On the Decay of the Meanders of Eastward Currents

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ABSTRACT

The separation of western boundary currents from the coast and their eastward extensions into the open ocean are characterized by the presence of quasi-stationary, large-scale meanders. These meanders result when the western boundary current overshoots the latitude of zero wind-stress curl. The wavelength and the scale of decay of those meanders have previously been estimated by considering the superposition of a westward Rossby wave and an eastward mean flow. While observations indicate that the wavelength of the meanders is in good agreement with theory, the decay scale is much shorter than that indicated by scaling arguments. The purpose of this article is to show that the rapid decay of the meanders of eastward currents can be related to the effect of the meridional shear of the current.

1. Introduction

The separation of western boundary currents from the coast and their eastward extensions into the open ocean are characterized by the presence of large-scale, quasi-stationary meanders. Figure 1 shows examples associated with two western boundary currents of the South Atlantic Ocean, the Brazil Current, and the North Brazilian Coastal Current. Moore (1963) regarded such meanders as damped Rossby waves that match a western boundary current to a Sverdrup interior flow.

In the steady state the wavelength of the stationary meanders (L_S) can be estimated to be (Pedlosky 1987)

$$L_S = 2\pi\left(\frac{U}{\beta}\right)^{1/2}, \qquad (1)$$

where U is a representative value of the mean flow velocity and β is the meridional gradient of the planetary vorticity. Observations indicate that (1) is a good estimate of the dominant scales of the meanders shown in Fig. 1; mean flow speeds of 30–50 cm s^{-1} imply 800–1000-km wavelengths as observed for the meanders (Gordon et al. 1978; Mueller-Karger et al. 1988). If we assume that the decay of the meanders' amplitude is due to friction then the decay scale (L_D) can be estimated to be (Pedlosky 1987)

$$L_D = C_{gx}t_D \approx \frac{\beta L_s^4}{A_H(2\pi)^4} = \frac{U^2}{\beta A_H}, \qquad (2)$$

where C_{gx} is the speed at which the wave energy propagates, t_D is a dissipative time scale $t_D = (k^2 A_H)^{-1}$, and A_H is the coefficient of Laplacian mixing of momentum. If U varies between 30 and 50 cm s^{-1} and A_H is O(10^6 cm^2 s^{-1}), then L_D varies between 50 000 to 90 000 km. The decay scales predicted by (2) are at least an order of magnitude larger than those observed in Fig. 1. Barnier et al. (1991) also noted that the decay scales observed in numerical simulations of wind driven gyres are shorter than the values expected from simple frictional arguments. They attributed the discrepancy to an increase in mixing due to eddy processes. While this is a plausible explanation for a highly nonlinear flow it can be shown that even in the linear case the scale L_D is far shorter than the value estimated from (2).

To obtain (2) it is necessary to assume that an eastward mean flow exists that has no meridional shear of velocities. Under this hypothesis and in the linear limit, the only effect of the mean flow on the westward propagating waves is a Doppler shift of their frequencies. In a more realistic case, however, the wave propagation can be inhibited by the presence of critical layers associated with the shear of the mean flow. The purpose of this article is to further investigate this possibility as an explanation for the rapid decay of the meanders of eastward currents. Following this introduction in section 2, we make a WKB analysis of the potential vorticity equation to show that for linear flows Rossby waves released in an eastward current are trapped near the source. The analytical results are complemented in section 3, by numerical experiments using a shallow-water model. Finally, in section 4 all the results are summarized and discussed.

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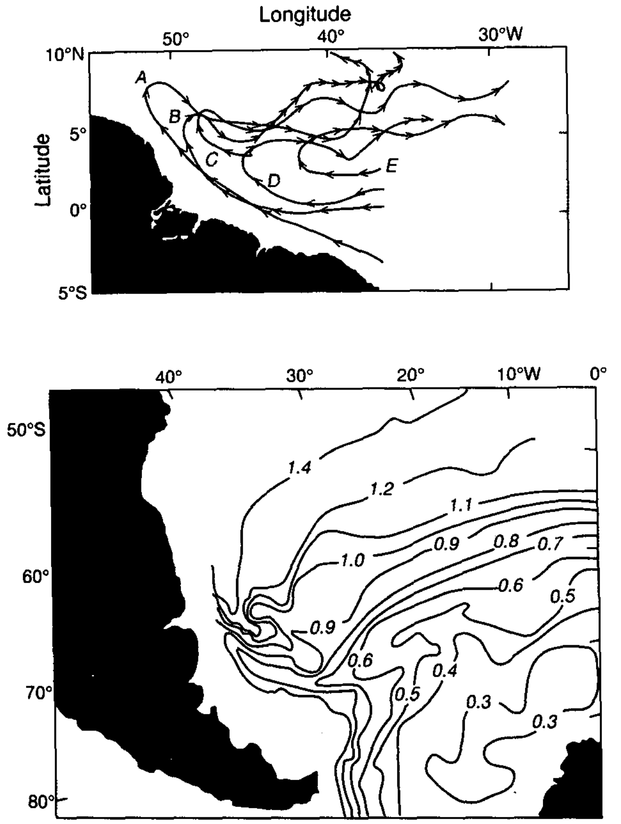

FIG. 1. Examples of quasi-stationary meanders associated with the eastward flow of western boundary currents in the South Atlantic Ocean. The upper panel shows buoy drifter trajectories associated with the path of the North Brazilian Coastal Current [adapted from Richardson and Reverdin (1987)]. The lower panel shows the geopotential height of the sea surface elevation referred to the 1000-db level in the southwestern Atlantic [adapted from Gordon et al. (1978)].

2. A WKB analysis of Rossby waves in an eastward mean flow

Consider the potential vorticity equation for a quasigeostrophic, barotropic fluid linearized about a zonal mean flow. In the absence of forcing or dissipation it can be written as

$$\left[\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right]\nabla^2\phi + \left(\beta - \frac{d^2U}{d^2y}\right)\frac{\partial\phi}{\partial x} = 0, \qquad (3)$$

where ϕ is the streamfunction of the perturbations. If the ocean is unbounded in the x direction then, we look for a solution to (3) of the form

$$\phi(x, y, t) = \mathcal{A}(y)\exp\{i(kx - \sigma t)\},$$

where k and σ denote the zonal wavenumber and frequency, respectively. Substitution in (3) gives the Schrödinger equation

$$\frac{d^2\mathcal{A}}{d^2y} + l^2(y)\mathcal{A} = 0, \qquad (4)$$

where

$$l^2(y) = \frac{\left(\beta - \dfrac{d^2U}{d^2y}\right)}{U - \dfrac{\sigma}{k}} - k^2. \qquad (5)$$

The character of the solutions to (4) depends on the value of the meridional wavenumber l^2 (Killworth 1979). If l^2 is greater than zero, then the solutions are propagating. If l^2 is less than zero, they are evanescent. Propagating and evanescent regions are separated by lines where l^2 vanishes. These lines are known as turning latitudes. Waves reaching these latitudes are supposed to be reflected back without loss of energy. If $U(y) = \sigma/k$, then $l^2 \to \infty$, and the solutions of (4) become singular. Regions with this property are known as critical layers. In a linear dissipative model it can be shown that the wave energy is absorbed though the physical relevance of this process has been questioned (Tung 1979).

An approximate solution to (4) can be found using the WKB technique (Bender and Orzag 1978). Write

$$\mathcal{A}(y) = M\exp\{iq(y)\},$$

where M is a constant. The approximate solution for $q(y)$ is

$$q(y) = \int l(y)dy + \frac{1}{2}i\ln l(y).$$

Then $\mathcal{A}(y)$ can be written as

$$\mathcal{A}(y) = Ml^{-1/2}\exp\left\{i\int l dy\right\}. \qquad (6)$$

Although (6) is valid neither at the turning nor the critical latitudes the WKB expression to the north and south of such layers is correct to the leading order (Dickinson 1968). Since $\mathcal{A}(y)$ is proportional to $l^{-1/2}$, a wave approaching a turning latitude will experience an increase in its amplitude; on the other hand, near a critical layer the wave amplitude is reduced. By adding a linear damping term to the right-hand side of the vorticity equation and then taking the limit of vanishing friction it can be shown that such a linear dissipative critical layer absorbs incident Rossby waves (Held 1983).

Equation (6) gives an approximate description of the wave amplitude. To study the trajectories of wave packets released in a mean flow it is advantageous to use the ray tracing theory. For this purpose let us rewrite the local dispersion relation (5) as

$$\sigma = Uk - \frac{(\beta - d^2U/d^2y)k}{k^2 + l^2}.$$

The group velocity, which is related to the speed of propagation of the energy of the wave, is given by

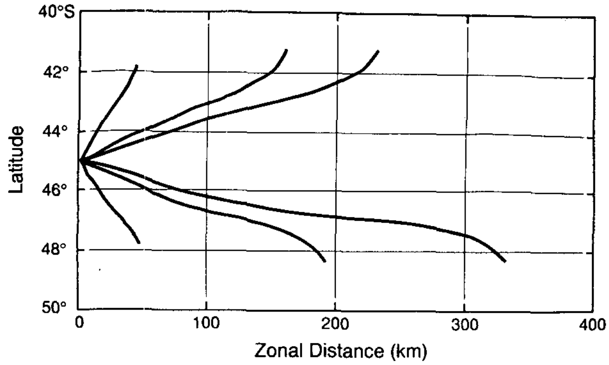

FIG. 2. Ray paths of stationary waves propagating in an eastward current. The mean current is represented by a Gaussian jet with a maximum speed of 50 cm s^{-1}. Two rays are produced for each value of the zonal wavenumber k. The asymmetry between the northward and southward rays is produced by the latitudinal dependence of the gradient of planetary vorticity.

$$C_{gx} = \frac{\partial \sigma}{\partial k} = \frac{\sigma}{k} + \frac{2(\beta - d^2U/d^2y)k^2}{(k^2 + l^2)^2},$$

$$C_{gy} = \frac{\partial \sigma}{\partial l} = \frac{2(\beta - d^2U/d^2y)kl}{(k^2 + l^2)^2}.$$

Defining a ray to be everywhere in the direction of the local group velocity, the ray equations are

$$\frac{dx}{dy} = \frac{C_{gx}}{C_{gy}} = \frac{k}{l}.$$

Given the initial conditions the previous equations can be solved to find the trajectory of a wave packet.

As an application of these ideas consider the case of an eastward Gaussian jet represented by

$$U(y) = U_0 \exp\left\{-\frac{(y - y_0)^2}{d^2}\right\}. \qquad (7)$$

We force waves of zero frequency at the center of the jet. In this case there are two critical lines located north and south of the center of the jet where the mean flow vanishes. Figure 2 shows the ray paths computed for various values of the zonal wavenumber. At the origin two rays are produced for each value of the zonal wavenumber k. The asymmetry observed between the northward and the southward rays is produced by the variation of β with latitude. Since the mean flow is independent of x and t, the values of k and σ must be constant along a ray. As the wave train propagates downstream, the meridional wavenumber adjusts to satisfy the local dispersion relation producing a refraction of the ray path. When the wave approaches the critical latitude, $l \to \infty$ and $C_{gx}/C_{gy} \to 0$, so that the rays are oriented in the north–south direction. Since the wave amplitude is inversely proportional to the meridional wavenumber, as the wave approaches the critical latitudes its amplitudes decrease, and the wave energy is absorbed.

In a zonal flow, without shear and on a beta plane, Rossby waves propagate only in the east–west direction. Their decay is related to a small amount of friction and the meanders associated with these waves can extend for long distances from its source. The sphericity of the earth and the presence of a meridional shear of the velocity field bends the wave rays causing an intersection with critical layers where the wave energy is absorbed. This prevents the waves from propagating far from their origins and reduces the lateral extension of the associated meanders in the zonal mean current.

3. Numerical experiments

To corroborate the hypothesis that the rapid decay of the meanders of eastward currents is related to the meridional shear of the current, we have designed a numerical experiment in which a zonal current is perturbed at some longitude. Two experiments are run: a "no shear" experiment, where a zonal mean flow that does not change with latitude is imposed, and a "shear" experiment, where the mean flow is represented by a Gaussian jet.

The numerical model used in these experiments solves the shallow-water equations on a rotating sphere using the scheme proposed by Sadourney (1975).

The model domain is a zonal channel that extends from 40° to 50°N and has a longitudinal extent of 132° (Fig. 3). The model has a grid resolution of 1/3° and 2/3° in the meridional and zonal directions, respectively. Cyclic boundary conditions are imposed at the lateral sides. To prevent wave reflection or spurious contamination due to the cyclic conditions, buffer regions were added near the walls and to the west of the longitude where the Rossby waves were excited (Fig. 3). In those regions the coefficient of bottom friction was increased, using a smooth function, from its value of 1/100 day^{-1} in the open domain to 1/0.5 day^{-1} in the center of the buffer zone. The form of the perturbation is

$$v'(x_0, y, t) = KU(y) \quad \text{for} \quad t \geq 0, \qquad (8)$$

where K is a constant and $U(y)$ is the background mean flow. In the linear case the qualitative character of the

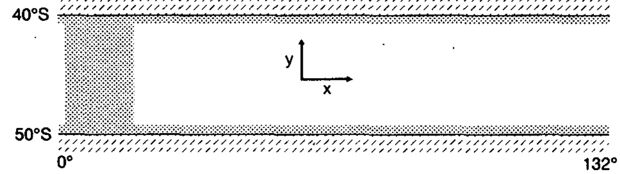

FIG. 3. Schematical representation of the domain of the numerical experiment described in section 3. It is a zonal channel on a rotating sphere that extends from 40°N to 50°N and has a longitude of 132°. The stippled areas represent regions where the coefficient of bottom friction has been increased to avoid wave reflections or contaminations due to the existence of cyclic boundary conditions.

results is independent of the value of this constant. The effect of (8) is to trigger vorticity waves by displacing fluid parcels northward of their equilibrium position. At difference of the ray tracing calculations we have chosen to force the numerical model along a longitudinal line instead at an isolated grid point. This approach helps to reduce the computational noise associated with the generation of small-scale features. It should be noted that in the case of a no shear flow $v' = 0$ on the channel walls. This small distortion of the forcing function has no effect on the interior solution because of the presence of sponge layers near the walls.

In the first experiment, we analyze the propagation of Rossby waves in a zonal current without shear. The specified current profile is $U = U_0$, where $U_0 = 50$ cm s^{-1}; it should be noted that for linear flows the qualitative character of the response is independent of the magnitude of U_0. Figure 4 shows the perturbed variables (observed minus mean value) after the model reached steady state. The results are basically those predicted by the classical Rossby wave theory. At the origin, the perturbation (8) generates waves of all wavelengths that are carried eastward by the mean flow. In the steady state only those waves whose phase speed match the mean flow velocity remain. The wavelength of the stationary wave train, of approximately 1100 km, is in good agreement with the value predicted by (1). The meridional bending of the wave path associated with the latitudinal changes in the gradient of planetary vorticity is easily identifiable in the plot of the zonal component of the velocities (Fig. 4c). The distortions observed near the walls are related to the presence of the buffer zones, which have no other effect on the numerical experiments. The alongchannel decay of the wave amplitude is related to the effect of the bottom friction.

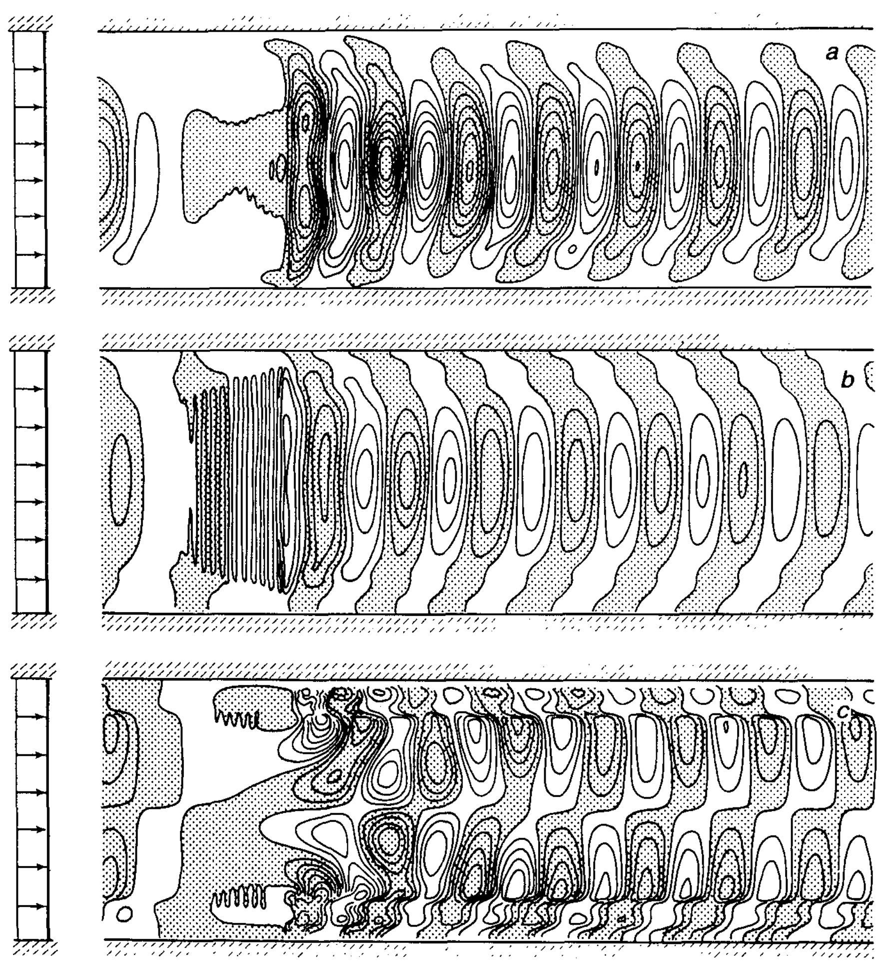

FIG. 4. Steady state of the experiment in which a zonal mean flow without meridional shear of velocities is perturbed with a time invariant forcing. (a) Perturbation (observed minus mean value) of the free surface elevation, (b) idem for the meridional velocity component, and (c) idem for the zonal velocity component. The wavelength of the stationary wave train is approximately 1100 km. The distortion observed near the walls is due to the presence of the sponge layers.

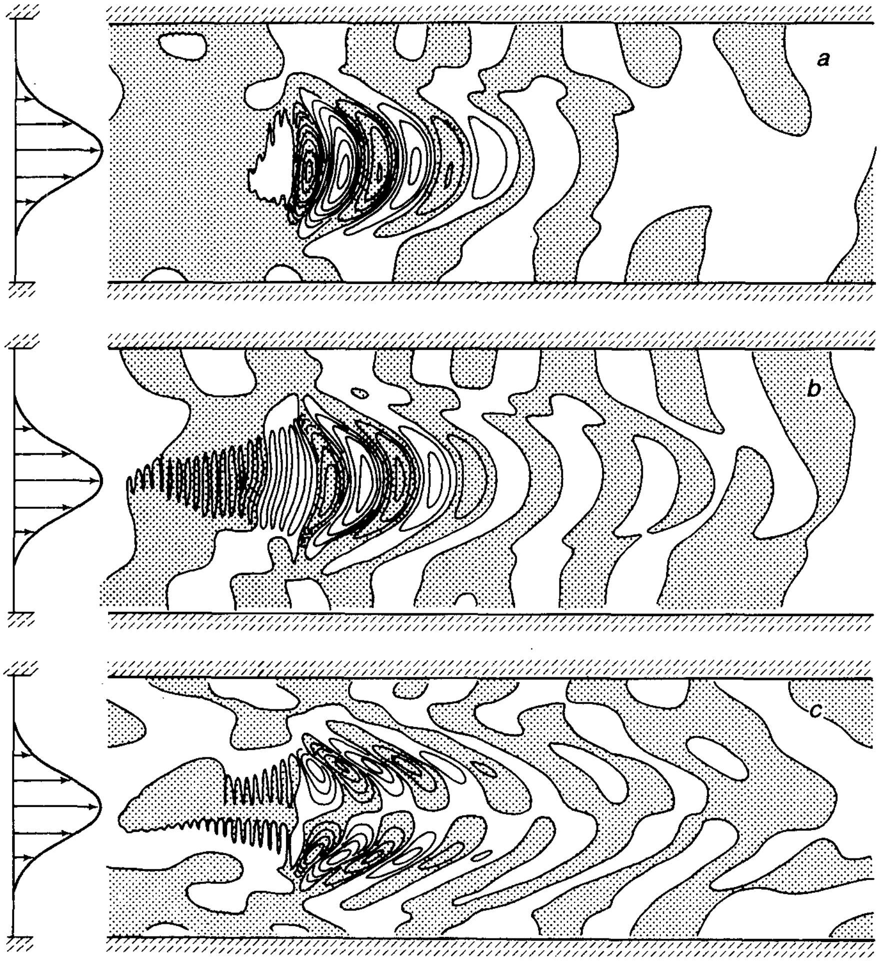

FIG. 5. Steady state of the experiment in which a Gaussian jet is perturbed with a time invariant forcing. (a) Perturbation (observed minus mean value) of the free surface elevation, (b) idem for the meridional velocity component, and (c) idem for the zonal velocity component.

In the next experiment we investigate the effect of a meridional shear on the wave propagation. The current profile in this case is a Gaussian jet with a latitudinal width of 4° and a maximum speed of 50 cm s^{-1}. Using (8) as the forcing function, the model was run until it reached a steady state. Figure 5 shows the model results for the perturbed variables in the steady state. In contrast to the previous experiment, where the perturbations generated at the origin propagated all along the channel, in this case the waves appear strongly trapped near their source with no downstream propagation. Since the coefficient of bottom friction remains unchanged from the previous experiment, the differences between Figs. 4 and 5 are solely due to the effect of the meridional shear of the mean flow. The strong inhibition of the wave propagation due to the existence of a current shear is consistent with the existence of critical layers that absorb the wave energy and hinder further wave propagation. The closeness between the WKB predictions of the previous section and the model results can be further corroborated by comparing Fig. 5c, which shows the contours of the perturbed value of the u velocity, with Fig. 2, which shows the rays of the stationary waves in a Gaussian jet. Waves that originate at the center of the current propagate toward the flanks of the jet; near the critical latitudes the waves are oriented in a north–south direction, with a marked decrease in their amplitude.

Figure 6 shows the free surface displacement in a longitudinal cross section along the center of the channel for both experiments. In the case of a constant mean flow (heavy line), the decrease in wave amplitude is due solely to the effect of bottom friction, and the waves extend all along the channel. In the case of a current with a meridional shear (thin line), the existence of critical layers inhibits the downstream propagation of waves, which are mostly confined to the source region.

4. Summary and discussion

The objective of this study was to to offer an explanation to the rapid offshore decay of the meanders of eastward currents. In earlier discussions of these features it has been assumed that the mean flow has no latitudinal dependence. In that case waves can only be attenuated by friction, with decay scales that are at least one order of magnitude larger than those indicated by observations. A WKB analysis of the potential vorticity equation and numerical experiments indicate that if the meridional gradient of velocities is taken into account then the wave propagation is inhibited by the presence of critical layers.

In the previous sections we have limited our discussion to the case of stationary waves; however, the present results can be easily generalized to waves with other frequencies. Let us consider the adjustment of a zonal current in a channel. The unforced potential vorticity equation in such a case is again (4) with the boundary condition $\mathcal{A}(0, t) = \mathcal{A}(L, t) = 0$. This is now an eigenvalue problem with $C = \sigma/k$ as the eigenvalue. This system contains the case of barotropic instability as a subset (Kuo 1949). Since we are only concerned with stable waves, we will restrict ourselves to cases where $\beta > d^2U/dy^2$. The eigenmodes can be divided into two groups. Those associated with $C \neq U(y)$ are called the discrete set. If $\beta \gg d^2U/dy^2$, they can be thought of as simple Rossby waves propagating in a mean flow. For those modes the meridional shear has no major effect. Although the discrete eigenmodes can be infinite, they are not complete (Case 1960). To represent an arbitrary function the discrete modes have to be completed by a continuum of modes, each of which has a critical layer where $C = U(y)$. The discrete modes together with the continuum of modes form a complete set. It means that the forced version of (3) can be solved by expanding the forcing in these eigenmodes. Chang and Philander (1990) discuss the application of these ideas to some simple forcing. The important point is that the character of the oceanic response is going to be dependent on whether the discrete or the continuum of modes are excited. If the discrete modes are excited then, for the case of weekly sheared flows ($\beta \gg d^2U/dy^2$), the oceanic response is as if there were no shear. However, if the forcing is mainly projected onto the continuum part of the spectrum, then the modes excited have critical layers associated with them and the zonal propagation is strongly inhibited. For an eastward flow all the discrete modes travel westward and all the continuum modes to the east. This means that only wind oscillations of long period, which have small wavenumber and westward propagation, will escape from the forcing region. This situation is represented schematically in Fig. 7, where a zonal mean current flowing in a channel is perturbed (by a wind burst for example) in the area indicated by the stippled rectangle (Fig. 7a). The points A and B mark the location of two hypothetical wave meters. If the zonal current has no shear, after a while both wave meters will be able to record the perturbations that propagate from the forcing region (Fig. 7b). If, however, the current has a meridional shear of velocities there will be only a signal propagating to the west of the forcing (Fig. 7c). In the particular case of the adjustment near a western boundary, where the westward propagation is inhibited, all the modes are associated with critical layers, which implies that all the fluctuations originated in this area are going to be dissipated in critical layers and, therefore, zonally trapped.

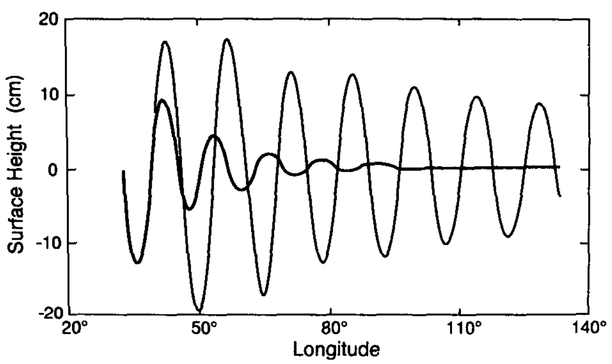

FIG. 6. Sea surface elevation in a cross section along the center of the channel for the steady state of a zonal current without meridional shear of velocities (thin line) a Gaussian jet (heavy line).

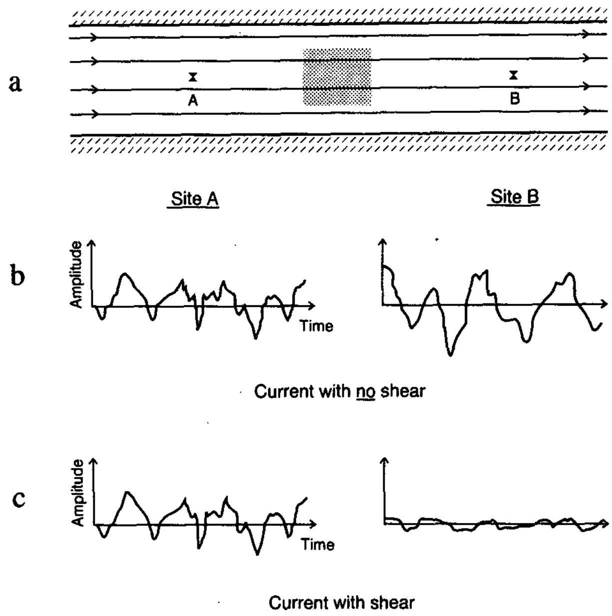

FIG. 7. Schematical representation of the adjustment of an eastward mean current to a wind burst [marked by the stippled area in panel (a)]. A and B mark the hypothetical position of two wave meters. (b) Time series of the two wave meters for a mean current without a meridional shear of velocities; (c) idem for a mean current that has a meridional shear of velocities.

The results presented in this article are only valid in the linear limit, that is, when the waves and the mean flow do not interact with each other. A reviewer correctly pointed out to us that the wave–mean flow interaction may alter the present results significantly. Killworth and McIntyre (1985) noted that in the weakly nonlinear limit the effect of the waves on the mean flow is to mix the potential vorticity gradient near the critical layer and to drive it to zero as time tends to infinity. In such a case the absorbing layer becomes a reflecting line. In our case it implies that the zonal current will become a waveguide for perturbations that no longer will be trapped near their source. Whether such a limit is ever reached in the ocean is uncertain. Observations indicate that the strong decay of meanders near the western margins of the oceans is a robust characteristic of the ocean circulation (see Fig. 1). Such a decay is also evident in numerical studies of eddy–mean flow interaction (Barnier et al. 1991), but it is unclear at this stage whether the cause is enhanced mixing by an eddy process, as suggested by Barnier et al. (1991), or the effect of critical layers.

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