatorial structure are relatively unimportant. Inspection of the structure of the eigenfunctions reveals a hybrid coastal-equatorial wave structure only in the vicinity of the two lines along which the zonal component of the group velocity vanishes (see Figures 2 and 3). At points far from these lines the

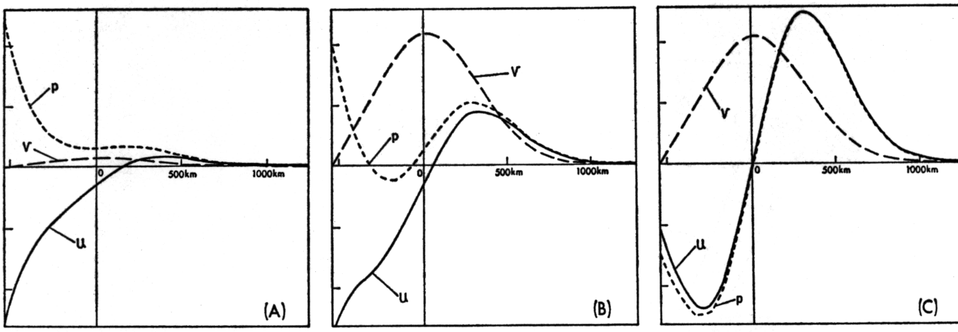

Figure 3. Latitudinal structure of the meridional and zonal velocity components corresponding to points indicated on Fig. 2. These figures correspond to eigenvalues h = 60 cm, $\nu_0^{\pm} = .045$ and periods T and wavelengths L with the following values: A: T = 47 days, L = 9,300 km; B: T = 16.5 days, L = 2,300 km; C: T = 4.7 days, L = 1,400 km. For point B′, the u, v and p fields are identical to the p, v and u fields, respectively, for point B. (After Hickie, 1977.)

structure of the mode is essentially that of a coastally trapped Kelvin wave (points A and A′ for example) or an equatorially trapped Rossby-gravity wave (points C and C′ for example). In Figure 3A the pressure and zonal velocity component decay nearly exponentially with increasing distance from the coast and the zonal component of the velocity is much larger than the meridional component. In the case of a coastal Kelvin wave v is identically zero. In Figure 3C v is nearly a Gaussian and u and p practically have the structure of H_1, the first Hermite Function. This is essentially the structure of a Rossby-gravity wave. Because the hybrid coastal-equatorial waves have nearly zero zonal group velocity it follows that in an impulsive initial value problem the response of the ocean can, initially, be described in terms of coastally and equatorially trapped waves. Only after initial disturbances have propagated away and those with zero group velocity remain, will the hybrid equatorial-coastal waves be significant.

3. Forced waves

If N is a function of depth then it can not be assumed that solutions are of the form given in (2.2). Write

$$U = Z(z)\, V(y)\, e^{i(kx-\sigma t)} \qquad 3.1$$

then (2.1) yields the two equations (2.3) and

$$\frac{\partial}{\partial z}\left(\frac{1}{N^2}\frac{\partial Z}{\partial z}\right) + \frac{1}{gh} Z = 0 \qquad 3.2$$

where h is the constant of separation, referred to as the equivalent depth. It is clear from (3.2) that h is related to the vertical wavelength through (2.4). Solutions to (3.2) must satisfy

$$Z_z = 0 \quad \text{at } z = -H, \text{ the ocean floor} \tag{3.3a}$$

$$Z_z + \frac{N^2}{g} Z = 0 \quad \text{at } z = 0, \text{ the ocean surface.} \tag{3.3b}$$

One of equations (2.3, 3.2) should have a forcing term on the right-hand side. We return to this matter shortly.

Hickie (1977) solved equation (2.1) by first determining a complete set of eigenfunctions Z (the barotropic plus baroclinic modes) and eigenvalues h (the equivalent depths, all of which are positive) from (3.2). The forcing function in (2.1), which is assumed to be a body force in a mixed surface layer, can then be projected onto these vertical modes and it remains to solve (2.3) with h specified and with the appropriate component of the forcing function on the right-hand side. This can be done by exploiting the completeness of the eigenfunctions in (2.9a). In this approach the vertical wavenumbers are a priori discretized. Equation (2.7) can readily be solved since for a given vertical mode (i.e. given value of h) and given position of the coast (i.e. given value of L) η_0 has a constant value. The dispersion relation (2.6) then yields the relation between σ and k for a given vertical mode.

As was pointed out in the introduction, it is sometimes more convenient to regard the frequency σ and zonal wavenumber k as specified by the forcing function and to determine the vertical structure (and the value of h) that is consistent with this σ and k. It is now more difficult to solve (2.7) for ν since η_0 is a function of h (see 2.8) which in turn is a function of ν (see 2.6). The solution can be found graphically as follows. The dispersion relation (2.6) can be solved for h as a function of ν:

$$\sqrt{gh} = [-\beta(2\nu_l+1) \pm \{\beta^2(2\nu_l+1)^2 + 4\left(\frac{\beta k}{\sigma} + k^2\right)\sigma^2\}^{\frac{1}{2}}]/2\left(k^2 + \frac{\beta k}{\sigma}\right) \tag{3.4}$$

Substitution of this expression into (2.8) gives η_0 as a function of ν

$$\eta_0 = \left(\frac{\beta^2}{gh}\right)^{\frac{1}{4}} L \equiv \eta_0(\nu)$$

A second equation for η_0 as a function of ν is provided by (2.7). The intersection of the curves given by these two equations give the desired values of ν_l. The latitudinal eigenfunctions are now known from (2.9). The forcing function can be projected onto these modes whereafter it remains to solve equation (3.2) with the appropriate component of the forcing term on the right-hand side. (Philander (1977) has discussed the solution to this equation in detail). For h of the order of 1 cm or less it is convenient to think of waves propagating vertically through a fluid with variable stratification. In regions of high stability (such as the thermocline) the waves become short and their vertical shear becomes large so that wavebreaking is likely. If the value of h is much larger than 1 cm it is still convenient to think of vertically propagating waves should the ocean floor be so rough as to scatter all waves incident on it. In the case of a reflecting ocean floor, equivalent depths of the order of 20 cm or more will correspond to vertically standing modes. The oceanic response will be resonant if the value of the equivalent depth happens to coincide with that of one of the natural baroclinic modes. The approximate equivalent depths (on the basis of measurements on the equator at 10°W in June, 1974) are 60, 20 and 8 cm for the first three baroclinic modes. For the barotropic mode the equivalent depth is the depth of the ocean.

Equation (3.4) gives two sets of solutions h^+ and h^-. With each member of each set is associated a value of ν_l so that we shall refer to pairs (h_l^+, ν_l^+) and (h_l^-, ν_l^-). From the example shown in Figure 1 it is clear that

$$\nu_l^- >> 1 \qquad \text{when } l >> 1 . \tag{3.5}$$

It then follows from (3.4) that

$$\sqrt{gh_l^-} \rightarrow \frac{\sigma^2}{\beta(2\nu_l+1)} \qquad \text{when } l >> 1 . \tag{3.6}$$

For values of $|y|$ greater than that corresponding to a turning latitude y_T

$$y_T^2 = (2\nu_l+1)\sqrt{gh}/\beta \tag{3.7}$$

equation (2.3) has exponentially decaying solutions. In the limit (3.6)

$$y_T^2 \rightarrow \sigma^2/\beta^2 . \tag{3.8}$$

The turning latitude is simply the inertial latitude for the given frequency. We therefore identify the (h_l^-, ν_l^-) set with inertia-gravity waves.

The (h_l^+, ν_l^+) set is associated with Rossby waves and comes into play only when the values of σ and k are such that

$$\frac{\beta k}{\sigma} + k^2 < 0 \tag{3.9}$$

This condition, which ensures that $\sqrt{gh^-} > 0$, is necessary if the argument of the eigenfunctions (2.9) is to be real. Values of σ and k in the shaded region of Figure 4 satisfy this condition.

Since

$$\nu_l^+ >> 1 \qquad \text{for } l >> 1$$

$$\sqrt{gh_l^+} \rightarrow -\frac{\beta}{k^2 + \dfrac{\beta k}{\sigma}}(2\nu_l^+ + 1) - \frac{\sigma^2}{\beta(2\nu_l + 1)} . \tag{3.10}$$

The turning latitude y_T of equation (2.3) therefore increases as l increases (see 3.7). The equatorial β-plane therefore gives an infinite set of Rossby modes for given

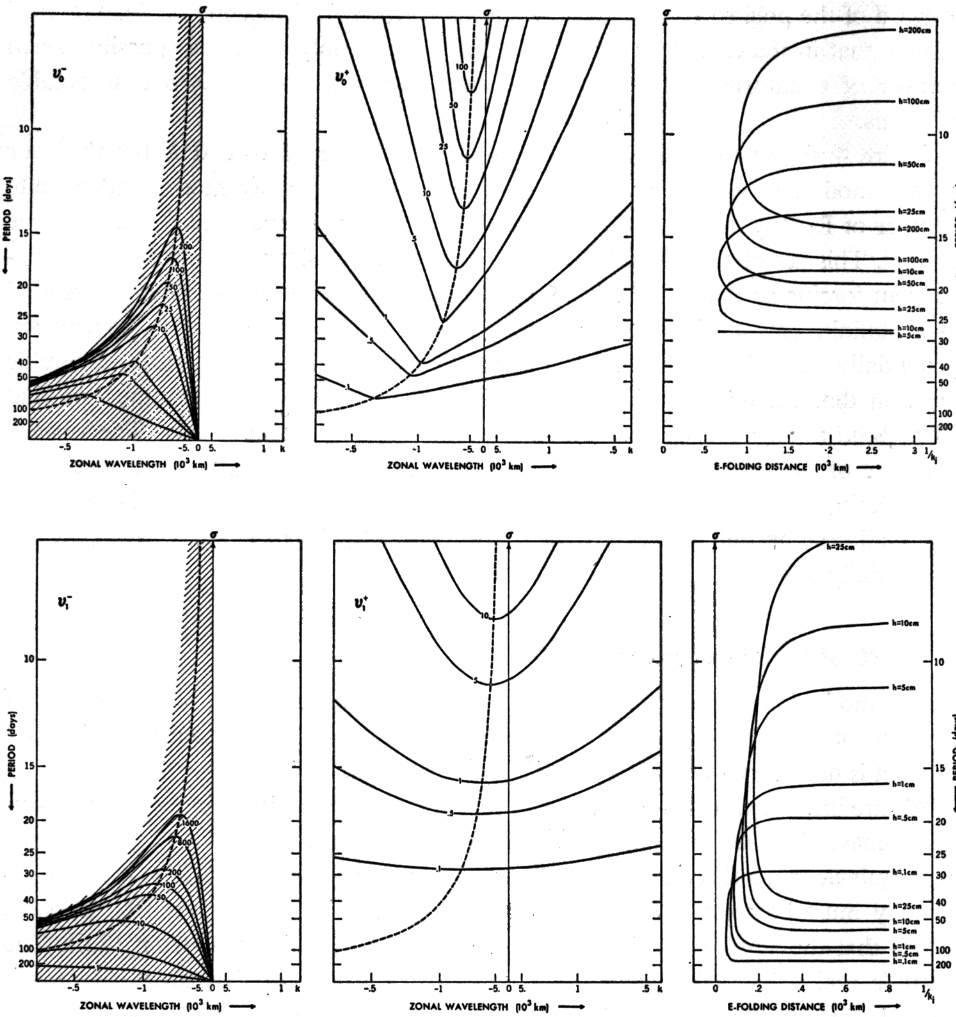

Figure 4. Equivalent depths, as a function of σ and k for the (a) $l = 0$ and (b) $l = 1$ modes. The right-hand figures are described in the text.

values of σ and k that satisfy (3.9). All but the gravest have their turning latitude y_T 'beyond the pole'. (In other words $y_T > D$, where D is distance from the equator to the pole, for all but the gravest modes.)

A similar situation occurs in the absence of a zonal coast; in such a case the equatorial β-plane also gives an infinite number of Rossby modes. However, the more accurate solutions for a spherical geometry (Longuet-Higgins, 1968) show only a *finite* number of Rossby modes. These modes are approximated on an equatorial β-plane by the finite number of modes that have their turning latitude equatorward of the pole so that $y_T < D$ (see Philander, 1977, Lindzen, 1967). We shall assume that in the case where there is a zonal coast, only the finite (possibly zero) number of equatorial β-plane Rossby modes for which $y_T < D$, are acceptable solutions.

Figure 4 shows the equivalent depths as a function of σ and k for the four gravest modes. In general an infinite number of inertia-gravity modes and a finite number of Rossby modes (if the values of σ and k are in the shaded area) will be excited. This set of eigenfunctions is not complete. Consider, for example, x independent forcing so that $k = 0$. The eigenfunctions available in that case correspond to an infinite set of inertia-gravity waves. Since all the eigenfunctions decay exponentially beyond the inertial latitude, they cannot be used to represent a forcing function that extends beyond that latitude. For completeness, eigenfunctions that correspond to negative values of the equivalent depth h are necessary. The vertical structure of the oceanic response then decays exponentially with depth (see 3.2). These latitudinal modes cannot be described by a semi-infinite equatorial β-plane. We shall confine our attention here to the vertically propagating waves that are equatorially trapped.

4. Effects of meridional coasts

The motivation for this study is its possible relevance to the oceanography of the Gulf of Guinea. Since this Gulf is bounded to the east by a nearly north-south coast, it is necessary to consider the effect of that coast.

We are interested in a wave, with an eastward group velocity, incident on a north-south coast. All the waves introduced by the coast must have the same frequency and equivalent depth (vertical structure). If a reflected wave is excited, its group velocity must be westward. Given σ and h (of the incident wave), the possible values that the zonal wavenumber k can have may be calculated from (2.6). The possible real values of k, which are associated with waves that propagate freely toward and away from the north-south coast, are shown in Figure 5a. For a given value of ν_l (see the $\nu_l = .01$ curves, for example) there is a range of frequencies for which there are no entirely real values of k. In this frequency range k is complex so that the associated waves are trapped against the north-south coast. The real part of the complex k is simply

$$k_R = -\beta/2\sigma$$

which corresponds to the dotted line in Figures 4 and 5a. Figure 5b shows the e-folding distance of these trapped waves for a fixed value of h. The right-hand parts of Figure 4 show the e-folding distance for different values of h for the ν_0 and ν_1 modes. For the gravest mode this distance is of the order of 800 km. The higher the mode (the larger the value of l), the smaller this distance becomes.

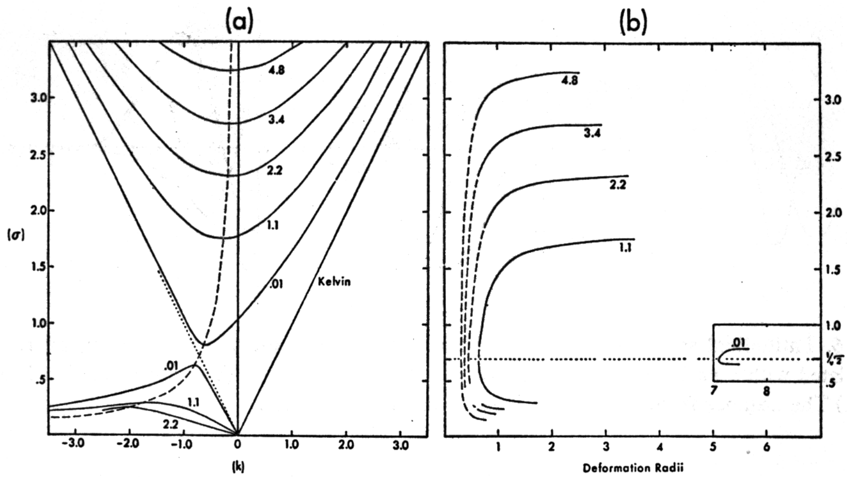

Figure 5. (a) This is a $\sigma - K$ section from Figure 2 and is a dispersion diagram for a given value of h (= 20 cm). The values for ν are indicated on the figures. The frequency σ and wavenumbers K have been nondimensionalized relative to a time scale $(\beta^2 gh)^{-\frac{1}{4}}$ = 2 days and a length scale $\left(\frac{gh}{\beta^2}\right)^{\frac{1}{4}}$ = 250 km, which is the radius of deformation.
(b) This shows the e-folding distance of the (discrete) set of waves trapped against the north-south coast, all with the same equivalent depth h (= 20 cm).

It is evident from Figure 5 that for a certain frequency range ($\Delta\sigma$ say) centered on σ_0 where

$$\sigma_0{}^2 = \beta\sqrt{gh}/2$$

the only zonally propagating wave is an eastward equatorial Kelvin wave. If its frequency is in this frequency range and if it is incident on a north-south coast, then no reflected waves, only coastally trapped waves, are excited. The energy that is propagated eastward by the Kelvin wave ends up in the southern hemisphere because the coastally trapped waves represent a poleward propagating coastal Kelvin wave in extra-equatorial regions (Moore, 1968).

Consider next waves outside the above-mentioned frequency range $\Delta\sigma$. Figure 5 shows that there is now more than one wave with eastward group velocity, and that a north-south coast introduces reflected waves (with westward group velocity). The gravest modes trapped against the north-south coast cease to play a role. In general, the more the frequency under consideration differs from σ_0 the more important are reflected waves and the less important are waves trapped against the north-south coast.

5. Examples

There is evidence of several atmospheric waves, with well-defined frequencies σ

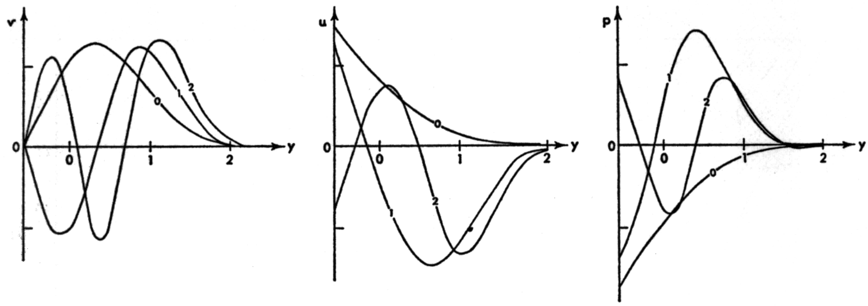

Figure 6. Latitudinal structure of the velocity and pressure fields, of the three gravest modes, associated with a period of 2 days and wavelength of −2000 km. (The normalization is arbitrary.) The distance from the equator is given in units of 10^3 km.

and wavenumbers k, over the Gulf of Guinea. These waves will excite an oceanic response that includes an infinite set of inertia-gravity waves and a finite set of Rossby waves, which propagate in the vertical each with a distinct equivalent depth (vertical wavelength) and all with the same values of σ and k. (The higher the order l of the waves, the smaller the equivalent depth in the case of inertia-gravity waves and the larger the equivalent depth in the case of Rossby waves). For each of these vertically propagating waves there may be a finite set of propagating waves excited at the north-south coast, and an infinite set of waves trapped against that coast. Here we describe the gravest modes ($l \sim 0(1)$) that are likely to be excited by known atmospheric waves. For none of the examples to be considered is the gravest ($l = 0$) wave trapped against the north-south coast excited. Effects of this coast will therefore be felt only up to distances of the order of 250 km from the coast.

a. 2 day wave. Orlanski (1976) and Orlanski and Polinsky (1977) have described atmospheric oscillations with a period of 2 days. Over the Gulf of Guinea these westward propagating waves have a wavelength of about 2000 km. Byshev and Ivanov (1969) also report a 2-day peak in spectra of surface winds at Ascencion Island (8°S, 10°W).

The following are the numerically determined eigenvalues for westward travelling waves with a period of 2 days and a wavelength of 2000 km:

l	:	0	1	2
$(\nu^-, h^-\text{cm})$	:	(.33, 1227)	(1.76, 676)	(3.2, 398)

Figure 6 shows the latitudinal structure of the three gravest modes. The $l \geqslant 1$ modes are essentially equatorially trapped inertia-gravity waves. The equivalent depths of the $l = 1$ and 2 modes are so large that their vertical structure will resemble that of the first baroclinic mode. The gravest (l=0) latitudinal mode is effectively a coastally trapped Kelvin wave modified by the variability of the Coriolis parameter. Measurements to determine whether or not these modes actually get established will be of great value. Their turning latitude is so far from the equator (near 15°S) that measurements may show packets of waves propagating along rays rather than oscillations with a modal structure.

b. 4 day wave. Westward propagating disturbances with a period of about 4 days and a wavelength of approximately 3000 km are common in the vicinity of the Intertropical Convergence Zone (ITCZ) which, at certain times of the year, is close to the equator. The eigenvalues associated with the gravest oceanic modes excited by these waves are the following. (No ν^+ modes enter into consideration since condition (3.9) is not satisfied).

l	0	1	2
(ν^-, h^-cm)	(.24, 545)	(1.4, 136)	(2.52, 58)

The latitudinal structure of the eigenfunctions is quite similar to that shown in Figure 6 but the latitudinal scales are somewhat smaller. Thus the gravest mode ($l = 0$) is again a modified coastal Kelvin wave and the *e*-folding distance of the exponentially decaying u velocity component is about 700 km. The higher order ($l \geqslant 1$) modes are more strongly equatorially trapped than those in example (a) above. The equivalent depths are such that a combination of the first baroclinic and barotropic modes will describe the vertical structure. Note that the third mode ($l = 2$) may be resonant since its equivalent depth is close to that of the first baroclinic mode.

c. 14 day wave. An analysis of time-series records of sea-surface temperature and sea-surface height as measured at coastal stations has revealed the presence of an energetic wave with a 14 day period, 900 km wavelength and a westward phase speed in the Gulf of Guinea (Picaut and Verstraete, 1976). Current measurements close to the northern coast of the Gulf of Guinea confirm the presence of such a wave (Houghton and Beer, 1976). Rinkel (1969) on the basis of measurements on several occasions in the vicinity of the equator at 8°W, describes meridional oscillations of the Equatorial Undercurrent with a period of about 14 days. We investigate the structure of waves with this period and a wavelength of about −900 km, in the Gulf of Guinea.

From Figure 4 we learn that for $\sigma = 2\pi/14$ days and $k = 2\pi/900$ km no modes corresponding to (ν_l^+, h_l^+), in other words no Rossby type modes, are excited because we are not in the shaded part of Figure 4 and are therefore not satisfying condition (3.9). For the gravest inertia-gravity Kelvin type modes

l	0	1
(ν_l^-, h_l^-cm)	(0, 6.)	(1, 1.4) .

The values for ν_l are nearly those for an equatorial ocean in the absence of a zonal coast. We therefore expect the ν_0^- mode to be a coastally trapped Kelvin wave that is little affected by the proximity of the equator. The higher order ν_l^- modes are of no interest since they are inertia-gravity waves with turning latitudes near 2° latitude. Their latitudinal and vertical shears are so large that they are almost certainly unstable.

The structure of the ν_0^- mode is essentially that of a coastally trapped Kelvin wave. The zonal velocity component decays exponentially with increasing distance from the coast. The associated *e*-folding distance is approximately 70 km. The meridional velocity component is very small and has a maximum at the equator. It is extremely unlikely that the v component is sufficiently large to cause the earlier mentioned oscillations of the Equatorial Undercurrent described by Rinkel (1969). These oscillations of the Undercurrent therefore appear to be a phenomenon independent of the coastal waves unless the oscillations at the equator have an eastward group velocity in which case the presence of the north-south coast will cause the excitation of westward propagating coastally trapped waves (with the same period).

If the observed wave is a coastal Kelvin wave then it would have to be locally forced (all along the coast) because its equivalent depth does not coincide with that of a standing vertical mode (which could propagate freely along the coast). It seems more probable that the observed wave is freely propagating rather than locally forced, in which case it must be a topographic shelf wave and not a Kelvin wave.

d. 45 day wave. Madden and Julian (1972) describe an eastward propagating oscillation of the tropical atmosphere with a period of 40 to 45 days. Such disturbances could excite equatorial Kelvin waves in the ocean. Picaut and Verstraete's (1976) analysis of coastal time-series records reveals an oceanic oscillation with a period of 45 days but it appears to be a standing mode (without phase propagation). The relation between this oceanic mode and the propagating atmospheric wave is unclear.

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