MITIGATING ELEVATION-INDUCED ERRORS IN SATELLITE TELEMETRY LOCATIONS

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Abstract: Errors in Argos satellite telemetry locations are likely if transmitter elevation is specified incorrectly, but often it is impossible to specify the correct elevation a priori. To enable ex post facto corrections, I developed a geometric model of directions (θ_E) and magnitudes (r_E) of elevation-induced satellite telemetry errors. In field tests, the model explained 96–100% of the variance in θ_E ($P < 0.001$), but tended to underestimate θ_E by 1–3° ($P \leq 0.01$). The model explained 53–95% of the variance in r_E ($P < 0.001$), but tended to underestimate r_E by 41–188 m, or 3–9% of the log-normal mean error ($P \leq 0.03$). After using modeled estimates of r_E and θ_E to calculate corrected locations, I evaluated correction efficacy. Scatterplots and 95% confidence ellipses for corrected locations were qualitatively indistinguishable from those for locations calculated with no elevational error. Overall bias ($P > 0.05$) and precision ($P > 0.43$) of corrected locations did not differ from values expected in the absence of elevational error; still, some residual bias and imprecision were likely. Optimal performance is best ensured by specifying the correct transmitter elevation a priori. Where elevational errors do occur, however, the model presented here offers a tool for mitigating resulting location errors.

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The Argos satellite telemetry system requires that the elevation of a transmitter, or platform transmitter terminal (PTT), be estimated before a location is calculated. Extreme errors can occur if the estimated elevation is incorrect (Harris et al. 1990, Keating et al. 1991), but reliable a priori estimates are not always possible. For example, it may be necessary to examine calculated locations before PTT elevation can be estimated, data may have been processed before users could easily input changes in elevation, or elevation may have been incorrectly specified due to misinformation (Serv. Argos 1984) about effects of elevation on performance. A method for mitigating elevation-induced errors ex post facto is needed.

I developed a geometric model to estimate directions and magnitudes of elevation-induced errors. I evaluated the model's goodness-of-fit in controlled tests, then used modeled estimates of errors to calculate corrected locations. I evaluated correction efficacy graphically, and by testing the null hypothesis that bias and precision of corrected locations were the same as for locations calculated with zero elevational error.

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METHODS

The Error Model

To correct an elevation-induced telemetry error, its direction (θ_E) and magnitude (r_E) must be estimated. I modeled these variables based on the geometry of satellite telemetry location calculations, as described by Service Argos (1984) and R. Liaubet (Serv. Argos, Toulouse, Fr., pers. commun.).

Signals from a PTT, transmitting at about 401.650 MHz, are received by 2 satellites in

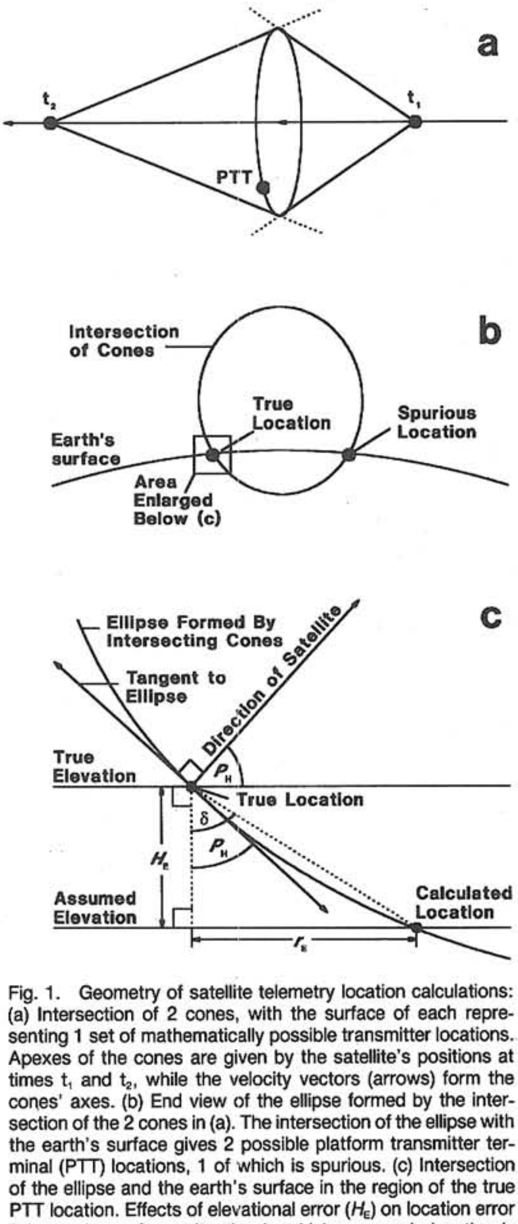

Fig. 1. Geometry of satellite telemetry location calculations: (a) Intersection of 2 cones, with the surface of each representing 1 set of mathematically possible transmitter locations. Apexes of the cones are given by the satellite's positions at times t_1 and t_2, while the velocity vectors (arrows) form the cones' axes. (b) End view of the ellipse formed by the intersection of the 2 cones in (a). The intersection of the ellipse with the earth's surface gives 2 possible platform transmitter terminal (PTT) locations, 1 of which is spurious. (c) Intersection of the ellipse and the earth's surface in the region of the true PTT location. Effects of elevational error (H_E) on location error (r_E) are shown for a situation in which assumed elevation is less than true elevation. In this case, error occurs toward the satellite (relative to true PTT location) and is given by $r_E = H_E \tan(\delta)$, where δ is the angle between the calculated location, the true location, and the vertical projection of the true location onto the surface of the assumed elevation. Maximum satellite pass height (in degrees above the horizon) is given by P_H.

circumpolar orbit, whose positions above the earth's surface (ground tracks) are known. As a satellite passes the PTT, the signals are Doppler-shifted due to the relative motion of the satellite. Thus, the frequency recorded by the satellite is higher than the transmitted frequency as the satellite approaches the PTT, and lower as the satellite recedes. Given the satellite's velocity vector, and the received and emitted frequencies, Service Argos calculates the angle between the velocity vector and the PTT. Projecting this angle in 3 dimensions defines a cone, with the satellite at the cone's apex and the satellite's velocity vector forming the cone's axis. Multiple messages enable calculation of multiple cones. The surface of each represents 1 set of mathematically possible PTT locations (Fig. 1a).

In the simplest case, 2 transmissions enable calculation of 2 angles and, thus, 2 cones (Fig. 1a). Movement of the PTT during the overpass is assumed to be negligible. Thus, the PTT must be located along the intersection of the 2 cones, represented mathematically by an ellipse (Figs. 1a and b). The intersection of this ellipse and the earth's surface (World Geod. Syst. 1984 ellipsoid) defines 2 possible PTT locations; 1 real, 1 spurious (Fig. 1b). Service Argos uses results of consecutive passes to choose between them. In practice, >2 signals usually are needed to estimate emitted frequency, which may vary from the theoretical value of 401.650 MHz.

When PTT elevation is specified incorrectly, the intersection between the 2 cones and the earth's surface is altered; location error results. For example, consider a PTT whose actual elevation is greater than assumed (Fig. 1c). The angle between the PTT and the satellite's velocity vector will be calculated correctly. Thus, the cones and the intersection of the cones also will be calculated correctly. However, the intersection between the cones and the ellipsoid (i.e., the earth's surface) will be calculated for a surface that is lower than the one that the PTT actually occurs on. This causes the calculated location to be displaced toward the satellite, relative to the PTT's true position (Fig. 1c). Defining elevational error (H_E) as the estimated minus the true elevation, it follows that elevation-induced errors occur toward the satellite when $H_E < 0$ and away from the satellite when $H_E > 0$. Thus, I modeled directions of elevation-induced errors as follows. For $H_E < 0$,

$$\hat{\theta}_E = \theta_S, \tag{1a}$$

and for $H_E > 0$,

$$\hat{\theta}_E = \theta_S - 180°, \tag{1b}$$

where $\hat{\theta}_E$ was the estimated direction of the location error and θ_S was the direction of the satellite at its maximum pass height, relative to the PTT's calculated location.

To model magnitudes of errors (r_E), I assumed that the ellipsoid describing the earth's surface is flat over the distance r_E (Fig. 1c). Letting δ be the angle formed by the calculated location, the true location, and the vertical projection of the true location onto the surface of the earth at the assumed elevation (Fig. 1c), it follows that

$$\hat{r}_E = |H_E|\tan(\delta), \quad (2)$$

where $\hat{r}_E$ is the estimated magnitude of the error.

Evaluating the Model

To evaluate goodness-of-fit and correction efficacy, I deployed 10 PTTs (Model ST-3, Telonics, Inc., Mesa, Ariz.) in 2 phases. In phase 1, I placed PTTs in a circle (radius = 2 m) around a second-order National Geodetic Survey marker near Scalplock Lookout, Glacier National Park, Montana. Given National Geodetic Survey standards (D. R. Doyle, Natl. Geod. Surv., Rockville, Md., pers. commun.), I considered horizontal and vertical controls for the site accurate to within ±0.1 m. When calculating observed location errors, I assumed PTTs were directly on the survey marker, as I considered the 2 m difference trivial. In phase 2, I placed the same PTTs on domestic dogs, horses, and mules, confined to 6 areas with radii <20 m. I disregarded data collected when animals were indoors or removed from test sites. I surveyed phase 2 locations using a Pathfinder II global positioning system (Trimble Navig., Ltd., Sunnyvale, Calif.) in autonomous mode. Position dilution of precision values were <6, so that survey errors were estimated to be <10 m for horizontal controls and <15 m for vertical controls (A. E. Jasumback, U.S. For. Serv., Missoula, Mont., pers. commun.). Elevation for 1 of the 6 sites was estimated from survey data for a nearby site and was believed to be within 20 m of the true elevation. All test sites were within 48°1′ and 48°31′ north, and 113°33′ and 114°15′ west. To introduce elevational errors, records from phases 1 and 2 were processed 5 times by Service Argos; assumed elevation was altered each time, such that the difference (H_E) between assumed and actual elevations was 0, 500, 1,000, 1,500, and 2,000 m.

Coordinates for satellite telemetry locations were reported as latitude and longitude, referenced to the World Geodetic System 1984 ellipsoid. I converted these to North American Datum 1927 equivalents, then to Universal Transverse Mercator coordinates, using NADCON4 (Dewhurst 1989) and UTMS (Carlson and Vincenty 1989) software. I also converted test site coordinates before calculating observed directions and magnitudes of telemetry errors. Calculated locations were assigned a location quality index, or *NQ*-value, by Service Argos. I considered only those locations where $NQ \geq 1$.

The error model (eqs. [1a and b] and [2]) requires estimates of H_E, θ_S, and δ. In this study, H_E was known and θ_S was estimated using Telonics Satellite Predictor software and ephemeris data from National Aeronautics and Space Administration (NASA) Prediction Bulletins. Unfortunately, δ cannot be estimated directly from information typically available to satellite telemetry users. However, δ should vary with P_H (Fig. 1c) and, in this study (where r_E and H_E were both known), δ could be estimated by rearranging equation (2):

$$\delta = \arctan\left(\frac{r_E}{H_E}\right). \quad (3)$$

Therefore, I used least squares regression to model δ as a linear function of P_H: $\hat{\delta} = b_0 + b_1 P_H$, where $\hat{\delta}$ is the regression estimate of δ, and b_0 and b_1 are regression coefficients. I estimated P_H using Telonics Satellite Predictor software and ephemeris data from NASA Prediction Bulletins. Employing cross-validation (Neter et al. 1990), I developed the regression model of δ using one-half of phase 1 data; those data were then excluded from ensuing tests of the error model.

After estimating H_E, θ_S, and δ, I estimated (eqs. [1b] and [2]) the magnitudes ($\hat{r}_E$) and directions ($\hat{\theta}_E$) of elevation-induced errors, and compared estimated and observed values. To measure overall performance, I calculated coefficients of variation (r^2) for $\hat{r}_E$ and $\hat{\theta}_E$. For $\hat{r}_E$, I calculated r^2 according to Zar (1984:271). For $\hat{\theta}_E$, I calculated r^2 as the proportion of the angular variance explained by the model

$$r^2 = 1 - \frac{s^2(\hat{\theta}_E - \theta_E)}{s^2(\theta_E)},$$

where $s^2()$ is angular variance, such that $s^2() = 2(1 - MV)$, and MV is the mean vector (Batschelet 1981:10):

$$MV = \frac{1}{n}\sqrt{[\Sigma\cos()]^2 + [\Sigma\sin()]^2}.$$

I further examined goodness-of-fit by evaluating null hypotheses 1 and 2, which follow. For tests of these hypotheses and for calculations of the r^2-values described above, I measured observed values of r_E and θ_E relative to the calculated location when $H_E = 0$ m; thus, by definition, θ_E was undefined and $r_E = 0$ m when $H_E = 0$ m. This isolated the elevation-induced portion of the error, so that tests of goodness-of-fit were not influenced by errors due to other causes.

Null Hypothesis 1.—Estimated directions of elevation-induced errors were unbiased. To test null hypothesis 1, I first calculated residual directional errors (θ_R) as $\theta_R = \hat{\theta}_E - \theta_E$, and the mean of the θ_R as (Batschelet 1981:10)

$$\bar{\theta}_R = \begin{cases} \arctan(\bar{y}/\bar{x}) & \text{if } \bar{x} > 0 \\ 180° + \arctan(\bar{y}/\bar{x}) & \text{if } \bar{x} < 0, \end{cases} \quad (4)$$

where

$$\bar{x} = \frac{1}{n}(\cos \theta_{R_1} + \cos \theta_{R_2} + \ldots + \cos \theta_{R_n}),$$

$$\bar{y} = \frac{1}{n}(\sin \theta_{R_1} + \sin \theta_{R_2} + \ldots + \sin \theta_{R_n}),$$

and n is sample size. I then used a bootstrap procedure (Manly 1991) to test the null hypothesis that $\bar{\theta}_R = 0°$. For each dataset, I calculated bootstrap estimates of expected θ_R-values (θ_R^*) for each of n randomly selected samples as $\theta_R^* = \hat{\theta}_E - \theta_E - \bar{\theta}_R$. I then estimated the expected mean of the θ_R^* ($\bar{\theta}_R^*$) (eq. [4]). I calculated 1,000 such $\bar{\theta}_R^*$-values for each dataset examined and, from the resulting distribution, estimated the probability that $|\bar{\theta}_R^*| \geq |\bar{\theta}_R|$.

Null Hypothesis 2.—Estimated magnitudes of elevation-induced errors were unbiased. To test null hypothesis 2, I calculated residual magnitudes of errors (r_R) as $r_R = \hat{r}_E - r_E$, then used a t-test to evaluate the null hypothesis that $\bar{r}_R = 0$ m.

Using modeled estimates of error directions and magnitudes, I calculated corrected location coordinates. Correction efficacy was evaluated by comparing scatterplots of corrected and uncorrected locations, and by testing 2 additional null hypotheses. For tests of these hypotheses, observed values of r_E and θ_E were measured relative to the true location.

Null Hypothesis 3.—Corrected location coordinates were unbiased. Keating et al. (1991) reported that bias of satellite telemetry locations increased with elevational error (H_E), but that locations were unbiased when $H_E = 0$ m. Therefore, effective correction for elevation-induced errors should eliminate location bias. I used Hotelling's 1-sample test (Batschelet 1981) to evaluate the null hypothesis that the mean location of the corrected coordinates was unbiased (i.e., $[\bar{x}_R, \bar{y}_R] = [0, 0]$, where $\bar{x}_R$ and $\bar{y}_R$ are the mean residual location errors in the east-west and north-south directions, respectively).

Null Hypothesis 4.—The precision of corrected locations was the same as the precision of locations calculated with zero elevational error. Elevational errors reduce precision of satellite telemetry locations (Harris et al. 1990, Keating et al. 1991). If corrections are effective, then precision of corrected locations should be unrelated to original elevational errors and indistinguishable from precision of locations originally calculated with no elevational error. Therefore, I used a Kruskal-Wallis test (Zar 1984) to evaluate the null hypothesis that, following corrections, distributions of r_E did not vary with elevational error (H_E), for H_E = 0–2,000 m. To provide measures of observed precisions, differences in precisions, and confidence limits surrounding those differences, I used a bootstrap procedure (Manly 1991). I took log-normal mean error ($\bar{r}_{E,ln}$) as the measure of precision because the r_E were approximately log-normally distributed. For each dataset, I calculated bootstrap estimates of expected log-normal mean error ($\bar{r}_{E,ln}^*$) from n randomly selected samples as

$$\bar{r}_{E,ln}^* = \frac{\sum_{i=1}^{n} \ln(r_{E,i} + 1)}{n},$$

where $r_{E,i}$ is the observed value of r_E for the ith randomly selected sample, and n is the original sample size. I then calculated bootstrap estimates of expected differences in precision ($\Delta\bar{r}_{E,ln}^*$) as

$$\Delta\bar{r}_{E,ln}^* = \exp(\bar{r}_{E,ln}^*) - \exp(\bar{r}_{E,ln,null}^*),$$

where $\bar{r}_{E,ln}^*$ is the bootstrap estimate of log-normal mean error for the dataset being examined (i.e., for H_E = 500–2,000 m), and $\bar{r}_{E,ln,null}^*$ is the bootstrap estimate of log-normal mean error for control data ($H_E = 0$ m). I calculated 1,000 such estimates of $\bar{r}_{E,ln}^*$ and $\Delta\bar{r}_{E,ln}^*$ for each dataset. I calculated final estimates of precisions and of differences in precisions as the means of those 1,000 values. I calculated 95% confidence limits for differences in precision directly from the distribution of the $\Delta\bar{r}_{E,ln}^*$. Except for bootstraps,

Table 1. Coefficients of determination (r^2) and biases of estimated directions ($\hat{\theta}_E$) and magnitudes ($\hat{r}_E$) of elevation-induced location errors, by elevational error (H_E). Bias of $\hat{r}_E$ is listed in meters and as a proportion of the log-normal mean error of the uncorrected telemetry locations (bias/$\bar{r}_{E,ln}$). Phase 1 data were gathered from transmitters placed at a survey marker, while phase 2 data were from transmitters on domestic animals. *P*-values listed below indicate the probability that bias was zero (bootstrap test for $\hat{\theta}_E$, *t*-test for $\hat{r}_E$). *P*-values associated with r^2-values are not shown, but were all <0.001.

	$\hat{\theta}_E$			$\hat{r}_E$				
H_E (m)	r^2	Bias (°)	*P*	r^2	Bias (m)	Bias/$\bar{r}_{E,ln}$	*t*	*P*
Phase 1 (n = 389 locations each)								
500	0.956	−0.9	0.065	0.534	−45.9	−0.09	−2.528	0.012
1,000	0.968	−1.1	0.014	0.834	−40.9	−0.04	−2.343	0.020
1,500	0.975	−1.2	0.002	0.912	−40.8	−0.03	−2.245	0.025
2,000	0.986	−1.1	<0.001	0.950	−31.1	−0.02	−1.738	0.083
Phase 2 (n = 121 locations each)								
500	0.974	−3.0	<0.001	0.891	−56.6	−0.08	−3.078	0.003
1,000	0.996	−2.4	<0.001	0.899	−100.4	−0.07	−2.899	0.004
1,500	0.996	−2.2	<0.001	0.902	−147.0	−0.07	−2.890	0.005
2,000	0.997	−2.2	<0.001	0.906	−188.1	−0.07	−2.842	0.005

I used SYSTAT and SYGRAPH (Wilkinson 1990*a*,*b*) for statistical analyses. Significance was defined at $\alpha = 0.05$.

RESULTS

Parameter Estimation and Goodness-of-Fit

I calculated δ (eq. [3]) and P_H for one-half of the phase 1 locations where $H_E > 0$ m (n = 1,556), then modeled δ using linear regression as ($r^2 = 0.94$, $P < 0.001$):

$$\hat{\delta} = 18.473° + 0.757P_H. \quad (6)$$

Substituting this into equation (2), I estimated directions and magnitudes (eqs. [1b] and [2]) of elevation-induced errors for the remaining 1,556 phase 1 and 484 phase 2 locations where $H_E >$ 0 m. The error model explained 96–100% of the variance in observed directions of elevation-induced errors and 53–95% of the variance in observed magnitudes of such errors (Table 1).

I rejected the null hypothesis that estimated directions of elevation-induced errors ($\hat{\theta}_E$) were unbiased. For phase 1 locations where $H_E = 500$ m, $\hat{\theta}_E$ was unbiased ($P = 0.065$). Otherwise, $\hat{\theta}_E$ tended to underestimate θ_E by about 1° for phase 1 locations and 2–3° for phase 2 locations (Table 1). I also rejected the null hypothesis that estimated magnitudes ($\hat{r}_E$) of elevation-induced errors were unbiased. For phase 1 locations where $H_E = 2{,}000$ m, $\hat{r}_E$ was unbiased ($t = -1.738$, 388 df, $P = 0.083$). For other phase 1 locations, $\hat{r}_E$ tended to underestimate r_E by 41–46 m, or 2–9% of the log-normal mean error (Table 1). For phase 2 locations, $\hat{r}_E$ tended to underestimate r_E by 57–188 m, or 7–8% of the log-normal mean error (Table 1).

Correction Efficacy

I calculated corrected location coordinates for 1,556 phase 1 locations and 484 phase 2 locations. Graphical comparisons (Fig. 2) revealed no differences between dispersions of corrected locations and dispersions of locations calculated with zero elevational error.

I failed to reject the null hypothesis that corrected locations were unbiased (i.e., $[\bar{x}_R, \bar{y}_R] = [0, 0]$). I observed bias only for corrected phase 1 locations where $H_E = 2{,}000$ m (Table 2). However, for corrected phase 2 locations where $H_E \geq 500$ m, tests of the univariate null hypothesis that $\bar{x}_R = 0$ indicated a westerly bias ($F = 4.900$–5.179; 1, 120 df; $P = 0.03$). I concluded that

Table 2. Hotelling's 1-sample test of the null hypothesis that corrected location coordinates were unbiased (i.e., $[\bar{x}_R, \bar{y}_R] = [0, 0]$, where $\bar{x}_R$ and $\bar{y}_R$ are the mean residual errors along the east-west and north-south axes, respectively). Results (*F*-statistic and *P*-value) are listed by dataset and elevational error (H_E). Phase 1 data were gathered from transmitters placed at a survey marker, while phase 2 data were gathered from transmitters on domestic animals.

	$\bar{x}$ residual error			
H_E (m)	$\bar{x}_R$ (m)	$\bar{y}_R$ (m)	*F*	*P*
Phase 1 (n = 389 locations each)				
500	29.2	15.4	0.168	0.845
1,000	30.0	50.4	0.736	0.480
1,500	32.4	77.1	1.609	0.201
2,000	34.1	114.4	3.363	0.036
Phase 2 (n = 121 locations each)				
500	−382.7	−45.7	2.511	0.085
1,000	−395.8	−6.7	2.431	0.092
1,500	−416.6	21.7	2.490	0.087
2,000	−448.8	61.8	2.754	0.068

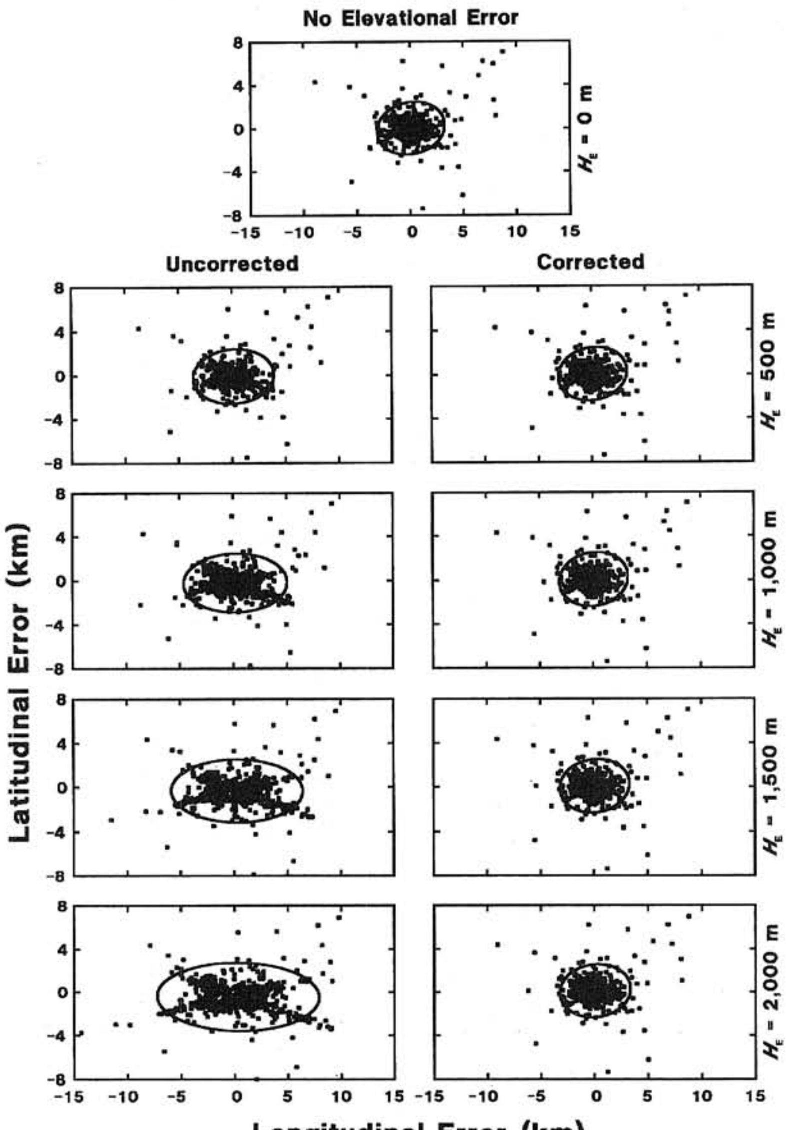

Fig. 2. Dispersion of uncorrected and corrected locations of 10 platform transmitter terminals placed on the ground (phase 1) at a second-order National Geodetic Survey marker, with 95% confidence ellipses for the distributions (n = 389 locations for each graph). The dispersion of locations calculated with zero elevational error appears at the top. Dispersions of uncorrected locations show reduced precision with increased elevational error (H_E). Dispersions of corrected locations show little difference among elevation classes, an indication of correction efficacy. Phase 2 locations (not shown here) showed greater dispersion overall, but exhibited a nearly identical pattern of correction efficacy.

corrections mitigated biases due to elevational errors, but that some residual bias was likely.

I failed to reject the null hypothesis that the precision of corrected locations was the same as the precision of locations calculated with zero elevational error. Following corrections, precision did not vary with elevational error (H_E = 0–2,000 m) for either dataset (phase 1, Kruskal-Wallis = 3.812, 4 df, P = 0.43; phase 2, Kruskal-Wallis = 0.284, 4 df, P = 0.99). However, differences between precisions of corrected locations and locations calculated with zero elevational error tended to increase with elevational error (Table 3). I concluded that corrections mitigated loss of precision due to elevational errors, but some residual imprecision was likely.

Table 3. Estimated precisions of corrected telemetry locations (calculated as the log-normal mean error, $\bar{r}_{E,ln}$, of the corrected locations) and of differences ($\Delta\bar{r}_{E,ln}$) between those precisions and the precision ($\bar{r}_{E,ln,null}$) of locations calculated with zero elevational error. For phase 1, $\bar{r}_{E,ln,null}$ = 542.8 m; for phase 2, $\bar{r}_{E,ln,null}$ = 954.8 m. Each estimate was calculated as the average of 1,000 bootstrap estimates, with each bootstrap estimate being calculated from a bootstrap sample of size *n*. Phase 1 data were gathered from transmitters placed at a survey marker, while phase 2 data were gathered from transmitters on domestic animals.

			95% CI for $\Delta\bar{r}_{E,ln}$ (m)	
H_E (m)	$\bar{r}_{E,ln}$ (m)	$\Delta\bar{r}_{E,ln}$ (m)	Lower	Upper
Phase 1 (n = 389)				
500	542.8	1.7	−85.1	84.7
1,000	544.4	9.7	−70.6	89.9
1,500	552.5	35.3	−50.6	117.2
2,000	616.7	73.9	−8.5	155.2
Phase 2 (n = 121)				
500	954.8	43.2	−181.6	282.0
1,000	954.7	43.3	−186.1	275.8
1,500	998.2	86.6	−140.6	317.7
2,000	1,023.0	112.0	−111.4	353.2

DISCUSSION

Elevation must be specified before a satellite telemetry location is calculated, and is a major determinant of location accuracy and precision (Harris et al. 1990, Keating et al. 1991), but it is not always possible to specify the correct PTT elevation a priori. Keating et al. (1991) presented empirical models of the directions (θ_E) and magnitudes (r_E) of location errors, and suggested that such models might enable mitigation of elevation-induced errors ex post facto. Using a mechanistic model, I confirmed the feasibility and efficacy of model-based corrections. The mechanistic model explained most of the observed variance in θ_E and r_E ($\hat{\theta}_E$, r^2 = 0.96–1.00; $\hat{r}_E$, r^2 = 0.53–0.95), and overall (bivariate) bias and precision of corrected locations were, in most cases, indistinguishable from bias and precision of locations that had originally been calculated with zero elevational error. Limitations of the model were indicated by a tendency to underestimate θ_E and r_E, a residual westerly (univariate) bias, and the likelihood of residual imprecision when elevational errors were especially large. Also, the sometimes low r^2-values for $\hat{r}_E$ suggested the potential for further refinement of the model, possibly by improving the estimate of angle δ.

The error model presented in this study derives from a mechanistic understanding of how elevation estimates influence location calculations, improving the empirical model of Keating et al. (1991) in 2 ways. First, the mechanistic model describes the relationship between r_E and elevational error in a single equation, whereas the earlier model required consideration of 10 equations. Second, the mechanistic model estimates magnitudes of elevation-induced errors only, whereas the earlier model estimated magnitudes of errors due to all causes. Elevation-induced errors should be distinguished because we presently have no basis for estimating the directions of (and, hence, for correcting) errors due to other causes.

MANAGEMENT IMPLICATIONS

Use of the error model requires that magnitudes and directions of the elevational errors themselves be estimated. In aquatic or low-relief environments, this is a straightforward but trivial problem. For species in high-relief environments, estimating elevational errors may be problematic. However, work with bighorn sheep (*Ovis canadensis*) (Keating, unpubl. data) suggests that digital elevation models and geographic information systems can aid in estimating at least large elevational errors (≅1,500 m) for PTTs on animals in precipitous terrain and that subsequent corrections for elevation-induced errors can improve interpretability of satellite telemetry data. In cases where elevational error cannot reasonably be estimated, the model offered here will be of little use.

Users should strive to specify PTT elevation correctly before locations are calculated. Doing so remains the best assurance of optimizing system performance and minimizing data manipulation. However, when elevational errors do occur and can reasonably be estimated, the model presented in this study is recommended for mitigating the resulting location errors.

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