

## NOTES AND CORRESPONDENCE

## A Test of Convergence of a Numerical Calculation of the Wind-Driven Ocean Circulation

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Numerical calculations based on a nonlinear model of a wind-driven ocean are described in an earlier paper published in the *JOURNAL OF THE ATMOSPHERIC SCIENCES* (Bryan, 1963). An initial value problem is solved for a shallow, barotropic ocean in a rectangular basin on the  $\beta$ -plane. The ocean is kept in motion by a simplified pattern of wind stress. The model, first proposed by Munk *et al.* (1950), has many shortcomings, but it represents a significant first step toward a hierarchy of models to describe the ocean circulation in increasing detail.

A nondimensional form of the governing vorticity equation may be written,

$$\zeta_t + R_o[\psi_x \zeta_y - \psi_y \zeta_x] + \psi_z = -\sin \frac{\pi}{2} y + \frac{R_o}{R_e} \Delta \zeta, \quad (1)$$

where

$$\zeta = \Delta \psi. \quad (2)$$

The two parameters of the problem are a Rossby number and a Reynolds number, where

$$R_o = V^* / \beta L^2,$$

$$R_e = V^* L / \nu.$$

$\beta$  is the first order variation of the coriolis force with respect to latitude and  $\nu$  is an effective lateral viscosity. The scale velocity is defined in such a way that the wind stress term,  $\sin \frac{\pi}{2} y$ , is of order unity. For further details the reader is referred to the original paper (Bryan, 1963).

The pattern of the transport stream function obtained for a case corresponding to a Rossby number equal to  $3.2 \times 10^{-4}$  and a Reynolds number of 100 is shown in Fig. 1. This calculation was carried out using a grid of uniform density with  $60 \times 120$  points. As pointed out by Stommel (1965, p. 210), in spite of the very large number of points involved, this net is still not dense enough to completely specify the very thin western boundary current. Since the amount of calculation in the original investigation was considerable, it is extremely difficult to greatly increase the resolution of the numerical grid. As a way of partially overcoming this problem, a graded grid is introduced in the present calculations which increases the resolution only in the western boundary region. The basin is divided into three areas as shown by the dashed lines of Fig. 1. From left to right the distance between grid points is increased by a factor of 2 in each area. In the area adjacent to the western boundary the resolution is 1/100 compared to 1/60 in the original calculations. This spacing represents a compromise between the need to increase resolution and at the same time avoid excessive computation. The numerical scheme is exactly the same as outlined in the previous study. Particular care is taken to join the different grids so that the advection and diffusion of vorticity are exactly continuous across the inner boundaries. It was judged that the distance between grid points should not differ by more than a factor of 2 between adjacent areas in order to avoid sharp transitions.

As before, the forcing function is applied to the system initially at rest. The variation of kinetic energy with respect to time is shown in Fig. 2. The original calcula-

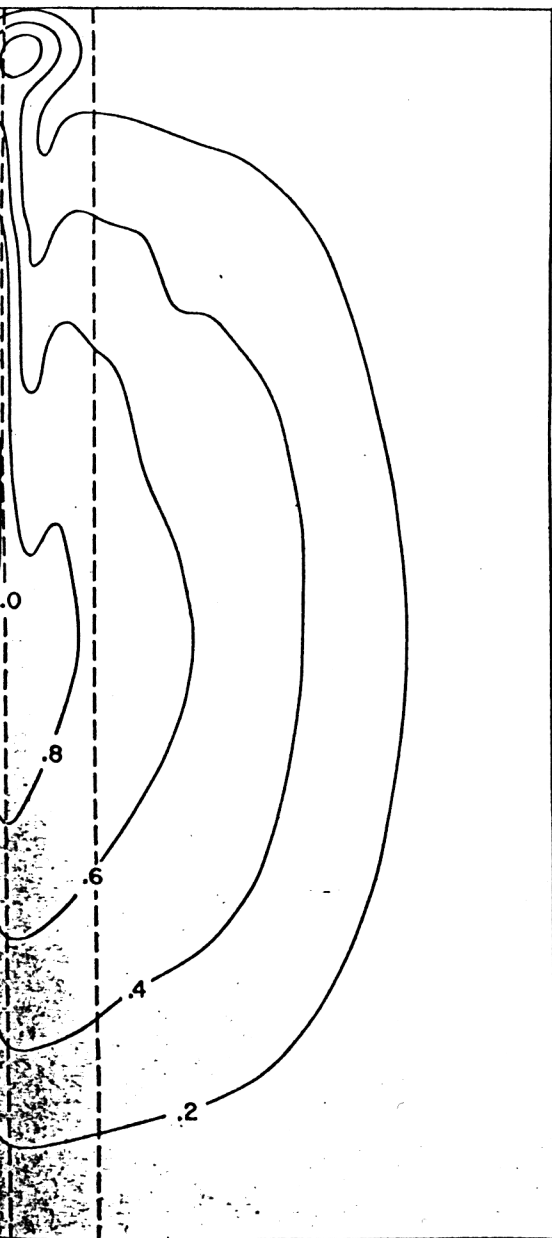


FIG. 1. The time-averaged pattern of the transport stream function for the case of  $R_0$  equal to  $3.2 \times 10^{-4}$ , and  $R_1$  equal to 100.

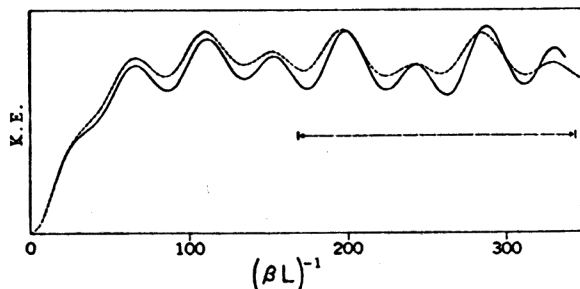


FIG. 2. The change of total kinetic energy with respect to time. The parameters are the same as in Fig. 1.

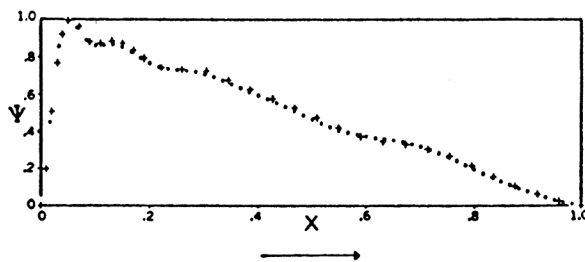


FIG. 3. The profile of the transport stream function along a line separating the north and south half-basins. The new calculation is shown by crosses.

tion is shown as a solid line and the new calculation with a graded grid is shown with a dotted line. In Fig. 3 a profile of the transport stream function is plotted along a line separating the north and south half-basins. These values represent a time average made over the interval indicated by arrows in Fig. 2. The results of the new calculation are shown by crosses and the original calculation by dots. Quite satisfactory agreement is indicated.

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REFERENCES

Bryan, K., 1963: A numerical investigation of a nonlinear model of a wind-driven ocean. *J. Atmos. Sci.*, 20, 594-606.  
 Munk, W. H., G. W. Groves and G. F. Carrier, 1950: Note on the dynamics of the Gulf Stream. *J. Mar. Res.*, 9, 218-238.  
 Stommel, H., 1965: *The Gulf Stream; A physical and dynamical description*. Cambridge, Cambridge University Press, 2nd ed., 243 pp.