NOAA
Professional Paper NOS 2

NORTH AMERICAN DATUM OF 1983

Charles R. Schwarz
Editor

National Geodetic Survey
Rockville, MD 20852
December 1989

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration

DR. WIILLIAM BOWIE
1872-1940

Chief of the Geodesy Division
United States Coast and Geodetic Survey
1909-36

In 1913 William Bowie persuaded the geodesists of Canada and Mexico to formally connect the surveys of their respective countries to the existing United States Standard Datum. Because of the new international character, and at Bowie's suggestion, the Superintendent of the United States Coast and Geodetic Survey directed that the name be changed to the North American Datum.

During Bowie's tenure as chief of the Geodesy Division the decision was reached to carry out a general readjustment of the triangulation networks. Bowie is credited with both designing the method of computation and supervising the work which resulted in the North American Datum of 1927. Soon after the 1927 readjustment, the geodesists of Canada and Mexico also readjusted their triangulation to make it consistent with the new datum, so that the continental character was preserved.

Thus William Bowie was largely responsible for both the original North American Datum and the North American Datum of 1927. This report, describing the third horizontal geodetic datum of continental extent in North America, is dedicated to him.

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EDITOR'S PREFACE

This report is written primarily from the viewpoint of the National Geodetic Survey (NGS), a component of the Office of Charting and Geodetic Services, National Ocean Service, National Oceanic and Atmospheric Administration. NGS represented the United States and served as coordinator for the international project which resulted in the North American Datum of 1983 (NAD 83). Some sections of the report describe the project from the point of view of other participants.

At the National Geodetic Survey, this undertaking was called the New Datum Project, since the new datum was not actually named until the project was well underway. It was the largest single activity at NGS from 1974 until 1986, consuming most of the resources of the Horizontal Network Branch and significant resources from other parts of NGS. The project was an object of management interest and attention throughout its lifetime.

This report covers the background of the project. It also describes the actual execution, including the inventory of data used, the long laborious task of building the data base, the computations themselves, and the datum implementation activities. It does not include the actual coordinates, which must be ordered from the National Geodetic Information Branch, NOAA.

The report is intended to serve as a record of what was actually done during the new datum project. Much of the material has already been released as technical papers, presentations, and journal articles. This report collects together much of that literature, augments it with new material, and adds the benefit of hindsight.

The authors and editors of this report have attempted to emphasize the many decisions that were made during the planning and execution of the project. They describe many of the alternatives that were considered and provide the rationale for many of these decisions.

The authors also describe the project from the point of view of those people who were actually working on it. Several sections describe the computer programs that were written and the tasks that were actually performed. For many of those people, this report will explain how their contributions fit into the overall project.

Many activities had to be coordinated to carry out the NAD 83 project. No one person had in-depth knowledge of all of them. Therefore, this report has been assembled from the contributions of several authors. As editor, I have attempted to coordinate the various sections and to achieve some consistency of style.

I thank the many authors for their contributions. It was not always easy for the authors to remember and articulate decisions and actions that occurred over a span of many years. Without their cooperation this report could not have been assembled.

I also thank Joseph F. Dracup, B. K. Meade, and Charles A. Whitten for their reviews of the typescript. Each of these individuals had played an important role at the National Geodetic Survey during the period when the New Datum Project was being established. Each retired before the project was completed. Fortunately, each was able to play another important role at the end of the project by adding their wisdom and experience to this report.

The tasks of copy editing and production of the entire document were in the very capable hands of Eleanor Andrée.

The new adjustment of the North American Datum spanned an entire decade. During this period more than 300 persons committed themselves to the completion of this immense task. Unfortunately, the contributions of each employee cannot be acknowledged individually, but this publication is a testimony to their dedication, cooperation, and perseverence.

Charles R. Schwarz
Rockville, Maryland
January 1989

FOREWORD

The redefinition of the horizontal geodetic control network in North America was a large project which spanned 12 years, produced a large set of accurately positioned control points, and resulted in significant savings to the taxpayer. The savings occurred because the project's scope, originally defined to include only those data essential for the redefinition, was immediately broadened to include the automation, checking, and management of all the information then resident in the archives of the National Geodetic Survey. Because all data would have had to be automated eventually, a massive economy of scale in digitizing, editing, and managing these data was achieved by including all data in the automation phase of the NAD project. For example, digitization of narrative information (descriptions) associated with control stations would not have been included in the original scope of the NAD project. Automation of these data was a major project in itself, and it would have been considerably more expensive had it been undertaken as an independent effort. These data are now automatically retrievable and fully integrated with the positional data.

It is most satisfying to all of us involved with the NAD project that both redefinition and total automation have been completed and that this was accomplished in a manner quite close to the way we originally intended. By enlarging the scope of the project at its outset, we defined a large and ambitious task. It is important to understand that in 1974 formats for keying the various data were not even defined, let alone the strategy for the adjustment, international aspects, introduction of new technologies, and dozens of other complicated issues and considerations. The details about how this was accomplished are the subject of this report.

I was named project manager in 1974 and began to hold a series of highly enjoyable "NAD staff" meetings. It was in those meetings that most of the fundamental decisions were made. These included decisions to make the datum geocentric, to include all the existing data in the adjustment (this decision was made almost entirely on economic considerations given the earlier decision to convert all the data to computer-readable form), to obtain an elevation for every point, to use the originally observed data (abstracts) instead of combined data (summaries), and many others. Major decisions were highly consensual with the opinion leaders involved in the project playing their appropriate and typically forceful roles. In instances where issues were unclear and polemic, the NAD staff resorted to an unstated but implicit rule that, if forced to paper, would read something like this: "In the absence of compelling scientific or economic information on which to base a decision, we will do what we believe is best in the long term, incorporating both scientific and economic considerations." Time will tell whether these decisions can be considered optimum. Given this process, it is still clear that ultimately one individual had to be responsible, and I was that individual. I take full responsibility for all the decisions made prior to my relinquishing the project manager position in 1983. The project managers who followed me, John Gergen (1983-84) and Libby Wade (1984-86), had the task of completing the project. They too had to make many hard decisions right up to the project's completion in July 1986.

This report is the capstone of the NAD 83 project, a project that is a testimony to the perseverance, dedication, and excellence of a large number of Federal employees, both in the field and in the office. It is my hope that the search for quality and excellence inherent in the NAD 83 project will continue and flourish in the future.

John D. Bossler
Columbus, Ohio
February 1989

OVERVIEW

The North American Datum of 1983 (NAD 83) is the third horizontal geodetic datum of continental extent in North America. It is intended to replace both the original North American Datum and the North American Datum of 1927 (NAD 27) for all purposes. Both were established by the U. S. Coast and Geodetic Survey (C&GS), predecessor of the National Ocean Service (NOS).

The establishment of NAD 83 was the result of an international project involving the National Geodetic Survey (NGS) of the United States, the Geodetic Survey of Canada (GSC), and the Danish Geodetic Institute (responsible for surveying in Greenland). The geodetic data in Mexico and Central America were collected by the Inter American Geodetic Survey and validated by the Defense Mapping Agency Hydrographic/Topographic Center.

The fundamental task of NAD 83 was a simultaneous least squares adjustment involving 1,785,772 observations and 266,436 stations in the United States, Canada, Mexico, and Central America. Greenland, Hawaii, and the Caribbean islands were connected to the datum through Doppler satellite and Very Long Baseline Interferometry (VLBI) observations.

The computations were performed with respect to the ellipsoid of the Geodetic Reference System of 1980 (GRS 80), recommended by the International Association of Geodesy (IAG).

The parameters of this ellipsoid are

$$a = 6378\ 137 \text{ meters (exactly)}$$
$$1/f = 298.257\ 222\ 101 \text{ (to 12 significant digits)}$$

The ellipsoid is positioned in such a way as to be geocentric, and the orientation is that of the Bureau International de l'Heure (BIH) Terrestrial System of 1984 (BTS-84). In these respects, NAD 83 is similar tö other modern global reference systems, such as the World Geodetic System of 1984 (WGS 84) of the U.S. Defense Mapping Agency (DMA).

The BTS-84 system was realized by applying a shift in Z of 4.5 m, a rotation around the Z axis of -0.814 arc seconds, and a scale correction of -0.6 parts per million, to Doppler-derived coordinates in the Naval Surface Warfare Center (NSWC) 9Z-2 system.

Within the United States, the new datum project was treated as a simultaneous adjustment of all data in the horizontal control network. In practice, this meant that NGS attempted to validate all data in its holdings, regardless of order, class, purpose, or geographic location of the survey, and that all validated data were used in the adjustment. In Canada, Mexico, Central America, and Greenland, only the framework surveys were used, with the intention of fitting lower order surveys into the adjusted framework at a later time.

More than 95 percent of the stations included in the adjustment are within the United States. This is partially a result of the difference in approach and partially due to the greater amount of survey activity within the United States.

The NAD readjustment project involved a detailed analysis of crustal deformation in those areas of California, Nevada, Alaska, and Hawaii where horizontal crustal motions were thought to be significant. Models for these motions were developed and used to replace observed values with estimates of values that would have been observed on December 31, 1983. Thus essentially all historical observations were used in the adjustment. Significant information concerning crustal motion was gained in the process.

The readjustment project also included the computation of geoid heights and deflections of the vertical at all 193,241 occupied control points. Most of the deflections were computed by the method of astro-gravimetric leveling, using approximately 1.4 million gravity points and 5,000 observed astro-geodetic deflections.

Because the chosen reference ellipsoid is geocentric and best-fitting only globally, geoid heights with respect to NAD 83 are large. However, predicted values for both geoid heights and deflections of the vertical were computed and their effects were fully accounted for in the computations.

The mathematical model for the NAD readjustment was the height-controlled three-dimensional system. This formulation is fully equivalent to the projection method of survey adjustment (with estimated geoid heights and deflections). Furthermore, it is conceptually simpler than the classical model for observation equations on the ellipsoid and is therefore easier to program for computers. Most important, it facilitates the combination of terrestrial data with space systems data (such as Doppler positions and VLBI position differences) in a straightforward way.

In addition to the expected latitude and longitude coordinate unknowns, the solution included scale factor unknowns for many groups of Electronic Distance Measuring Instruments (EDMI). Additional parameters related the coordinate systems of the terrestrial observations, the Doppler data, and the VLBI data to the final coordinate system.

The U.S. portion of NAD 83 contains 258,982 stations, classified as follows:

First order	39,460
Second order	95,013
Third order	60,821
Intersected landmarks	63,234
unclassified	454

These stations were connected by the following inventory of terrestrial data:

First-order directions	392,426
Second-order directions	467,763
Third-order directions	400,912
Fourth-order directions	279,989
Astronomic azimuths	4,470
EDM distances (lightwave instruments)	124,328
EDM distances (microwave instruments)	25,642
Taped distances	38,659

In the conterminous United States and Alaska there were 655 Doppler position observations (with three components each) at 612 stations (some stations were occupied more than once). An additional 11 Doppler position observations at 10 stations were used to position the Hawaiian network.

The adjustment also involved 112 VLBI position difference observations (with three components each) involving 45 stations. These observations were taken in 26 groups. Initially each group was treated as a separate coordinate system, but in the final solution all the coordinate system unknowns were set equal to a single set of VLBI coordinate system parameters.

At sites where Doppler or VLBI observations were taken, it was sometimes necessary to relate the reference points of the various observing systems with three-dimensional eccentric ties. There were altogether 45 such ties, 40 accomplished by conventional surveying and 5 performed with GPS observations.

The participating network in Canada consisted of 7,454 stations and 44,347 observations. These were composed of:

Directions	28,460
Distances	10,333
Azimuths	398
Doppler position components	726
Doppler position difference components	4,430

The participating network in Greenland consisted of about 400 stations and included the following observations:

Doppler satellite and GPS stations	200
First-order directions	883
Astronomic azimuths	12
EDM distances (microwave instruments)	153
Taped distances	11

There exist survey ties between Canada and the northwest coast of Greenland. However, these belong to the second-order network in Canada and did not participate in the fundamental adjustment. Therefore, the Greenland networks are brought into NAD 83 only through the Doppler and GPS observations.

The horizontal survey network in Mexico, Central America, and the Caribbean islands, exclusive of Puerto Rico and the U.S. Virgin Islands, consisted of 1,884 stations established by first-order triangulation and traverse methods. Observations among these stations included 9,970 directions, 82 Laplace azimuths, 55 base lines (Invar and Geodimeter) and 4,000 km of traverse. These observations were transferred to NGS, where they were revalidated and merged into the NGS data base. They were thereafter treated as part of the U.S. network.

The least squares adjustment generated a system of 928,735 simultaneous linear normal equations in 928,735 unknowns. Fortunately, the coefficients of these equations were extremely sparse, so that an exact solution of this very large system was feasible. The formation and solution of these equations were partitioned according to the Helmert blocking method. NGS divided the continental U.S. network into 161 first-level blocks (plus two blocks for Hawaii). These were combined according to a binary strategy, resulting in a total of 321 blocks. Interior unknowns were eliminated at each level, leaving junction points along the U.S.-Canada border.

The Geodetic Survey of Canada processed its data according to a similar scheme, partitioning the Canadian primary framework into 17 blocks of terrestrial data and 3 blocks of GPS and Doppler data. Interior unknowns were eliminated, leaving a block of 953 junction unknowns along the U.S.-Canada border.

At the end, reduced normal equations generated from the U.S. and Canadian data sets were combined and the combined equations were then solved for the coordinates of the junction points. These computations were carried out in parallel by both NGS and GSC, each as a check on the other.

In the United States the computations were carried out on an IBM 3081 computer, using an automated system of computation, scheduling, and data management. The specialized software for this purpose was written by NGS programmers in the PL/1 language.

The iterative solution process was carried through three cycles of linearization to ensure convergence. Small data corrections were also allowed after the first and second solutions. The three cycles of linearization and solution required more than 940 hours of computer CPU time (IBM 3081). By the last solution, after all data problems had been resolved, the entire cycle could be accomplished in 3 to 4 weeks.

The datum shifts between NAD 27 and NAD 83, as shown in figures 21.1 through 21.8, can be as large as 100 m. This change is large enough that it must be considered in most applications using coordinates. It is detectable on large scale maps and charts, such as the NOS series of harbor and small craft charts and the 1:24,000 scale maps of the U.S. primary topographic mapping program.

The differences in coordinates have both smooth and random components. The part of the coordinate change which is due to a change of reference ellipsoid is systematic and smooth, while the part which arises from the removal of the distortions in NAD 27 is random and unpredictable. The latter part can amount to more than 15 m. It is the presence of this random component which means that the coordinate differences cannot be predicted or exactly represented by mathematical formulas. Instead, they must be represented by tables or graphs.

When the project began, almost no data were in machine-readable form. Furthermore, the survey data had been acquired over a period of 150 years and were stored in a variety of hard copy formats. The initial tasks involved finding the old data, assessing the usefulness of each survey project, placing all data in machine-readable form, and validating the entire data set. To manage the data NGS constructed a Geodetic Data Base Management System. This was the major tool for merging different data types, for providing a single consistent view of all data, and for providing global seamless access to the data. The geodetic data base environment also provided many of the validation tools. All relevant data were loaded into the geodetic data base before the actual adjustment began.

Validation of the data set took place in three major stages. First, each of the approximately 5,000 survey projects was adjusted as an independent entity. Each of these was a minimum constraint adjustment. The purpose of these adjustments was (1) to ensure that the observations necessary to connect all the stations in the network were present, and (2) to detect keying and other blunders which would manifest themselves as large residuals. The coordinates resulting from these adjustments were not used. Several hundred projects were dropped and several hundred others were added during project validation, resulting in a data set of 4,997 projects.

The second stage was block validation. The data set was rearranged into 843 geographic blocks of 300 to 500 stations each. The block boundaries were drawn without respect to project boundaries, and as a result many observations crossed block boundaries. Each block was adjusted as a separate entity, with the same purposes as the project validation adjustments.

The third and last stage of validation was the continental adjustment itself. The first linearization and solution established that the normal equations could be solved and that the network therefore was properly connected. A few remaining data problems, mostly involving observations that crossed block boundaries, still had to be resolved at this point.

The data set which was validated included observations to azimuth marks and reference objects, since prior to validation it was not known which of these marks could be positioned and thereby become part of the fundamental network. The validated data set contained approximately 2.5 million observations, of which 1.7 million participated in the fundamental adjustment.

After completion of the adjustment, the new coordinates were loaded back into the geodetic data base. This is now the basis for computer-assisted publication of the new coordinates, which are being provided to users in a variety of formats and media.

The NAD 83 project extended from July 1, 1974, to July 31, 1986. During these 12 years it was the largest single project at NGS. The cost to NGS was approximately $37 million. Of this, the largest single cost was the building of the data base, which included keying and extensive validation of archival data. New surveys and their processing accounted for less than 25 percent and the actual Helmert block adjustment computations accounted for less than 10 percent of the total cost.

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Part IV: Project Execution and Results (continued)

Part V. Implementation Issues

1. EARLY HISTORY AND FORMULATION OF THE PROJECT

1.1 THE IDEA OF A NEW ADJUSTMENT

The idea of performing a general adjustment of the horizontal control networks in North America began as an increasing awareness of the inadequacies of the existing North America Datum of 1927 (NAD 27). These inadequacies were ascribed to several causes, rooted both in the sparsity of the data used in the 1927 adjustment and in the way the network had grown since then.

The network became inadequate because it was weak in relation to the increasing demands that were placed upon it. The weaknesses became apparent in several ways. Surveyors were buying accurate electronic distance measuring (EDM) equipment and finding unexplainable discrepancies between the existing control network and the distances measured by their new instruments. Missile ranges and satellite tracking systems demanded their own independent surveys. The geodesists of the National Geodetic Survey experienced increasing difficulty in fitting new urban surveys into the existing NAD 27 system.

By the late 1960s it was widely recognized that the existing datum could not be easily repaired and that a new network adjustment was required. In 1967, as a means of obtaining support for the idea of a new adjustment, Capt. Leonard S. Baker, Chief of the Geodesy Division, sent questionnaires seeking the opinion of other Federal agencies, the private sector, and the academic community. This step was followed in 1968 with a seminar which allowed invited representatives of these groups an opportunity to discuss the issues. Geodesists in the United States discussed the need for a new datum with their counterparts in Canada, who were experiencing similar difficulties with the old system (Whitten and Burroughs, 1969). There was soon widespread agreement that a continental readjustment was needed.

Details of how such a continental readjustment might be carried out were not set at that time, but it was clear that it would be a major project, beyond the capabilities of the ongoing network maintenance program. A budget enhancement, as well as significant reprogramming of existing activities, was needed. Dr. Charles Whitten had recently been appointed U.S. Chief Geodesist, and was in a position to propose a major project. Therefore, he and others prepared a series of proposals and issue papers requesting budget authority to embark on this project.

Capt. Baker initiated the request from the Environmental Science Services Administration (predecessor to NOAA) to the National Academy of Sciences (NAS) for advice concerning the benefits of a new adjustment. A special study group (National Academy of Sciences/National Academy of Engineering, 1971) reviewed the need for a new adjustment and provided the endorsement of an independent agency. Furthermore, the report was prepared by a panel with strong representation from the academic and engineering communities and therefore credibly expressed the needs of the users of the geodetic control networks.

By the time the NAS report was issued, the general outlines of the project were taking form. For instance,

1. The readjustment was to be continental in extent, making it necessarily international in scope.
2. The objective was to provide an entirely new set of horizontal coordinates, completely replacing NAD 27 for all points and for all surveying, mapping, and engineering purposes.
3. The old survey data were still valid and would be used. Some new surveys would also be performed and new data sources would play an important role.
4. The new datum would make use of the data produced by the significant investments being made in satellite geodesy and would be consistent with the satellite systems of the future.
5. The new datum should be a part of a world geodetic system, using a geocentric best fitting ellipsoid as a reference surface.
6. The determination of geoid heights and deflections of the vertical should be a part of the project.

Some aspects of the project described in the NAS report were later modified. For instance,

1. The original plan called for the completion of the North American Densification Project of the Satellite Triangulation Program (also called the BC-4 program). By 1973, the BC-4 data source was replaced by the rapidly accumulating set of more accurate surveys based on Doppler satellite tracking.
2. The original plan placed great emphasis on the Transcontinental Traverse surveys. Some geodesists felt that this implied a hierarchical approach, in which the Transcontinental Traverse would be adjusted by itself, as a kind of "super first-order" or "zero-order" control. First-order networks would then be adjusted to the traverse, and second-order surveys would be adjusted to the first-order points. A competing concept was the simultaneous adjustment of all data, in which each observation would be used according to its individual weight. The NAS report was actually silent on this point, but the latter concept was eventually selected as the most effective way of accomplishing the final objectives.

The findings expressed in a second influential document, "Report of the Federal Mapping Task Force on Mapping, Charting, Geodesy and Surveying" (Office of Management and Budget, 1973), agreed that a new adjustment was necessary, although not endorsing the C&GS proposed approach. Moreover, the report stated, "10 years is too long to wait" and proposed instead an accelerated 5 year program using a hierarchical approach. By 1973 the effectiveness and efficiency of the Doppler method of satellite geodesy had become clear. The OMB report ignored the BC-4 data and recommended instead using Doppler surveys.

1.2 FUNDING THE NEW ADJUSTMENT

As often happens, there were several different estimates of what the new adjustment of the North American Datum might cost. The 1971 National Academy of Sciences report described the following incremental costs:

	($M)
Satellite triangulation (BC-4)	8.7
Transcontinental Geodimeter traverse	3.9
Conventional base lines and azimuths	1.3
Computations, analysis, and adjustment	5.0
Total	18.9

The satellite triangulation portion dominated this budget because the completion of the BC-4 program would require the purchase and launch of a new balloon satellite (Echo I and Echo II were no longer usable). The office task of building the data base from existing observations was not listed, even though it was estimated that 200 staff-years would be required for the preparation and re-evaluation of about 2 million old field observations. Apparently this cost was considered to be part of the ongoing base program of network maintenance, and therefore not an incremental cost.

By 1973 the C&GS proposal included the use of Doppler satellite surveying instead of extending the BC-4 satellite triangulation program. This significantly reduced the overall costs. The 1973 OMB report reflected this decision. It described a program with a total cost of about $10.8 million, of which $8.8 million would be spent in the first 5 years. This budget was also dominated by the cost of field work. The cost of computations, analysis, and adjustment was reduced to $4 million. There was no consideration of the incremental cost of building a computer-readable data base.

The new adjustment finally appeared in the NGS budget for fiscal year 1975, which actually began July 1, 1974. This is taken as the official beginning of the new adjustment project, although NGS had been engaged in preparation for several years. The FY 75 budget contained an increase of $1.5 million per year to the national geodetic control network program. This increase was to continue for 5 years and be supplemented by $1.7 million per year of reprogrammed funds, for a total program cost of $15.7 million. NOAA management reduced the funding increase to $750,000 per year and extended the program out to 8 years.

With the commencement of the project, John D. Bossler was appointed project manager by the NGS Director. He continued to be involved with the project as he later occupied successively more responsible positions as Deputy Director of NGS, Director of NGS, and finally as Director of the Office of Charting and Geodetic Services, the parent organization of NGS. He was succeeded in the position of project manager by John G. Gergen and Elizabeth B. Wade.

Based on the 8-year program, more detailed project plans were prepared. These projected that the computations would be completed by late 1982, with the publication of the results scheduled for 1983 (Bossler, 1978). It was therefore agreed that the new datum would be called the North American Datum of 1983 (NAD 83). This decision was officially announced in the *Federal Register of* June 29, 1979 (Office of Federal Register, 1979). By that time completion was "expected in 1983-84, with publication of the results to take another 12 months."

With the official beginning, more attention was given to the office tasks of building and validating the data base and carrying out the actual adjustment computations. These tasks involved new activities. It was difficult to estimate their costs because there was very little applicable experience on which to build. However, it quickly became apparent that the office costs would be substantial and might well dominate the project.

By 1978 the cost of the project in the United States was estimated at $20.7 million, including both new and reprogrammed funds (Bossler, 1978). The Transcontinental Traverse was completed, but the amount of additional field work was reduced.

Table 1.1 contains an early estimate describing the costs of the office work based on the number of points surveyed. Since the network was estimated to contain 250,000 points, this estimate projected a cost of $23 million for office costs alone.

TABLE 1.1.—*Estimated new adjustment costs per point*

Task	Labor ($)	Computer ($)
Data preparation, keying of observations, and project level validation	25	25
Keying and validation of descriptions	10	2
Block validation	20	5
Helmert blocking adjustment	3	2

The picture of the true costs of the new adjustment emerges in the above table. The Helmert block adjustment itself was a minor cost. The real cost was in forming and validating the data base, primarily the labor of the analysts who prepared the data and resolved the many data problems that arose.

Over the years the New Datum Project was delayed and extended for a variety of reasons. Although there were many follow-on and implementation activities, the official end of the project came with the last iteration of the solution on July 31, 1986. By then the project had actually lasted 12 years. Table 1.2 reflects the costs of the new adjustment in the United States. By the end of project, about 75 percent of the total had been spent for office activities. The allocation of office costs, as projected in table 1.1, turned out to be quite accurate.

TABLE 1.2.—*NAD 83 costs by fiscal year*

Year	Cost ($K)
1975	3,407
1976	3,558
1976 T[1]	720
1977	3,260
1978	3,020
1979	2,989
1980	2,658
1981	2,342
1982	3,315
1983	3,586
1984	3,771
1985	3,358
1986	1,239
Total	37,223

[1] A 3-month transitional fiscal year.

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2. NORTH AMERICAN DATUM OF 1927

2.1 THE NEED FOR A NEW DATUM

Prior to 1927 the horizontal reference system of the United States was the original North American Datum. This was essentially an extension and renaming of the U.S. Standard Datum, which is described in *Special Publication* 19 (Bowie, 1914). On some charts, particularly those of the Great Lakes, this was referred to as the North American Datum of 1902.

Within a few years of the adoption of the original datum, the same problems were arising that would arise again in 1969. As stated at the time:

> The [original] geodetic datum was adopted when the triangulation system consisted of a meager skeleton joining the arc near the Atlantic coast to that near the Pacific coast. As this mere outline was supplemented by new work, the discrepancies in the closures of loops were adjusted into the new arcs, the part already adjusted being held fixed. . . . As the country became more divided up by many closed arcs of triangulation, this method of adjustment became more and more objectionable because often comparatively short arcs were forced to absorb loop closures that were out of all proportion to their lengths, and as a result the corrections that had to be applied to them were unduly large. (Adams, 1930a)

Furthermore, the final arcs for the framework in the western part of the country had been completed in 1926. Most new work would consist of filling in the framework. This was an appropriate time for a general adjustment of the entire network.

2.2 NAMING THE NEW DATUM

The coordinates resulting from the adjustment of the western half of the network were published in a series of hardcover reports, one for each state. In the first of these state reports we find what appears to be the first use of the phrase "North American Datum of 1927":

> The date is appended to the name of the new datum to distinguish it from the old North American Datum. . . . Only positions on the North American datum of 1927 should be used hereinafter. . . . (Adams, 1930b)

2.3 THE 1927 DATUM PARAMETERS

In the 1927 readjustment one station was held in position. This station, MEADES RANCH, was assigned the same position that it had in the original North American Datum. The following reasoning was given:

> After a careful analysis of the agreements and disagreements of the geodetic and astronomic longitudes and latitudes at many stations of the existing triangulation, the late Dr. John F. Hayford, then in charge of the geodetic work of the United States, selected a longitude and latitude for a triangulation station called MEADES RANCH, in Kansas near the geographical center of the United States. The coordinates thus selected approached the ideal datum which would make the sum of the squares of the differences between the astronomic and the geodetic latitudes and longitudes a minimum. . . . (Bowie, 1928)

Furthermore,

> In selecting a datum for the United States Hayford decided that the Clarke spheroid of 1866, as expressed in meters, which had been used for many years by the U.S. Coast and Geodetic Survey for its triangulation, was the most practicable one for the new datum. (Bowie, 1928)

Also,

> The orientation in the new adjustment is controlled by the various Laplace azimuths distributed through the network of arcs. The position of MEADES RANCH, together with the Laplace azimuths included in the arcs, serve to define the North American Datum of 1927. (Adams, 1930b)

2.4 THE METHOD OF COMPUTATION

The task of carrying out the computation was assigned to Oscar S. Adams. He wrote *Special Publication* 159, which described the computations for the western half of the country. Details of the computational aspects for the adjustment of the eastern half were described in various reports but never in as complete a form as that for the western half. The entire process occupied 5 years from 1927 to 1932.

A true simultaneous least squares adjustment of the entire network was far beyond the computational capabilities available in 1927. The network was therefore partitioned. The western half was adjusted first, since that part contained the most recent surveys and the work most in need of readjustment. The eastern half (without the eastern seaboard network) was adjusted later. All the control points on the 98th meridian arc were held fixed at their values from the western half adjustment. Figure 4.2 shows the survey network used in the 1927 adjustment.

Adams estimated that a true least squares adjustment of just the western region would still require the solution of more than 3,000 simultaneous linear equations. This was considered to be possible, but it would require much more work than could be justified economically.

An economically feasible method for making the adjustment was worked out by Dr. William Bowie, then chief of the Division of Geodesy. By the Bowie method the network is adjusted by analogy with level networks. Junction figures, consisting of the survey figures where arcs of triangulation intersect, are treated as single junction points. Arcs of triangulation between the junction figures play the role of sections. The misclosures of latitude and longitude around a loop are computed and distributed in much that same way that elevation misclosures are distributed in an adjustment of leveling. Figure 2.1 shows how the arcs of triangulation were arranged as loops.

The step-by-step instructions for the Bowie method are as follows:

1. Identify the junction figures. In general all the survey work at the junction between arcs is included in such a way that an arc of triangulation connects to the junction figure only along a single line, involving only two points. However, it is also necessary that each junction figure include at least one azimuth and at least one distance. It may be necessary to extend the junction figure somewhat out along an arc in order to include a measured azimuth and a measured distance. If this cannot be reasonably done, then an azimuth and/or distance is computed from measurements in the surrounding arcs and these computed quantities may be treated as observations. Figure 2.2 shows an example of a junction figure.
2. Perform a free adjustment of each junction figure, without regard to surrounding work. In general, it is expected that this adjustment will be done by condition equations, so that no fixed point is needed. However, it is important that the best available value for the longitude of astronomic points should be used so that the Laplace correction to the astronomic azimuths will be correct. Compute the length and azimuth at each line where the junction figure connects to an arc of triangulation.

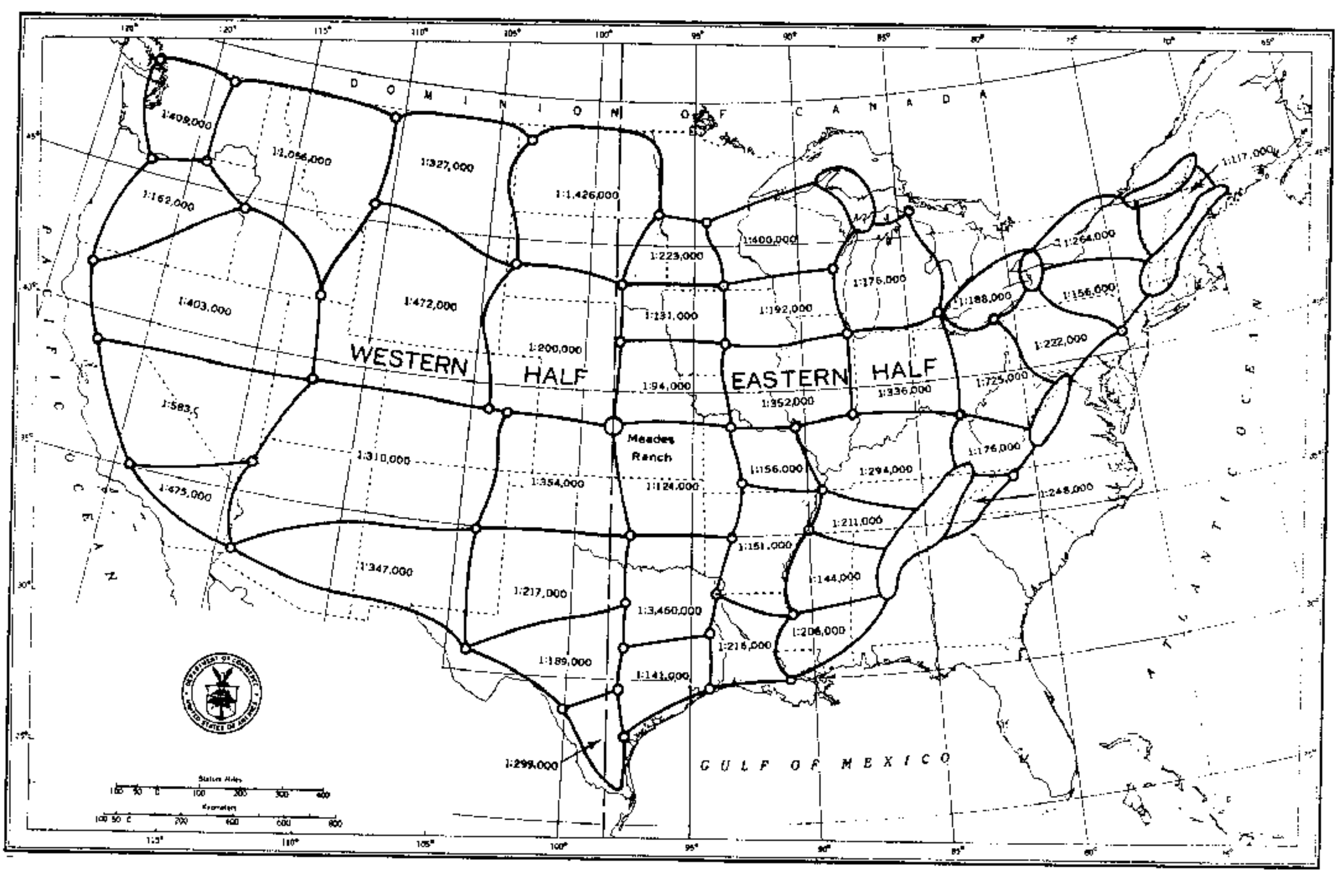

Figure 2.1. Adjustment closures for the North American Datum of 1927.

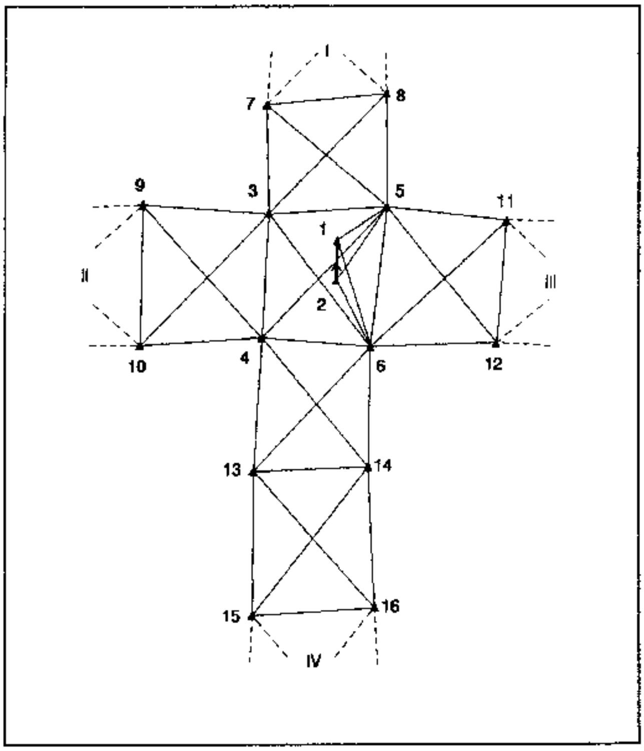

Figure 2.2. Simple junction of two arcs of triangulation.

3. Adjust the sections of arcs between figures. In these adjustments fix the length and azimuth of each line where the section connects to a junction figure, using the values from the previous step. There will thus be at least one length and one azimuth condition for each section, with an additional condition for each distance or azimuth observation. It is expected that this adjustment will also be done by condition equations. The best available longitudes will be needed for the Laplace corrections at astronomic stations. From this adjustment compute the differences in latitude and longitude between one of the points on the beginning line and one of the points on the ending line of the section.
4. The computed differences in latitude and longitude for each section are used as observations in the junction point adjustment. Adams performed this adjustment by observation equations, so that the values of the unknown parameters were obtained directly. In this adjustment each junction figure is held fixed in shape and allowed to move only in latitude and longitude. These coordinate corrections were the unknown parameters.
5. Recompute the corrections to latitude and longitude for all points within each junction figure, accounting for the changes in Laplace azimuths arising from the changes in coordinates.
6. Recompute the coordinates of points within each section, holding fixed the coordinates of the points in the junction figures.

2.5 THE NAD 27 NETWORK ADJUSTMENT

The network adjustment involved the quantities shown in table 2.1. Each section generated a latitude and longitude observation and each junction figure had a latitude and a longitude unknown. Thus the adjustment of the western half involved 84 observations in 52 unknowns. The observations were weighted according to the length of the section. However, the "observed" latitude and longitude differences for a section were treated as statistically independent quantities; no covariances between latitude and longitude differences were considered. Under these conditions, the adjustment simplifies into separate adjustments for the latitude and longitude corrections. Thus Adams describes the solution as two separate sets of 26 simultaneous equations in 26 unknowns.

TABLE 2.1.—*Quantities used in the NAD 27 adjustment*

	Base lines	Azimuths	Junctions	Sections	Loops
Western half	50	74	26	42	16
Eastern half	62	101	29	55	26

2.6 SHORTCOMINGS OF THE NAD 27 NETWORK ADJUSTMENT

The Bowie method produces an approximate least squares adjustment. This may be seen clearly from the following instructions, which describe the Bowie method as a modified Helmert block adjustment:

1. Perform a geographic partitioning of the network. To do this place a point somewhere in the interior of each junction figure (it doesn't matter where). Place a point somewhere in the interior of each loop, and connect each such point with the points in the junction figures around that loop. For each junction figure on the outside boundary of the network, also connect the junction figure point to the neat line of the map. The lines just drawn partition the map into exactly as many regions as there are sections in the network.
2. Identify the junction points according to the rules of chapter 13. If all the points which were part of junction figures in the Bowie method are not already junction points, make them special junction points. All the points which belonged to sections in the Bowie method will then be interior points. In figure 2.2, points 4 and 6 will be junction points for block IV.
3. Devise a Helmert blocking strategy with only two levels. All the interior points in all the sections are eliminated at the first level; all the partial reduced normal equations are combined and all the junction point coordinates are solved for at the second level.

4. Perform a free adjustment of the junction figures, as in the Bowie method. All base lines and azimuths are held fixed and all directions are equally weighted. Use the resulting coordinates as approximate coordinates in the network adjustment.
5. Begin the Helmert block adjustment. In the partial reduced normal equations that are passed forward from each block, ignore all off-diagonal terms that relate a latitude to a longitude unknown.
6. In the highest level block, solving for the coordinates of all the junction points, add the constraints that all stations of a junction figure must take on the same corrections to the approximate values of the latitude and longitude. These constraints may be used to reduce the number of unknowns to one latitude and one longitude for each junction figure. If the constraints are handled this way, the set of reduced normal equations obtained is exactly the set that would be obtained in the Bowie method.

The approximations enter the solution explicitly in steps 5 and 6. The exact numerical effect of these approximations on the NAD 27 adjustment has not been computed. It has been speculated that if the observations used in 1927 were recomputed by present day practices, the positions would differ by 0.01 arc second at the most.

There were other approximations in the adjustment method. For instance, the coefficients of the various condition equations were computed only approximately.

The larger difficulties with the NAD 27 adjustment are those discussed in chapter 3. There was no geoid model, the network contained an insufficient number of base lines and azimuths, and the loops especially were much too large.

Adams, writing at the completion of the western half of the NAD 27 adjustment, said: "The whole network has therefore been fitted together in a rigid system without undue strain in any of its parts. Any short arc that may be observed in the future between sections of this framework should fit into the general scheme with comparatively small closure in position." (Adams, 1930: p. 31)

This statement was certainly true, but even those small closures were greater than could be tolerated by the increased demands that were being place upon the network by 1969, when the need for another adjustment became clear.

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3. THE NEED FOR A NEW ADJUSTMENT

A new adjustment of the North American Datum was necessary as users realized that existing NAD 27 coordinates were inadequate to meet many of the demands placed on them. These demands were varied, and so were the ways in which the weaknesses of the NAD 27 were noticed. Modern surveying methods were demanding a network in which the relative coordinates of points were reliably known to at least one part in 100,000. Some applications required even greater accuracy. The relative coordinates in the NAD 27 system, however, were sometimes in error by as much as one part in 15,000. Errors and distortions in the system, which occurred in unpredictable places and in unpredictable ways, left many of the coordinates unreliable. Fixing the errors that had been discovered so far would not be sufficient. Furthermore, it was known that errors affected the relative coordinates of widely separated as well as nearby points. Obviously, all data needed to be readjusted in a consistent way.

3.1 ACCURACY OF THE NAD 27 SYSTEM

The first-order surveys that were adjusted in 1927 were designed to produce accuracies of at least 1 part in 25,000. This number described the maximum propagation of scale error between base lines. It also described the maximum propagation of position error between stations. It was understood somewhat loosely to describe the uncertainty of the relative coordinates of pairs of points. There were indications that this accuracy specification was not always met.

3.2 CAUSES OF INCONSISTENCIES IN THE NAD 27 SYSTEM

The inconsistencies in the NAD 27 coordinates were ascribed to several reasons: The major cause was the fact that the network grew without readjustment. Other causes included the sparsity of data used in the 1927 adjustment, the computational procedure used to carry out the adjustment, and the effects of earthquakes and other forms of crustal motion upon the network.

3.2.1 Lack of Simultaneous Adjustment

The main cause of distortion was the manner in which the network had grown (Whitten, 1958). The network in 1927 provided a framework, consisting mostly of very large loops. Most surveys since 1927 had provided densification of control within those loops. To illustrate, suppose that loop ABCDA in figure 3.1 was one of the large loops in the NAD 27 adjustment. Suppose that a new chain of triangulation EFIGH is now established. This new chain must be adjusted into the existing network by holding the points E,F,G, and H fixed. The problem is that holding fixed these relative coordinates is unwarranted. Point G is not perfectly known relative to point E. In fact, the uncertainty of their relative coordinates in the existing system must be computed by considering the distance EBCG through the network, rather than the direct distance EG. When this is done, it may well be found that the existing relative coordinates are even less accurate than those produced by the new survey. Yet the former coordinates are held fixed as the new survey is fitted to the old control network.

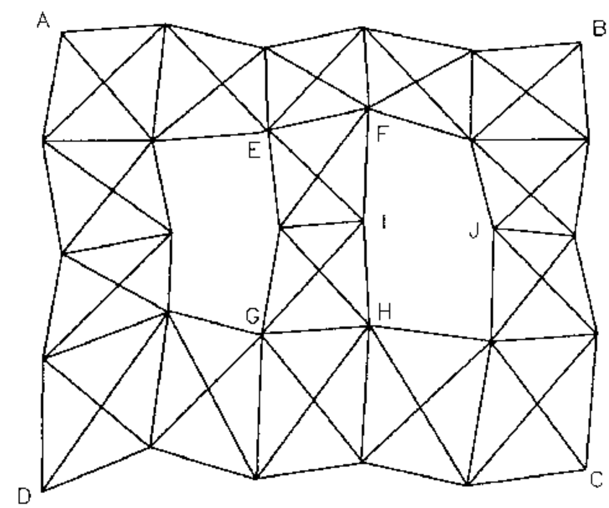

Figure 3.1. Growth of the network and error propagation.

Throughout the period of growth of the networks it was the practice to perform new surveys such as EFIGH to first-order specifications, with the intention of fitting the new survey into the existing network and producing first-order results. Sometimes this worked well and other times it did not. In the latter case there would be a large discrepancy between the relative positions of points at the two ends as computed from the existing coordinates and as computed through the new survey. Since the existing coordinates could not easily be changed, the most common practice was to distribute this discrepancy through the new survey. This often caused the residuals to the observations in the new survey to be large and systematic in nature.

The computational tools and resources needed to compute the true accuracies of the adjusted coordinates were not available during this period. In modern terms, the adjustment of the new survey is seen to be sub-optimal because there are unestimated parameters (the coordinates of fixed control points). A proper computation of error propagation would take account of the covariance matrix of these unestimated parameters.

The problem was compounded with the repetition of the densification process. For instance, a traverse might be run from I to J. In the adjustment of this traverse both points would be held fixed. The error in the existing coordinates of points I and J would be unknown because no proper error analysis had been done. Whatever its size, the entire error would be absorbed by the adjustment of the new survey.

The problem, then, was that the concept of a hierarchy of surveys, consisting of a framework network that is filled in by a series of densification surveys, is not really viable. Its error propagation properties are largely unknown. What is known, both from experience and from modern methods of error analysis, is that the network can produce unexpectedly large errors in unexpected places.

The Coast and Geodetic Survey responded to this problem in two ways. First, to lessen the magnitude of the distortion, the minimum standard of accuracy for first-order triangulation was raised to 1 part in 50,000 (Whitten, 1958). This increased accuracy of course also increased the cost of the surveys. Second, when the misclosure was too large, the fixed end points would be relaxed and a portion of the existing network would be readjusted. This was done at the cost of the confusion caused when readjusted coordinates were issued for previously published points in the network.

3.2.2 Lack of Data

All of the observations used in the 1927 adjustment and those added since then were considered to be still valid in 1983. However, the data set lacked an adequate number of base lines and azimuths, since these observations were the most difficult to obtain. Therefore, the plan for establishing NAD 83 included the observation of a number of new base lines and azimuths, as described in chapter 7.

Two other areas suffered from lack of data in the 1927 system. Survey observations from the Atlantic seaboard were not included. In Alaska, the survey observations were connected to the national network by only a single arc of triangulation along the Alaska Highway, providing only marginally adequate position control for the region.

3.2.3 Crustal Motion

Some areas, notably California and Alaska, had experienced earthquakes and other forms of crustal motion, such as creep along geologic faults. The earlier coordinates in these areas were no longer valid. Although some surveys had been performed to determine the magnitude and extent of the crustal motion, no consistent set of currently valid coordinates existed.

3.2.4 Method of Adjustment

The 1927 adjustment had been performed according to "The Bowie Method of Triangulation Adjustment." This was an approximate method, not a true least squares method. It was selected because of the limited computing means available. Chapter 2 discusses the relation between the Bowie method and Helmert blocking. The numerical effects of the approximations in the adjustment method are difficult to compute, but these approximations were never identified as a major cause of error in the 1927 adjustment.

It is also known that the 1927 adjustment was performed in two parts. The western United States was adjusted first, holding only MEADES RANCH fixed. The eastern half was then fitted to the western half, holding fixed all the junction points along the 98th meridian. Although an approximation, this is not thought to be a significant source of error.

At least one apparent oversight occurred in the 1927 adjustment. Shortly after the eastern portion of the adjustment was completed, and before the results were published, it was discovered that the position of a station of the U.S.-Canada boundary survey in northern Michigan had not been held fixed as originally intended. The discrepancy amounted to approximately 10 m in latitude. A portion of the network in Minnesota, Wisconsin, and Michigan was later readjusted to resolve this discrepancy. Although the major part of this discrepancy was corrected, small effects still existed throughout the network.

Another shortcoming of the 1927 adjustment was that it was done according to the development method, rather than the projection method. In general, values for geoid heights and deflections of the vertical were not known. Distances were therefore reduced to the geoid, not to the ellipsoid. Angles were not corrected for deflections. The neglect of geoid heights is known to lead to regional distortions. The datum origin (MEADES RANCH) was chosen in such a way that the geoid heights with respect to NAD 27 were small, making the distortions also small. They were, nevertheless, systematic. The geoid heights tended to have little effect on the relative position of nearby points, but instead caused errors in the relative positions of points separated by hundreds or thousands of kilometers.

3.3 OTHER INADEQUACIES OF NAD 27

In addition to known and suspected distortions, the NAD 27 datum was inadequate because control points were too far away from where they would be used. More densification was needed, especially in areas of economic development where engineering surveys were being performed. The 1971 NAS report discussed guidelines for densification. The 1973 OMB report recommended that the new adjustment project include 10,500 miles of new triangulation arcs and traverses. This was eventually scaled back to the amount described in chapter 7.

3.4 EFFECTS OF THE DISTORTIONS

The distortions and inconsistencies in the NAD 27 system were felt in a variety of ways. Perhaps the most serious was that the control network could no longer fulfill its primary role of serving to control local surveys. Most local surveyors had been in the habit of using the control network as a standard. They would begin and end surveys at control points. Any misclosure was ascribed to the new survey. By the 1960s surveyors were experiencing more and more difficulty with this concept. Misclosures as large as 1 m in 15 km were occasionally found.

Many surveyors in this period were buying and using more accurate instruments, especially EDM, and misclosures of 1:15,000 could not be credibly ascribed to instruments or surveying practices. Confidence in the network eroded. Some surveyors refused to distort their work to fit the published data. As a result, a great deal of excellent surveying work, often referenced only to a local datum, was excluded from the national network. Other surveyors revised the published coordinates to agree with their own data or devised computational practices. This often led to serious problems when adjoining surveys were performed (Dracup, 1978).

As these problems became more widespread and severe, there was actually very little NGS could do to improve the situation aside from readjusting portions of the network. These expedient attempts at a solution rarely resolved existing problems. Through this process the distortions were merely redistributed over larger areas only to reappear as newer work was fitted to the existing network.

Before the 1960s, complaints received from network users usually concerned the frequency with which coordinates for network points were being revised. Since then, surveyors have been using 1-second theodolites and electronic distance measuring instruments at nearly all levels of the profession. As a result, a large number of horizontal control network users became even more discouraged when they encountered closure problems using modern instrumentation and sound observing procedures.

NGS also felt the effects of the distortions directly. By the 1960s NGS was in a period of executing and adjusting surveys of large urban areas. In many cases the new surveys did not fit the published values of the existing control points. When this happened the common practice was to spread out the discrepancies by readjusting the network in the area along with the new data. The size of the area to be readjusted became larger and larger, and NGS found itself spending more and more time on these local adjustments.

As a result of these difficulties, the time lag between the completion of a survey and the publication of the NAD 27 coordinates increased. NGS allowed field-adjusted and preliminary coordinates to be used for many purposes. This further confused the situation, leading to litigation over land boundary disputes and other economic losses.

3.5 THE GROWTH OF NEW DEMANDS ON THE NETWORK

Historically, development of the geodetic reference system has paralleled the needs of traditional uses, namely, mapping, charting, boundary determination, and large-scale engineering endeavors, such as railroad and highway construction, dams, and irrigation and inland waterway systems. After 1960, however, there was a tremendous increase in the number and types of programs dependent on reliable position data. These include earthquake-hazard-reduction programs, satellite data collection, electronic navigation systems, offshore boundary extensions, definition of offshore lease blocks, missile defense systems, environmental management, natural resource development and management, coastal zone management, urban and regional planning, and hazardous waste disposal programs.

The list of those persons who depend on sound geodetic reference data grew from surveyors, cartographers, and engineers to include legislators, economists, environmentalists, policy analysts, attorneys, social scientists, planning specialists, and a variety of others. As the number and types of network users increased, so did their accuracy needs. Since 1927, the relative-position accuracy needs of a great many users had increased from one part in 50,000 to one part in 100,000.

Rapid population growth and economic development placed new demands on the geodetic reference system. The needs of growing population centers include accurate maps for tax assessment and land-use planning, and the construction and maintenance of sewer and water supply lines, highways, bridges, tunnels, telephone lines, pipelines, and power transmission lines, among many other related services. The problems faced by Ada County, ID, and El Paso, TX, were typical.

A class-action suit by the taxpayers of Ada County, ID, resulted in a court order requiring the County to update its tax maps. A prerequisite of a modern tax-mapping system is an up-to-date and reliable geodetic control network. The lack of a satisfactory local network prompted Ada County officials to cooperate with the National Oceanic and Atmospheric Administration in a special project to develop and update the geodetic network there.

The City of El Paso, TX, reported slightly different growing pains in its request for NGS assistance to develop the national network there. Land records in that city were based on five separate reference systems—a situation that hindered city officials in providing basic services and maintaining public utilities. Again, updating and strengthening the national network were needed to resolve discrepancies among the five independent systems.

Another class of new demands resulted from military and space activities. Missile ranges needed highly accurate survey networks so that tracking instruments could be accurately located. Satellite tracking activities required that tracking stations thousands of kilometers apart be accurately located, both with respect to each other and with respect to the center of mass of the Earth. It quickly became apparent that NAD 27 coordinates were not adequate for this purpose, and a primary task of geodetic satellite tracking programs was the determination of better tracking station coordinates.

The appearance of portable Doppler satellite tracking systems clearly showed the inadequacy of NAD 27. The DoD Geoceiver test was an attempt to evaluate these new receivers. Part of the evaluation compared Doppler-derived coordinates to NAD 27 coordinates. Discrepancies of almost 10 m were found (Defense Mapping Agency, 1972). Other evidence indicated that the Doppler coordinates were more nearly correct. The NAD 27 coordinates could no longer serve as a standard of comparison for the Doppler-derived coordinates; instead, the Doppler system quickly became the standard by which distortions were detected in NAD 27.

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4. HISTORY OF HORIZONTAL GEODETIC CONTROL IN THE UNITED STATES

Joseph F. Dracup

The history of the horizontal control network in the United States is little known, even among members of the geodetic community. Few have access to the historical documents which record, often in minute detail, the trials and tribulations, as well as the successes. There have been many of each. Geodetic surveying is not a glamorous profession and does not attract the attention of those with literary bent. As a result, much of the story will remain buried in Government archives.

4.1 THE BEGINNING

On February 10, 1807, during the presidency of Thomas Jefferson, Congress enacted legislation which authorized the President ". . . to cause a survey to be taken of the coasts of the United States, in which shall be designated the islands and shoals, with the roads or places of anchorage. . . ." With these simple but direct words, the Survey of the Coast was born. Shortly thereafter, President Jefferson sought proposals for carrying out this act from several qualified persons, and finally accepted the plan set forth by Ferdinand R. Hassler. Funds were eventually appropriated, and in 1811 Hassler went to London and Paris to acquire equipment and instruments which were not available in the United States. Hassler remained in Europe during the War of 1812, returning to the United States in 1815 (Jeffers, 1953).

In the following year, Hassler began geodetic operations in the vicinity of New York City. He measured two base lines, one along the shoreline of Gravesend Bay in the present-day Coney Island section of Brooklyn, and the other near Englewood, NJ. In 1817 he executed a small triangulation network consisting of 11 stations. The first triangulation station, named WEASEL, was located in Passaic County, NJ, about 2 miles south of Paterson (Reynolds, 1933). Hassler made the first observations at this point on July 16, 1817. With the completion of this small project no further geodetic surveys were undertaken for a rather long period, because the Survey of the Coast was transferred from the Treasury Department to the Navy by an Act of Congress in 1818. This act prohibited the employment of other than military personnel in carrying out the activities of the bureau. All civilians, including Hassler, were discharged.

Little geodetic work was accomplished by the Navy during the ensuing years. However, many enlightened individuals were aware of the need for geodetic control required to produce accurate maps and charts so essential to the development of the country. They continued to press the legislative and executive branches to reactivate the Survey.

In 1832 Congress restored the Act of 1807, returning the Survey of the Coast to the Treasury Department, and Hassler was again named to direct the bureau. Hassler collected his equipment and instruments and immediately began a reconnaissance survey, extending eastward from his 1816-17 net along the Connecticut and New York shorelines. Although Hassler was 62 years old when he began this work, he attacked the effort with the vigor of a man half his age.

By the late spring of 1833, Hassler was ready to begin his observations. The first station was named BUTTERMILK, occupied on June 11, 1833 (Dracup, 1976). This station is still in existence and is located on the Rockefeller estate in Westchester County, NY. While Hassler's assistants were carrying out secondary surveys along the New York-Connecticut shoreline, he continued the primary survey southward, making all the observations himself. This was the beginning of what was later to be known as the Eastern Oblique Arc, which eventually extended from Calais, ME, to New Orleans, LA, following the trend of the Appalachian mountains southwesterly to the Gulf coast near Dauphin Island, AL, then westward to New Orleans, a distance of 1,623 miles (Bowie, 1928).

Progress was slow. By late 1843 the arc was completed only to Salem County in southern New Jersey. It was here at station BURDEN that Hassler made his last observations. He died shortly thereafter, following an injury sustained while trying to protect his instruments during a severe late fall storm which hit his campsite in Delaware. Thus an era ended.

The Fire Island base line on Long Island was the only one established during this period and was measured using four 2-meter iron bars placed end to end. Astronomical observations were limited to a few latitude and azimuth determinations.

Although the progress on the primary arc was somewhat slow, the Coast Survey, as the Survey of the Coast had been renamed in 1836, made excellent progress in extending secondary triangulation east to Rhode Island and south to the head of the Chesapeake Bay. By 1843, more than 1,200 stations had been established, covering an area of 9,000 square miles.

4.2 THE PERIOD 1844-1900

A new era began under the direction of Alexander Dallas Bache, a great-grandson of Benjamin Franklin (Wraight and Roberts, 1957). During Bache's superintendency (1843-67) progress was continued on the Eastern Oblique Arc, but on a reduced scale, because control surveys for hydrographic purposes on the Atlantic, Pacific, and Gulf coasts were given a higher priority. These engineering surveys were often secondary in character and often based on independent astronomic datums.

George Davidson, one of Bache's most trusted assistants, was sent to carry out surveys in California. He accomplished some primary triangulation, but for the most part lower order surveys—sufficient to control the mapping and hydrography—were predominant. Except for the Civil War years and one or two foreign assignments, he spent most of his 50-year Coast Survey career in California. The great trigonometric figures extending over the Sierras, which were observed later in the century, bear his name: "Davidson's Quadrilaterals." These figures contain many sides exceeding 100 miles in length, the longest being 192 miles between Mount Shasta and Mount Helena.

Benjamin Peirce followed Bache as Superintendent of the Coast Survey, serving from 1867-74. During his term of office, Congress authorized an arc of triangulation along the 39th parallel, connecting the Atlantic and Pacific coasts. This great arc, perhaps the longest executed by a single agency, extends from Cape May, NJ, to Point Arena Lighthouse, CA, a distance of 2,750 miles. The Transcontinental or 39th Parallel Arc, as this net is identified, was observed during the period 1871-97. Westward from central Colorado the arc is composed of figures of immense size. In addition to "Davidson's Quadrilaterals," there is another figure known as the "Great Hexagon." This figure has Wheeler Peak in Nevada at its center and covers a wide area from the Wasatch Mountains near Salt Lake City, UT, almost to central Nevada. Most sides exceed 100 miles. Due to the remoteness of the station sites and the short working season, it took almost 10 years to complete the observations.

Another significant arc, completed in 1875, spanned the Mohawk Valley of New York. It connected the triangulation in New England with the work of the U.S. Lake Survey near Rochester, NY. The U.S. Lake Survey, a branch of the Corps of Engineers, carried out extensive first-order surveys in the vicinity of the Great Lakes primarily during the period 1864-1900. In addition, this organization also observed a connection southward from Chicago to the Transcontinental Arc in eastern Illinois.

By the end of the 19th century, the Eastern Oblique Arc had been completed, Davidson had extended first-order control to the vicinity of Los Angeles, and an extension northward on the 98th meridian from the 39th Parallel Arc in Kansas was initiated. Work in Alaska was progressing, and in 1890 an arc of triangulation along the west coast between Mexico and Canada was completed. This latter survey was principally made up of second- and third-order work. It was not until the first decade of the 20th century that a first-order arc following a more inland route was observed.

The cardinal longitude of the United States was first determined at the Harvard Observatory, Cambridge, MA. The determination was based on the chronometer method, and used 1,065 exchanges between Liverpool, England, and Boston, MA, during the period 1843-55. Determinations were made also by moon culminations and other astronomical phenomena, beginning in 1838 and employing observations at American and European observatories. Once the telegraphic method became operational and the transatlantic cable was in place, the earlier procedures were largely abandoned. Telegraphic expeditions in 1866, 1870, and 1872 fixed the longitude at the Harvard Observatory. Further observations were made in 1880 when two additional cables were laid and the subsequent adjustment of the U.S. longitude net changed this value slightly. Another adjustment in 1885 changed this determination by 0.001 second of time.

4.3 THE ERA OF GREAT ARCS 1900-40

With the completion of the Eastern Oblique and 39th Parallel Arcs, plans were made to extend arcs of first-order triangulation north to south and east to west at about 100-mile intervals, covering the entire country in a checkerboard pattern. Further breakdowns would then be made by establishing second-order arcs so spaced that no place in the conterminous States would be more than 25 miles from a first- or second-order station. Eventually the areas in between would be covered by third-order networks.

This plan seemed to fit the needs of the country and was within budgetary considerations. However, as with most long-range programs, radical changes are often dictated in order to retain the primary concepts and to meet ever changing conditions.

The first of these changes surfaced during World War I, when the rising cost of lumber prohibited the building of high wooden structures, necessary for elevating instruments and signals above obstacles. To continue the program, traverse was substituted for triangulation in relatively flat and, in some instances, heavily forested areas of the South and Midwest where extensive railroad routes and expanding highway systems provided corridors for establishing such surveys. In one case, the frozen surface of the Rainy River between the Lake of the Woods and an arc of established triangulation in northeastern Minnesota was utilized to complete a section of the United States-Canada boundary control. During a period of about 10 years (1917-26), several thousand miles of first-order traverse were measured in Virginia, North and South Carolina, Georgia, Florida, Mississippi, Louisiana, Indiana, Illinois, Wisconsin, Minnesota, and South Dakota.

A second major change occurred when Jasper Bilby developed the portable steel tower in 1926 (Bowie, 1933). The preference for triangulation was restored and the use of traverse procedures was relegated to special purpose surveys, occasional city surveys, and a few low-order surveys along the coast. This was the case until the advent of electronic distance measuring instruments in the 1950s, when traverse procedures were once again instituted for primary surveys. Early investigations indicated that first-order traverse was equivalent in accuracy to the same class of triangulation. However, as the network was developed, the structural weaknesses of these earlier traverses were brought to light, and in several instances blunders were uncovered.

Prior to the invention of the Bilby tower, several great arcs of triangulation had been accomplished. Among the most notable are the 98th Meridian Arc, which extends 1,720 miles from the Rio Grande River in Texas to the Canadian border; the 49th Parallel Arc, accomplished in cooperation with the Geodetic Survey of Canada, which straddles the international boundary between the 98th Meridian Arc and Point Roberts in northwestern Washington; and the Texas-California Arc from the 98th Meridian Arc in Texas to the first-order network in southern California, a distance of 1,207 miles. Other great arcs include the survey along the 104th meridian from the Texas-California net to the Canadian border; triangulation following the 35th parallel from the 98th Meridian Arc in Oklahoma joining the Texas-California and 112th Meridian Arcs in southwestern Arizona; and several projects establishing control in Idaho, Oregon, and Washington, including the California-Washington arc. In the East, few first-order triangulation surveys were carried out during the period between 1917 and 1927. Once the Bilby towers came into use, however, work was accelerated in that section of the country.

Triangulation was observed along the Gulf coast and inland in numerous states, including the long Mississippi River Arc, where towers as high as 150 feet were required and towers more than 100 feet in height were commonplace. The last of the truly great arcs in the conterminous United States followed the Atlantic coast from Providence, RI, terminating at Key West, FL, a distance of perhaps 1,600 miles. During the Great Depression of the 1930s, funds were made available to aid the unemployed by providing jobs in public works. The work of establishing control surveys benefited greatly from this policy. Very large field parties roamed the land extending geodetic control. Some of these parties included more than 150 employees, with as many as 12 observing units deployed on 1 night by a single field party. In 1935 the Coast and Geodetic Survey had almost 3,000 employees in the field (U.S. Coast and Geodetic Survey, 1935).

Progress was rapid. Major William Bowie, who was then chief of the Geodesy Division, left no stone unturned in his effort to complete what he considered to be the fundamental framework of the United States. Requirements had changed since the original plan had been drawn up, and the need for more closely spaced control to a higher accuracy became evident. Few arcs to second-order specifications and no third-order area networks were actually observed by geodetic field parties. While some lessening of the first-order specifications was permitted in performing surveys classified as second-order, the great majority of the work fell within the anticipated accuracy for primary surveys. With the addition of strategically placed base lines and Laplace azimuths, this accuracy could be assured.

The period of establishing great arcs of triangulation drew to a close about 1940. The primary network was essentially complete, and geodetic surveys entered the epoch of densification.

4.4 THE TIME OF GREAT ADVANCEMENTS 1940-75

By the middle decades of the 20th century some area-type networks had been observed, but arc systems were still the general rule. Supplemental stations were frequently established to provide additional control. Little attention was given to the fact that on numerous occasions stations determined in other projects had already been established nearby, with the result that nearby points were often not connected. This was to lead to problems at a later date, when many locally accomplished surveys could not tolerate the inconsistencies brought on by such situations. At the time, no solution was considered, as an almost total effort was directed toward military related activities.

Perhaps the greatest geodetic achievement during the wartime period was the completion of an arc of triangulation from Skagway in Southeast Alaska to Whitehorse in Canada. The arc extended via the Alcan Highway to the major land mass of Alaska, thus tying this vast land to the North American Datum of 1927. Prior to this time, triangulation in Alaska had been computed on several independent datums; it was not until the 1950s that all surveys were finally positioned on a single datum.

The job of filling in the uncontrolled areas now began. Although the surveys were classed as second-order, the specifications were only slightly modified from those required for first-order work. Some party chiefs ignored the modifications and continued to employ first-order specifications for directions. Most of these projects could be upgraded to first-order by the addition of a few base lines and Laplace azimuths.

Although extension of the horizontal control network continued during the 1950s and 60s, much of the effort was directed toward projects in high-density population areas. These surveys were scaled by numerous measured distances and oriented by sufficient Laplace azimuths to assure that the requested accuracies were maintained throughout the entire network. More than 50 such projects were accomplished by the Coast and Geodetic Survey and the National Geodetic Survey between 1960 and 1975.

With few exceptions, modern surveys have been carried out in the nonconterminous states and possessions. Much work still remains to be done in Alaska. Much of Hawaii has sufficient and reasonably new control. Molokai and a few of the smaller islands are the exceptions. Puerto Rico and American Samoa have been recently surveyed, but new surveys on the Virgin Islands are still to be carried out.

By the 1960s, it became evident that extremely accurate surveys would be required to support missile and satellite activities. Lansing C. Simmons, then the Chief Geodesist of the Coast and Geodetic Survey, conceived a unique plan for an ultraprecise traverse which was expected to produce relative accuracies of one part in one million. The survey which became known as the Transcontinental Traverse was initiated in 1961 in Florida. It was completed in 1976 with a total length exceeding 22,000 km. Several sections of this traverse were observed by survey parties attached to the Defense Mapping Agency and its predecessors, which also contributed financial assistance.

4.5 CHANGES IN INSTRUMENTS AND METHODOLOGY

In the last half of the 19th century base lines were measured using iron bars and rods and a variety of compensating and contact type base apparatus. Steel tapes were employed in measuring several bases near the end of the period. After 1907 all primary base lines were measured using tapes made of a nickel-steel alloy, commonly called Invar.

Developments in electronics created a tremendous break-through. In the late 1940s, Dr. Bergstrand of Sweden built the first distance-measuring device. This instrument, which employed visible light, was to revolutionize geodetic surveying because the time-consuming practice of measuring base lines was reduced from weeks to a few hours. In addition, terrain restrictions were lifted.

The concept of using light for measuring distances was not new. Professor Albert A. Michelson had carried out experiments in the early 1920s to determine the speed of light. The Coast and Geodetic Survey became interested in the problem and to aid Michelson measured perhaps the most accurate taped base line ever on the hopes that the experiments might lead to the development of a distance measuring instrument. Unfortunately, the experiments were not a total success. Michelson passed away within a few years, and with his passing the idea was abandoned.

Late in the 1950s, distance measuring equipment utilizing microwave sources came into being. These instruments had a much greater range than light wave equipment, but this advantage was offset by the effect of humidity which could produce less-accurate measurements. Instruments utilizing infrared as the carrier beam were later introduced, and while this type of equipment can produce very accurate measurements, its range is rather short.

Numerous astronomic determinations were made for latitude and azimuth. Longitude observations lagged behind despite the perfecting of the use of telegraph lines for transmitting time signals simply because the stations needed to be located near these lines. Development of the telegraph method began in 1846 and was used until it was replaced by the use of radio time signals in 1922.

The strength of the networks was increased measurably by a decision at an early stage (about 1845) that the principal triangulation consist only of complete quadrilaterals (both diagonals observed) or central point figures. This policy was rigorously followed until the 1970s. Only the earliest work and some surveys made by the U.S. Lake Survey prior to 1900 were observed as chains of single triangles.

4.6 GEODETIC DATUMS

The early surveys were established as separate, independent networks. Each was based on one or more astronomical determinations of latitude, longitude, and azimuth. These separate pieces of triangulation were extended until they touched or overlapped (Dracup, 1980).

With the completion of the Transcontinental Arc around 1900, it was possible to compute the net as a single coordinated survey and to replace the previous independent systems which, of course, did not fit together properly at the junctions. The recomputation of all triangulation that had been completed up to that time would have been a fairly heavy piece of work. Considerable thought was given to devising the best possible method in order to adopt a datum that could be held fixed for a long time into the future. After careful study, it was decided to extend the datum that had been used in New England and along the Atlantic coast between 1880 and 1901 through the entire network. This decision avoided much recomputation and, at the same time, gave an almost ideal datum for the Nation.

The origin of the New England Datum was station PRINCIPIO in Maryland. Its position had been determined in a computation using all astronomic latitudes, longitudes, and azimuths which had been observed in the eastern triangulation.

The position of PRINCIPIO was retained for the new datum, which became known as the United States Standard Datum. The geographic position of station MEADES RANCH was said to define the origin, but this position was actually computed through the triangulation from PRINCIPIO.

In 1913 the same datum was adopted by Canada and Mexico for the geodetic networks of those countries. In recognition of its new continental character, the name was changed to the North American Datum. This was the first time anywhere that international cooperation had led to a common datum of continental extent.

All computations were carried out on the Clarke spheroid of 1866, which had been adopted by the Coast and Geodetic Survey in 1880. In 1924, the International Association of Geodesy adopted a new ellipsoid for use by all member countries that might be in a position to recompute their triangulation nets. This international ellipsoid was based on dimensions that had been derived in 1909 by John Hayford of C&GS. By the time of its adoption, the positions of thousands of stations in the United States were based on the Clarke spheroid, and numerous tables had been computed and published. Because the ellipsoid already in use differed only slightly from the new one, it was decided that no change would be made.

In a 5-year period beginning in 1927, the national control network was recomputed. Although the Clarke spheroid and the position originally adopted for MEADES RANCH were still satisfactory, the station positions were far from ideal. Chapter 2 gives a synopsis of the 1927 adjustment.

The name of the datum was changed from North American Datum to North American Datum of 1927. This change was to guard against confusing the new positions with old positions. MEADES RANCH was the only station where the position remained the same. The changes were small in the vicinity of MEADES RANCH but were fairly large at greater distances. In the State of Washington, for example, the change of position was slightly over 1 second in latitude and nearly 1.4 seconds in longitude.

Table 4.1 shows that the size of the horizontal control network grew by almost an order of magnitude between the major datum adjustments in the United States. Figures 4.1, 4.2, and 4.3 depict the growth of the horizontal control network from 1901 to 1981.

TABLE 4.1.—*Growth of the networks in the United States*

Year	Datum	Stations
1901	U.S. Standard Datum	5,000
1927	North American Datum	25,000
1983	North American Datum	272,000

4.7 ORGANIZATIONAL CHANGES

As the survey networks grew, the agency responsible for the primary geodetic control networks underwent several changes in its organization. The Survey of the Coast became the Coast Survey in 1836 and was renamed the Coast and Geodetic Survey in 1871. The name was changed to emphasize the increased importance placed on geodetic surveys and geodesy in general.

In October 1970, as part of a general reorganization, the Coast and Geodetic Survey became the National Ocean Survey. This was further reorganized and renamed the National Ocean Service in December 1982. The Geodesy Division became the National Geodetic Survey Division.

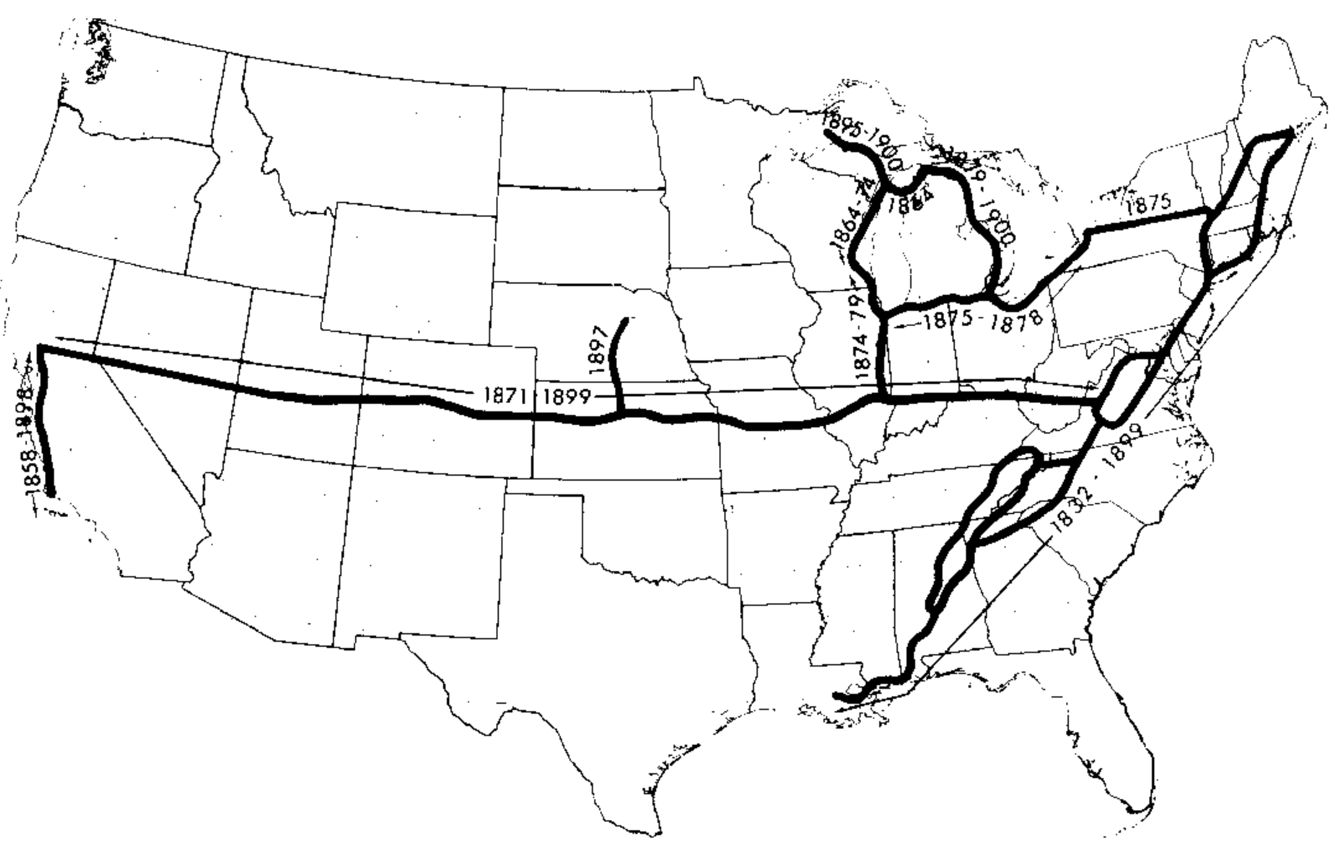

Figure 4.1. U.S. horizontal control network in 1900.

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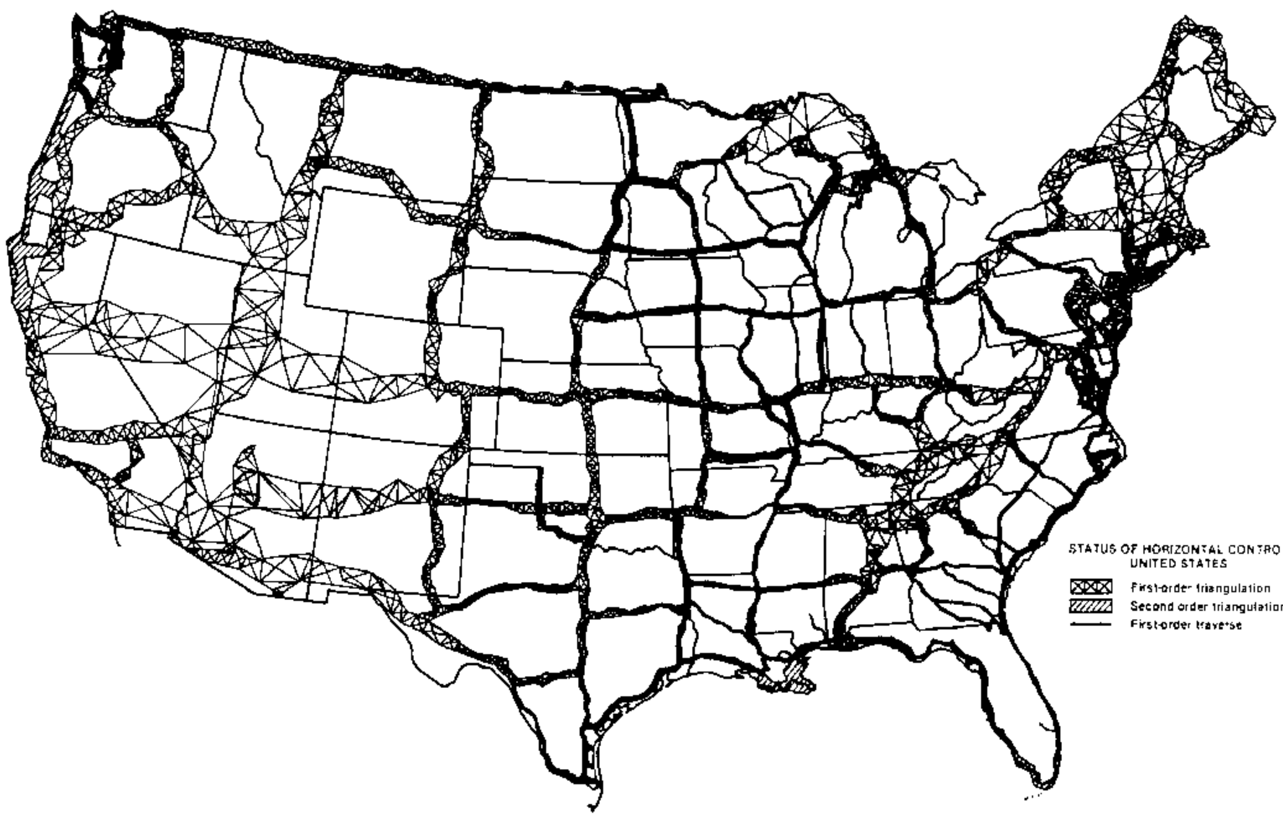


Figure 4.2. U.S. horizontal control network in 1927.

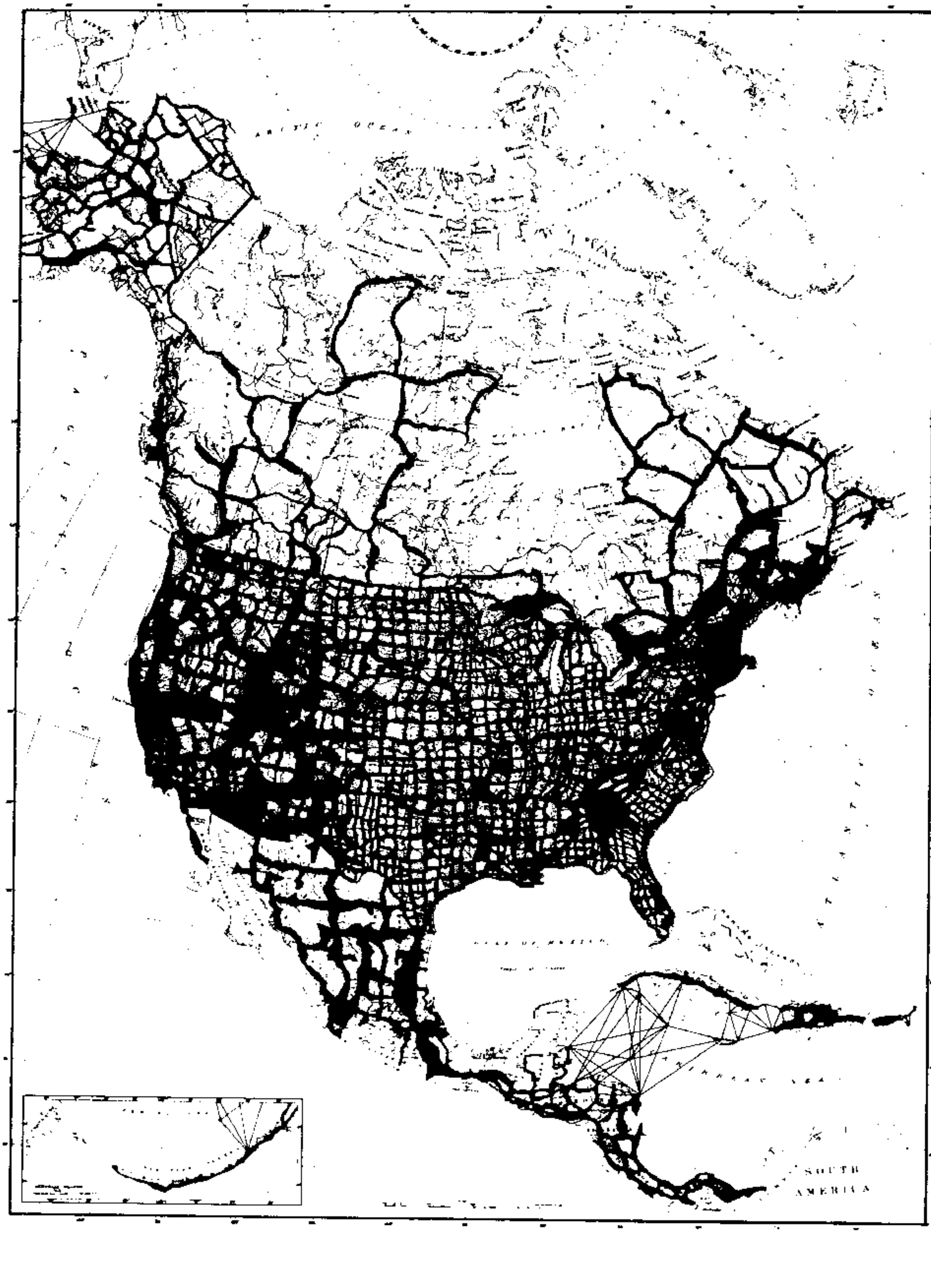


Figure 4.3. Status of geodetic control in North American in 1981.

5. INTERNATIONAL ASPECTS

5.1 THE CONTINENTAL CHARACTER

In 1901 the U.S. Coast and Geodetic Survey (C&GS) adopted for the triangulation system of the United States what was called the United States Standard Datum. At this time major triangulation projects were also being carried out in Canada and Mexico. William Bowie, then chief of the C&GS Geodesy Division, discussed these projects with the survey directors of Canada and Mexico, W. H. King and Pedro Sanchez respectively. By 1913 he had persuaded them to formally connect the surveys of their respective countries to the U.S. Standard Datum (Whitten, 1975). Because of the new international character, the C&GS Superintendent directed that the name be changed to the North American Datum (Bowie, 1914: p. 80).

In 1927 the North American Datum was readjusted, resulting in new coordinates for all stations. Soon afterward, the geodesists of Canada and Mexico also readjusted their triangulation to make it consistent with the new datum. Subsequent surveys in Central America, extending to the border between Panama and Colombia, were also connected to this datum.

5.2 ROLE OF OTHER COUNTRIES IN NEW ADJUSTMENT

The new adjustment of the North American Datum was always perceived as an international effort. Capt. Leonard S. Baker, NOAA, had held discussions with Louis A. Gale of the Geodetic Survey of Canada and with ING. J. A. Villasana of the Geodetic Survey of Mexico seeking their cooperation. Letters were sent to the survey directors of all the republics of Central America. In the case of Canada, this resulted in a formal intergovernmental agreement. In most other cases agreements were made between the geodetic agencies.

5.2.1 Canada

The participation of Canada in the new adjustment was coordinated by the Geodetic Survey of Canada, a division of Surveys and Mapping Branch, Department of Energy, Mines and Resources. The (U.S.) National Geodetic Survey and the Geodetic Survey of Canada both recognized the need for a new adjustment as early as 1969 and proposed appropriate programs to their respective governments. Both agencies coordinated their new adjustment activities with each other. Although the surveying communities were pressing for network improvements in both countries, there was never any serious possibility of either country performing a datum readjustment without the participation of the other.

The Geodetic Survey of Canada chose a hierarchical approach to the adjustment of the geodetic networks in Canada. The primary network formed part of the continental network adjusted by the Helmert block method. After the July 1986 completion of the fundamental NAD 83 adjustment, Canada still had the task of integrating more than 200,000 stations contained in regional and local secondary networks into the continental system (Parent and Pinch, 1988). Most of the second- and lower-order surveys are held by provincial and other Federal survey agencies, and the adjustment of these networks was planned in conjunction with those agencies.

5.2.1.1 Survey Networks in the New Adjustment

The Canadian portion of the continental network consists of about 7,500 stations and 44,347 observations. It consists mainly of first-order work in chain triangulation, traverses, and area triangulation.

As in the United States, the existing geodetic control network was strengthened by additional observations in preparation for the new adjustment. In Canada, about 600 lengths and 65 Laplace azimuths were measured for this purpose. The primary network was further strengthened and extended by first-order traverse, by the addition of a basic satellite Doppler network having stations spaced from 200 to 500 km across the country, and by additional triangulation and Doppler surveys in the more densely populated areas (McLellan, 1980).

Geoid heights and deflections of the vertical were also fully accounted for in the processing of the Canadian observations.

5.2.1.2 Helmert Blocking Solution

The Geodetic Survey of Canada was completely responsible for the computation of all observations in Canada. Program GHOST, which implemented the height-controlled three-dimensional mathematical model, was used for these computations. For each iteration, the Geodetic Survey of Canada and NGS exchanged partial reduced normal equations. Both agencies computed the final combined solution, each as a check on the other.

The exchange of normal equations required considerable detailed coordination. The Geodetic Survey of Canada drew the border used to separate the U.S. and Canadian geodetic networks in the Helmert blocking system. Canada also helped to develop the detailed format and procedures used for the exchange of normal equations.

It was desirable that dividing lines in the Helmert blocking strategy be drawn through weak areas of the network, since this would lessen the number of junction points. The geodetic boundary between the U.S. and Canada was therefore drawn north of the political boundary. (See fig. 18.4.)

5.2.2 Mexico, Central America, and the Caribbean

The North American Datum has become the conventional datum for surveying and mapping, not only for Mexico but also for the republics of Central America and for the Caribbean area. Most of these countries had participated with the United States through collaborative programs under the Inter American Geodetic Survey (IAGS). By the time of the new adjustment, IAGS had become a component of the Defense Mapping Agency (DMA).

The horizontal surveys in these areas were collected, validated, and merged by the DMA Hydrographic/Topographic Center (DMAHTC). This process is described by Skaggs (1980). The resulting horizontal survey network consisted of 1,884 stations established by first-order triangulation and traverse methods. Observations among these stations included 9,970 directions, 82 Laplace azimuths, 55 base lines (Invar and Geodimeter) and 4,000 km of traverse.

Prior to the new adjustment, station positions had been computed by DMA on NAD 1927 by using border ties with the United States. The adjustments were carried out in successive blocks from Mexico to Panama. There were many known weaknesses in these networks, due to the sparsity of surveys, the long extension from the U.S. border, and lack of an adequate geoid profile.

The new adjustment provided an opportunity and impetus for the strengthening of the geodetic networks in North America. New first-order surveys were added by Mexico to tie together gaps in the existing triangulation arcs. Doppler-derived coordinates were established throughout the network on existing stations, using an average spacing of 200 km between Doppler stations.

DMAHTC constructed a geoid profile to cover Mexico, Central America, and the Caribbean. Predictions of the deflections of the vertical and the geoid height at each station were made by least squares collocation. The geoid computations were based on:

1. a satellite-derived Earth model,
2. 1,693 1- by 1-degree mean free air gravity anomalies,
3. 73,512 point gravity observations,
4. geoid heights directly observed by satellite altimetry,
5. 83 geoid heights directly observed by Doppler positioning, and
6. observed deflections at 96 astronomic stations.

Most of the previously processed horizontal observation data for the area existed at DMAHTC before the new adjustment, and most of these data were already in machine-readable form. DMAHTC retrieved and revalidated this data set. New observational data were added and the entire network was validated by the DMAHTC horizontal network adjustment program. Finally, the entire observation data set was transformed to the Trav-deck format and transferred to NGS in machine-readable form. NGS treated this as any other project which had been validated in Trav-deck form, and the data were eventually processed through block validation and loaded into the geodetic data base.

5.2.3 Denmark

Denmark became a participant in the new adjustment in 1974 because Greenland was at the time an administrative district of Denmark. The Danish Geodetic Institute was responsible for geodetic and mapping activities in Greenland. It had been left out of the initial NAD planning simply by an oversight. Soon formal relationships were established between the geodetic agencies and Denmark was a fully participating country.

5.2.3.1 Methodology

The Danish Geodetic Institute was one of the few geodetic agencies which had experience solving large sparse systems of equations. Much of the methodology used in the Danish adjustment programs (Poder and Tscherning, 1973; Poder and Madsen, 1978) was adopted by NGS. In addition, several extended visits were held between geodesists of the two agencies to discuss details of the Helmert blocking method.

Geodesists of the Danish Geodetic Institute also described and advanced the method of collocation for predicting geoid heights and deflections of the vertical (Tscherning and Forsberg, 1978).

5.2.3.2 Greenland Surveys

A first-order network exists along the west coast of Greenland approximately from latitude 60 degrees to latitude 77 degrees. The single chain network consists of about 200 stations. All types of geodetic measurements, including directions, distances, and Laplace azimuths, have been performed according to first-order standards. The observations are adjusted on the Qornoq Datum, which originally was a local datum for the central part of the west coast of Greenland. Extensions have been established to southern Greenland and northwestern Greenland. Additional independent network adjustments were performed at two places on the east coast of Greenland, but the lack of geodetic connections to the Qornoq Datum forced both areas to be established on separate datums, i.e., the Angmagssalik Datum and Scoresbysund Datum.

Densification of all primary networks in Greenland has been performed from time to time. It was decided that all observations that would improve the accuracy of the secondary networks would be adjusted to the framework at a later date. All observations are in computer readable form. The number of stations totals about 4,300.

Doppler satellite surveying was used for two main purposes: to support the connection of the existing networks to the North American Datum of 1983 and to establish geodetic control in northern and eastern Greenland. Since 1974, 28 Doppler stations have been established in the existing networks with an average distance of 200 km between stations. Four of the stations were established in cooperation with the Geodetic Survey of Canada and NOAA/NOS National Geodetic Survey. The remaining stations were surveyed by using the two Doppler receivers purchased by the Danish Geodetic Institute in 1976 and 1977. Seventeen stations exist on the Qornoq Datum, while the Scoresbysund Datum has nine stations and the Angmagssalik Datum two stations. About 170 satellite stations are distributed over the remaining coastal areas of Greenland, which has virtually no classical triangulation networks.

Until recently the unsurveyed areas in Greenland covered approximately 500,000 km^2. Geodetic control has now been introduced by means of the Doppler satellite technique. The Doppler stations are observed with a spacing of about 60 km, and supplementary control is established using classical traverses. GPS observations were introduced in 1986 in cooperation with the Defense Mapping Agency Hydrographic Topographic Center (DMAHTC). Figure 5.1 pinpoints the satellite stations.

Coordinates for satellite stations were calculated on NAD 83 by using the transformation parameters given in chapter 11. These satellite-derived coordinates are then introduced into the network adjustment as coordinate observations together with estimated variances.

The second-order network in the northwestern part of Greenland has connections to the network in the eastern part of Canada's Northwest Territories. These networks will be adjusted as a common block. It is expected that this adjustment will be completed during 1989. The remaining networks will thereafter be adjusted and the NAD 83 will be fully implemented throughout Greenland in 1990.

A major mapping project of northern Greenland is being conducted by the Danish Geodetic Institute. These maps will be produced on the new datum.

5.3 ROLE OF THE INTERNATIONAL ASSOCIATION OF GEODESY

The International Association of Geodesy (IAG) is the natural organization to coordinate an international geodetic project such as the new adjustment of the North American Datum. Prior to 1975, the scientific readjustment of the European Datum (known as the RETRIG Project) was coordinated by Commission X of the IAG. At the 1975 General Assembly, Commission X was reorganized into several subcommissions. One of these is the subcommission for North America. This subcommission continues to provide an additional channel of communication for those individuals and agencies who are concerned with the NAD as a continental network.

The IAG also passed a formal resolution in 1975 recommending that a new datum be developed for North America. This resolution recognized and provided international approval of the plans and work already in progress.

The IAG played an important role in the new North American Datum adjustment by recommending the ellipsoid that would be used for the new adjustment. This ellipsoid was part of the Geodetic Reference System of 1980 (GRS 80), adopted by the IAG (and by its parent group, the International Union of Geodesy and Geophysics) at the General Assembly of 1979.

The IAG had already recommended a standard Geodetic Reference System in 1967 (GRS 67). The numeric values of the fundamental parameters recommended in 1967 were soon rendered out of date by the rapidly improving results from satellite geodesy. Although it was clear throughout the 1970s that the 1967 values were no longer the most current, it was not clear which values should be adopted. It was the task of Special Study Group 5.39 of the IAG to analyze the many determinations of the fundamental constants and to recommend the best values. The situation was not well settled in 1979, and in the normal course of events it might not have been the right time to recommend a new reference ellipsoid. However, the agencies preparing to perform the new NAD adjustment needed and were prepared to accept a new ellipsoid. The Study Group and the IAG selected the new ellipsoid in a manner that was completely acceptable to all the parties concerned.

5.4 THE NAD SYMPOSIA

The new adjustment was primarily an operational project. However, it contained many technical problems that needed to be resolved. Some of these problems concerned the correct mathematical model for combining terrestrial and satellite observations. A different set of problems concerned the numerical aspects of the adjustment. For the first time sufficient computer power was now available to perform a simultaneous least squares adjustment of the entire network. However, an adjustment of such a large network had never been performed. No one was sure what problems might arise.

The technical problems associated with the new adjustment attracted worldwide interest among geodesists. The two NAD symposia provided an opportunity for extensive discussions of these problems.

The first International Symposium of Problems Related to the Redefinition of the North American Geodetic Networks was held on the campus of the University of New Brunswick in Fredericton, N.B., Canada, May 20 to 25, 1974. It was organized primarily by the Department of Surveying Engineering of the University. Figure 5.2, taken from the symposium proceedings, identifies the participants and their respective countries.

The following technical sessions were held:

1. Data Inventory and Assessment
2. Datum Definition
3. Mathematical Models for the Networks
4. Statistical Treatment of Models
5. Problems arising from Redefinition

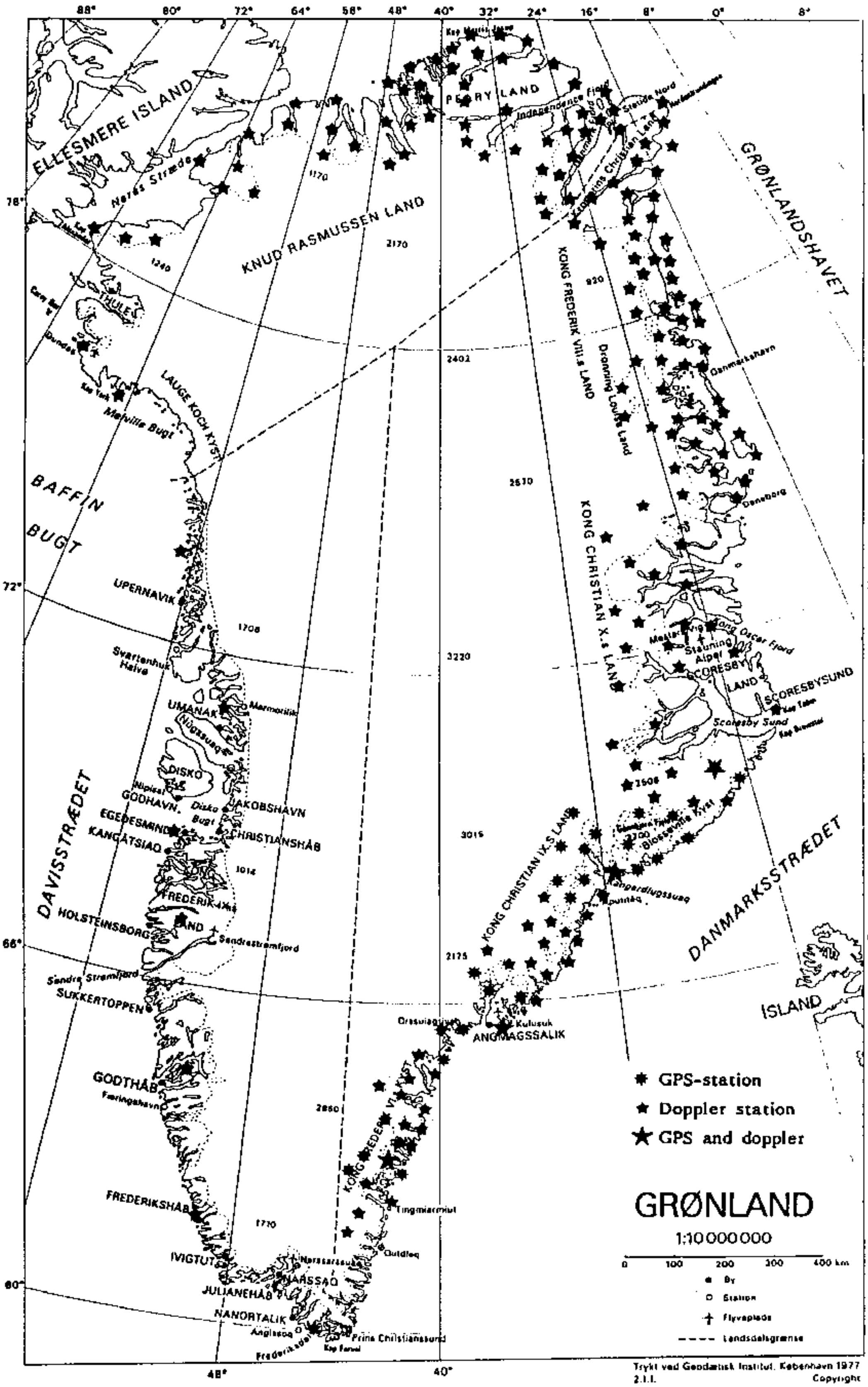
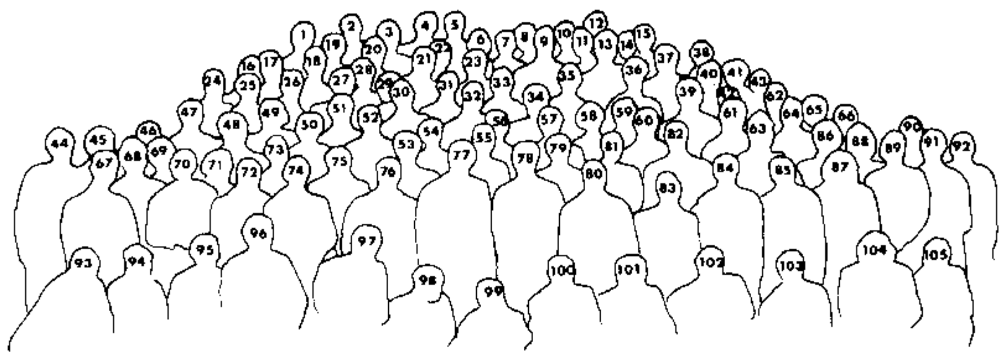


Figure 5.1. Geodetic satellite stations used to position Greenland on NAD 83.



Participants at the Symposium

Alberda, J. E., Holland (40)
Allman, J. S., Australia (78)
Anderle, R. J., U.S.A. (43)
Antoun, C., Ethiopia (47)
Arvizu, D. R., Mexico (73)
Ashkenazi, V., Great Britain (48)

Baker, L. S., U.S.A. (17)
Bishop, W., U.S.A. (12)
Blaha, F., Canada (66)
Blaha, G., Canada
Blais, J. A. R., Canada (67)
Bossler, J. D., U.S.A. (55)
Breton, L. W., Canada (92)

Caldwell, B., Canada
Canellopoulos, N., Canada (88)
Carlin, C. B., Canada
Castonguay, R., Canada
Chamberlain, C., Canada
Chovitz, B., U.S.A. (37)
Chrisodoulidis, D., U.S.A. (69)
Chrzanowski, A., Canada (91)
Coker, R. O., Nigeria (63)

Derenyi, E., Canada (30)
Dermanis, A., U.S.A. (71)
Detollis, J., Canada (27)
Doyle, F. J., U.S.A.
Dracup, J. F., U.S.A. (21)
Dreger, B., Canada (29)

Faig, W., Canada
Fila, K., Canada
Finnegan, E. W., U.S.A. (9)
Fischer, I., U.S.A. (58)
Fischer, E., U.S.A. (59)
Flemming, N., Canada
Foreman, J., U.S.A. (77)
Fubara, D. M., U.S.A. (60)

Gadegbeku, S. H., Ghana (93)
Gagnon, P., Canada (103)
Gale, L. A., Canada (57)
Gemael, C., Brazil
Gergen, J., U.S.A (96)
Gilchrist, C. F., U.S.A. (15)
Gloss, G., Canada (98)
Gomez, R. M., Venezuela (54)
Grafarend, E., W. Germany (8)
Gray, D., Canada (2)
Gregerson, L. F., Canada
Groot, R., Canada (90)
Groten, E., West Germany
Gunn, B., Canada (56)
Gunther, T. L., U.S.A. (44)

Hadi, S., Indonesia (83)
Hajela, D. P., U.S.A. (64)
Hamelin, P., Canada (7)
Hamilton, A. C., Canada (81)
Henderson, P., Canada (14)
Henneberg, H. G., Venezuela (70)
Herat, S. T., Sri Lanka (31)
Hittel, A., Canada (100)
Holdahl, S. R., U.S.A. (52)

Isner, J. F., U.S.A. (10)

Jones, H. E., Canada (85)

Kahar, J., Indonesia
Khosla, K. L., India (33)
Klinkenberg, H., Canada (3)
Kouba, J., Canada (101)
Krakiwsky, E. J., Canada (86)
Krukoski/Wilson, R. M., Brazil (42)

Lachapelle, G., Austria (6)
Latimer, J., U.S.A. (5)
Leclerc, J. G., Sweden (22)
Lefebvre, M., France
Linkwitz, K., West Germany
Long, J. T., U.S.A. (35)

Mancini, A., U.S.A. (76)
Martusewicz, J., Canada (68)
Masry, S. E., Canada
Mather, R. S., Australia
Matthew, T., Canada
McKay, E., U.S.A. (41)
McLaughlin, A., Canada
McLellan, C. D., Canada (36)
Meade, B. K., U.S.A. (28)
Meissl, P., Austria
Merry, C., Canada
Mock, C. J., Panama (24)
Monaghan, J. W. L., Canada
Moritz, H., Austria (50)
Mosienko, N., Canada (89)
Mueller, I. I., U.S.A. (39)

Nagy, D., Canada (53)
Nassar, M., Canada
Ndyetabula, S. L. P., Canada (72)
Noailles, C., Canada (80)

Obenson, G., Nigeria (61)
Ogilvie, M., Canada
Oriol, R., Haiti
Orlin, H., U.S.A. (75)

Parent, R., Canada (97)
Parker, D., Great Britain (45)
Pelletier, L., Canada (20)
Peterson, A., Canada (4)
Pinch, M., Canada (18)
Plouffe, R., Canada (26)
Pope, A. J., U.S.A. (1)
Porter, A., Canada

Rais, J., Indonesia (84)
Roberts, W. F., Canada
Robertson, L. P., Canada (94)
Rogers, A., Canada (23)

Sanchez, R., Argentina (32)
Sayn-Wittgenstein, L., Canada
Schwarz, C., U.S.A. (74)
Seppelin, T. O., U.S.A. (51)
Smith, P. A., U.S.A.
Soekotjo, T., Indonesia
Staszak, T., Canada (25)
Steeves, P., Canada
Stem, J., U.S.A. (105)
Strange, W., U.S.A. (65)

Tessari, E. J., Canada (34)
Thomson, D. B., Canada
Tscherning, C. C., Denmark (46)
Tsivos, V., Canada (87)
Turner, L. G., Australia (95)

Ugarte, P. G., Honduras
Uotila, U. A., U.S.A. (13)

Vanicek, P., Canada (82)
Veis, G., Greece (49)
Villasana, A., Mexico (99)
Vinklers, J., Canada

Wade, R., Canada (102)
Walker, S., Canada (62)
Wells, D. E., Canada
Whalen, C., U.S.A. (79)
Whiting, M. C. S., U.S.A.
Whitten, C. A., U.S.A. (19)
Wigen, S. O., Canada (38)
Wilson, P., West Germany (11)
Wray, T., Canada (16)

Young, G., U.S.A. (104)

Figure 5.2. Participants at the first NAD symposium (*Canadian Surveyor*, 1974)

Figure 5.3. Attendees at second NAD symposium (National Geodetic Survey, 1978).

The Second International Symposium on Problems Related to the Redefinition of the North American Geodetic Networks was held in Arlington, VA, April 24 to 28, 1978. Organized primarily by the National Geodetic Survey, the second symposium attracted 150 scientists from 17 countries. Figure 5.3, reprinted from the proceedings, shows most of the attendees.

By the time of the second symposium the scope of interest had grown to include computational problems, data management problems, and computer programming problems. The technical sessions included:

1. Status Reports and Test Adjustments
2. Datum Definition, Ellipsoid, and Geoid
3. Employment of Extra-terrestrial Methods
4. Data Preparation and Management
5. Helmert Block Adjustment System—I
6. Helmert Block Adjustment System—II
7. Postadjustment Considerations
8. Other Topics of Interest.

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6. TERRESTRIAL DATA

John G. Gergen

6.1 DATA DEFINITION

Prior to the middle of the 20th century, almost all geodetic surveying was done by measuring directions. The theodolite, capable of measuring a direction with great precision, had been a mature instrument for more than 100 years. Because distances could be measured only by slow and expensive technologies, such as with high precision tapes, they were measured infrequently.

The dominant surveying scheme was triangulation. Since the primary purpose was to construct a geodetic framework covering large distances, triangulation was most often performed in arcs. An arc of triangulation consists of a series of braced quadrilaterals laid end to end. (See fig. 6.1.) Central point quadrilaterals and polygons were also used occasionally. The quadrilaterals were made as large as possible, consistent with the need to be able to see from one station to the other. Surveyors also attempted to locate stations so that the quadrilaterals would be close to rectangular. Within each quadrilateral 12 directions were measured resulting in 8 angles (2 at each vertex). A base line was measured at every 8 to 10 quadrilaterals, sometimes even 25 quadrilaterals.

Once the primary arcs of triangulation were in place, it was possible to fill in the areas between the arcs with area triangulation. (See fig. 6.2.) Area triangulation may span less distance, but it results in the establishment of many more points than arc triangulation. With area triangulation,-there were fewer restrictions on the geometry of the individual figures, but the dominant figure was still the triangle.

Geodetic observations of triangulation began at the beginning of the 19th century. Progress was slow at first. During the first 100 years, the majority of geodetic observations were made by the Coast & Geodetic Survey (C&GS). Later, triangulation surveys were carried out by the U.S. Army Corps of Engineers, primarily in the Great Lakes region. In the 20th century, the highway departments of many states contributed significantly to the densification of the networks by the method of traversing. (See fig. 6.3.) The U.S. Geological Survey (USGS) also performed many area triangulation and traverse surveys in support of mapping projects.

The 1960s saw the beginning of modern urban surveys. This scheme was an attempt to establish a moderately dense control network for a metropolitan area. As cities had expanded, it became evident that errors had been introduced into the coordinate system by the piecemeal addition of surveys to the existing framework. In some cities, the NGS primary survey had lacked strength. Urban surveys rectified the many deficiencies of the control networks in exactly those areas where demand for the use of the control networks was strongest. These surveys tended to be large, with 1,000 points or more, and were adjusted by computer.

Preparations for the new adjustment began in the early 1970s. At that time, NGS had an inventory of about 5,000 horizontal survey projects. These formed the nucleus of the data set that was eventually built for the adjustment. Arrangements were made with the

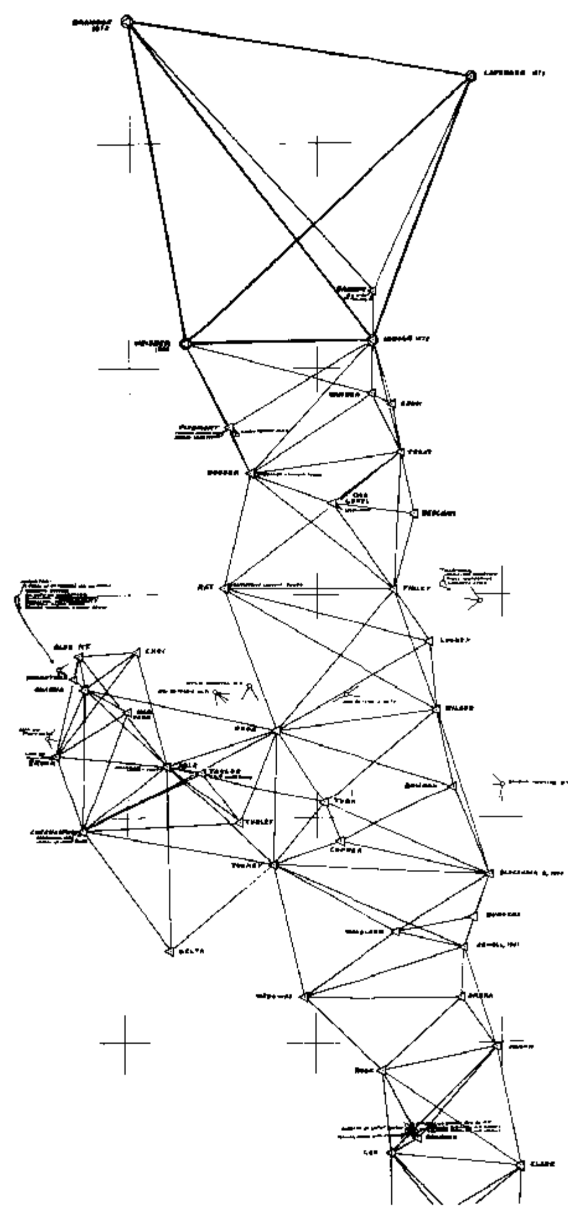

Figure 6.1. A typical arc of triangulation.

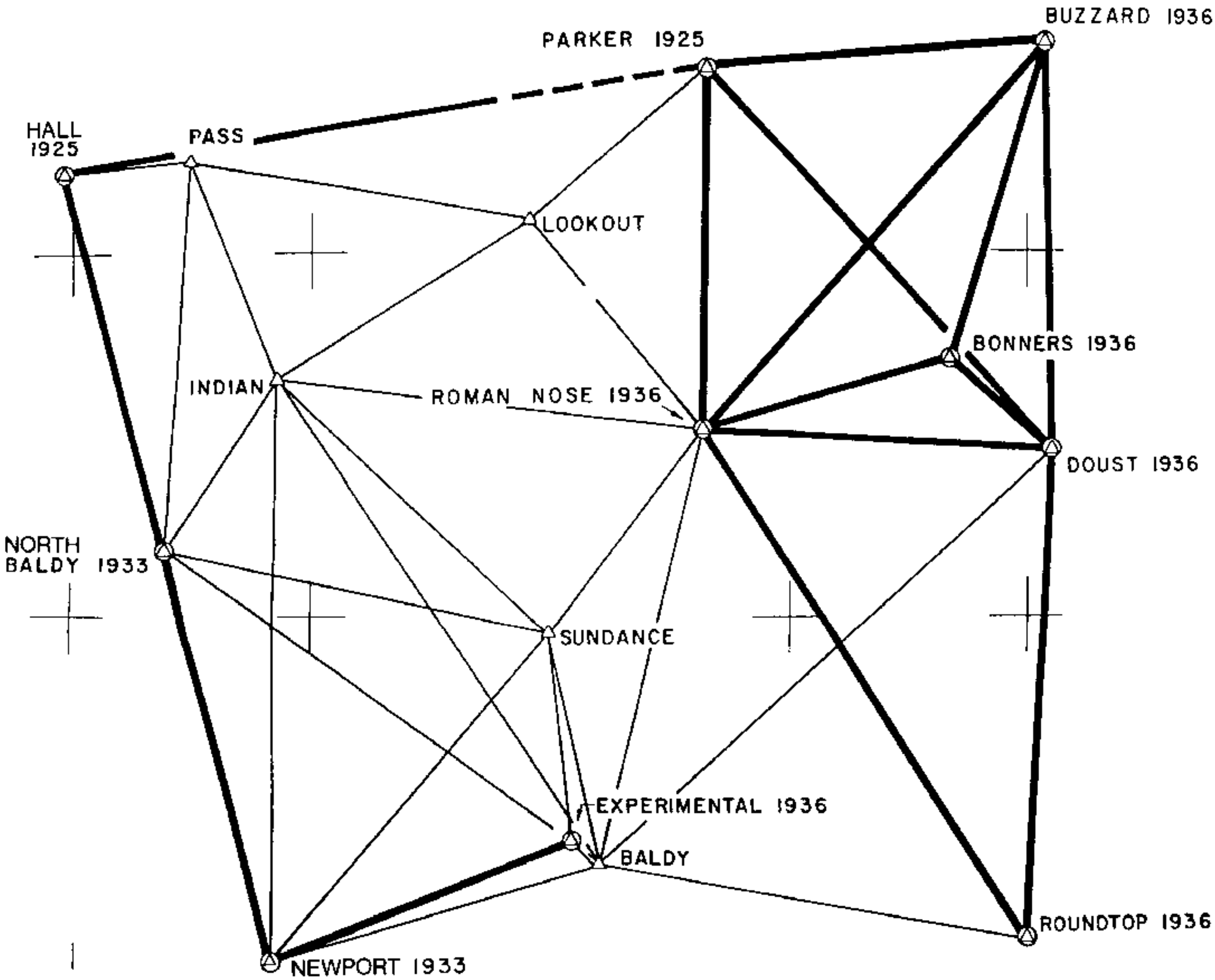

Figure 6.2. Area triangulation.

USGS National Mapping Division for the evaluation and identification of those USGS surveys which were of appropriate accuracy to be used in the new adjustment. USGS placed these projects in machine-readable form according to NGS specifications. A total of 134 USGS projects were transferred in time to be included in the NAD 83 adjustment.

At the beginning of the New Datum Project NGS was also receiving about 100 new projects every year from various sources. For each of the new projects, there was not only the need to put the data into computer-readable format, but coordinates also had to be computed for all new network points. This was an elaborate process involving the constrained adjustment of the project into the network. By 1981 it was realized that the agency was being swamped by new projects that were being submitted for adjustment. A decision was made that new projects submitted after July 1, 1981, would not be processed until after the completion of the NAD 83 project.

On the average, a horizontal survey project contained 70 points. However, the number varied greatly, and some had more the 1,000 points.

6.1.1 Observables

The observations most relevant to the new datum were those associated with classical survey operations. These consisted of horizontal directions, zenith distances, astronomic azimuths, and distances. The first three were determined using theodolites. The distances consisted of older tape measurements and more modern electronic distance measurement (EDM) observations.

6.1.1.1 Horizontal Directions

The horizontal directions constituted the vast majority of the material in the NGS files. The NGS practice had always been to record horizontal directions rather than angles. Directions may be interpreted as an azimuth relative to an arbitrary (but unknown) azimuth of the zero mark on the horizontal circle of the theodolite. Angles can always be computed by differencing directions. However, the directions more closely correspond to the actual act of making an observation, and are therefore more likely to be statistically independent.

Horizontal direction observations were stored in several ways: in the field books as Abstracts of Directions (fig. 6.4), and as Lists of Directions (fig. 6.5). The means of each single observation (direct and reverse) were stored in the field books. In general, single observations were made with the horizontal circle of the theodolite in several different positions to lessen the effect of errors in the manufacture of the circle. The mean of these single observations at different positions was generally computed in the field book. Both the single observations and the means were copied onto the Abstract of Directions. The means constituted a single list of directions, which gave a single direction to each observed object for that station occupation. If more than one occupation occurred at a station, which could happen if the station was occupied as part of more than one survey project, then the various lists would be collected together into a combined List of Directions.

The single observations which were meaned together in an abstract were generally a homogeneous data set. Each round of directions at a particular setting of the horizontal circle contained exactly the same objects, and all observations were made with the same instrument. This was not true for the combined Lists of Directions. Two different lists would often contain different objects and might contain observations with different instruments. This practice created unwanted biases which made combined lists less than desirable.

Early in the New Datum Project, NGS faced the question of whether the single lists from the abstracts or the combined Lists of Directions should be placed into machine-readable form and used in the network adjustment. Using combined lists would demand less keying of data, would result in a smaller data base, and would allow the network software to assume only a single list of directions at each occupied station. On the other hand, the combination of directions into lists had not always been done in a consistent manner, so that each combination would have to be verified. Furthermore, NGS was aware that it had been mathematically demonstrated that the means in a single list are statistically independent, while those in a combined list

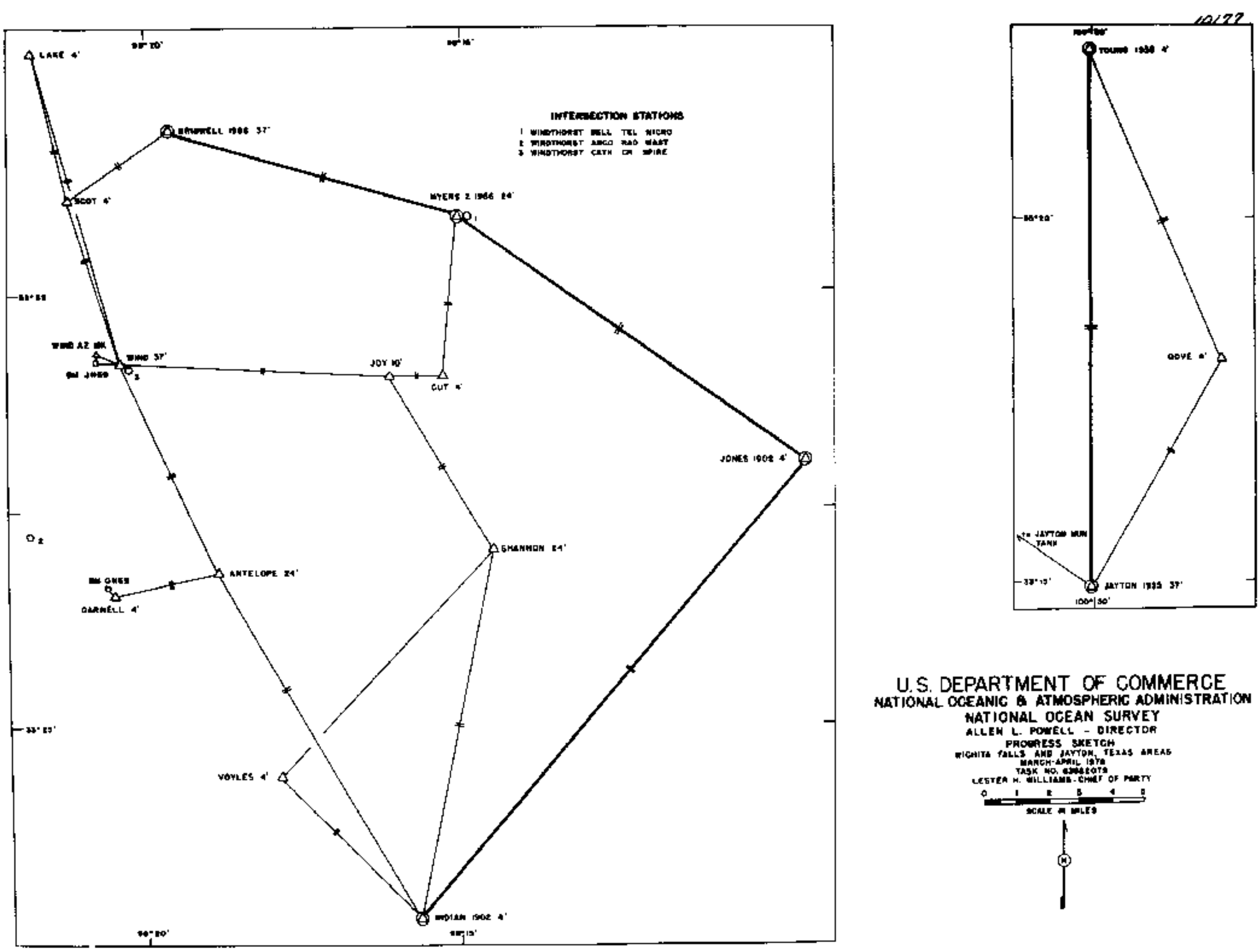

Figure 6.3. A typical traverse layout.

are in general correlated (McKay, 1973). After considering these alternatives, NGS decided to use the single lists from the abstracts. In general, therefore, "a horizontal direction" is taken to mean the direction found on the bottom of an abstract, which is the mean of forward and reverse readings at several positions of the horizontal circle.

The NGS files were estimated to contain 2.5 million horizontal direction observations, an average of more than 10 observed directions per occupied station. The number of single directions making up the observation is a function of the desired precision of the result. For first-order triangulation stations, it was customary to observe 16 individual directions from which a mean direction was computed on the abstract. Landmarks, such as water tanks, church spires, and other conspicuous objects, were observed with only four directions. On occasion, in modern urban survey projects, eight directions were observed for these same objects. The precision of observed horizontal directions to landmarks was correspondingly lower.

The precision of horizontal directions, in general, is given as a function of the observing procedure used. The observing procedure involves (among other things)

STATION: JUNIPER USC & GS	NOAA FORM 76-86 (11-72)	U. S. DEPARTMENT OF COMMERCE NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION	
	ABSTRACT OF DIRECTIONS		
STATE: AZ	COMPUTED BY: D. MORELAND	DATE: 10-20-87	VOLUME NO. 11
OBSERVER: E. OBIDINSKI	CHECKED BY: J. JORDAN	INSTRUMENT NO. T-3 74405	SHEET 1 OF 1

POSITION NO.	GRASSY USC&GS	MULE USC&GS	GOVERNMENT USC&GS			AZIMUTH MARK	RM#2	RM#1
	(INITIAL) 0° 00′	72 02	225 07	° ′	° ′	64 44	237 24	331 28
	″	″	″	″	″	″	″	″
1	0.00	21.8	51.4			34.9	42.1	50.9
2	0.00	27.4 (30.2)R	54.0 (58.4)R					
3	0.00	25.1	47.2					
4	0.00	23.5	53.9					
5	0.00	23.3	51.7			41.9	38.5	56.2
6	0.00	24.4	46.3					
7	0.00	21.9	46.3					
8	0.00	24.8	44.7 (44.8)R					
9	0.00	23.7	47.1			33.5	37.2	55.3
10	0.00	24.5	49.7					
11	0.00	23.1	51.9					
12	0.00	23.1 (29.1)R	53.9					
13	0.00	27.7	52.4			36.6		
14	0.00	25.9	49.9					
15	0.00	26.5	50.4					
16	0.00	23.3	51.5					
SUM,		389.0	806.3			146.9	117.8	162.4
MEAN,		24.31	50.39	—		36.7	39.3	54.1
COR. FOR ECC.,								
DIRECTION,								

Figure 6.4. Sample of completed form, "Abstract of Directions."

NOAA FORM 76-72 (8-71)

U. S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL OCEAN SURVEY

LIST OF DIRECTIONS

Station: JUNIPER	State: AZ	Height of tower: 1.55
Chief of party: K. SUTTLES	Date: 10-20-87	Eccentricity: NONE
Observer: E. OBIDINSKI	Instrument: T-3 74405	Recorded in volume: 11
		Computed by: D. MORELAND
		Checked by: J JORDAN

OBSERVED STATION	Observed direction ° ′ ″	Eccentric reduction ′ ″	Corrected direction with zero initial ° ′ ″	Sea level reduction ″	Adjusted direction ″
GRASSY	0 00 00.00		0 00 00.00		
MULE	72 02 24.31				
GOVERNMENT	225 07 50.39				
AZ MARK	64 44 36.7				
RM#1 K E.	331 28 54.1				
22.01 FT					
6.708 M					
RM#2 L NNW	237 24 39.3				
12.52 FT					
3.817 M.					

Figure 6.5. Sample of completed form, "List of Directions."

the precision of the theodolite used and the number of settings of the horizontal circle. The observing procedure selected generally depended on the intended order of the two endpoints of the line. Table 6.1 gives the standard deviation of a single observation and the average number of telescope positions for each order of observation.

TABLE 6.1.—*Precision of direction observations*

Order of observation	Standard deviation of a single observation (arc sec)	Number of circle positions	Resulting standard deviation (arc sec)
High-precision	2.0	[1]32	0.35
First-order	2.4	16	0.60
Second-order	2.8	16	0.70
Third-order	3.4	8	1.20
Intersection	6.0	4	3.00

[1] Sixteen positions on 2 separate nights.

As an example, consider a direction observation from a second-order station to a first-order station: The lower order observation takes precedence. Thus assuming 16 positions of the telescope, the standard deviation of the mean of 16 direction observations will be $2.8/\sqrt{16} = 0.70$ arc second.

6.1.1.2 Astronomic Azimuths

Astronomic azimuths were required to provide orientation to the network. The NGS files contained about 5,000 astronomic azimuths between points of the horizontal control network, taken over a period of 130 years. About 75 percent of these azimuths were observed since 1960, mostly as part of the Transcontinental Traverse surveys.

Precision estimates of astronomic azimuths have been found to be poor indicators of accuracy. In the late 1970s, a systematic analysis was performed by members of the Gravity, Astronomy, and Satellite Branch of NGS to determine the error budget of astronomic data, including astronomic azimuths. (See chapter 8.) The development of correct accuracy estimates involved statistical evaluation of repeat measurements and a careful attempt to identify error sources. Whereas the internal precision of astronomic azimuths was 0.3 arc second, the analysis of the total error budget showed that the accuracy of these observations is actually about 1.1 arc second.

6.1.1.3 Taped and EDM Distances

At the beginning of the New Datum Project, the NGS files contained several hundred base lines measured with great care by precise tapes. Their precision was estimated to be 1 in 500,000. Almost all of these taped base lines had been observed prior to 1960.

The measurement of base lines by taping was quickly abandoned when EDM equipment was introduced. Suddenly relatively long distances could be measured routinely with great ease. Projects executed with EDM contained hundreds of distance measurements. The old triangulation with only a few base lines was transformed into triangulation with saturation level distance observations. What old triangulation had been lacking, scale, was suddenly present in abundance. Large redundancies in distance observations contributed to an increase in accuracy for modern triangulation projects. The 19th-century estimated accuracy of triangulation was 1 in 25,000, whereas with the advent of EDM the estimated accuracy of modern triangulation projects increased to 1 in 200,000.

In September 1974, NGS had the following inventory of distance observations:

Type	Number
EDM distances (lightwave instruments)	120,000
EDM distances (microwave instruments)	25,000
Taped base lines (NGS)	500

Unfortunately, the EDM measurements which formed the bulk of the holdings of distance measurements had been taken in a period when great changes occurred in instruments, observing procedures, and processing procedures. As a result, the data had not been treated consistently. Most of the measurements had been partially processed and were stored as sea level distances (i.e., distances along the geoid).

The following variations in processing were found:

- The most current value for the speed of light was not always used.
- The second velocity correction was not always applied.
- The beam curvature correction was not always applied.
- Different refraction models were used.
- The computations for transforming lengths from instrument-to-instrument down to mark-to-mark were not always done correctly.
- The arc-to-chord correction was not always applied.
- The most recent frequency count for an instrument on record was not always applied as a correction to the measuring frequency.
- The delay line conversion data for Geodimeter observations were not always computed and applied in the same way.
- Not all computations were carried out using double precision arithmetic.

Because of these variations, it was decided that all EDM distances in the NGS files should be recomputed, starting with the field notes. For each measurement, the computed quantity was the mark-to-mark distance. A series of measurements made on a single night was meaned to produce a single observation, but observations carried out on different nights were kept as separate observations.

A large number of the EDM observations had been taken as part of the high precision Transcontinental Traverse (TCT) surveys of the 1960s and 1970s. In these surveys all observations—the directions, astronomic azimuths, and distances—were executed twice. Observations were made on 2 different nights, with different instruments and different observers. On some long lines, meteorological observations at the midpoint of the line were made with the help of balloons. On occasion, airplanes were flown along the line of sight to collect meteorological data to be used for correcting the observations for atmospheric refraction. The TCT constituted an independent survey network. When adjusted by itself, the TCT network resulted in coordinates estimated to have an accuracy of 1 part in 1 million. Furthermore, the TCT surveys used existing points along the survey path whenever possible. As a result, existing portions of the network were strengthened and strong connections between previously unconnected arcs of triangulation were created.

Electronic distances were determined primarily with Geodimeters. During later years, laser Geodimeters were used exclusively. NGS has always considered the class of instruments operating at lightwave frequencies to be superior to the class of instruments operating at microwave frequencies. For high precision work NGS used Geodimeters exclusively. This extensive experience has contributed to a clear understanding of the technology, including the determination of the error budget. Geodimeters have determined distances to a precision of 1 to 3 mm plus 1 part per million (ppm).

6.1.2 Supporting Data

6.1.2.1 Zenith Distances

Zenith distances were measured in order to determine the elevation differences between pairs of points. Older specifications did not require knowledge of the heights of horizontal control stations. For many older horizontal control stations, the height is known only by barometric measurements, by photogrammetric measurements, or by scaling from topographic maps. Modern specifications require that a number of points be connected to bench marks—points for which elevations are known to high precision—while the remaining heights should be surveyed by zenith distances. The modern specifications were instituted to support the precise EDM measurements. Precise distance observations require reasonably good elevations for the reduction to the geoid (and ultimately to the ellipsoid). The standard deviation of the mean for zenith distances is computed to be 5.0 arc seconds + 1.0 arc second/km.

6.1.3 NAD 27 Geodetic Positions

The preliminary and adjusted NAD 27 positions of control points were often stored in the project folders. These served the following tasks:

- associate observations with positions,
- provide positions during the project validation phase of the NAD project, and
- use as preliminary coordinates (after transformation to the proper new datum) for the adjustment.

6.2 FORMAT DEFINITION

Having identified the terrestrial data to participate in the new adjustment, it was necessary to define an appropriate format so that the data could be placed into machine-readable form. The dominant input technology at the time was the 80-column punched card. It was necessary that any input format be constrained to 80-character records and that projects be assembled as physical card decks. The format selected was one that had already been developed for a computer program named TRAV01. This computer program had been written for the purpose of performing least squares adjustments of TCT observations, and this purpose was the source of the name. However, the program was equally useful for adjusting other projects. It became the standard tool for validating projects, and its input format became the standard for placing terrestrial data into machine-readable form. The original program was replaced by a series of new versions named TRAV02, TRAV03, etc., eventually ending with TRAV10 (Schwarz, 1978). (Programs named TRAV11 and TRAV12 were written but never placed in production.) Through all the program versions, the computerized input format, called a Trav-deck, remained essentially the same.

The Trav-deck represented an interim storage mode for "automated" projects. All 5,000 individual survey projects were keyed in this format. Eventually the data in the Trav-decks were entered into the NGS data base and the decks themselves were no longer of importance.

By 1976, NGS defined its "Input Formats and Specifications of the National Geodetic Survey Data Base" (FGCC, 1989). This represented a more general and comprehensive format and subsequently became the standard. By the end of the New Datum Project, the Trav-deck was superseded for all purposes. Its importance is historic, only representing the storage format for horizontal network projects during the initial validation phase of the NAD 83 project.

6.3 HISTORIC DOCUMENTS

6.3.1 Project Archive

Individual field projects, as well as the accompanying office computation folder, were stored in cahiers—the contents of standard folders—and were classified by state.

The archive was classified according to library methods resulting in a system of abbreviations as follows:

A ASTRONOMY
G GEODESY
  GA Descriptions of stations
  •
  •
  GTZ Triangulation
H HYPSOMETRY (PRECISE LEVELING)

Certain suffixes added the following meanings:
R reconnaissance
Z computations
°office computations

For example, GTZ° simply means GEODESY, TRIANGULATION, OFFICE COMPUTATIONS.

The majority of cases involved the field portion of the project—the GTZ—stored in one or more cahiers. The office computation cahier—the GTZ°—was usually separate. Both the GTZ and the GTZ° would coexist in parallel. In some cases, as for example when the documents of a small project would fit into one cahier, the GTZ would be changed to a GTZ° by adding the office computations into the field records cahier.

The most complex case occurred when a large geographic area was readjusted. All GTZ cahiers as well as GTZ° cahiers in that respective geographic area would be collected and the resulting set of cahiers would be different from the input.

The identification of individual observations with the appropriate archive number—the GTZ number—was one of the goals of the automation process. On occasion, when this was difficult, exceptions had to be made. For example, there were cahiers which contained abstracts of directions from more than one project. In such cases, the GTZ° number was used instead of the GTZ number.

In 1974 a committee of senior geodesists was formed to review archival projects. The purpose was to identify and delete those survey projects which were of little or no value to the network adjustment. This group was named the "Committee to Review Archival Projects." Several hundred projects were identified and removed over the years. In rare cases it was found that individual points did not connect to the network, and thus had to be removed.

6.3.2 Coding of Documents and Assembly of Projects

The Trav-deck format contained separate sections for the different data types, such as directions, azimuths, and distances. The material in the folders was separated and sent for keypunching according to data type. NGS developed a document called "Trav-deck Procedures," which provided standard operating procedures for each class of documents. These procedures covered the review of documents in the folders, preparation of documents for keypunching, quality control of the key entry process, and assembly of the punched cards into Trav-decks (National Geodetic Survey, 1979).

Some data elements were included in the Trav-decks but were not used directly in the new datum adjustment. Among these were the order and class of the observing station, and the state plane coordinate zone of the observing stations. The date of the observation was used only to compute crustal motion corrections in areas of significant motion, according to the model in chapter 17. These were included because they were judged to be potentially useful to investigations and procedures that might take place after the new datum adjustment. The effort required to include these additional elements was minimal as long as they were coded along with the other elements required for the new datum adjustment.

A great effort was expended to ensure the correct determination of station order and type, the coding of the observations proper, and the correct assignment of the standard deviations of the observations (National Geodetic Survey, 1979).

6.4 VALIDATION OF ASSEMBLED PROJECTS

6.4.1 Guidelines and Procedures

The coding and subsequent keying of projects resulted in a computer-readable file containing a Trav-deck. An unconstrained least squares adjustment was performed with each and every such Trav-deck. The main purpose of this adjustment was verification of the coding of the observations. Bad or improper coding would result in unacceptable results from the unconstrained adjustment. This procedure was iterated until all parameters came within the specified tolerance. A single project, therefore, might be adjusted up to four times before the project was judged to be acceptable.

6.4.2 Software

The Trav-deck validation was performed by program TRAV10, which was capable of performing a least squares adjustment of all horizontal observations in a project (Schwarz, 1978). TRAV10 performed a two-dimensional adjustment on the ellipsoid. It used the Cholesky solution method with a variable band storage scheme. The normal equations were partitioned into variable sized blocks stored on a random access device and recalled into memory when needed. TRAV10 also used a station reordering scheme in order to reduce both normal equation storage and the number of arithmetic operations. During the period that Trav-decks were being keyed and validated, the TRAV10 program was run as many as 30 times per day.

6.5 SPECIAL CONSIDERATIONS

6.5.1 Data Selection

Within NGS, considerable discussions took place on the subject of exactly which observations should be used in the NAD 83 adjustment. Some said that only the most precise observations should be used in the adjustment because the inclusion of less precise observations would contaminate the precise ones. Others said that the adjustment should contain all observations as long as the precision of each observation was properly identified.

The alternative selected was the latter: all observations (250,000 stations) were used. A number of arguments contributed to this decision:

- Considerable effort had been expended in determining the precision of the observations. As a result, it was possible to let the adjustment program combine the observations, assigning weights based on the precision of each observation.
- The composition of each of the 5,000 projects precluded a clear separation between the precise and less precise observations. Separation was difficult and clumsy. Even if the precise part was separable, what was left of the project was no longer an adjustable entity by itself. It depended heavily on the higher order survey that had been removed.
- Performing the new adjustment on only a subset of the network was unacceptable because no one could ensure an orderly continuation of the project once the main solution was in hand.

6.5.2 Special Parameters

During the period that projects were being put into machine-readable form and validated, it became clear that some of the projects submitted to NGS by the highway departments of some states presented special problems. Records indicated that not all highway departments had calibrated their EDM instruments properly and reliably. As a result, scale differences up to 10 parts per million (ppm) between the NGS measurements and those of some state highway departments were not uncommon. The larger differences generally involved microwave-frequency EDM instruments.

Vidal Ashkenazi, who at the time was a Visiting Senior Scientist at NGS, suggested that it should be possible to solve for these scale differences in the overall network adjustment. As a result of this suggestion, additional parameters, called "observation class deck" parameters, were introduced into the mathematical model. These were used almost exclusively to represent the scale error of highway department instruments. During the project validation phase, NGS identified 30 state highway department instruments for which it had observations. Special scale parameters were added to the adjustment for each. The solution given in chapter 18 shows that many of these parameters are indeed significantly different from zero and that their inclusion is therefore important.

6.6 CONCLUSION

The coding, keying, and analysis of 2.5 million observations constituted a major task within the NAD 83 project. This effort proved to be the most laborious and expensive task of the entire project, at times involving as many as 35 people. The positive benefits were twofold: In addition to the successful completion of the NAD 83 project, the observations are now in computer-readable format accessible for current applications. Before the project began, these observations were on paper in 5,000 different cahiers.

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7. STRENGTHENING THE NETWORK THROUGH FIELD SURVEYS

Edward J. McKay

7.1 REVIEW OF NETWORK DEFICIENCIES

In 1971, prior to beginning the new adjustment of the North American Datum, a comprehensive review of the U.S. horizontal control network was performed. This review was undertaken to determine the number and extent of new field surveys that would be required to ensure that the relative distance accuracies between pairs of adjacent points did not exceed one part in 100,000. Although this review had a specific objective, it was part of a more general plan titled "Objectives for Geodetic Control" (Coast and Geodetic Survey, 1968). This plan detailed the spacing and accuracy requirements for continuing and completing the horizontal network based on a priority system.

At the time of the comprehensive review, the observational data had not been converted to computer-readable form; hence, a rigorous statistical network analysis was not possible. Instead the review was performed by visually examining geodetic network diagrams and locating areas where the network was judged to be poorly configured or lacked sufficient scale and orientation. The criterion used to determine where the network was poorly connected was essentially the 20-percent rule, as given in the Federal Geodetic Control Committee (FGCC) publication, *Specifications to Support Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys* (FGCC, 1980). Similarly, the criterion used to determine where additional base lines were needed was to count the number of figures between existing base lines. A new base line was added if it substantially exceeded the FGCC specification for first-order triangulation. A similar procedure was followed to determine where additional astronomic azimuth observations were needed. In general, the rule of thumb for such networks dictates a base line and azimuth about every fourth figure, or every 40 miles.

7.2 REASSESSMENT

The original evaluation of the network indicated that about 10,000 km of new arcs, or in some instances traverses, should be observed. A subsequent reassessment of the network showed that the accuracy requirements could be satisfied if 6,000 km of new surveys were observed at strategic locations, along with the addition of satellite Doppler positions.

A priority system was established, based on the following criteria:

1. Ensure that all areas of known weakness were strengthened.
2. Complete or tie-off partially completed surveys.
3. Prioritize surveys according to population density and economic and national resources development.
4. As resources allowed, provide a general strengthening of the network.

7.3 ACTUAL NEW SURVEYS

The beginning of the NAD 83 project placed a sense of urgency on the plans for strengthening the network. The new surveys would have much greater value if they could be used in the fundamental adjustment. This required that all new surveys be completed by the date at which the data set for the adjustment was frozen (1981).

Only about 50 percent of the planned field work (6,000 km) was actually completed. Most of the supporting arcs and traverses originally planned for the portion of the United States east of the Mississippi River were observed, but the number of arcs and traverses planned for the western portion was reduced.

7.4 BASE LINES, ASTRONOMIC AND GRAVITY OBSERVATIONS

In addition to entire field surveys needed to connect parts of the network that were designated as "weak," individual new base lines and azimuths were measured. The original evaluation performed in 1971 proposed the measurement of about 600 new base lines and 600 new azimuths. Of these, only about 60 new base lines and 60 new astronomic azimuths were actually observed.

Another field surveying effort involved the observation of astronomic latitude and longitude at 106 stations. These observed values were used to form observed deflections of the vertical, which were then used to control the astro-gravimetric prediction of deflections at other network points. (See chapter 16.) The criterion used by NGS was that astronomic positions would be observed at any station where the correction to any observed horizontal direction due to the deflection of the vertical could reach 0.5 arc second.

No new gravity surveys were performed to support the computation of the deflections of the vertical. NGS' gravity data base had sufficient data, a distribution of at least one point for each 5 arc-minute square, to support this aspect of the NAD 83 project.

7.5 THE TRANSCONTINENTAL TRAVERSE

A field surveying program that was not begun expressly for the NAD program, but had a major impact on NAD 83 results, was the high-precision Transcontinental Traverse (TCT). This project officially began in 1961 to support the Satellite Triangulation Program by providing very long (continental) base lines. TCT is comprised of extremely accurate length, angle, and azimuth measurements in somewhat rectangular loops spanning the continental United States. Originally using a specially designed observing scheme of elongated polygons, which was later revised to elongated triangles and finally to single line traverse, TCT provided position control of approximately one part in 1,000,000 between connected stations. Two different observers measured each line on different nights, using at least two high-accuracy electronic distance measuring instruments.

As stated in chapter 6, all distance measurements for the TCT project were made from towers at least 10 m in height to obtain a representative value for the refractive index along each line. Atmospheric pressure, temperature, and humidity values were recorded at the endpoints of the lines, with mid-line temperatures obtained for some lines. The sight paths of several of the longer lines were flown, to obtain meteorological values along the entire line. In addition, first-order astronomic position and azimuth observations were made at the initial and go-ahead stations of the particular configuration used. The azimuth observations were also taken on 2 nights with a different observer using a different instrument each night. Observed horizontal directions to all adjacent stations were generally included with the direction to Polaris.

The TCT project (22,000 km of ultra-precise measurements), combined with the Doppler satellite positioning program, provided a uniformly high standard of accuracy for the network in all regions of the country.

7.6 TODAY'S HORIZONTAL GEODETIC NETWORK

While the field effort in direct support of NAD 83 fell short of the recommended amount of new first-order surveys, base lines and astronomic azimuths, the new observations that were accomplished contributed substantially to the new adjustment. The overall horizontal network meets the stated accuracies for the individual control points. There are still areas of the United States that lack adequate geodetic control and those which should be upgraded with new field surveys using the Global Positioning System. This new work represents NGS' post-NAD 83 surveying activities.

7.7 REFERENCES

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8. EXTRATERRESTRIAL DATA

William E. Strange

8.1 ASTRONOMIC OBSERVATIONS

8.1.1 Introduction

The NAD 83 adjustment made use of astronomic observations taken over a 130-year period at approximately 5,000 stations. Although these measurements extended over a long period, most observations are relatively recent. More than half are associated with the establishment of the Transcontinental Traverse in the 1960s. The most important application of astronomic measurements was to control locally the orientation of the NAD 83 network by establishment of Laplace stations, stations where astronomic azimuth and longitude were observed. Astronomic latitude was also measured at most stations. (See fig. 8.1.)

Astronomic latitudes and longitudes also entered the height controlled three-dimensional mathematical model directly. (See chapter 12.) The measured astronomic positions were a primary data source in the astro-gravimetric process which formed estimates of deflections and geoid heights at all occupied stations. (See chapter 16.) For the majority of stations the astro-gravimetric interpolation was sufficient. However, where horizontal angles were measured over lines with vertical angles in excess of 7 degrees greater accuracy was needed. Therefore, a special observation program was undertaken to obtain astronomic latitude and longitude observations at stations with vertical angles greater than 7 degrees. This special observation program involved approximately 115 stations.

Considerable effort was expended in analyzing the astronomic observations before they were used in the adjustment. Analysis focused on two subjects: (1) derivation of the corrections required to relate the measurements made over the entire 130-year period to a common coordinate system, and (2) determination of satisfactory estimates of observational error.

8.1.2 Astronomic Azimuths

The majority of astronomic azimuth measurements made by NGS used the "direction method," as described in *USC&GS Special Publication* 237 (Hoskinson and Duerksen, 1947). Observations on 2 nights with 16 positions of Polaris observed on each night (usually with a different observer on each night), resulting in a formal standard error of ±0.45 arc second, was required for a first-order azimuth.

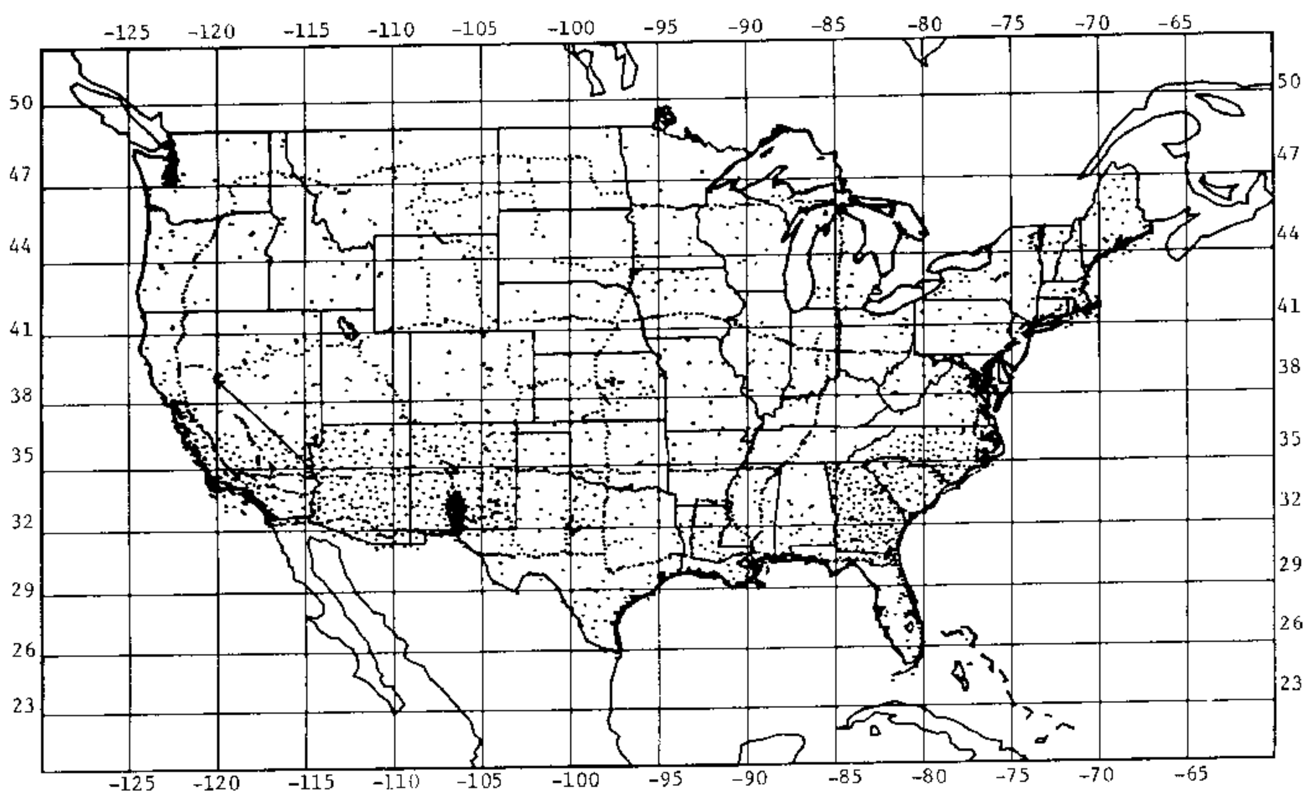

Figure 8.1. Astronomic latitudes and longitudes included in NAD 83.

Although the formal standard error of a first-order azimuth determination is ±0.45 arc second, repeat determinations have demonstrated that this formal error overestimates the accuracy. Thus in the mid-1970s a study was undertaken to determine realistic estimates of astronomic azimuth error. The results of this study have been reported in detail elsewhere (Carter et al., 1978) and will only be summarized here.

Estimates of astronomic azimuth accuracy were determined using analysis of variance (ANOVA) methods with various sets of repeat observations at a station. The error sources investigated were random error, individual instrument bias, observer bias, and differences in instrument types and observing procedures. An investigation of the latitude dependence of azimuth error was also conducted since such a latitude dependence was to be expected due to the increasing elevation angle of Polaris with increasing latitude.

Analysis of 88 pairs of repeat daily azimuth determinations extending over a latitude range of 16 degrees (32° to 48°) indicated an increase in the standard error of a 16-position daily determination of azimuth of +0.012 arc second per degree increase in latitude. This was in agreement with an estimate of Mueller (1969) of a standard error increase of ±0.011 arc second per degree increase in latitude over the range 30 to 50 degrees. The conclusion was that, over the range in latitude in the conterminous 48 States where the direction method of azimuth determination is used, the latitude dependence of azimuth standard error was negligible and could be ignored.

The conclusion of the analysis of repeat astronomic azimuth determinations was that the formal standard error associated with astronomic azimuth determinations was not satisfactory. The formula recommended for determination of the standard error for a single night's observations by a single observer was

$$\sigma_A = \left[\frac{(\sigma_R)^2}{N} + (\sigma_S)^2 \right]^{1/2} \tag{8.1}$$

where:

σ_R = random error of single position
σ_S = systematic error associated with the night's observations
σ_A = Standard error of the night's observations
N = number of observations.

The values derived for the error components were

σ_R = ±1.7 arc seconds
σ_S = ±1.4 arc seconds

In this case the standard error for a typical 1 night observation involving 16 observations of position would be about ±1.5 arc seconds.

The analysis indicated that the primary source of systematic error was a personal equation of the observers with a generally much smaller contribution from systematic instrument bias. Thus the proposed standard error to be assigned to a first-order azimuth determination (2 nights of observations with 16 positions of Polaris determined each night) was estimated to be:

±1.4 arc seconds - when observations were made on both nights by the same observer using the same instrument.
±1.1 arc seconds - when a different observer and instrument were involved each night.

8.1.3 Historical Summary of Astronomic Latitude Determination

Systematic observations of usable astronomic latitudes by the USC&GS began in 1851. Between 1851 and 1914 zenith telescopes, transit telescopes, and meridian telescopes were used. During the period 1914 to 1959 the Bamberg broken telescope transit instrument was used. Since 1960 the Wild T4 universal theodolite has been used. Almost all latitude determinations have been made using the Horrebow-Talcott method. Prior to 1910 it was common to have observations extend over a number of nights and to make multiple observations of star pairs. During the period 1900-10 studies made by Bowie (1917) determined that single observations of 15 to 25 star pairs on a single night were adequate. Since 1910 single night, single observations of 15 to 25 star pairs have been used for astronomic latitude determination. The standard used in recent decades is single observations on a single night of 16 star pairs for a first-order astronomic latitude and of 8 star pairs for a modified first-order latitude. Modified first-order latitudes have been observed primarily in connection with the high accuracy Geodimeter Transcontinental Traverses carried out in the 1960s and 1970s.

8.1.4 Coordinate System Corrections

The implied coordinate system for astronomic latitude determinations is not dependent upon highly accurate timing. Therefore, satisfactory reduction of astronomic latitudes to a common coordinate system required only the relating of the various star catalogue declination systems used to the proper declination system for NAD 83 and application of proper polar motion corrections.

Table 8.1 lists the star catalogues used for astronomic latitude determinations. The information for this table and all subsequent tables except table 8.2 was taken directly from Pettey and Carter (1978). Prior to 1908 some 48 star catalogues were used in latitude determination. A list of these catalogues appears in *USC&GS Special Publication* 110 (Beall, 1925). During the period 1851 to 1907 many latitudes were recomputed as improved star catalogues became available. No attempt was made to correct pre-1908 latitude determinations by recomputation using star declinations updated to the FK-4 star system. To do this would have required going back to the original observations, identifying the stars used, and on a star-by-star basis determining each star's FK-4 declination using the Smithsonian Astrophysical Observatory (SAO) Star Catalog (Smithsonian Astrophysical Observatory, 1966). The improvement was not considered worth the effort involved since: (1) the formal standard errors of latitudes computed using these older catalogues were not greatly different than those obtained with modern catalogues, implying the internal consistency of the older catalogues was about equal to that of modern catalogues, and (2) comparisons of 1851-1907 latitudes with recent repeat measurements that were reduced using FK-4 declinations indicate no gross systematic declination differences between the older catalogues and FK-4.

TABLE 8.1.—*Catalogues used in latitude determination*

(Pettey and Carter, 1978)

Period	Stations determined	Catalogues used	Catalogue system for latitudes
1846 to 1907	1,003	Various (48)	Various
1908 to 1939	885	PGC (Boss)	FK3 (quasi)
1940 to 1967	1,693	GC (Boss)	FK3 (quasi)
1968 to present	1,270	SAO	FK4 (quasi)

Between 1908 and 1939 the Boss Preliminary General Catalogue (PGC) was used for latitude determinations, between 1940 and 1967 the Boss General Catalogue (GC), and since 1968 the SAO Star Catalog. Corrections were made to the latitudes computed between 1908 and 1967 to place them on the FK-3 declination system. For the period 1910 through 1940 the tables of Nowacki (1935) were used, for the period 1940 through 1962 the tables of Kopff (1937), and for the period 1963 through 1967 the tables of Borsche. In each case the conversion tables divided the sky into right ascension-declination zones and gave a mean correction for each zone. These mean corrections were applied on a star-by-star basis to convert the latitudes to the FK-3 declination system.

A study was made to determine if additional corrections should be made to convert from the FK-3 to the FK-4 declination system. However, it was determined that the FK-3/FK-4 declination system differences were so small as to make the correction unnecessary, averaging less than 0.01 arc second. The SAO Star Catalog that was used to reduce observations taken since 1968 is a quasi FK-4 catalogue in that stars not in the FK-4 catalogue have been corrected for systematic declination differences using zone corrections to place them in the FK-4 system. Most of the stars used for latitude determinations since 1968 have had their FK-4 positions determined in this way rather than using stars referenced in the FK-4 catalogue itself.

From the above discussion it can be seen that the astronomic latitudes observed since 1907 should be expected to show no significant systematic differences from the FK-4 declination system. However, the random errors will be larger than would be the case if only stars in the FK-4 catalogue had been observed.

To verify that the quasi FK-4 system defined by the SAO catalog gave astronomic latitudes in the desired declination system, NGS teams made latitude observations directly at the International Latitude Service (ILS) station at Gaithersburg and at the Bureau International de l'Heure (BIH) station at the U.S. Naval Observatory. Forty star pairs were observed on each of 3 nights at the two stations. Table 8.2 compares the BIH and NGS latitudes at the two sites. Clearly the declination system used by NGS is sufficiently close to the BIH system that no significant error occurred.

TABLE 8.2.—*Comparison of NGS and BIH latitudes*

Observatory	Latitude (deg)	(min)	(sec)	Uncertainty (arc sec)
Gaithersburg International Latitude Service Station				
NGS	39	08	13.15	±0.08
BIH	39	08	13.10	
U.S. Naval Observatory Photographic Zenith Tube				
NGS	38	55	16.95	±0.05
BIH	38	55	16.86	

8.1.5 Estimation of Errors of Astronomic Latitude Determinations

To investigate the change in random error of a single star pair latitude determination, formal standard deviations were computed for various time periods involving different instruments and/or star catalogues. The formal standard deviation of a single star latitude was derived by comparisons of the single star latitudes with the grand mean of all latitudes computed during a setup. Table 8.3 gives the formal standard deviations. As shown, the random error associated with a single star-pair observation has not changed significantly since latitude observations began.

TABLE 8.3.—*Statistical variability of latitude determinations*

(Pettey and Carter, 1978)

Period	Instrument	Star catalogue	σ Star pair (arc sec)
1847-1907	ZT, MT, VT	Various	0.65
1908-1914	ZT	PGC	.50
1914-1939	Bamberg	PGC	.70
1940-1956	Bamberg	GC	.72
1957-1967	T-4	GC	.71
1968-present	T-4	SAO	.62

To test for systematic error a Model II analysis of variance was carried out for repeat observations at 20 stations where latitudes had been determined on 2 or more nights during the period 1913 to 1976. Data taken on 54 nights were examined.

The results were:

σ_p = 0.714 arc second = within-sets standard deviation

σ_B = 0.267 arc second = between-sets standard deviation

The within-sets variance is in excellent agreement with the data given in table 8.3.

A similar Model II analysis of variance was undertaken using data from 13 stations where initial observations were made during the period 1851 to 1906 and repeat observations from the period 1957 to 1975.

For these data the results showed:

σ_ρ = 0.729 arc second = within determination standard deviation
σ_β = 0.259 arc second = between determination standard deviation.

Given that accurate polar motion data were not available for the pre-1900 data and the pre-1908 data had no corrections applied to correct to a common coordinate system the value of σ_ρ is unexpectedly small. The most logical explanation is that, on the average, failure to make corrections to the pre-1908 data did not significantly impact the accuracy of the results.

Based on the above analysis it was decided to assign the astronomic latitudes used in the NAD 83 standard errors given by

$$\sigma_\phi = \left[\frac{(\sigma_\rho)^2}{N} + (\sigma_\beta)^2\right]^{1/2} \tag{8.2}$$

where

σ_ρ = 0.72 arc second
σ_β = 0.26 arc second
N = number of star-pair observations.

8.1.6 Astronomic Longitude Determination

8.1.6.1 Historical Summary

Since the beginning of astronomic longitude observations in the late 1840s, changes and improvements have occurred involving instrumentation, star catalogues used, methods of obtaining time, accuracy of time signals, and observing programs. These changes have introduced considerable complexities in relating all longitudes to a common longitude origin, causing accuracies to change substantially with time.

From the beginning of astronomic longitude determinations through 1922, the telegraphic method of longitude determination was used, except for a few stations in Alaska. By means of the telegraphic method differential longitudes were determined with the telegraph used to synchronize timing at the two stations involved. A detailed description of these methods can be found in Bowie (1917). Significant instrumentation changes prior to 1922 included replacement of meridian transits by the smaller, more portable, broken telescope transit at about the turn of the century and the introduction of the tracking micrometer in 1904. With the telegraphic method, connections to the longitude origin were made using differential measurements between Greenwich, England, and North America by way of St. Pierre Island and Newfoundland. Approximately 300 longitudes were determined using the telegraphic method through adjustments of the differential longitude measurements obtained.

Beginning in 1922 the wireless (radio) method of longitude determination was initiated. Time signals were recorded and longitudes of stations were determined directly using the recorded time information and catalogues of star positions. From 1922 to 1962 NGS used time signal information furnished by the Time Service Division of the U.S. Naval Observatory (USNO). Since 1962 time information was obtained from the BIH. Three star catalogues were used by NGS in the post-1922 time period for astronomic longitude determinations: the Eichelberger Catalogue during 1922 to 1939, the FK-3 catalogue from 1940 to 1961, and the FK-4 catalogue for the post-1961 period.

Additional complexities were introduced when NGS related its longitudes to a desired common origin of longitude. These resulted from (1) changes in star catalogues and stations used by USNO during the period 1922-62 to derive time information, and (2) the need to relate USNO longitude origin and BIH longitude origin.

8.1.7 Telegraphic Longitude Analysis

To improve the accuracy of telegraphic longitudes prior to the NAD 83 adjustment, a free adjustment of all useful longitude differences obtained prior to 1922 was undertaken. The objectives of this readjustment were to minimize distortions in the network and to reference these longitudes as accurately as possible to the BIH reference system. To achieve these objectives, additional longitude differences obtained in recent years were included in the readjustment. These recent measurements served the purposes of strengthening network geometry and connecting the longitude network to the USNO PZT station. Knowledge of the astronomic longitude of the USNO station in the BIH system was then used to more accurately relate the telegraph longitude network to the BIH longitude origin.

8.1.8 Coordinate System Corrections

Longitudes observed by NGS since 1961 required no corrections since BIH time and the FK-4 star catalogue were used and the longitudes obtained by NGS were correctly referred to the BIH longitude origin. For the period 1922-61 the following three types of corrections were required to relate the required astronomic longitudes to the correct longitude origin:

1. Corrections to account for the fact that between 1922 and 1962 NGS did not use the FK-4 star catalogue in reducing its observations.
2. Corrections to account for the fact that between 1922 and 1962 the USNO did not use the FK-4 star catalogue when determining the times provided by its time service.
3. Corrections to account for the fact that the astronomic longitudes assigned to the Washington, DC, and the Richmond, FL, sites by the USNO for purposes of time computations during the 1922-62 time period were not the correct astronomic longitudes for these sites in the BIH system.

The total corrections to be applied to the computed astronomic longitudes available at NGS for correct astronomic longitudes referred to the proper longitude origin, taking into account all three corrections described above, can be obtained using the formulas:

$$\Delta\Lambda_T = (\Delta\Lambda_{NGS} - \Delta\Lambda_{USNO}) + (\Lambda_I - \Lambda_o) \quad \text{(1922 through 1949)} \qquad (8.3)$$

$$\Delta\Lambda_T = (\Delta\Lambda_{NGS} - \Delta\Lambda_{USNO}) + \tfrac{1}{3}\,[(\Lambda_I - \Lambda_0) + 2\,(\Lambda_{II} - \Lambda_{00})] \quad \text{(1950 through 1961)} \qquad (8.4)$$

where

$$\Delta\Lambda_{USNO} = \Delta\Lambda_W \quad \text{(1922 through 1949)} \qquad (8.5)$$

$$\Delta\Lambda_{USNO} = \tfrac{1}{3}\,[\Delta\Lambda_W + 2\Delta\Lambda_R] \quad \text{(1950 through 1961)} \qquad (8.6)$$

The symbols are defined as:

Λ_I = longitude of Washington Clock Room of the USNO in the BIH coordinate system as determined by NGS

Λ_0 = longitude of the Washington Clock Room of the USNO used by the USNO when producing time information.

Λ_{II} = longitude of the Richmond, FL, PZT of the USNO in the BIH coordinate system as determined by NGS.

Λ_{00} = longitude of the Richmond, FL, PZT of the USNO used by the USNO when producing time information.

$\Delta\Lambda_T$ = total correction to be added to NGS astronomic longitudes to refer them to the correct longitude origin.

$\Delta\Lambda_{NGS}$ = correction to account for differences between the star catalogue used by NGS for data reduction and the FK-4 star catalogue.

$\Delta\Lambda_W$ = correction to account for differences between the PZT star catalogue used by USNO for its Washington PZT and the FK-4 star catalogue.

$\Delta\Lambda_R$ = correction to account for differences between the PZT star catalogue used by USNO for its Richmond PZT and the FK-4 star catalogue.

The change in formulas since 1950 takes into account that during 1922 through 1949 the USNO used only observations from its Washington, DC, site to generate time information. From 1950 through 1962 the USNO generated time using observations from both the Washington, DC, and Richmond, Florida, PZT observations, with the Richmond observations given double the weight of the Washington observations.

During the 1922-62 time period, the USC&GS used two star catalogues in making astronomic longitude observations—the Eichelberger Catalogue during the period 1922-39 and the FK-3 catalogue during 1940-62. The USNO used the Eichelberger catalogue during 1922-33, the Washington, DC, PZT catalogue during 1934-49, and a combination of the Washington, DC, and Richmond, FL, PZT catalogues during 1950-62. Table 8.4 lists the values for the various correction factors used by USC&GS and USNO to convert the catalogues to the FK-4 star catalogue. Table 8.4 also shows the final combined corrections for star catalogue differences relative to the FK-4.

To relate the astronomic longitudes observed by the Coast and Geodetic Survey during the period 1922-62 to the correct BIH longitude origin, not only corrections for star catalogue differences but also a systematic correction to account for corrections to the adopted Washington, DC, and Richmond, FL, PZT longitudes were needed to relate them to the BIH longitude origin. To obtain the required corrections, observations made by NGS astronomic field parties at the Washington, DC, site in 1966, 1975, and 1976 and at the Richmond, FL, site in 1976 were analyzed. Table 8.5 summarizes the corrections made to these observations.

TABLE 8.4.—*Systematic catalogue corrections*

(Pettey and Carter, 1978)

Period	$(FK4\text{-}CAT)_{USNO}$	$(FK4\text{-}CAT)_{NGS}$	$\overline{\Delta\alpha}_{USNO}$ (arc sec)	$\overline{\Delta\alpha}_{NGS}$ (arc sec)	$\Delta\Lambda_{CAT}$ (arc sec)
1922-1934	FK1-EICH	FK4-EICH	+0.09	+0.09	+0.00
1934-1940	FK4-PZT	FK4-EICH	−0.18	+0.16	−0.34
1940-1950	FK4-PZT	FK4-FK3	−0.18	+0.03	−0.21
1950-1962	FK4-PZT	FK4-FK3	+0.18*	+0.04	+0.14

* Based on weighted catalog differences of USNO and NOTSS.

$\overline{\Delta\alpha}_{USNO}$ = USNO catalogue correction

$\overline{\Delta\alpha}_{NGS}$ = NGS catalogue correction

$\Delta\Lambda_{CAT} = \overline{\Delta\alpha}_{USNO} - \overline{\Delta\alpha}_{NGS}$ = Total catalogue correction

TABLE 8.5.—*Systematic longitude corrections*

(Pettey and Carter, 1978)

Period	$\Delta\Lambda_R$ (arc sec)
1922-1950	−0.33
1950-1962	−0.61

Table 8.6 shows the total corrections applied to the astronomic longitudes to relate them to the FK-4/BIH system for the period 1922-62. These corrections were used to obtain final astronomic longitude values for inclusion in the NAD 83 adjustment.

TABLE 8.6.—*Total longitude corrections*
(Pettey and Carter, 1978)

Period	ΔA_T (arc sec)
1922-1934	−0.33
1934-1940	−0.67
1940-1950	−0.54
1950-1962	−0.47
Nominal correction	−0.50

8.1.9 Accuracy of Longitude Observations

For longitudes obtained by the telegraphic method (observations prior to 1922) the geodesist deals with independent differential measurements that form loops. In this situation the standard errors resulting from the free adjustment were considered an adequate measure of the error. Therefore, the standard error of unit weight from this adjustment, ±0.52 arc second was used as the accuracy estimate of the pre-1922 astronomic longitudes.

To determine the proper error estimates to be assigned to the post-1922 observations numerous analyses of variance (ANOVA) studies were carried out. Initial Model II ANOVA studies were aimed at determining if systematic components of error were present. These would not be apparent from looking at the standard error of a star observation obtained from observations made by a single receiver using a single instrument during a single night. These analyses indicated that no significant systematic errors occurred due to between-night or between-instrument components. However, a substantial observer-related systematic error was found, no doubt due to an observer's personal equation.

Within-determination variances σ_W were computed for five different time periods involving significant differences in the accuracy of the timing information used in reducing observations. During the period 1922-34 visual star observations were used by USNO to produce time information. In 1934 the PZT was introduced. In 1948 WWV was introduced as a means of distributing time information. In 1962 BIH replaced USNO as a source of time information. In mid-1975 digital recorders replaced chronographs for recording time information at observing sites. Table 8.7 gives the results of within-determination variance values.

During the 1948-62, 1962-1975.5, and post-1975.5 periods, sufficient repeat observations were made, enabling between-determination components of error σ_β to be computed using Model II ANOVA methods. Table 8.7 also shows these results. The between-determination variance values are not significantly different from one time period to another. This is a reasonable result, given that the primary cause for this between-determination variance is believed to be a personal equation. There is no reason to believe that a personal equation error would change between time periods. Thus it was decided to pool the results from all three time periods for the between-determination component. This gave $\sigma_\beta = 0.37$ arc second. This value of σ_β was then used for all five time periods.

The values of σ_W in table 8.7 decrease with time. During the post-1961 period it was felt that the only event significantly affecting within-determination accuracy was the introduction of the digital recorder in mid-1975. However, during the 1922-62 period events other than the two noted (introduction of PZTs and adoption of WWV) may have had an impact upon the within-determination accuracy. These events were related to various improvements in catalogues, time signals, and field instrumentations. However, not enough observations were available to break the 1922-62 time frame into more periods. Thus it was decided to represent within-determination accuracy during 1922-62 as a linear function of time. Table 8.8 lists the final accuracy estimators for astronomic longitudes used in the NAD 83 analyses.

TABLE 8.7.—*Secular trend of statistics associated with longitude determinations*
(Pettey and Carter, 1978)

Period	$\hat{\sigma}^2_\beta$ (s^2)	Degrees of freedom (DOF)	σ^2_W (s^2)	DOF	Significance of period
1975.5	0.000494	13	0.000247	196	Introduction to digital recorder
1962-1975.5	0.000671	95	0.000577	242	Adoption of 1968 BIH's
1948-1962	0.000544	68	0.000780	267	Adoption of WWV signals
1934-1948	------		0.001283	84	Introduction of PZT
1922-1934	------		0.001170	148	

TABLE 8.8.—*Longitude accuracy estimators*

(Pettey and Carter, 1978)

Period	$\hat{\sigma}_A = [(\sigma_W)^2/n + (\hat{\sigma}_B)^2/k^1]^{1/2}$ $\hat{\sigma}_W$ (arc sec)	$\hat{\sigma}_B$ (arc sec)
1922-1962	$\pm[(0.57)^2-(0.07)^2 (T_{epoch}-1922)]^{1/2}$	± 0.37
1962-1975.5	± 0.36	± 0.37
1975.5	± 0.24	± 0.37

T_{epoch} = time of observation.

8.2 DOPPLER MEASUREMENTS

8.2.1 Background

In April 1973, NGS began an observational program to establish a network of Doppler stations to support the NAD 83 adjustment. (See fig. 18.12.) The objectives were threefold. A uniformly distributed set of Doppler stations in the conterminous 48 States, in conjunction with the Transcontinental Traverse, was used to ensure that long wavelength deformations in NAD 83 would be at the submeter level. For islands such as Hawaii, Puerto Rico, Virgin Islands, the Aleutians, and parts of Alaska where conventional networks were weak, the Doppler stations ensured connections to NAD 83 at the meter level or better. Finally, by reliance upon the Doppler data and the use of collocated Doppler stations to provide connections to Very Long Baseline Interferometry (VLBI), satellite laser ranging (SLR), and Lunar laser ranging (LLR) sites, the NAD 83 coordinate system and scale were established.

The NAD 83 Doppler observational program was essentially completed by the end of 1978. A total of 599 stations were established in the 50 States, Puerto Rico, the Virgin Islands, and other U.S. territories. In establishing the NAD 83 Doppler network observational support was provided by the Defense Mapping Agency Hydrographic Topographic Center (DMAHTC). Also, a number of stations established by DMAHTC in support of its own programs were used in the NAD 83 adjustment as well as stations established by the U.S. Geological Survey and the Bureau of Land Management (BLM) in Alaska. Figure 18.12 shows the location of the Doppler stations used in the NAD 83 adjustment.

8.2.2 Observation and Reduction Methods

Standard field procedures were employed to observe at least 40 passes of satellites where the satellites rose more than 10 degrees above the horizon during a pass. Only those satellites which possessed precise ephemerides were observed. Initially, ephemeris data were available for either one or two satellites. However, during the last half of the observation program precise ephemeris data were available from three to five satellites. Where ephemeris data were available for two satellites, 5 to 6 days were normally required to obtain 40 passes. Where data from one satellite were available, 10 to 12 observation days were required. Most observations were made using the AN/PRR-14 (Geoceiver) instrument. Dots for a few stations were obtained using the Magnavox MX1502 and JMR Doppler receivers. Offsets between the Geoceiver antenna measurement point and the ground monument at each station were determined before and after each observing session. Weather data (temperature, pressure, and humidity) were recorded during each pass for use in making tropospheric refraction corrections.

It was necessary to undertake repeat observations at a number of stations during the observation program due to instrument malfunction. The bulk of these malfunctions was caused by instrument oscillator instability. They were detected at the time of data reduction because of the effect on estimated bias parameter values (Hothem, 1975).

Data reduction was carried out using the point positioning method in conjunction with a precise ephemeris computed after the fact using tracking data from a Department of Defense, 20-station worldwide tracking network. Prior to 1976 this precise ephemeris was provided to NGS by the Naval Surface Warfare Laboratory (NSWL); beginning in 1976 it was provided by DMAHTC. These precise ephemerides were produced, beginning in January 1973, by using the NSWL 10E gravity field (Anderle, 1976) and the NSWL 9D tracking station coordinates. In June 1977, a change was made to the NSWC 10E-1 gravity field and the NSWC 9Z-2 station coordinates to reflect small refinements in gravity field and station coordinates. These refinements had no significant effect on the defined coordinate system (Leroy, 1982).

Data reduction was undertaken using the Doppler computer program developed at DMAHTC (Smith et al., 1974) which had been converted to the NGS computer with minor modifications such as representing the orbit through a polynomial fit to values of *X*, *Y*, *Z* taken at 1-minute intervals rather than at 2-minute intervals as was done by DMAHTC (Jenkins et al., 1982). Briefly, this program holds fixed the input ephemeris and solves for station position and several bias parameters using as input integrated Doppler counts. The estimated bias parameters are receiver delay, satellite-receiver clock time offset, and satellite-receiver oscillator frequency offset.

Beginning with initial estimates of approximate values of receiver position and bias parameters, the program used an iterative procedure, differentially correcting the unknowns during each interation. Iteration continued until the sum of corrections in the *X*, *Y*, and *Z* coordinates of the station during an iteration were less than 1 m. Iteration then stopped after one additional iteration. After each iteration those integrated Doppler counts having residuals greater than three times the standard deviation of the observational residual were rejected before the subsequent iteration. Except for occasions when entire passes were rejected due to interference from the signal emitted by another Doppler satellite, the amount of data rejected was typically 2 to 3 percent. It rarely exceeded 5 percent.

8.2.3 Accuracy of Doppler Results

In the NAD 83 adjustment, the scale and orientation of the reference network were supplied by VLBI and SLR results. Thus to a large degree, leaving aside coordinate system origin, long- and short-term precision (i.e., repeatability) can be considered as nearly equivalent to accuracy. Estimated Doppler precision and accuracy were derived from comparisons of repeat measurements and from comparison with external standards such as VLBI and SLR results. Comparisons of repeat measurements at a number of stations over time periods up to 5 years have been made (Strange and Hothem, 1976; Strange et al., 1982). They indicate that both random and systematic variations occur in station position determinations. The random scatter of determinations was in the range of 20 to 30 cm in latitude, 30 to 40 cm in longitude, and 30 to 40 cm in height. Also systematic variations with time were noted. These were of the order of 20 to 30 cm in latitude and longitude and 50 to 100 cm in height. These systematic variations were in the form of both secular trends and variations with a yearly period. The periodic variations are believed to be related to orbit error. The secular trends are believed to be related not only to orbit error but also to increases in ionospheric refraction effects between 1974 and the time of solar maximum in the 1980-81 time frame.

A more definitive evaluation of Doppler positioning accuracy can be found from comparisons with external standards [Strange et al. (1975), Strange and Hothem (1976), Hothem et al. (1978), Hothem (1979), Strange and Hothem (1980)]. Perhaps the most definitive comparisons with other space systems are those resulting from the special intercomparison tests carried out in 1978 and reported in Hothem et al. (1978), Hothem (1979), and Strange and Hothem (1980). The general conclusion is that, after removal of systematic differences related to scale and coordinate system orientation, comparisons of Doppler differential positions and those obtained from other space systems agree at the 30- to 60-centimeter level.

8.2.4 Coordinate System Relations

Numerous investigations have been carried out to relate the orientation of the Doppler coordinate system to that of VLBI. The VLBI system was used to define the BIH/International Earth Rotation Service (IERS) terrestrial coordinate system, which was selected to define the orientation of the NAD 83 coordinate system. These early studies [Strange et al. (1975), Hothem et al. (1978), Hothem (1979), Strange and Hothem (1980), Hothem et al. (1982)] showed the orientation difference between the Doppler and VLBI systems to be between 0.75 and 0.85 arc second. These results are in good agreement with the final Doppler-VLBI orientation difference found in the final NAD adjustment of 0.77 arc second. Results of White and Huber (1979) involving comparison of astro-Doppler and gravimetric deflections indicate that a Doppler longitude rotation of 0.86 ± 0.10 arc second was required to bring the Doppler coordinates in agreement with the optical astronomy coordinate system. This would imply that the optical star system and the VLBI system longitude orientations are nearly the same. This would be expected given the known relationships between the radio star and optical star systems.

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9. DATA BASE FORMATION

Charles R. Schwarz

From the very first discussions, the existence of computers and their potential application played a large role in the planning for the new adjustment of the North American Datum. Although some changes had occurred in surveying instrumentation and practice since 1927, the most dramatic improvements involved computational practices. By the beginning of the New Datum Project in 1974, computers had become widespread and several programs were used to adjust horizontal networks.

The use of computer programs for processing data required that the data be in machine-readable form. Prior to 1974, the National Geodetic Survey had used a variety of computer programs for several purposes, but there was no policy or common practice concerning the treatment of machine-readable data. The idea of treating such data as an agency resource had not yet emerged. Data were treated as an adjunct to a program (i.e., the data deck was what you put in back of the program deck). Programs were shared on occasion, but almost no thought had been given to the sharing of data. Data decks were usually discarded after their use; when a few individuals thought that it might be a good idea to save the data they had prepared, it was often in the form of boxes of punched cards stashed under the desk.

The beginning of the New Datum Project completely changed the former concepts and practices. It was clear that there would be something corresponding to a data deck for the new adjustment, but it was equally clear that this data set would be far too big to be physically realized as a single deck of punched cards. Furthermore, the preparation of this data set would involve much more than keypunching.

Not only did NGS not have even the beginnings of a machine-readable data base in 1974, but problems were experienced with the traditional paper files. Two large permanent files were thought to be particularly pertinent to the new datum effort. The first was the file of published stations and NAD 27 coordinates. This file was handled by the National Geodetic Information Center (NGIC), which was in the midst of transforming the file from organization by state to organization by 30 minute quadrangle. The second was the file of observations. These were organized by survey project and stored in cahiers, usually one cahier per survey project. The following major problems were evident:

1. The two files were not entirely consistent. Some geodetic stations appeared in one but not the other. Other stations appeared in both but with some variation in name or position.
2. Although the two files had many data elements in common, there was no standardized meaning of the data elements. Latitude and longitude were expressed clearly enough, but there were several inconsistent rules for naming stations.
3. Both files contained more errors than could be tolerated for computer processing.
4. Neither file was in machine-readable form.

At the same time generalized data base management systems (DBMS) were emerging in the computer software market. Most of these software packages were directed toward commercial applications, but the application to numerical and scientific data was apparent. Furthermore, the relation of agency or corporate management to the management of data was being defined at this time. The idea of a corporate data base, managed by a data base administrator, became popular.

In this environment, NGS made the decision to build both an integrated data base and a data base management system to manage it. This was considered to be a decision of considerable importance and implication. Since no other geodetic agency used a true data base management system at that time, no models existed on which to base the effort.

9.1 OBJECTIVES OF THE DATA BASE EFFORT

Several objectives entered into the decision to build a data base and its management system. The first was to achieve data consistency. By keeping only a single copy of data items for which there had previously been many copies, inconsistencies could be eliminated. Such data items included, for example, station names, latitudes, longitudes, heights, astronomic positions, and geoid heights. Futhermore, with a single centralized file, it would be possible to concentrate the agency's resources on the editing, validation, and verification of that file.

A second objective was to build much stronger access methods than had previously existed. Essentially, this meant access to records or groups of records by keys and indices and access to fields within records by field name rather than field position. The purpose was to enable many different programmers to access only those fields they needed from the data base. Programmers and end users could be shielded from details about the access methods and from concern with data items in which they had no interest. Since access would be by field name, the actual structure of the data base could be changed with no effect on the many existing application programs.

A third objective was to construct an interactive query language which would serve as a friendly interface between the geodesists of NGS and the operating system of the computer. The idea was to access the data base in geodetic terms, such as geographic areas, rather than in terms of tape numbers, device names, and record positions.

A fourth objective was to accelerate the development of computer programs and procedures for geodetic applications by relieving application programmers of the responsibility for data management. Tasks such as extracting the needed records from large data sets, merging data fields from various files, and organizing the data for the specific task at hand were seen as occupying a significant amount of the programmer's attention. Production processes often contained several steps devoted solely to data management tasks, such as migrating data from tape to disk, and backing up data files.

A fifth objective was to create an environment in which NGS management could exert configuration control over the programs and procedures being used for production. Programs and procedures which were intended for use by groups of people (rather than individuals) would be brought into the data base environment and executed by means of the query language. Within this environment, it would be possible to control the number of programs, versions, and processing options. This would ensure both that all data were processed in a consistent manner and that all employees had access to the proper set of programs and procedures.

Although clear analogies emerged between the data management objectives of NGS and those objectives which were being addressed by data base management systems in commercial applications, there were also some significant differences:

1. The projected cost of providing permanently mounted on-line storage for all data was considerable in 1974. Although data base management was considered to be an important activity, its value was not considered to be sufficient to justify this cost. Therefore, the initial design included procedures to migrate data from tapes to on-line disk as the data were needed. The on-line disk space was released after use. As the cost of on-line disk storage decreased, this design was modified. In the final design, the most active data (station names, positions, and observations) were kept permanently on-line, while the less active and more voluminous station descriptions were kept on mountable disk packs.
2. Transactions tended to involve significant batches of data, such as an entire survey project. Commercial systems typically accessed only a few records for each transaction.
3. The data base was dominated by update activity rather than by retrievals because NGS was in the process of building the data base during most of the life of the project. Features which tended to optimize performance for retrievals were therefore largely irrelevant.

Other geodetic agencies had constructed file management systems as well as application systems for retrieving data based on geographic and other keys. The NGS data base development effort differed from earlier efforts in the following respects:

1. The amount of data to be managed was much greater. Including descriptions, the total size of the NGS data base was originally estimated to be 12 gigabytes. This estimate included several large data sets held by other agencies. Only a few of these data sets actually materialized, and the total data base size was reduced to 3-4 gigabytes.
2. NGS set out to build a system which would provide more than one logical view of the data. The perspective of the Horizontal Network Branch, which was concerned mostly with survey projects, was different from the perspective of the NGIC, which was concerned with the publication of station names, positions, and descriptions. This property of multiple logical views, or subschemas, distinguished the NGS data base from file management systems.

9.2 THE COMPUTER ENVIRONMENT

In 1974, the National Geodetic Survey was one among many users of the NOAA central computer facility. As was the common situation at the time, this facility offered batch processing on large main frame computers on a shared basis. Interactive computing was considered to be an expensive resource to be used sparingly. Not being a dominant user of the computer center, NGS could not strongly influence the choice of system hardware, language systems, or system level data base management system.

Although generalized data base management systems were appearing on the market, these packages were still immature, prone to failure, and lacking many of the features that would appear later. Most importantly, they lacked the interfaces to programs written in FORTRAN and PL/1, the languages used for application programs at NGS.

Given this environment, NGS chose not to use a generalized DBMS but to build its own system. In the end, the geodetic DBMS consisted of approximately 30,000 lines of PL/1 code. The data base programs actually ran in batch mode, but an interactive user interface was provided by an additional 20,000 lines of code written in the SUPERWYLBUR text editing language. These programs managed the prompting and interactive dialogue with the user and prepared jobs to be run in the background, but could not actually access the data base. When the workload on the computer was relatively light, the background jobs would execute quickly and the system would provide the response expected of true interactive computing; many steps and processes could be executed in a single session. When the computer was heavily loaded, the user could initiate a data base management task, end the interactive session, and come back to examine results in a later session.

9.3 DESIGN DECISIONS

The geodetic data base management system was never intended to be a generalized DBMS and therefore lacked many of the features associated with such systems. Specifically, it had no data manipulation language. Instead, all programs which accessed the data base were brought into the data base environment and could be executed only through the interactive user interface. Secondly, there was no separate schema. The structure of the data base was coded into those programs which actually accessed the data.

9.3.1 Unique Identifiers

The lack of unique station identifiers suitable for computer processing posed an early problem for the horizontal network stations. The stations were not numbered and station names were not unique. A new identifier had to be assigned to each station. The scheme selected was the 13-character Quad Identifier/Quad Station Number (QID/QSN), which had already been partially implemented by the NGIC for the purpose of data publication. The QID is nine characters long and describes a 7½-minute quadrangle. (See fig. 9.1.) It is composed of: (a) a single character hemisphere code (N or blank or 0 for north; S or 1 for south), (b) two characters denoting the degrees of latitude of the southeast corner of the quadrangle, (c) three characters denoting the degrees of west longitude, and (d) three characters denoting the selection of 30-minute, 15-minute, and 7½-minute quadrangle respectively, according to the numbering scheme of figure 9.1.

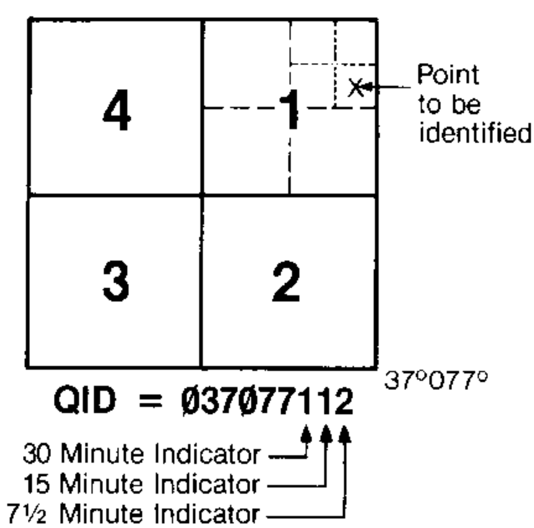

Figure 9.1. Quad identifier definition.
Numbering sequences are in clockwise direction;
range is from 1 to 4.

The QSN is a four-character numeric subfield which is assigned sequentially by the DBMS as new stations are loaded. The QSN uniquely defines a station within a 7½-minute quad.

When a new point was loaded into the data base it was assigned the appropriate QID, based on its NAD 27 geodetic position, and the next available QSN for that quad. Once a station was entered into the data base its QID/QSN was never changed, even if the station's position was changed as a result of correction or adjustment. Thus the QID was a good guide but not a precise indicator of a station's position. If a station was deleted from the data base (which happened only rarely), its QSN was not reused. Thus the concatenated QID/QSN was able to serve as a unique data base identifier for each station.

Because of this scheme it was necessary that a station with its positional information be loaded before any other information, such as the observations or descriptions. The normal method of supplying this information was the 3-card format. (See chapter 20.) It was also possible to initialize a non-publishable station with skeleton 3-cards containing only a name and a position. (See chapter 10.)

9.3.2 Data Structure

The geodetic DBMS was a hierarchical system, based on the QID/QSN. The subfields of this identifier were used to address a hierarchy of indices. The highest level was the 1-degree quadrangle index. If the quad contained a large amount of data, this was broken down to 30-minute, 15-minute, or 7½-minute indices. The lowest level was the station index, containing an entry for each QID/QSN. This index pointed into the data base itself.

The data for a single station were broken into the following detailed records:

- position and associated information
- horizontal observations
- gravimetrically determined quantities
- astronomic positions and azimuths
- Doppler-determined positions
- a cross-reference list containing all stations which observe to this station
- station descriptions
- historical data (superseded positions)
- associated stations such as reference objects and azimuth marks together with observations from these stations.

Different detail records for the station could be distributed among various data sets. Thus it was possible that the station description could be stored on an off-line mountable disk pack while the station position was in an on-line data set. Some of the attributes, especially those that could be represented by a few bits or bytes, were stored in the station index, so that retrievals qualified by these attributes could be satisfied by searching the index rather than the data base itself.

The only key in the geodetic data base was some form of the QID/QSN, which necessitated that all data base transactions specify a geographic window. Data within the geographic area specified could be further qualified based on attributes.

The original geodetic DBMS design had called for indices to be built on a variety of attribute fields, such as station order and type, to facilitate retrievals based on these attributes (Alger and Gurley, 1975). However, demand for these facilities never materialized. Almost all transactions in the geodetic data base selected data records based only on geographic area.

9.3.3 Query Language

The original design of the geodetic DBMS called for a command or verb-oriented query language, such as:

GET (record types) KEY = (qualifiers based on location and attributes)

(See Alger and Gurley, 1975.) This concept was temporarily replaced by a prompting language, since the interactive language being used for the user interface was not well suited to parsing commands. Later, prompting was felt to be a preferable style, less powerful but more suitable for the occasional user of the system. The query system is described in Alger (1981a and 1981b).

9.3.4 Data Content

The geodetic data base was designed to support publication activities and analysis of historical data, as well as the NAD adjustment. This meant that it stored records of geodetic data that might be useful for any purpose. The logical views to which a station belonged were specified by flags in the station record. Thus a station could be specified as publishable/non-publishable or adjustable/non-adjustable. A station record was almost never physically deleted from the data base, even if it was unsuitable for some purpose. Thus a station which was marked as non-adjustable because it did not have appropriate observations connecting it to the network could later be made adjustable if suitable observations were found.

The geodetic data base contained horizontal observations and stations descriptions as well as station positions. Astronomic positions, astronomic azimuths, and Doppler positions were also included, since these were considered to be attributes of horizontal stations. The original intent had been to integrate the horizontal with the vertical and gravity networks. However, this was not accomplished until after the NAD 83 adjustment.

The only requirement for including a point in the data base was that it have a position, otherwise it could not be assigned a QID. Unpositioned points, such as reference marks and azimuth marks, had to be associated with a parent station that was positioned. All information for such marks, including observations, was stored in the parent station record. This approach limited the data base size in terms of the number of station records. However, there were situations where new observations were found so that a station, previously treated as a reference mark, became a positioned point. The process of separating such a station and all its associated observations from the parent station was cumbersome. Fortunately, this situation did not arise often.

Rejected observations (those containing apparent blunders) were also carried in the data base. The decision to reject them could always be undone. Out-of-date station descriptions and recovery notes were carried for historical purposes.

9.4 DATA ENTRY FACILITIES

Throughout most of the life of the geodetic DBMS the primary emphasis was on the building of data entry facilities. Each organizational unit within NGS was responsible for the entry of its own data. Data entry procedures were built for the following separate data types:

1. Positions, names, and associated publication data for existing stations.
2. Station descriptions for existing stations.
3. Astronomic positions and azimuths.
4. Horizontal directions for archival projects.
5. Archival distances (which had been reprocessed).
6. Geoid heights.
7. Deflections of the vertical determined by astro-gravimetric leveling.
8. Complete survey project data sets for current surveys, for which data were recorded in machine-readable form and validated in the field by the TENCOL system (Safford, 1978).

One problem common to all of these data types was the need for data validation procedures. Standards were developed for each data type and later translated into editing and validation programs that became part of the data entry procedure.

A major function of the data entry procedures was to merge the data type with the data types already in the data base. This was difficult only because most data did not yet contain a data base identifier. In each case the task was to find the QID/QSN of the data base record with which the new data should be merged. This was done by matching on one or more elements which were common to the data base and the new data. The elements used most often for matching were station name and position. However, these were imperfect elements. The assignment of station names had not always been done the same way. Some variations were also found in the assignment of station position.

Once positional data entry was complete (within an area), the other data types could be loaded. Figure 9.2 illustrates the process of matching a given data type. The first step was to scan the input data to determine its geographic distribution. (This was only possible for those types that contained positions in some form.) Within that geographic window, the appropriate "match records" were retrieved from the data base. These records contained the match data elements for that data type and the station QID/QSN (Alger, 1976; Alger, 1978).

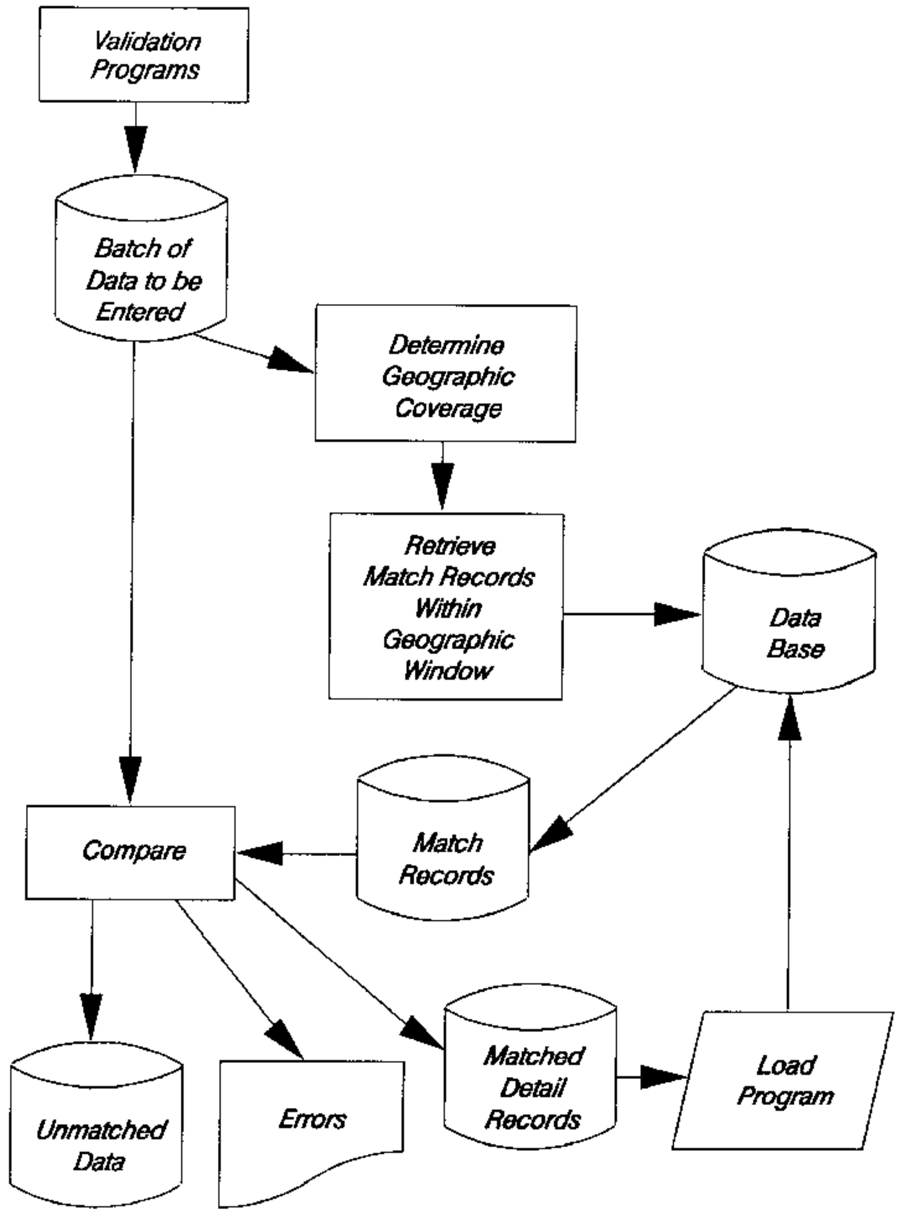

Figure 9.2. Typical data entry path.

The data to be entered were compared to the match records. If a unique and unequivocal match could be made, then the QID/QSN of the match record was transferred to the new data and the detail record for that data type was ready for loading.

This match was more than a simple merging process: it was the final validation of different data types prior to data base entry. The matching process identified missing data, incorrect identifiers, and the existence of duplicate data. The programs displayed the unmatched data for resolution by the analyst. For some data entry paths the data which had been matched were loaded into the data base while the unmatched data were being resolved. For other data types no data base loading was performed until a complete clean run, free of unmatched data, could be obtained.

In general, the complete new detail record replaced the current contents of that detail record in the data base. However, a notable exception was made for the data found in the Trav-decks. In this case, new detail records were synthesized by selecting appropriate data elements from both the new data and the current data base contents. (see chapter 10.)

Even the positional data were put through this matching process. Each new batch of stations to be entered was compared to the stations already in the data base. This process identified duplicate positional records and historical data records as well as updates to the existing data base contents. The matching program also compared the positional data being entered to itself in order to detect duplicate input records. Records which were positionally close (within 0.3 arc second) were displayed for manual resolution.

The analyst could select one of the following four actions for each displayed input record:

1. Input—The record refers to a station which is different from any nearby station.
2. Delete—The record refers to a station for which a data base record already exists.
3. Update—The record refers to a station which already exists in the data base, but some of the attribute information is to be changed.
4. History—The record refers to a station for which positional data exist in the data base. The data are stored directly into a historical record for that station.

Directives were set indicating the appropriate action for each station in a batch of potential duplicates. The directives were interpreted by the actual data base loading programs.

Cross-reference lists were not actually loaded but were computed from the observations. These lists were updated whenever observations were loaded or deleted. The cross-reference list was a redundant data item. It was controlled and kept consistent at all times by the geodetic data base management system.

9.5 DATA BASE LOCKING SYSTEM

The geodetic data base was designed so that different individuals could be performing data entry concurrently. However, it was necessary to prevent two users from entering data for the same detail record at the same time. The major problem was to prevent the contents of the detail record from changing between the time the user looked at the data base to begin the matching process and the time the new data were actually loaded. Since all operations were actually performed with batches of data, this process could take several days, especially if many problems needed to be resolved. The data base management system provided the system of locks shown in figure 9.3. A lock affected only users who wanted to query the data base for pre-entry information, or who wanted to load matched data. All other data base operations were able to proceed normally.

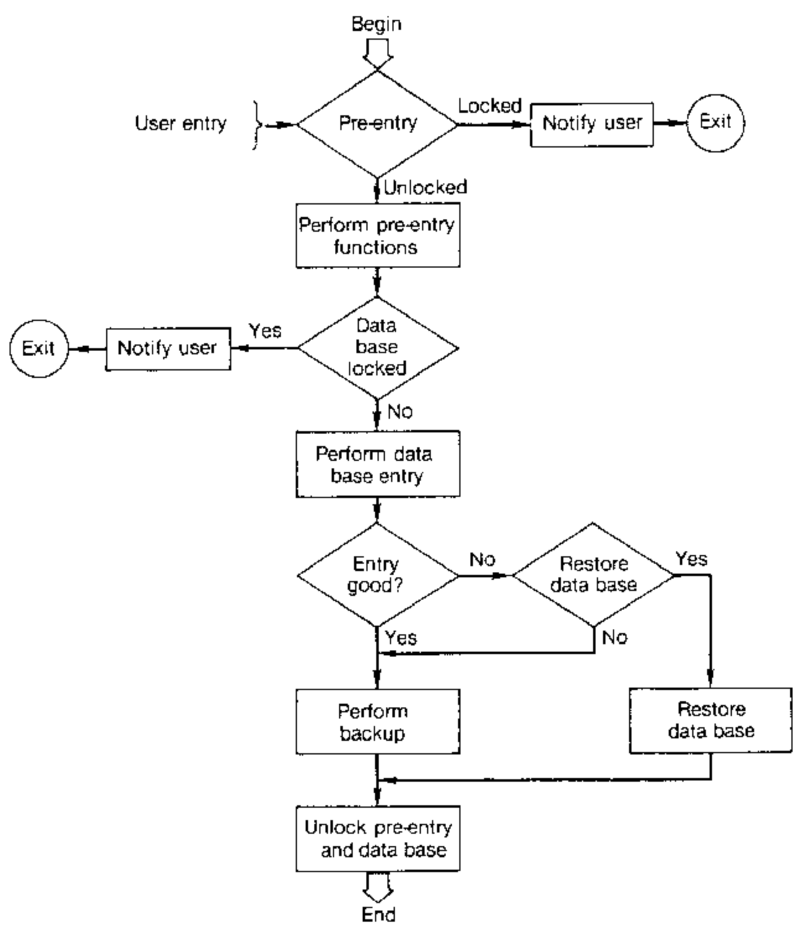

Figure 9.3. Pre-entry lock and data base lock.

9.6 DATA BASE FUNCTIONS FOR NAD 83

Many of the functions vital to the new adjustment of the North American Datum were performed inside the data base environment. Although each such function might involve several programs and intermediate files, each was invoked as a single function through the data base query language. The following are some of the major NAD 83 functions:

- Retrieve Data Base Deck. This was a major step in Block Validation.
- Horizontal Block Load. This was the last step of Block Validation, and resulted in the actually loading of observations into the data base.
- Retrieve RESTART File. This file was built with all points inside a defined block boundary, all observations from these points, all points outside the boundary which are seen from inside the block, all points outside the block which see into the block, and the actual observations from outside the block to the inside. The latter information was built from the cross-reference list and used to determine interior and junction points with respect to the block boundary.
- Analyze Strategy. This caused an entire Helmert blocking adjustment to be simulated within the data base environment.
- Retrieve RESTART-83. This retrieved a RESTART file with parameters taken from an Adjustment Project File. It was the beginning of the actual Helmert blocking adjustment.

9.7 DATA BASE SIZE AND ACTIVITY

Both the data base and the facilities of its management system grew throughout the NAD 83 project. By the time it was fully loaded, the data base contained approximately 275,000 station records and used about 3 gigabytes of storage. During the loading phase, as many as 20 data base transactions were executed per day. Most of these were retrieval operations, although 20 to 30 percent were update or initial entry operations.

9.8 USER'S SERVICES

9.8.1 User's Assistance Desk

Almost 100 registered users accessed the geodetic data base. Most of these had data retrieval but not update privileges. Even though most of the users were NGS employees, not all were personally acquainted with the programming staff. In response to this situation, a "data base user's assistance desk" was established within the Data Base Management Branch. The programmers staffed this desk on a rotating basis. The desk dealt with a variety of problems including not only the use of the data base facilities but also the use of application programs and the computer system utilities. Easy problems were resolved immediately; difficult problems were referred to the appropriate data base programmer or to the systems programming staff at the computer center.

9.8.2 Interactive Access for Outside Users

Individuals and organizations outside of NGS were registered as data base users with retrieval-only privileges. This allowed these users to have access to the data base via telecommunications on an almost interactive basis. The data base resided on a commercial time-sharing computer system. Outside users established their own accounts with the computer center, which billed them for any use of computer resources. NGS made no further charges for access to the data, since the data were already considered to be in the public domain.

9.9 RETROSPECTIVE

The process of "building the data base" took almost 10 years. This is certainly more than originally expected, but the final form of the data base satisfied many more requirements than originally planned. Ninety percent of the effort was expended on the actual data, less than 10 percent on the DBMS. At most times during the project, the programming team was able to write and install new data base features just fast enough to keep up with new requirements.

As a result of this 10 year effort, NGS has eliminated most of the redundancy and inconsistency in its data holdings. Integrity of the data has been guaranteed through validation procedures and security facilities. This has prompted the sharing of data among the operating units, while at the same time reaching a balance of their conflicting requirements.

This experience was not without difficulties. As expected, the institutional resistance was substantial. The centralization of data management took control away from operating units which had previously "owned" parts of the data, creating many small "turf battles." The installation of a data base changed many personal relationships in the organization, with those people skilled in the use of the data base achieving increased status.

By the end of the project in 1986, it had become obvious that many things could have been done differently. Certainly the decision to build rather than buy a data base management system was among them. It had also become apparent that the number of data base applications had been limited by the lack of a data manipulation language. Programmers outside the data base development group were unable to access the data base directly. Applications were written to execute inside the data base environment, but in reality that meant that such application programs could be written only by those individuals who understood that environment.

9.10 REFERENCES

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10. BLOCK VALIDATION

Maralyn L. Vorhauer

10.1 INTRODUCTION/PURPOSE

Block validation was the second step of the three step process of forming and validating that data set which would be adjusted to yield the new datum.

In the first step of the process, individual data sets had been placed into machine-readable form and validated as far as possible. The primary activity at this level had been the conversion of the survey projects in the National Geodetic Survey archives into machine-readable form, as discussed in chapter 6. These projects contained approximately 250,000 horizontal control points and 2.5 million observations, including the azimuth and reference mark measurements which would not actually participate in the adjustment. Validation at this level assured consistency within a project. (See fig. 10.1.) The validated projects were stored as 4,997 separate Trav-decks.

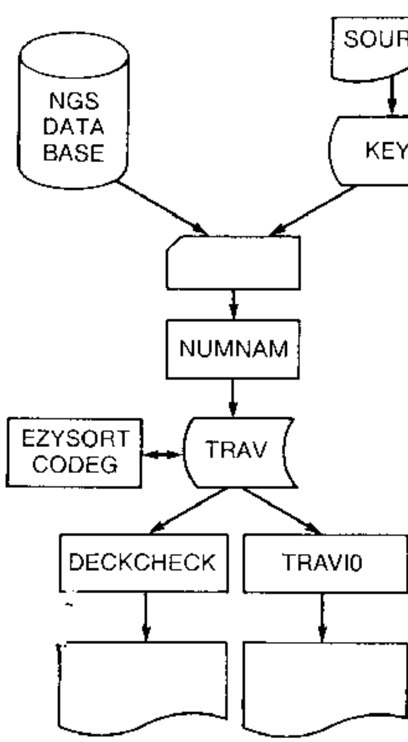

Figure 10.1. Network validation.

Other data sets that had been placed into machine-readable form included the electronic distance measurement (EDM) lengths, i.e., the so-called length data set (LDS) measurements (discussed in chapter 6), astronomic positions and azimuths (discussed in chapter 8), and published station identifiers and positions (discussed in chapter 20). (See fig. 10.2.)

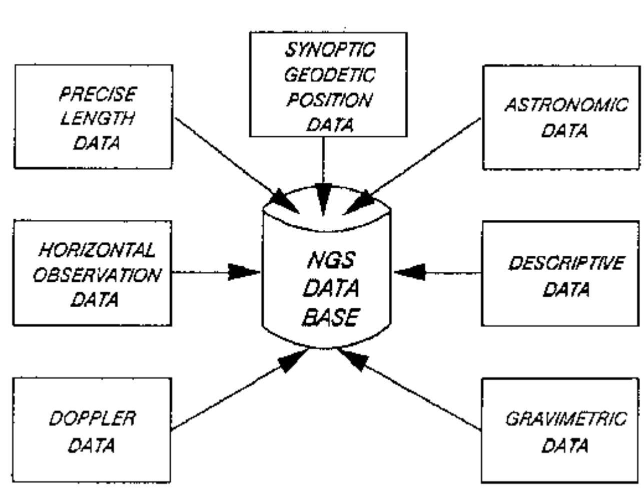

Figure 10.2. NGS data base entry path.

Block validation covered two major tasks:

1. Merge all of these different data sets.
2. Ensure that the merged data set was both correct and capable of being adjusted.

It was anticipated that a large number of data problems would surface during the merge process, since many data items appeared in more than one data set. For instance, positions, astronomic azimuths, and EDM lengths appeared in the project Trav-decks as well as in their individual data sets. It was expected that inconsistencies would arise between sources of information, each of which would need to be researched and resolved.

It was also anticipated that there would be difficulties fitting the different survey projects together. It was known, for example, that the naming of control stations had not always been consistent, so identification problems would occur. It was also known that some observations had appeared in more than one project; these observations would appear as duplicates in the merged data set.

There were several possible approaches to the merging and validating of the different data sources. One was to bring each data source into the data base, one by one, and revalidate the combined data set after the addition of each source. Another possibility was to load all sources into the data base and then sort out the various duplicates, misidentifications, and other problems. A third possibility was to validate everything before data base loading. For a variety of reasons these possibilities were rejected. The alternative selected was a combination of the last two. The project Trav-decks were validated outside of the data base environment before loading. Other data were loaded with only a minimum of data checks. Combined data sets, containing merged Trav-decks and other sources, were then extracted so that the merged data could be further validated.

Because of the large volume of data involved and the number of anticipated problems, the merge and validation process was broken up into blocks. The problem of validating observations that crossed block boundaries was postponed until later. This would be the third step of the three step process, and would not be fully accomplished until the first least squares network solution was completed.

10.2 PLANNING AND PREPARATION

Originally, all descriptions and recovery notes were to be checked concurrently with the project data analysis during the block validation effort. Estimates of the work involved showed, however, that this task could be accomplished more efficiently if it was done as a separate task during slack periods in the adjustment effort or by other NGS branches. Therefore, in the interest of making progress with the main task of the new adjustment, the checking of descriptions was deferred.

The block validation effort began in March of 1982 with the establishment of a small group to test the programs and develop the procedures. It continued until the last block of data was loaded into the data base in April 1985. The production rate was roughly 1.2 blocks/person/month. (See fig. 10.3.)

Earlier testing of the Helmert blocking system of adjusting large blocks of data had been carried out on a 3- by 5-degree rectangular area in Kentucky and Tennessee (Timmerman, 1978). The area was broken into four sub-blocks, each containing 750 to 1,000 stations, for the purpose of data validation. The experience indicated that blocks of this size were too cumbersome; too many problems had to be solved at once and the paper computer listings were so large as to be unwieldy. Smaller blocks, each containing approximately 300 to 500 stations, were selected for the actual block validation project.

MONTH	ASSIGNED THIS MONTH	ASSIGNED TO DATE	PREVALIDATION IN PROGRESS	AWAITING REASSIGNMENT	DRAGNET IN PROGRESS	STADJUST IN PROGRESS	NEMO IN PROGRESS	DATABASE LOADED THIS MONTH	DATABASE LOADED TO DATE
1982									
MARCH	5	5	5	--	--	--	--	--	--
APRIL	4	9	2	2	5	--	--	--	--
MAY	1	10	3	2	--	5	--	--	--
JUNE	9	19	11	3	--	5	--	--	--
JULY	23	42	34	2	1	3	2	--	--
AUGUST	22	64	52	6	1	2	3	--	--
SEPTEMBER	10	74	33	18	12	3	3	5	5
OCTOBER	18	92	36	13	22	13	3	--	5
NOVEMBER	13	105	28	18	29	16	14	-5	0
DECEMBER	8	113	24	13	24	26	26	0	0
1983									
JANUARY	48	151	47	17	27	19	21	30	30
FEBRUARY	27	188	34	31	34	22	21	16	46
MARCH	48	236	38	46	40	22	24	20	66
APRIL	9	245	17	44	37	29	25	26	92
MAY	1	246	3	36	45	30	31	9	101
JUNE	15	262	3	27	34	45	25	27	128
JULY	23	285	3	15	40	33	29	37	165
AUGUST	26	311	2	9	45	26	29	34	199
SEPTEMBER	52	363	0	12	53	39	20	40	239
OCTOBER	44	407	0	9	66	30	23	40	279
NOVEMBER	28	435	0	8	47	43	31	27	306
DECEMBER	34	469	0	3	74	34	13	39	345

MONTH	ASSIGNED THIS MONTH	ASSIGNED TO DATE	PREVALIDATION IN PROGRESS	AWAITING REASSIGNMENT	BLOCKS IN PROGRESS	DATABASE LOADED THIS MONTH	DATABASE LOADED TO DATE
1984							
JANUARY	32	501	0	3	110	43	388
FEBRUARY	48	549	0	3	118	40	428
MARCH	64	613	0	1	135	50	478
APRIL	43	656	0	1	131	46	524
MAY	38	694	0	1	125	44	568
JUNE	33	727	0	0	119	40	608
JULY	35	762	0	0	110	44	652
AUGUST	18	780	0	0	95	32	684
SEPT	24	804	0	0	89	31	715
OCT	16	820	0	0	83	22	737
NOV	19	839	0	0	79	24	761

Figure 10.3. Example of a monthly status report on block validation.

The primary tools in the selection of the block boundaries were a map and a listing of the number of stations in the geodetic data base (by 7½-minute quadrangle). The block boundaries were chosen such that the boundaries were rectangular and the blocks contained 300 to 500 stations each. (See fig. 10.4.) No consideration was given to survey project boundaries or to minimizing the number of cross boundary connections (Wade, 1982). These cross-boundary connections brought in additional stations from outside the block which had to be analyzed along with the stations inside the block. As a result, the total number of stations per block was not uniform and some blocks were associated with considerably more than 500 stations.

Altogether 843 blocks were formed, as illustrated in figures 10.5 and 10.6. Each block was given an identifier, usually the two-character state abbreviation, plus a six-character region name. A master computer file was created which listed each block and the boundaries of the area covered. By matching this file against the master file of all Trav-decks and their locations, the decks in each block could be identified and the master block file so updated. (See fig. 10.7.)

10.3 PROCEDURAL OVERVIEW

Block validation was conceived as a fairly simple process: It was a stepping-stone between project level validation and the final validation of the entire data set for the NAD adjustment. However, as the software was completed and procedures were put in place, block validation became more complicated. To a large degree this was caused by deferring the resolution of many problems from earlier processes, on the assumption that more data and better analysis tools would be available at the time of block validation.

As finally configured, block validation combined the Trav-decks with the previous contents of the data base and ended by updating the data base (Milbert, 1981). (See fig. 10.8.) The following major steps were emphasized, as highlighted in figure 10.9.

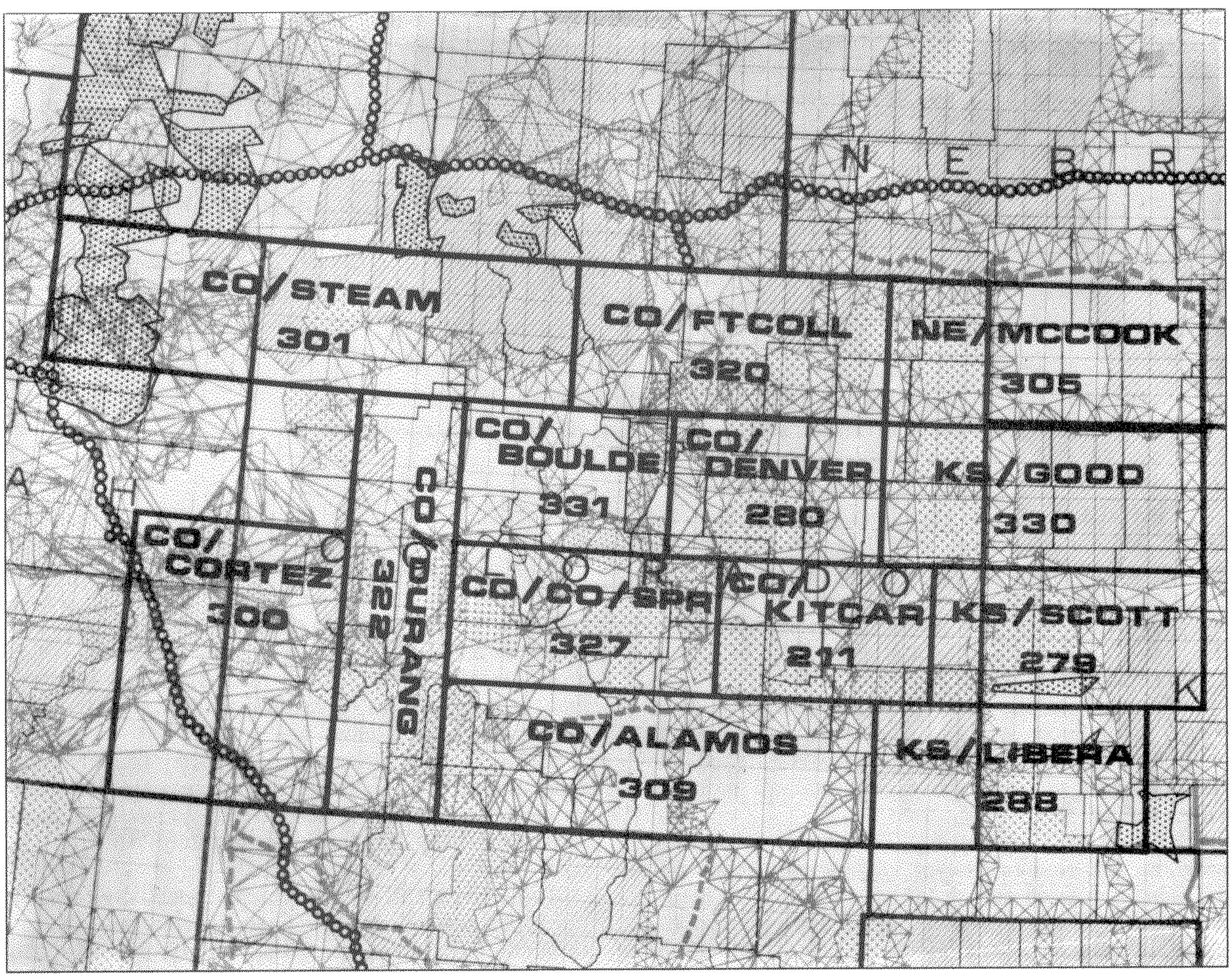

Figure 10.4. Validation blocks for Colorado.

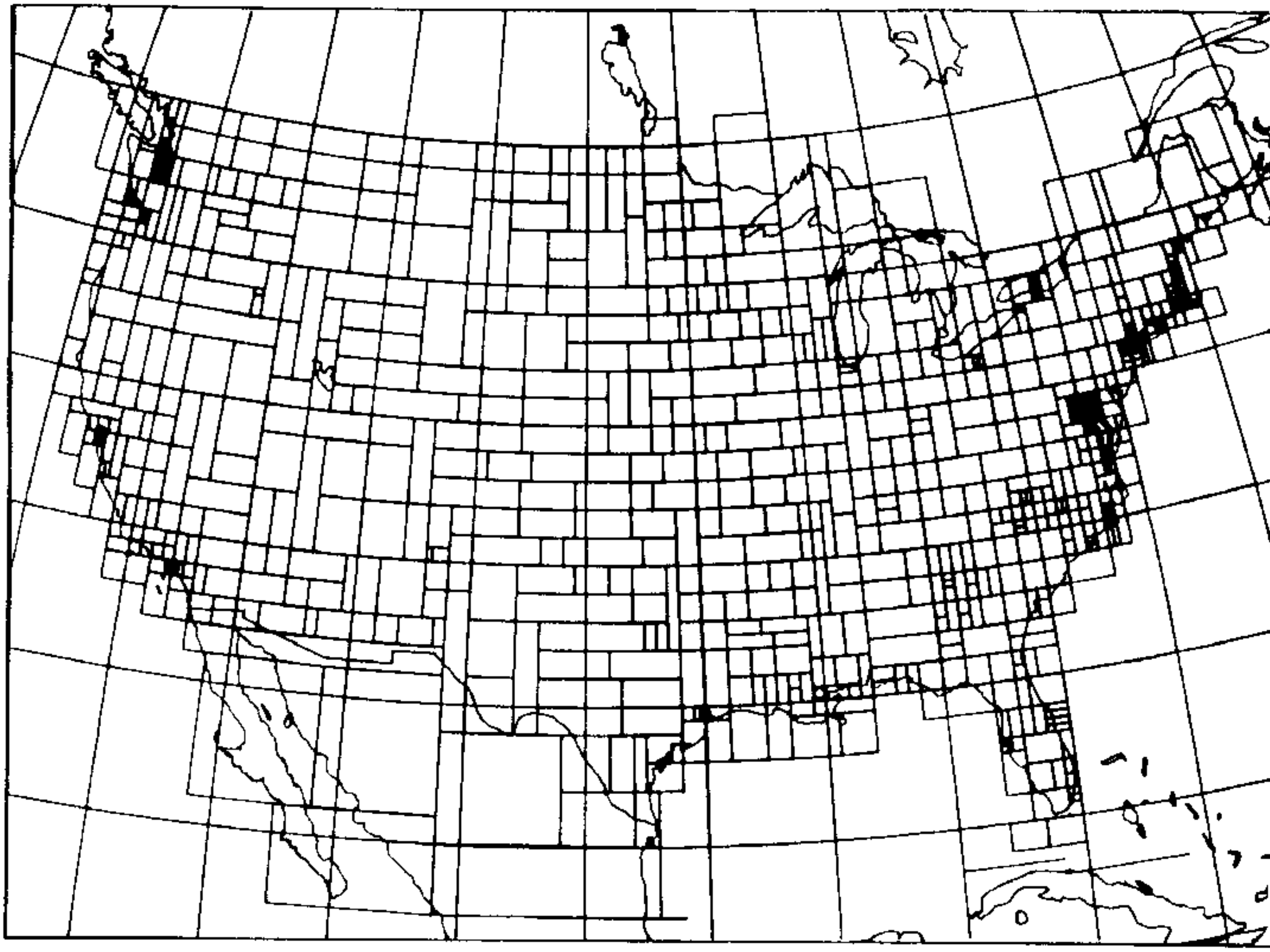

Figure 10.5. Boundaries of blocks of data used in block validation.
(Darken areas reflect status of project as of 1981.)

1. Prevalidation. The guidelines and programs for keying and editing project level data had been refined during the 10 years of automation, but the projects completed in the earlier years had never been reviewed for compliance with later rules. This step brought all the data up to a consistent set of standards.

2. DRAGNET processing. The DRAGNET program merged all the data for a given block and detected most inconsistencies, duplicates, and missing data. The problems it detected were researched and resolved. The program was rerun until all problems were resolved.

3. STADJUST processing. The station adjustment program was used to analyze all observations at each individual occupied station in a last attempt to ensure correct identification of all occupied and observed stations. It detected more problems which had to be researched and resolved. This program was also rerun until all problems were resolved.

4. NEMO processing. This computer program performed a least squares adjustment of a block. It was used to verify that the observations on the inside of a block held together as an adjustable network. Large residuals in the residual analysis phase would also indicate misidentified observations. Problems which arose at this step were also researched and resolved, and the program was rerun until there were no remaining unresolved problems.

5. Data base horizontal data load. The pre-entry programs detected inconsistencies between a block and its neighbors. These problems were also researched and resolved. Finally, the block was actually loaded to the data base.

These five steps were executed sequentially for each block. In general, a single individual carried a block through the entire process up to the point of actual data base loading. This last step was handled by a single individual for all blocks.

Quad	Block	# Sta
410713	RIWESTER	35
410714	CTPUTNAM	36
410721	CTWILLIM	40
410722	CTLONDON	39
410722	NYSHELTE	41
410723	CTNHAVEN	42
410724	CTHARFOR	43
410731	CTDANBUR	46
410732	CTDANBUR	46
410733	CTSTAMFO	53
410733	NYPEEKSK	61
410734	NYPOUGH	54
410741	NYMIDDLE	66
410742	NYMIDDLE	66
410743	NYMIDDLE	66
410744	NYMIDDLE	66
410751	NYMIDDLE	66
410752	NYMIDDLE	66
410753	PASCRANT	84
410754	PASCRANT	84
410761	PASCRANT	84
410762	PASCRANT	84

Figure 10.6. Sample listing of blocks and quadrangles covered by the blocks.

The major file generated during the block validation process was the RESTART file illustrated in figure 10.8. This was the output of a successful DRAGNET run and was the source of data for the STADJUST and NEMO programs. Other programs were written to update the RESTART file and to report on its contents. Finally, the RESTART file was loaded to the geodetic data base. A RESTART file for a block existed only for the period of time that a block was being validated; this could range from several days to more than a month.

10.4 PROCESSING AND ANALYSIS SPECIFICATIONS

10.4.1 Block Definition, File Management, and Preparation

As the data for each field project had been placed into machine-readable form, checked and provisionally adjusted, the files had been stored on magnetic tapes in the Trav-deck format. At the same time a master computer file listing of each Trav-deck was updated for file name, storage location, and area covered. (Specifically, all 7½-minute quadrangles covered by the project were generated from the geodetic positions in the deck.)

A folder was established for each block to hold the extensive log notes that would be generated as part of the analysis. Every decision and resultant change were to be thoroughly documented. Also, all previously published data were obtained, including the descriptions and the appropriate 1- by 2-degree geodetic control diagrams.

The first step in the block validation process was the retrieval of all the Trav-decks associated with the assigned block from tape storage to on-line computer files. A procedure called BLOCKOUT was executed to migrate the Trav-decks from tape and to create job control statements for running subsequent steps in the block validation analysis. If a Trav-deck had already

```
B VITHOMAS 1700 6400 1900 6500  271      EEC 0882  EEC 0882  EEC 0982  EEC 0183 L
T VITHOMAS G12709    G52485    G73713    G81161    G82156    STCROIX
B PRGRANDE 1800 6500 1900 6600  395      EEC 1082  EEC 1182  EEC 1282  EEC 0283 L
T PRGRANDE G15010    G15109    G15714    G16307    G32       G4928     G51715
T PRGRANDE G12709    G12735    G12938    G13438    G13447    G13707    G13744
T PRGRANDE G53038    G53127    G54258    G62950    G68729    G73713    G81273
T PRGRANDE G82161    G82159    G81558    G13938    G53018    G52485    G51716
B PRISABEL 1700 6500 1800 6700  194      EEC 1182  EEC 1182  EEC 1282  EEC 0283 L
T PRISABEL G16307    G32       G51298    G51835    G51845    G53018    G53038
T PRISABEL CERRILOS  G13744    G15010    G15109    G15714    STCROIX   G92281
T PRISABEL G9
B PRSJUAN  1800 6600 1900 6700  368      EEC 0982  EEC 1082  EEC 1082  EEC 0183 L
T PRSJUAN  G51298    G51835    G51845    G53038    G53145    G54258    G55
T PRSJUAN  G11121    G12735    G13447    G13744    G15109    G16292    G16307
T PRSJUAN  CERRILOS  G10028    G50940    G4404     G4275     G92281    G9
T PRSJUAN  G62904
B METOPS   4500 6600 4530 6800 1500      KOM 0684  KOM 0684  KOM 0684  KOM 0984 L
T METOPS   CANTRV16  CANTRV17  CROIXCAL  EMTDESE   EOBLIQ4   G12087    G15694
T METOPS   IBC16546  IBC16592  IBC16608  NEMAINE   PASCABAY  SCROCAL   STCRINLN
T METOPS   TAA16933  STCROI    G4933
B METOPS2  4530 6600 4600 6800 1000      KOM 0684  KOM 0684  KOM 0684  KOM 0984 L
T METOPS2  CANTRV16  CANTRV17  CROIXCAL  EMTDESE   EOBLIQ4   G12087    G15694
T METOPS2  IBC16546  IBC16592  IBC16608  NEMAINE   PASCABAY  SCROCAL   STCRINLN
T METOPS2  TAA16933  STCROI    G4933
B MEHOULT  4600 6600 4800 6800  170      KOM 0484  KOM 0484  KOM 0484  KOM 0684 L
T MEHOULT  CANTRV15  CANTRV16  G11957    G12087    G12299    G14910
T MEHOULT  G15066    G15656    G4933     G5369     MMME      NEMAINE   STCRINLN
```

Figure 10.7. Sample listing of BT (Block Trav-deck) cards.

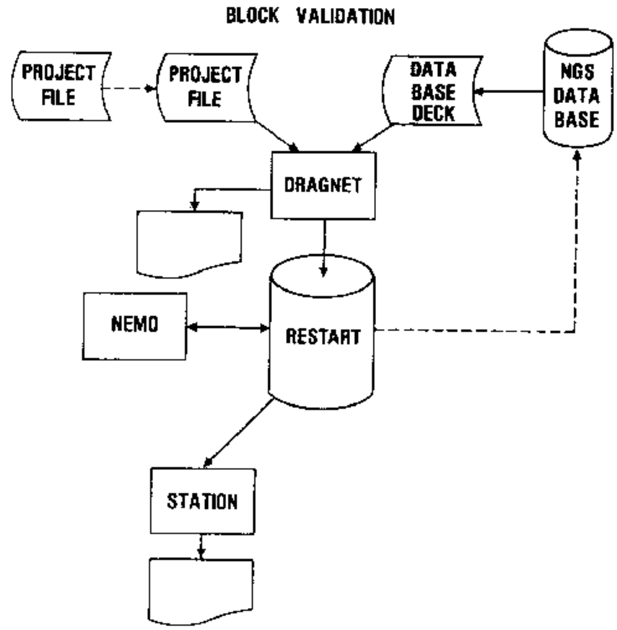

Figure 10.8. Graphic depiction of block validation process.

Define Block Boundaries

Get Trav-decks for Block

Prevalidation

Get Data Base Deck for Block

DRAGNET Processing:
Detect inconsistencies between Trav-decks and the Data Base Deck. Research and resolve inconsistencies.

STADJUST Processing:
Analyze all data at a station for correct identification. Research and resolve inconsistencies.

NEMO Processing:
Detect internal inconsistencies between Trav-decks. Assure adjustability. Research and resolve errors. Rerun until no errors.

Data Base Load:
Detect inconsistencies between this block and neighboring blocks. Research and resolve errors. Rerun until no errors.

Figure 10.9. Processing steps for block validation and horizontal data loading.

been brought on-line by another person, the control statements would simply establish access to that person's file. Thus only one active version of a Trav-deck was stored on-line. Also, since the macro procedure automatically created the control statements for all future program runs, the chance of errors in accessing files was virtually eliminated.

The retrieval procedure also updated the master block file by adding the initials of the responsible individual. This ensured that each block was only processed once and that a permanent record was maintained of each block processed. Statistical programs were developed to monitor progress and produce requested management reports from this file. Extensive guidelines were developed to ensure that the analysis and decisions were consistent throughout the branch (Horizontal Network Branch, 1984).

10.4.2 Prevalidation of Trav-decks

Prior to block validation, each individual project level Trav-deck was processed by a final checking program named DEKCHECK. This final check was performed to assure that each deck met the same standards. The program made checks to Trav-decks that were generally not made during the later steps of the block validation process, such as checking that correct standard errors were being used for the order of the project and that deck structure was correct. Therefore, the program was also run when a substantial number of changes had been made to assure that no blunders occurred during editing.

10.4.3 DRAGNET Processing

A computer program was designed to merge together all the various sources of data for a single block. These sources included the Trav-decks and all the information already loaded into the data base. The concept was to gather up all the files, large and small, and then to sort out what should be kept. The program was named DRAGNET, after a similar practice in fishing.

10.4.3.1 DRAGNET Files

Figure 10.10 shows the major aspects of the DRAGNET data flow.

10.4.3.1.1 Combined Trav-decks.

The primary input file to DRAGNET was the combined data from those Trav-decks which had been identified as being either wholly or partially contained within the block. These files were stacked one behind the other in a single run, temporary data set.

10.4.3.1.2 Data base deck.

A second major input file was the data base deck, built from the information already loaded in the data base. The intent was to extract and reformat all of the

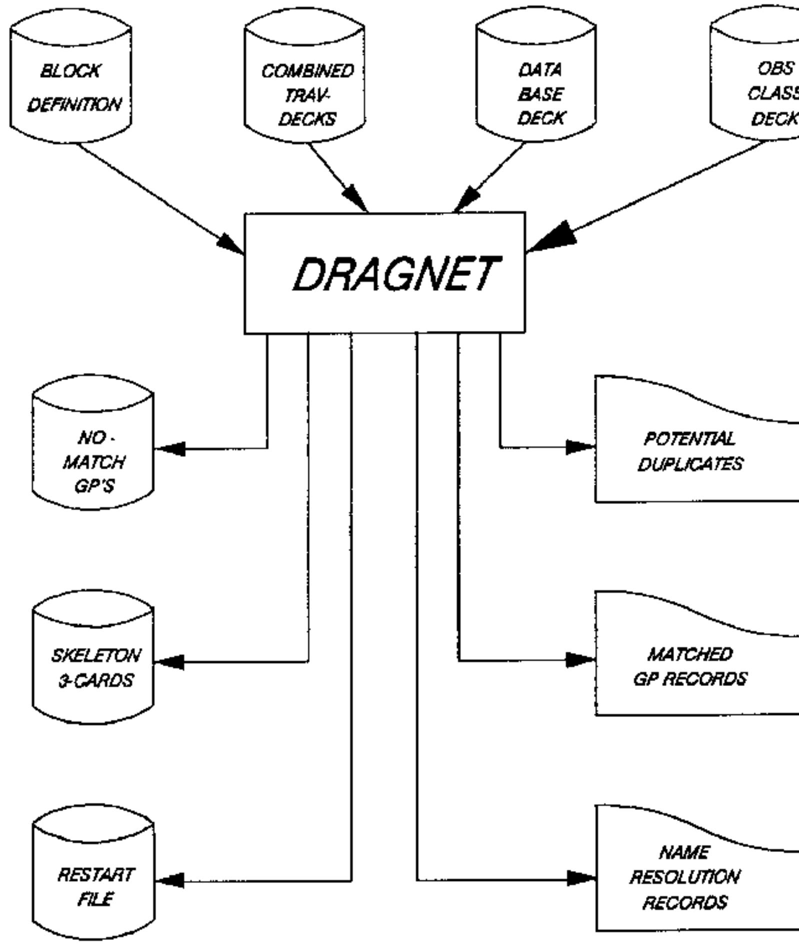

Figure 10.10. Data flowchart for DRAGNET files.

data base information that might be relevant to the merging process. The assumptions for this process were that the control point information (positions and identifiers), azimuths, and EDM lengths were completely loaded into the data base for all blocks, but that all other observations had not necessarily been loaded. Clearly, the control point information for all stations inside the block definition was placed into the data base deck; however, it was also necessary to find the control point information for those stations which were outside the block definition but participated (at one end or the other) in observations which crossed the block boundary. The observations stored in the data base were not sufficient to identify these stations, since not all observations had necessarily been loaded. The Trav-decks associated with the block contained all the observations, but proper station identifiers had not yet been assigned, so stations in the Trav-decks could not be unambiguously associated with stations in the data base. This problem was solved by placing a buffer area around the block definition. (See fig. 10.11.) The definition of the buffer area was computed by a program that scanned all of the Trav-decks associated with the block, examined those stations outside the block that had observations crossing the boundary, and determined the maximum distance of any such station from the block boundary. The buffer area definition was then input to the data base deck retrieval process along with the block definition.

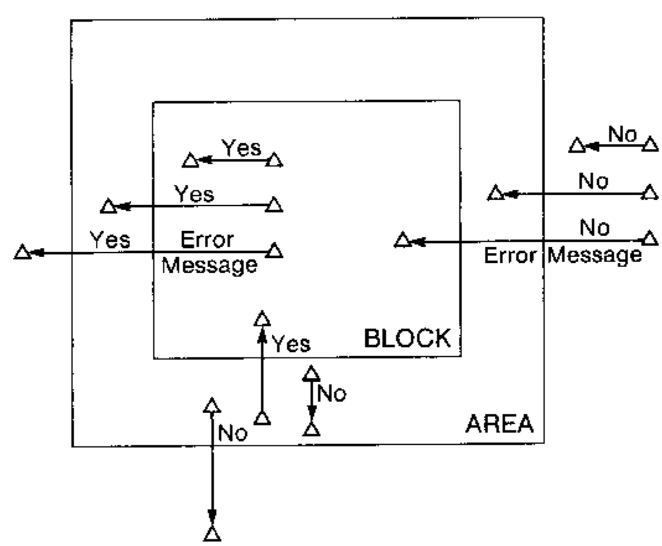

Figure 10.11. Block showing which distances, directions, and azimuths were to be placed in the data base deck.

The data base deck contained station information for all stations within the total area definition; all observations from stations inside the block definition; and all observations from stations outside the block, but inside the area, to stations inside the block definition. Observations between a station inside the block and another station outside the area were not expected, and caused error messages to be printed. (See fig. 10.12.)

The data elements extracted from the data base for each station included all identifiers, the geodetic position (on NAD 27), the astronomic position (if it had been observed), a "reconstructed" astronomic position computed from the gravimetric determination of the deflection of the vertical described in chapter 16, and the orthometric elevation. The observations extracted included electro-optical distance measurements and astronomic azimuths (each of which had been automated, validated, and loaded separately) and any previously loaded (and therefore validated) horizontal direction and taped or microwave distance observations. Again, error messages were generated if observations crossed outside the area retrieved. (See fig. 10.12.)

A similar data base retrieval process was planned for the formation of Helmert blocks for the actual adjustment, and this process would include a datum shift to approximate NAD 83 coordinates. For the purpose of block validation, however, all positions were left on NAD 27.

The data base deck file was temporary and deleted after validation of the block was completed.

10.4.3.1.3 RESTART files.

The major output of the DRAGNET process was a RESTART file. It was created only if the DRAGNET run completed without finding major errors. It contained a complete representation of the block and all its observations. When the RESTART file was loaded to the data base, the directions and taped distances it contained overrode previously loaded data. Most station synoptic information, astronomic azimuths, and electro-optical distances were not affected. These data types were corrected, when necessary, through their own data base editing routines.

The RESTART files were used by several other processes, such as the station adjustment program STADJUST and the network adjustment program NEMO (fig. 10.8). A program was available to apply updates to RESTART files. This facility was used sparingly and only under controlled circumstances, since corrections applied to RESTART files did not get reflected back into the Trav-decks.

RESTART files were configured as individual data bases, managed by the SYSTEM 2000 Data Base Management System (DBMS). This DBMS was used for its indexed access methods and retrieval-by-name features. Most of the other facilities normally associated with the management of a data base were not used. Thus the geodetic data base (which was permanent) was managed by application code written by NGS programmers, while the RESTART files (which existed only during the validation of a block) were managed by a commercial DBMS. Although unusual, this arrangement provided NGS with the best facilities for its particular environment.

Appendix 10.A describes the data elements stored in the RESTART file and their formats.

10.4.3.2 DRAGNET Phases

DRAGNET had two major phases: the analysis of the geodetic positions and the analysis of the observations. In both phases the program made decisions concerning which data should be deleted, what should be kept, and how they should be properly identified. The person assigned the block then analyzed these decisions for correctness.

10.4.3.2.1 Geodetic position (GP) analysis.

The control station records from the data base and from the Trav-decks were analyzed based on geodetic position. It was recognized that the position of a station as recorded in a Trav-deck might not be the same as the one in the data base, since positions had been

```
THE FOLLOWING OBSERVATIONS PREVENT THIS DATABASE DECK FROM BEING CORRECT!!

THESE OBSERVATIONS CROSS THE BOUNDARIES OF THE BLOCK AND THE AREA!

THE AREA IS NOT LARGE ENOUGH TO ENCOMPASS BOTH ENDS OF THE OBSERVATIONS.


OBSERVATION AND TYPE

2169740651403330001            XXX9090641404440003          1601A00000000011 G16974       HZTL OBS DIRECTION
3169740651403330001            XXX9090641404440004          2001 00022333011 G16974       HZTL OBS DIRECTION
2169740651403330001            XXX9090641411110003          1601 02517097011 G16974       HZTL OBS DIRECTION
2169740651403330001            XXX9740641404440003          1002A00000000011 G16974       HZTL OBS DIRECTION
2169740651403330001            XXX9740641404440003          1003A00000000011 G16974       HZTL OBS DIRECTION
2153670651403330001            1789740641404440003          1001 00000000011 RESUR141     HZTL OBS DIRECTION
2153670651403330001            1789740641404440003          1002 00000000011 RESUR141     HZTL OBS DIRECTION
HZTL OBS FROM 0641404440003 TO 0651403330001 OF TYPE 2                                    HZTL OBS XREFERENCE
HZTL OBS FROM 0641404440004 TO 0651403330001 OF TYPE 3                                    HZTL OBS XREFERENCE
HZTL OBS FROM 0641411110003 TO 0651403330001 OF TYPE 2                                    HZTL OBS XREFERENCE
HZTL OBS FROM 0641404440003 TO 0651403330002 OF TYPE 4                                    HZTL OBS XREFERENCE
HZTL OBS FROM 0641411110003 TO 0651403330002 OF TYPE 4                                    HZTL OBS XREFERENCE
HZTL OBS FROM 0641411140001 TO 0651403330002 OF TYPE 4                                    HZTL OBS XREFERENCE
HZTL OBS FROM 0651412320001 TO 0651403330002 OF TYPE 4                                    HZTL OBS XREFERENCE
3169740651403330003            XXX9090641404440003          3401 00000000011 G16974       HZTL OBS DIRECTION
3169740651403330003            XXX9090641404440001          3401 00300050011 G16974       HZTL OBS DIRECTION
3169740651403330003            XXX9090641411110003          3401 02307206011 G16974       HZTL OBS DIRECTION
HZTL OBS FROM 0641404440001 TO 0651403330003 OF TYPE 3                                    HZTL OBS XREFERENCE
HZTL OBS FROM 0641404440003 TO 0651403330003 OF TYPE 3                                    HZTL OBS XREFERENCE
```

Figure 10.12. Sample printout of observations crossing area boundary.

obtained from a variety of sources and preliminary positions had been used in many cases. Positions which fell within a 2-second square area of latitude and longitude were considered to be in the same match group. This relationship was transitive, so that if station B was within 2 seconds of station A, and C was within 2 seconds of B, then all three were placed in the same match group. The match groups of geodetic positions were classified according to table 10.1.

TABLE 10.1.—*Classification of geodetic position (GP) by match groups*

Number of data base GPs in match group	Total number of GPS in match group: 1	>1	
0	1	3	match status
1	2	4	
>1	5	5	

Match status 4 was the expected situation. This meant that one or more Trav-deck GPs had been matched with one data base GP. It was possible that the GP records in each Trav-deck were slightly different from the position stored in the data base. Even so, all the GPs in the match group became associated with the data base record. The winning Trav-deck was defined as the one in which this particular station had the highest order and type. An output record was synthesized according to table 10.2.

TABLE 10.2.—*Synthesis of output GP record by DRAGNET*

Field	Source
Accession number	Data base deck
Name	Data base deck
Latitude/longitude	Data base deck
Elevation	Most precise elevation in the group
Order/type	Winning Trav-deck
Plane coordinate zone	Data base deck
Block classification	Winning coordinates
Quad identifier/quad station No.	Data base deck

All GP records in Trav-decks, as well as all observations in those Trav-decks, were then identified with the data base identifier—quad identifier/quad station number (QID/QSN)—of the synthesized record.

Match status 1 or 3 indicated that there were stations in Trav-decks that could not be associated with a data base record, and therefore could not be assigned a data base identifier. This was an error condition, since it was assumed that the data base was completely loaded with respect to positions. For the purpose of continuing the analysis, a GP record was synthesized according to table 10.2, but the fields that would otherwise come from the data base deck were taken from the first (or only) Trav-deck GP record. If the station was inside the area definition, then a set of skeleton synoptic records (3-cards) were written to an output file. If the analyst determined that this station should indeed be in the data base, then he/she would leave the DRAGNET process, complete the 3-cards using a text editor, and load the 3-cards into the data base using the station entry path.

Match status 2 indicated a station in the data base that did not appear in any Trav-deck. This was also an error condition. Any station which would appear in the new adjustment would need to be connected to the network by observations. This analysis was intended to ensure that these observations were found. It was possible that the observations involving this station had been misplaced or misidentified. To aid the analyst, the match group was written to the NO-MATCH output file.

Match status 5 was also an error condition. It indicated that two stations in the data base were much closer together than had been expected. The program could not select the position with which to identify the observations.

Astronomic positions existed both in Trav-decks and in the data base deck, since Trav-deck automation guidelines required their addition to the deck and the astronomic positions had already been loaded into the data base as a separate data type. The astronomic positions in the data base were considered to be the definitive source, but the two sources were compared as an added check. Messages for consideration by the analyst were produced when:

1. an astronomic position in a Trav-deck could be identified with an astronomic position in the data base (because the corresponding GPs were in the same match group), but the difference in either coordinate was greater than 30 seconds;
2. an astronomic position was found in a Trav-deck and could be associated with a station in the data base, but no astronomic position record existed in the data base; or
3. an astronomic position was found in a Trav-deck, but could not be associated with a station in the data base deck.

A similar analysis of astronomic azimuths was performed. The azimuths in the data base were the definitive source, but messages were produced when:

1. an azimuth existed in a Trav-deck but not the data base, or
2. an azimuth in the Trav-deck did not have a corresponding geodetic position in the deck but a position existed in the data base with a similar name with which it might possibly be matched. This happened most often when an astronomic azimuth was observed to a newly set azimuth mark but no position was computed for the mark until a later date when a distance was measured.

10.4.3.2.2 Observation analysis.

The purpose of this analysis was to determine the data base identifier of both the occupied and observed station for each observation in each Trav-deck. Once this was done, duplicates could be identified and each observation loaded into the proper place in the data base.

The analysis was complicated by several factors. One was the earlier decision that the data base was not to contain station synoptic entries for nonpositioned points, such as reference marks, azimuth marks, and witness marks. These were considered to be ancillary to a parent station (so-called supernumeraries). On the other hand, observations from and to such marks were to be stored in the data base. Such marks could later become positioned points, and the observations from and to them would then be needed. Furthermore, the data base was being built for a broader purpose than just the new adjustment, and there was a strong desire to keep all data if possible. The azimuth marks helped to establish station identification. They are published because they provide starting orientation for new surveys.

A second complicating factor was that within a Trav-deck the association of an occupied or observed station with a GP record was made only through the station names. The rules for making such associations were complicated, since the names of ancillary stations did not always directly match the name of the parent station. Furthermore, the Trav-decks contained observations to points which had not been identified.

A third complicating factor involved stations that had been given different names in different Trav-decks. This meant that the association of the name of an occupied or observed station with a data base identifier was a two-step process: The name was first associated with a name in a GP record in its own Trav-deck, and that name was then associated with a data base GP through the results of the global GP analysis described above.

Several attempts were made to express the logic of the classification of observations in a form in which the responsible parties could understand and agree upon the logic. Table 10.3 displays the final form. Y means "yes," N means "no," and - means "don't care."

The conditions in table 10.3 were defined as follows:

C1. This is a synthetic observation, defined as one which had been borrowed from another Trav-deck or simply made up. It was identified by all zeroes in the accession number field or all blanks in the date field.

C2. The occupied point is a positioned station.

C3. The occupied station is a reference mark, azimuth mark, or witness point that can be associated with a parent positioned station.

C4. The observed point is a positioned station.

C5. The observed station is an ancillary point that can be associated with a parent positioned station.

C6. The observation was rejected during project level validation (but is still carried in a Trav-deck because it might later be rescued).

C7. The observation is a distance.

C8. The "from associated station" is inside the block boundary.

C9. The "to associated station" is inside the block boundary.

C10. "Keep flag" means that the observation was coded as an observation to a reference or azimuth mark, or that a program option was selected to force retention of all observations.

An occupied or observed station was determined to be positioned if its name matched exactly with a name in the GP section of its Trav-deck and that record had been matched with a data base GP. If the name of the station could not be matched, then a new search name was formed by stripping off any ending characters such as "RM","RM1","RP","WP","AZ MK", etc. If this modified search name could be matched to a name in the GP section of its Trav-deck, then the occupied or observed station was considered to be ancillary to a parent station.

The "from associated station" in condition C8 is the occupied station, if it is positioned; otherwise it is the parent of the occupied station, if it exists. A similar definition is used for the "to associated station" in condition C9.

The classification codes resulting from the application of this decision table had the following meanings:

1. This is an observation between positioned points, both interior to the block.

TABLE 10.3.—*Decision table for the classification of observations in DRAGNET*

C1: Synthetic	Y	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
C2: From sta pos	-	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
C3: From parent pos	-	-	-	-	-	-	-	-	-	-	-	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N
C4: To sta pos	-	Y	Y	Y	Y	N	N	N	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	N	Y	N	N	N	N
C5: To parent pos	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	N	Y	Y	Y
C6: Rejected	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
C7: Distance	-	-	-	-	-	-	-	-	Y	Y	Y	N	N	N	Y	Y	Y	N	N	N	N	N	Y	Y	Y	Y
C8: From assoc. inside	-	Y	N	Y	N	Y	Y	N	-	-	-	Y	Y	N	Y	Y	N	Y	Y	N	-	-	-	-	-	-
C9: To assoc. inside	-	N	Y	Y	N	-	-	-	Y	Y	N	-	-	-	-	-	-	-	-	-	-	-	-	Y	Y	N
C10: K flag	-	-	-	-	-	Y	N	-	Y	N	-	Y	N	-	Y	N	-	Y	N	-	-	-	-	Y	N	-
Classification	8	2	3	1	8	4	8	8	5	8	8	6	8	8	6	8	8	6	8	8	8	8	8	7	8	8

2. The observation crosses the block boundary from inside to outside.
3. The observation crosses the block boundary from outside to inside.
4. This is an observation from a positioned station to a reference or azimuth mark.
5. This is an observation from a reference or azimuth mark to a positioned station.
6. The occupied station is an ancillary point and the parent is positioned.
7. The occupied station cannot be identified, but the observed station is an ancillary point and the parent is positioned.
8. The observation is to be deleted.

Observations with classification codes 1, 2, and 3 were to be stored in the data base record of the occupied station, with cross references set up in the records of the observed stations. These were the observations that would participate in the continental adjustment. Codes 4 and 6 were stored in the "reference mark obs" section of the data base record of the occupied station, and codes 5 and 7 were stored in the corresponding record of the observed station. Observations with classification 8 were deleted without further consideration.

Once an observation was classified, it was stored in the appropriate section of the working files for eventual storage in the generated RESTART file and the data base. At the time of storage, it was compared with other observations in the same storage area so that potential duplicates could be detected.

If the classification of an observation was greater than 3 (meaning that it was not possible to position both the occupied and observed station), then one more search was made. This was a search of the global name table to see if the occupied or observed station could be matched with a name that was in another Trav-deck.

10.4.3.3 Analysis of DRAGNET Output

DRAGNET produced printed output and several machine-readable files. The comparison of the Trav-deck data file and the data base file began with the analysis of geodetic positions. (See fig. 10.13.) A description of each match group was printed and each group was analyzed. The synthesized output record was shown whenever it could be produced.

If any match group generated a set of "skeleton 3-cards," these records could be used for direct loading into the data base, although they contained only the minimum information required for data base insertion. The accession number (GTZ°) given to all these positions was 17020. In this way positions added as a result of block validation could be easily identified. Notes in the block folder assured future documentation of the reasons for any additions or deletions.

Geodetic positions were found to be missing from the data base for a variety of reasons:

1. A blunder—the keypuncher simply missed the position.
2. An entire project was not loaded into the data base due to a blunder or a change in publishability, e.g., secret to unrestricted.
3. Positions were not missing but because of a mispunching were not within the matching tolerance.
4. Additional positions were needed for solvability which had not been previously added to the data base, e.g., a reference mark position computed to provide a tie between nearby stations.

Positions were found to be missing from the Trav-decks because:

1. projects were not put into machine-readable form,
2. observations which supported the positions were never received,
3. observations were misidentified,
4. observations to support a particular position were not keyed because they were not part of the standard hard copy stored with the project data, or
5. the Committee for the Review of Archival Projects had determined that the entire project was to be discarded, since it contained observations of insufficient quality for inclusion in the NAD 83 adjustment. Some positions determined in these projects were still found in the data base.

Because GPs falling within a 2 second square area were listed together, possible duplicate data base positions could be spotted. The criteria for matching were exact name or position, but the program could not provide a clear match between stations having very similar names and nearly identical positions. The analyst had to make this determination on a case by case

```
2.  16056 Y 55 AMS 1966        32 43 08.52521 111 50 35.75698  432.57 000.0 042      15 DATABASE  IN
++  16056 Y 55 AMS 1966        32 43 08.52521 111 50 35.75698  432.57 000.0 042      15           IN            389++
**********************************************************************************************************
1.  16056 Y 56 AMS 1966        32 42 15.90028 111 50 35.92851  440.93 000.0 042      15 DATABASE  IN
2.  16056 Y 56 AMS 1966        32 42 15.90028 111 50 35.92851  440.93 -15.1 042      15 CASAGRAN  IN      303
++  16056 Y 56 AMS 1966        32 42 15.90028 111 50 35.92851  440.93 000.0 042      15           IN            390++
**********************************************************************************************************
1.  16056 Y 50 AMS 1966        32 47 28.48706 111 50 36.09441  425.56 -15.0 042      15 CASAGRAN  IN      297
2.  16056 Y 50 AMS 1966        32 47 28.48708 111 50 36.09441  425.56 000.0 042      15 DATABASE  IN
++  16056 Y 50 AMS 1966        32 47 28.48706 111 50 36.09441  425.56 000.0 042      15           IN            391++
```

Figure 10.13. Sample DRAGNET page.

basis and "force" correct matches by having identical names or positions for each point.

In all, approximately 2,000 of the stations in the data base were marked for removal from the NAD adjustment as a result of the DRAGNET analysis. These were stations for which no observations were available or the observations were of insufficiently high quality. On the other hand, approximately 2,000 other stations were found in the Trav-decks and added to the data base.

In a few cases, stations were marked for deletion from the data base altogether. By design, this was a somewhat cumbersome process. To delete a geodetic station, all associated data, including descriptive data, had to be deleted first.

In most cases, the proper action was to set a nonadjustability flag. When this flag was set, data for the point would not be retrieved for any NAD 83 program. Such stations were invisible to the NAD adjustment, but remained in the data base and were available for other purposes. Furthermore, this flag could easily be reset if new data came to light which changed the previous decision.

While the determination of the need to set a flag was made by the person doing the analysis, the actual data base edit was performed by supervisory personnel. Similarly, the actual process of loading and modifying the data base was carried out by a single individual. This assured that control was maintained on all data base interactions.

Data elements other than geodetic positions were also displayed. The results of applying the rules of table 10.2 were shown for all data elements.

Normally, only a cursory investigation was done of data elements other than positions. However, of these, the elevation was the most critical to the adjustment, and so anomalies between sources (generally differences >10 m) were investigated. Initially, all bench mark elevation values were checked, but this became too time-consuming and did not yield sufficient changes to warrant the effort.

The next section of the DRAGNET printout listed possible resolutions of unknown "to" or "from" records. This was a list of observations in Trav-decks that did not have GPs in the Trav-deck for the "from" or "to" station but appeared to have GPs in the data base, based on the name. This may have happened, e.g., when an azimuth mark was positioned after the Trav-deck was created. It could have happened because an incorrect or very similar name was used in the Trav-deck and should not have matched a data base name. These problems were resolved by either adding a geodetic position to the Trav-deck, or by changing the name sufficiently to prevent a program match.

Another section paired observations from different Trav-decks that were considered potential duplicates. (See fig. 10.14.) Direction observations were paired if the "from" station, the "to" station, and the observed value were the same on both records. Distances were paired if they had the same "from" and "to" station. The analyst checked the values and dates to determine the existence of real duplicates.

Occasionally an entire Trav-deck was found to be a duplicate. More often, a few observations would have been coded twice. Early (1920s and 1930s) adjustments used combined lists of directions. These were frequently retyped from originals in several sources, or had older directions typed onto the combined direction list, but were not identified or possibly not recognized as such by inexperienced coding personnel. This happened even more frequently for reference and azimuth mark observations. To resolve this problem, codes were changed in the Trav-deck to indicate that such observations were borrowed and therefore should not be included in the combined file.

Another source of frequent duplication was the precise taped base line data. The original Trav-deck guidelines called for these distance measurements to be included in each deck. This was later revised in favor of the creation of separate Trav-decks containing only taped base lines but spanning several blocks. This resulted in a large number of duplicates which needed to be removed from the Trav-decks.

Another section listed matches (fig. 10.15) or no matches between the astronomic and length observations in the data base and the Trav-decks. As a result of this analysis, some observations found in Trav-decks had to be loaded separately to ensure their inclusion in the adjustment. This often happened for astronomic azimuths to previously unpositioned azimuth marks which had been observed as part of the mark maintenance program. Furthermore, astronomic azimuths had not been submitted to the Gravity and Astronomy Branch unless astronomic positions had also been observed, and this was seldom the case with mark maintenance data. With the development of the ability to predict deflections of the vertical at any point and, therefore, the ability to compute the LaPlace correc-

```
DRAGNET                          AZCASAG                                          84/03/01     08.37.17
POTENTIAL DUPLICATES
M107495SIERRA 1960        101973SIERRA 1960 RM 1              5680  TRAVDECK  MMAZ       2510
M145895SIERRA 1960        048971SIERRA 1960 RM 1              5676  TRAVDECK  TIONMAZ    1490

M107495SIERRA 1960        101973SIERRA 1960 RM 2              8461  TRAVDECK  MMAZ       2511
M145895SIERRA 1960        048971SIERRA 1960 RM 2              8452  TRAVDECK  TIONMAZ    1491

M125886CHUI 1936          035960CHUI 1936 RM 1                8507  TRAVDECK  AZHSGILA    800
M160986CHUI 1936          164966CHUI 1936 RM 1                8498  TRAVDECK  CASAGRAN   5459
```

Figure 10.14. DRAGNET printout of potential duplicate observations.

```
DRAGNET                                AZCASAG                                  84/03/01      08.37.17

OBSERVATION MATCHES


S16322MEN AZDT 1978           066979TAK AZDT 1978           30010 10698898   DATABASE
X16322MEN AZDT 1978           066979TAK AZDT 1978           15 10 10698910   TRAVDECK  AZ16322     429

S16322MEN AZDT 1978           066979TAK AZDT 1978           30010 10698904   DATABASE
X16322MEN AZDT 1978           066979TAK AZDT 1978           15 10 10698910   TRAVDECK  AZ16322     429

S16322MEN AZDT 1978           066979TAK AZDT 1978           30010 10698916   DATABASE
X16322MEN AZDT 1978           066979TAK AZDT 1978           15 10 10698910   TRAVDECK  AZ16322     429

S16322MEN AZDT 1978           066979TAK AZDT 1978           30010 10698933   DATABASE
X16322MEN AZDT 1978           066979TAK AZDT 1978           15 10 10698910   TRAVDECK  AZ16322     429

S14590BURRO RM A 1971         039971PASS 1960               30010 14333472   DATABASE
X14590BURRO RM A 1971         039971PASS 1960               17 10 14333476   TRAVDECK  TTDNMAZ    1607
```

Figure 10.15. Length and azimuth observation matches: Trav-decks vs. data base.

tion without astronomic longitudes, astronomic observations could be loaded as long as both ends of the line were positioned stations.

DRAGNET formed and printed pairs of observations such that one member was from the data base, the other was from a Trav-deck, both sources matched exactly, and the observation crossed the boundary between blocks. (This was possible only if the adjacent block(s) was loaded.) Errors occurred when observations were changed in the RESTART file and loaded from one block but not changed in the Trav-deck, and hence appeared differently in the adjoining block. The procedure for actually loading observations into the data base required that such pairs be exact matches (except for rejections or standard error changes), and so these discrepancies had to be resolved before the block under consideration could be loaded. The program also listed observations crossing the boundary which appeared in only one source. These included observations in the data base but not in the RESTART file or vice versa. (See fig. 10.16.)

All of the error conditions detected by the program required analysis and resolution. In most cases the resolution required that one or more Trav-decks be modified. This was easily done with a text editor. Since only one copy of each Trav-deck was on-line, there was no possibility of inconsistencies existing between different copies of the same Trav-deck.

In other cases the resolution required modifications to data that were already in the data base for the block being validated. For instance, it might be necessary to add or delete stations or to modify the astronomic position, azimuth, or an electro-optical distance.

It was also possible that a change was needed to a neighboring block which had already been loaded. This was an unusual situation, since a block that had been loaded would have been validated and usually involved observations crossing the boundary. However, the process of updating previously loaded observations was not difficult. A small RESTART file covering only the affected area was retrieved from the data base. This could then be edited to reflect the change and reloaded into the data base. It was not necessary to reload the entire block that was found to contain the error. In fact, the geodetic data base was seamless, and observations, once loaded, lost all identification with a block.

The final DRAGNET listing, illustrated in figure 10.17, was a tabulation of all the stations associated with the block. It shows the elements of table 10.2 for each station plus its station number in the block and its status in the block (e.g., inside or inside junction). It should be noted that because the geoid height model was not finalized at this point in the project, all geoid heights were zero filled in the RESTART file during the block validation analysis.

Sometimes the DRAGNET process had to be rerun several times before all discrepancies were resolved. At this point, a RESTART file was created which could be loaded into the data base. However, before loading, additional procedures for checking and analysis of the data were carried out.

```
DRAGNET                                AZCASAG                                  84/03/01      08.37.17

NOMATCH OBSERVATIONS
216322MEN AZDT 1978           334978TAK AZDT 1978              7 1V199033470  TRAVDECK  AZ16322     222

S16322MEN AZDT 1978           066979TAK AZDT 1978           30010 10698898   DATABASE

S16322MEN AZDT 1978           066979TAK AZDT 1978           30010 10698904   DATABASE

S16322MEN AZDT 1978           066979TAK AZDT 1978           30010 10698916   DATABASE

S16322MEN AZDT 1978           066979TAK AZDT 1978           30010 10698933   DATABASE

412197OCOTILLA 1960           029960FLOY FEDERAL COMPRESS CO N TK   2 18407043   TRAVDECK  AZHSGILA    495

412197OCOTILLA 1960           029960FLOY FEDERAL COMPRESS CO S TK   2 18256430   TRAVDECK  AZHSGILA    494
```

Figure 10.16. Sample printout showing no-match observations crossing the boundary.

```
GLOBAL NAMES

**    16974 ARCH                        66 05 22.06627 141 06 31.46128 1067.30 000.0 022     44  0661412220002     83**
**    11087 ARCTIC IBC 1910             66 27 34.42705 140 53 11.24800 1272.27 000.0 570     21  0661403440001     99**
**    08665 ARDEN OF THE BOUNDARY       65 21 55.54405 140 59 56.12106 1111.9  000.0 022     71  0651404330002     51**
**    11087 AURORA IBC 1912             65 15 05.35605 140 49 54.96800 1447.90 000.0 570     21  0651403420002    243**
**    11087 BACK IBC 1909               65 09 45.91006 140 57 47.32300 1575.63 000.0 570     11  0651403340010     24**
**    15367 BACK 2 1974                 65 09 45.69563 140 57 47.34027 1575.51 000.0 570     21  0651403340011     23**
**    11087 BACKHOUSE IBC 1912          65 21 54.60400 140 47 20.47600 1103.62 000.0 570     21  0691403420001    248**
**    11067 BARNEY IBC 1905             65 05 53.42100 140 52 38.91500 1256.57 000.0 570     11  0651403330008     16**
**    15367 BARNEY 2 IBC 1974           65 05 53.42019 140 52 38.94317 1256.69 000.0 570     21  0651403330009     15**
**    16106 BARNEY2                     65 05 53.61961 140 52 39.23232 1253.96    .0 570     21  0000000000000     17**
**    11087 BATTLE IBC 1910             66 59 47.79000 140 53 47.24400 0920.43 000.0 570     21  0661404440001    121**
**    11087 BLUE IBC 1910               66 02 00.47300 141 05 15.96500 1073.53 000.0 022     21  0661412220001     79**
**    16106 BUG IBC 1912                69 27 41.67265 140 50 24.33461 0454.46 000.0 570     21  0651403410001    254**
**    16106 BUG TOPO                    69 26 37.73761 140 54 50.25670 0439.27 000.0 570     11  0651403440004    253**
**    08665 C 1 OF THE BOUNDARY         65 03 41.65700 140 59 56.27600 1348.9  000.0 022     71  0651403330006      9**
**    11087 CAIRN NEAR RAMPART IBC 1953 67 21 42.52000 140 56 36.75000 0672.50 000.0 570     45  0671403430001    138**
```

Figure 10.17. Global name listing.

10.4.4 Station Adjustment

The first additional step was designed to rescue observations which had been marked for deletion because the forepoint was misidentified and to identify other inconsistent, miscoded, or misidentified observations. This was done by combining all the separate lists of directions at a particular station (commonly called a station adjustment) and printing the results. (See fig. 10.18.)

In areas where earthquakes had occurred, an additional program, CRUSPROC, was run prior to the station adjustment. This program applied a correction to the observations to put all data in the same epoch. (This model and its use are fully discussed in chapter 17.)

The station adjustment program (STADJUST) could be run on all stations in the entire block (the option usually selected for the first run) or for selected stations within the block (usually selected to check corrections made as a result of the initial analysis). The analyst also had the choice of the radius to be used to define the limits of the region to search for possible matches with unidentified or rejected observations. In areas of limited line of sight (as in the eastern United States) or high station density, a value of 25,000 m was usually sufficient. Larger values were used in other areas.

If an unsuccessful combination occurred, as evidenced by large residuals on the combined observations, inconsistent values of observations coded to the same points in different abstracts could be resolved and corrected. (See fig. 10.19.) Lists were combined through common observations that could be to either a published station or an azimuth mark. Each observation to an unpublished point, i.e., those coded with a "N" or "U", were treated as separate observations, as were previously rejected directions. These observations were not used in the combination of directions even if one direction matched another.

If significantly large residuals occurred on observations (other than reference or azimuth marks) the possibility of a misidentification was investigated. In general, rejections were not made at this stage, but were deferred to the analysis of the least squares adjustment results.

Large residuals on azimuth or reference marks that were matched by name often indicated different marks had been coded with the same name or that discrepancies existed between values observed in different years. These discrepancies were usually resolved by using the station descriptions. If, for example, a recovery note stated that earlier observations were incorrect, the earlier observations were rejected.

If no errors were noted in the description or recovery notes, the following formula was used to determine the maximum allowable residual on an observation to a reference mark:

$$\text{Maximum allowable residual} = 1800/s,$$

where s = distance in meters. If the residual exceeded this value, the earlier observations were rejected.

Because the mark maintenance program was still active at this time, a memorandum would also be generated to the appropriate field person to investigate the problem. Often measurements to reference marks appeared to be switched and the correct orientation could not be determined. Again the oldest observations would be rejected and notes made in the block log.

Another situation which indicated a possible error occurred when more than one component was formed by the observations. (See fig. 10.19.) In this case, it was necessary to look for an observation which could provide the tie between the components. The tie could be provided by a misnamed azimuth mark, an unnecessary rejection, or a misidentified observation. Occasionally the best solution to the problem was to transfer observations from one list to another. If the rotation to combine the observations into one list could be made only through a "N" or "U" station, then the observations in such a list were NOT transferred using these common directions. Rejected azimuth mark observations were not used for rotation nor were directions to reference or other nearby marks. Because multiple components might result in a weak geometrical connection to the network at this station, this situation was studied carefully for a resolution.

The second section of the station adjustment output printed the adjusted combined list. The program also printed a comparison of these combined observations with those determined by inverse. There were two types of computed directions. The first was the direction determined by using the identified position of the

```
                                                                                                        MARCH  2,1984
STATION ADJUSTMENT                                                                                                    STATION  318

 JOB   STATION NAME                                LATITUDE        LONGITUDE        HEIGHT GEOID HT  PLANE COORD  ORDER     QID/QSN

12917  SIERRA 1960                             32 50 41.74090  111 46 43.59635   454.9      0.0    042            15    0321114428005

USER-DEFINED RADIUS = 25000.


*****************************************************************************************************************************
SIERRA 1960                    ************************************************************************************ COMPONENT  1
*****************************************************************************************************************************

STATION OBSERVED                 STA OBS  G-NUM   DATE LIST  LN  C71  UESD DIRECTION  SE      RESID*L   SIGMA V RESID*L/ ADJ DIRECTION
                                COND CODE                                             (SEC)                     SIGMA V
                                                    DECKNAME = AZHSGILA
GRAVEL 1960                         3    12197  046960  1    631  V      0  0  0.0     -        0.0      0.000              0  0  0.0
SIERRA NORTH BASE 1960              3    12197  046960  1    632  V      8  2 49.60    -        0.0      0.000              8  2 49.590
CASA GRANDE 2 1960                  2    12197  046960  1    633  V     13 14  5.90   1.0       0.0      0.000             13 14  5.890
SIERRA SOUTH BASE 1960              3    12197  046960  1    634  V     48 27 17.80    -        0.0      0.000             48 27 17.790
CHUI 1936                           2    12197  046960  1    635  V     82 32 41.80   1.0       0.0      0.000             82 32 41.790
BIANCO 1960                         2    12197  046960  1    636  -    148 36 11.50   1.0       0.0      0.000            148 36 11.500

                                                    DECKNAME = AZHSGILA
GRAVEL 1960                         3    12197  046960  2    637  V      0  0  0.0    2.0     -0.095     0.084  -1.138
SIERRA 1960 RM 1                    R    12197  046960  2    638  R    189 23 37.50    -      37.320    57.950   0.644  189 24 14.915
SIERRA 1960 RM 2                    R    12197  046960  2    639  R    249  7 53.50    -      48.820    57.950   0.842  249  8 42.415

                                                    DECKNAME = AZHSGILA
GRAVEL 1960                         3    12197  046960  3    640  V      0  0  0.0    2.0      0.095     0.084   1.130
CASA GRANDE MUNICIPAL TANK          4    12197  046960  3    641  V    296 46 59.00   3.5      0.0       0.000          296 46 58.905
CASA GRANDE RAD STA KPIN MAST       4    12197  046960  3    642  V    324 55 17.00   3.5     -0.395     0.347  -1.139  324 55 16.518

                                                    DECKNAME = AZHSGILA
GRAVEL 1960                         3    12197  046960  4    643  V      0  0  0.0     -        0.0      0.000
BENCH MARK Z 92 1936                2    12197  046960  4    644  V    196 29 26.50   1.0       0.0      0.000          196 29 26.500

                                                    DECKNAME = MMAZ
CASA GRANDE RAD STA KPIN MAST       4    10749  101973  1   1841  V      0  0  0.0     -        0.317    0.278   1.139
SIERRA 1960 RM 1                    R    10749  101973  1   1842  R    224 39 28.30    -      -89.568   57.950  -1.546
SIERRA 1960 RM 2                    R    10749  101973  1   1843  R    294 13 22.80    -        3.432   57.950   0.059
SIERRA 1960 RM 3                    Z    10749  101973  1   1844  Z    308  3 57.80    -        0.0      0.000          272 59 13.983

                                                    DECKNAME = TTONMAZ
BIANCO 2 1971                       1    14589  076971  1   1105  A      0  0  0.0    0.7      0.0      0.000          147 24 38.374
SIERRA 1960 RM 1                    R    14589  076971  1   1106  R     41 56 44.30    -      52.251    43.904   1.190
CASA GRANDE ATT MICROWAVE TWR       4    14589  076971  1   1107  V     49 55 58.90    -       0.001     0.000          196 20 37.265
SIERRA 1960 RM 2                    R    14589  076971  1   1108  R    101 44 56.30    -     -52.249    43.904  -1.190
CASA GRANDE MUNICIPAL TANK          4    14589  076971  1   1109  V    149 21  7.20    -       0.001     0.000          296 45 45.565
BIANCO 2 RM A 1971                  1    14589  076971  1   1110  V      0 14 35.00   0.7      0.0       0.000          147 39 13.374


                          VARIANCE OF UNIT WEIGHT FOR     3 DEGREES OF FREEDOM =    1.60631
                          TOTAL NO. OF NON-SPUR OBS =    1?
```

Figure 10.18. Station combination from STADJUST.

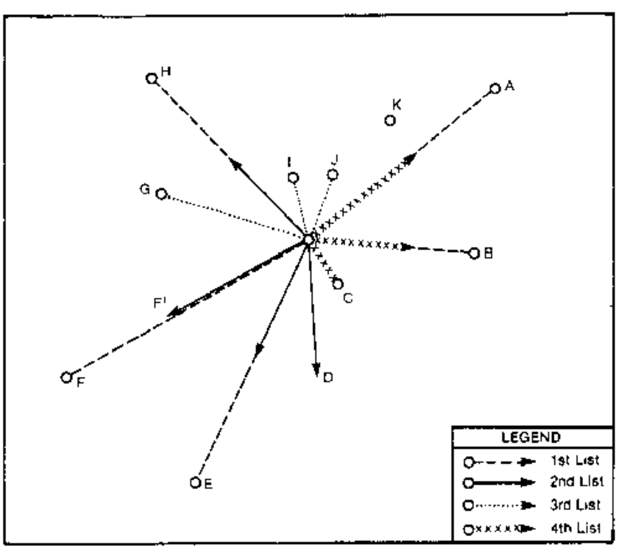

Figure 10.19. Station adjustment validation.

forepoint. A second possible computed direction match occurred with a data base station inside the user-defined radius to which the direction was not coded.

The difference between the combined, observed, and computed direction was also printed. Differences greater than 10 seconds with a linear error greater than 0.5 m were flagged for investigation.

The last value printed was the elevation difference between the standpoint and forepoint. If the magnitude of the sight angle was greater than 5 degrees, then the difference was flagged for investigation.

Many data problems were revealed by this listing. Inconsistent spelling of the names of unpositioned reference and azimuth marks may have caused a mark to be treated as two different entities. This could be resolved by a simple name change.

A frequent situation involved a match between a direction that had been coded with a "N" or a "U" and a data base computed direction. Many directions that had been miscoded were rescued by this process.

Directions were flagged by the program as possible matches with the data base using the following formula:

Tolerance (in arc seconds) = (0.5)(206265)/(distance in meters).

Verification of the identification of a miscoded or unidentified observation was based on similarity of names, descriptions and, if necessary, previous NAD 27 adjustments.

Another problem resolved was a large difference between the combined adjusted direction and the computed direction. Differences greater than 10 seconds with linear errors greater than 0.5 m were flagged. Rejections were made only if a direction was clearly misidentified. Otherwise, the computed-observed terms generated in the least squares adjustment of the Trav-deck for the project, the value of the geodetic position, and the coding of the observations were investigated. If a large difference in elevation was flagged, the DRAGNET printout was investigated for a possible error in the elevation passed to the RESTART file.

The next section of STADJUST listed all the distances observed from the station and compared them to the inversed distances. As with the directions, there were two types of computed (inversed) values which could be generated. The first was an inversed distance using the GP identified with the observation; the second, if found, was one within the user defined radius which matched the observation within 0.1 m but was not identified with it. Any such distance found was flagged. The difference between the observed and inversed value was flagged if it exceeded 0.5 m. Large differences might occur if elevations were incorrect or distances had been incorrectly reduced to sea level.

Also printed were stations within a given distance of the occupied station (usually 100 m) that had no observations and therefore could potentially indicate missing data. (See fig. 10.20.) If no direct observations were made, a search was made for a commonly observed nearby mark (often a reference mark) which, when positioned, could provide a tie between the neighboring stations.

Corrections resulting from the station adjustment analysis were made to the Trav-decks and to the data base as necessary. The steps to be rerun were determined by which files were affected by the changes, e.g., a data base change necessitated reretrieving a data base deck, rerunning DRAGNET, and recreating a RESTART file. Station adjustments were reran, however, only on those stations that needed corrections. A special program to edit the RESTART file was used for limited types of changes and generally only used for the least squares adjustment rejections and standard error changes.

A station report program, STREPORT, listed the observations and all associated data at a particular station in an easily viewed format. (See fig. 10.21.)

10.4.5 Earthquake Area Analysis

Some special considerations were necessary for the analysis of blocks overlapping earthquake areas. Prior to the NAD 83 adjustment, stations whose position changed because of crustal motion had a value published for each epoch that could be identified from the observations available. This frequently resulted in several geodetic positions associated with each station. Since in the NAD 83 adjustment only one position would be associated with each point (corrections would be applied to the observations to put them all in the same time epoch), numerous positions had to be removed from the data base.

In general four types of GPs might be associated with a station in an earthquake area:

1. pre-earthquake
2. post-earthquake
3. pre-earthquake constrained
4. post-earthquake constrained

It was desirable to keep the latest position (post-earthquake constrained if available), so the research centered on identifying these positions and then identifying all the positions that should have non-adjustability flags set. Because the shift was often greater than the default tolerance for clustering GPs in DRAGNET, additional care had to be taken to identify all positions either by increasing the tolerance for the cluster or by manually identifying all positions by name.

10.4.6 Least Squares Adjustment Analysis

In the final analysis step, the least squares adjustment program NEMO was run along with the post-processor program POSTPROC. In contrast to the TRAV10 least squares adjustment program, which had been used in the analysis of individual projects, NEMO used a height-controlled three-dimensional model (as would subsequently be used in the Helmert block adjustment of the entire continent). The RESTART file format was its input.

A major purpose of the NEMO adjustment was to ensure that the interior of the block held together as a network. (See fig. 10.22.) The "observational summary" section of the printout listed the number of "from" and "to" observations for each station. If too few observations were present to position the point, it was flagged with a "U" (for undetermined) in this section. A search was then made for additional observations by checking the original computations of the position for the point. If no additional observations

STATIONS WITHIN 100. METERS OF THIS STATION
*MEANS NO DISTANCE OBSERVATION

JOB	STATION NAME	LATITUDE	LONGITUDE	HEIGHT	GEOID HEIGHT	PLANE COORDINATE	ORDER	QID/QSN
16056	AF 48 AMS ECC 1966	32 49 15.34683	111 43 16.24860	479.86	0.0	042	15	0321114130061

Figure 10.20. Sample list showing possible missing connections between closeby stations.

```
                                                                                            JULY       30, 1985
STATION REPORT                                                                                                          STATION  532

 JOB   STATION NAME                         LATITUDE         LONGITUDE        HEIGHT GEOID HT  PLANE COORD  ORDER    QID/QSN

16614  QUINCY DOPPLER STA 51213         39 58 23.64324  120 56 24.80290  1083.91      0.0    061            TT    0391204440013  IN

GRAVIMETRIC RECORD
     0                                          39 58 31.47000  120 56 17.84000    -24.5210

DIRECTIONS
 DATE    JOB      C  TS#    TO-STATION NAME                   COND   LN  V    OBSERVATION      CM       SE          DK          LN

241978 15859     1   503   ARGENTINE 1949                            1  -      0  0  0.00    0.127   0.6       QUINCY01
241978 15859     4   504   ARGENTINE ROCK LOOKOUT 1949                  -      0  9  4.20    0.128    -        QUINCY01
241978 15859     1   529   CLAREMONT USGS 1949                          -     76 42 32.90   -0.020   1.0       QUINCY01
241978 15859     4  1233   MOUNT HOUGH LOOKOUT TOWER 1949               -    291 35 13.90    0.010    -        QUINCY01
241978 15859     1   531   QUINCY STA 7051 1974                         -    327 35 29.10    0.115   0.8       QUINCY01
241978 15859     1   530   QUINCY STA 7051 ECC 1974                     -    354 19 39.17    0.121   0.8       QUINCY01

323979 15859     1   503   ARGENTINE 1949                            2  -      0  0  0.00    0.098   0.6       QUINCY01
323979 15859     4   504   ARGENTINE ROCK LOOKOUT 1949                  -      0  9  3.11    0.098   2.1       QUINCY01
323979 15859     1   529   CLAREMONT USGS 1949                          -     76 42 31.38   -0.015   0.7       QUINCY01
323979 15859     1   533   QUINCY ARIES 1979                            -    169 14 20.58    0.089   0.8       QUINCY01
323979 15859     4  1233   MOUNT HOUGH LOOKOUT TOWER 1949               -    291 35 14.52    0.008    -        QUINCY01
323979 15859     1   531   QUINCY STA 7051 1974                         -    327 35 27.76    0.088   0.8       QUINCY01
323979 15859     1   530   QUINCY STA 7051 ECC 1974                     -    354 19 40.81    0.093   0.8       QUINCY01

324979 15859     1   503   ARGENTINE 1949                            3  -      0  0  0.00    0.098   0.8       QUINCY01
324979 15859     1   533   QUINCY ARIES 1979                            -    169 14 26.06    0.089   0.8       QUINCY01
324979 15859     1   531   QUINCY STA 7051 1974                         -    327 35 29.24    0.088   0.8       QUINCY01
324979 15859     1   530   QUINCY STA 7051 ECC 1974                     -    354 19 36.00    0.093   0.8       QUINCY01

DISTANCES
 DATE    JOB      C  TS#    TO-STATION NAME                   COND            OBSERVATION      CM       SE          DK          LN

324979 15859     T   533   QUINCY ARIES 1979                                       34.704              1.4 1.0  QUINCY01
322979 15859     X   533   QUINCY ARIES 1979                                       34.719              5.0 9.9  QUINCY01
322979 15859     X   533   QUINCY ARIES 1979                                       34.721              5.0 9.9  QUINCY01
241978 15859     X   530   QUINCY STA 7051 ECC 1974                                60.966             17.0 1.0  QUINCY01
241978 15859     X   531   QUINCY STA 7051 1974                                    65.454             17.0 1.0  QUINCY01
```

Figure 10.21. Sample STREPORT listing.
(LN—list No., CM—crustal motion, SE—standard error, DK—Trav-deck name)

```
***OBSERVATIONAL SUMMARY

RFSN ELIM  CMP            NAME                   DIR     AZI DIS     RFSN ELIM  CMP            NAME                   DIR     AZI DIS
                                               FRM  TO                                                              FRM  TO

   1    1    1 728006                            [illegible]  0  0  0 *U      2   28    2 CASTLE IBC 1909            12  11   0   0 *
   3   20    2 CONE SHAPED PEAK                  [illegible]  6  0  0 *       4   19    2 PINNACLE                    4   4   0   0 *
   5   39    2 D 1 OF THE BOUNDARY               3  8  0  0 *                 6   30    2 MONUMENT 102 1909           5   5   0   0 *
   7   32    2 C 1 OF THE BOUNDARY               9  7  0  0 *                 8   22    2 MONUMENT 103 1909           6   6   0   0 *
   9   31    2 BARNEY IBC 1909                   7  7  0  0 *                10   37    2 BARNEY 2 IBC 1974           6   4   0   0 *
  11  324    3 SHARP PEAK E OF BACK              [illegible]  0  0  0 *U     12   39    2 GAME IBC 1909               8   8   0   0 *
  13   52    2 EAST                              [illegible]  6  0  0 *      14  323    4 HIGHEST PINNACLE W OF GRUB  0   0   0   0 *U
  15   53    2 TALUS                             4  4  0  0 *                16   54    2 MONUMENT 100 1909           3   1   0   0 *
  17   33    2 MONUMENT 101 1909                 4  4  0  0 *                18   49    2 GRUB IBC 1909               9   7   0   0 *
  19   42    2 GRUB 2 IBC 1974                   2  4  0  1 *                20   44    2 BACK IBC 1909              15  13   0   0 *
  21   40    2 BACK 2 1974                       4  4  0  0 *                22   47    2 SLIDE IBC 1909              9   7   0   0 *
  23   41    2 SLIDE 2 IBC 1974                  4  2  0  0 *                24   70    2 LOST IBC 1910               7  10   0   0 *
  25  326    5 HIGH ROCKY POINT                  [illegible]  0  0  0 *U     26   59    2 LIME IBC 1910               6   7   0   1 *
  27   56    2 VIEW NE IBC 1909                  9 12  0  0 *                28   58    2 MONUMENT 98 1910            0   2   0   0 *N
```

Figure 10.22. Sample of original printout of NEMO observational summary.

were located, then the station would be deleted from the adjustment by setting the appropriate flag in the data base.

Because positions could not be deleted from the RESTART file, deleting a station was somewhat cumbersome. It was necessary to delete the GP from any Trav-deck in which it appeared, code the observations to it with a "N" or "U" (indicating that it had no associated position), set the flag in the data base to nonadjustable, and rerun all steps necessary to recreate the RESTART file.

NEMO provided a list of the specific connections between all points in the block. (See fig. 10.23.) Each point's internal station number was listed followed by all those stations to which it was directly connected. This was especially useful in determining all observations to a particular interior point when investigating large residuals.

NEMO performed a solution holding junction points fixed. This section printed the numbers of singular (or nearly singular) interior unknowns plus all stations having observations to or from these points. If the stations which were originally used to determine a singular station could be identified, then figure 10.23 could be used to find out which observations were coded and which were not. To resolve the singularity, it was necessary to either locate additional connections or to reclassify the point as unadjustable. Singularities did not prevent NEMO from running to completion, however. The coordinates of such stations were fixed at the input values and the process continued.

```
NEIGHBORS OF ALL POINTS

<<<+1>>><<<+2>>>+3+7+8+9+10+71+72+73+75-284-295-288<<<+3>>>+2+71-284-288-289-297<<<+4>>>+71+75-283-284-288<<<+5>>>+6+7+9+17+20+22+29
+73+76<<<+6>>>+5+9+12+75+76<<<+7>>>+2+5+5+20+76-282-284-285-296<<<+8>>>+2+7+71+75-284-288<<<+9>>>+2+5+6+12+20+73+76<<<+10>>>+2+12+21
+72+75+77<<<+11>>><<<+12>>>+6+9+10+20+21+69+76+77<<<+13>>>+15+16+18+22+27+29+68<<<+14>>><<<+15>>>+13+16+27+68-295-296<<<+16>>>+13+15
+68<<<+17>>>+5+20+75+76<<<+18>>>+13+20+22+27+32+68+70-295-296<<<+19>>>+21+23+27-295-296<<<+20>>>+5+7+9+12+17+18+22+27+29+32+69+70+75
+76-296<<<+21>>>+10+12+19+23+69<<<+22>>>+5+13+18+20+29+32+69+70-296<<<+23>>>+19+21+69-296<<<+24>>>+26+30+34+40+42+43+44+64+65+67
<<<+25>>><<<+26>>>+24+27+30+34+65+67-295<<<+27>>>+13+15+18+19+20+26+29+30+57+68-295-296<<<+28>>>+30+67<<<+29>>>+5+13+20+22+27+30+32
+69+70<<<+30>>>+24+26+27+28+29+31+34+35+37+43+44+65+67<<<+31>>>+30<<<+32>>>+18+20+22+29+69-296<<<+33>>>+34+65<<<+34>>>+24+26+30+33
+35+37+65+67<<<+35>>>+30+34+37+38<<<+36>>>+37+43+64<<<+37>>>+30+34+35+36+38+43+64+65+67<<<+38>>>+35+37+65<<<+39>>><<<+40>>>+24+41+44
+63+64+65<<<+41>>>+40+42+44+46+49+51+62+63-292-293<<<+42>>>+24+41+43+44-293<<<+43>>>+24+30+36+37+42+44+64+65<<<+44>>>+24+30+40+41+42
+43+45+48+49+54+59+63+65-292-293<<<+45>>>+44<<<+46>>>+41+51+54-292-293<<<+47>>>+48+49+63<<<+48>>>+44+47+49+63-293<<<+49>>>+41+44+47
+48+59+63-293<<<+50>>><<<+51>>>+41+46+54+55+56+59-292-293-294<<<+52>>>+55+59-292<<<+53>>>+54+59<<<+54>>>+44+46+51+53+59-293<<<+55>>>
+51+52+59<<<+56>>>+51+61+80+81+83+123-292-294-311<<<+57>>>+58+85+123<<<+58>>>+57+59+80+85+123-294<<<+59>>>+44+49+51+52+53+54+55+58
+80+81+83+85+123-292-293-294-311<<<+60>>><<<+61>>>+56-292-294<<<+62>>>+41+63-292-293<<<+63>>>+40+41+44+47+48+49+62+64-293<<<+64>>>
```

Figure 10.23. Sample list of connections between block points.

Observations were listed when the value of the computed-minus-observed term exceeded 30 seconds or the associated linear error exceeded 5 m. (See fig. 10.24.) For convenience sake, these differences are called "misclosures." Each such term was compared with the value from the Trav-deck adjustment in which it appeared. If the same term appeared there and was resolved in the adjustment (a sign of only a poor starting position), no action was taken. If new misclosures appeared, then additional investigation was necessary to find the cause—usually in changes to observations or positions during the block validation analysis. Large misclosures for no-check observations were usually not investigated.

If the maximum position shift was less than 0.03 m and there were no excessive misclosures, the solution was considered to have converged. (See fig. 10.25.) (This criterion was adopted in January 1984; previously the criteria had been 0.003 m.)

If the adjustment did not converge then the reason was investigated. The problem may have been large position shifts at some stations, which would require more iterations to meet the convergence criteria. These poor preliminary positions would be recomputed, the data base updated, and a new RESTART file created. (Positions in the RESTART file could not be changed, deleted, or added.)

Nonconvergence also occurred when a very weakly determined station was held fixed and the reordering routine placed this station last in the order of elimination. In this fairly rare case, manually fixing another station would result in the adjustment meeting the convergence criterion in two iterations.

Large elevation differences between nearby stations (e.g., two stations 3 m apart but differing in elevation by 10 m) could also cause the solution not to converge.

When a large number of positions in the block were poorly determined, more than two iterations might be needed to achieve convergence. In this case the position shifts from one run of the program could be retained to temporarily update the positions for the next run.

The solution might also diverge—i.e., the shifts in position might increase from one iteration to the next. When this occurred, the cause was usually due to incongruous observations. A common example of this situation occurred when the observations to two nearby intersection stations were reversed.

***ITERATION [illegible]

***TROUBLESOME OBSERVATIONS

FROM	TO	CC1	IDENTIFICATION	FROM	TO	LIST	OBS	C-O	DIST	LINEAR
2	286	2	16974	XXX909 CASTLE IBC 1909	CHIEF IBC 1909	01	025170969	-36.76	10375.355	1.8
2	3	4	16974	XXX909 CASTLE IBC 1909	CONE SHAPED PEAK	01	069225300	-131.76	5208.915	3.3
2	8	3	16974	XXX909 CASTLE IBC 1909	MONUMENT 103 1909	01	086461000	-33.73	6241.187	1.0
2	7	3	16974	XXX909 CASTLE IBC 1909	C 1 OF THE BOUNDARY	01	108105509	65.81	8261.243	2.6
2	9	2	16974	XXX909 CASTLE IBC 1909	BARNEY IBC 1909	01	151345929	188.83	10131.816	9.3
2	75	2	16974	XXX909 CASTLE IBC 1909	SKOOK IBC 1974	01	099085719	52.46	11139.579	2.8
2	73	2	16974	XXX909 CASTLE IBC 1909	SQUAW IBC 1909	01	104472719	55.06	8296.048	2.2
2	75	2	16974	XXX974 CASTLE IBC 1909	SKOOK IBC 1974	02	099085100	58.66	11139.579	3.2
2	10	3	16974	XXX974 CASTLE IBC 1909	BARNEY 2 IBC 1974	02	151345189	110.63	10137.928	5.4
2	72	3	16974	XXX974 CASTLE IBC 1909	SQUAW 2 IBC 1974	02	104473939	53.30	8295.776	2.1
2	72	3	16974	XXX974 CASTLE IBC 1909	SQUAW 2 IBC 1974	03	104473719	55.50	8295.776	2.2
2	75	2	16974	XXX974 CASTLE IBC 1909	SKOOK IBC 1974	03	099085119	58.46	11139.579	3.2
2	75	2	15367	178974 CASTLE IBC 1909	SKOOK IBC 1974	01	099085097	58.68	11139.579	3.2
2	72	2	15367	178974 CASTLE IBC 1909	SQUAW 2 IBC 1974	01	104473941	53.28	8295.776	2.1
2	10	2	15367	178974 CASTLE IBC 1909	BARNEY 2 IBC 1974	01	151345188	110.64	10137.928	5.4
2	75	2	15367	178974 CASTLE IBC 1909	SKOOK IBC 1974	02	099085121	58.44	11139.579	3.2
2	72	2	15367	178974 CASTLE IBC 1909	SQUAW 2 IBC 1974	02	104473720	55.49	8295.776	2.2
2	10	2	15367	178974 CASTLE IBC 1909	BARNEY 2 IBC 1974	02	151345150	111.03	10137.928	5.5
4	75	3	16974	XXX909 PINNACLE	SKOOK IBC 1974	01	151513409	-39.89	7086.832	1.4
7	2	3	16974	XXX909 C 1 OF THE BOUNDARY	CASTLE IBC 1909	01	317025869	-48.71	8261.243	2.0
8	2	3	16974	XXX909 MONUMENT 103 1909	CASTLE IBC 1909	01	278045600	-147.51	6241.187	4.5
9	20	2	16974	XXX909 BARNEY IBC 1909	BACK IBC 1909	01	150172239	-74.56	8255.649	3.0
9	73	2	16974	XXX909 BARNEY IBC 1909	SQUAW IBC 1909	01	053364150	-75.43	7515.024	2.7
9	6	3	16974	XXX909 BARNEY IBC 1909	MONUMENT 102 1909	01	078303300	-75.84	5851.960	2.2
9	76	2	16974	XXX909 BARNEY IBC 1909	HI YU IBC 1909	01	079463150	-75.37	10721.870	3.9

Figure 10.24. Sample of an original printout of troublesome observations using NEMO.

```
472 -   431


***CONVERGENCE CRITERIA

PVV                                 3.40146407E+03
DEGREES OF FREEDOM                            2654
VARIANCE OF UNIT WEIGHT             1.28163680E+00

MAXIMUM POSITION SHIFT (METERS)     2.13787595E-01
ITERATIONS FAILED TO CONVERGE
```

Figure 10.25. Convergence criterion.

Figure 10.26 shows the shifts *dx* and *dy* and the vector shift. These shifts could be used to identify particularly large changes in a position that might represent a misidentification.

The postprocessor program, POSTPROC, printed tables of statistics about groups of observations based on each project and each observation type within that project. (See fig. 10.27.) Statistics tabulated were: 1) mean absolute residual, 2) sum of the weighted residuals squared (PVV), 3) normalized residual, and 4) sample size. This section was intended to be used to identify the need for scaling projects or observations. The person doing the block validation analysis did not routinely use these statistics unless a combination of large variance and small residuals indicated a need to scale an entire project.

All weighted residuals exceeding 2.5 in absolute value were listed in descending order showing type of observation (i.e., distance, direction, or azimuth), "from" and "to" internal station number, the number of the list on which the observation appeared, the name of the Trav-deck in which it appeared, and the value of the normalized residual. (See fig. 10.28.) This listing ensured that all of the largest residuals were resolved. It was often marked with the resolution in each NEMO run for inclusion in the block folder.

POSITION SHIFTS FOR ITERATION 2

RFSN	NAME	DELTA X	DELTA Y	VECTOR	
0001	CATALINA 1910	7.42410498E-03	0.00000000E+00	7.42410498E-03	-
0002	BALDY USGS 1910	0.00000000E+00	0.00000000E+00	0.00000000E+00	-
0003	BLACK MOUNTAIN 1920	1.28688331E-02	0.00000000E+00	1.28688331E-02	-
0004	TORTILLA	1.02201332E-02	0.00000000E+00	1.02201332E-02	-
0005	HELMET PEAK MINERAL HILL	-2.06605657E-03	0.00000000E+00	2.06605657E-03	-
0006	WASSON 1920	3.62484248E-03	0.00000000E+00	3.62484248E-03	-
0007	SAMANIEGO USGS 1920	0.00000000E+00	0.00000000E+00	0.00000000E+00	-
0008	STACK 1935	8.79324207E-03	0.00000000E+00	8.79324207E-03	-
0009	ROSKRUGE 1919	7.10265707E-03	-7.35057773E-03	1.02214836E-02	-
0010	WATER 1936	8.29606674E-03	0.00000000E+00	8.29606674E-03	-
0011	PASS 1960	7.10527208E-03	-7.35314529E-03	1.02251473E-02	-
0012	NEWMAN 1935	7.10509236E-03	-7.35369272E-03	1.02254161E-02	-
0013	BIG MOUNTAIN	7.10804289E-03	-7.35104820E-03	1.02255652E-02	-
0014	PICACHO PEAK	2.50440117E-02	0.00000000E+00	2.50440117E-02	-
0015	AIRWAY BEACON ON PICACHO PEAK	5.09716233E-03	0.00000000E+00	5.09716233E-03	-
0016	POSTEN 1935	7.10494905E-03	-7.35323451E-03	1.02250188E-02	-
0017	SASCO 1935	-3.32649629E-03	0.00000000E+00	3.32649629E-03	-
0018	PICACHO 1935	4.42928954E-03	0.00000000E+00	4.42928954E-03	-
0019	HOLE 1935	1.19139874E-02	0.00000000E+00	1.19139874E-02	-
0020	MEN AZDT 1978	5.10473457E-03	0.00000000E+00	5.10473457E-03	-
0021	OCOTILLA 1960	1.52635992E-03	0.00000000E+00	1.52635992E-03	-
0022	ELKS AZDT 1975	9.00818660E-03	0.00000000E+00	9.00818660E-03	-

Figure 10.26. Program NEMO showing *dx* and *dy* coordinate shifts and vector shift.

RESIDUALS ANALYSIS OF POSTPRT FILE MARCH 02, 1984

[illegible] NORMALIZED V

GNUM	1	2	3	4,6,Z	[illegible]	[illegible]	J	C,G,S,X	F,M,T,U	E,Y	ALL
A3371	0.000	0.000	0.000	0.000	0.207	0.000	0.000	0.000	0.000	0.000	0.207
A4233	0.000	0.000	0.000	0.000	0.407	0.000	0.000	0.000	0.000	0.000	0.407
A4234	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SP110	0.000	0.000	0.000	0.000	0.000	0.213	0.000	0.000	0.000	0.000	0.213
10747	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10749	0.000	0.000	0.220	0.552	0.000	0.000	0.000	0.000	0.223	0.000	0.352
12197	0.000	1.132	1.392	0.816	0.000	0.000	0.000	0.000	0.000	0.000	1.095
12586	0.000	0.000	0.000	0.056	0.000	0.000	0.000	0.000	0.399	0.000	0.399

Figure 10.27. Residual statistics using POSTPROC.

DECEMBER 10, 1984

	F#	T#	LIST	DECK	VSQP
DST	67	295		DATABASE	8.542
DST	67	295		DATABASE	8.479
DST	67	295		DATABASE	8.479
DST	67	26		DATABASE	8.417
DST	67	26		DATABASE	8.385
DST	67	26		DATABASE	8.159
DST	67	26		DATABASE	8.030
DST	234	235		G16974	6.200
DIR	271	272	4	BARDEMAR	5.785
DST	221	226		CANTRV25	5.490
DIR	71	74	2	FAIRCIRC	-5.125
DST	251	344		CANTRV25	4.415
DST	77	38		G16974	4.266
DST	261	344		CANTRV25	4.067
DST	67	84		G16974	4.000
DST	251	344		CANTRV25	3.652
DIR	165	170	1	G16974	2.625
DST	251	344		G16974	2.597

Figure 10.28. List of highest residuals in POSTPROC.

The main portion of the POSTPROC printout listed all observations and residuals for each station that had one or more observations with a weighted (normalized) residual greater than 2.5 seconds or a rejected observation. (See fig. 10.29.)

The following criteria were used to reduce the number of normalized residuals greater than 3.0 seconds:

1. If a collinear observation existed which was not used only for orientation, the observation was rejected.
2. If no such collinear observation existed, the standard error was doubled until the normalized residual was under 3.0 seconds.
3. If a direction observation to an intersection station had a high normalized residual, it could be rejected if at least three usable observations from different stations were left.
4. If not enough observations existed to meet this criteria, the standard error was doubled as before.
5. If all the residuals to á particular station were large, it usually indicated that the observations were not intersecting correctly and the point might have to be removed. If the observations fell into two groups several years apart, the station may have been rebuilt. In this case only the latest observations were kept.
6. Normalized residuals above 3.0 seconds on the length data set or astronomic observations required a data base update for resolution. The analysis and criteria used for determining the changes to these observations were the same as for the other observations. However, after the updates were made to the data base, all the block validation procedures had to be rerun leading to the creation of a new RESTART file.

In general, throughout these procedures if two sources of data were inconsistent and no clear preference for one or the other could be found, then the newer source was kept. Extensive logs documented all investigations and changes. In addition, changes that affected the descriptions and recovery notes were forwarded to the National Geodetic Information Center for data base editing.

RESIDUALS ANALYSIS OF RESTART FILE MARCH 02, 1984

ALL RESIDUALS AT STATIONS ASSOCIATED WITH WEIGHTED RESIDUALS GREATER THAN 2.5

YUMADIST	437	1	24	431	SILVER BELL 1919	XXX920	SIERRA PRIETA 1920	0.510	0.669	0.478	
YUMADIST	438	1	24	3	SILVER BELL 1919	XXX920	BLACK MOUNTAIN 1920	0.510	-1.139	-0.814	
YUMADIST	439	1	24	6	SILVER BELL 1919	XXX920	WASSON 1920	0.510	1.109	0.792	
YUMADIST	440	1	24	9	SILVER BELL 1919	XXX920	ROSKRUGE 1919	0.510	-0.629	-0.449	
YUMADIST	441	2	24	431	SILVER BELL 1919	XXX920	SIERRA PRIETA 1920	2.039	-0.139	-0.199	
YUMADIST	442	2	24	3	SILVER BELL 1919	XXX920	BLACK MOUNTAIN 1920	0.826	0.789	0.718	
YUMADIST	443	2	24	1	SILVER BELL 1919	XXX920	CATALINA 1910	0.390	-0.949	-0.593	
YUMADIST	444	3	24	3	SILVER BELL 1919	XXX920	BLACK MOUNTAIN 1920	0.510	0.029	0.021	
YUMADIST	445	3	24	1	SILVER BELL 1919	XXX920	CATALINA 1910	0.510	-0.029	-0.021	
YUMADIST	446	4	24	3	SILVER BELL 1919	XXX920	BLACK MOUNTAIN 1920	1.561	-0.349	-0.437	
YUMADIST	447	4	24	1	SILVER BELL 1919	XXX920	CATALINA 1910	1.561	0.349	0.437	
YUMADIST	448	5	24	431	SILVER BELL 1919	025920	SIERRA PRIETA 1920	2.039	0.219	0.314	
YUMADIST	449	5	24	156	SILVER BELL 1919	025920	SAWTOOTH	0.107	0.000	0.000	
YUMADIST	450	5	24	209	SILVER BELL 1919	025920	CASA GRANDE MOUNTAIN	0.081	0.000	0.000	
YUMADIST	451	5	24	13	SILVER BELL 1919	025920	BIG MOUNTAIN	0.105	0.000	0.000	
YUMADIST	452	5	24	14	SILVER BELL 1919	025920	PICACHO PEAK	0.080	0.000	0.000	
YUMADIST	454	5	24	3	SILVER BELL 1919	025920	BLACK MOUNTAIN 1920	0.510	0.859	0.614	
YUMADIST	455	5	24	4	SILVER BELL 1919	025920	TORTILLA	0.110	0.000	0.000	
YUMADIST	456	5	24	6	SILVER BELL 1919	025920	WASSON 1920	0.591	0.189	0.146	
YUMADIST	457	5	24	5	SILVER BELL 1919	025920	HELMET PEAK MINERAL HILL	0.110	0.000	0.000	
YUMADIST	459	5	24	9	SILVER BELL 1919	025920	ROSKRUGE 1919	0.591	-1.709	-1.315	
YUMADIST	460	5	24	332	SILVER BELL 1919	025920	MOUNT DEVINE N COMOBABI 1919	0.000	-99.979	0.000	R
YUMADIST	461	5	24	414	SILVER BELL 1919	025920	SANTA ROSA	0.081	0.000	0.000	
YUMADIST	462	5	24	51	SILVER BELL 1919	025920	COMOBABI PEAK	0.110	0.000	0.000	
YUMADIST	464	6	24	431	SILVER BELL 1919	XXX920	SIERRA PRIETA 1920	2.039	-0.250	-0.357	
YUMADIST	465	6	24	459	SILVER BELL 1919	XXX920	SOUTH MOUNTAIN 1920	2.039	0.000	0.000	
YUMADIST	466	6	24	53	SILVER BELL 1919	XXX920	KITTS 1920	2.039	0.250	0.357	

Figure 10.29. Sample printout of POSTROC residuals.

10.5 HORIZONTAL DATA ENTRY

With the completion of the analysis of an individual block, the data contained in the RESTART file were ready for data base entry. Only data included in the file were loaded at any given time. Hence, observations in an individual Trav-deck might be split among several blocks and loaded at different times. Because the block boundaries were arbitrary, many observations crossed them and would appear in two blocks. The following rule applied: any such observations appearing in one block must also appear in the other block. Furthermore, the two appearances of such observation records had to be identical, except for the standard error or rejection field. The data base programs insisted on consistency.

Data base observation entry performed two specific checks. First, it verified that the positions in the RESTART file matched those in the data base. Second, it checked those observations that crossed the block boundary. If the neighboring block had already been loaded, then the two sources were cross-checked for consistency. (See fig. 10.30, observations B and C.) Fatal error messages included: "The following observation was expected, but not found in the RESTART file"; and "The following unexpected observation was found in the RESTART file." Warning messages included: "SE or rejection flag differs in the DB and RESTART obs." Information concerning observations to stations in blocks that had not yet been loaded were also accumulated. (See fig. 10.30, observations A and D.) This double entry accounting system localized any errors due to noncontiguous boundaries or inconsistent observation identification.

As explained earlier, additional positional information also resulted from the block validation analysis. The positional accuracy information of the best order of the station and the types of observations (as reflected in the winning "order" and "type" of the station) used to best determine the position were updated. New elevations were also obtained from the information in the observational data.

The observation loading took place from January 1983 to April 1985. Many of the 843 blocks were loaded with no problems. Only 327 needed corrections made to the previously loaded observations or to themselves. Most of the changes were made to the last blocks loaded as they were generally in the more congested areas of the country.

Upon completion of the loading procedure, a program was run to migrate all the Trav-decks back to tape. All the on-line files (including the RESTART files) were then scratched.

The block validation log folders were archived, since it was assumed that the results of the analysis would be relevant to any future use of the data. There was a great deal of variation in the amount of work required to validate an individual block. For some blocks, many actions had been taken and the folders were quite large.

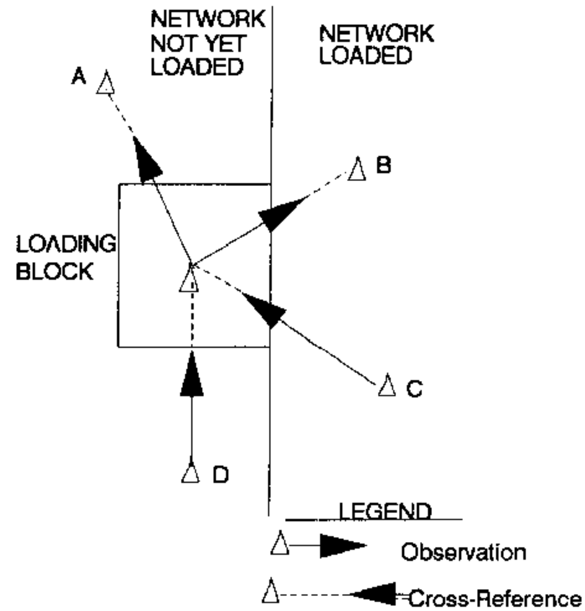

Figure 10.30. Data base observation entry validation.

10.7 REFERENCES

Horizontal Network Branch, 1984: "Block Validation Guidelines." National Geodetic Survey, NOAA, Rockville, MD 20852, 32 pp.

Milbert, Dennis G., 1981: "Validation of Horizontal Observations for the National Geodetic Survey Data Base." *Proceedings of the Symposium on Management of Geodetic Data*, Copenhagen, Denmark. Geodetic Institute of Denmark, pp. 122-133.

Timmerman, Edward L., 1978: A "Test of the North American Datum Horizontal New Adjustment System." *Proceedings of the Second International Symposium on Problems Related to the Redefinition of the North American Geodetic Networks*, Arlington, VA, April 21-24. National Geodetic Information Branch, NOAA, Rockville, MD, pp. 447-455.

Wade, Elizabeth B., 1982: "The Horizontal Block Validation Phase of the North American Datum Project." *Proceedings of the American Congress on Surveying and Mapping and American Society of Photogrammetry*, Denver, CO, pp. 381-410.

APPENDIX 10.A
RESTART FILE STRUCTURE

1 **BLKNAM: Block Name** A9 format
An arbitrary name used by the block assignment program BLOCKOUT to identify the output files from DRAGNET and to identify the data base input deck.

2 **DATUM: Datum Used** I9 format
The following datum codes are defined:
- NAD 27
- PNAD83
- MR78
- NAD 83

3 **SE: Table of default standard errors to be used when the standard error is not given explicitly. Inserted by CREAPROC.**

122	SECODE - ccl observation code	A1 format
123	SESE1 - components needed to compute default	F4.1 format
124	SESE2 - standard errors as in TRAV10	F5.3 format

4 **PROJ: Project Name** A9 format

5 **BL: Record Containing Block Boundary Information**

6	BLBD1 - minimum degrees of latitude	I3 format
7	BLBM1 - minimum minutes of latitude	I2 format
8	BLBS1 - minimum seconds of latitude	I2 format
9	BLLD1 - minimum degrees of longitude	I3 format
10	BLLM1 - minimum minutes of longitude	I2 format
11	BLLS1 - minimum seconds of longitude	I2 format
12	BLBD2 - maximum degrees of latitude	I3 format
13	BLBM2 - maximum minutes of latitude	I2 format
14	BLBS2 - maximum seconds of latitude	I2 format
15	BLLD2 - maximum degrees of longitude	I3 format
16	BLLM2 - maximum minutes of longitude	I2 format
17	BLLS2 - maximum seconds of longitude	I2 format

25 **ST: Station Record**

26	STGN - accession number	A5 format
27	STNAME - station name	A32 format
28	STBD - degrees latitude	I3 format
29	STBM - minutes latitude	I2 format
30	STBS - seconds latitude	F8.5 format
31	STLD - degrees longitude	I3 format
32	STLM - minutes longitude	I2 format
33	STLS - seconds longitude	F8.5 format
34	STGHT - geoid height	F5.1 format
35	STHTCD - height code	I1 format
36	STHT - elevation	F7.2 format
37	STZ1 - plane coordinate zone 1	A3 format
38	STZ2 - plane coordinate zone 2	A3 format
39	STZ3 - plane coordinate zone 3	A3 format
40	STQID - QID (quad identifier)	A9 format
41	STQSN - QSN (quad sequence #)	A4 format
42	STOT - station order type	A2 format
421	STIN - interior station	I1 format
422	STJUN - junction station	I1 format
44	STISN - internal station number	I4 format
45	STBSE - standard error of latitude	F8.5 format
46	STLSE - standard error of longitude	F8.5 format
47	STBLCV - covariance of latitude and longitude	F13.10 format

48 **STSHFT: Record of Adjusted Positions**

49	STITER - number of iterations	I2 format
50	STDB - latitude shift-seconds	F8.5 format
51	STDL - longitude shift-seconds	F8.5 format

56 AP: Record Containing Astro Positions

(If both astro positions and gravimetric positions exist, they should match to within 0.2″)

57	APCODE - ccl code	A1 format
58	APG - accession number	A5 format
581	APPD - degrees of latitude	I3 format
59	APPM - minutes of latitude	I2 format
60	APPS - seconds of latitude	F8.5 format
61	APED - degrees of longitude	I3 format
62	APEM - minutes of longitude	I2 format
63	APES - seconds of longitude	F8.5 format
64	APST - state code	A2 format

113 GR: Record Containing Gravimetric Information

(If both astro positions and gravimetric positions exist, they should match to within 0.2″)

114	GRCODE- ccl code	A1 format
115	GRPD - degrees of latitude	I3 format
116	GRPM - minutes of latitude	I2 format
117	GRPS - seconds of latitude	F8.5 format
118	GRED - degrees of longitude	I3 format
119	GREM - minutes of longitude	I2 format
120	GRES - seconds of longitude	F8.5 format
121	GRGHT - geoid height	F9.3 format

65 AB: Record of Abstracts

66	ABG - accession number	A5 format
67	ABDATE - date	A6 format
670	ABCNT - # of usable obs. on abstract	I3 format
673	ABDK - TRAV deck name	A9 format
68	ABLIST - list number	I2 format

69 DR: Record of Direction Observations

70	DRCODE - ccl code	A1 format
71	DRSE - standard error	F3.1 format
72	DRVIS - visibility code	A1 format
73	DRD - degrees	I3 format
74	DRM - minutes	I2 format
75	DRS - seconds	F5.2 format
751	DRCM - crustal motion correction	F7.3 format
81	DRNUM2 - internal "to" station # (zero if nonpositional)	I4 format
82	DRNAM2 - name of "to" station	A30 format
821	DRSTCD - status code (cc66, U,N, or null)	A1 format
83	DRREJ - rejection code	I1 format
	1-rejected	
	0-not rejected	
831	DRV - residual	F5.2 format
832	DRVSE - standard error of residual	F5.2 format

84 AZ: Record of Azimuths

85	AZCODE - ccl code	A1 format
86	AZG - accession number	A5 format
87	AZDATE - date	A6 format
88	AZSE - standard error	F3.1 format
89	AZD - degrees	I3 format
90	AZM - minutes	I2 format
91	AZS - seconds	F5.2 format
911	AZCM - crustal motion	F7.3 format
95	AZNUM2 - internal "to" station #	I4 format
96	AZNAM2 - name of "to" station	A30 format
961	AZSTCD - status code	A1 format
97	AZREJ - rejection code	I1 format
	1-rejected	
	0-not rejected	
971	AZV - residual	F5.2 format
972	AZVSE - standard error of residual	F5.2 format

98 DS: Record of Distances

99	DSCODE- ccl code	A5 format
100	DSG - accession number	A5 format
101	DSDATE - date	A6 format
102	DSSE1 - standard error	F4.1 format
103	DSSE2 - standard error	F3.1 format
104	DSDIS - distance	F10.3 format
125	DSCM - crustal motion	F7.3 format
106	DSDK - deck name	A9 format
110	DSNUM2 - internal "to" station #	I4 format
111	DSNAME2- "to" station name	A30 format

	165 DSSTCD - status code	A1 format
	112 DSREJ - rejection code 1-rejected 0-not rejected	I1 format
	126 DSV - residual	F10.3 format
	127 DSVSE - standard error of residual	F10.3 format
128	**SU: Record of Supernumerary Observations**	
	129 SUNAME - supernumerary station name	A30 format
130	**SUAB: Record of Supernumerary Abstracts**	
	131 SUABG - accession number	A5 format
	132 SUABDA - date	A6 format
	134 SUABDK - TRAV deck name	A9 format
	135 SUABLI - list number	I2 format
136	**SUDR: Record of Supernumerary Directions**	
	137 SUDRCD - ccl code	A1 format
	138 SUDRSE - standard error	F3.1 format
	139 SUDRVI - visibility code	A1 format
	140 SUDRD - degrees	I3 format
	141 SUDRM - minutes	I2 format
	142 SUDRS - seconds	F5.2 format
	143 SUDRNA - "to" station name	A30 format
	1431 SUDRNU - "to" station number	I4 format
	166 SUDRSC - status code	A1 format
	144 SUDRRE - rejection code 1-rejected 0-not rejected	I1 format
145	**SUAZ: Record of Supernumerary Azimuths**	
	146 SUAZCD - ccl code	A1 format
	147 SUAZG - accession number	A5 format
	148 SUAZDA - date	A6 format
	149 SUAZSE - standard error	F3.1 format
	150 SUAZD - degrees	I3 format
	151 SUAZM - minutes	I2 format
	152 SUAZS - seconds	F5.2 format
	153 SUAZNA - name	A30 format
	1531 SUAZNU - "to" station #	I4 format
	167 SUAZSC - status code	A1 format
	154 SUAZRE - rejection code 1-rejected 0-not rejected	I1 format
155	**SUDS: Record of Supernumerary Distances**	
	156 SUDSCD - ccl code	A1 format
	157 SUDSG - accession number	A5 format
	158 SUDSDA - date	A6 format
	159 SUDSS1 - standard error	F4.1 format
	160 SUDSS2 - standard error	F3.1 format
	161 SUDSDI - distance	F10.3 format
	162 SUDSDK - TRAV deck name	A9 format
	163 SUDSNA - "to" station name	A30 format
	1631 SUDSNU - "to" station number	I4 format
	168 SUDSSC - status code	A1 format
	164 SUDSRE - rejection code 1-rejected 0-not rejected	I1 format

11. DATUM DEFINITION

Bernard H. Chovitz

11.1 INTRODUCTION

A datum may be defined as a set of specifications of a coordinate system for a collection of positions on the Earth's surface. Although the NAD 83 adjustment was primarily concerned with *horizontal* positions (thus involving only two coordinates), many of the supporting observations were three-dimensional in nature (e.g., Doppler, VLBI). Furthermore, a two-dimensional reference surface is naturally embedded in three-dimensional space. Therefore, a three-dimensional coordinate system is assumed.

More specifically, we must define both:

(1) a reference surface to which the latitude and longitude coordinates are referred, and
(2) a three-dimensional Cartesian coordinate system, the origin, orientation, and scale of which must fit the coordinates of physical points in the system.

For both the reference surface and the coordinate system, we must consider:

(1) issues of philosophy or principle (reasons for the chosen attributes of the datum), and
(2) issues of materialization (how these attributes are achieved).

In a hybrid adjustment like the NAD 83, we have several groups of observations, such as terrestrial surveys, Doppler positions, and VLBI baselines. Each group is specified with respect to its own (possibly preliminary) coordinate system. A common new datum is obtained by defining the transformation from each individual coordinate system to the final system.

Because the transformations are in the form of linear least squares solutions (any actual non-linearity being accounted for by iteration), the mathematical form of the transformation between three-dimensional Cartesian coordinate systems is given by:

$$\mathbf{Y} = \mathbf{A} + k\mathbf{R}\mathbf{X} \qquad (11.1)$$

where $\mathbf{Y}$, $\mathbf{A}$, and $\mathbf{X}$ are 3×1 vectors, k is a scalar, and $\mathbf{R}$ is a 3×3 orthogonal rotation matrix. In this equation, $\mathbf{X}$ represents a position on the preliminary datum, $\mathbf{Y}$ is the position on the new datum, $\mathbf{A}$ defines the origin shift, $\mathbf{R}$ defines the change in orientation of the coordinate system, and k defines the change in scale.

Both $\mathbf{X}$ and $\mathbf{Y}$ are assumed to be expressed in Cartesian coordinates. These can be transformed to ellipsoidal latitude, longitude, and height by a transformation that involves the semimajor axis and flattening of the ellipsoid.

Equation (11.1) contains seven parameters—a shift in each coordinate, a rotation around each coordinate axis, and a scale change. Some observational types, such as angles measured by theodolite, are independent of any coordinate system, and are not changed in any way by the transformation (11.1). Consequently, they give no information about the parameters in (11.1). For this type of group, it would be necessary to constrain all parameter corrections to zero.

Some observational types, like Doppler data, are transformed into positional coordinates before being used. In this case the derived coordinates are treated as observations and observation equations are written for them. Such positional observations are dependent on the parameters of the coordinate system in which their numerical values are expressed. For groups containing these observation types, the parameter corrections must be determined in the adjustment.

Still other observational types fall somewhere in between: distances depend only on the scale of the coordinate system, not its origin or orientation; position differences depend on scale and orientation but not on the coordinate system origin.

Table 11.1 shows the parameters that actually appeared in the observation equations used in the NAD 83 adjustment. Of these, three translations, three rotations, and a scale could be arbitrarily chosen, specifying the coordinate system of the adjustment. It was not necessary that all seven parameters arbitrarily chosen belong to the same group.

TABLE 11.1.—*Global parameters*

Parameter	Terrestrial	Doppler	VLBI
X shift		F	
Y shift		F	
Z shift		F	
X rotation		F	A
Y rotation		F	A
Z rotation	F	A	A
Scale	A	F	A

F= parameter fixed at a priori value in the final solution.
A= parameter solved for in the final solution.

No global rotation parameters around the X and Y axes were used for the terrestrial data. The only observational types that could be affected by these parameters would be the astronomic azimuths. It was deemed that these observations were properly referred to the Bureau of International de l'Heure (BIH) pole, so that it would not be useful to carry corrections to these quantities as unknown parameters.

In the last iteration, the fixed parameters were the origin shifts, X and Y rotations and scale for Doppler data, as well as the Z rotation for terrestrial data. Numerical values were obtained for the other parameters at the highest level of the Helmert block adjustment. These are described in chapter 18.

11.2 CHOICE OF ORIGIN

From the earliest discussions, it was proposed that the coordinate system for the NAD 83 datum should have its origin at the center of mass of the Earth. This had been done before only for systems used for special applications and restricted audiences, such as the WGS 72 system used for military applications. It had not previously been carried out for a datum used for civilian surveying and mapping.

11.2.1 Philosophy

Regardless of the value of a geocentric (as opposed to a local) origin, this option was not practical before the satellite era. Afterwards, although favored for satellite tracking, military applications, and other global activities, there was still the question of whether it was the right choice for NAD 83. The documents leading to the funding of the new datum project had recommended and assumed a geocentric origin [see, for instance, National Academy of Sciences/National Academy of Engineering (1971)], and a geocentric origin was implicitly assumed within the NAD project. However, the wisdom of this choice continued to be a matter of discussion for several years. [See, for instance, Baker (1973), Chovitz (1973), Rice (1973), and Moritz (1978).]

The major reason for choosing a geocentric origin was universal compatibility, in particular with the aforementioned military and satellite systems. It was anticipated that eventually many surveyors would use satellite surveying equipment like Global Positioning System (GPS) receivers. In their normal configuration, these instruments produce positions in a geocentric system. Although the computers used with these receivers could be programmed to produce coordinates in a local non-geocentric system, there would always be some uncertainty as to whether this was done correctly. The surveying and mapping community would be best served if there were no chance for confusion.

A major objection to a geocentric coordinate system was that geoid heights would be larger than had been the case for NAD 27. But this is not necessarily a disadvantage of a geocentric system. It is desirable to keep geoid heights small only if one intends to ignore them. If these quantities are known and properly considered in the computations, their size is immaterial. Therefore, the uncertainty of the geoid heights, not their absolute magnitude, is the important consideration. The determination of a detailed geoid to support the NAD 83 adjustment had already been planned. Thus geoid heights could be published with the new NAD 83 coordinates, and surveyors who wished to achieve high accuracy could consider these quantities in their computations.

There was thus a choice between which surveyor was to suffer the potential confusion caused by the switch to a new datum—the one utilizing a satellite survey system or the one attempting to observe and compute highly accurate conventional surveys with theodolites and electronic distance measuring instruments. The burden was placed on the latter, because it was expected that there would eventually be more of the former. The effect on truly local surveys, those confined to a city or county, did not play a role in this decision. The accuracy requirements are so modest, and the effects of geoid heights so limited, that local surveys would not be seriously impacted by either choice.

The introduction of a geocentric origin does not actually worsen the range of geoid heights in North America, but introduces a bias which increases the average value. Coordinate differences between NAD 27 and NAD 83 are sufficiently large to require recasting of the map graticule for some U.S. Geological Survey topographic maps. This inconvenience was known from the beginning and was again pointed out during the last stages of the project. However, compatibility with the new Department of Defense datum, WGS 84, to which GPS would be referred, was the prime factor in proceeding with a geocentric origin.

11.2.2 Materialization

In the late 1960s the Navy Transit system became operational, providing Doppler observations referred to a geocentric origin. The accuracy and convenience of these measurements supplied the means for establishing a geocentric system. In the 1970s, an even more accurate measuring system tied to a geocentric origin—satellite laser ranging (SLR)—was developed and put into operation. Because this latter system contained very few observing stations compared to the Doppler network, it was not feasible to utilize it alone to connect the terrestrial data; the Doppler observations were still essential in this respect. But the relation between Doppler and SLR could be determined on a global basis, and could serve to refine the Doppler geocenter.

The numerical values of the parameters defining NAD 83 had to be selected in 1985, when the highest level of the Helmert Block solution was reached. The most authoritative and universally accepted source at that time was the BIH. Its computations defined the "BIH System" or "BIH Terrestrial System" (BTS). The most current numerical values were those in its 1984 Annual Report (Bureau International de l'Heure, 1985). This listed the difference between the NSWC 9Z-2 Doppler system and the BIH reference system (BTS 84) as (BIH minus NSWC): -0.106 m in X, $+0.697$ m in Y, and $+4.901$ m in Z. These were based largely on SLR observations, but were accepted as authoritative.

Using preliminary values of these figures and rounding, the Defense Mapping Agency (DMA) had already selected 0.0 m in X and Y and $+4.5$ m in Z as the origin shift from NSWC 9Z-2 to WGS 84. The BIH determinations were further refined in later years as shown in table 11.2. Since the errors in these determinations are of the order of decimeters, the rounded values selected by DMA were considered sufficiently accurate. Moreover, compatibility dictated that the same parameters be selected for WGS 84 and NAD 83. Thus the geocentric origin of NAD 83 was defined by adding 4.5 m to the Doppler NSWC 9Z-2 Z coordinates, and using the resultant set of Doppler coordinates in the highest level solution.

TABLE 11.2.—*Datum shift parameters, NSWC 9Z-2 to BTS*

Designation	Shift (in meters) X	Y	Z	Reference
BTS 84	−0.106	0.697	4.901	BIH (1985)
BTS 85	−0.061	0.363	4.732	BIH (1986)
BTS 86	0.167	0.212	4.314	BIH (1987)
IERS 87	−0.071	0.509	4.666	IERS (1988)
Adopted	0	0	4.5	

11.3. REFERENCE SURFACE

11.3.1 Philosophy

The change in NAD 27 coordinates due to a change in ellipsoidal parameters is small compared to the change caused by the origin relocation to the center of mass. Hence, the choice of a new ellipsoid to replace the Clarke 1866 was not controversial.

Special Study Group 5.39 of the International Association of Geodesy (IAG) was established in the mid-1970s to review and recommend fundamental geodetic constants. Its chairman, Helmut Moritz, asked John Bossler, the director of NGS, if NGS would use a new reference ellipsoid were it to be recommended and adopted by IAG. The answer was affirmative, and the desirability of such an action was discussed in (Bossler, 1979). The new standard ellipsoid (along with other parameters) was adopted by the IAG as the Geodetic Reference System 1980 (GRS 80) (Moritz, 1980) replacing the previous reference system GRS 67. The IAG recommended that henceforth GRS 80 be used as an official reference for geodetic work.

11.3.2 Materialization

NGS adopted the GRS 80 fundamental and derived parameters exactly as published by the IAG. DMA, in computing the parameters of the ellipsoid to be used with WGS 84, converted the GRS 80 dynamical form factor (second zonal harmonic of the equipotential ellipsoid) to normalized form and truncated to eight significant digits before computing the flattening of the ellipsoid. This caused the flattening of the two ellipsoids to differ beyond the eighth significant digit and the semiminor axes to differ beyond the 10th significant digit. This discrepancy is negligible for practical purposes.

11.4 ORIENTATION

11.4.1 Philosophy

Since the Doppler station network furnished the bulk of the framework for datum definition, its orientation was the starting point for NAD 83. As in the case of the determination of the origin, the most accurate orientation was judged to be provided by other means, in this instance VLBI and the latest stellar astronomic data. The relationship between Doppler and the latter two was published in the BIH 1984 Annual Report.

The orientation of the pole, represented by rotations around the X and Y axes, is well-constrained, since the pole is a naturally defined physical position. On the other hand, the origin of longitude, represented by rotation around the Z axis, has no such physical tie. One choice is not intrinsically better than another. However, the relationship between differing origins of separate systems must be known in order to connect them. Again, a predominant factor in choosing orientation parameters was to maintain compatibility between WGS 84 and NAD 83.

In a classical adjustment of terrestrial survey data on the ellipsoid, the Laplace equation relating astronomic and geodetic azimuths and longitudes guarantees that the pole of the ellipsoid is parallel to the astronomic pole. In the height-controlled three-dimensional observation equations, the Laplace equation does not appear explicitly; nevertheless the assumption that the axes are parallel is still present, so that the use of at least one (and preferably more) measured astronomic azimuth will ensure that the pole of the ellipsoid is parallel to the astronomic pole.

11.4.2 Materialization

Astronomic azimuths at approximately 5,000 stations were available for the NAD 83 adjustment. Astronomic longitudes were also measured at most of these stations and astronomic latitudes at many of them. These astronomic observations had been taken over a long period of time and were referred to the classical star catalogs. (See chapter 8.)

The astronomic longitudes were based on the adopted longitude of the U.S. Naval Observatory, which provided the time signals used in the measurements. This was presumed to be consistent with the BIH meridian, so that the astronomic longitudes could be taken as the standard. The NAD adjustment contained a parameter to shift the Doppler longitudes to the astronomic longitudes (i.e., rotate around the Z axis to move the intersection of the Doppler X-axis on the equator). However, rotations around the Doppler X and Y axes were constrained to zero.

The adjustment also contained VLBI observations in the VLBI coordinate system. This system is based on extragalactic radio source positions and certain defined connections between these positions and the classical astronomic system (U.S. Naval Observatory, 1983: appendix 12). All three VLBI rotational parameters were determined freely in the adjustment.

In the final solution for NAD 83 there were actually two parameters for the Doppler longitude rotation—one for the determination from U.S. Doppler stations and one for Canadian Doppler stations. The numerical values obtained in the solution were respectively −0.455 and −0.443 second of arc. These two values were judged to be insignificantly different and were averaged to produce a common value of −0.449 second of arc. The longitude rotation from the VLBI system to the astronomic system, obtained in the same solution, was +0.375 second. Table 11.3 shows other numerical values obtained in this solution.

TABLE 11.3.—*NAD 83 parameters obtained in the final solution*

Parameter	Terrestrial	Doppler	VLBI
X shift (meters)	—	0	—
Y shift (meters)	—	0	—
Z shift (meters)	—	4.5	—
X rotation (arc sec)	—	0	0.020
Y rotation (arc sec)	—	0	0.020
Z rotation (arc sec)	0	−0.449	0.375
Scale (part per million)	−0.237	−0.6	−0.075

On the other hand, the BIH Annual Report for 1984 listed the numerical values of the rotations from the individual terrestrial systems to the BIH meridian (in seconds of arc) as: −0.8137 for the NSWC 9Z-2 (Doppler) coordinate system, and −0.0057 for the VLBI system. Thus the determination of the rotation from the Doppler system to the VLBI system was −0.808 from the BIH determination, and −0.824 from the NAD 83 adjustment. These two determinations were judged to be sufficiently close, and thus mutually confirming. However, there was an apparent discrepancy of about −0.365 arc second between the BIH meridian as determined by the BIH and as determined from the astronomical longitudes in the NAD 83 adjustment. This situation is depicted in figure 11.1.

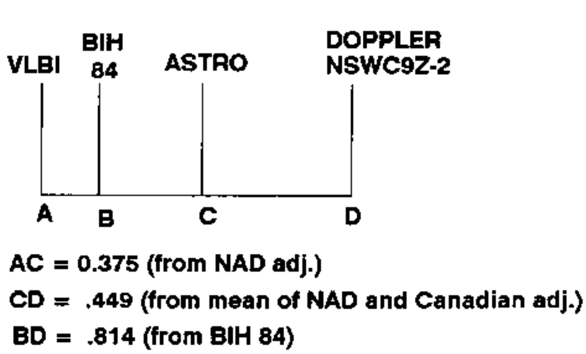

Figure 11.1. Relationships of the meridians.

Elimination of the apparent discrepancy and consistency with the BIH required that all longitudes obtained from the NAD 83 adjustment be further rotated by −0.365 second. NGS proposed this to DMA (Kaula, 1986a) and to the Geodetic Survey of Canada (Kaula, 1986b). Each agreed (Vander Els, 1986; O'Brien, 1986). If this rotation is applied to both the astronomic and geodetic longitudes, there will be no change to the deflection of the vertical in the prime vertical, and thus none to Laplace azimuths. The final NAD 83 parameters, after this conversion, are listed in table 11.4.

TABLE 11.4.—*NAD 83 parameters after correction to BTS-84*

Parameter	Terrestrial	Doppler	VLBI
X shift (meters)	—	0	—
Y shift (meters)	—	0	—
Z shift (meters)	—	4.5	—
X rotation (arc sec)	—	0	0.020
Y rotation (arc sec)	—	0	0.020
Z rotation (arc sec)	−0.365	−0.814	0.010
Scale (part per million)	−0.237	−0.6	−0.075

There was considerable speculation on possible causes of the −0.365 arc second discrepancy between the BIH determination of the longitude origin and the determination from terrestrial data. These conjectures involved sources such as observational and systematic errors in the optical star catalog. In the end, there was no clear explanation; a rotation of −0.814 arc second from the Doppler system to the BTS was simply adopted.

11.5 SCALE

11.5.1 Philosophy

The most accurate determinations of scale are provided by SLR and VLBI. The BIH obtained a scale change of −0.604 ppm of Doppler-derived lengths in comparing them to SLR and VLBI measurements (Bureau International de l'Heure, 1985). DMA decided to adopt the rounded value of −0.6 ppm as the transformation from NSWC 9Z-2 to WGS 84. NGS agreed to the same scale change for its Doppler observations, primarily for reasons of compatibility, but also because the BIH result includes laser ranging and is therefore more comprehensive than the VLBI determination alone.

11.5.2 Materialization

In the first two solutions of the NAD 83 adjustment, a parameter was included for the Doppler scale change, but the VLBI scale was held fixed. The results were −0.65 and −0.53 ppm (mobile VLBI observations were added in the second solution). For the last solution, a scale change of −0.6 ppm was *fixed* for the Doppler positions.

The last solution, unlike the first two, included a parameter for VLBI scale shift: −0.075 ppm. Thus the relation between Doppler and VLBI from the solution is −0.525 ppm, which checked out almost exactly with the previous solution.

Because VLBI scale is more accurate than Doppler, it would have been intrinsically more correct to have proceeded as in the first two solutions: adjust the Doppler scale and hold the VLBI scale fixed. The greatest possible distortion allowed is the change in VLBI lengths of 0.075 ppm. However, this has no real bearing on the accuracy of the NAD 83 positions, because, first, the change induced is much smaller than the general accuracy of the adjustment (about 3 ppm), and, second, a scale shift has negligible effect on horizontal coordinates, which are the only officially published results of NAD 83.

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12. MATHEMATICAL MODELS

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and
C. R. Schwarz

The NAD 83 adjustment was based on the height-controlled three-dimensional model. This chapter summarizes the model, describes how it was actually applied, and discusses some of its important properties.

12.1 GENERAL CONCEPT

The models for the various kinds of observations used in the adjustment are based on the principles of three-dimensional geodesy. All observations are processed mathematically in three-dimensional space, without reduction to a chosen reference ellipsoid. This approach turns out to be simpler to understand and to put into effect. It does not place any restrictions on the lengths of the lines, nor on the extent of the network. It is thus much "cleaner." These features have their particular usefulness when space systems observations are to be combined with terrestrial data in the same adjustment.

The major unknowns in the model are the corrections (shifts) to the coordinates of points, expressed in the geodetic horizon system of each point. At most points, the geodetic height cannot be well determined from the observations. The correction to the height at such stations is therefore constrained to zero (and eliminated as an explicit parameter).

As in all adjustments by variation of parameters, we need some approximate coordinates at all stations. These are transformed to conventional terrestrial (geocentric) coordinates X, Y, Z. The adjustment is iterated until final coordinates are obtained.

12.2 OBSERVABLES

The observables (the quantities used as observations) are divided conceptually into two categories: scalar and vector. The scalar part includes the classical terrestrial obsevations such as astronomic azimuths, unoriented directions, and straight-line distances in space between the ground marks of the points. The vector part comprises Doppler (point positioning), VLBI vectors, and vectors derived from three-dimensional adjustments of terrestrial observations over short lines to connect VLBI sites with the rest of the network. Since most of the vector observations involve satellite or space geodesy, they are often called space systems observations. [For the observables in the Canadian part of NAD 83 and for the methods of their handling see Steeves (1984).]

Certain original observations, which had previously been processed and for which the results were accepted as correct, are not classified here as observables. These include, but are not limited to, spirit leveling, vertical angles, gravimetric measurements (with the attendant geoid heights and deflections), and astronomic latitudes and longitudes.

There had never been any intention of including vertical angles in the NAD 83 adjustment as observations. The task of assessing the quality of the vertical angles was judged to be far larger than could be justified by any expected improvement in the results. It was also known that without the vertical angles, or some suitable substitute, the heights of most points cannot be well determined. The solution to this problem was to set the corrections to all height unknowns to zero in the observation equations for the terrestrial observations. This means that the heights were fixed as previously established. This model has thus acquired the designation "height-controlled three-dimensional adjustment."

At some points a height can be determined from space systems observations. For these points an elevation unknown was included, but only in the observation equations for the space systems data. At these stations there were actually two numerical values of the height: the value which was determined from some form of leveling and which was used in the processing of the terrestrial observations, and the value of the parameter in the space systems equations. This situation came to be known as a "dual height system." This does not imply that these points have two positions in space but only that the two heights are referred to two slightly different surfaces: the ellipsoid as defined by terrestrial heights and the ellipsoid implied by space systems observations. If all observations were perfect, there would be no difference between them.

The height-controlled three-dimensional model requires that astronomic latitude and longitude be known at all stations from which theodolite observations were made. Of course this is also true of the equations of classical geodesy when they are rigorously applied, so the height-controlled approach really makes no greater demands than the classical approach.

Whenever the astronomic latitudes and longitudes were observed at a station, they were used as observed. At all other stations they were computed by astrogravimetric methods, as described in chapter 16.

For derivations of the appropriate equations see Vincenty and Bowring (1978), Vincenty (1980, 1982), and Steeves (1984), where further references are given.

12.3 NOTATION

- X,Y,Z Cartesian coordinates in the equatorial system
- ϕ,λ astronomic latitude and longitude (ground level values), positive north and east respectively
- S spatial distance
- A astronomic azimuth, clockwise from north
- V vertical angle, positive upwards from the astronomic horizon
- B,L geodetic latitude and longitude, positive north and east respectively
- H height above the ellipsoid
- a,e equatorial radius and first eccentricity of the reference ellipsoid
- M,N radii of curvature in the meridian and in the prime vertical.

12.4 FUNDAMENTAL EQUATIONS

To transform geographic coordinates to Cartesian coordinates in the equatorial system we have as usual

$$X = (N + H) \cos B \cos L$$
$$Y = (N + H) \cos B \sin L$$
$$Z = [N(1 - e^2) + H] \sin B.$$

The vector between two points in space is simply

$$\Delta X = X_2 - X_1$$
$$\Delta Y = Y_2 - Y_1$$
$$\Delta Z = Z_2 - Z_1.$$

The components of this vector in the astronomic horizon coordinate system of the first station are

$$\begin{bmatrix} p_1 \\ q_1 \\ t_1 \end{bmatrix} = \mathbf{R}_{A1} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

where

$$\mathbf{R}_{A1} = \begin{bmatrix} -\sin\phi_1 \cos\lambda_1 & -\sin\phi_1 \sin\lambda_1 & \cos\phi_1 \\ -\sin\lambda_1 & \cos\lambda_1 & 0 \\ \cos\phi_1 \cos\lambda_1 & \cos\phi_1 \sin\lambda_1 & \sin\phi_1 \end{bmatrix}$$

Written explicitly, this is

$$p_1 = -\sin\phi_1 (\cos\lambda_1 \, \Delta X + \sin\lambda_1 \, \Delta Y) + \cos\phi_1 \, \Delta Z$$
$$q_1 = -\sin\lambda_1 \, \Delta X + \cos\lambda_1 \, \Delta Y$$
$$t_1 = \cos\phi_1 (\cos\lambda_1 \, \Delta X + \sin\lambda_1 \, \Delta Y) + \sin\phi_1 \, \Delta Z.$$

The horizontal component of this vector is given by

$$r_1^2 = p_1^2 + q_1^2.$$

It should be noted that p_1 is the northing coordinate of point P_2 in the local horizon system with origin at P_1, q_1 is the easting coordinate, and t_1 is the height of P_2 above the horizon of P_1 along the line parallel to the direction of gravity at P_1. This comprises a left-handed coordinate system.

The inverse solution in space is derived by expressing these spatial coordinate differences in spherical coordinates. The results can be written as follows:

$$S^2 = \Delta X^2 + \Delta Y^2 + \Delta Z^2 = t_1^2 + r_1^2$$
$$A_1 = \tan^{-1} (q_1/p_1)$$
$$V_1 = \sin^{-1} (t_1/S) = \tan^{-1} (t_1/r_1).$$

The unoriented direction from P_1 to P_2 is then

$$d_1 = A_1 - z,$$

where z is the orientation unknown common to a round of directions at P_1.

To obtain the corresponding values for the reverse direction, the subscripts are changed to 2 and the signs of ΔX, ΔY, and ΔZ are reversed.

The equations for azimuth, distance, and direction provide the nonlinear observation equations for the terrestrial observations. The equation for vertical angle is included here for completeness. It was not actually used in the NAD 83 adjustment program, since there were no vertical angle observations.

Note that the above inverse formula in space uses astronomic latitude and longitude at the standpoint. If geodetic values were used instead, this would produce an azimuth very similar to the azimuth of the geodesic, that is, the normal section azimuth referred to the forepoint at its height rather than to its projection on the ellipsoid. This would immediately require the use of Laplace corrections to azimuths, and other customary corrections to azimuths and directions that are known from classical geodesy.

12.5 PARAMETERS AND DIFFERENTIAL RELATIONSHIPS

The parameters of interest are ultimately the latitude and longitude of all stations. However, it is more convenient to work in terms of the linear coordinate corrections dx and dy in the geodetic horizon system.

The differentials of the observed quantities can be written in terms of the coordinate corrections in the astronomic horizon system at the standpoint dp_1, dq_1, and dt_1. These in turn can be related to coordinate corrections in the geodetic horizon system at each point by

$$\begin{bmatrix} dp_1 \\ dq_1 \\ dt_1 \end{bmatrix} = \mathbf{R}_{A1} \begin{bmatrix} d\Delta X \\ d\Delta Y \\ d\Delta Z \end{bmatrix}$$

and

$$\begin{bmatrix} d\Delta X \\ d\Delta Y \\ d\Delta Z \end{bmatrix} = \begin{bmatrix} dX_2 \\ dY_2 \\ dZ_2 \end{bmatrix} - \begin{bmatrix} dX_1 \\ dY_1 \\ dZ_1 \end{bmatrix}$$

with

$$\begin{bmatrix} dX_2 \\ dY_2 \\ dZ_2 \end{bmatrix} = \mathbf{R}_{G2}^{T} \begin{bmatrix} dx_2 \\ dy_2 \\ dH_2 \end{bmatrix}$$

and

$$\begin{bmatrix} dX_1 \\ dY_1 \\ dZ_1 \end{bmatrix} = \mathbf{R}_{G1}^{T} \begin{bmatrix} dx_1 \\ dy_1 \\ dH_1 \end{bmatrix}$$

and where

$$\mathbf{R}_{Gi} = \begin{bmatrix} -\sin B_i \cos L_i & -\sin B_i \sin L_i & \cos B_i \\ -\sin L_i & \cos L_i & 0 \\ \cos B_i \cos L_i & \cos B_i \sin L_i & \sin B_i \end{bmatrix}$$

is the matrix which rotates geocentric coordinates into the geodetic horizon system of point i.

Note that the third coordinate correction in the geodetic horizon system is the correction to the geodetic elevation. These corrections are set to zero in the observation equations arising from terrestrial observations. This forces the adjustment to coordinates to take place in the geodetic horizon of each station.

The shifts dx and dy are converted to geographic equivalents by

$$dB = dx/(M + H)$$
$$dL = dy/[(N + H) \cos B].$$

12.6 AUXILIARY PARAMETERS

Auxiliary parameters are those that are included in the adjustment in addition to the station coordinate shifts dx, dy, dH. A familiar example is the station orientation unknowns for individual sets of unoriented directions. In the height-controlled three-dimensional model, these are handled in the same way as is done in classical adjustments. Another example is the scale factor for a group of distance observations which are thought to share the same (unknown) scale error.

Many auxiliary parameters also enter the models for space systems observations. An example is the translation unknowns of one positioning system with respect to another (dX_0, dY_0, dZ_0). If the adjustment had been done in a local system, with only one terrestrial position held fixed in all three coordinates (such as MEADES RANCH on NAD 27), then translation components could be generated for any other positioning system. Since the adjustment was done in a geocentric system, without holding any position fixed, and because Doppler (as translated a priori by 4.5 m in Z) was the only source of positioning, the translation unknowns were not used. The remaining auxiliary parameters are orientation and scale unknowns. These deserve special consideration.

The observations contributed by many space systems contain orientation and scale information. One of these may be accepted as correct and used to define the orientation and scale of the coordinate system of the adjustment. One can then solve for the orientation and scale of the others. In the NAD 83 adjustment the Doppler observations defined the coordinate system of the adjustment. The VLBI system was scaled and oriented to the Doppler. Thus the VLBI data contributed only to the *shape* of the network.

In principle, one can also determine the relative orientation between space systems and the terrestrial network. However, only the rotation around the Z-axis can be determined well. For example, Doppler positions received a rotation around the Z-axis from the orientation implied by astronomic azimuths, while the other two rotation angles were set to zero values.

Scale correction parameters were carried for both the terrestrial distances and for most space systems observations. However, these two groups of parameters are not directly related.

For space systems observations, a scale correction is applied to all three coordinates equally. For these systems, a correction to scale means that the distance of stations from the center of the coordinate system is changed but all angles remain the same.

For terrestrial distances, a scale error really means an inconsistency exists between the unit of length in which distances are measured and the unit of length by which the size of the reference ellipsoid is measured. This occurs most commonly when there are systematic errors in geodetic elevations. However, the effect of a scale change to terrestrial distances is quite different from a scale change to three-dimensional observations.

A change to the scale of terrestrial distances (which could arise because of a change to the calibration of all EDMI, because of a systematic change to heights, or because of a change to the adopted value of the size of the reference ellipsoid) would have the effect of changing the areal extent of the survey network (on the surface of the reference ellipsoid). The angles between pairs of stations (measured at the center of the ellipsoid) would change. The scale of terrestrial distances is thus determined most strongly from the space systems information concerning the *shape* of the network.

On the other hand, the scale of the terrestrial network in three-dimensional space depends not on the terrestrial distances but on the heights (together with the adopted size of the reference ellipsoid). This scale information could be used to determine the scale of the three-dimensional observations if all heights were forced to agree with terrestrial fixed heights. However, the accuracy of the terrestrial fixed heights is largely unknown. To the extent it is known, these heights are judged to be less accurate and less consistent than the scale information contained in the space systems observations. Therefore these two sources of scale information are kept separate by the dual height system.

Since the terrestrial network is not a good source of scale for the three-dimensional systems, the scale of one of these systems must be fixed. In the NAD 83 adjustment, the Doppler scale (after correction by −0.6 parts per million to the BIH Terrestrial System) was held fixed.

In the NAD 83 adjustment, every EDM distance observation equation carried a scale correction unknown. There were altogether 30 separate scale corrections. (See chapter 18.) The observation class deck identified as "Geodimeter" included all of the Geodimeter measurements performed by NGS, including all the distances measured on the Transcontinental Traverse (TCT) project. Since it contained the most accurate and by far the largest number of observations, the scale correction for this observation class deck is taken to be the terrestrial scale correction.

12.7 OBSERVATION EQUATIONS

1. *Astronomic azimuth*

The observation equation for an observed astronomic azimuth is

$$v_A = a_1dx_1 + a_2dy_1 + a_3dx_2 + a_4dy_2 + K_A$$

where

$$a_1 = a_1' + m_1a_2', \quad a_2 = a_2' - m_1a_1'$$
$$a_3 = a_3' + m_2a_4', \quad a_4 = a_4' - m_2a_3'$$
$$m_i = \sin\phi_i \sin(\lambda_i - L_i) \qquad i = 1,2$$

and

$$a_1' = q_1/r_1^2$$
$$a_2' = -p_1/r_1^2$$
$$a_3' = -[q_1(\sin\phi_1 \sin\phi_2 \cos\Delta\lambda + \cos\phi_1 \cos\phi_2) + p_1 \sin\phi_2 \sin\Delta\lambda]/r_1^2$$
$$a_4' = (p_1 \cos\Delta\lambda - q_1 \sin\phi_1 \sin\Delta\lambda)/r_1^2$$

and where p_1, q_1, and r_1 are the same as in the space inverse formula previously shown. Here K_A is the constant term, that is, computed minus observed value, and v_A is the residual.

2. *Unoriented directions*

The observation equation for an unoriented direction is the same as that for an astronomic azimuth, with addition of a station orientation unknown dz common to a set of directions, with coefficient of −1. Explicitly,

$$v_d = a_1dx_1 + a_2dy_1 + a_3dx_2 + a_4dy_2 - dz + K_d$$

where K_d is now the computed minus observed value of the direction and v_d is the residual.

3. *Straight-line distances*

The observation equation for a straight line distance is

$$v_s = b_1dx_1 + b_2dy_1 + b_3dx_2 + b_4dy_2 - Sds_k + K_s$$

where

$$b_1 = b_1' + m_1b_2', \quad b_2 = b_2' - m_1b_1'$$
$$b_3 = b_3' + m_2b_4', \quad b_4 = b_4' - m_2b_3'.$$

The m's are the same as in the azimuth observation equation and

$$b_1' = -p_1/S \quad b_3' = -p_2/S$$
$$b_2' = -q_1/S \quad b_4' = -q_2/S.$$

The symbols p_2 and q_2 denote values computed using ϕ and λ at the forepoint. In a distance observation equation, K_s denotes computed minus observed distance, v_s is the residual, and ds_k is the scale correction unknown for the observation class to which this observation belongs.

4. *Doppler Positions*

The full observation equations for Doppler X, Y, Z coordinates are

$$\begin{bmatrix} v_x \\ v_y \\ v_H \end{bmatrix} = \begin{bmatrix} dx \\ dy \\ dH \end{bmatrix} - \mathbf{R}_G \begin{bmatrix} dX_0 \\ dY_0 \\ dZ_0 \end{bmatrix} - \mathbf{R}_G\mathbf{U} \begin{bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{bmatrix}$$
$$- \mathbf{R}_G \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} ds + \mathbf{R}_G \begin{bmatrix} X-X' \\ Y-Y' \\ Z-Z' \end{bmatrix}$$

where

$$\mathbf{U} = \begin{bmatrix} 0 & -Z & Y \\ Z & 0 & -X \\ -Y & X & 0 \end{bmatrix}$$

and $\mathbf{R}_G$ is the matrix which rotates coordinate differences in the equatorial system into the geodetic horizon system, as defined previously. Here X', Y', and Z' are the observed values and X, Y, and Z are the computed values.

The equation was programmed in this form, but the analyst was given the option to constrain any unknown parameter correction. As actually used for the NAD 83 adjustment, all three translations and the first two rotation angles were set to zero. Furthermore, the Doppler scale correction ds was also constrained to zero in the last iteration.

The shifts dx, dy in the geodetic horizon system have the same meaning here as in the terrestrial observation equations. For the Doppler observations, we also have the elevation correction parameter dH, which was absent in the terrestrial observation equations.

5. *Coordinate differences*

The model allows for multiple groups of coordinate difference observations. The full observation equations for the difference between the three-dimensional coordinates of stations i and j are

$$\begin{bmatrix} v_{\Delta X} \\ v_{\Delta Y} \\ v_{\Delta Z} \end{bmatrix} = -\mathbf{R}_{Gi}^{\mathrm{T}} \begin{bmatrix} dx_i \\ dy_i \\ dH_i \end{bmatrix} + \mathbf{R}_{Gj}^{\mathrm{T}} \begin{bmatrix} dx_j \\ dy_j \\ dH_j \end{bmatrix} - \Delta \mathrm{U} \begin{bmatrix} \omega_{Xk} \\ \omega_{Yk} \\ \omega_{Zk} \end{bmatrix}$$

$$- \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} ds_k + \begin{bmatrix} \Delta X - \Delta X' \\ \Delta Y - \Delta Y' \\ \Delta Z - \Delta Z' \end{bmatrix}$$

where

$$\Delta U = U_j - U_i$$

Here ω_{Xk}, ω_{Yk}, and ω_{Zk} are the orientation parameters and ds_k is the scale correction for group k. Also, $\Delta X'$, $\Delta Y'$, *and* $\Delta Z'$ are the observed coordinate differences and ΔX, ΔY, and ΔZ are the corresponding computed differences.

As used for the NAD 83 adjustment, each campaign of observations between mobile VLBI observations was treated as a group, with separate orientation and scale unknowns. The set of observations between the fixed VLBI stations was also a group.

A few true three-dimensional surveys were included in the adjustment. These were mostly local surveys performed to connect different observing systems at a single site. These three-dimensional surveys were reduced to three-dimensional coordinate difference observations. For these local surveys, the auxiliary orientation and scale unknowns were omitted.

12.8 REFERENCES

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13. HELMERT BLOCKING

Charles R. Schwarz

13.1 SELECTION OF THE METHOD

Prior to the new adjustment, the National Geodetic Survey already had considerable experience in carrying out least squares adjustments of horizontal networks. The largest network that had been adjusted to that time was the one in Alaska, which contained 1,847 stations, requiring the solution of 3,348 simultaneous linear equations.

It was known that in principle the adjustment of the entire continental horizontal network should be carried out by similar methods, only scaled up. The prospect of solving a system of approximately 900,000 simultaneous linear normal equations was nevertheless quite daunting, since this had not previously been done. On the other hand, it was known that the normal equations would be quite sparse and that a solution should therefore be feasible. There was never any consideration of using an approximate solution.

At the beginning of the project, considerable literature already existed on how to deal with large sparse systems of equations (see, for instance, Ried, 1971; Rose and Willoughby, 1972; Tewerson, 1973; Bunch and Rose, 1976). The question of intense concern was how to choose and apply a method which would best take advantage of the sparseness of the geodetic normal equations.

Among the methods that would produce a simultaneous least squares solution, Helmert blocking was always the leading candidate. The idea of partitioning a network adjustment was already well known to NGS. It had been used, for example, in the adjustment of the European triangulation in 1950 (Whitten, 1952). It had more recently been expressed as a computer algorithm and used in the RETRIG project (the scientific adjustment of the European datum).

NGS briefly considered other alternatives, such as arranging the solution algorithm for a parallel processor or some other supercomputer that would solve the total set of equations in one pass. The Helmert blocking approach was found to be more advantageous than these for the following reasons:

1. It takes advantage of the sparseness of the normal equations at least as well as any competing scheme.
2. No separate algorithm is necessary to determine the ordering of the blocks. Other approaches require elaborate algorithms to reorder the unknowns.
3. Helmert blocking provides a natural checkpoint and restart capability. It was assumed that the entire solution would be long and expensive, no matter what computer was used. Some form of checkpoint and restart capability would be necessary. In the Helmert blocking scheme this capability is built in.
4. Checkpoints are taken at places that allow a meaningful analysis to be made. Each block is a geographic area containing a subnetwork. If desired, the solution for this subnetwork can be carried to completion (holding the junction points fixed) and the statistics of this subnetwork can be examined.
5. It allows work on different blocks to proceed in parallel. NGS expected to have a considerable workforce of experienced analysts after the project validation and block validation phases of the project. In the Helmert block scheme these people would contribute by preparing and analyzing individual blocks.

Within the Helmert blocking scheme there were still many choices and decisions to be made. For example,

1. How should the partitioning be done?
2. How big should the blocks be?
3. How should the blocks be combined?
4. What algorithms should be used to form and solve individual blocks?

These issues are addressed in detail in the next chapter. The Helmert blocking scheme was applied to blocks that already contained as many as 2,000 stations. For exploiting the sparseness of the normal equations in the lowest level blocks, NGS selected the variable bandwidth storage structure and profile minimization scheme that had been previously used (Schwarz, 1978; Snay, 1976).

13.2 HELMERT BLOCKING DESCRIPTION

Helmert blocking is a procedure for adjusting large geodetic networks by partitioning the network into geographic blocks. F. R. Helmert, for whom the procedure is named, never had the opportunity to apply the method. However, as described by Wolf (1978), Helmert (1880) gave the following instructions:

1. Establish the normal equations for each partial net separately.
2. Eliminate the unknowns for all those points (inner points) that do not have any observational connection with the neighboring partial nets. The reduced normal equations so obtained then contain only the "junction unknowns" which are in common with the neighboring blocks.

3. Add together all these reduced normals, term by term, so that the "main system" is established.
4. Solve this main system for all junction unknowns. Subsequently, the unknowns for the inner points are obtained from the back solution as performed within the various partial nets.

Helmert's description was not mathematical in nature. Nevertheless, it was recognized that the method is entirely equivalent to a simultaneous solution of the entire set of normal equations arising from the network.

There are many variations of the Helmert blocking approach. For instance, the Canadian Section Method (Pinch and Peterson, 1974) and the "Method of Divided Normals" (Bomford, 1971) may both be considered to be variations of Helmert blocking.

13.3 MATHEMATICAL DEVELOPMENT

Mathematical explanations of the Helmert blocking method are usually derived from first principles (see, for instance, Hanson, 1974; and Wolf, 1978). However, it is more instructive (and ultimately easier) to derive Helmert blocking as an application of recursive block partitioning, a widely used method of dealing with certain sets of large sparse systems. (See Mikhail, 1976: sec. 11.6.)

13.3.1 Block Partitioning

13.3.1.1 Partitioning of Observations.

Let the entire set of observation equations be written

$$\mathbf{AX} = \mathbf{L} + \mathbf{V}$$

where $\mathbf{X}$ contains the unknown parameter corrections, $\mathbf{L}$ contains the "observed minus computed" terms, and $\mathbf{V}$ contains the residuals. The normal equations are then

$$\mathbf{NX} = \mathbf{U}$$

where

$$\mathbf{N} = \mathbf{A}^T\mathbf{WA} \text{ and } \mathbf{U} = \mathbf{A}^T\mathbf{WL}$$

and $\mathbf{W}$ is the weight matrix.

Now partition the observations into n groups. A group may be a single observation. The observation equations are then written

$$\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \bullet \\ \bullet \\ \bullet \\ \mathbf{A}_n \end{bmatrix} \mathbf{X} = \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ \bullet \\ \bullet \\ \bullet \\ \mathbf{L}_n \end{bmatrix} + \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \bullet \\ \bullet \\ \bullet \\ \mathbf{V}_n \end{bmatrix} .$$

Partition the weight matrix similarly, and assume that the partitioned weight matrix is block diagonal:

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & \mathbf{0} & \mathbf{0} & \bullet & \bullet & \bullet & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_2 & \mathbf{0} & \bullet & \bullet & \bullet & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{W}_3 & \bullet & \bullet & \bullet & \mathbf{0} \\ \bullet & \bullet & \bullet & \bullet & & & \bullet \\ \bullet & \bullet & \bullet & & \bullet & & \bullet \\ \bullet & \bullet & \bullet & & & \bullet & \bullet \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \bullet & \bullet & \bullet & \mathbf{W}_n \end{bmatrix} .$$

Then

$$\mathbf{N} = \Sigma\, \mathbf{A}_k^T\mathbf{W}_k\mathbf{A}_k \quad \text{and} \quad \mathbf{U} = \Sigma\, \mathbf{A}_k^T\mathbf{W}_k\mathbf{L}_k$$

or

$$\mathbf{N} = \Sigma\, \mathbf{N}_k \quad \text{and} \quad \mathbf{U} = \Sigma\, \mathbf{U}_k$$

where

$$\mathbf{N}_k = \mathbf{A}_k^T\mathbf{W}_k\mathbf{A}_k \quad \text{and} \quad \mathbf{U}_k = \mathbf{A}_k^T\mathbf{W}_k\mathbf{L}_k$$

Equations of the form

$$\mathbf{N}_k\mathbf{X} = \mathbf{U}_k$$

are called partial normal equations, since they arise from only part of the total set of data. This development establishes that the final set of normal equation coefficients can be accumulated by summing over the partial normal equations.

13.3.1.2 Partitioning of Unknowns.

Let the unknown parameter corrections be partitioned into two groups

$$\mathbf{X} = \begin{bmatrix} \dot{\mathbf{X}} \\ \ddot{\mathbf{X}} \end{bmatrix}$$

and partition the observation equation coefficients similarly

$$\begin{bmatrix} \dot{\mathbf{A}} & \ddot{\mathbf{A}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{X}} \\ \ddot{\mathbf{X}} \end{bmatrix} = \mathbf{L} + \mathbf{V}.$$

Then the normal equations may be written

$$\begin{bmatrix} \dot{\mathbf{N}} & \bar{\mathbf{N}} \\ \bar{\mathbf{N}}^T & \ddot{\mathbf{N}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{X}} \\ \ddot{\mathbf{X}} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{U}} \\ \ddot{\mathbf{U}} \end{bmatrix}$$

where

$$\dot{\mathbf{N}} = \dot{\mathbf{A}}^T\mathbf{W}\dot{\mathbf{A}}$$
$$\bar{\mathbf{N}} = \dot{\mathbf{A}}^T\mathbf{W}\ddot{\mathbf{A}}$$
$$\ddot{\mathbf{N}} = \ddot{\mathbf{A}}^T\mathbf{W}\ddot{\mathbf{A}}$$
$$\dot{\mathbf{U}} = \dot{\mathbf{A}}^T\mathbf{WL}$$
$$\ddot{\mathbf{U}} = \ddot{\mathbf{A}}^T\mathbf{WL}.$$

This is a system of two simultaneous matrix equations in two matrix unknowns. Eliminating the second set of unknowns (by the method of elimination) yields

$$(\dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^T)\dot{X} = (\dot{U} - \bar{N}\ddot{N}^{-1}\ddot{U})$$

which may be written

$$\tilde{N}\dot{X} = \tilde{U}.$$

These are called *reduced* normal equations, because they contain a reduced number of unknown parameters.

3.3.1.3 Partitioning of Both Observations and Unknowns.

Let the observation equations be written

$$\begin{bmatrix} \dot{A}_1 & \ddot{A}_1 \\ \dot{A}_2 & \ddot{A}_2 \\ \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \\ \dot{A}_n & \ddot{A}_n \end{bmatrix} \begin{bmatrix} \dot{X} \\ \ddot{X} \end{bmatrix} = \begin{bmatrix} L_1 \\ L_2 \\ \bullet \\ \bullet \\ \bullet \\ L_n \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ \bullet \\ V_n \end{bmatrix}$$

Then the partitions of the normal equations are

$$\begin{aligned} \dot{N} &= \Sigma\, \dot{A}_k^T W_k \dot{A}_k \\ \bar{N} &= \Sigma\, \dot{A}_k^T W_k \ddot{A}_k \\ \ddot{N} &= \Sigma\, \ddot{A}_k^T W_k \ddot{A}_k \\ \dot{U} &= \Sigma\, \dot{A}_k^T W_k L_k \\ \ddot{U} &= \Sigma\, \ddot{A}_k^T W_k L_k \end{aligned}$$

which establishes that the partitions of the reduced normal equations can be accumulated by summing over the observations.

13.3.1.4 Patterned, Sparse, Normal Equations.

Now suppose that the second set of unknowns is further subdivided and that the observations are ordered so that the observation equations have the form

$$\begin{bmatrix} \dot{A}_1 & \ddot{A}_1 & 0 & \bullet & \bullet & \bullet & 0 \\ \dot{A}_2 & 0 & \ddot{A}_2 & \bullet & \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \dot{A}_n & 0 & 0 & \bullet & \bullet & \bullet & \ddot{A}_n \end{bmatrix} \begin{bmatrix} \dot{X} \\ \ddot{X}_1 \\ \ddot{X}_2 \\ \bullet \\ \ddot{X}_n \end{bmatrix} = \begin{bmatrix} L_1 \\ L_2 \\ \bullet \\ \bullet \\ L_n \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ V_n \end{bmatrix} \tag{13.1}$$

The corresponding normal equations are

$$\begin{bmatrix} \dot{N} & \bar{N}_1 & \bar{N}_2 & \bullet & \bullet & \bullet & \bar{N}_n \\ \bar{N}_1^T & \ddot{N}_1 & 0 & 0 & \bullet & \bullet & 0 \\ \bar{N}_2^T & 0 & \ddot{N}_2 & 0 & \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bar{N}_n^T & 0 & 0 & 0 & \bullet & \bullet & \ddot{N}_n \end{bmatrix} \begin{bmatrix} \dot{X} \\ \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_2 \\ \bullet \\ \ddot{X}_n \end{bmatrix} = \begin{bmatrix} \dot{U} \\ \ddot{U}_1 \\ \ddot{U}_2 \\ \bullet \\ \bullet \\ \ddot{U}_n \end{bmatrix} \tag{13.2}$$

where the partitions of the normal equations are now

$$\begin{aligned} \dot{N} &= \Sigma\, \dot{A}_k^T W_k \dot{A}_k \\ \bar{N}_k &= \dot{A}_k^T W_k \ddot{A}_k \\ \ddot{N}_k &= \ddot{A}_k^T W_k \ddot{A}_k \\ \dot{U} &= \Sigma\, \dot{A}_k^T W_k L_k \\ \ddot{U}_k &= \ddot{A}_k^T W_k L_k. \end{aligned}$$

Furthermore, we can write

$$\dot{N} = \Sigma \dot{N}_k \text{ and } \dot{U} = \Sigma \dot{U}_k$$

where

$$\dot{N}_k = \dot{A}_k^T W_k \dot{A}_k \text{ and } \dot{U}_k = \dot{A}_k^T W_k L_k.$$

We can now take advantage of the sparsity of this pattern by noting that $\ddot{N}$ is a block diagonal matrix, and its inverse is simply the block diagonal matrix containing the inverses of the diagonal blocks. Thus,

$$\bar{N}\ddot{N}^{-1}\bar{N}^T = \left[\bar{N}_1 \bar{N}_2 \bullet\bullet\bullet \bar{N}_n\right] \begin{bmatrix} \ddot{N}_1^{-1} & 0 & \bullet & \bullet & \bullet & 0 \\ 0 & \ddot{N}_2^{-1} & \bullet & \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \ddot{N}_n^{-1} \end{bmatrix} \begin{bmatrix} \bar{N}_1^T \\ \bar{N}_2^T \\ \bullet \\ \bullet \\ \bullet \\ \bar{N}_n^T \end{bmatrix}$$

$$= \Sigma \bar{N}_k \ddot{N}_k^{-1} \bar{N}_k^T$$

and the coefficients of the reduced normal equations are

$$\begin{aligned} \tilde{N} &= \dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^T \\ &= \Sigma\, \dot{N}_k - \Sigma\, \bar{N}_k \ddot{N}_k^{-1} \bar{N}_k^T \\ &= \Sigma\, (\dot{N}_k - \bar{N}_k \ddot{N}_k^{-1} \bar{N}_k^T). \end{aligned}$$

Similarly,

$$\tilde{U} = \Sigma\, (\dot{U}_k - \bar{N}_k \ddot{N}_k^{-1} \ddot{U}_k).$$

With

$$\tilde{N}_k = (\dot{N}_k - \bar{N}_k \ddot{N}_k^{-1} \bar{N}_k^T)$$

and

$$\tilde{U}_k = (\dot{U}_k - \bar{N}_k \ddot{N}_k^{-1} \ddot{U}_k)$$

this is written

$$\tilde{N} = \Sigma \tilde{N}_k \text{ and } \tilde{U} = \Sigma \tilde{U}_k.$$

The equation

$$\tilde{N}_k \dot{X} = \tilde{U}_k$$

is called a partial reduced normal equation.

Block partitioning can be expressed as a processing algorithm by following the basic rule:

> Process the observations in order, accumulating the contributions to the various partitions of the normal equations. When all of the observations involving a particular group of unknowns $\ddot{\mathbf{X}}_k$ have been processed, then the diagonal block $\ddot{\mathbf{N}}_k$ of the normal equation coefficient matrix corresponding to that group is complete. That group of unknowns may then be eliminated.

This means that the partial reduced normal equation term $\tilde{\mathbf{N}}_k$ is computed and added to similar terms from other groups.

The algorithm can be put into a form appropriate for computer processing if the observations are ordered as shown in eq. 13.1. Here all those observations which contain $\ddot{\mathbf{X}}_1$ come first, followed by those which contain $\ddot{\mathbf{X}}_2$, and so forth, ending with the observations which contain $\ddot{\mathbf{X}}_n$. By hypothesis, these sets are disjoint so that this partitioning of the observations is possible.

The flowchart in figure 13.1 describes the computational process. Here the symbols denote storage locations rather than matrices with fixed values, and the left arrow is read "gets" or "receives." It is necessary to set aside storage space for only one each of the partitions $\dot{\mathbf{A}}_k$, $\ddot{\mathbf{A}}_k$, $\mathbf{L}_k$, $\dot{\mathbf{N}}_k$, $\bar{\mathbf{N}}_k$, and $\dot{\mathbf{U}}_k$. After they are used to accumulate the contributions of one group of observations, they are reinitialized and reused for the next group. Only the partitions $\tilde{\mathbf{N}}$ and $\tilde{\mathbf{U}}$ remain for the entire process.

After the reduced normal equations are accumulated they are solved for $\dot{\mathbf{X}}$. Then the other unknowns may be found by solving the elimination equations

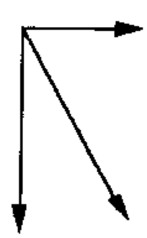
$$\ddot{\mathbf{N}}_k\ddot{\mathbf{X}}_k = \ddot{\mathbf{U}}_k - \bar{\mathbf{N}}_k^T\dot{\mathbf{X}}.$$

Block partitioning provides the following advantages:

1. It takes advantage of a priori knowledge of the location of blocks of zeroes in the normal equations—these zeroes are neither stored nor involved in computations.
2. It is necessary to set aside only enough computer memory for the reduced normal equations and only one set of the partitions associated with individual groups.

13.3.2 Recursive block partitioning

The patterned matrix of normal equation coefficients described above can be represented as



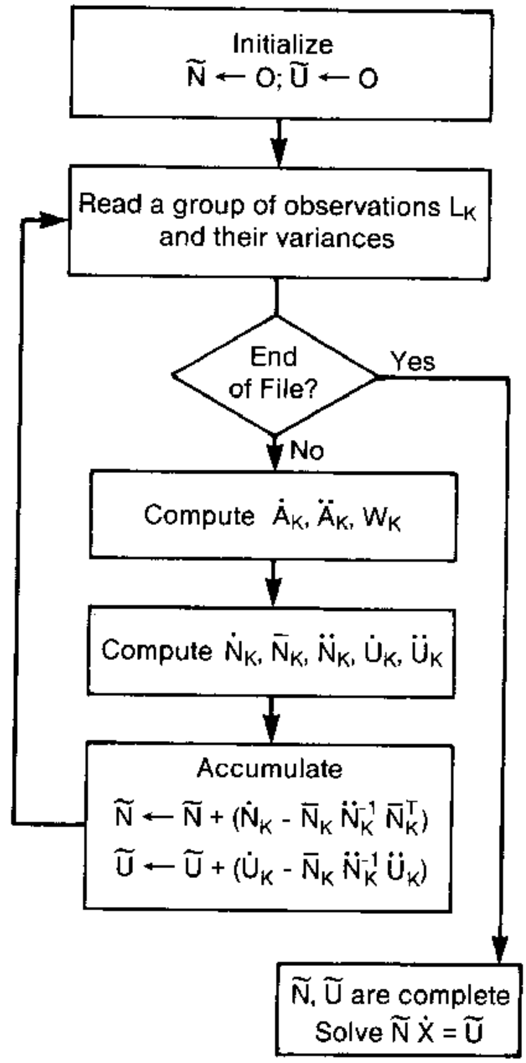
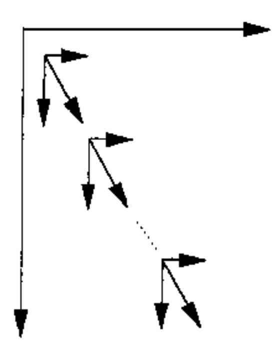


Figure 13.1. Accumulation of partial reduced normal equations.

where the arrows give a general impression of the location of the nonzero terms in the coefficient matrix.

In many problems there is even more structure in the normal equations. Of particular interest is the case where the $\ddot{\mathbf{N}}_k$ terms are themselves sparse and the unknowns can be arranged such that the $\ddot{\mathbf{N}}_k$ have the same pattern as the orignal set of coefficients $\mathbf{N}$. This is shown symbolically as



Now each $\ddot{\mathbf{N}}_k$ can be processed by the method of block partitioning, resulting in a smaller system of equations to be added to the main system. The system indicated symbolically above is called a *second-order partitioned system.*

In practice, the entire set of normal equations is never formed explicitly. Instead, the solution proceeds from the bottom up by applying the processing rule recursively to the subgroups within each group. The reduced normal equations corresponding to each $\ddot{\mathbf{N}}_k$ are accumulated; then the contribution $\tilde{\mathbf{N}}_k$ to the main system is computed and added to the other contributions.

This idea can be extended to even more levels. In fact, it is not even necessary that each diagonal submatrix have the same number of partitions. The processing rule is applied recursively to subgroups within other groups to as many levels as one wants. Each time it is applied, some unknowns are eliminated and the computer space that had been used to accumulate the corresponding partitions of the normal equations becomes available for the next group.

13.4 GEOGRAPHIC PARTITIONING

The pattern in eq. 13.1 arises naturally in many problems in satellite geodesy and photogrammetry. It is less natural, but still possible, to find this pattern in the observation equations that arise in surveying applications. This is, in fact, the basis for the Helmert block method.

Classical geodetic observations, such as directions, distances, azimuths, and elevation differences, always connect exactly two stations, never more. Furthermore, one station is always identified as the standpoint and the other as the forepoint. We say that the standpoint "sees to" the forepoint and that the forepoint is "seen" by the standpoint. With theodolite observations this identification is natural; in the case of distances or elevation differences the identification may be arbitrary.

In Helmert blocking, the first step is to identify the stations inside the block boundary. Then the observations are partitioned. The observations which belong to a block are those for which the standpoint is an inside station. The stations are then classified. Any point within the block boundary which is "seen" by one or more points outside the boundary is classified as an inside junction point for that block; otherwise it is classified as an interior point. Those points outside the boundary which are "seen by" inside points are classified as outside junction points.

Figure 13.2 shows a geodetic network laid out geographically. The directed lines indicate the observations, so that the standpoint and forepoint of each observation may be determined. The dashed line divides the network into two blocks. The observations are partitioned and the stations are classified according to the rules above. Then the unknown coordinates of the junction stations are denoted by $\dot{\mathbf{X}}$, those of the interior stations for block I are denoted $\ddot{\mathbf{X}}_1$, and those of the interior stations for Block II are denoted $\ddot{\mathbf{X}}_2$. Furthermore, the observations are ordered so that those belonging to Block I come first, followed by those belonging to Block II. Then the observation and normal equations are

$$\begin{bmatrix} \dot{\mathbf{A}}_1 & \ddot{\mathbf{A}}_1 & \mathbf{0} \\ \dot{\mathbf{A}}_2 & \mathbf{0} & \ddot{\mathbf{A}}_2 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{X}} \\ \ddot{\mathbf{X}}_1 \\ \ddot{\mathbf{X}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

and

$$\begin{bmatrix} \dot{\mathbf{N}} & \bar{\mathbf{N}}_1 & \bar{\mathbf{N}}_2 \\ \bar{\mathbf{N}}_1{}^{\mathrm{T}} & \ddot{\mathbf{N}}_1 & \mathbf{0} \\ \bar{\mathbf{N}}_1{}^{\mathrm{T}} & \mathbf{0} & \ddot{\mathbf{N}}_2 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{X}} \\ \ddot{\mathbf{X}}_1 \\ \ddot{\mathbf{X}}_2 \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{U}} \\ \ddot{\mathbf{U}}_1 \\ \ddot{\mathbf{U}}_2 \end{bmatrix}$$

These are instances of eqs. 13.1 and 13.2, so the block partitioning algorithm can be applied. After the observations for block I have been processed the partitions for that block are complete and the partial reduced normal equations for block I can be formed. Then the partial reduced normal equations for block II are formed and added to those for block I, producing the complete set of reduced normal equations.

The computations in the two blocks are actually independent of each other, so that there is no requirement to order the computations in time. The partial reduced normal equations for block II may actually be computed before, after, or simultaneously with those of block I. In the end, the partial reduced normal equations from the two blocks must be added together to produce the complete set of reduced normal equations.

The classification rules above produce some effects that are not intuitively obvious. For instance, station 2 in figure 13.2 is an interior station in block I, even though it lies at the junction between the two blocks and has a connection across the boundary to block II. Nevertheless, all of the observations involving station 2 belong to block I.

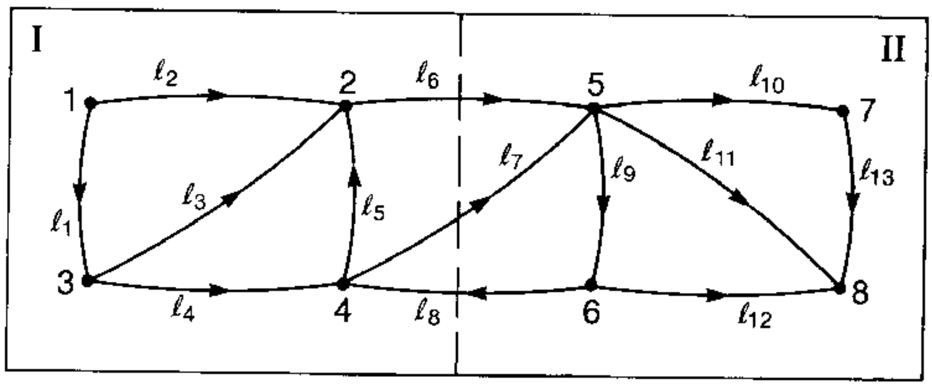
In figure 13.3, station 5 is a junction station for block II, even though there are no observations involving station 5 in block II. This means that the partial reduced normal equations for block II will contain zeroes in those locations corresponding to the unknown coordinates of station 5. However, when the partial reduced normal equations from the two blocks are added together these locations will be filled in.

Some authors have proposed different classification rules which would classify point 5 as interior to block I and avoid storing these zeroes. These more complicated rules were not chosen for the NAD 83 adjustment, since the issue is only one of a small amount of temporary storage space, not of extra computational effort.

The first classification rule states that an inside station is classified as a junction point if it is "seen" by an outside point. Such an observation would not belong to the block being processed. The application of this rule therefore requires that the total data set be searched to see if any such observations exist. Searching through the entire data set is much more difficult and time-consuming than searching through a single block. Not only is the entire data set large, but it does not exist as a single computer file.

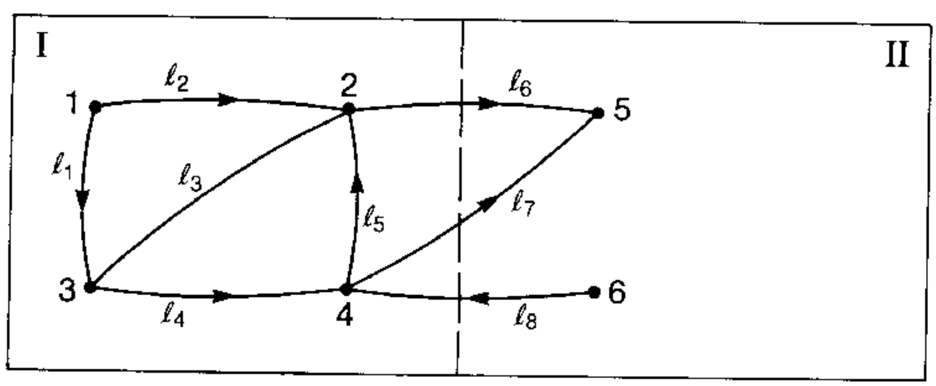
The task of finding the observations from outside to inside a block was assigned to the geodetic data base system, which was the only place with a global view of the entire data set. The data base program did not actually search the entire data base for these observations; instead it relied on the cross-reference lists that were carried in each station record. Each such list contained the identifiers of all those other stations from which the station is "seen." These lists were clearly redundant data items, but the data base management system ensured that they were updated whenever a block of observations was loaded or deleted, and were thus kept consistent at all times.

When a Helmert block was defined, all the data belonging to that block were retreived by a geodetic data base application function and stored in external format known as a RESTART file. (See appendix A of chapter 10.) The RESTART file for a single block contained all observations originating within the block as well as all observations from outside to inside the block. These are exactly the observations which are necessary to classify the stations associated with the Helmert block.



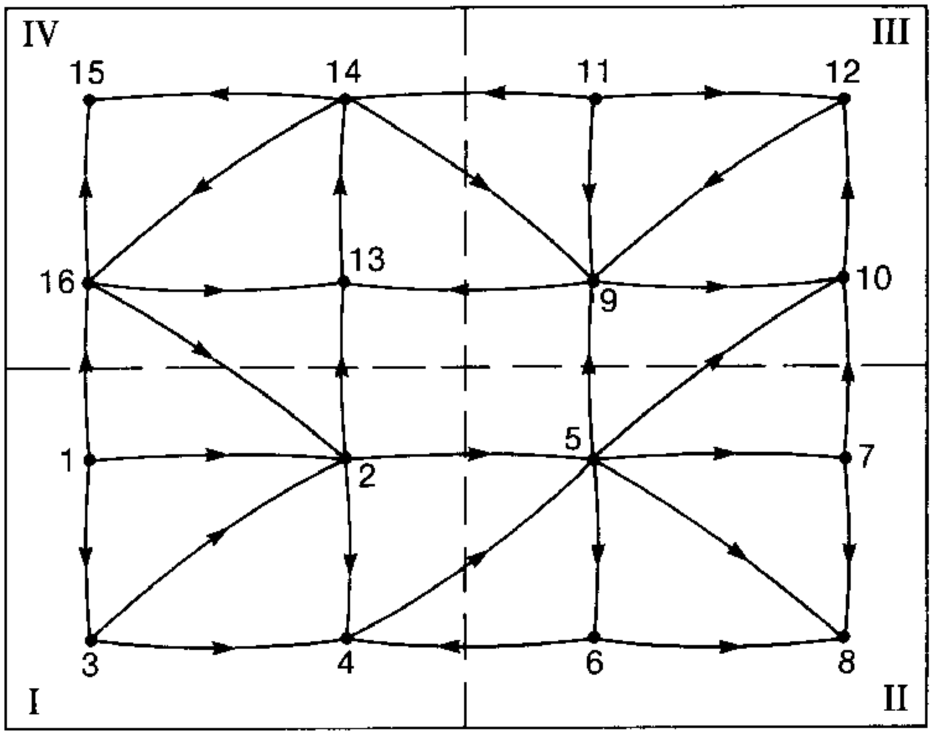
Block	I	II
Inside Stations	1, 2, 3, 4	5, 6, 7, 8
Observations	ℓ_1, ℓ_2, ℓ_3, ℓ_4 ℓ_5, ℓ_6, ℓ_7	ℓ_8, ℓ_9, ℓ_{10} ℓ_{11}, ℓ_{12}, ℓ_{13}
Interior Stations	1, 2, 3	6, 7, 8
Junction Stations	4, 5	4, 5

Figure 13.2. Geographic partitioning.



Block	I	II
Inside Stations	1, 2, 3, 4	5, 6,
Observations	ℓ_1, ℓ_2, ℓ_3, ℓ_4 ℓ_5, ℓ_6, ℓ_7	ℓ_8
Interior Stations	1, 2, 3	6
Junction Stations	4, 5	4, 5

Figure 13.3. Classification of stations.



Block	I	II	III	IV
Inside Stations	1, 2, 3, 4	5, 6, 7, 8	9, 10, 11, 12	13, 14, 15, 16
Interior Stations	1, 3	6, 7, 8	11, 12	15
Junction Stations	2, 4, 5	5, 4, 9, 10	9, 10, 13, 14	13, 14, 16
	13, 16			9, 2

Figure 13.4. Classification of stations for four blocks.

13.5 GEOGRAPHIC PARTITIONING (MANY BLOCKS)

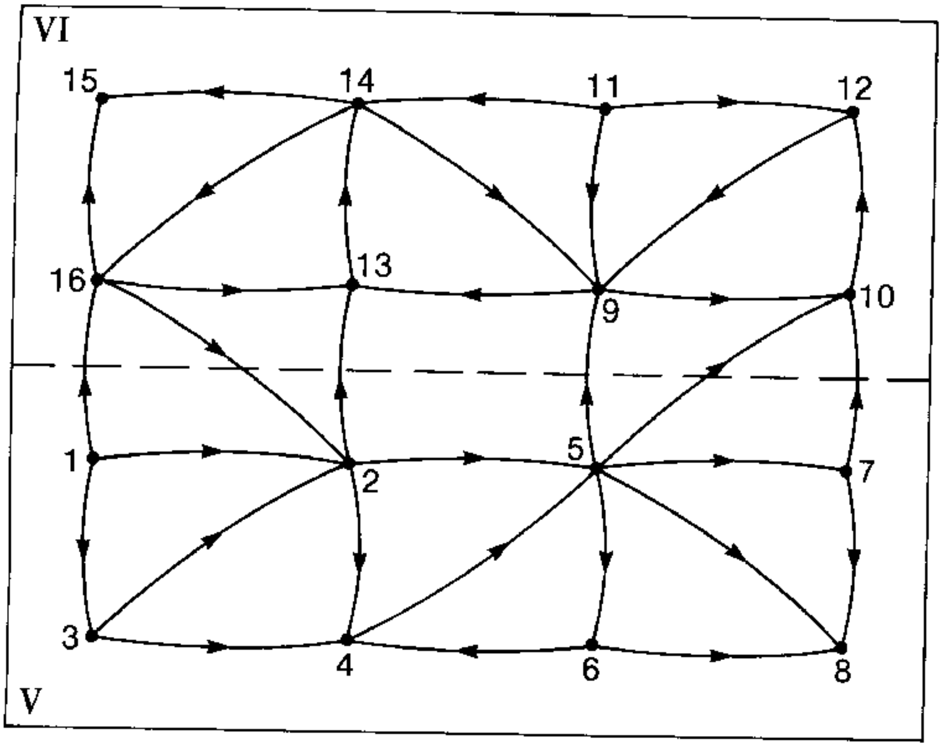
Figure 13.4 shows a larger network partitioned geographically into four blocks. The observations are partitioned and the stations are classified using the rules above.

Within the Helmert blocking method, there are several ways to proceed with this network. First, this can be treated as a first-order partitioned network. The union of all the junction points (stations 2, 4, 5, 9, 10, 13, 14, 16) is denoted $\dot{\mathbf{X}}$. The interior point unknowns for block I are denoted $\ddot{\mathbf{X}}_1$, and so forth. The complete reduced normal equation coefficient matrix still contains many zeroes. One is tempted to apply other algorithms to exploit the presence of these zeroes.

Another alternative is to approach this as a second-order partitioned system. The geodesist combines blocks I and II into a new block V, and combines blocks III and IV into a new block VI. Figure 13.5 shows the classification of the stations with respect to these combined blocks. We treat the partitioning in figure 13.5 as a first-order partitioned system, identifying $\dot{\mathbf{X}}$ with the junction point unknowns and $\ddot{\mathbf{X}}_1$ and $\ddot{\mathbf{X}}_2$ with the interior unknowns of blocks V and VI. Within block V, the observations are ordered so that those belonging to block I come first, followed by those belonging to block II. Furthermore, we divide the total set of unknowns in block V into those that are junction points for either block I or block II, those that are interior to block I, and those that are interior to block II. With this partitioning, we see that $\ddot{\mathbf{N}}_1$ has the pattern of a first-order partitioned system, and the total set of normal equations therefore has the pattern of a second-order partitioned system.

The plan for putting the blocks together is called a Helmert blocking strategy. If the geographic partitioning was developed recursively, according to some criteria, then it is natural to put the blocks together by moving back up through the recursion tree. Alternatively, if one begins the analysis at the point where geographic partitioning already exists, then the decisions about which blocks to combine together may be arbitrary. The issue of strategy development is discussed in detail in the next chapter.

By convention, the smallest geographic units are called first-level blocks and the combined ones are called higher level blocks. The entire project area is called the highest level block. In a recursively par-



Block	V	VI
Inside Stations	1, 2, 3, 4 5, 6, 7, 8	9, 10, 11, 12 13, 14, 15, 16
Interior Stations	1, 3, 4, 5, 6, 7, 8	11, 12, 14, 15
Junction Stations	2, 9, 10, 13, 16	2, 9, 10, 13, 16

Figure 13.5. Classification of stations for second-level Helmert blocks.

titioned matrix representation, it is only the coefficients of the interior unknowns in the first level blocks that are not further subdivided.

13.6 APPLYING HELMERT BLOCKING IN LARGE MULTILEVEL SYSTEMS

In a large network Helmert blocking is usually applied in such a way that each combining of blocks is a separate computer run. This allows the entire equation solving process to be partitioned into many computer runs. It also provides a natural checkpoint and restart capability. It is expected that the blocks are large enough so that a reasonable amount of work will be accomplished in each computer run, but small enough that no computer run will be excessively long.

The steps to be accomplished in each computer run are:

1. Construct a description of the boundary of the combined block.
2. List all the unknowns which appear in the combined block. This is the union of those which appear in the partial reduced normal equations to be combined; that is, the unknowns which appear at this level are all those which were junction unknowns at the previous level.
3. Reclassify all the unknowns as interior or junction unknowns. The station coordinate unknowns are classified using the basic rules described in section 13.4. Denote the junction unknowns by $\dot{X}$ and the interior unknowns by $\ddot{X}$. Similarly, the partitions of the new set of partial normal equations are denoted $\dot{N}$, $\bar{N}$, $\ddot{N}$, $\dot{U}$, $\ddot{U}$.
4. Accumulate the contributions of each of the constituent blocks to the new set of partial normal equations. The unknowns in the new set may be ordered differently than they were in any of the constituent partial reduced normal equations. Therefore, the proper location for each normal equation element must be computed.
5. Perform the matrix reduction step

$$\tilde{N} = (\dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^{T})$$

and

$$\tilde{U} = (\dot{U} - \bar{N}\ddot{N}^{-1}\ddot{U})$$

leaving the partial reduced normal equation terms to be passed on to the next step. This has the effect of eliminating the unknowns which were classified as interior for this block, leaving only the junction unknowns.

At the highest level, the normal equations that are accumulated are complete. They may therefore be solved directly for the unknowns they contain. The values of the junction point unknowns for each of the constituent blocks are found in this solution. The interior unknowns for each of the constituent blocks may then be found by solving the elimination equations

$$\ddot{\mathbf{N}}_k \ddot{\mathbf{X}}_k = \ddot{\mathbf{U}}_k - \bar{\mathbf{N}}_k{}^T \dot{\mathbf{X}}.$$

The decision as to which blocks to combine can be made according to a preset plan, or on an ad hoc basis. A preset plan is available if the geographic partitioning was developed recursively, and this is the approach that was adopted for the new datum adjustment.

Each computer run which combines blocks requires only that the partial reduced normal equations from the constituent blocks be available. It is otherwise independent of any knowledge of the entire network.

13.7 COMPUTATIONAL CONSIDERATIONS

13.7.1 Computer Resources

At the beginning of the New Datum Project, it was not known whether there would be sufficient computer resources to accomplish the adjustment. Table 13.1 shows the estimates prepared in 1974. These estimates assumed that many blocks would be combined at each level. They also indicated that a larger number of smaller blocks would be advantageous. Most importantly, they showed that the adjustment could be accomplished with about 250 hours of CPU time per iteration. This was significantly more CPU time than NGS had normally used, yet it indicated that the equations could be solved if the runs were spaced out over several weeks or months.

The actual experience of the NAD 83 adjustment validated this analysis. There were 161 first-level blocks and 321 total blocks in the U.S. terrestrial network. (See chapter 18.) The first-level blocks averaged more than 1,000 stations each, of which about 20 percent were junction points. The computation times were

Iteration	CPU hours	Elapsed time
0	431	6 months
1	226	4 months
2	283	4 months

13.7.2 Growth of Roundoff Error

Another concern in any large computational process is the growth of numerical roundoff error. This was particularly true at the beginning of the new datum adjustment, since there was no prior experience in solving such a large system of simultaneous linear equations. It was assumed that the computations would be performed on a mainframe computer using double-precision floating point arithmetic with 14 to 16 decimal digits. Numerical roundoff error had been a problem in some poorly conditioned adjustments in the past, and there was concern that in the new datum adjustment it might grow so large that there would be no significant digits left in the solution. To ensure that the computations would not be entirely worthless, it might be necessary to use extended precision arithmetic (at greater cost) or even a specially designed processor.

The behavior of the roundoff errors was analyzed by Professor Peter Meissl of the Technical University of Graz during his visit to NGS as a Visiting Senior Scientist (Meissl, 1980). He considered two classes of floating-point processors: one which performs true rounding (in binary) and one which truncates the results after every arithmetic operation. The latter design is actually the most common.

Professor Meissl estimated that about 2×10^{11} elementary arithmetic operations would be performed during a single solution of the normal equation system, even with the efficiencies afforded by the Helmert block approach and by the reordering of interior unknowns at the first level. The effect of roundoff error in each of these elementary operations was treated as the response of a linear system to an impulse-type disturbance.

The analysis showed that in a uniform network roundoff error can be treated as a random variable whose standard deviation grows only as the logarithm of the size of the network. For machines that truncate rather than round, there is also a bias in the result of about the same magnitude. The bias arises because all of the operations in forming the diagonal terms of the Cholesky factor of the normal equations have the same algebraic sign, so positive and negative truncation errors do not tend to balance out.

The non-uniform aspects of geodetic networks required special analysis. Observations over very short lines, such as taped distances, often have very high weights. These can cause "numerical singularities," and have a deleterious effect on roundoff error propagation. Observations relating widely separated distances, such as the Geodimeter lines of the Transcontinental Traverse and Doppler observations, have a very favorable effect.

The most important result of this study was the unambiguous conclusion: "It can be guaranteed that . . . at least 2-3 leading decimal digits of the largest coordinate shift will be recovered correctly during one iteration. With a small probability of error it can be predicted that about two more decimal digits will be correct. Relative positions of closely situated stations, i.e., the differences between their latitudes and between their longitudes, will be even more accurate. . ." (Meissl, 1978).

TABLE 13.1.—*Effects of junction station percentage and stations per block on a 190,000 station NAD readjustment*

Junction stations (%)	Stations per block	Block level	No. of blocks	Computer runs	cpu[1] hours
5	500	1	380	760	127
	500	2	19	38	6
	475	3	1	1	0.2
Totals + 25% for reruns:			500	999	166
5	1000	1	190	380	190
	950	2	10	20	10
	475	3	1	1	.5
Totals + 25% for reruns:			251	501	250
10	500	1	380	760	127
	500	2	38	76	13
	475	3	4	8	1
	190	4	1	1	.2
Totals + 25% for reruns:			529	1056	176
10	1,000	1	190	380	190
	1,000	2	19	38	19
	950	3	2	4	2
	190	4	1	1	.5
Totals + 25% for reruns:			265	525	264

[1] Assuming 10 min/500 station run and 30 min/1,000 station run.

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14. STRATEGY: DESIGNING THE ADJUSTMENT PROCEDURE

Charles R. Schwarz

14.1 INTRODUCTION

A Helmert blocking *strategy* is a plan for combining Helmert blocks to form larger blocks. Figure 14.1 shows a geographic area which has been divided into Helmert blocks and a strategy for combining the blocks. The geographic area is assumed to contain the entire network to be adjusted (there are no points outside the outer boundary). The strategy is represented as a tree data structure, with the leaves at the bottom and the root at the top. The blocks formed in the geographic subdivision are the leaves, and the tree describes how these blocks are to be combined. By convention, and probably because of the pictorial representation, the leaves are called first-level or bottom-level or lowest level blocks and the root is called the highest level block.

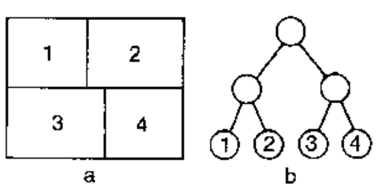


Figure 14.1. A division of a network into Helmert blocks and a strategy for combining blocks. Blocks 1 and 2 are combined into a higher level block, and blocks 3 and 4 are combined into another higher level block. Finally the two higher level blocks are combined.

After the boundaries of the geographic blocks are drawn, the points are identified according to the block in which they fall and classified as interior or junction points. Figure 14.1 assumes that points which are interior to the first-level blocks have already been eliminated but that junction points are left along all of the interior borders. The strategy specifies that blocks 1 and 2 are to be combined into a higher level block (which could also be given a label). All the points in the combined block are then reclassifed. Points along the border between blocks 1 and 2, which become interior to the new combined block, are then eliminated. Similarly, blocks 3 and 4 are combined and the points which become interior to that combined block are eliminated. The only junction points which are then left are those which appear along the border of the two higher level blocks. When these two blocks are combined into the block at the root of the tree, these points also become interior and can be eliminated. Since no junction points are left, this elimination step provides the solution for the highest level block. The solution is then propagated back down the tree.

The tree representation clearly identifies which processes can occur in parallel. For instance, blocks 1 and 2 can be combined in a process that is independent of, and can therefore run in parallel with, the combination of blocks 3 and 4. Furthermore, if the strategy itself is represented in machine-readable form, then the geodesist can have the computer carry out the entire adjustment without human intervention. For instance, the computer may be programmed to examine the files available to it. When it finds that the files containing the partial reduced normal equations for blocks 1 and 2 are available, it can dispatch a new job to combine these two blocks, eliminate the points which become interior, and store the resulting new set of reduced normal equations in a new file. Having done that, the program can look for other blocks to combine. If this program runs continuously or even periodically, then eventually the whole adjustment will be accomplished. This provides the possibility of achieving as much work as possible at one time while still retaining the advantageous natural checkpoint and restart capabilities of the Helmert blocking process. This scheme was implemented by the DISPATCHER program written for the NAD 83 adjustment.

14.2 DESIGN CRITERIA

For an adjustment using Helmert blocks to run automatically, the strategy must be completely designed before the adjustment is initiated. To do this requires not only data structures and programs, but also agreement about what constitutes a good or a bad strategy. Figure 14.2 shows some alternative strategies that could be used with the Helmert blocking scheme of figure 14.1a. The relative advantages and disadvantages of these alternatives are not immediately apparent. Some rules guiding the selection of a strategy can be developed, but it is first necessary to fix the nature of the blocks themselves and the computing environment in which the adjustment is to be carried out.

For the NAD 83 project, most of the technical issues were analyzed before the adjustment began. The four decisions discussed next were made early in the project.

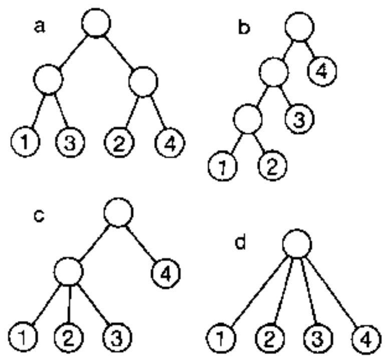


Figure 14.2. Some alternative strategies for the Helmert blocking scheme of figure 14.1a.

14.2.1 Block Size

No Helmert block should be allowed to exceed a limit of 2,000 to 3,000 stations (total of interior and junction points), and most should be considerably smaller. The main reason for this limitation concerned human factors. NGS felt that even with a fully validated data base, the task of analyzing a step in the Helmert block process might be difficult. We considered that a network of about 2,000 stations was about all that an analyst could comprehend. The paper listing of the observations in such a block would be about as bulky as could be conveniently handled. Another reason for limiting the size of the blocks was that we felt the time any step ran on the computer needed to be limited to a few hours. The Helmert blocking process provides a natural checkpoint and restart capability between steps, but a computer failure within a step would cause that step to be lost. Therefore, it was not prudent to rely on the computer running more than a few hours without failure. Yet another reason for limiting the size of the blocks was that we could not be sure that the adjustment would ultimately be run on a machine with virtual memory, and that we might therefore be limited by the size of memory available for arrays of station identifiers, coordinates, and related information.

14.2.2 Method of Subdividing Blocks

All Helmert blocks would be simply connected geographic areas, and all dividing lines between blocks would be drawn along 7½ minute graticule lines. With this stipulation, we could assign a point to a block simply by using the point's QID/QSN from the data base, without recourse to the actual coordinates and without the necessity of using point-in-polygon tests.

For most applications, geographic areas were represented as simple lists of quadrangles. For some applications it was necessary to represent the area internally by a list of 7½ minute quads. For other applications it was preferable to amalgamate the basic quads into larger quadrangles, and in still other applications, areas were represented internally by their boundaries. In all cases, the representation used by the system to communicate with the user was a list of amalgamated quadrangles.

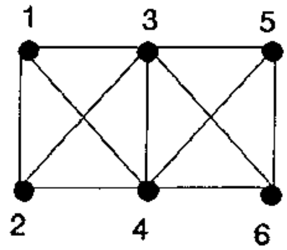
14.2.3 Treatment of Orientation Unknowns

Orientation unknowns would be eliminated first, before any other elimination step. The Schreiber equation (Jordan-Eggert, 1935: secs. 100 and 110) would be used for this operation, so that the orientation unknowns would never appear explicitly. We compared this approach with the alternative of carrying the orientation unknowns explicitly. In the latter case we assumed that the orientation unknowns for a station were carried together with the station latitude and longitude in the list of unknowns, and that we could identify blocks in the normal equation coefficient matrix corresponding to the set of unknowns at a station. The use of the Schreiber equation was perceived to have the following advantages.

1. With the Schreiber equation, each station has only two unknowns and the location of a station block in the normal equations can be found simply by multiplying the relative station number by two. With explicit orientation unknowns, each station would have a variable number of unknowns and an additional index would be necessary to locate the unknowns corresponding to a given station in the normal equations.
2. With the Schreiber equation, each station has a full 2×2 block on the diagonal of the normal equations. With explicit orientation unknowns, the diagonal block corresponding to the unknowns of a given station will have embedded zeroes whenever the station has more than one round of directions. Without further modifications, the adjustment will fail to take advantage of the a priori knowledge of the location of these zeroes.
3. Eliminating orientation unknowns at the outset with the Schreiber equation greatly reduces the total size of the normal equation coefficient matrix which must be dealt with in the Helmert block process. Since the orientation unknowns do not appear in the normal equations, we are not tempted to solve for them nor to compute their uncertainties.

The effect of eliminating orientation unknowns at the outset results in more complex rules governing connections between stations. Station i is now connected to station j whenever there is an observation from i to j or from j to i, or i and j are connected by a Schreiber equation. This occurs whenever i and j both appear in a round of directions. For instance station 1 is connected to station 5 in figure 14.3c because they both appear in the round of directions taken at station 3 (and also the round of directions taken at station 4). The greater number of connections that are generated at the first level might be seen as a disadvantage of the Schreiber equation, since the first-level matrices are less sparse and therefore harder to solve. However, this is not a real disadvantage, since

(a)



(b)

$\mathbf{P}_1$	$\mathbf{Z}_1$	$\mathbf{P}_2$	$\mathbf{Z}_2$	$\mathbf{P}_3$	$\mathbf{Z}_3$	$\mathbf{P}_4$	$\mathbf{Z}_4$	$\mathbf{P}_5$	$\mathbf{Z}_5$	$\mathbf{P}_6$	$\mathbf{Z}_6$
x	x	x	x	x	x	x	x	0	0	0	0
	x	x	0	x	0	x	0	0	0	0	0
		x	x	x	x	x	x	0	0	0	0
			x	x	0	x	0	0	0	0	0
				x	x	x	x	x	x	x	x
					x	x	0	x	0	x	0
						x	x	x	x	x	x
							x	x	0	x	0
								x	x	x	x
									x	x	0
										x	x
											x

(c)

$\mathbf{P}_1$	$\mathbf{P}_2$	$\mathbf{P}_3$	$\mathbf{P}_4$	$\mathbf{P}_5$	$\mathbf{P}_6$
x	x	x	x	x	x
	x	x	x	x	x
		x	x	x	x
			x	x	x
				x	x
					x

Figure 14.3 Connectivity diagrams for the case of pure triangulation: (a) Where it is assumed that direction observations are made in both directions along each line. The connectivity diagrams indicate which blocks or elements in the normal equations are known to contain zeroes. Blocks of zeroes are indicated by **0**'s and nonzero blocks are indicated by **x**'s. $\mathbf{P}_1$, $\mathbf{P}_2$, . . ., $\mathbf{P}_6$ indicate the position (latitude and longitude) unknowns associated with points 1, 2, . . . 6, while $\mathbf{Z}_1$, $\mathbf{Z}_2$, . . ., $\mathbf{Z}_6$ indicate the orientation unknowns for the six rounds of directions. (b) Connectivity diagram when orientation unknowns are not eliminated. (c) Connectivity diagram when orientation unknowns have been eliminated.

the alternative is to carry the orientation unknowns in the normal equations and eliminate them numerically later. In this case the same connections would be generated numerically in the higher level matrices.

14.2.4 Special Junction Points

Some stations would be carried as "special junction points," whose coordinates would not be solved for until the highest level. This category would include all points at which Doppler observations were made, since the Doppler observations would not be processed until the highest level. Other points could be included as well. The inverse of the normal equations obtained at the highest level would provide a covariance matrix of the total set of junction points. By ensuring that the special junction points are geographically well distributed, it would be possible to obtain enough information from the top level solution to discern the pattern of error propagation in the overall adjustment.

14.3 IMPLEMENTING HELMERT BLOCKING

Even after these early decisions, there still remained several questions concerning the implementation of the Helmert blocking procedure. For instance,

> How exactly should the blocks should be formed? Is there some "natural" way to divide the network into blocks? Is a small number of large first-level blocks, as in figure 14.4a, preferable to a larger number of smaller first-level blocks, as in figure 14.4b?
>
> Can or should Helmert blocking be combined with other schemes for exploiting the sparseness of the normal equation coefficient matrix?
>
> Is a broad tree with few levels, as in figure 14.2d, preferable to a deep tree with less branching, as in figure 14.2a?
>
> Are balanced trees, as in figures 14.2a and 14.2d, preferable to the unbalanced trees of figures 14.2b and 14.2c?

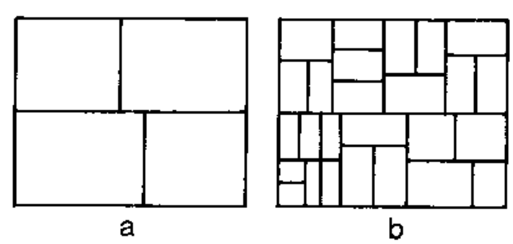


Figure 14.4. (a) Subdivision of an area into a small number of large first-order blocks. (b) Subdivision of the same area into a large number of smaller first-order blocks.

In considering these questions, we were influenced by the work of Alan George and the method he called "nested dissection." Figure 14.5 exemplifies the idea of nested dissection for an idealized geodetic network with a fairly regular distribution of points and observations. The individual points are not shown here. Instead, groups of points are labelled according to their order of elimination. The groups of points marked "1" are interior points at the lowest level; all other points are junction points at the first level. The points marked "2" are eliminated at the second level, those

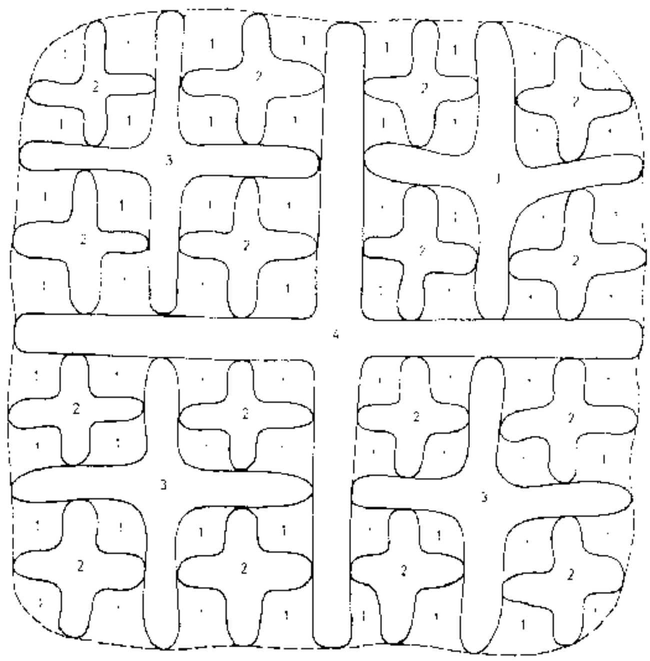


Figure 14.5. The concept of nested dissection. Clusters of points are numbered according to the order in which they are eliminated.

marked "3" at the third level, and those marked "4" at the fourth and highest level. This is clearly a Helmert blocking procedure. In this case, there is a definite plan to the forming of the blocks. The blocks are formed by first drawing one horizontal and one vertical line through the center so as to divide the network into quarters. Points are then identified as interior or junction points for this dissection. With the assumption that points are connected only to close-by other points, the junction points for this first dissection are those marked by "4." By the definitions of interior and junction points, none of the points interior to any one quadrant is connected to any point in another quadrant. In effect, the junction points marked by "4" form a "barrier" between the quadrants. This subdivision scheme is applied recursively to each quadrant, so that the dissection is "nested."

George (1973) shows that if the stations are ordered so that the unknown parameters are eliminated in the indicated order, then the total number of nonzero coefficients (original plus fill-in) is bounded by

$$\text{const } n \log n.$$

Meissl (1980: p. 23) shows that this is asymptotically superior to both bandwidth minimization and variable bandwidth (profile) minimization schemes. George (1973) further showed that no ordering algorithm can improve upon nested dissection asymptotically by more than a constant factor. Meissl (1980) notes further that nested dissection is also superior when comparing the number of arithmetic operations needed to form the Cholesky factor. Since the number of arithmetic operations was expected to be a major cost factor in the NAD 83 adjustment, we found this analysis to be important.

George's "nested dissection" scheme was presented as an ordering scheme to be applied to an adjustment which was carried out in one computer run, completely in computer central memory, and using an individual element storage structure. This was clearly different from the adjustment of the North American Datum, for which an in-core solution was not contemplated. However, from the similarities between George's dissection scheme and the Helmert blocking procedure, it was clear that the best results would be obtained by starting with the whole network and recursively dividing it into quarters. The number of numerical operations would be least if the subdivision were carried out recursively until there were no interior points left in any first-level block. This suggested a large number of extremely small first-level blocks.

Although the number of arithmetic operations was important, it was not the only cost factor to be considered in planning the computational procedures for the NAD adjustment. Some type of file management scheme had to be created to manage the numerous files of partial reduced normal equations which would arise. These files would likely be backed up on tape as well as disk. In addition, there would be many aspects of file management that would require considerable human effort. People, not machines, would be expected to initiate actions to assign file names, restore damaged and lost files, and handle similar details. This consideration suggested that it was better to have fewer, rather than more, Helmert blocks, and that the first-level blocks should therefore be larger rather than smaller.

The compromise was to aim for a first-level block of a few hundred points. This quantity is small enough to still require several levels of the Helmert block procedure, but also large enough to invite some scheme for exploiting the sparseness of the normal equations at the lowest level. To exploit the sparseness we selected the variable bandwidth storage structure and profile minimization scheme that had previously been used at NGS (Schwarz, 1978; Snay, 1976).

The nested dissection scheme appears to suggest that best results are obtained if the analyst combines four blocks at a time in the Helmert blocking procedure. It suggests a preference for the broad tree strategy of figure 14.2d over the deep tree strategy of figure 14.2a. However, the analysis described by George (1973) is actually neutral on this question. George implies a preference for minimum degree ordering to determine the order of elimination of all points at a given level of elimination, but this preference is not a critical factor.

Since the question of whether to choose a broad tree or a deep tree strategy could thus not be resolved on the basis of counts of mathematical operations, we compared the operational advantages of the two approaches. Figure 14.6 describes the operations which occur when a system of two Helmert blocks is combined two at time, while figure 14.7 describes the same system solved by combining four blocks at a time. The following characteristics were noted:

1. The deep tree strategy allows more operations to proceed independently and in parallel. In the example of figure 14.6, blocks 1 and 2 can be combined and reduced as soon as they are formed, without waiting for the formation of blocks 3 and 4. As an operational matter, it is always better to get something done sooner rather than wait until later. Later, the computer may be clogged up with other work.
2. Combining more blocks in a broad tree strategy requires the formation of larger blocks (before elimination of interior unknowns). In figure 14.7 the total number of unknowns input to the top level is larger than in figure 14.6. This means that the broad tree strategy would be the first to be constrained by a program that limits the total number of stations that can be handled at one time.
3. In addition to being larger, the higher level blocks formed with a broad tree strategy are more sparse than those formed with the deep tree strategy. Use of a broad tree would therefore invite the use of a sparse matrix storage structure and reordering algorithm at the higher levels, while the use of a deep tree strategy would largely obviate the need for such algorithms.
4. Combining four blocks at a time results in a total of five Helmert blocks to be formed, reduced, and stored, while combining two blocks at a time results in seven such blocks. For a large number of first-level Helmert blocks, the ratio of the total number of blocks for these two strategies approaches 2/3. In fact, if even more blocks are combined at a time, the advantage is even sharper. If we have N first-level blocks that are combined k blocks at time, then the total number of blocks to be formed, reduced, and stored is $(k\ N-1)/(k-1)$. As N becomes large, the ratio of this number to the case of $k=2$ approaches $k/(2\ k-2)$.

We felt that the first three characteristics, which suggest a preference for a deep tree strategy, outweigh the last characteristic, which suggests a preference for as broad a tree as possible.

All of these considerations provided the following guidance for setting up a strategy and carrying out an adjustment:

1. The partitioning of the network into first-level blocks should be carried out completely and a strategy should be designed before the adjustment is begun. This eliminates any surprises.
2. The strategy should be stored in machine-readable form, available to the adjustment and file handling programs. Since the strategy needs to be available for the duration of an adjustment project, which could last anywhere from 1 day to several months, we formed a named Adjustment Project File (APF) for each adjustment to be carried out. These files also held other information important to the adjustment. By using distinct names we could have more than one large Helmert block adjustment project in progress at a time.
3. The system of Helmert blocks should be developed by binary dissection, in which each block is divided into two subblocks. This also implies that the blocks should be combined two at a time, resulting in a deep tree strategy.
4. The subdivision should be done from the top down, starting with the entire network and subdividing into smaller subblocks. The subblocks should be further subdivided until all first-level (undivided) blocks have between 500 and 2,000 stations. The actual adjustment is performed from the bottom up.
5. At each subdivision, a block should be divided into two subblocks of approximately equal size. The dividing line should be drawn through weakly connected areas of the network, so that only a relatively few junction points are formed. Dividing lines must also follow 7½ minute graticule lines. Within these constraints, we should choose dividing lines which run roughly north-south or east-west.

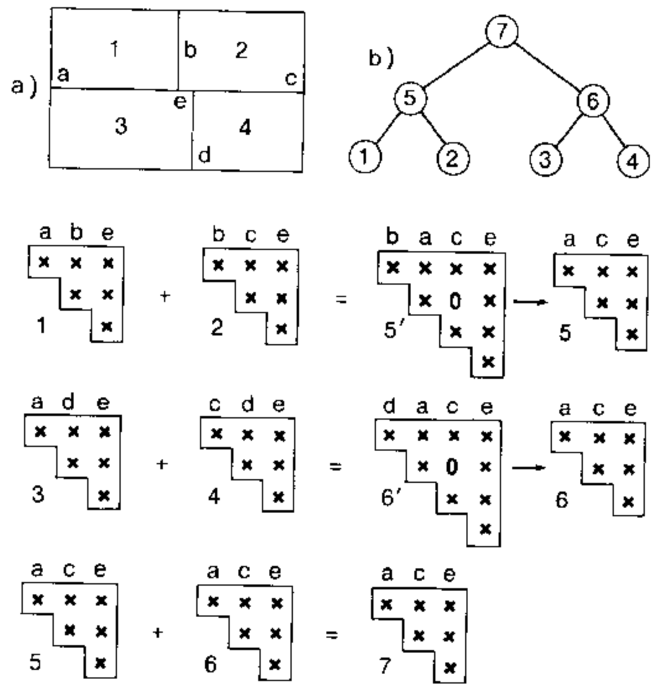


Figure 14.6. Combination of blocks by a deep tree strategy.

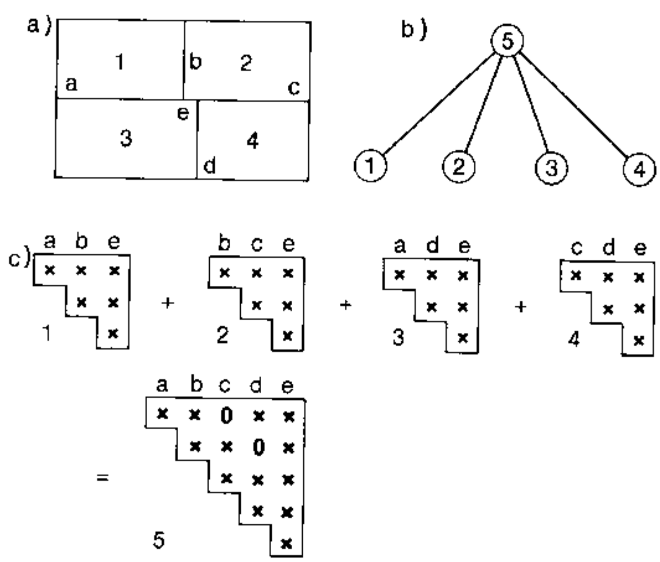


Figure 14.7. Combination of blocks by a broad tree strategy.

6. The most critical dividing line is the first, which divides the entire network in two, since this line is likely to create the greatest number of junction points. If we are able to draw this line in such a way that the total number of points to be handled at this level fits within the limitations of the programs, then we are unlikely to have problems with program limitations at lower levels.

14.4 STRATEGY DESIGN TOOLS

A set of software functions was developed to aid in the design of a Helmert block strategy. These software tools allowed for the possibility that we might wish to develop more than one candidate strategy, and that such strategies might be related. Figure 14.8 shows interactions of the Strategy Development program. The following major directives are implemented by this program:

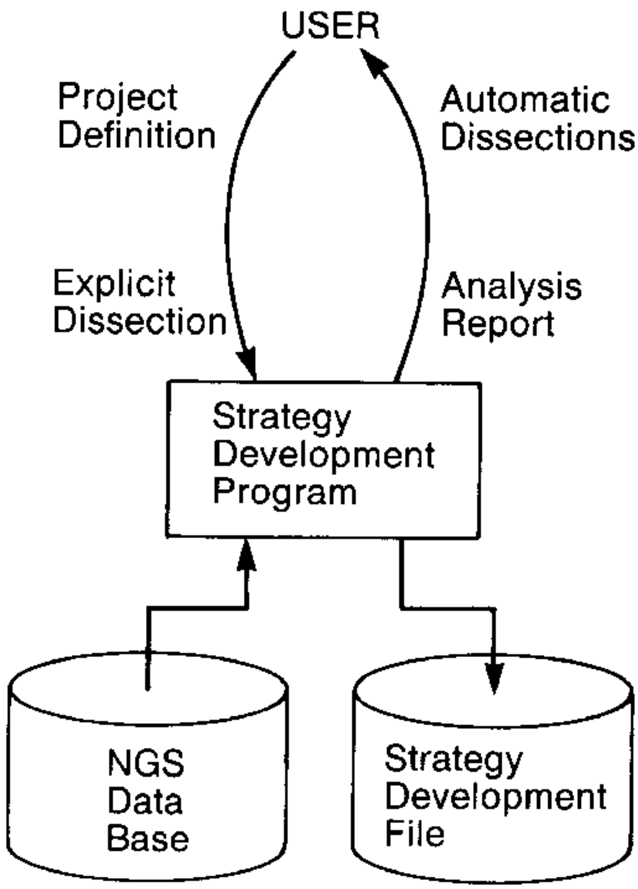


Figure 14.8. Interactions of the Strategy Development program with the user and with its major files.

1. CREATE STRATEGY. The user must define the entire geographic area of the network, and this becomes the definition of the area of the highest level block. Since the user defines a geographic area in terms of multiple rectangles bounded by maximum and minimum latitude and longitude, this directive checks the user's definition for proper form. The network area must be simply connected.

2. SAVE STRATEGY. This directive creates a STRATEGY DEVELOPMENT FILE and saves the strategy currently under development.

3. RESTORE STRATEGY. This directive reads a STRATEGY DEVELOPMENT FILE and makes it the strategy under development.

4. SUBDIVIDE A BLOCK. The block to be subdivided must be specified. Three methods are provided for specifying the dividing line:

   a. EXPLICIT SEPARATOR. The user explicitly specifies a dividing line by listing its vertices. The line must run along 7½ minute graticule lines, and must also begin and end on the existing block boundary.

   b. MEAN LATITUDE. The mean of the latitudes of all the points in the block is computed and rounded to the nearest multiple of 7½ minutes. This graticule line becomes the dividing line.

   c. MEAN LONGITUDE. The mean of the longitudes of all the points in the block is computed and rounded to the nearest multiple of 7½ minutes. This graticule line becomes the dividing line.

   As a result of this directive, two new subblock area definitions are created (and presented to the user in multiple rectangle form). The strategy tree under development is updated to include these new areas.

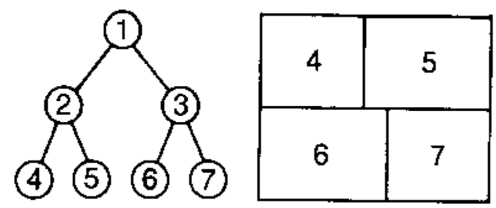
5. ANALYZE. The strategy under development is analyzed by counting the number of points which must be handled at each level. The geodetic data base is read to determine the connections between points. A report in the form of figure 14.9 is produced. This software function simulates the entire Helmert block procedure. It must be used with discretion, since it can easily result in a long computer run.

6. CONTINUE. Form a strategy automatically. All existing first-level blocks which contain at least 200 points are subdivided by the mean latitude separator, creating a new set of first level blocks. Of these new first-level blocks, those which contain at least 200 points are divided by the mean longitude separator. This process is continued recursively until no block contains more than 200 points.

7. DELETE. A node of the strategy under development, together with all of descendants, is deleted. This enables the user to restart the development of a strategy at some prior stage.

14.5 EXPERIENCE

The Strategy Development program allowed the user to examine several alternative strategies and to select the one which would result in the smallest total number of points passed forward through the Helmert block procedure. In practice, we found that most analysts preferred the explicit separator to the other alternatives. By examining even a generalized network diagram, the analyst was able to select a dividing line that produced acceptable results.



REPORT

Block No.	Separator code	Son 1	Son 2	Inside points	Input points	Junction Points Inside	Junction Points Outside	Points Elim.	Points Fwd.
1	1	2	3	2,000	86	0	0	86	0
2	3	4	5	1,050	210	40	44	126	84
3	3	6	7	950	130	32	49	49	81
4	0	0	0	550	690	72	140	478	212
5	0	0	0	500	566	81	66	419	147
6	0	0	0	450	502	62	52	388	114
7	0	0	0	500	577	46	77	454	123
TOTALS								2,000	961

Figure 14.9. Example of a report formed by analyzing a network of 2,000 stations divided into four Helmert blocks by binary dissection. The figure of merit for this strategy is the total of the points passed forward, shown in the last column.

14.6 REFERENCES

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15. HELMERT BLOCKING COMPUTER PROGRAMS

Edward H. Herbrechtsmeier

15.1 INTRODUCTION

In the following discussion we shall speak of "Helmert blocks" or just "blocks." Properly, a block consists of a geographical area definition, a set of normal equations that corresponds to this area, tables of diagonal terms, preliminary values, unknown identifiers, and assorted bookkeeping information. We may, at times, use the term block rather more loosely to refer to only the area or the set of normal equations. These distinctions should be clear in context.

We shall also speak of the "Helmert blocking system." This had two different meanings. The broader meaning was the total system of programs which performed the adjustment, once the data base was established. Specifically, it included the programs which generated observation equations and those which computed and analyzed residuals. The more narrow meaning referred only to the system of programs, files, and procedures which solved the normal equations. The equation solver was written so that it could be used with any system of equations. It has no feature which depends of the type of equations that are being solved, so it can handle normal equations which arise from a leveling network just as well as it handled the NAD horizontal network.

The user interfaced with the system via procedures, most of which were written in an interactive command language. The procedures generally prompted the user for the necessary parameters and input file names. Most procedures executed one or more computer programs. In some cases the programs were executed in the foreground, all within a single computer session. In other cases the programs were executed in the background as a separate computer run. In many cases a program or computer run would initiate another computer run, causing execution of one or more other programs. Thus a single user session could result in many computer runs and program executions.

The "Helmert block system," in both the broad and narrow sense, was made up of procedures and programs. The programs in turn relied heavily on subroutine packages, the abstractions, which were made up of individual subroutines that implemented the Helmert blocking algorithms.

15.2 DESIGN GOALS

The primary task was to form and solve a system of equations that was considerably larger than anything NGS or any other geodetic agency had previously handled. The Helmert blocking method provided an appropriate approach. However, the price to be paid was an increase in complexity; i.e., it became necessary to track and coordinate a large number of interrelated blocks instead of a single system. Furthermore, "disassembling" a single system into many subsystems generated problems that would not exist in a larger single system. It became clear at an early stage that controlling complexity had to be one of the primary concerns of the design.

An objective of the design was to have the computer system manage most of the generated coordination problems. The user was to be shielded from the added complexity which arose when a single process was partitioned into many computer runs. To the degree possible, the Helmert block system was to be no harder to use than those programs which performed a network adjustment in a single computer run. At NGS, the standard for comparison was the TRAV10 program (Schwarz, 1978).

One of the more obvious of the "generated" problems is that of retaining systems of equations between jobs. Previous adjustment procedures used various data structures such as arrays and files to represent a system of equations. These structures had no "life" beyond a single computer run; they were created as needed during the computer run and disposed of when the run terminated. The most obvious way to save equations between runs was to make run temporary files into permanent files and to write the contents of various program variables (e.g., arrays) to other permanent files. This approach would convert each system of equations to a set of files. Its difficulty is that it converts one entity (a system of equations) into many entities (a set of files) and thereby increases the complexity of the system.

The solution we chose for this problem was to put all of the parts of a system of equations into a single file. In addition to the normal equations, this file contained the preliminary values associated with each unknown, the unreduced diagonal terms of the equations, a table of the names of the unknowns, and some identification information. A file of this type was often referred to as a Helmert block.

There are a number of problems involved in coordinating various parts of the system. One of these is to track the state of all of the blocks involved. The geodesist needs to know whether or not a block exists and whether it is a reduced block, a solved block, an inverted block, or a block that is "in process." The NGS Helmert blocking programs used a single file, called the Adjustment Project File (APF), to contain the control information for the entire system. This file had several partitions. It contained the strategy, the lock and phases for each node in the strategy, and the definition of the geographical area for each node in the strategy. It also served as the repository for a number of parameters that were global to the adjustment.

Another problem was dealing with the necessity to keep track of all the computer files that are temporary to the solution process but permanent to the computer system, since they exist between computer runs. The solution was to provide a large number of cataloged files that are known to the Helmert blocking system. Three such files were created for each Helmert block, although the user was largely unaware of their use and existence. These files could be stored on either tape or disk.

The Helmert blocking system was also provided with an automatic mode. In this mode, the computer system schedules and dispatches all the necessary computer runs. The user can walk away; the system stops only when its task is completed or there is an error.

The major task of the user is to provide the Helmert block system with all the input Helmert blocks. These are the blocks of partial normal equations corresponding to the leaf nodes. After these are registered with the system, the system may be placed in automatic mode. The user may monitor the progress of the adjustment and return to manual control at any time.

15.3 PROGRAMMING METHODOLOGY

The fundamental software development techniques used in the design and implementation of the Helmert blocking system were informal specification and data abstraction. The term informal specification means that the methodology used is a relaxed form of the formal specification method. The methods employed in data abstraction are much the same as those employed in object-oriented programming.

A data abstraction is defined as a collection of data objects (probably in machine-readable form) and a collection of operations on the objects such that the behavior of the objects can be completely specified in terms of the operations. An object is created or transformed only as a direct result of an operation on the object, never as a side effect of another operation. Both procedural and representational detail are suppressed. All such detail is inside the code which implements the abstraction and cannot be seen from outside.

The operations on an object are separated into two groups. Each operation in the "O" group causes an object to undergo a change of state. Each operation in the "V" group causes no change of state but allows some aspect of the current state to be viewed from the outside. The state of an object (or at least the externally visible component of its state) is simply the collective results of all V-operations. The specification, then, need only indicate the effect of each O-operation on the result of each V-operation.

The use of data abstractions was advanced at NGS by John Isner, who singlehandedly produced most of the computer programs that make up the Helmert blocking system. His ideas are further described in Isner (1982).

Nine data abstractions were written and became the core of the Helmert blocking system. Each was implemented as a package of subroutines available to the main programs and to other subprograms. The abstractions were written first in Univac ASCII Fortran, which allowed internal subroutines. Later, when the system was ported to an IBM mainframe, all the programs, including the abstractions, were rewritten in PL/1.

ATYPE is the area data abstraction. Its primary use was in the development and use of the strategy. It was used to represent the geographical areas that corresponded to each node in the strategy. ATYPE provides operations for creating an area, dividing an area, adding two areas together, and checking whether or not a point, quad, or an area is inside an area.

BTYPE is the bag data abstraction. A bag is defined to be a file system object that is used to store other objects (including other BTYPE objects). Many of the other data abstractions provide operations for storing and retrieving their objects into and from bags. For example, ATYPE provides BAGA to put a copy of an ATYPE object into a bag and UNBAGA to retrieve an ATYPE object from a bag. All of the data that were stored between runs in the Helmert blocking system were kept in BTYPE objects. Thus the Adjustment Project File and all of the Helmert blocks were BTYPE objects.

ETYPE is the equivalence class data abstraction. An equivalence class is a set of data points that are said to be "equivalent" in some user-defined sense.

FTYPE is the direct-access, file data abstraction. An FTYPE object is a file containing variable length records of any size. It provides considerably more flexibility in handling direct access files than that normally found in higher level languages. Its primary use was in NTYPE for the out-of-core storage of normal equation elements.

GTYPE is the graph data abstraction. This handles objects that are graphs in the mathematical sense, i.e., sets of vertices connected by edges. GTYPE was used in network analysis to detect unobserved and no-check stations. It was also used to reorder unknowns in the normal equations to reduce the profile.

NTYPE is the normal equations data abstraction. This was a central focus of the Helmert blocking system. It provides an extensive set of normal equation operations. NTYPE was used to form, partially or fully reduce, solve, or invert normal equations. It implements the inner product form of the Cholesky method of solving a symmetric system of linear equations as described by Hanson (1974). It handles large systems by partitioning the normal equation elements into pages that normally reside out of core and are brought into core memory as needed, as described by Poder and Tscherning (1973). The innermost loop (the computation of partial inner products) is coded in Assembly Language.

TTYPE is the table data abstraction. TTYPE implements two-column tables. One of the columns is known as the "key" column and the other as the "value" column. The data in each row of the key column are required to be unique. There is no restriction on what is contained in the value column. TTYPE is used extensively in the many name-matching and cross-referencing operations of the Helmert blocking system. It provides a method that is both fast and easy to use.

WTYPE is the stopwatch data abstraction. It is used for timing various portions of programs.

YTYPE is the tree data abstraction. Representation of the Helmert blocking strategy is the primary use of YTYPE.

15.4 THE PROGRAMS

The Helmert blocking system consists of a total of 63 programs, most of which implement the automatic equation solver. The major programs are described below. Figure 15.1 shows the major data and control flows.

15.4.1 Systems Creation

CRAPF is used to create an Adjustment Project File (APF) and thereby initialize a Helmert blocking adjustment project. Its major input is a strategy in machine-readable form. (See chapter 14.) Other inputs include a list of special junction points and the definition of the observation class decks.

The APF becomes a permanent file associated with a named adjustment project. There is a program to provide a formatted report of the state of an APF and several utilities to modify an APF.

15.4.2 Data Base Programs

The major data base procedure is RETRIEVE_RESTART_83. This executes several programs inside the geodetic data base environment. It retrieves data for a Helmert block and stores them in a RESTART File (described in chapter 10), outside the data base envi-

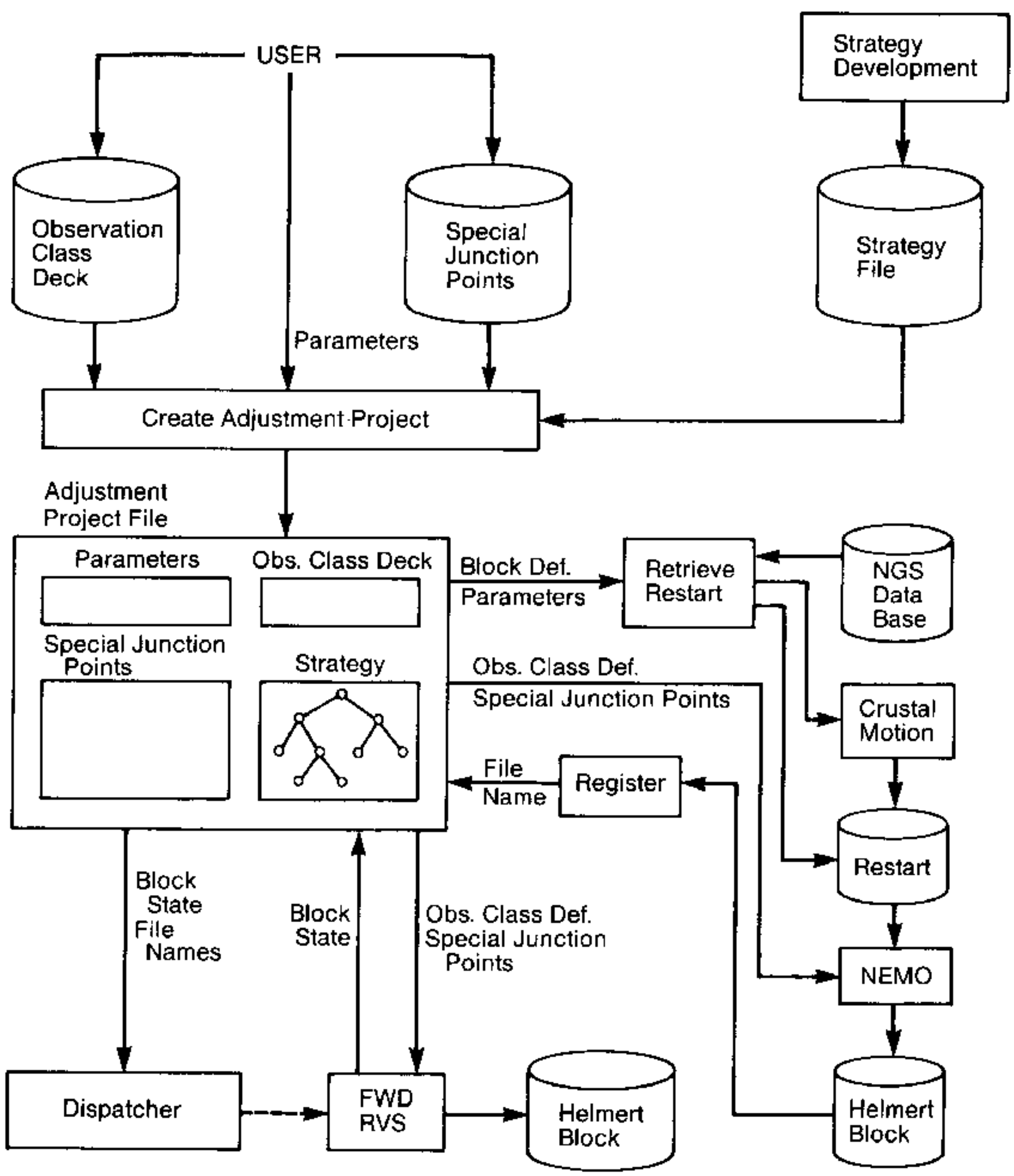

Figure 15.1. Helmert blocking system.

ronment. The Helmert block boundaries are taken from the Strategy section of the APF. A fixed set of parameters is used in this data base retrieval. The NAD 27 coordinates in the data base are transformed to preliminary NAD 83 coordinates, using a digital representation of the graphs described by Vincenty (1979). Observed values in areas of crustal motion are transformed to values that would have been observed on Dec. 31, 1983, using the methodology described in chapter 17. All observations which cross the block boundary in either direction are included. The coordinates of the outside stations that participate in these observations are also included.

15.4.3 Lowest Level Programs

HBNEMO is the Helmert block version of the NEMO program described in chapter 10. It is the major lowest level program for terrestrial (scalar) observations. Its major input is a RESTART File that contains data corresponding to a Helmert block. It computes observation equations and partial normal equations. It identifies interior and junction points. Interior unknowns are reordered using Snay's algorithm (Snay, 1976) to minimize the matrix profile. Junction point unknowns are placed at the end of the system of unknowns, with no special ordering. For each junction point the program examines the block definitions in the APF and computes the block in which the point will become interior. HBNEMO generates observation equations according to the three-dimensional height-controlled model described in chapter 12. The default weighting scheme is described in chapter 18.

OMEN is the reverse of HBNEMO. It receives parameter corrections computed by the normal equation solver and computes updated coordinates and residuals. The RESTART file for the block is updated.

POSTPROC analyzes the residuals for terrestrial observations. Reports are prepared concerning the largest residuals and the rms residuals for various subsets of the data.

SOAP computes observation and partial normal equations for all space system observations (Doppler, VLBI, and local three-dimensional surveys). It implements the vector observation equations described in chapter 12. It includes a variety of options concerning which global parameters are to be free and which are to be constrained to some a priori value. It also includes a reverse mode in which it updates coordinates and computes residuals.

STREPORT prepares a station report from a RESTART file, listing all data (including observations and residuals) associated with a single station.

15.4.4 Mid-Level Programs

FWD performs the forward reduction of a Helmert block. It combines two or more input Helmert blocks containing partial reduced normal equations. The union of all the station coordinates and other parameters in all the input blocks is formed. If a particular unknown parameter appears in more than one block, the programs must check that the same approximate value of the unknown was used in each case. All the unknowns are reclassified as either interior or junction. Normal equation elements are accumulated. The interior unknowns are eliminated from the system of equations and the new Helmert block, containing reduced normal equation terms, is written out.

RVS performs the back solution for a Helmert block. Given the normal equations for a block and the solution for the junction points, the solution for the interior points is computed.

INV computes the matrix inverse of the normal equation coefficients for a Helmert block. Only the inverse terms within the matrix profile of the original normal equation coefficient matrix are computed using the algorithm described by Hanson (1978).

15.4.5 Highest Level Programs

When the analyst reaches the highest level of the Strategy and all the blocks have been combined, only a single Helmert block is left. All the unknowns now become interior. The user can proceed directly to a solution of that block. However, this block contains the global unknowns and special junction point unknowns. It was assumed that there would be a desire to experiment with different constraints at this level. Therefore, the highest level programs were taken outside the system of automatic job scheduling and execution.

HLS2 is the highest level system program. It can be used to add, factor (fully or partially), solve, or invert Helmert blocks.

STOAT computes the solution of a Helmert block for which there is a complete forward solution. It differs from RVS in that it operates outside of the automatic job scheduling and execution system.

VOLE computes the inverse of a Helmert block at the highest level. It differs from INV in that it operates outside of the automatic job scheduling and execution system.

15.4.6 Utility Programs

Major utility programs are described below.

APFRPT generates a formatted report on the state of the APF (and hence the state of the Helmert blocking system).

HBRPT generates a formatted report on the contents of a Helmert block.

HBCOPY copies a Helmert block from tape to disk.

HBDUMP dumps a Helmert block to an ASCII file. The file is formatted according to the transfer structure agreed upon by NGS and the Geodetic Survey of Canada.

HBLOAD is the reverse of HBDUMP. It creates and populates a Helmert block by loading the data from a transfer file.

HTRPT generates a report on the Helmert block tape management system, which is meaningful if Helmert blocks are being stored on tape instead of disk.

LOCK is used to lock and unlock nodes in the Helmert block strategy.

15.5 ALGORITHMS

The Helmert blocking system solves the normal equations by the Cholesky algorithm. Most of the work is performed in the forward reduction program FWD. After the unknowns have been classified as either interior or junction, they are ordered so that the interior unknowns come first, but the ordering is otherwise arbitrary. Once the ordering of the unknowns is determined, each normal equation coefficient and constant term has an assigned row and column number. Normal equation elements are accumulated in their proper locations. The upper triangular part of the combined system is depicted as

$$\begin{array}{ccc} \ddot{\mathbf{N}} & \bar{\mathbf{N}}^T & \ddot{\mathbf{U}} \\ & \dot{\mathbf{N}} & \dot{\mathbf{U}} \\ & & \mathbf{S} \end{array}$$

The data abstraction NTYPE transforms this to

$$\begin{array}{lll} \ddot{\mathbf{T}} & (\ddot{\mathbf{T}}^T)^{-1}\bar{\mathbf{N}}^T & (\ddot{\mathbf{T}}^T)^{-1}\ddot{\mathbf{U}} \\ & \dot{\mathbf{N}} - \bar{\mathbf{N}}\ddot{\mathbf{N}}^{-1}\bar{\mathbf{N}}^T & \dot{\mathbf{U}} - \bar{\mathbf{N}}\ddot{\mathbf{N}}^{-1}\ddot{\mathbf{U}} \\ & & \mathbf{S} - \ddot{\mathbf{U}}^T\ddot{\mathbf{N}}^{-1}\ddot{\mathbf{U}} \end{array}$$

where $\ddot{\mathbf{T}}$ is the upper triangular Cholesky factor of $\ddot{\mathbf{N}}$.

The highest level system takes a system of reduced normal equations depicted as

$$\begin{array}{cc} \tilde{\mathbf{N}} & \tilde{\mathbf{U}} \\ & \mathbf{S} \end{array}$$

and calls on NTYPE to transform this to

$$\begin{array}{ll} \tilde{\mathbf{T}} & (\tilde{\mathbf{T}}^T)^{-1}\tilde{\mathbf{U}} \\ & \mathbf{S} - \tilde{\mathbf{U}}^T\tilde{\mathbf{N}}^{-1}\tilde{\mathbf{U}} \end{array}$$

where $\tilde{\mathbf{T}}$ is the upper triangular Cholesky factor of $\tilde{\mathbf{N}}$.

This completes the forward reduction. The back solution is begun with the completely reduced normal equations at the highest level. STOAT uses NTYPE to transform this system into

$$\begin{array}{ll} \tilde{\mathbf{T}} & \tilde{\mathbf{T}}^{-1}(\tilde{\mathbf{T}}^T)^{-1}\tilde{\mathbf{U}} \\ & \mathbf{S} - \tilde{\mathbf{U}}^T\tilde{\mathbf{N}}^{-1}\tilde{\mathbf{U}} \end{array}$$

and the upper right corner is

$$\tilde{\mathbf{T}}^{-1}(\tilde{\mathbf{T}}^T)^{-1}\tilde{\mathbf{U}} = \tilde{\mathbf{N}}^{-1}\tilde{\mathbf{U}} = \tilde{\mathbf{X}},$$

the solution for the unknowns at the highest level.

The solution is then propagated back through the Helmert blocks by RVS. For each block the term at the second row and third column is replaced by the solution for the junction point unknowns from the next higher level, giving

$$\begin{array}{lll} \ddot{\mathbf{T}} & (\ddot{\mathbf{T}}^T)^{-1}\bar{\mathbf{N}}^T & (\ddot{\mathbf{T}}^T)^{-1}\ddot{\mathbf{U}} \\ & \dot{\mathbf{N}} - \bar{\mathbf{N}}\ddot{\mathbf{N}}^{-1}\bar{\mathbf{N}}^T & \dot{\mathbf{X}} \\ & & \mathbf{S} - \ddot{\mathbf{U}}^T\ddot{\mathbf{N}}^{-1}\ddot{\mathbf{U}}. \end{array}$$

The back solution is continued, transforming this system to

$$\begin{array}{lll} \ddot{\mathbf{T}} & (\ddot{\mathbf{T}}^T)^{-1}\bar{\mathbf{N}}^T & \ddot{\mathbf{T}}^{-1}\left[(\ddot{\mathbf{T}}^T)^{-1}\ddot{\mathbf{U}} - (\ddot{\mathbf{T}}^T)^{-1}\bar{\mathbf{N}}^T\dot{\mathbf{X}}\right] \\ & \dot{\mathbf{N}} - \bar{\mathbf{N}}\ddot{\mathbf{N}}^{-1}\bar{\mathbf{N}}^T & \dot{\mathbf{X}} \\ & & \mathbf{S} - \ddot{\mathbf{U}}^T\ddot{\mathbf{N}}^{-1}\ddot{\mathbf{U}} \end{array}$$

The term in the upper right corner is the solution for the interior unknowns $\ddot{\mathbf{X}}$.

The lower right corner of the triangular system of normal equations contains a scalar. This location is used to accumulate the weighted sum of squares of residuals, according to the equation

$$\mathbf{V}^T\mathbf{W}\mathbf{V} = \mathbf{L}^T\mathbf{W}\mathbf{L} - \mathbf{U}^T\mathbf{N}^{-1}\mathbf{U},$$

where $\mathbf{L}^T\mathbf{W}\mathbf{L}$ is the weighted sum of squares of observation equation constant terms. For each Helmert block at a leaf node of the strategy, the weighted sum of squares of the constant terms of the observation equations in that block is placed in the lower right corner. As the Helmert block solution progresses, the contributions from different blocks are added together. At the same time, the second term, $\mathbf{U}^T\mathbf{N}^{-1}\mathbf{U}$, is partially computed as each block is reduced. At the end of the forward reduction, the term in the lower right corner is

$$\mathbf{S} - \tilde{\mathbf{U}}^T\tilde{\mathbf{N}}^{-1}\tilde{\mathbf{U}} = \mathbf{V}^T\mathbf{W}\mathbf{V}.$$

At the end of every forward run, the term in the lower right corner is interpreted as the sum of squares of residuals that would be obtained if all remaining junction points were held fixed at their current approximate values. The input and output values of this term are stored in the Log file for use by the analyst.

15.6 CONTROL MECHANISMS

15.6.1 Courses and Phases

The computation of the Cholesky factor of the normal equations (including the constant column) is said to be the *forward* course of the solution, while the back substitution is said to be the *reverse* course. In a Helmert block solution the forward course is accomplished over a number of separate and independent computer runs. The same is true of the reverse course. Furthermore, the complete least squares solution may require several iterations of the forward and reverse courses.

Each node of the Strategy carries an attribute called the *phase* and represented by an integer number. An odd numbered phase signifies that the forward reduction of the Helmert block belonging to the node has been accomplished. All the subblocks have been combined and all interior unknowns have been eliminated. An even numbered phase means that the back solution for the block has been accomplished. The normal equations associated with the block contain the corrections to the unknowns in that block. If P is the phase number, then (P − 1) (modulo 2) is the current iteration number for the block.

The following rules are used for assigning phase numbers:

1. Initially, all nodes have phase zero.
2. When a node is registered (see below), its phase is increased by 1.
3. When a forward job completes successfully, the parent node obtains the same phase as its children.
4. When a reverse job completes successfully, the child nodes obtain the same phase as the parent.

When all the children of a node have an odd phase number which is one greater than that of the parent, then the job of extending the forward solution to the parent is said to be *enabled*. The child blocks have all been reduced; the reduced normal equations can now be combined and the unknowns which are interior to the parent block can be eliminated.

When all the children of a node have an odd phase number which is one less than that of the parent, then the job of extending the reverse solution from the parent to the children is said to be *enabled*. The Helmert block associated with the parent node contains the solution for all parameters in that block; these are junction point unknowns in the child blocks. The solution for the interior unknowns in each child block can now be computed.

Figure 15.2 shows a typical strategy where all the phases are initially zero.

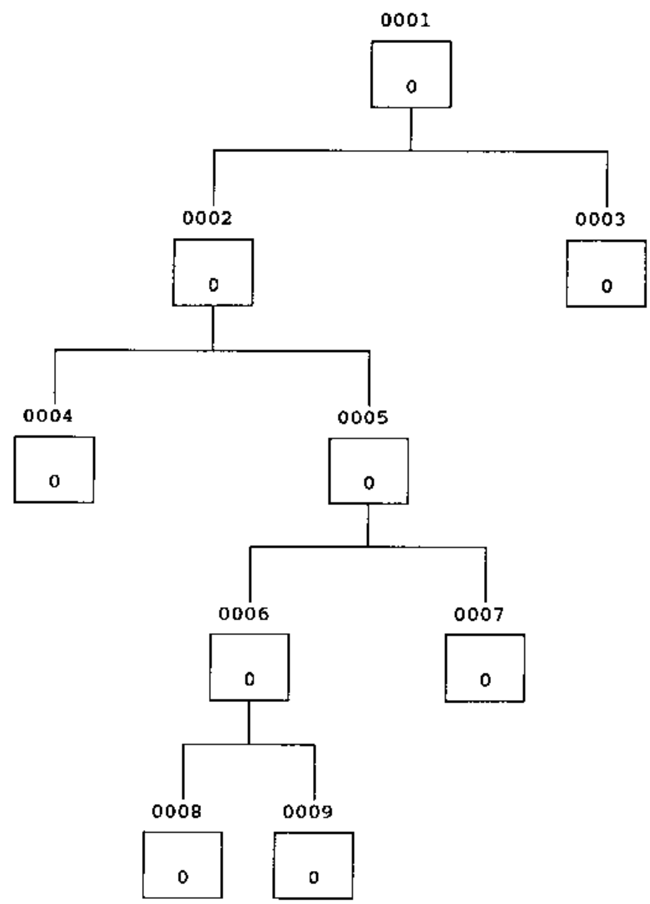

Figure 15.2. A Helmert blocking strategy in the initial state. Node number appears above the box and phase number within the box.

15.6.2 Registration

A Helmert block (set of partial normal equations) is introduced into an adjustment project by a process called *registration*. The equation solver of the Helmert blocking system makes a copy of the normal equations for its own use, stored in files which belong to the system and are largely invisible to the user. The phase of the node of this block is increased by one. The file name of the internal file which contains this Helmert block is an attribute of the node.

Figures 15.3 (a,b,c,d,f) shows the changes to the strategy of figure 15.2 as a result of registering Helmert blocks for nodes 4,3,9,8, and 7, respectively.

15.6.3 The Dispatcher

The DISPATCHER procedure and program initiate all jobs that are enabled. Dispatcher does this by submitting procedures containing executions of the FWD and RVS programs to the operating system. Each of these jobs runs independently and possibly concurrently.

Each job which changes the phase of some node, such as a registration of a Helmert block or the successful completion of a forward or reverse run, submits another job which calls the Dispatcher. If the change of phase has caused some new job to be enabled, then the Dispatcher will submit that job. By this means the Helmert blocking system will continue to run as long as it can find useful work to do. At times there may be several jobs active on the computer. When no more jobs are enabled the system stops.

The automatic mode of the Dispatcher can be stopped by the Inhibit function. The INHIBIT parameter in the APF is set to true and the Dispatcher is suppressed. If the Inhibit function is performed while jobs are active, the active jobs will be allowed to complete, but no new jobs will be created.

The automatic dispatching mode can be restored by the Uninhibit function. An execution of the Dispatcher will cause the system to "wake up" and begin dispatching enabled jobs.

In figure 15.3, after the Helmert blocks for nodes 8 and 9 have been registered, the forward reduction of block 6 is enabled. If the Dispatcher is executed at that point and the forward job completes successfully, then the state of the adjustment will be changed to that of figure 15.3(e). At that point no more work can be done, there are no new jobs to be dispatched, and the system "goes to sleep." After block 7 of figure 15.3(f) is registered, the forward reduction of block 5 is enabled. When the Dispatcher is executed this job will be submitted to the operating system. When it completes normally, as shown in figure 15.3(g), the forward reduction of block 2 will be enabled. If the system is still in automatic (uninhibited) mode at that time, then the FWD job for block 2 will also be submitted. When it completes successfully the system

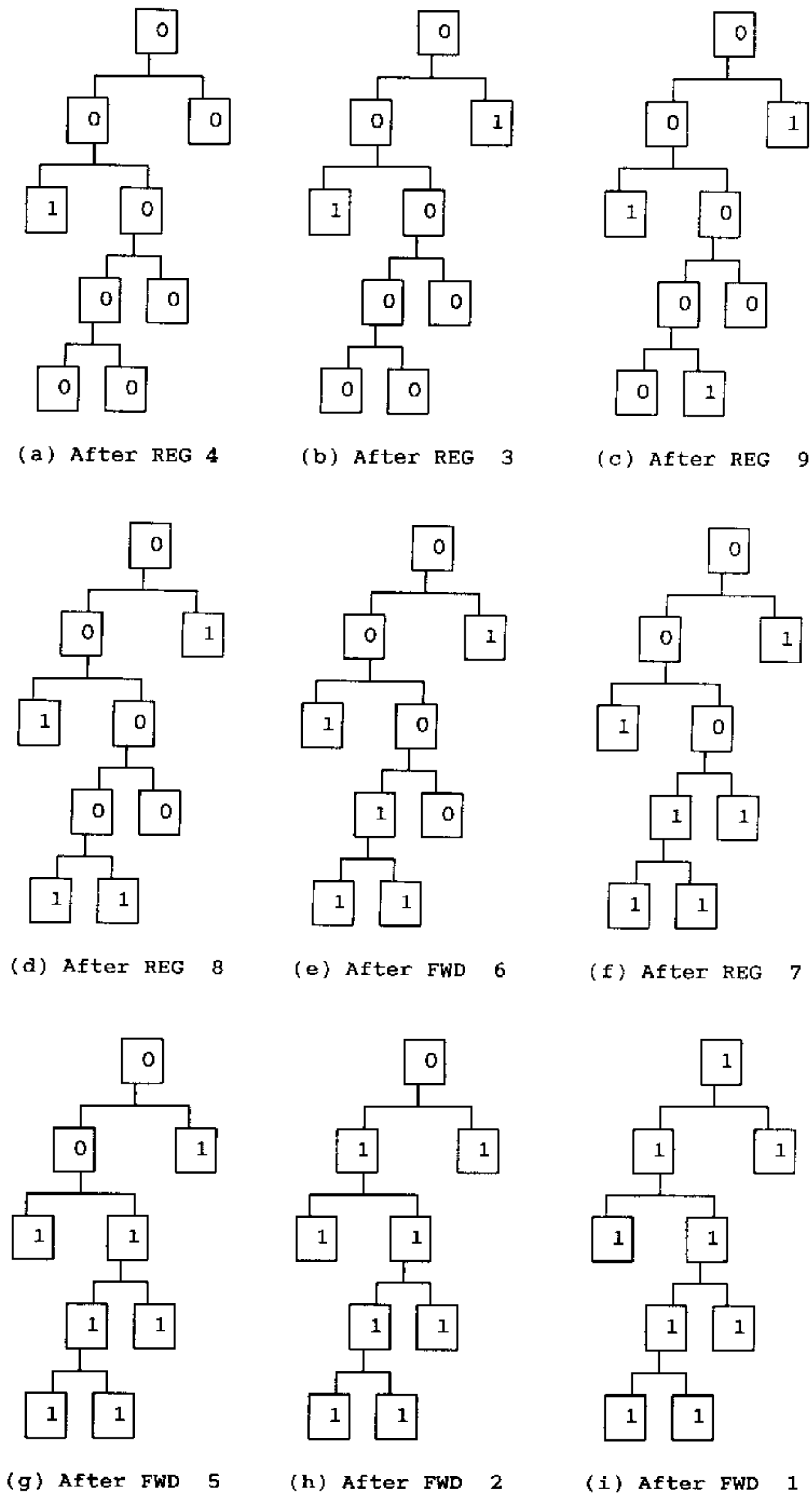

Figure 15.3. Changes to the phases in the strategy in figure 15.2 as a result of registration and processing.

will be in the state depicted in figure 15.3(h). Eventually the system reaches the state shown in figure 15.3(i); the forward course of the adjustment is complete, there is no more useful work to do, no more jobs are dispatched, and the system goes to sleep.

The solution of the highest level block is performed outside the automatic dispatching system. After this solution is available it is registered with the system and the state of the system is changed to that shown in figure 15.4(a). The reverse jobs for blocks 2 and 3 are now enabled. If the system is in automatic mode then the Dispatcher, when executed, will begin to submit jobs. If all jobs complete successfully, then the system will pass through the states shown in figure 15.4, finally reaching a state where the reverse course has been completed; no more work remains to be done, and the system goes to sleep again.

At the end of the reverse course of the zeroth iteration all nodes will be in phase 2, as illustrated in figure 15.4(e). The only thing remaining to be done to complete this stage of the adjustment is to transfer the solution now existing in each of the first-level Helmert blocks back into the corresponding RESTART files. This takes place outside of the automatic adjustment system.

When all analyses are complete and changes have been made to the RESTART files, the forward course of the next iteration can begin. Procedurally, the next iteration is identical to the first; the only observable difference will be in the phase numbers (which now go from 2 to 3 in the forward course, and from 3 to 4 in the reverse).

15.6.4 Locks

The Inhibit and Uninhibit functions provide global control of the system. Finer-grained control of events is possible using the Locks function. This function allows the selective placement of "locks" on nodes of the strategy, so that any enabled jobs involving those nodes will not be dispatched. For example, a lock could be placed on the strategy of Figure 15.2 to prevent the system from dispatching the forward job for node 2. The resulting state of the strategy is shown in figure 15.5, where the locked node is shown as a box made of X's.

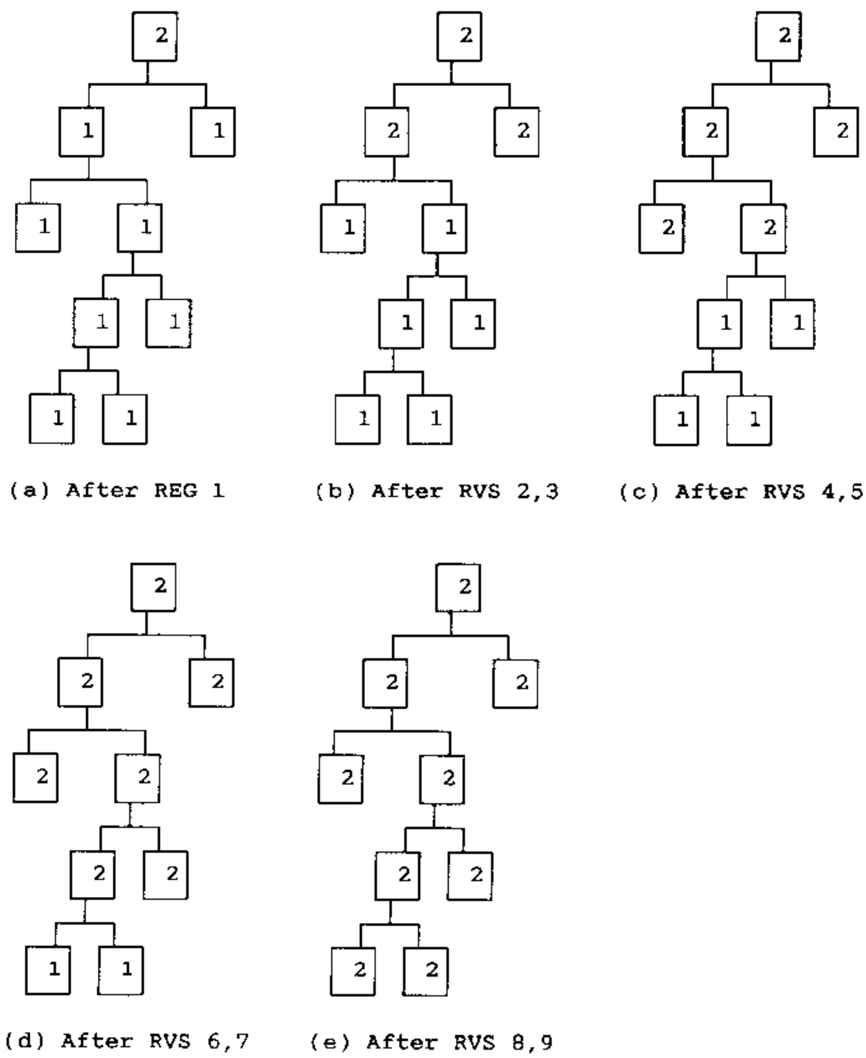

Figure 15.4. The reverse course of iteration 0.

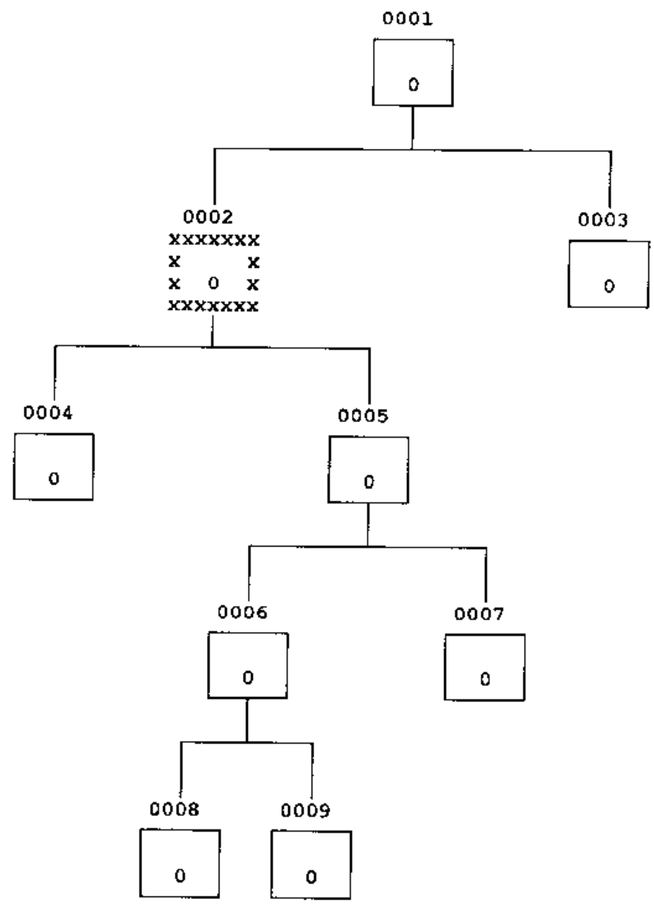

Figure 15.5. A strategy with node 2 locked.

The Locks function can also be used to unlock nodes which were previously locked.

In addition to the explicit locking and unlocking of nodes done by the user, there is a large amount of locking and unlocking performed by the adjustment system itself as part of its operation. When a forward or reverse job begins executing, it checks that all nodes which it needs are unlocked. If not, it aborts; otherwise, it places locks on all of the needed nodes. When it completes normally, the job removes the locks. When no jobs are active (i.e., not currently executing or waiting in the input queue), all nodes should normally be unlocked.

The system uses locks for two purposes. First, they prevent multiple redundant jobs from being dispatched. When a job has been dispatched and is running, it is still enabled (the phase is not updated until the job completes). The lock prevents another execution of the Dispatcher from submitting such a job a second time. Second, the presence of locked nodes in a project with no active jobs may indicate a prior system failure. For example, suppose that a forward job has failed because it ran out space in one of its scratch files. Since the job does not complete, the strategy does not change and the job remains enabled. The locks prevent the system from futilely dispatching the same job again, since the job would only fail again.

15.6.5 Setbacks

We have seen that registration may, as a side effect, enable forward or reverse jobs (depending on whether a first-level or highest level block is registered). Enabled jobs may be dispatched and may, in turn, enable other forward or reverse jobs, and so on. To put it simply, registration may cause the adjustment to "progress" one or more steps. In the normal course of events, where every job completes normally, the phase of a parent block is always equal to, one less than the phase of a child (during the forward course), or one greater than the phase of a child (during the reverse course). There is, however, the possibility that an error can be discovered in one or more basic Helmert blocks (leaf nodes). The user may then decide to re-register a block that has already been registered. Any progress that depended on the contents of the "old" block will then be lost. However any progress that did not depend on the old block need not be lost. We refer to the loss of progress due to such an untimely registration as a "setback." There are two kinds of setback:

LOCAL SETBACK	A first-level block is re-registered before the reverse course has begun.
GLOBAL SETBACK	(a) A first-level block is re-registered after the reverse course has begun. (b) The highest-level block is re-registered.

In a local setback, all Helmert blocks on the path from the block being re-registered to the highest level block must be "thrown away." In iteration zero, this is equivalent to resetting phases to zero along this path.

In global setback, any Helmert blocks containing a solution contaminated by information from the "old" version of the block must be thrown away. In iteration zero, all blocks in phase 2 must be reset to phase zero.

It is possible that at a given time two nodes in the same adjustment may be in different courses of different iterations. Figure 15.6 illustrates this point. In figure 15.6(a), node 3 has not yet received the back solution for iteration 0, while node 2 has not only received the back solution, but it has begun iteration 1. In this case, a decision to re-register node 3 would entail a global setback and the phases would be reset as in figure 15.6(b). Resetting node 2 to the beginning of iteration 1 would entail only resetting its phase to 2. Re-registering the block would then reproduce the state shown in figure 15.6(a).

15.7 INTERNAL ADJUSTMENT SYSTEM FILES

During the forward and reverse course of an adjustment, many Helmert blocks are processed. Each stage of transformation of each Helmert block is stored in a file that belongs to the adjustment system.

The default storage medium for internal files is tape. In the default case, one internal file (representing one Helmert block) occupies a single 6250 BPI 9-track labeled tape. The use of tape simplifies "storage management" considerably, because an unlimited supply of tapes is available and a tape can always hold a single Helmert block (even a very large one). Unfortunately, tapes have to be mounted, and this causes delays.

Using disk as the storage medium for internal files eliminates waiting for tape mounts, but creates a number of storage management problems associated with disk. When several files share a single disk pack, concerns are raised about device capacity and fragmentation. Unfortunately, these problems are not easy to solve automatically.

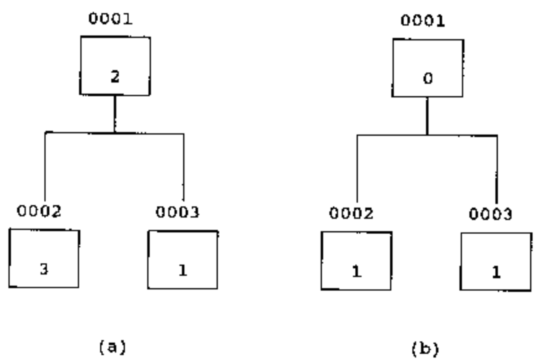

Figure 15.6. The effect of a setback.

As a compromise, the adjustment system provides a way to designate disk as the storage medium for internal files, but places responsibility for storage management on the user. When disk storage is used, Helmert blocks are copied onto disk at registration, and both forward and reverse jobs would create new Helmert blocks on disk. If the adjustment is large, the disk storage might eventually fill up with files, leaving some job with insufficient space for its output Helmert block. If this were to happen to a forward job, it would abort, but only after using considerable CPU time. Although the aborting job leaves its nodes locked, other active jobs would not be prevented from running, and they too would abort. This could result in a great waste of computer time. When using disk, therefore, the user must carefully monitor the space available.

Although internal adjustment files are managed by the adjustment system, and can ordinarily be ignored, it is nevertheless important for the user to understand conflicts. It is also helpful for the user to have some familiarity with the role of internal files in order to be able to interpret system messages concerning the files in the event of problems.

Three file names are catalogued for each Helmert block. These are Axxxx, Bxxxx, and Cxxxx, where xxxx is the number of the node. When the actual files are created by the adjustment system, the catalog will be updated to reflect an actual file location (e.g., tape volume or disk volume).

Internal adjustment system files are assigned names from the reserved set according to their usage. Files with "A" prefix names are used to store Helmert blocks for the forward course in EVEN iterations, files with "B" prefix names are used to store Helmert blocks for the forward course of ODD iterations, and files with "C" prefix names are used to store Helmert blocks for the reverse course of both odd and even iterations. Thus, for example, we can deduce that a file named C0003 contains a Helmert block with a back solution for node 3. Figure 15.7 shows the complete internal file usage picture for a simple 3-node strategy. Because of this arrangement, a block can always be set back as far as the forward course of the previous iteration.

15.8 USING THE SYSTEM

The Helmert blocking system was written to be used in a computer environment that provided a Conversational Remote Batch Entry (CRBE) system, but no true time sharing. The CRBE system supported a programming language (SUPERWYLBUR) which could be used interactively to prompt the user for data and which could submit jobs to the batch-oriented operating system (MVS) for scheduling and execution. SUPERWYLBUR could also perform a variety of file maintenance operations, including the listing and editing of text files. However, the interactive language had no significant numerical processing capability, and programs written in a true processing language could not be run interactively.

Because of this computer environment restriction, all user functions in the Helmert blocking system are made available by means of macros, which are programs written in the interactive language. Some macros accomplish their functions directly, but most create one or more batch jobs which accomplish the desired function in the background. When a macro submits a batch job it tells the user what job number was assigned. The user may wait to be notified that the batch job has completed, at which point he or she may examine the output text files with the interactive editor. Alternatively, the user may terminate the interactive session and allow the batch job(s) to continue to run.

User functions are divided into two groups, consistent with the plan that the labor in a large adjustment project, such as the NAD 83 adjustment, will be divided among many individuals. The first group contains functions that require overall knowledge of the adjustment project in order to be used safely. Functions in this group are capable of creating system tasks, monitoring system status, and managing system resources, and may affect the actual numerical results of the adjustment. Consequently, such functions should be performed by a single individual designated as project leader. The second group consists of those functions that may be performed safely by a subordinate "project member" who does not necessarily have a purview of the project. Functions in this group are not capable of affecting the adjustment directly (they operate "outside of the adjustment system"), and include routine file maintenance (backup/recovery of RESTART files) and reporting/analysis of first-level results. The highest level of an adjustment is also carried out by means of project member functions.

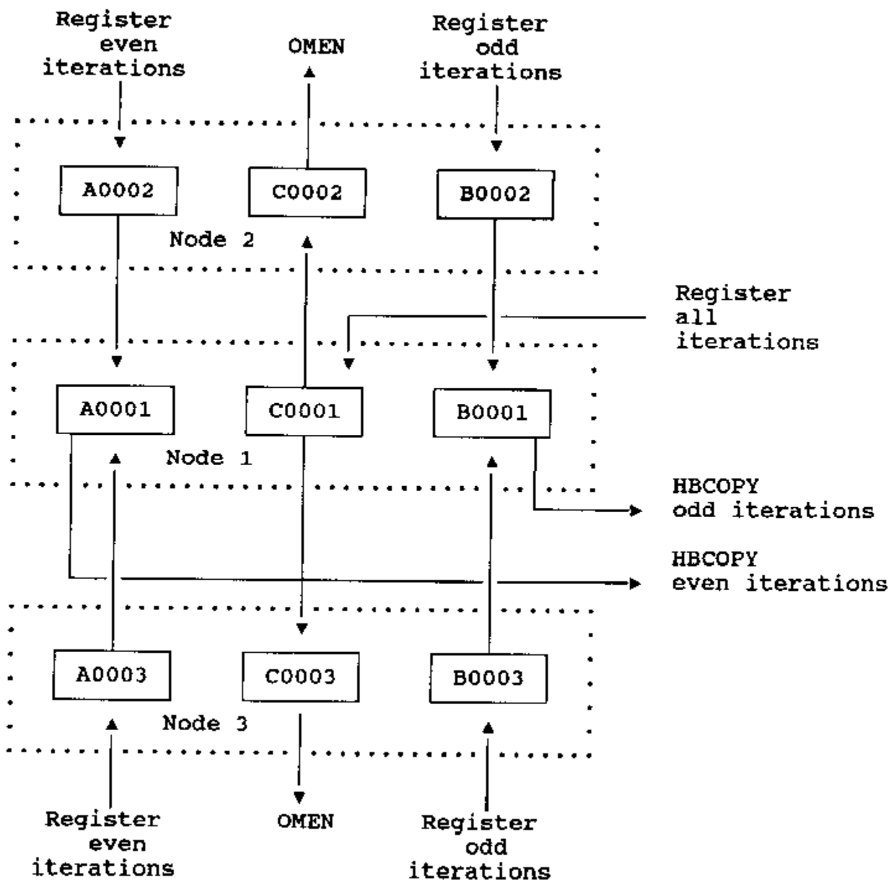

Figure 15.7. The use of internal file names.

The adjustment project is associated with the user ID (computer user's account) of the project leader. The APF and all the internal files are catalogued under the project leader's directory. Since the system is designed to be used for very large adjustments requiring many staff-years of data preparation and analysis, it is not contemplated that the project leader would have more than one adjustment project in process at any one time. Should this occur, the project leader would need to open an additional computer account.

The first task of the project leader is to design the adjustment project. This means:

1. Assuring that the observations in the geographical area of interest have been validated and entered into the data base (the entire data base need not be loaded).
2. Defining special junction points
3. Defining observation classes
4. Defining crustal motion parameters (where appropriate)
5. Selecting an ellipsoid
6. Deciding whether the adjustment result will be stored back into the data base
7. Acquiring resources (e.g., personnel, disk space)
8. Designing a binary tree strategy for Helmert blocking.

Of these items, strategy design is the most important, expensive, and difficult. Chapter 14 describes the tools that were developed to assist with this task.

At the end of the adjustment design phase, several items exist in machine-readable form:

1. The strategy will reside in a "Strategy Development File"
2. Special junction point QID/QSNs will exist in a text file
3. An observation class deck will exist in a text file
4. If needed, a data base of crustal motion parameters will exist.

An adjustment project may now be created by means of the "Create Adjustment Project" function, invoked by the CRAPF macro. CRAPF is a project leader macro which asks for the location of the various machine-readable products of the design phase, and asks a number of questions pertinent to the future administration of the project. CRAPF submits a batch job that sets up two on-line files:

1. The "Adjustment Project File" (APF)
2. The "Project Log" (LOG).

All the machine-readable products of the project design phase are stored in the APF. These contents will be displayed in a report generated by the batch job submitted by the CRAPF macro. The log is an ordinary text file which will be updated by all future jobs that modify the adjustment state; initially it will contain a single line giving its creation date and time.

The next task is to begin the retrieval of data from the data base. The result of this retrieval is a RESTART file, which is then used as input to a HBNEMO run. While the design and project creation tasks must be carried out in strict order by the project leader, the retrieval and subsequent phases may overlap. Given a group of project members of varying speed and ability, it may be an advantage to the project leader to exploit this potential for overlap by allowing some project members to move into later phases while others remain active in earlier phases. Each project manager is assigned one or more Helmert blocks.

For each Helmert block, the project manager must retrieve data from the data base and run HBNEMO. When an error-free run of HBNEMO is achieved, the project leader registers the resulting partial normal equations. Once registered, the normal equations are copied into the equation solver's internal files. The copy of the normal equations held by the project manager is no longer relevant and may be discarded. The RESTART file, however, should be saved, since it will be updated at the end of each iteration. If the adjustment is expected to last weeks or months, it would be sensible to store RESTART files on tape between iterations.

Although the events within an adjustment project are normally driven by the availability of data, the project leader may desire to exercise some control over events. The project leader might wish to allow HBNEMO runs and registration to be performed during the day (when project members are present to carry out these manual tasks), and restrict the computationally intensive forward and reverse jobs to execute overnight. Several project leader functions are available for this purpose. The INHIBIT macro inhibits the automatic dispatching of enabled jobs. Finer-grained control of events is possible using the project leader macro LOCKS. This allows the selective placement of locks on nodes of the strategy, so that any enabled jobs involving those nodes will not be dispatched.

The project leader monitors the progress of the adjustment by examining the LOG file. Each registration, forward reduction step, and reverse step records its beginning and ending time in the log. The project leader should look for jobs that begin but do not end, since this is evidence of abnormal termination. The forward program also records in the log any apparent singularities in the portion of the normal equations corresponding to the interior unknowns. Such apparent singularities must be investigated, since they can indicate data errors which somehow remain even after block validation.

The project leader may also examine the state of the adjustment by requesting an APF report from time to time. This should be examined for nodes which are locked even when no jobs are active, since this indicates that some job has failed. The reason for the job failure must be determined and fixed. Common causes are running out of run temporary file space and computer system crashes. Once the problem has been fixed, the node may be unlocked. Since the job is still enabled, it will be submitted as soon as the Dispatcher is run.

The project leader fixes problems as necessary. If necessary, the phases of the blocks can be manually modified with the APFFIX program. Some data problems may require setbacks, which are an option of the REGISTER macro.

The project leader also determines whether the adjustment system should use tape or disk for its internal storage of Helmert blocks. The storage medium can be changed at any time with the USETAPE and USEDISK macros. It was suggested that tape be used in the forward course and disk in the reverse course. The reason was that reverse jobs use much less CPU time yet require many more tape mounts than the forward jobs.

Eventually the adjustment reaches the top level. The Helmert block containing the reduced normal equations for the junction points at the highest level are taken out of the automatic system with the HBCOPY macro (which submits a job that executes the HBCOPY program). The project leader, possibly working with a small group, has several tools to solve these equations. They may be solved directly with HLS2, they may be first combined with normal equations from space system (vector) observations prepared by SOAP, and they may be combined with normal equations imported from other agencies.

Once the highest level solution is available, the project leader registers it with the system and monitors the reverse course. When the reverse course has completed successfully, the project leader uses the OMEN macro to transfer the solution now existing in each first-level block back into the corresponding RESTART file. Updated parameters and residuals are computed and analyzed by the project member assigned to the block.

OMEN is designed to operate on one (RESTART file—Helmert block) pair at a time. OMEN and all subsequent first-level activities (e.g., POSTPROC, STREPORT, RESTART file editing) take place outside of the automatic adjustment system.

When all analyses are complete and necessary changes to RESTART files have been made, the forward course of the next iteration can begin. The project members again run HBNEMO (with the updated RESTART files as input) and the project leader registers the resulting partial normal equations.

If an adjustment project has been created with the intention of saving adjustment products (coordinates, covariances), then the final adjusted values in the RESTART files should be transferred back into the data base after convergence of the iterations. Furthermore, if any changes were made to observed values during project execution, then these are recorded in the RESTART file and should also be transferred to the data base.

15.9 DESIGN CHANGES

In the final Helmert blocking system, some features were not written exactly as designed and planned. Two of these are mentioned below.

Orientation unknowns were not handled by the Schreiber equation as discussed in chapter 14. Instead they appeared explicitly in the normal equations. They all became interior unknowns and were eliminated at the first level. The major price to be paid occurred at the end of the adjustment, when the uncertainties of the latitudes and longitudes were desired. Since the orientation unknowns were interspersed among the coordinate unknowns at the lowest level, it became necessary to compute the matrix inverse terms corresponding to the orientation unknowns as well.

The rule for classifying stations with respect to a block boundary was modified. The rule stated, "If there is an observation crossing the boundary, either from inside to outside or from outside to inside, then the stations at both ends of the line are classified as junction stations. Any inside stations which are not junction stations are classified as interior stations." This rule resulted in slightly more stations being classified as junctions than would have been the case had the rule in section 13.4 been applied.

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16. GEOID HEIGHTS AND DEFLECTIONS

Rudolf J. Fury

16.1 INTRODUCTION

An early decision in planning for the new adjustment of the North American Datum specified that deflections of the vertical and geoid undulation were to be associated with every occupied network station in the horizontal data base (Bossler, 1978; Strange and Fury, 1977). Previously, astronomic deflections had been observed at only 2 percent of the occupied triangulation stations. Similarly, geoid undulation estimates had been based on fitting polynomial surfaces to sparsely distributed astrogeodetically determined undulations. Because gravity data had recently become sufficient for geodetic parameter estimation, deflections of the vertical and geoid heights were predicted by gravimetric methods for the remaining 98 percent of the network stations. This chapter describes the computational methodology employed and numerical results achieved in the prediction of parameters (Fury, 1984).

16.2 GEODETIC PARAMETER ESTIMATION BY GRAVIMETRIC METHODS

The classical methods of Stokes and Vening Meinesz were adopted for the computation of geoid undulations and deflections of the vertical, respectively (Strange and Fury, 1977). These geodetic parameters are derived in a geocentric reference system as defined by the gravity anomalies. To provide for quality control of estimated parameters by direct comparison with astronomically derived values, the deflections of the vertical were transformed into the NAD 27 geodetic reference system.

16.2.1 Prediction of Deflection of the Vertical

16.2.1.1 Deflections on the geoid.

The integral for the calculation of deflections of the vertical is the Vening Meinesz formula (Heiskanen and Moritz, 1967)

$$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = \frac{1}{4\pi\bar{g}} \iint_\sigma \Delta g(\alpha, \psi) \frac{dS(\psi)}{d\psi} \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} d\sigma \tag{16.1}$$

where

$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix}$ = deflection components at a given point on the geoid,

$\iint_\sigma \ldots d\sigma$ = integration over the global sphere,

$\frac{dS(\psi)}{d\psi}$ = the Vening Meinesz function, also $S'(\psi)$

$\Delta g(\alpha,\psi)$ = free-air gravity anomalies on the geoid derived from surface observations,

α,ψ = azimuth and spherical distance of variable point in the integration relative to the given point, and

$\bar{g}$ = the average (global) value of gravity.

The vertical deflection components are represented as the sum of three terms (Strange and Fury, 1977)

$$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = \frac{1}{4\pi\bar{g}} \int_0^{2\pi}\int_0^{\pi} \Delta g^\circ S'(\psi) \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} \sin\psi d\psi d\alpha + \frac{1}{4\pi\bar{g}} \int_0^{2\pi}\int_0^{\psi_0} \widetilde{\Delta g}\, S'(\psi) \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} \sin\psi d\psi d\alpha + \begin{Bmatrix} d\xi \\ d\eta \end{Bmatrix} \tag{16.2}$$

The first term expresses the long wavelength (global) components of the deflections which can be obtained using a harmonic series representation (Δg_n) of the gravity field

$$\Delta g^\circ(\phi, \lambda) = \sum_{n=2}^{L} \Delta g_n(\phi, \lambda) \tag{16.3}$$

where Δg^0 designates boundary (geoid) values, L is the degree of truncation of the series and ϕ, λ represent geodetic position. The second term of eq. (16.2) represents the short wavelength components of the total deflection superimposed on the global field. Therefore, it is computed from the residual gravity anomaly field,

$$\widetilde{\Delta g} = \Delta g - \Delta g^\circ \tag{16.4}$$

where Δg is obtained from observations. Although the integration should be extended over the global sphere in principle, it is limited to a spherical cap ($0 \rightarrow \psi_0$) for practical considerations. The error thus committed is represented by the third term ($d\xi$, $d\eta$), known as the truncation error (Fell and Karaska, 1981; Hagiwara, 1973).

16.2.1.2 Deflections at station height.

The vertical deflections calculated via the Vening Meinesz formula are at the geoid, i.e., mean sea level. These are not directly comparable with astronomically determined (observed) values unless the latter are reduced to the geoid by applying plumb line curvature corrections. However, the calculation of these corrections is involved and the results can be uncertain (Groten, 1981). It is better to obtain the vertical deflections at station height. This was accomplished through the extension of the Vening Meinesz formula to points exterior to the geoid via Pizzetti's generalization of the function $S'(\psi)$, (Heiskanen and Moritz, 1967: eqs. 6-30, 6-46b). The resulting formula for the short wavelength components of the deflections of the vertical is

$$\begin{Bmatrix}\xi\\ \eta\end{Bmatrix}_s = \frac{1}{4\pi g}\int_0^{2\pi}\int_0^{\psi_0} \widetilde{\Delta g}\,(\alpha,\psi)\,S'(r,\psi)\begin{Bmatrix}\cos\alpha\\ \sin\alpha\end{Bmatrix}\sin\psi\, d\psi\, d\alpha \tag{16.5}$$

where the variable r indicates radial distance from the geocenter to the physical surface, subscript s designates the short wavelength term, and $\widetilde{\Delta g}$ is now computed at the physical surface rather than at the geoid.

16.2.2 Prediction of Geoid Undulation

Undulations of the geoid relative to the reference spheroid were calculated by Stokes' formula (Heiskanen and Moritz, 1967)

$$N = \frac{R}{4\pi g}\iint_\sigma \Delta g\,(\alpha,\psi)\,S(\psi)\,d\sigma \tag{16.6}$$

where N is the geoid undulation, $S(\psi)$ represents Stokes' function, and Δg are gravity anomalies on the geoid.

In the same fashion as the deflection calculation, the geoid undulation can also be expressed as a sum of three components in which the first term is the global component, and is modeled with a harmonic series similar to the method used for modeling deflections. The second term in the sum is the short wavelength component of the total undulation

$$N_s = \frac{R}{4\pi g}\int_0^{2\pi}\int_0^{\psi_0} \widetilde{\Delta g}\,(\alpha,\psi)S(\psi)\sin\psi\, d\psi\, d\alpha \tag{16.7}$$

The third term (dN) represents the truncation error.

16.2.3 Computation of Global Components of the Parameters

A set of spherical harmonic coefficients (truncated GEM-10) was chosen to calculate the global components (Strange and Fury, 1977) of the parameters

$$\xi_g = \frac{1}{R\gamma}\frac{GM}{r}\sum_{n=2}^{L}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}[(\bar{C}_n^m-\bar{\bar{C}}_n^\circ)\cos(m\lambda) + \bar{S}_n^m\sin(m\lambda)]\frac{d\bar{P}_n^m(\sin\phi)}{d\phi} \tag{16.8}$$

$$\eta_g = \frac{1}{R\gamma\cos\phi}\frac{GM}{r}\sum_{n=2}^{L}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}[-(\bar{C}_n^m-\bar{\bar{C}}_n^\circ)\sin(m\lambda) + \bar{S}_n^m\cos(m\lambda)]m\,\bar{P}_n^m(\sin\phi) \tag{16.9}$$

$$N_g = \frac{1}{\gamma}\frac{GM}{r}\sum_{n=2}^{L}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}[(\bar{C}_n^m-\bar{\bar{C}}_n^\circ)\cos(m\lambda) + \bar{S}_n^m\sin(m\lambda)]\bar{P}_n^m(\sin\phi) \tag{16.10}$$

where, ξ_g, η_g, N_g are the deflections of the vertical and geoid undulation, respectively.

- GM = product of gravitational constant and mass of the Earth,
- γ = normal gravity at latitude,
- r = radial distance to geoid,
- a = mean equatorial radius of the Earth,
- $\bar{P}_n^m(\sin\phi)$ = spherical harmonic (Legendre) functions (normalized),
- $\frac{d\bar{P}_n^m(\sin\phi)}{d\phi}$ = derivatives of harmonic functions,
- $\bar{C}_n^m$, $\bar{S}_n^m$ = coefficients of spherical harmonic expansion (normalized),
- $\bar{\bar{C}}_n^0$ = coefficients of reference field which are functions of flattening ($\bar{\bar{C}}_n^0 \neq 0$ only for $n = 2$ and $n = 4$ to an accuracy of 4th power in the second eccentricity), and
- L = indicates the degree of truncation (L = 22) for computations in eqs. (16.8), (16.9) and (16.10).

The normalized Legendre functions and their derivatives were calculated recursively through the relations given in appendix 16.A.

A remark is appropriate concerning the computation of the radial distance (r) to the geoid. This value is

$$r = R + N$$

where R is the radial distance to the spheroid and N is the geoid undulation. However, N is initially not known. Therefore, the evaluation of double sums, i.e., eq. (16.10), is initially iterated with N = 0. Convergence is usually reached in two iterations.

As indicated, the double sums are evaluated first to obtain the global components of the deflections of the vertical and geoid undulation at network stations. However, they are also utilized in calculating the gravity reference field, i.e., eq. (16.3). When performed many times, the evaluation of the double sums is a time-consuming computation, even though the algorithm was optimized as much as possible. The large number of computations is necessitated by the need to calculate gravity anomaly residuals ($\widetilde{\Delta g}$) at a large number of area elements when integrating over the spherical cap for short wavelength components, using eqs. (16.5) and (16.7) as will be discussed in the next section.

Since the gravity field produced by the satellite-derived spherical harmonic model is smooth, point anomalies on the geoid were calculated only at five locations in the vicinity of the station through the harmonic series

$$\Delta g^\circ = \frac{GM}{r^2}\sum_{n=2}^{L}(n-1)\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}[(\bar{C}_n^m-\bar{\bar{C}}_n^\circ)\cos(m\lambda) + \bar{S}_n^m\sin(m\lambda)]\,\bar{P}_n^m(\sin\phi) \tag{16.11}$$

These five reference values then provided the basis for linear interpolation of anomalies at other points on the geoid. (See appendix 16.B.)

16.2.4 Computation of Short Wavelength Components of Parameters

The practical evaluation of the integrals for the short wavelength terms is achieved through numerical integration. An important consideration in such calculations is the subdivision of the spherical cap (i.e., integration region in the vicinity of the stations) into area elements. The method chosen is a combination of circular sectors [Rice-circles (Rice, 1952)] and geographic quadrangles. (See fig. 16.1.) In the immediate vicinity of the station (0 – 235 m), a gradient circle is used to evaluate the effect of the gravity field (Shultz et al., 1974). From this circular area (Rice-ring no. 5) to 45′ in latitude and 45′/cosϕ in longitude (Rice-ring no. 42) the mean anomalies ($\widetilde{\Delta g}$) of circular sectors are calculated by averaging the interpolated values at sector corners. The remaining area of the spherical cap (ψ = 10°) is divided into three concentric zones over a geographic lattice formed by meridional and parallel spherical arcs: the first zone extending from the circular sectors out to 2° from the station is overlayed with 5′ by 5′ blocks, from this boundary to 5° with 15′ by 15′ blocks, finally to 10° with 1° by 1° blocks. The mean anomalies ($\widetilde{\Delta g}$) for this geographic lattice were precalculated using observed values from the NGS gravity data bank. There is a small error committed in matching the circular outer boundary of sectors with the rectangular inner boundary of geographic lattice. This error is minimized by first moving the rectangular boundary to the even 5′ grid line in the vicinity of outermost circle (i.e., 45′ from the station in latitude and 45′/cosϕ in longitude); secondly, the summation includes only those sectors whose center points fall within this rectangular area (fig 16.1). The truncation limit (ψ = 10°) was chosen as a compromise between the goal for achievable accuracy (±1 arcsec) and computational cost (Bossler, 1978). The global harmonic geoid model used in the computations (GEM-10) was truncated to L = 22 for computational economy. Considering that the estimated resolution of this harmonic model in terms of wavelength is 360°/22 ≈ 16°, the spherical cap radius should have been 16°. However, the moderate gain in accuracy (Strange and Fury, 1977: fig. 2) versus the very significant increase in computational cost did not justify the effort.

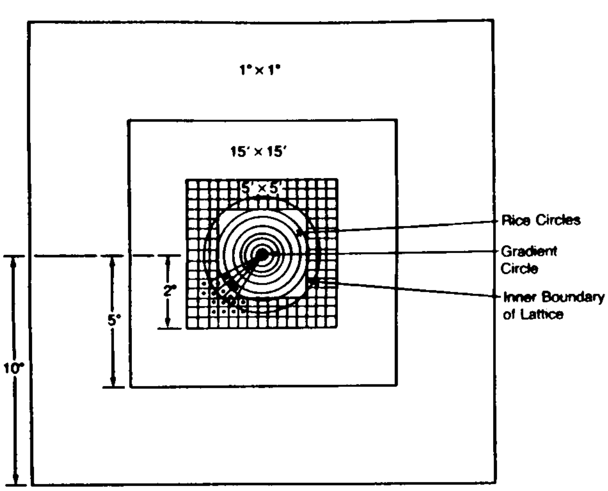

Figure 16.1. Gravity anomaly integration scheme.

16.3 COMPUTATION OF MEAN FREE-AIR GRAVITY ANOMALIES

Mean free-air gravity anomalies had been computed for the solution of the Stokes and Vening Meinesz integrals. Since the long wavelength components of geodetic parameters were calculated directly from harmonic series, mean anomalies are needed for the calculation of short wavelength components only. Further, the short wavelength components are superimposed on the global model, which implies a residual gravity field for numerical integration. The appropriate mean anomaly residuals are then

$$\widetilde{\Delta g} = \Delta g - \sum_{n=2}^{L} \Delta g_n \tag{16.12}$$

16.3.1 Geodetic Reference Field

The point free-air anomalies stored in the NGS gravity data bank had been computed on the Geodetic Reference System 1967 (GRS 1967)

$$\Delta g(\phi,\lambda) = g(\phi,\lambda) - \gamma(\phi,\lambda) \tag{16.13}$$

where $g(\phi,\lambda)$ is an observed value reduced to mean sea level (geoid), and $\gamma(\phi,\lambda)$ is the theoretical (normal) gravity at the surface of the spheroid. The spheroid parameters are

$$\begin{aligned} GM &= 0.398603 \times 10^{15} \quad cm^3/sec^2 \\ a &= 6378160 \text{ m} \\ \omega &= 0.72921151467 \times 10^{-4} \text{ rads/sec} \\ J_2 &= 1082.7 \times 10^{-6} \text{ (exact) } [J_2 = -\bar{\bar{C}}_2^{\,0}] \end{aligned}$$

Derived parameters:

$$\begin{aligned} J_4 &= -2.3712644 \times 10^{-6} \\ \gamma_e &= 0.97803187 \times 10^{6} \text{ mgal} \\ 1/f &= 298.2472 \end{aligned}$$

where ω is the angular velocity of the earth, 1/f is the reciprocal flattening of the spheroid, γ_e is the equatorial normal gravity, and the other symbols have already been identified.

The GRS 1967 constants were substituted into the harmonic series (eqs. 16.8, 16.9, 16.10, 16.11). As a result, the long wavelength components of the parameters are then referenced to this field (i.e., $\bar{C}^0_2 - \bar{\bar{C}}^0_2 = \bar{C}^0_4 - \bar{\bar{C}}^0_4 = 0$). It was desirable to obtain the geoid undulations as close as possible to the GRS 80 system planned by NGS for geometric reference. However, since no final parameters were yet adopted, the following zonal terms (normalized) were substituted into the harmonic series

$$\Delta\bar{C}^o_2 = \bar{C}^o_2(\text{GEM-10}) - \bar{\bar{C}}^o_2(\text{GRS1967}) = +0.340074 \times 10^{-7}$$
$$\Delta\bar{C}^o_4 = \bar{C}^o_4(\text{GEM-10}) - \bar{\bar{C}}^o_4(\text{GRS1967}) = -0.253549 \times 10^{-6} \quad (16.14)$$

Some very small effects seep into the higher harmonic terms by this substitution due to weak correlations. Since the harmonic coefficients of GEM-10 had been derived from least squares solution, they are not entirely independent, i.e., orthogonality relations are not perfect.

The computed geoid undulations (full value) were compared with values derived from Doppler tracking data at 10 stations. (See table 16.1.) Geoid undulations determined by Doppler tracking were transformed into the GRS 80 system. (See appendix 16.C.) The test stations are well distributed in the conterminous United States. Agreements of the two sets of values indicate that the substitution of $\Delta\bar{C}^0_2$ and $\Delta\bar{C}^0_4$ was appropriate.

16.3.2 Observed Gravity Reduction

Free-air anomalies on the geoid (boundary values) are needed for the solution of the third boundary-value problem of physical geodesy, i.e., the prediction of geoid undulation. Free-air anomalies at the physical surface are also needed for the computation of the disturbing potential (Heiskanen and Moritz 1967, p. 233) and for its derivatives, i.e., deflections of the vertical at station height (Heiskanen and Moritz 1967, p. 235). When surface anomalies are corrected for the effect of the terrain, the formalization becomes equivalent to the solution of Molodensky's boundary-value problem, assuming that free-air anomalies are linearly correlated with topographic elevations. In giving a physical interpretation to such solution, Moritz (1968, p. 35) shows its relation to the disturbing potential of a surface layer which may be obtained through the "condensation reduction" of Helmert (Heiskanen and Moritz 1967, p. 145). A "co-geoid" surface thus defined is a "single-layer free-air geoid" (Bjerhammar, 1967), which is obtained when all masses of topography are condensed in a layer at mean sea level. A significant feature of this co-geoid is the fact that to a linear approximation the predicted deflections of the vertical are invariant with respect to the condensation of topographic masses.

The masses to be removed were estimated via a Bouguer plate using the topographic height of the gravity station for plate thickness and a density (ρ) of 2.67 g/cm^3. Corrections were applied for the deviations of topography from the Bouguer plate (Goad, 1981; Dimitrijevich, 1972). The infinite Bouguer plate approximation to the topographic masses carries a significant error (Moritz, 1968), but this is of no great consequence in this application since its utility is limited to the smoothing of the gravity field for interpolation.

Following the removal of masses the observation (g) was reduced to sea level using the uniform free-air gradient of 0.3086 mgal/meter,

$$\Delta g(\phi,\lambda)^S = g(\phi,\lambda,h) - A_t + 0.3086h - \gamma(\phi,\lambda) \quad (16.15)$$

where

$\Delta g(\phi,\lambda)^S$ = gravity anomaly at sea level,
$g(\phi,\lambda,h)$ = observed gravity at station,
A_t = the effect of removed masses,
$0.3086h$ = reduction from station height to sea level in free space, and
$\gamma(\phi,\lambda)$ = gravity at the spheroid.

The condensation reduction of Helmert may be viewed as a limiting case of isostatic reduction of the Pratt-Hayford type when the depth of condensation (D) is zero (Heiskanen and Moritz 1967, p. 145). Accordingly,

$$\Delta g(\phi,\lambda) = \Delta g(\phi,\lambda)^S + A_C \quad (16.16)$$

in which A_C represents the effect of restored topography calculated with constant density (ρ = 2.67g/cm^3) considering the fact that A_t was obtained through a Bouguer reduction (Heiskanen and Moritz, 1967: p. 138). The "direct effect" ($-A_C + A_t$) is a small quantity since "the attraction of the Helmert layer nearly compensates that of the topography" (Heiskanen and Moritz, 1967: p. 145),

TABLE 16.1.—*Comparison of geoid undulations predicted by gravimetric methods and computed from Doppler satellite tracking*

Sta.	Latitude ° ′ ″	Longitude ° ′ ″	H (m)	Doppl. N(m)	Pred. N(m)	Diff. (m)
10028	30 34 4.34	86 12 58.92	36.00	−26.46	−26.37	−0.09
10055	37 29 53.63	122 29 50.24	53.82	−33.57	−33.19	−0.38
10070	47 7 16.58	122 29 20.36	95.21	−22.45	−23.52	+1.07
51041	41 38 26.87	101 35 56.21	1179.40	−19.98	−20.44	+0.46
51057	40 23 42.05	115 12 25.13	1856.00	−20.23	−19.76	−0.47
51081	46 18 30.44	85 27 23.69	260.62	−36.34	−36.62	+0.28
53114	38 26 13.65	79 49 55.37	822.26	−30.65	−29.88	−0.77
51134	32 51 55.56	117 14 59.06	76.21	−37.58	−37.18	−0.40
51960	39 8 16.36	123 12 38.69	197.92	−30.69	−30.61	−0.08
51014	27 57 25.32	80 33 28.02	7.26	−30.16	−29.81	−0.35

$$A_t = 2\pi\, G\rho h_P \approx A_c = 2\pi\, G\rho \bar{h} \qquad (16.17)$$

where h_P represents the topographic height of gravity station, and $\bar{h}$ is the mean height of template compartments derived from the digitized topographic heights.

Mean anomalies were precomputed for the geographic lattice from data in the NGS gravity observations data bank. Three data sets were generated for 5′ by 5′, 15′ by 15′, and 1° by 1° geographic blocks. These anomalies were considered boundary values on the co-geoid, i.e.,

$$\Delta g(\phi,\lambda) = g(\phi,\lambda,h) + 0.3086h - \gamma(\phi,\lambda) \qquad (16.18)$$

since the direct effect may be neglected in the distant zones. Because the indirect effect of condensation reduction is even smaller than the direct effect (e.g., 1 m per 3 km of average topographic height), its estimation was not considered.

The condensation anomalies may be regarded as sea-level, free-air anomalies which could have been obtained by linear approximation of downward continuation of surface gravity anomalies (Heiskanen and Moritz, 1967: p. 329). This implies that "modern" methods of physical geodesy are applicable in computing deflections of the vertical at the physical surface. Indeed, this reasoning was followed in calculating deflections for the vertical at station height (Heiskanen and Moritz, 1967: p. 320).

16.3.3 Gravity Anomaly Interpolation

Although there is an abundance of gravity in most areas of the United States, sizeable gaps or areas with sparse coverage still remain. Therefore, interpolation and extrapolation (prediction) are basic requirements in parameter estimation.

Least squares collocation has been used successfully for gravity anomaly predictions and error estimation (Tscherning, 1975). The method of least squares collocation for the prediction of gravity anomalies (Δg^s_p) and their error variances ($\sigma^2_{\Delta g^s_p}$)are represented by the formulas (Lachapelle, 1978)

$$\Delta g^s_p = \bar{C}^T_{\Delta g^s, \Delta g^s_p} \cdot \left[C_{\Delta g^s, \Delta g^s}\right]^{-1} \cdot \overline{\Delta g}^s \qquad (16.19)$$

$$\sigma^2_{\Delta g^s_p} = \sigma^2_{\Delta g^s} - \bar{C}^T_{\Delta g^s, \Delta g^s_p} \cdot \left[C_{\Delta g^s, \Delta g^s}\right]^{-1} \cdot \bar{C}_{\Delta g^s, \Delta g^s_p} \qquad (16.20)$$

where $\overline{\Delta g}^s$ is a vector of the gravity anomalies derived from observations ("observed" gravity $\Delta g(\phi,\lambda)^s$ was "centered" on a reference plane); $[C_{\Delta g^s,\Delta g^s}]$ represents a covariance matrix of observed anomalies, $\bar{C}_{\Delta g^s,\Delta g^s_p}$ is the cross-covariance (column) vector between observed and predicted anomalies, and $\sigma^2_{\Delta g^s_p}$ designates the variance of prediction. The covariance function of gravity anomalies was defined in terms of Legendre polynomials (Heiskanen and Moritz, 1967; Goad, 1981; Tscherning and Rapp, 1974)

$$C(\Delta g^s_Q, \Delta g^s_T) = C(\psi_{Q,T}) = \sum C_n \left[\frac{R_b}{r_Q\, r_T}\right]^{n+2} P(\psi_{Q,T})_n \qquad (16.21)$$

where the C_n are degree variances, r_Q and r_T are geocentric radii to points Q and T, and R_b is the radius of Bjerhammar sphere. The value of C_n was calculated from Goad (1981)

$$C_n = \frac{(n-1)^2}{R_b^2} K_n$$

where

$$K_n = \left(\frac{GM}{R_b}\right)^2 \frac{10^{-10}(2n+1)}{n^4} e^{2\alpha n}$$

and

$$\alpha = 0.876 \times 10^{-4}$$

This method of prediction is most applicable to a field of smooth anomalies. Therefore, the vector of observed anomalies ($\overline{\Delta g}^s$) was defined as "sea level anomalies" which are identical to "refined Bouguer-anaomalies" (i.e., terrain corrected) on land (Heiskanen and Moritz, 1967), and free-air anomalies on oceans (i.e., h = 0).

A data bank of prediction coefficients given by the product $[C_{\Delta g^s,\Delta g^s}]^{-1} \times \overline{\Delta g}^s$ is stored for predicting sea level anomalies at any point. The continental United States was partitioned into 1° by 1° geographic quadrangles. Each quadrangle was further subdivided into four 30′ by 30′ sectors for the calculation of local anomaly covariances. The prediction coefficients represent the sea-level anomaly surfaces within the sector boundaries. This requires the storage of a large number of coefficients for large numbers of observations. The problem was solved by iterative selection of those observed anomalies that significantly contributed (i.e., with dominant frequencies) to predicted sea-level anomalies. The maximum prediction error could therefore be kept to any desired level by storing a sufficient number of covariances for the sector. The iterative selection of data reduced the number of covariances to be stored by 30 to 60 percent.

16.3.4 Topographic Heights Interpolation

The mean heights of area elements in the numerical integration were obtained through the average point elevations at circular sector corners. The point elevations were computed via three-point interpolation from the NGS digitized topographic data bank. This data set contains a point elevation for every 30″ of latitude and longitude in the United States, extending into Canada, Mexico, and the oceans. The heights were computed from the three closest digitized values forming a triangle. (See appendix 16.D.)

16.4 ERROR ESTIMATION

The possibility of estimating geodetic parameter errors rigorously through error propagation was investigated. Testing indicated that it was not feasible to compute the errors by this method. A practical solution was implemented which consisted of comparing the predictions with values derived from observations.

16.4.1 Transformation of Deflection Components

The predicted deflections of the vertical are referenced to the modified GRS 1967 system and are not directly comparable to the astronomically derived values

$$\begin{aligned} \xi_A &= \Phi - \phi \\ \eta_A &= (\Lambda - \lambda)\cos\phi \end{aligned} \tag{16.22}$$

where ξ_A, η_A are the astrogeodetic deflection components, Φ, Λ are astronomic latitude and longitude, respectively, and ϕ, λ are the corresponding geodetic values referenced to the North American Datum of 1927. For the purpose of direct comparison, the predicted values were transformed into the NAD 27 system via differential transformation (λ positive east).

$$\begin{aligned} \delta\xi = -\frac{1}{M+H}[&-\sin\phi\cos\lambda\,\delta u - \sin\phi\sin\lambda\,\delta v + \cos\phi\,\delta w \\ &+ a e^2 \frac{\cos 2\phi(1-e^2\sin^2\phi) + e^2\sin\phi\cos\phi}{(1-e^2\sin^2\phi)^{3/2}} \\ &(\cos\lambda\,\delta\psi - \sin\lambda\,\delta\epsilon) \\ &+ \frac{e^2\sin\phi\cos\phi}{(1-e^2\sin^2\phi)^{1/2}}\delta a \\ &+ \sin\phi\cos\phi\,(2N+e'^2M\sin^2\phi)(1-f)\delta f] \end{aligned} \tag{16.23}$$

$$\begin{aligned} \delta\eta = -\frac{1}{(N+H)\cos\phi}[&-\cos\phi\sin\lambda\,\delta u + \cos\phi\cos\lambda\,\delta v - \\ &Ne^2\sin\phi\cos\phi(\sin\lambda\,\delta\psi + \cos\lambda\,\delta\epsilon)] \end{aligned}$$

where

$$e'^2 = \frac{e^2}{1-e^2};\quad N = \frac{a}{(1-e^2\sin^2\phi)^{1/2}};\quad M = \frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}}$$

δu, δv, δw	indicate shifts of ellipsoid (i.e., geocentric-geodetic)
δa, δf	are corrections to semimajor axis and flattening,
$\delta\epsilon$, $\delta\psi$, $\delta\omega$	are differential rotations,
a, e	are the semimajor axis and eccentricity of reference system,
M, N	are radii of spheroidal curvature in the meridian and prime vertical, respectively,
h	is the geodetic height of station, and
$\delta\xi$, $\delta\eta$	are corrections to transform geodetic into geocentric deflection components.

The gravimetrically predicted vertical deflections in the NAD 27 system are then

$$\begin{aligned} \xi_{NAD} &= \xi_{GRS67} - \delta\xi \\ \eta_{NAD} &= \eta_{GRS67} - \delta\eta. \end{aligned} \tag{16.24}$$

The following constants were used in the differential transformation (Vincenty, 1976)

$$\begin{aligned} a(\text{Clarke 1866}) &= 6378206.4 \text{ m} \\ 1/f(\text{Clarke 1866}) &= 294.9787 \\ \delta u &= -22 \text{ m} \\ \delta v &= +157 \text{ m} \\ \delta w &= +176 \text{ m}. \end{aligned}$$

The predicted geoid undulations are already very close to the GRS 80 system (table 16.1) adopted as preliminary reference for the geodetic network. Therefore, any further corrections may be applied regionally.

16.4.2 Interpolation and Error Estimation of Parameters

The general approach to quality control and error estimation was heuristic in nature due to the large computational effort which would have been required for error propagation. Assuming that the parameters derived from observations have very small errors as compared to prediction, any differences between predicted and observed values are attributed to errors in prediction. Therefore, the predicted values must be corrected to match the observations. A weighted interpolation scheme that has the characteristic of predicting the observed values at control stations was adopted. It is similar to astrogravimetric leveling (Heiskanen and Moritz 1967, p. 203), but is not limited to a profile. Instead, any number of observed parameters may be utilized. The interpolated parameters are then

$$\left\{\begin{matrix}\xi\\ \eta\\ N\end{matrix}\right\}_P = \left\{\begin{matrix}\xi\\ \eta\\ N\end{matrix}\right\}_P^p + \frac{1}{\sum\limits_m w}\sum_{m=1}^{M}\left(\left\{\begin{matrix}\xi\\ \eta\\ N\end{matrix}\right\}_m^\sigma - \left\{\begin{matrix}\xi\\ \eta\\ N\end{matrix}\right\}^p\right)w_m \tag{16.25}$$

where the superscripts σ and p indicate observed and predicted values, respectively, the subscripts P designate interpolated stations, while m designates the control stations. The weights (w) were chosen as the inverse distances between predicted and control stations; the summation limit was variable.

The errors of interpolated parameters were computed from two sources of information. The standard errors of observed values at control stations were summed with the weighted average of deviations

$$\sigma(\xi,\eta,N)_P = \left[\frac{1}{M}\sum_{m=1}^{M}\sigma^2(\xi,\eta,N)_m^A + \frac{1}{(\sum\limits_M w)^2}\sum_{m=1}^{M} w_m^2\Delta^2(\xi,\eta,N)\right]^{1/2} \tag{16.26}$$

where the σ indicates error estimates, the A superscript designates standard error for astronomic or Doppler observations, Δ is the residual difference between observed and predicted values following a regional correction (bias) to the latter, i.e., interpolated among neighboring control stations.

16.5 RESULTS OF PREDICTED PARAMETERS

Figure 8.1 displays the distribution of astronomic stations within the conterminous United States. It indicates that spacing of astronomic stations is inadequate to estimate the local variability of deflections, but can be used to remove regional distortions from the gravimetrically predicted deflections. The bar charts in figures 16.2, 16.3 and 16.4 show the statistical distributions of observed vertical deflections and Doppler-system derived geoid undulations. These have been compared to the predicted values for the calculation of regional distortions in the sampled gravity field. The distortions, of which distributions are shown in figures 16.5, 16.6, and 16.7, represent "calibration" values with which the predictions should be corrected to obtain deflections and geoid undulations in the respective reference system. A few large values suggest that the sampling of the gravity field at some regions is inadequate.

Some measure of the success of geodetic parameters prediction in the conterminous States was sought. The accuracy indicators could not be obtained by the regular computations of "calibrating" the predicted values at astronomic and Doppler stations because the interpolation method reproduces the observed values; i.e., the residual differences would be zero. Therefore, the observations were assumed unknown at the test stations, so that only surrounding values were used in interpolation for comparing observed and predicted deflections and geoid undulations. This approach for obtaining accuracy indicators is clearly inconsistent with the principles of interpolation technique used and tends to produce pessimistic estimates. Nevertheless, it provides a reasonable measure of the success of parameter estimation.

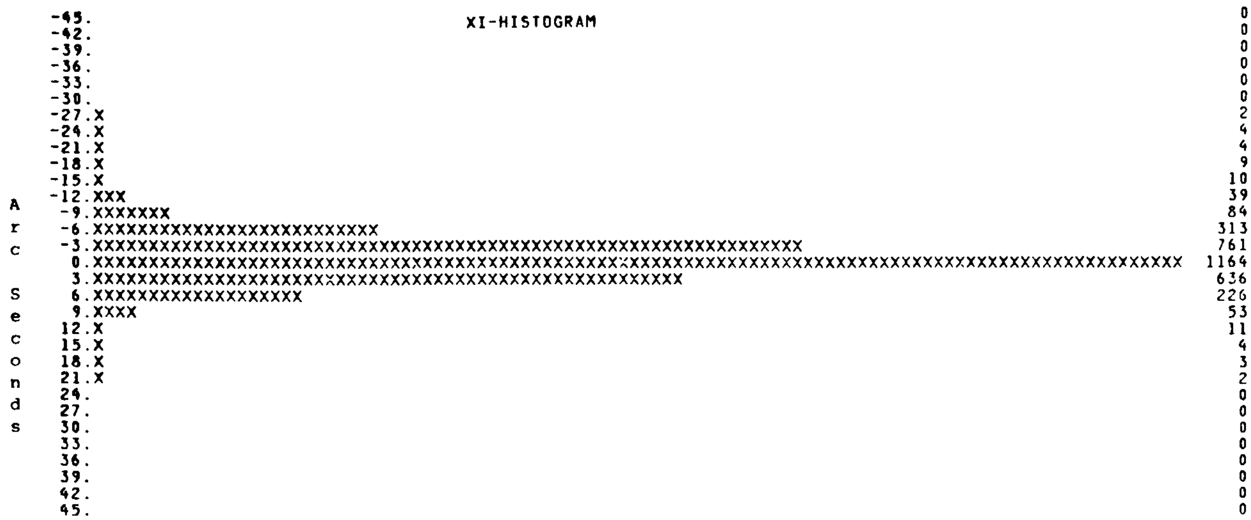

Figure 16.2. Distribution of the deflection of the vertical in the meridian.

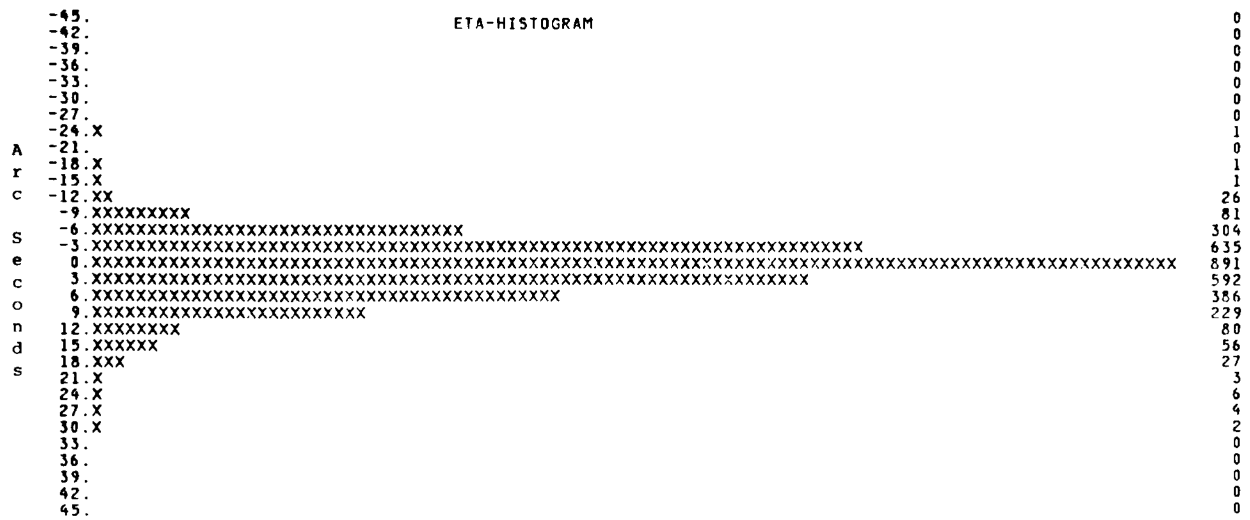

Figure 16.3. Distribution of the deflection of the vertical in the prime vertical.

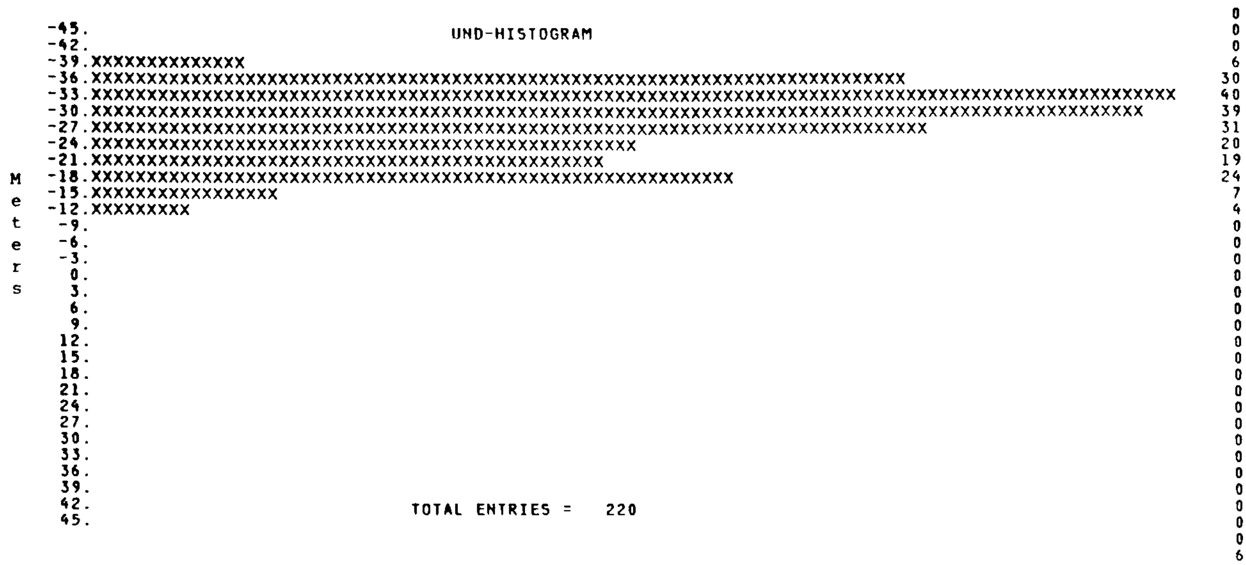

Figure 16.4. Distribution of Doppler system-derived geoid undulations. (Six values were greater than −45 m.)

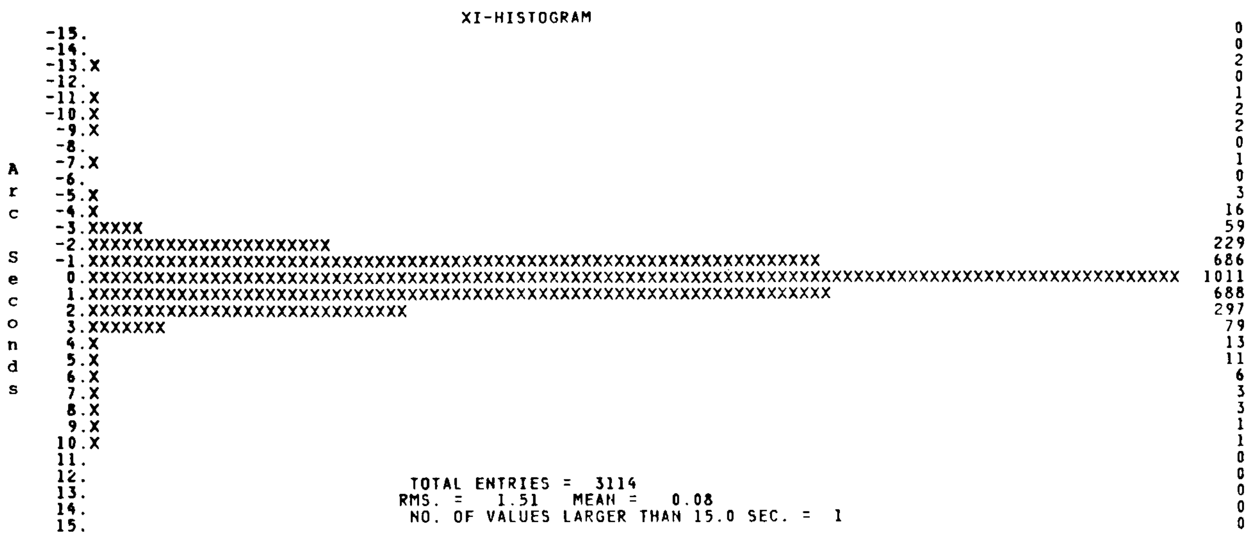

Figure 16.5. Distribution of regional distortions in the meridian.

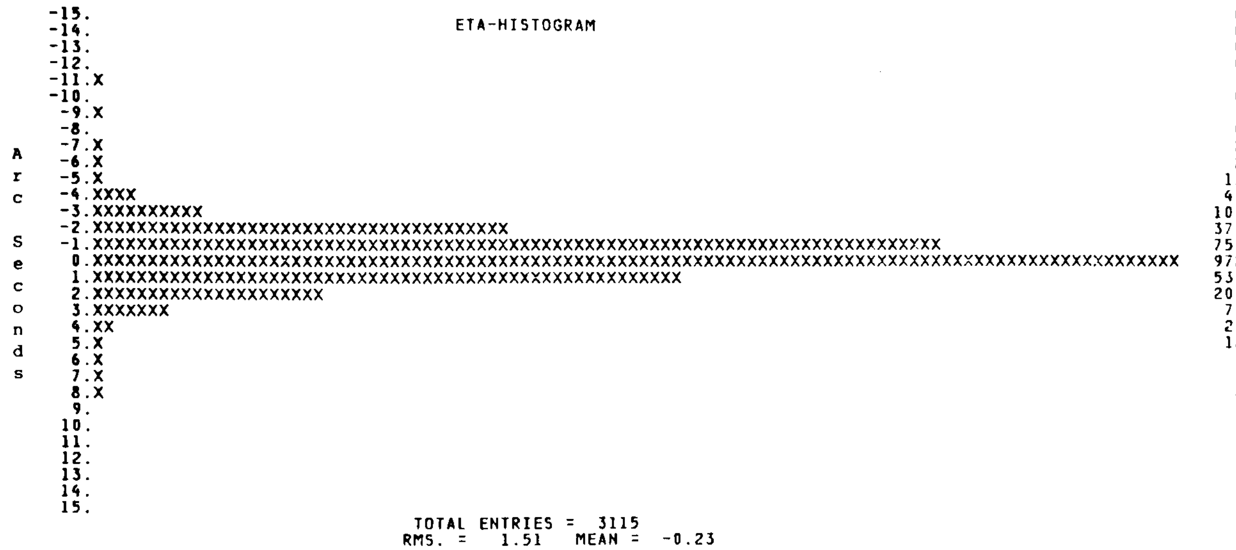

Figure 16.6. Distribution of regional distortions in the prime vertical.

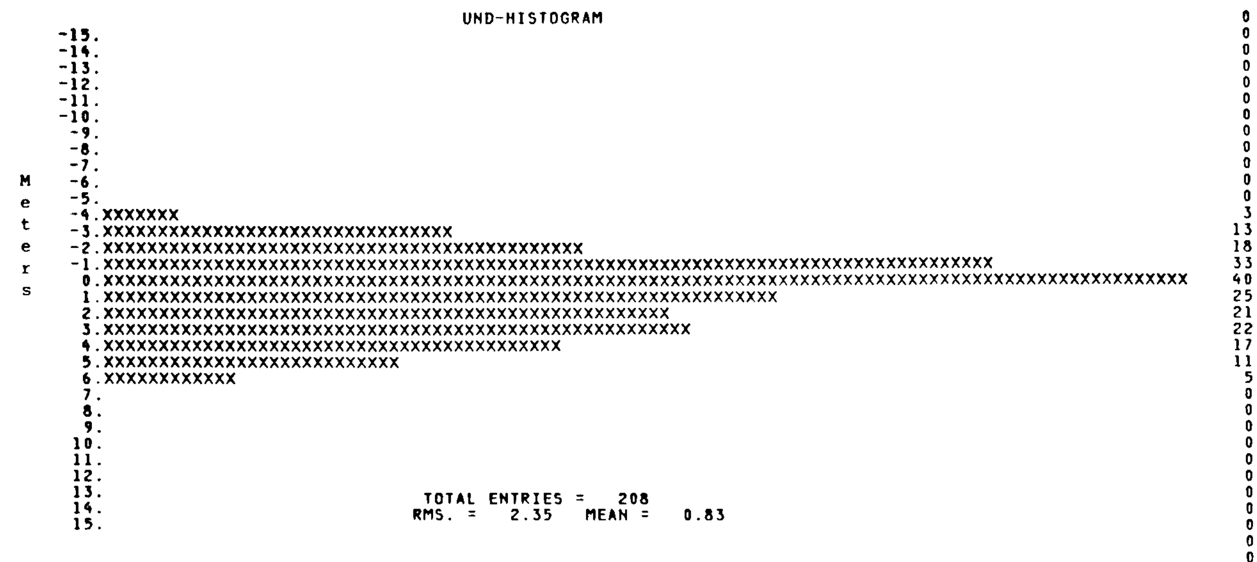

Figure 16.7 Distribution of regional distortions of geoid undulations.

16.5.1 Deflections of the Vertical

The results of tests for obtaining accuracy indicators for deflections of the verticals are given in figures 16.8, and 16.9 which show the results for the meridional and prime vertical components. Accordingly, the rms of residual differences at astronomic stations was ± 1.33 arc seconds in the meridian and ± 1.14 arc seconds in the prime vertical components. The residual differences $[\Delta(\xi,\eta)]$ were also substituted into the formula for computing error estimates of predicted geodetic parameters.

Figures 16.10 and 16.11 show distributions of the predicted vertical deflections and figures 16.12 and 16.13 the estimated errors. The predicted values are identical with the astronomically observed deflections at control stations, but they are different from the astronomically derived values when the regional distortion at an astronomic station was judged unreliable and not used in the interpolation.

16.5.2 Geoid Undulations

Figure 16.14 shows the accuracy indicators obtained for geoid undulations. The root mean square (rms) of residual differences was ± 1.40 m computed at 208 Doppler stations. Similarly, for computations performed for deflections, the residual differences $[\Delta(N)]$ were substituted into the formula for computing error estimates of geoid undulations. However, a much denser distribution of independently derived (Doppler) geoid heights would have been needed to provide a framework for reliable regional bias and error estimates computation. These will become available in the future as space techniques are improved.

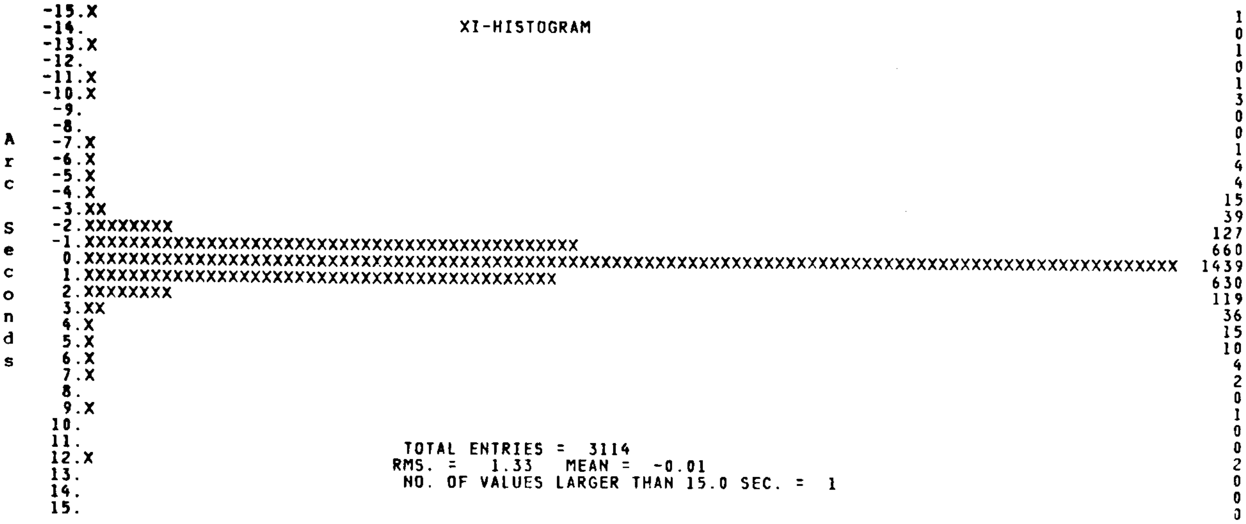

Figure 16.8. Distribution of residual differences in the meridian.

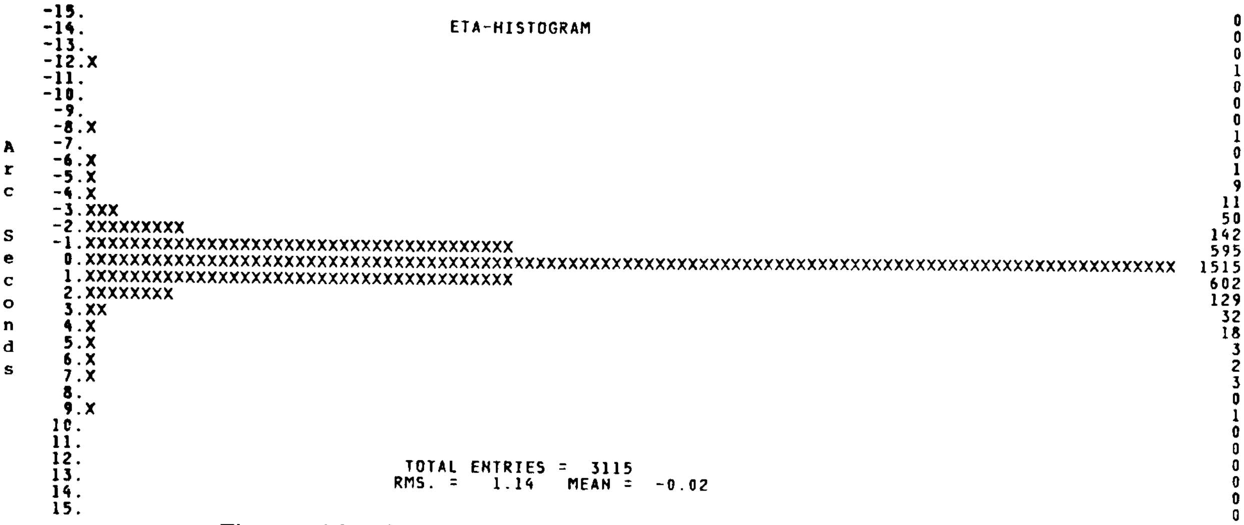

Figure 16.9. Distribution of residual differences in the prime vertical.

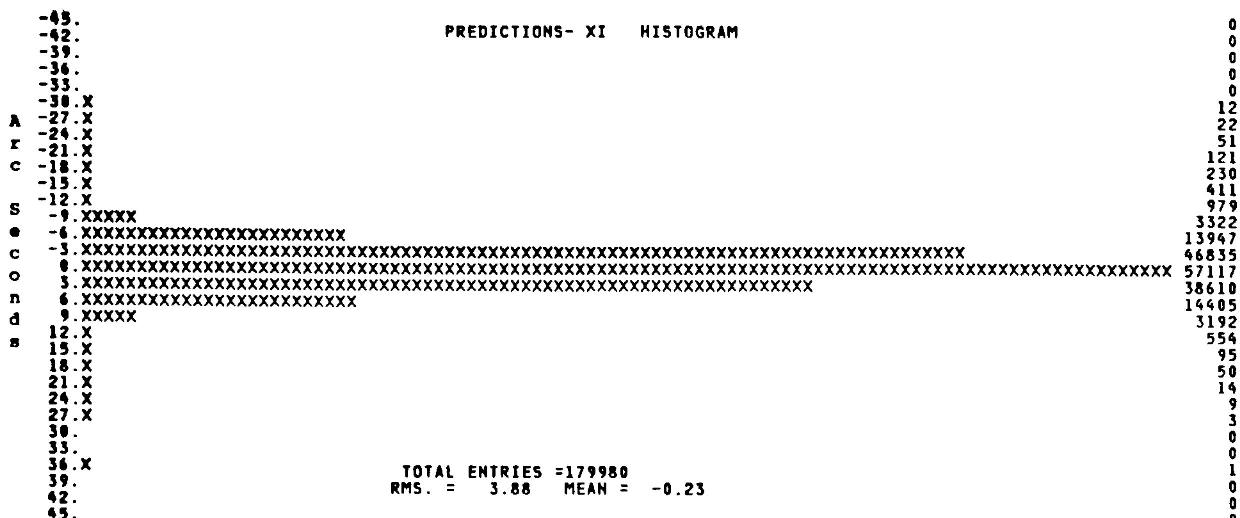

Figure 16.10. Distribution of predicted deflections in the meridian for the conterminous United States.

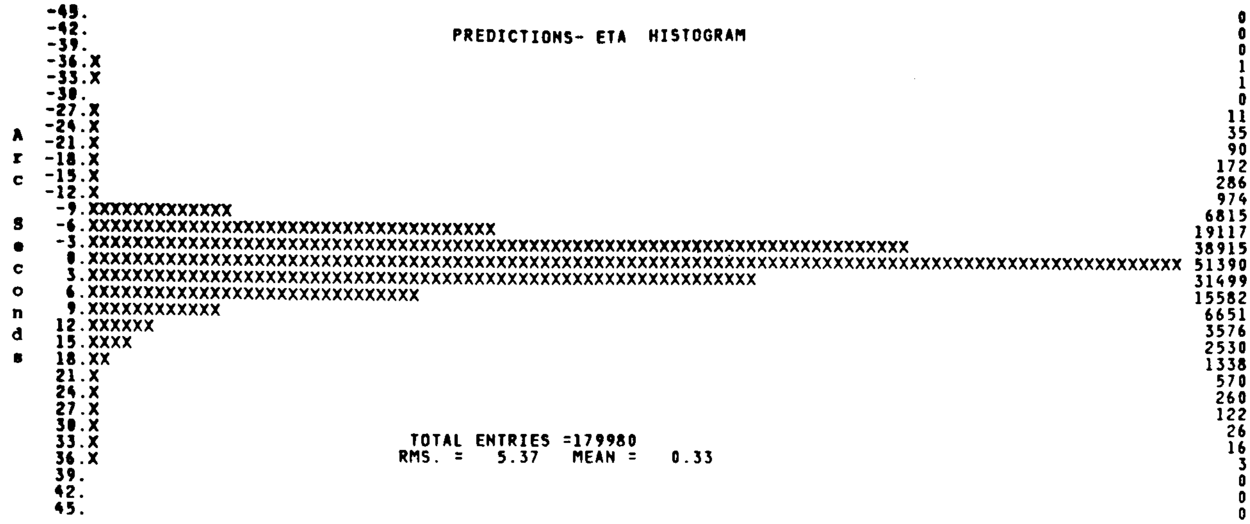

Figure 16.11. Distribution of predicted deflections in the prime vertical for the conterminous United States.

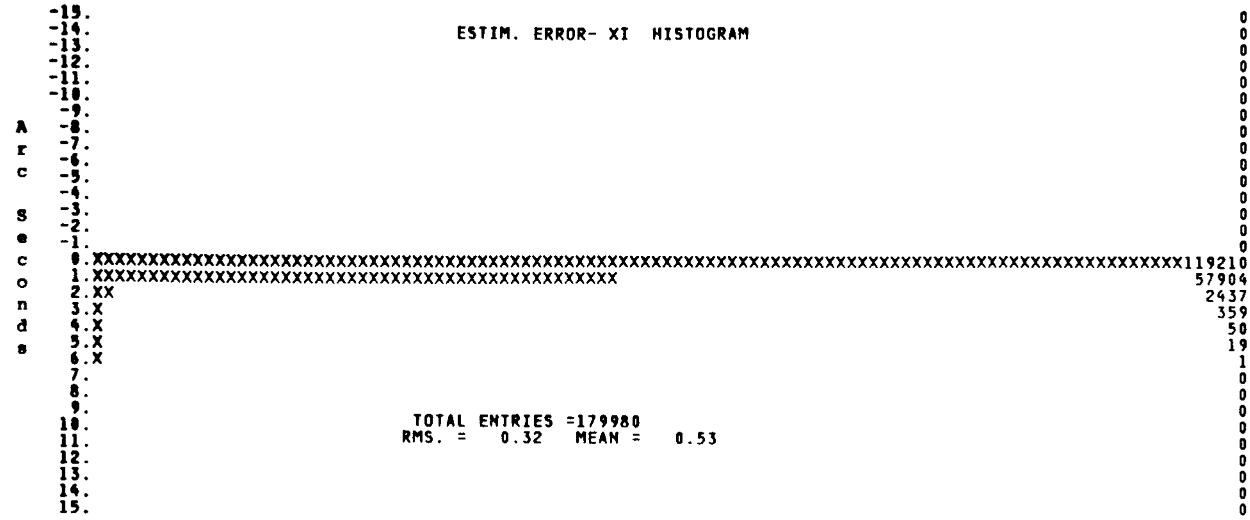

Figure 16.12. Distribution of estimated errors in the meridian for conterminous United States.

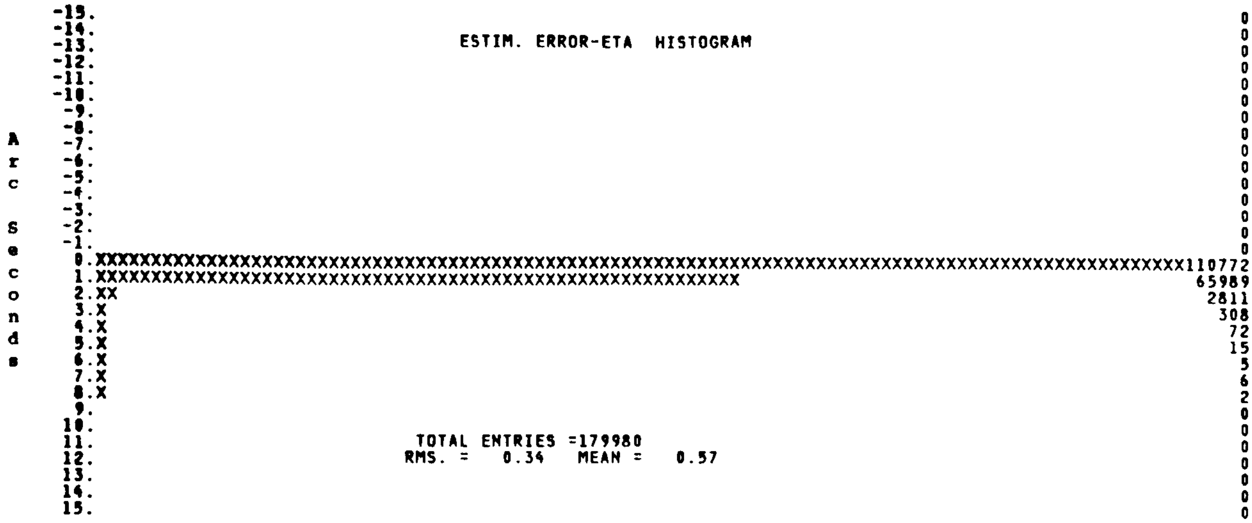

Figure 16.13 Distribution of estimated errors in the prime vertical for conterminous United States.

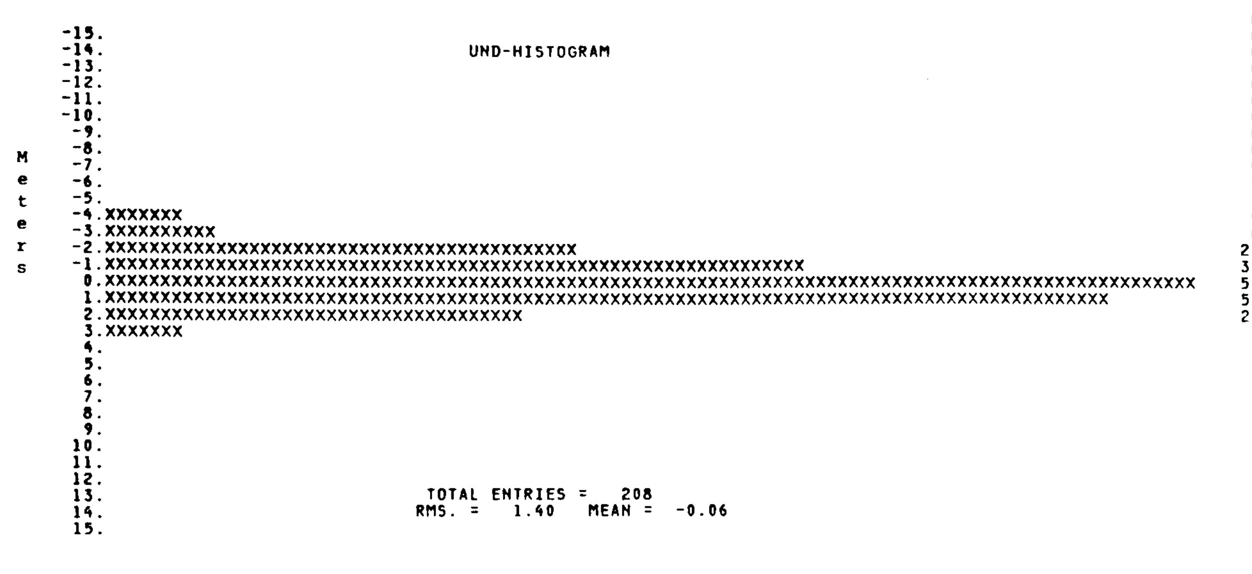

Figure 16.14. Distribution of residual differences of geoid undulations.

16.6 DATA ACCESS AND PROCESSING

The task of predicting vertical deflections and geoid undulations for the conterminous United States required four major processes:

1. Retrieval of network-, astronomic-, and Doppler-station data from the NGS data base.
2. Vertical deflections and geoid undulations prediction by gravimetric methods.
3. Vertical deflections transformation into the NAD 27 system.
4. Computation of accuracy estimates, error analysis, and entry of geoid parameters into the data base for the North American Datum.

The unprecedented large volume of geodetic and geophysical data, as represented by 180,000 network stations and 1.4 million gravity observations distributed over an area of approximately 55 million square kilometers, demanded a high degree of automation, powerful computational facilities, and modern data management techniques (Fury, 1981).

16.6.1 Gravity Data Banks

It has been shown in sections 16.2.4 and 16.3 that several sets of (residual) mean free air gravity anomalies are needed for numerical integration. These sets were obtained by two different methods. The first method entailed only the calculation of average free air anomalies over 5′ by 5′, 15′ by 15′ and 1° by 1° geographic lattices. The second required the generation of Bouguer anomaly "surfaces" for anomaly prediction via collocation.

The data bank of observed gravity values was established to satisfy the needs of various geodetic projects for measured or reduced gravity. Since most of the projects access data by geographic area, the major feature of the data management software provides for such operation. Special features aid quality control, general updates and provide data security against hardware failures. Mean anomaly data banks were constructed corresponding to the three geographic lattices in which the long wavelength components of gravity anomaly, calculated at the centers of lattice squares, were also retained. Data records were structured in array formats in which only anomalies are stored but in which positions are implied. The unit areas for data access are 1° by 1°, 2° by 2°, and 5° by 5°, corresponding to the 5′ by 5′, 15′ by 15′, and 1° by 1° geographic lattices. The single key access to arrays by geographic area, controlled through the data bank directory, has proven to be a very efficient access method for high frequency data retrievals.

16.6.2 Digitized Terrain Model

The computation of terrain effects on observed gravity necessitates the availability of a terrain model. The topographic elevations data bank of NGS represents such a model through the elevations which are digitized at every 30 seconds of latitude and longitude. Most of the point elevations were digitized from 1:250,000 scale maps. Therefore, the point positions have no relationship to either geodetic network stations or to the gravity stations. The gridded data set is amenable to the same array data structure as used for mean gravity anomalies, but the data density is higher by orders of magnitude. Consequently, the unit area of data access has been decreased to 30′ by 30′ geographic blocks. Due to the relatively high data density, three-point linear interpolation was considered adequate for mean height computations. The frequency of access for topographic elevations was high in both terrain effects computations and mean height calculations for Rice-circle compartments of numerical integration. The data bank management software, employing single key access by geographic area, responded readily to the demand for high frequency data access.

16.6.3 Gravity prediction coefficients data bank

As discussed in section 16.3.3, the interpolation of gravity data is a basic requirement in geodetic parameter estimation. This requirement was satisfied by generating a gravity data bank of "prediction coefficients," defined previously as the product of a vector of gravity anomalies derived from observations, and the inverse of the associated convariance matrix. The characteristic feature of retrieving sets of prediction coefficients by geographic area has been adopted from the gravity data banks. The unit area of definition of gravity anomaly "surfaces," represented by the coefficients, is 30′ by 30′ of geographic block. Random access to anomaly surfaces was achieved under the control of the data bank directory. In contrast to the fixed array size record structure of the mean gravity and terrain model data banks, the number of prediction coefficients per surface area varied, requiring special provisions in data bank design. Nonetheless, the method of single key retrieval by geographic area was retained, which facilitated efficient access and accurate gravity prediction.

16.6.4 Processing Facilities

The prediction of the deflections of vertical and geoid undulations by the classical method of numerical integration (Schwarz, 1978; Hopkins and McEntee, 1974) represented a large computational effort, necessitating powerful computing facilities. The majority of the computations were carried out on the IBM 360/195 mainframe operated by NOAA. At the same time, the geodetic data base, containing the coordinates of the network stations, was housed on an IBM mainframe operated by a commercial time-shared facility. First, the logistics of smooth data flow between the two facilities had to be resolved and, second, an automated batch processing software system had to be placed into operation which would provide automatic restart and processing recovery capability. The total project of predicting deflections of the vertical and geoid undulations was carried out from October 1980 to May 1982.

The magnitude of the project can be illustrated by the following statistics:

- Astronomic and network station records were stored on 173 magnetic tape files after retrieval from the data base (35 files hold records for Puerto Rico, Hawaii, and Alaska).
- The geographic area of the conterminous states was divided into 43 area projects for data sets of manageable size. The processing of these projects required the preparation, submittal, editing, and verification of approximately 4,000 prediction runs (computer jobs), 200 to 300 reruns, 150 to 200 transformation and error analysis runs, and the same number of data set backup runs.
- There were 179,980 vertical deflections and geoid undulations predicted and stored in the station records of the geodetic data base (some predictions at intersection stations were not entered into the data base). This required the processing and data base entry runs of 43 files corresponding to the area projects.

An indication of the success of the project may be given by the rms values of deviation between observed and predicted deflections, which were computed to be ± 1.33 arc second in the meridional and ± 1.15 arc second in the prime vertical components at 3,115 astronomic stations.

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APPENDIX 16.A
RECURSIVE RELATIONS OF LEGENDRE FUNCTIONS

Derived recursion relations of normalized Legendre functions:

$$\bar{P}_n^o(\sin\phi) = \frac{\sqrt{2n+1}}{n}\left\{\sqrt{2n-1}\,\sin\phi\,\bar{P}_{n-1}^o(\sin\phi) - (n-1)\sqrt{\frac{1}{2n-3}}\,\bar{P}_{n-2}^o(\sin\phi)\right\}$$

$$\bar{P}_n^m(\sin\phi) = \sqrt{\frac{2n+1}{(n+m)(n+m-1)}}\left\{\sqrt{\delta(2n-1)}\left[\cos\phi\,\bar{P}_{n-1}^{m-1}(\sin\phi) + \sqrt{\frac{(n-m)(n-m-1)}{2n-3}}\,\bar{P}_{n-2}^m(\sin\phi)\right]\right\}$$

Derived recursion relations of derivatives of normalized Legendre functions:

$$\frac{d\bar{P}_n^o(\sin\phi)}{d\phi} = \sqrt{\frac{2n+1}{n}}\left\{\sqrt{2n-1}\left[\sin\phi\,d\bar{P}_{n-1}^o(\sin\phi) + \cos\phi\,\bar{P}_{n-1}^o(\sin\phi)\right] - (n-1)\sqrt{\frac{1}{2n-3}}\,d\bar{P}_{n-2}^o(\sin\phi)\right\}$$

$$\frac{d\bar{P}_n^m(\sin\phi)}{d\phi} = \sqrt{\frac{2n+1}{(n+m)(n+m-1)}}\left\{\sqrt{\delta(2n-1)}\left[\cos\phi\,d\bar{P}_{n-1}^{m-1}(\sin\phi) - \sin\phi\,\bar{P}_{n-1}^{m-1}(\sin\phi)\right] + \sqrt{\frac{(n-m)(n-m-1)}{2n-3}}\,d\bar{P}_{n-2}^m(\sin\phi)\right\}$$

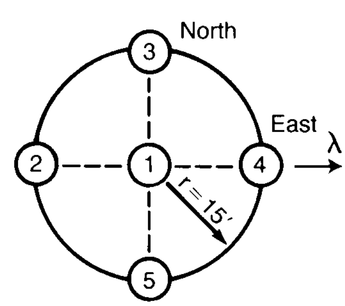
when $(m-1)>0$, then $\delta=1$, when $(m-1)=0$, then $\delta=2$; the functions are zero by definition when $n-2$, $n-1$, $m-1 \leqslant 0$.

APPENDIX 16.B
LINEAR INTERPOLATION OF GLOBAL COMPONENT OF GRAVITY ON THE GEOID

Values of Δg^0 (eq. 16.11) are obtained by spherical harmonic series at the network station (1), and at symmetrically located four points (i.e., 2, 3, 4, 5). Any other values of $\Delta g^0(\phi,\lambda)$ in the station's vicinity are obtained by linear interpolation:

$$\Delta g^o(\phi,\lambda) = \frac{\Delta g^o(3)-\Delta g^o(5)}{2r}\,\phi' + \frac{\Delta g^o(2)-\Delta g^o(4)}{2r}\,\lambda' + \Delta g^o(1)$$

where ϕ' and λ' are geodetic positions, and r is the distance of symmetrically located points from network stations in arc minutes.



Anomaly Computation Points on the Geoid

APPENDIX 16.C
GEODETIC REFERENCE SYSTEM OF 1980

$GM = 3.986005 \times 10^{14}\ cm^3/sec^2$
$a = 6378137$ meters
$J_2 = 1082.63 \times 10^{-6}$
$\omega = 0.7292115 \times 10^{-4}$ rads/sec
$1/f = 298.2572221(*)$

(*) Derived

APPENDIX 16.D
TOPOGRAPHIC HEIGHT INTERPOLATION

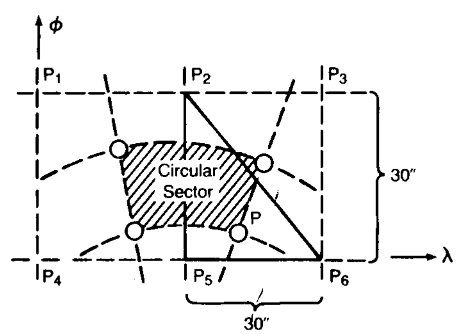
Topographic heights of circular sector corners were computed via three-point interpolation from evenly distributed ($P_1P_2, \ldots P_6$) digitized elevations in the NGS data bank. The point elevation h_P of sector corner P is interpolated from values at geographic grid intersections P_2, P_5, and P_6 where

$$h_p = Ah_{P_6} + Bh_{P_5} + Ch_{P_2}$$

$$A = 1 + (\phi_{P_5} - \phi_{P_2})(\lambda_P - \lambda_{P_6}) - (\lambda_{P_5} - \lambda_{P_2})(\phi_P - \phi_{P_6})$$

$$B = 1 + (\phi_{P_2} - \phi_{P_6})(\lambda_P - \lambda_{P_5}) - (\lambda_{P_2} - \lambda_{P_6})(\phi_P - \phi_{P_5})$$

$$C = 1 + (\phi_{P_6} - \phi_{P_5})(\lambda_P - \lambda_{P_2}) - (\lambda_{P_6} - \lambda_{P_5})(\phi_P - \phi_{P_2})$$



Topographic Height Interpolation

17. CRUSTAL MOTION MODELS

Richard A. Snay
Michael W. Cline
Edward L. Timmerman

17.1 INTRODUCTION

The project to model horizontal deformation for various tectonically active regions in the United States is identified as REDEAM (REgional Deformation of the EArth Models). Individual models were developed for 19 mutually disjoint geographic regions. Sixteen of these regions cover, in combination, the State of California (fig. 17.1). The three other regions are located in Nevada (fig. 17.1), Alaska (fig. 17.2), and Hawaii (fig. 17.3). This chapter is a condensation of the report by Snay et al. (1987) which documents the development and implementation of the models.

The REDEAM models were generated in support of the North American Datum (NAD) project, an international effort to redefine the geodetic reference

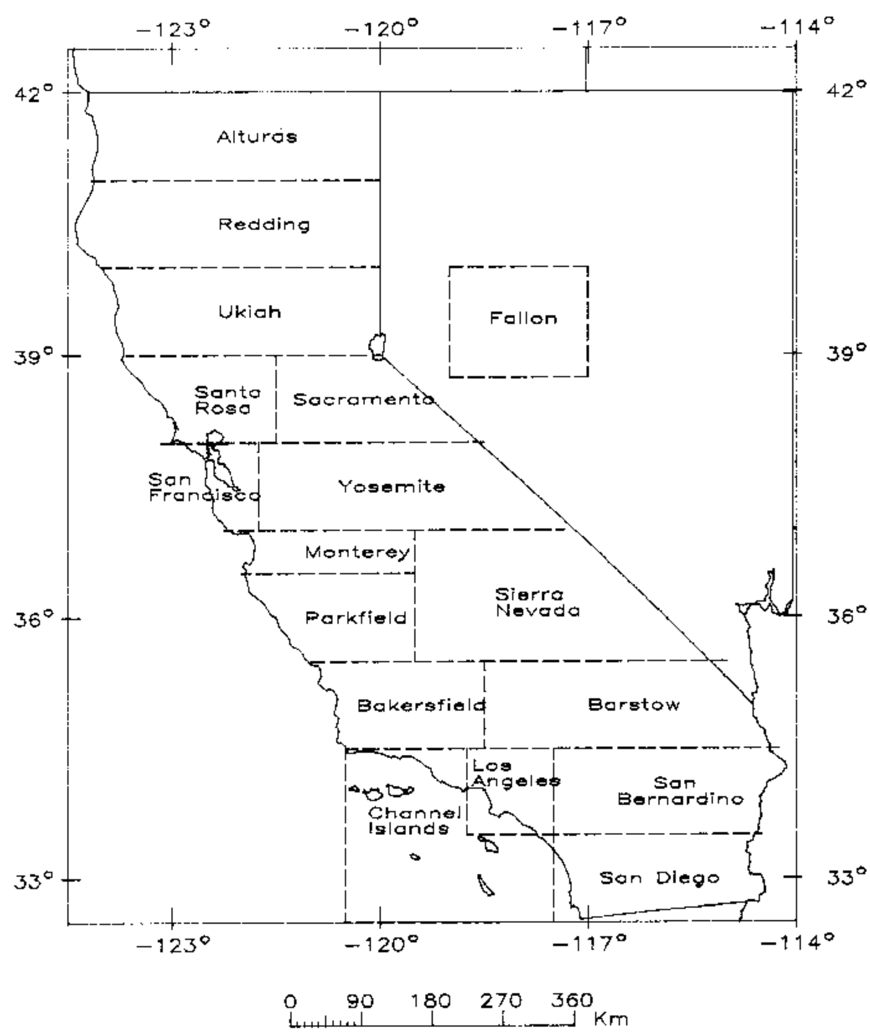


Figure 17.1. A model for historical crustal deformation was developed for each of 19 regions. Seventeen of these regions are pictured above. Other regions are located in Alaska and Hawaii.

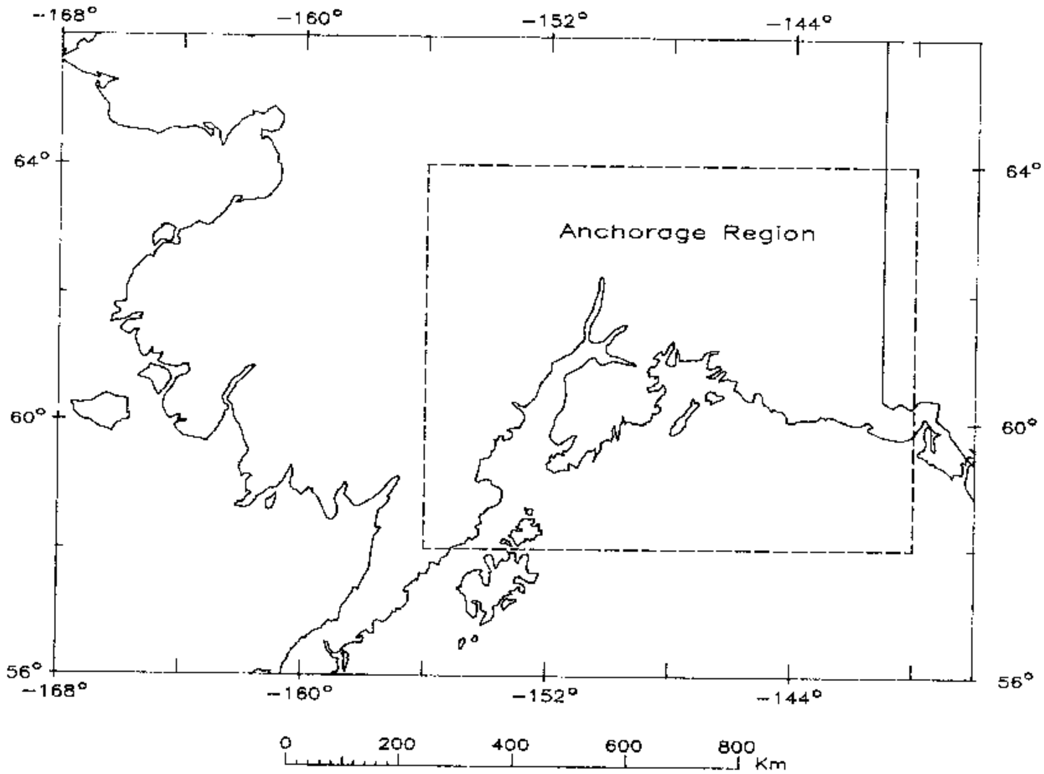


Figure 17.2. The crustal deformation model for Alaska's Anchorage region characterizes horizontal displacements associated with the 1964 Prince William's Sound earthquake.

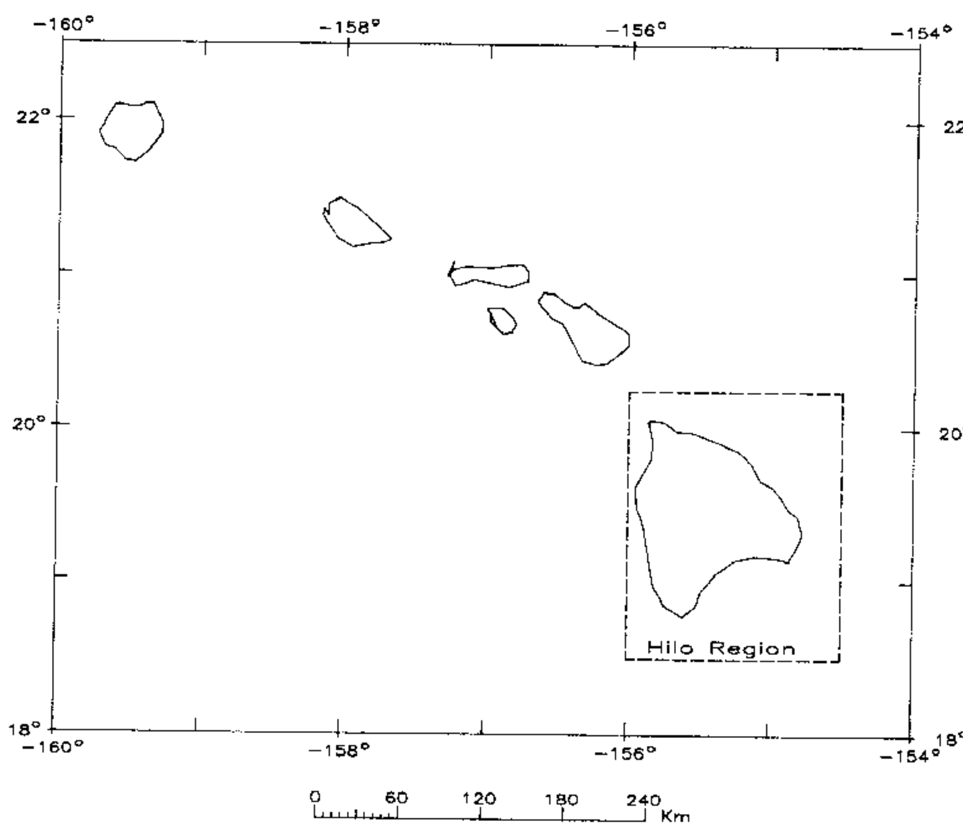


Figure 17.3. The crustal deformation model for Hawaii's Hilo region addresses horizontal motion associated with volcanic and seismic activity, especially the 1975 Kalpana earthquake (M = 7.1).

system used by map makers, engineers, land surveyors, and others for horizontal positioning in North America. For the NAD project, all appropriate geodetic observations were entered into a simultaneous solution to estimate positional coordinates (latitude and longitude) for the several hundred thousand monumented stations that comprise the North American horizontal reference network. Prior to entry into the solution, geodetic data in areas of suspected deformation were reduced, using REDEAM models, to a common date (December 31, 1983) to account for temporal variations of the positional coordinates. That is, for each geodetic observation in the deforming areas, the REDEAM models served to estimate the value that would be obtained if the observation were remeasured on December 31, 1983. The newly derived NAD positional coordinates thus correspond in time to this date.

According to plate tectonic theory, the latitudes and longitudes of monumented stations continually change. The rates of these motions have been estimated from geologic and seismic data by using models that assume that the Earth's surface consists of several rigid plates each rotating at a constant rate about a specific pole (Minster and Jordan, 1978). Although these models are acceptable on a global scale for motions averaged over millions of years, significant regional deviations develop when the motions are considered over a time period of decades. In particular, friction between adjacent plates retards relative plate motion and causes a gradual bending of the Earth's crust over a zone hundreds of kilometers in width. This regional bending is occasionally interrupted by the sudden displacements associated with earthquakes as elastic crustal elements rebound from their distorted states. The REDEAM models address both this slow regional bending and the rapid coseismic displacements.

17.2 GEODETIC DATA

Parameters for REDEAM models were estimated from geodetic data (directions, distances, and azimuths) contained in the archives maintained by NGS. This data base incorporates contributions from various Federal, state, and local organizations. The archives include numerous geodetic measurements in California which were performed explicitly to measure crustal motion. These crustal motion measurements include those performed by NGS and its predecessor agencies following most of the major earthquakes in the United States, including the San Francisco earthquake of 1906. These agencies have also repeatedly surveyed several geographic areas to monitor aseismic strain rate (fig. 17.4) and secular fault slip (fig. 17.5). The archived crustal motion measurements also include the regularly repeated line-length determinations performed by the California Department of Water Resources (1968) from 1959 to 1969 and the California Division of Mines and Geology from 1969 to 1979 (Bennett, 1980) and the U.S. Geological Survey (USGS) from 1970 to the present (Savage, 1983). (See fig. 17.6.) It is important to note, however, that most of the geodetic data used for project REDEAM were observed not to measure crustal motion but simply to position the marks that comprise the national geodetic reference network. The crustal motion information contained in this majority of the data results largely from past demands for additional marks whereby previously established marks were resurveyed to position the newer marks.

California's first geodetic data date back to the time of statehood, 1850. Most nineteenth century surveys, however, are concentrated along the coast as they were performed to aid navigation. California's interior network remained sparse until the introduction of Bilby towers around 1930. Because these 20- to 40-meter tall observation platforms are transportable and can be erected or dismantled in less than a day, they provided an economical means for seeing over trees, buildings, and other obstacles. Consequently, the 1930s represent the original epoch of data for much of California. With the exception of the San Diego region, only pre-1980 data were included in the modeling effort. This cutoff date reflects the status of NGS's automated data base in early 1982—the time when the data were organized for project REDEAM. The San Diego data set was updated subsequent to 1982 to model coseismic deformation associated with the Imperial Valley earthquake ($M = 6.6$) of 1979.

The regional data sets overlap. In particular, the model for each region was derived from not only data within the region but also extending to a distance of 16 km beyond the region's geographic span. This data overlap was engineered to provide a measure of spatial continuity among the various models.

A significant increase in the number of distance observations occurred around 1960 with the introduction of EDM (electronic distance measuring) instrumentation and again around 1970 with the start of USGS's strain monitoring program (Savage, 1983).

Only three California regions (Channel Islands, Los Angeles, and Bakersfield) include data that predate the San Francisco earthquake of 1906. For some regions (San Diego, San Bernadino, and Barstow) the pre-1906 data had not been automated when the corresponding models were derived. For the other 10 California regions, the pre-1906 data were intentionally excluded to avoid modeling the coseismic movement associated with the San Francisco earthquake.

17.3 MATHEMATICAL MODEL

The mathematical formulation of the REDEAM models includes parameters for both the secular and episodic components of motion.

Secular motion is represented by dividing the *region* to be modeled into a mosaic of *districts*. The words, region and district, convey specific meanings in this chapter. The geographic area pertaining to a specific model is called a region. A district is one of several specifically designated areas associated with a region. Each district is allowed to translate, rotate, and undergo spatially homogeneous deformation at a constant rate with respect to time. By approximating the known geologic faults with district boundaries, the relative motion between districts represents the relative movement across these faults. This is not to say that all district boundaries correspond to faults. Some districts have been introduced simply to increase the spatial resolution of the secular motion. Figure 17.7 identifies the 10 districts that comprise the San Diego region.

Modeled episodic motion corresponds to displacements associated with large earthquakes. For episodic motion the Earth is considered to be an isotropic, homogeneous, elastic halfspace whose bounding plane represents the Earth's surface; that is, the Earth is represented as the set of points (x,y,z) with $z \leqslant 0$. Rectangular planes of finite dimensions are embedded in the halfspace to represent seismically active faults. The motions associated with an earthquake correspond to the displacements that the elastic halfspace undergoes in response to slip along the rectangular surfaces. This motion is given by the equations of dislocation theory (Snay et al. 1987: appendix A). The displacements are a function of the location, size, and orientation of the rectangles, as well as the amount and sense of the slip. Figure 17.8 identifies the earthquakes modeled for the 16 California regions.

More specifically, the mathematical model expresses the geodetic latitude $\phi_M(t)$ (positive north) and longitude $\lambda_M(t)$ (positive west) of a station M in district i at time t by the equation

$$\begin{bmatrix}\phi_M(t)\\ \lambda_M(t)\end{bmatrix}=\begin{bmatrix}\phi_M(t_0)\\ \lambda_M(t_0)\end{bmatrix}+\begin{bmatrix}a_{\phi*}(i) & a_{\phi\phi}(i) & a_{\phi\lambda}(i)\\ a_{\lambda*}(i) & a_{\lambda\phi}(i) & a_{\lambda\lambda}(i)\end{bmatrix}\begin{bmatrix}1\\ \phi_M(t_0)-\bar{\phi}\\ \lambda_M(t_0)-\bar{\lambda}\end{bmatrix}(t-t_0)$$

$$+\sum_k r(t,t_k)\begin{bmatrix}A_k & B_k\\ C_k & D_k\end{bmatrix}\begin{bmatrix}s_k\\ d_k\end{bmatrix} \quad (17.1)$$

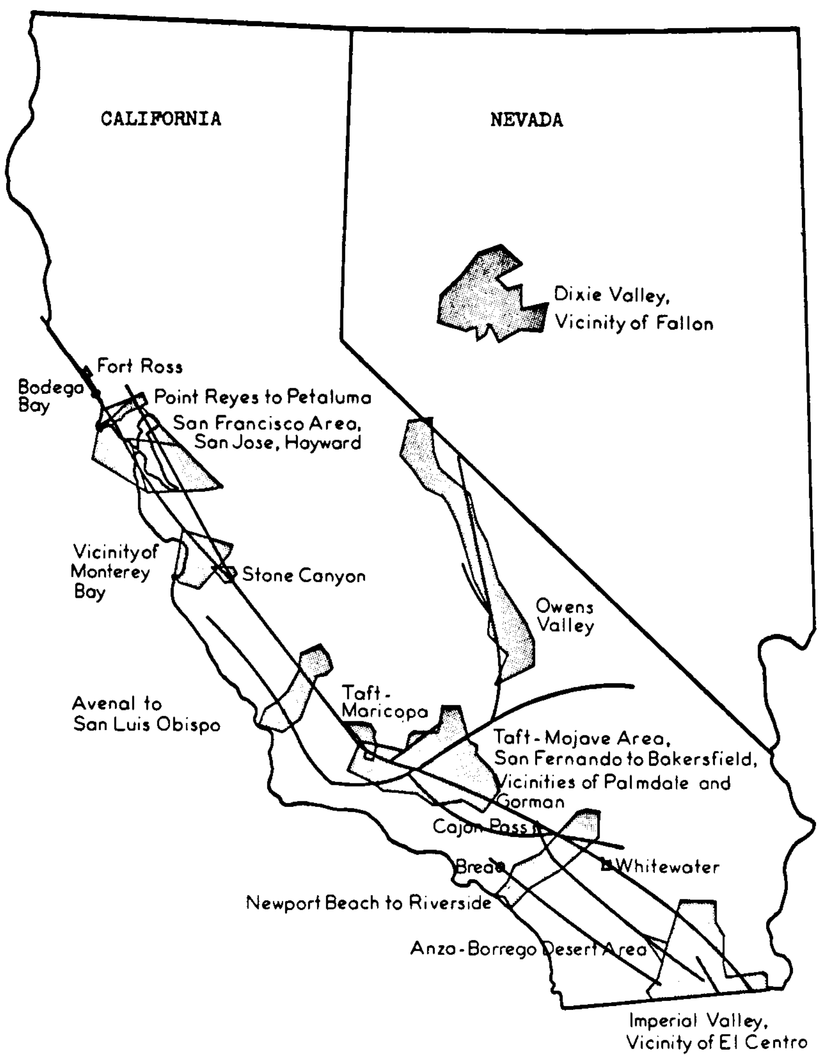


Figure 17.4. Meade (1971) published this figure identifying several areas that were being monitored before and during the 1960s using direction or triangulation observations. In the early 1970s, the more precise electronic distance measuring technique became sufficiently operational for monitoring deformation over large areas.

Here t_0 is a fixed time of reference. The preceding equation states that a station's coordinate values at time t equal its coordinate values at time t_0 plus a secular term, plus an episodic term.

In using eq. (17.1), we presume that secular motion is linear in time. In the secular term the variables ($\bar{\phi}$, $\bar{\lambda}$) represent reference coordinates that are selected prior to adjusting the model to the geodetic data. The variables $a_{\phi *}(i)$, $a_{\phi\phi}(i)$, $a_{\phi\lambda}(i)$, $a_{\lambda *}(i)$, $a_{\lambda\phi}(i)$, $a_{\lambda\lambda}(i)$ are parameters to be estimated for district i. Equation (17.1) implies that the secular motion interior to each district is essentially homogeneous strain plus rotation at a constant rate with respect to time.

The expression that involves the summation sign in eq. (17.1) corresponds to the episodic motion and gives the change of the station's latitude and longitude caused by strike slip s_k and dip slip d_k at time t_k on the k-th rectangle for each value of k. The episodic time dependence is embedded in the step function $r(t,t_k)$ defined by the conditions:

for

$$t_k < t_0 \qquad r(t,t_k) = \begin{cases} -1 & \text{if } t < t_k \\ 0 & \text{if } t > t_k \end{cases}$$

for

$$t_k > t_0 \qquad r(t,t_k) = \begin{cases} 0 & \text{if } t < t_k \\ 1 & \text{if } t > t_k \end{cases} \tag{17.2}$$

Equation (17.2) prescribes that the slip on the k-th rectangle occurs instantaneously at time t_k. In eq. (17.1) the quantities A_k, B_k, C_k, and D_k represent mathematical expressions involving the coordinates of station M as well as the location, orientation, and size of the k-th rectangle.

With eq. (17.1) observations are entered into a least squares process to estimate the unknown coordinates ($\phi_M(t_0)$, $\lambda_M(t_0)$) for all M, the unknown parameters $a_{\phi *}(i)$, $a_{\phi\phi}(i)$,...,$a_{\lambda\lambda}(i)$ for all values of i, and the slips s_k and d_k for all values of k. In the least squares process an observation β_t at time t (a direction, a distance, or an azimuth) is first corrected for known systematic errors such as refraction and it is then projected onto an ellipsoidal reference surface so that the "reduced" observation β_t' is expressible solely as a function h of mark coordinates. That is,

$$\beta_t' = h[\phi_P(t), \lambda_P(t), \phi_Q(t), \lambda_Q(t)] \tag{17.3}$$

where P and Q denote marks associated with the observation. Substituting into this equation from eq. (17.1), β_t' becomes a function of the coordinates ($\phi_P(t_0)$, $\lambda_P(t_0)$) and ($\phi_Q(t_0)$, $\lambda_Q(t_0)$), the parameters $a_{\phi *}(i)$, $a_{\phi\phi}(i)$..., $a_{\lambda\lambda}(i)$ for all i corresponding to the district(s) containing P and Q, and also a function of the slips s_k and d_k for all values of k. These expressions constitute the so-called "observation equations" of the least squares process. The observations are weighted in the solution equal to the squared inverse of their respective standard errors.

The observation-equation coefficients are computed by an application of the chain rule. That is, if β_t' is a reduced observation, say a distance, observed at time t, and if α is a parameter to be estimated, say a secular motion coefficient, then from eq. (17.3) the coefficient $\partial\beta_t'/\partial\alpha$ may be computed by the equation:

$$\left(\frac{\partial\beta_t'}{\partial\alpha}\right) = \left(\frac{\partial h}{\partial\phi_P(t)}\right)\left(\frac{\partial\phi_P(t)}{\partial\alpha}\right) + \left(\frac{\partial h}{\partial\lambda_P(t)}\right)\left(\frac{\partial\lambda_P(t)}{\partial\alpha}\right) + \left(\frac{\partial h}{\partial\phi_Q(t)}\right)\left(\frac{\partial\phi_Q(t)}{\partial\alpha}\right) + \left(\frac{\partial h}{\partial\lambda_Q(t)}\right)\left(\frac{\partial\lambda_Q(t)}{\partial\alpha}\right). \tag{17.4}$$

The partials $\left(\frac{\partial h}{\partial\phi_P(t)}\right)$, $\left(\frac{\partial h}{\partial\lambda_P(t)}\right)$, $\left(\frac{\partial h}{\partial\phi_Q(t)}\right)$, $\left(\frac{\partial h}{\partial\lambda_Q(t)}\right)$ in eq. (17.4) are exactly the coefficients that would be computed for a static horizontal network adjustment, and the appropriate formulas are given by Schwarz (1978). The other four partials on the right side of eq. (17.4), namely those involving partial derivatives with respect to α, may be computed by differentiating eq. (17.1) with respect to α. To compute these latter four partials, the following information must be specified: (1) the time of reference t_0; (2) the coordinates ($\bar{\phi}$,$\bar{\lambda}$) for the origin of reference; and (3) the dates of the earthquakes together with the various parameter values defining location, size, and orientation for the corresponding dislocation surfaces.

17.4 MODEL IMPLEMENTATION

The derived crustal motion models were used to "update" all archived horizontal data in the deforming areas of the United States to the common date December 31, 1983. Thus an observation measured in 1940, for example, was updated to approximate the value that would be expected if it were remeasured on December 31, 1983. These updated observations were then entered into a static horizontal network adjustment to determine latitudes and longitudes for the NAD 83 geodetic reference system. This section describes the algorithm used for computing the crustal motion "corrections" for updating observations. These correction are computed using the developed models whose parametric values may be found in Snay et al. (1987).

Crustal motion corrections were applied to all observations that involve two stations, namely, direction, azimuth, and distance observations. Crustal motion corrections were not applied to observations involving only a single station, namely, Doppler positioning observations, because the data used to generate the models are insensitive to "absolute" motion. The effect of not correcting the Doppler observations should be insignificant because all archived Doppler observations were performed after 1970 and because the horizontal components of these observations have meter-level uncertainties.

To correct an observation between two stations P and Q measured at time t_1 to its corresponding value at time t_2, approximate horizontal coordinates for P and Q at the preselected reference time t_0 = January 1, 1950 are needed. Because we are interested only in changes to observations from one time to another, the selection of t_0 was rather arbitrary (1950 corresponds to the approximate weighted midpoint in the observation dates), and the station coordinates at time t_0 did not need to be extremely accurate. For our purpose, the NAD-27 coordinates of P and Q constitute sufficiently accurate estimates for $\phi_P(t_0)$, $\lambda_P(t_0)$, $\phi_Q(t_0)$, and $\lambda_Q(t_0)$.

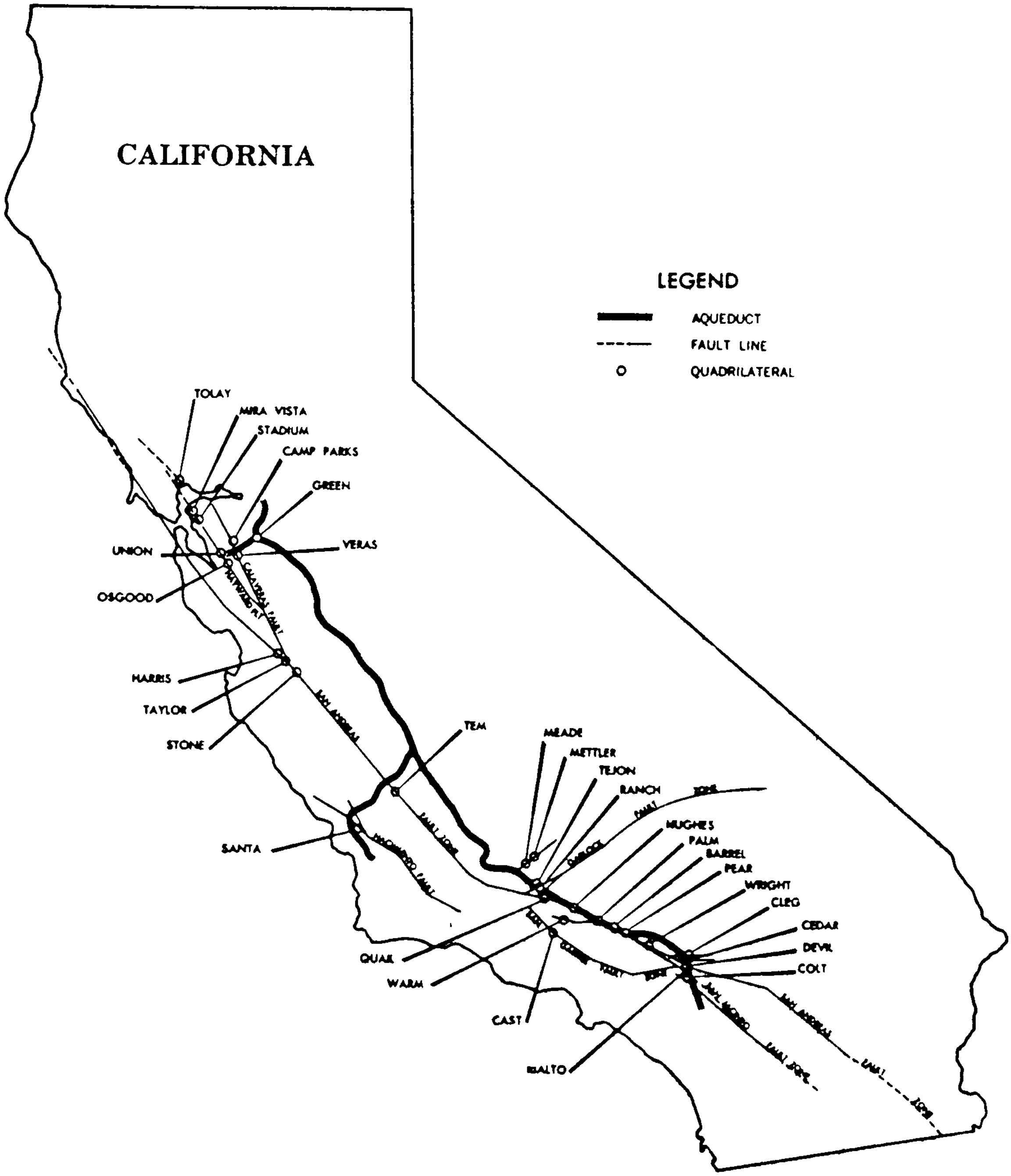


Figure 17.5. In the 1960s and 1970s, Federal agencies repeatedly surveyed 30 small geodetic networks to determine fault-slip rates in the vicinity of the California aqueduct. Most of these networks contained six to eight stations located within a kilometer of each other and with half of the stations to either side of the straddled fault (from Meade, 1971).

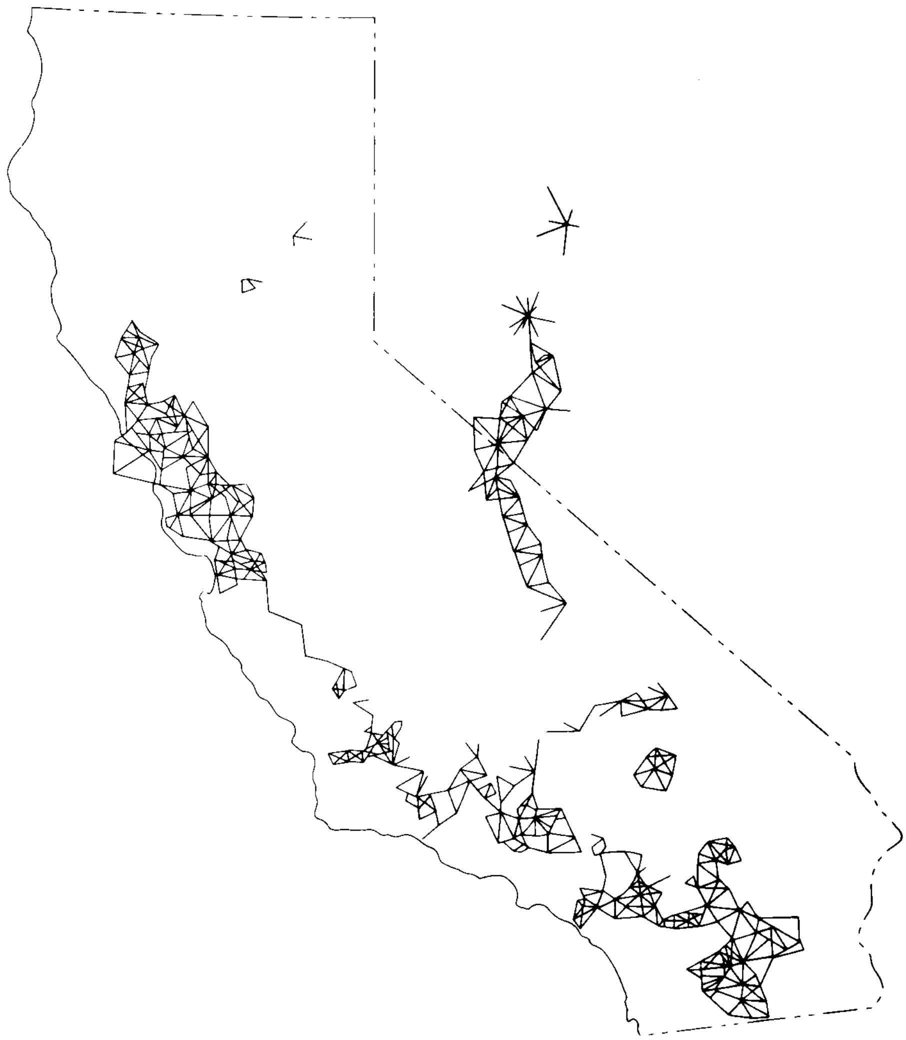


Figure 17.6. Since the early 1970s, Federal and State agencies have monitored crustal deformation using electronic distance measuring instrumentation. The figure identifies frequently measured lines located in California and Nevada.

Step 1: Determine the regions R_P and R_Q that contain stations P and Q respectively. Recall that crustal motion models were formulated for 19 mutually disjoint regions. Points in the United States that are not contained in any of these 19 regions are assigned to a twentieth, complementary region where the crustal motion model for region 20 is defined as the null model, that is, no motion.

Step 2: Let T denote the type of observation to be corrected. Use the model for R_P to compute $\phi_P(t_1)$, $\lambda_P(t_1)$, $\phi_Q(t_1)$, and $\lambda_Q(t_1)$ according to eq. (17.1), and let $b(t_1,R_P)$ denote the hypothetical observation of type T that would be measured at time t_1 between P and Q given these coordinates. Similarly use the model for R_P to compute $\phi_P(t_2)$, $\lambda_P(t_2)$, $\phi_Q(t_2)$, and $\lambda_Q(t_2)$ according to eq. (17.1), and let $b(t_2,R_P)$ denote the hypothetical observation of type T that would be measured at time t_2 between P and Q given these coordinates.

Step 3: Use the model for R_Q to compute, as in step 2, the hypothetical observations $b(t_1,R_Q)$ and $b(t_2,R_Q)$ of type T between P and Q at times t_1 and t_2, respectively.

Step 4: If b denotes the value of the actual observation measured at time t_1, then

$$\begin{aligned} b' &= b + 1/2\,[b(t_2,R_P) - b(t_1,R_P)] \\ &\quad + 1/2\,[b(t_2,R_Q) - b(t_1,R_Q)] \end{aligned} \qquad (17.5)$$

is the corrected observation corresponding to time t_2.

Note that the correction in eq. (17.5) represents the average of two estimates for the crustal motion between times t_1 and t_2: one estimate from the model for region R_P and the other from the model for region R_Q. This averaging process minimizes possible discrepancies between different models for observations that cross regional boundaries. Recall that regional boundaries are artifacts which, unlike most district boundaries, do not correspond to geologic faults.

Note also that the algorithm does not require that observation b be projected onto the reference ellipsoid to compute b', even though $b(t_1,R_P)$, $b(t_2,R_P)$, $b(t_1,R_Q)$, and $b(t_2,R_Q)$ may correspond to hypothetical observations on the reference ellipsoid. Moreover, any other data correction, for example, refraction, can be applied

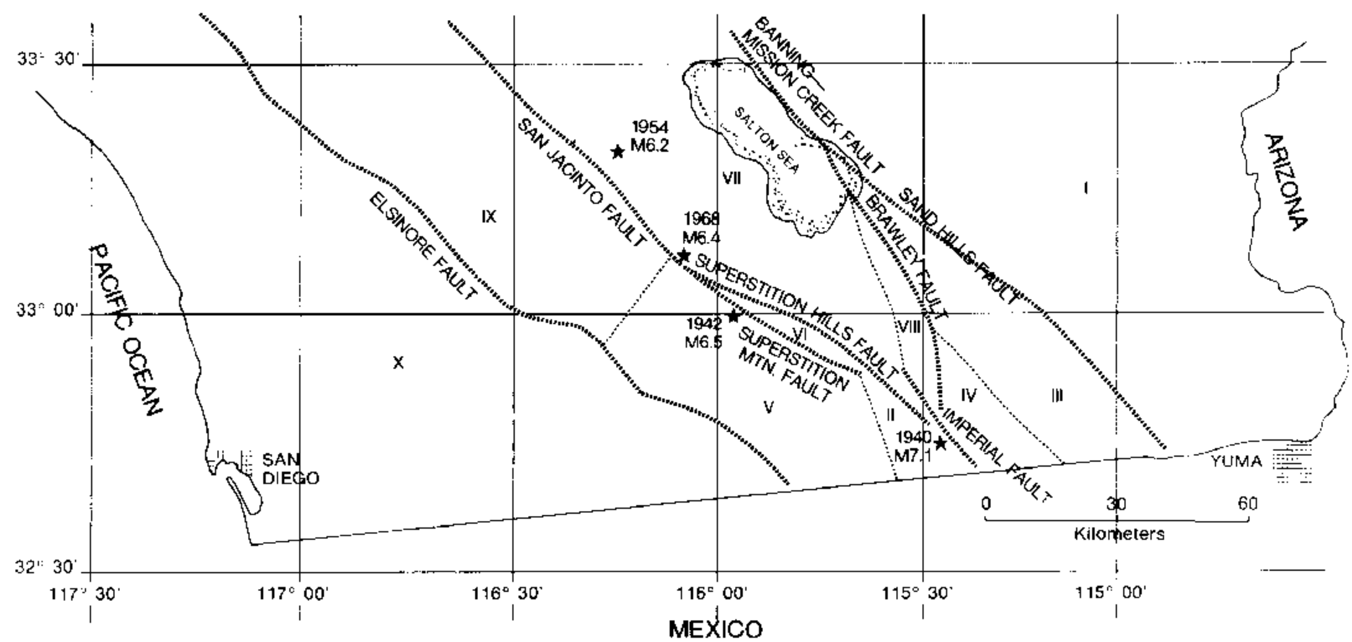


Figure 17.7. A mosaic of 10 districts is used to model the secular motion of the San Diego region. Each Roman numeral identifies a district that can individually translate, rotate, and undergo homogeneous deformation at a constant rate with respect to time. District boundaries usually approximate geologic faults (hatched lines) so that relative motion between the districts corresponds to secular slip. The dashed lines denote district boundaries that do not correspond to faults. Dislocation theory is used to model the episodic motion associated with major earthquakes. The stars locate four modeled earthquakes identified by year of occurrence and magnitude.

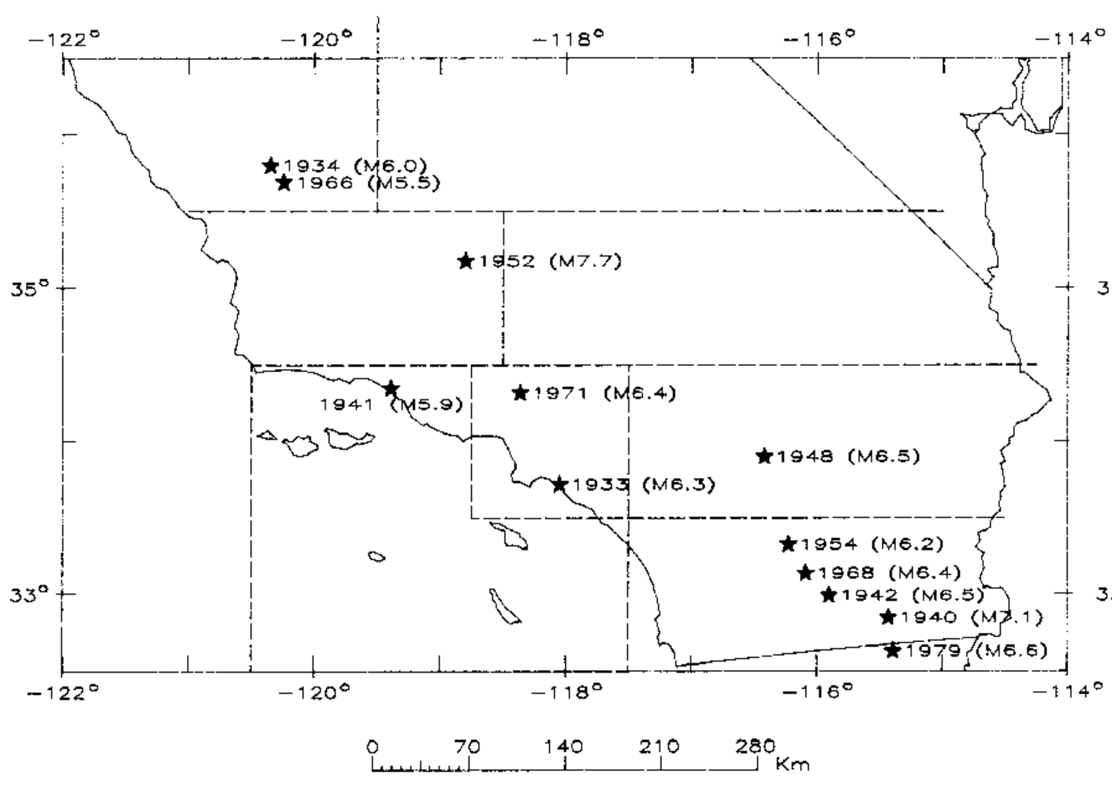


Figure 17.8. Modeled earthquakes in California identified by year of occurrence and magnitude.

either before or after the crustal motion correction, provided the value of the correction does not strongly depend on the date of observation or on the horizontal coordinate changes at the level of expected crustal motion.

Finally note that REDEAM models should not be used to compute positions that predate the 1906 San Francisco earthquake (April 18) for marks in California which are north of the 35.5 degree parallel of latitude.

17.5 MODEL EVALUATION

Evaluation of the REDEAM models constitutes an ongoing process. Five papers, in addition to the final report (Snay et al., 1987), have already appeared in print, and other studies are anticipated. Four of these five publications discuss specific models and compare these models both with results derived by independent investigators and with current geophysical theories. In particular, these publications discuss the model for the San Diego region (Snay et al., 1983), the model for the Los Angeles region (Cline et al., 1984), the models for the San Francisco, Santa Rosa, and Ukiah regions (Cline et al., 1985), and the model for the Fallon, Nevada, region (Snay et al., 1985). The fifth publication (Snay et al., 1986) presents an overall evaluation of the 16 regional models spanning California. The fifth paper also discusses ideas for improving the models. This section recaps some of the material appearing in these publications.

Figure 17.9 portrays the derived shear strain pattern for California. Because we have chosen to model the

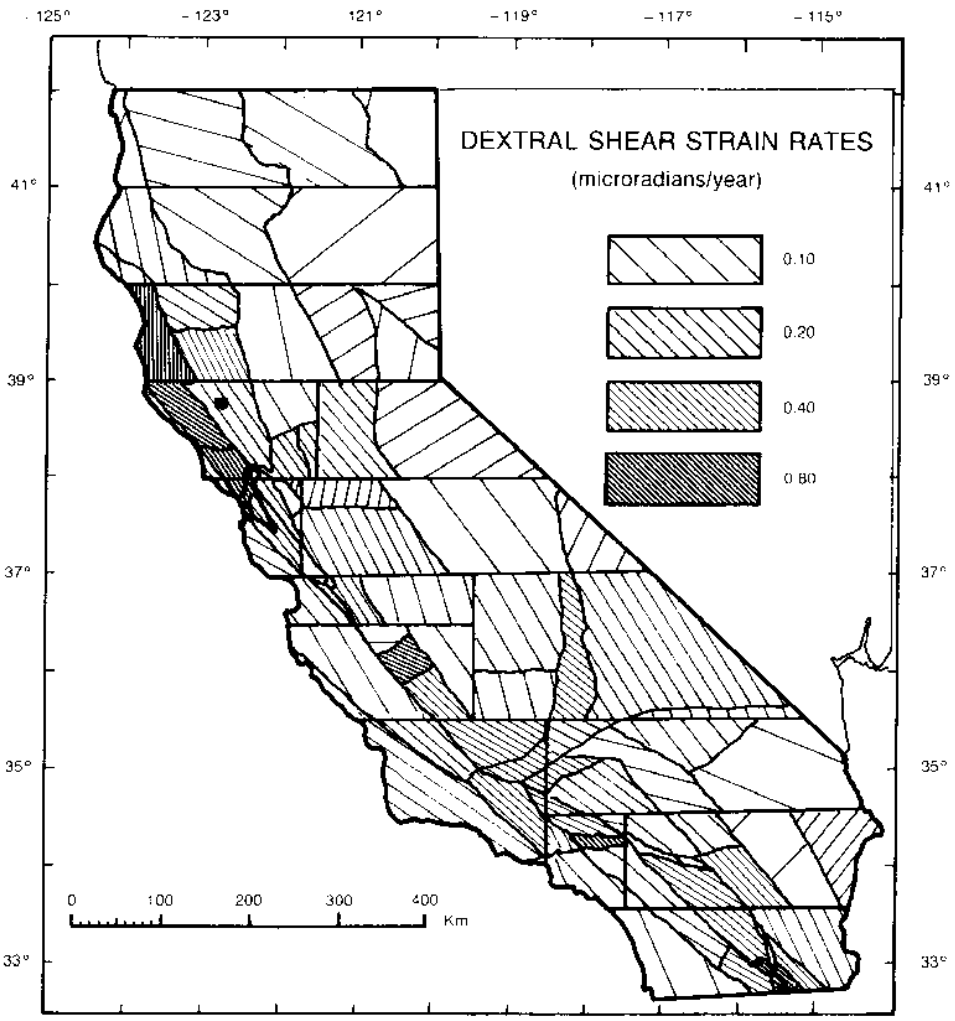


Figure 17.9. Secular shear strain pattern for California as derived from historical geodetic data. The line patterns designate directions and magnitudes (engineering units) of maximum dextral (right-lateral) shear strain rates for the mosaic of districts. Shearing between the North American and Pacific plates dominates the regional stress field producing an overall northwest-southeast trend for the direction of maximum dextral shear strain. The secular motion is assumed to be linear in time and excludes the movements associated directly with earthquakes of magnitude six and greater.

16 California regions independently, artificial discontinuities in secular velocity occur across regional boundaries. These discontinuities have an rms value of ~5 mm/yr even after individual models are adjusted to be mutually compatible. We partitioned California into 16 regions to limit the size of the individual data sets. Even with 16 regions the data sets were often too large for a person to become adequately familiar with all the data contained in a region. Also, we limited the sizes of the data sets because model development was largely a trial-and-error procedure: (1) the data sets had to be screened for blunders, (2) district boundaries had to be resolved, and (3) appropriate earthquake fault parameters had to be determined. It was not uncommon for us to perform as many as 10 adjustments of a region's data to various models in search of the best representation for regional crustal movement. Moreover, each such adjustment strained computer resources to the extent that frequently the computer could only execute the adjustment over the weekend. Now, having modeled the 16 regions, we feel sufficiently familiar with the data, the techniques, and the geophysics to undertake a simultaneous adjustment of all California data to a single model.

Figure 17.9 shows only the shear components of the secular deformation pattern. Data limitations render our estimates of the other components (rotation and dilatation) questionable.

The rotation estimates depend on the astronomic azimuth data. The geodetic archives contain less than 400 such azimuth observations for California, with each observation having a standard deviation of 7 microradians (1.4 arc sec) or greater. Moreover, the azimuth observations are distributed poorly through time, with approximately 80 percent observed since 1960. Consequently, rotation uncertainties (1σ) for districts are about 0.1 microradians per year. Said differently, for every 100 km separating two stations, an uncertainty of 1 cm/yr exists in the transverse component of the secular velocity between these stations. These velocity uncertainties are similar in magnitude to the expected secular velocities between stations on opposite sides of the state (Minster and Jordan, 1978). The near-future availability of space-based data, providing three-dimensional coordinate differences between stations to centimeter-level precision over lines exceeding 100 km in length, will enhance our estimates of regional rotation.

The dilatation component of the deformation depends on the collection of distance observations. Prior to the initial deployment of EDM equipment around 1960, distances were laboriously taped. Consequently, the set of distance observations spans essentially less than three decades. Moreover, unmodeled systematic errors, having magnitudes on the order of several parts in 10^6, are thought to contaminate much of the distance data. (See Snay et al., 1983: table 2; and Cline et al., 1984: table 4.) Considering the short time base, such errors could easily bias our estimated dilatation rates at the level of a few parts in 10^7 per year—a level that approximates in magnitude the dilatation rates that have been accurately measured with EDM by the USGS for selected areas of California (Savage, 1983). These USGS EDM data are more precise than most of the other archived distance measurements because the USGS flew aircraft over the observed lines-of-sight to obtain temperature and humidity profiles to better correct for refraction. For routine geodetic work, only endpoint meteorological readings are recorded. The highly precise USGS EDM data measured before 1979 were included in the REDEAM modeling project. These data profoundly helped to subdue biases in our dilatation-rate estimates, yet their effect was understandably limited to the areas that these data cover. The USGS monitoring program continues through time to the present and has expanded gradually to cover a greater area. Our planned second generation model will benefit from the inclusion of the USGS data measured since 1979.

Also, to better address the problem of systematic errors in the distance data, a future generation model will probably include several scale parameters. These parameters would be introduced on the premise that a large part of the systematic error manifests itself as scale factors, each such factor being common to a group of distances observed with the same instrument. Such an error could be caused, for example, by instrumental miscalibration. These scale parameters would be estimated simultaneously with other model parameters via the least squares process.

The secular shear strain pattern (fig. 17.9) is well determined for California because of the preponderance of triangulation data. The earliest data are from the 1850s, but most pre-1900 data are concentrated along the coast where they primarily supported navigational charting. For most California areas, then, the first geodetic data were observed in the 1930s, corresponding in time to the introduction of the Bilby tower. Consequently, the shear strain-rate estimates correspond essentially to a 50-year time interval, 1930-80. These estimates also represent spatial averages over several tens of kilometers; that is, the models presume that secular strain is spatially homogeneous within each district. A localized study of USGS EDM data in southern California (King and Savage, 1983) demonstrates the need for a model allowing greater spatial resolution. (See fig. 17.10.)

Figure 17.8 identifies the modeled earthquakes in California. The locations, dimensions, and orientations of the various rectangles that represent the fault planes were specified after reviewing the geologic and seismic literature. We scanned this literature for hypocenters, aftershock zones, focal mechanisms, and surface ruptures. The uncertainties (1σ) associated with the estimated components of the coseismic slip vectors imply decimeter-level resolution. We believe these uncertainties, however, are overly optimistic because the locations, dimensions, and orientations of the dislocation planes were introduced into the solution as if they were perfectly known. Also, the derived uncertainties are optimistic because of the ambiguities that exist in discriminating between coseismic and secular fault slip. Consequently, we contend that the existing geodetic data in general suffice only to resolve those coseismic slips exceeding a meter in magnitude. A meter of slip essentially requires an event with $M \geqslant 7$ (Bonilla and Buchanan, 1970), such as the 1940 Imperial Valley earthquake ($M = 7.1$) or the 1952 Kern County earthquake ($M = 7.7$). Only a few seismic events with $6 \leqslant M \leqslant 7$ have their coseismic slips well determined by existing horizontal data.

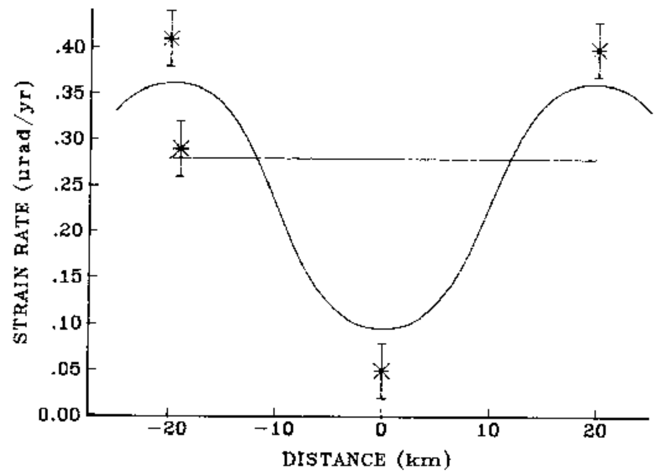


Figure 17.10. The San Bernardino regional model contains a district corresponding in area to the 40-kilometer-wide strip of land between the San Andreas and San Jacinto fault zones. The above graph represents a cross sections of this strip with the San Andreas fault zone located at +20 km and the San Jacinto at −20 km. The REDEAM model presumes that the secular shear strain rate is homogeneous across the strip. The horizontal line represents the REDEAM rate. Asterisks (with 1σ error bars) represent four localized strain-rate estimates that reveal significant spatial variation across the strip. The curve corresponds to a hypothetical representation of this variation as proposed by King and Savage (1983).

It may be unfair to attribute this poor resolution of coseismic slip completely to data limitations. Some of the problem, more than likely, rests with our employed mathematical representation of earthquake movement. Our technique assumes that coseismic slip is constant over a rectangle whose dimensions are typically on the order of tens of kilometers. Current theory for earthquake mechanics, however, favors the existence of significant spatial variations in slip over the rupture surface. The newer theory promotes the concepts of "asperities" and "barriers" that strongly influence the distribution of coseismic slip (Aki, 1984). Both terms refer to strong patches on the fault that are resistive to breaking. Considerable study is yet needed to identify and classify these fault features and then develop analytic expressions that will more realistically represent episodic motion.

17.6 SUMMARY

Crustal motion models were produced for 19 total regions, 16 of which combine to cover all of California, with one model each for parts of Nevada, Hawaii, and Alaska. The models address both the secular and episodic components of motion. For secular motion, each modeled region is partitioned into a mosaic of districts that are individually allowed to translate, rotate, and deform homogeneously as a linear function of time. Episodic movement corresponds to displacements associated with large earthquakes ($M \geqslant 6$), and is modeled in accordance with elastic dislocation theory. Prior to the NAD adjustment, the models were used to update all appropriate geodetic observations to the values that would be obtained if the observations were remeasured on December 31, 1983.

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18. PROJECT EXECUTION

Elizabeth B. Wade

18.1 OVERVIEW

The National Geodetic Survey performed two major data processing roles in the new adjustment of the North American Datum. NGS was responsible for the preparation of the normal equations from its own data holdings, and acted as a computing center by merging the normal equations of different participants.

In terms of computations, the Geodetic Survey of Canada was the only other participant. Greenland was brought into the project only through the Doppler observations, and these computations were totally independent of the networks on the remainder of the continent. Observations of the geodetic network in Mexico and Central America were gathered and furnished to NGS by the Defense Mapping Agency (DMA). These data were placed into the NGS data base and treated as part of the U.S. block. No attempt was made to draw a Helmert block boundary along the U.S.-Mexico border.

On the other hand, the observations made by space systems (Doppler, GPS, and VLBI) were treated as separate blocks, distinct from the very large block of U.S. terrestrial observations.

To NGS, the dominating task was to adjust the U.S. terrestrial observations by using the programs developed for that purpose, while combining those data with other data sources at the highest level. From this point of view, the NAD 83 adjustment involved the following major steps:

1. *Develop adjustment strategy*. The entire area of the contiguous United States, Mexico, Central America, Puerto Rico, the Virgin Islands, Alaska, and Hawaii was divided (STRATEGY) into 163 first-level blocks averaging 1,500 stations each. A binary combination scheme also was developed.
2. *Create Adjustment Project File (APF)*. A file to track the administration of the project was created (CRAPF) from the designated strategy.
3. *Retrieve first-level blocks*. All horizontal data were retrieved (RESTRT) from the NGS data base into RESTART files. Transformations to preliminary NAD 83 positions were made and crustal motion corrections computed where necessary.
4. *Create Helmert blocks*. Observational data in each RESTART file were used to form a system of partially reduced normal equations called a Helmert Block (HBNEMO). The creation of Helmert blocks was then registered with the APF (REGISTER).
5. *Run forward solution to highest level*. Helmert blocks were combined (FWD), two at a time. Unknowns which were interior to the combined block were then eliminated, creating a new Helmert block. This was aided by an automatic program (DISPATCH) which queried the APF to determine if additional combination runs were possible.
6. *Add additional observational data and perform highest level solution*. Doppler data and VLBI data were retrieved and additional Helmert blocks formed from the space system data (SOAP). These data were added to the terrestrial observation node 1 block. The Canadian data were then received from the Geodetic Survey of Canada and the highest level system of equations was solved (HLS).
7. *Analyze highest level solution*. Geodesists performed the following analysis: variance of unit weight, residuals on the Doppler observations (SOAP), residuals on selected terrestrial observations, and singularities at the highest level.
8. *Run back solution*. The back substitution for each highest level combination job was made (STOAT). The back substitution for each forward job was run (RVS) and the solution transferred from each first-level Helmert block into the corresponding RESTART file (OMEN).
9. *Analyze adjustment results*. The resulting terrestrial residuals and position shifts were evaluated (POSTPROC) and corrections made (RJSEPROC). If convergence had not been reached, the entire process was then iterated from the creation of the Helmert block (step 4).

18.2 STRATEGY DEVELOPMENT

The development of the strategy for forming initial level normal equations and their combination was accomplished between February and May 1985. The primary criterion was that each block contain between 1,200 and 2,000 stations. A graphic representation of the numbers of stations in the NGS data base served as the main tool. Each represented a 1-degree by 2-degree area. (See fig. 18.1.) The contiguous United States, Alaska, Central America, and Puerto Rico were divided into 161 blocks. These block boundaries were along 30-minute graticule lines. However, the blocks were not always chosen to be rectangular, as had been the practice in the Block Validation phase.

The smallest block in geographic size was a 30-minute by 1-degree area (fig. 18.2), while the largest blocks were in Alaska (fig. 18.3). However, geographic size was not closely correlated with the num-

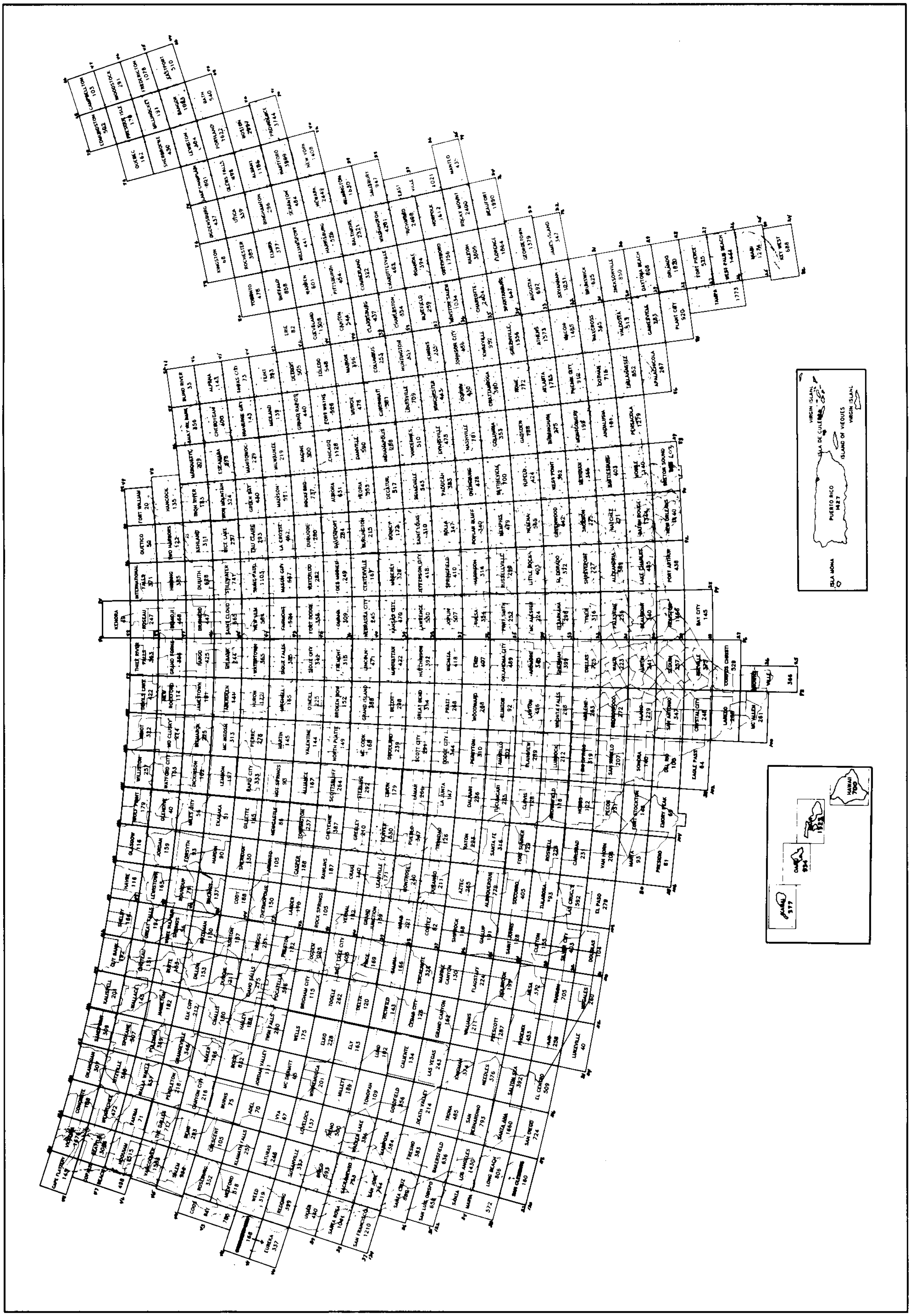

Figure 18.1. Horizontal control stations by specific area for NAD 83, as depicted on NGS Geodetic Control Diagrams.

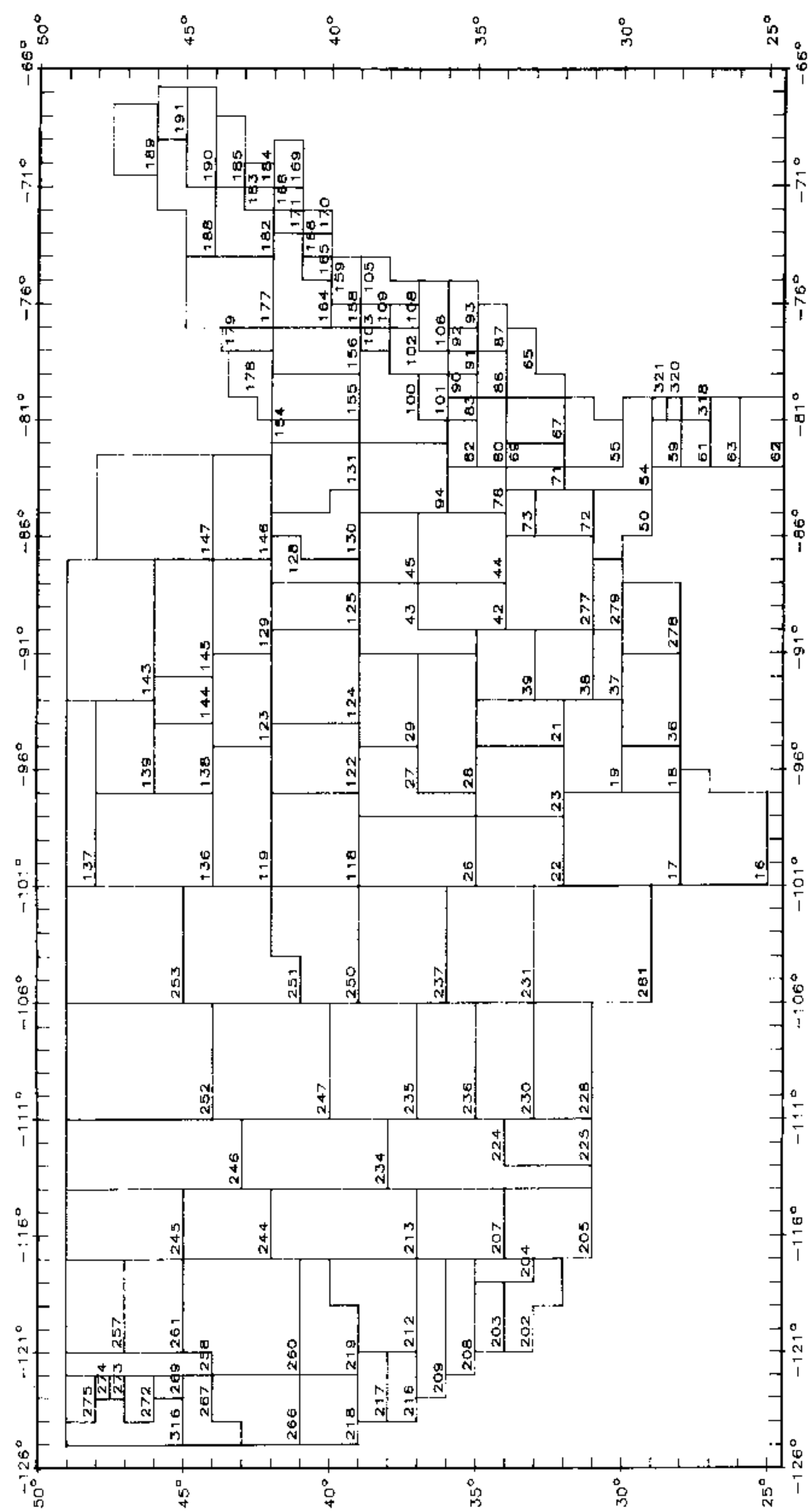

Figure 18.2. First-level Helmert blocks in the contiguous United States.

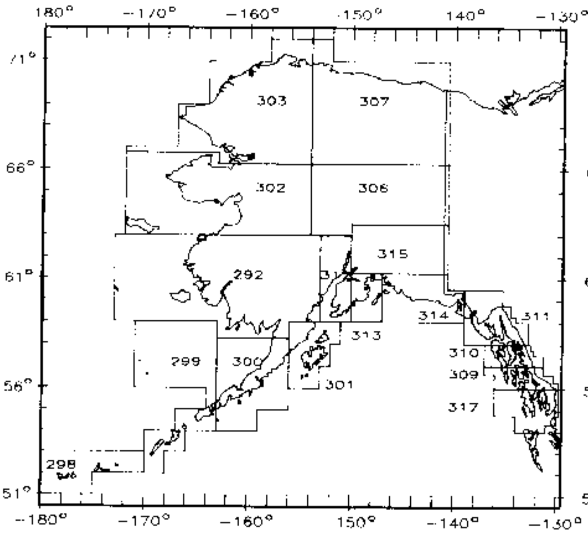

Figure 18.3. First-level Helmert blocks in Alaska.

ber of stations. By this measure, both the largest and smallest blocks were 1 degree by 1 degree in size: Block 318, in Florida, had the least number of stations (438), while block 171, in Connecticut, had the most (2,754).

Table 18.1 outlines the exact block definition areas. Some consideration was given to drawing the block boundaries through weak areas of the network, but this was not a dominant criterion. Using long straight boundaries to encode the areas easily was also considered important.

TABLE 18.1.—*Lowest level Helmert block definition areas*

Node 16					
	1.	027 00 00	096 00 00	028 00 00	101 00 00
	2.	025 00 00	097 00 00	027 00 00	101 00 00
Node 17					
	1.	029 00 00	097 00 00	031 00 00	101 00 00
	2.	028 00 00	097 00 00	029 00 00	101 00 00
	3.	031 00 00	097 00 00	032 00 00	101 00 00
Node 18					
	1.	028 00 00	095 00 00	029 00 00	097 00 00
	2.	029 00 00	095 00 00	030 00 00	097 00 00
Node 19					
	1.	030 00 00	095 00 00	031 00 00	097 00 00
	2.	031 00 00	093 00 00	032 00 00	097 00 00
	3.	030 00 00	093 00 00	031 00 00	095 00 00
Node 21					
	1.	032 00 00	093 00 00	035 00 00	095 00 00
Node 22					
	1.	032 00 00	098 00 00	035 00 00	101 00 00
Node 23					
	1.	032 00 00	095 00 00	035 00 00	098 00 00
Node 26					
	1.	035 00 00	098 00 00	039 00 00	101 00 00
Node 27					
	1.	035 00 00	097 00 00	039 00 00	098 00 00
	2.	037 00 00	095 00 00	039 00 00	097 00 00
Node 28					
	1.	035 00 00	093 00 00	037 00 00	097 00 00
	2.	035 00 00	091 00 00	037 00 00	093 00 00
Node 29					
	1.	037 00 00	091 00 00	039 00 00	093 00 00
	2.	037 00 00	093 00 00	039 00 00	095 00 00
Node 36					
	1.	029 00 00	091 00 00	030 00 00	095 00 00
	2.	028 00 00	091 00 00	029 00 00	095 00 00
Node 37					
	1.	030 00 00	090 00 00	031 00 00	093 00 00
Node 38					
	1.	031 00 00	090 00 00	032 00 00	093 00 00
	2.	032 00 00	090 00 00	033 00 00	093 00 00
Node 39					
	1.	034 00 00	090 00 00	035 00 00	093 00 00
	2.	033 00 00	090 00 00	034 00 00	093 00 00
Node 42					
	1.	034 00 00	088 00 00	035 00 00	090 00 00
	2.	035 00 00	088 00 00	037 00 00	090 00 00
Node 43					
	1.	037 00 00	088 00 00	039 00 00	091 00 00
	2.	035 00 00	090 00 00	037 00 00	091 00 00
Node 44					
	1.	034 00 00	086 00 00	035 00 00	088 00 00
	2.	034 00 00	085 00 00	037 00 00	086 00 00
	3.	035 00 00	086 00 00	037 00 00	088 00 00
Node 45					
	1.	037 00 00	085 00 00	039 00 00	086 00 00
	2.	037 00 00	086 00 00	039 00 00	088 00 00
Node 50					
	1.	030 00 00	084 00 00	031 00 00	087 00 00
	2.	029 00 00	084 00 00	030 00 00	086 00 00
Node 54					
	1.	029 00 00	080 00 00	030 00 00	082 00 00
	2.	029 00 00	082 00 00	030 00 00	083 00 00
	3.	029 00 00	083 00 00	030 00 00	084 00 00
	4.	030 00 00	083 00 00	031 00 00	084 00 00
	5.	031 00 00	083 00 00	032 00 00	084 00 00
Node 55					
	1.	030 00 00	081 00 00	031 00 00	083 00 00
	2.	031 00 00	080 00 00	032 00 00	083 00 00
Node 59					
	1.	028 00 00	082 00 00	029 00 00	083 00 00
	2.	028 00 00	081 00 00	029 00 00	082 00 00
Node 61					
	1.	027 00 00	082 00 00	028 00 00	083 00 00
	2.	027 00 00	081 00 00	028 00 00	082 00 00
Node 62					
	1.	024 00 00	080 00 00	026 00 00	082 00 00
	2.	024 00 00	082 00 00	026 00 00	083 00 00
Node 63					
	1.	026 00 00	080 00 00	027 00 00	082 00 00
	2.	026 00 00	082 00 00	027 00 00	083 00 00
Node 65					
	1.	033 00 00	077 00 00	034 00 00	079 00 00
	2.	032 00 00	079 00 00	034 00 00	080 00 00
Node 67					
	1.	032 00 00	080 00 00	034 00 00	082 00 00
Node 69					
	1.	032 00 00	082 00 00	034 00 00	083 00 00
Node 71					
	1.	032 00 00	083 00 00	034 00 00	084 00 00
Node 72					
	1.	031 00 00	084 00 00	032 00 00	086 00 00
	2.	032 00 00	084 00 00	033 00 00	086 00 00
Node 73					
	1.	033 00 00	084 00 00	034 00 00	086 00 00
Node 78					
	1.	034 00 00	083 00 00	036 00 00	085 00 00
Node 80					
	1.	034 00 00	082 00 00	035 00 00	083 00 00
	2.	034 00 00	080 00 00	035 00 00	082 00 00
Node 82					
	1.	035 00 00	082 00 00	036 00 00	083 00 00
	2.	035 00 00	081 00 00	036 00 00	082 00 00
Node 83					
	1.	035 00 00	080 00 00	036 00 00	081 00 00

TABLE 18.1.—*Lowest level Helmert block definition areas (continued)*

Node 86				
1.	034 00 00	079 00 00	035 00 00	080 00 00
2.	034 00 00	078 00 00	035 00 00	079 00 00
Node 87				
1.	034 00 00	077 00 00	035 00 00	078 00 00
2.	034 00 00	076 00 00	035 00 00	077 00 00
Node 90				
1.	035 00 00	079 00 00	036 00 00	080 00 00
Node 91				
1.	035 00 00	078 00 00	036 00 00	079 00 00
Node 92				
1.	035 00 00	077 00 00	036 00 00	078 00 00
Node 93				
1.	035 00 00	075 00 00	036 00 00	077 00 00
Node 94				
1.	036 00 00	082 00 00	039 00 00	083 00 00
2.	036 00 00	083 00 00	039 00 00	085 00 00
Node 100				
1.	038 00 00	078 00 00	039 00 00	079 00 00
2.	036 00 00	081 00 00	039 00 00	082 00 00
3.	037 00 00	079 00 00	039 00 00	081 00 00
Node 101				
1.	036 00 00	079 00 00	037 00 00	081 00 00
Node 102				
1.	036 00 00	078 00 00	038 00 00	079 00 00
2.	037 00 00	077 00 00	038 00 00	078 00 00
Node 103				
1.	038 00 00	077 00 00	039 00 00	078 00 00
Node 105				
1.	038 00 00	074 00 00	039 00 00	075 00 00
2.	037 00 00	075 00 00	039 00 00	076 00 00
Node 106				
1.	036 00 00	075 00 00	037 00 00	078 00 00
Node 108				
1.	037 00 00	076 00 00	038 00 00	077 00 00
Node 109				
1.	038 00 00	076 00 00	039 00 00	077 00 00
Node 118				
1.	039 00 00	097 00 00	042 00 00	101 00 00
Node 119				
1.	042 00 00	095 00 00	044 00 00	101 00 00
Node 122				
1.	039 00 00	094 00 00	042 00 00	097 00 00
Node 123				
1.	042 00 00	091 00 00	044 00 00	095 00 00
Node 124				
1.	039 00 00	090 00 00	042 00 00	094 00 00
Node 125				
1.	039 00 00	088 00 00	042 00 00	090 00 00
Node 128				
1.	039 00 00	087 00 00	042 00 00	088 00 00
2.	041 00 00	086 00 00	042 00 00	087 00 00
Node 129				
1.	042 00 00	087 00 00	044 00 00	091 00 00
Node 130				
1.	039 00 00	084 00 00	040 00 00	087 00 00
2.	041 00 00	085 00 00	042 00 00	086 00 00
3.	040 00 00	085 00 00	041 00 00	087 00 00
Node 131				
1.	039 00 00	082 00 00	041 52 30	083 00 00
2.	041 52 30	082 37 30	042 00 00	083 00 00
3.	039 00 00	083 00 00	041 00 00	084 00 00
4.	041 00 00	083 00 00	042 00 00	085 00 00
5.	040 00 00	084 00 00	041 00 00	085 00 00
Node 136				
1.	044 00 00	097 00 00	048 00 00	101 00 00
Node 137				
1.	048 00 00	099 30 00	049 15 00	101 00 00
2.	048 00 00	098 00 00	049 22 30	099 30 00
3.	048 00 00	097 37 30	049 15 00	098 00 00
4.	048 00 00	096 00 00	049 07 30	097 37 30
5.	048 00 00	094 30 00	049 30 00	096 00 00
6.	048 00 00	094 00 00	049 00 00	094 30 00
7.	048 00 00	093 00 00	048 45 00	094 00 00
Node 138				
1.	044 00 00	094 00 00	046 00 00	097 00 00
Node 139				
1.	046 00 00	094 00 00	048 00 00	097 00 00
2.	046 00 00	093 00 00	048 00 00	094 00 00
Node 143				
1.	048 00 00	089 00 00	048 45 00	093 00 00
2.	048 00 00	087 45 00	048 52 30	089 00 00
3.	048 00 00	087 30 00	048 45 00	087 45 00
4.	048 00 00	087 00 00	048 37 30	087 30 00
5.	046 00 00	087 00 00	048 00 00	093 00 00
Node 144				
1.	044 00 00	092 00 00	046 00 00	094 00 00
Node 145				
1.	044 00 00	087 00 00	046 00 00	092 00 00
Node 146				
1.	042 00 00	083 00 00	044 00 00	087 00 00
2.	042 30 00	082 22 30	044 00 00	083 00 00
3.	042 22 30	082 30 00	042 30 00	083 00 00
4.	042 15 00	082 45 00	042 22 30	083 00 00
Node 147				
1.	044 00 00	086 00 00	046 00 00	087 00 00
2.	048 00 00	086 00 00	048 30 00	087 00 00
3.	044 00 00	085 30 00	048 15 00	086 00 00
4.	044 00 00	085 00 00	047 52 30	085 30 00
5.	044 00 00	084 45 00	047 45 00	085 00 00
6.	044 00 00	084 30 00	047 15 00	084 45 00
7.	044 00 00	084 15 00	047 00 00	084 30 00
8.	044 00 00	084 00 00	046 37 30	084 15 00
9.	044 00 00	083 15 00	046 15 00	084 00 00
10.	044 00 00	083 00 00	046 00 00	083 15 00
11.	044 00 00	082 30 00	045 52 30	083 00 00
12.	044 00 00	082 00 00	045 00 00	082 30 00
13.	046 00 00	086 00 00	048 00 00	087 00 00
Node 154				
1.	039 00 00	081 00 00	042 00 00	082 00 00
Node 155				
1.	039 00 00	079 00 00	042 00 00	081 00 00
Node 156				
1.	040 00 00	077 00 00	042 00 00	079 00 00
2.	039 00 00	077 00 00	040 00 00	079 00 00
Node 158				
1.	039 00 00	076 00 00	040 00 00	077 00 00
Node 159				
1.	039 00 00	075 00 00	040 00 00	076 00 00
2.	039 00 00	074 00 00	040 00 00	075 00 00
Node 164				
1.	040 00 00	075 00 00	042 00 00	077 00 00
2.	041 00 00	073 00 00	042 00 00	074 00 00
3.	041 00 00	074 00 00	042 00 00	075 00 00
Node 165				
1.	040 00 00	074 00 00	041 00 00	075 00 00
Node 166				
1.	040 00 00	073 00 00	041 00 00	074 00 00
Node 168				
1.	041 00 00	071 00 00	042 00 00	072 00 00
Node 169				
1.	041 00 00	070 00 00	042 00 00	071 00 00
2.	041 00 00	069 00 00	042 00 00	070 00 00
Node 170				
1.	040 00 00	072 00 00	041 00 00	073 00 00
Node 171				
1.	041 00 00	072 00 00	042 00 00	073 00 00
Node 177				
1.	042 00 00	076 45 00	043 45 00	077 00 00
2.	042 00 00	076 30 00	044 07 30	076 45 00
3.	042 00 00	076 07 30	044 22 30	076 30 00
4.	042 00 00	076 00 00	044 30 00	076 07 30
5.	042 00 00	075 30 00	044 52 30	076 00 00
6.	042 00 00	075 00 00	045 07 30	075 30 00
7.	042 00 00	074 45 00	045 15 00	075 00 00
8.	042 00 00	074 30 00	045 22 30	074 45 00
9.	044 00 00	074 00 00	045 07 30	074 30 00
10.	042 00 00	074 00 00	044 00 00	074 30 00

TABLE 18.1.—*Lowest level Helmert block definition areas (continued)*

Node 178				
1.	042 00 00	080 00 00	042 30 00	081 00 00
2.	042 00 00	079 07 30	042 45 00	080 00 00
3.	042 00 00	078 00 00	043 30 00	079 07 30
Node 179				
1.	042 00 00	077 00 00	043 45 00	078 00 00
Node 182				
1.	042 00 00	072 00 00	044 00 00	074 00 00
2.	043 00 00	071 37 30	044 00 00	072 00 00
3.	043 00 00	071 00 00	044 00 00	071 37 30
Node 183				
1.	042 00 00	071 37 30	043 00 00	072 00 00
2.	042 00 00	071 00 00	043 00 00	071 37 30
Node 184				
1.	042 00 00	070 45 00	043 00 00	071 00 00
2.	042 00 00	070 30 00	043 00 00	070 45 00
3.	042 00 00	070 22 30	043 00 00	070 30 00
4.	042 00 00	070 00 00	043 00 00	070 22 30
Node 185				
1.	043 00 00	068 00 00	044 00 00	070 00 00
2.	043 00 00	070 45 00	044 00 00	071 00 00
3.	043 00 00	070 30 00	044 00 00	070 45 00
4.	043 00 00	070 22 30	044 00 00	070 30 00
5.	043 00 00	070 00 00	044 00 00	070 22 30
Node 188				
1.	044 00 00	071 37 30	045 07 30	074 00 00
2.	044 00 00	071 00 00	045 30 00	071 37 30
3.	045 00 00	070 45 00	045 30 00	071 00 00
4.	045 00 00	070 30 00	045 45 00	070 45 00
5.	045 00 00	070 22 30	046 00 00	070 30 00
6.	045 00 00	070 00 00	046 00 00	070 22 30
7.	045 00 00	069 30 00	046 00 00	070 00 00
8.	045 00 00	069 00 00	046 00 00	069 30 00
Node 189				
1.	046 30 00	070 22 30	047 00 00	070 30 00
2.	046 00 00	070 00 00	047 00 00	070 22 30
3.	046 00 00	069 30 00	047 15 00	070 00 00
4.	046 00 00	069 00 00	047 37 30	069 30 00
5.	046 00 00	067 30 00	047 30 00	069 00 00
Node 190				
1.	044 00 00	070 45 00	045 00 00	071 00 00
2.	044 00 00	070 30 00	045 00 00	070 45 00
3.	044 00 00	070 22 30	045 00 00	070 30 00
4.	044 00 00	070 00 00	045 00 00	070 22 30
5.	044 00 00	069 30 00	045 00 00	070 00 00
6.	044 00 00	069 00 00	045 00 00	069 30 00
7.	044 00 00	066 37 30	045 00 00	066 45 00
8.	044 00 00	067 30 00	045 00 00	069 00 00
9.	044 00 00	067 15 00	045 00 00	067 30 00
10.	044 00 00	067 00 00	045 00 00	067 15 00
11.	044 00 00	066 45 00	045 00 00	067 00 00
Node 191				
1.	045 00 00	067 30 00	046 00 00	069 00 00
2.	045 00 00	067 15 00	045 45 00	067 30 00
3.	045 00 00	067 00 00	045 30 00	067 15 00
4.	045 00 00	066 45 00	045 15 00	067 00 00
Node 202				
1.	032 00 00	117 00 00	033 00 00	119 00 00
2.	033 00 00	118 00 00	034 00 00	119 00 00
3.	033 00 00	119 00 00	034 00 00	121 00 00
Node 203				
1.	034 00 00	119 00 00	035 00 00	121 00 00
2.	034 00 00	118 00 00	035 00 00	119 00 00
Node 204				
1.	034 00 00	117 00 00	035 00 00	118 00 00
2.	033 00 00	117 00 00	034 00 00	118 00 00
Node 205				
1.	026 00 00	114 00 00	031 00 00	116 00 00
2.	031 00 00	114 00 00	032 00 00	117 00 00
3.	032 00 00	114 00 00	034 00 00	117 00 00
Node 207				
1.	034 00 00	114 00 00	037 00 00	117 00 00
Node 208				
1.	035 00 00	119 00 00	036 00 00	121 00 00
2.	035 00 00	121 00 00	036 00 00	122 00 00
3.	035 00 00	117 00 00	036 00 00	119 00 00
Node 209				
1.	036 00 00	119 00 00	037 00 00	121 00 00
2.	036 00 00	121 00 00	037 00 00	122 00 00
3.	036 00 00	122 00 00	037 00 00	123 00 00
4.	036 00 00	117 00 00	037 00 00	119 00 00
Node 212				
1.	037 00 00	119 00 00	039 00 00	121 00 00
2.	037 00 00	117 00 00	039 00 00	119 00 00
3.	039 00 00	117 00 00	040 00 00	119 00 00
Node 213				
1.	040 00 00	114 00 00	042 00 00	117 00 00
2.	037 00 00	114 00 00	039 00 00	117 00 00
3.	039 00 00	114 00 00	040 00 00	117 00 00
Node 216				
1.	037 00 00	121 00 00	038 00 00	122 00 00
2.	037 00 00	122 00 00	038 00 00	123 00 00
3.	037 00 00	123 00 00	038 00 00	124 00 00
Node 217				
1.	038 00 00	121 00 00	039 00 00	122 00 00
2.	038 00 00	122 00 00	039 00 00	123 00 00
3.	038 00 00	123 00 00	039 00 00	124 00 00
Node 218				
1.	039 00 00	122 00 00	041 00 00	123 00 00
2.	039 00 00	123 00 00	041 00 00	124 00 00
3.	039 00 00	124 00 00	041 00 00	125 00 00
Node 219				
1.	039 00 00	121 00 00	041 00 00	122 00 00
2.	040 00 00	117 00 00	041 00 00	119 00 00
3.	039 00 00	119 00 00	041 00 00	121 00 00
Node 224				
1.	035 00 00	111 00 00	036 00 00	114 00 00
2.	036 00 00	111 00 00	038 00 00	114 00 00
3.	026 00 00	113 00 00	031 00 00	114 00 00
4.	031 00 00	113 00 00	032 00 00	114 00 00
5.	032 00 00	113 00 00	035 00 00	114 00 00
6.	034 00 00	111 00 00	035 00 00	113 00 00
Node 225				
1.	026 00 00	111 00 00	031 00 00	113 00 00
2.	022 30 00	111 00 00	026 00 00	112 30 00
3.	031 00 00	111 00 00	032 00 00	113 00 00
4.	032 00 00	111 00 00	034 00 00	113 00 00
Node 228				
1.	026 00 00	106 00 00	031 00 00	111 00 00
2.	022 30 00	106 00 00	026 00 00	111 00 00
3.	032 00 00	106 00 00	033 00 00	111 00 00
4.	031 00 00	106 00 00	032 00 00	111 00 00
Node 230				
1.	033 00 00	106 00 00	035 00 00	111 00 00
Node 231				
1.	033 00 00	101 00 00	036 00 00	106 00 00
Node 234				
1.	040 00 00	111 00 00	043 00 00	114 00 00
2.	038 00 00	111 00 00	039 00 00	114 00 00
3.	039 00 00	111 00 00	040 00 00	114 00 00
Node 235				
1.	039 00 00	106 00 00	040 00 00	111 00 00
2.	037 00 00	106 00 00	039 00 00	111 00 00
Node 236				
1.	035 00 00	106 00 00	036 00 00	111 00 00
2.	036 00 00	106 00 00	037 00 00	111 00 00
Node 237				
1.	036 00 00	101 00 00	037 00 00	106 00 00
2.	037 00 00	101 00 00	039 00 00	106 00 00
Node 244				
1.	042 00 00	114 00 00	043 00 00	117 00 00
2.	043 00 00	114 00 00	044 00 00	117 00 00
3.	044 00 00	114 00 00	045 00 00	117 00 00

TABLE 18.1.—*Lowest level Helmert block definition areas (continued)*

Node 245				
1.	048 00 00	116 30 00	049 15 00	117 00 00
2.	048 00 00	114 00 00	049 22 30	116 30 00
3.	045 00 00	114 00 00	048 00 00	117 00 00
Node 246				
1.	048 00 00	112 00 00	049 15 00	114 00 00
2.	043 00 00	111 00 00	044 00 00	114 00 00
3.	048 00 00	111 00 00	049 22 30	112 00 00
4.	044 00 00	111 00 00	048 00 00	114 00 00
Node 247				
1.	040 00 00	106 00 00	044 00 00	111 00 00
Node 250				
1.	039 00 00	101 00 00	042 00 00	104 00 00
2.	039 00 00	104 00 00	041 00 00	106 00 00
Node 251				
1.	042 00 00	101 00 00	044 00 00	106 00 00
2.	044 00 00	101 00 00	045 00 00	106 00 00
3.	041 00 00	104 00 00	042 00 00	106 00 00
Node 252				
1.	048 00 00	107 00 00	049 22 30	111 00 00
2.	044 00 00	106 00 00	045 00 00	111 00 00
3.	045 00 00	106 00 00	048 00 00	111 00 00
4.	048 00 00	106 00 00	049 15 00	107 00 00
Node 253				
1.	045 00 00	101 00 00	048 00 00	106 00 00
2.	048 00 00	101 00 00	049 15 00	106 00 00
Node 257				
1.	048 00 00	117 00 00	049 15 00	121 00 00
2.	047 00 00	117 00 00	048 00 00	121 00 00
Node 258				
1.	048 00 00	121 00 00	049 15 00	122 00 00
2.	047 00 00	121 00 00	048 00 00	122 00 00
3.	044 00 00	121 00 00	047 00 00	122 00 00
Node 260				
1.	041 00 00	119 00 00	044 00 00	121 00 00
2.	041 00 00	121 00 00	044 00 00	122 00 00
3.	041 00 00	117 00 00	042 00 00	119 00 00
4.	043 00 00	117 00 00	044 00 00	119 00 00
5.	042 00 00	117 00 00	043 00 00	119 00 00
6.	044 00 00	117 00 00	045 00 00	121 00 00
Node 261				
1.	045 00 00	117 00 00	047 00 00	121 00 00
Node 266				
1.	041 00 00	122 00 00	044 00 00	123 00 00
2.	041 00 00	123 00 00	044 00 00	124 00 00
3.	041 00 00	124 00 00	043 00 00	125 00 00
Node 267				
1.	043 00 00	124 00 00	044 00 00	125 00 00
2.	044 00 00	122 00 00	045 00 00	125 00 00
Node 269				
1.	045 00 00	122 00 00	046 00 00	123 00 00
Node 272				
1.	046 00 00	122 00 00	047 00 00	124 00 00
Node 273				
1.	047 00 00	122 00 00	047 30 00	123 00 00
Node 274				
1.	047 30 00	122 00 00	048 00 00	123 00 00
Node 275				
1.	048 00 00	123 45 00	048 30 00	124 00 00
2.	048 00 00	122 00 00	049 07 30	123 45 00
Node 277				
1.	031 00 00	086 00 00	032 00 00	090 00 00
2.	032 00 00	086 00 00	034 00 00	090 00 00
Node 278				
1.	029 00 00	088 00 00	030 00 00	091 00 00
2.	028 00 00	088 00 00	029 00 00	091 00 00
Node 279				
1.	030 00 00	087 00 00	031 00 00	088 00 00
2.	030 00 00	088 00 00	031 00 00	090 00 00
Node 281				
1.	031 00 00	101 00 00	032 00 00	106 00 00
2.	029 00 00	101 00 00	031 00 00	106 00 00
3.	032 00 00	101 00 00	033 00 00	106 00 00
Node 282				
1.	025 00 00	101 00 00	029 00 00	106 00 00
2.	022 30 00	097 00 00	025 00 00	106 00 00
3.	019 00 00	096 00 00	022 30 00	103 00 00
4.	015 45 00	095 00 00	019 00 00	102 30 00
5.	014 00 00	091 00 00	019 00 00	095 00 00
6.	013 00 00	087 00 00	019 00 00	091 00 00
7.	012 00 00	083 00 00	018 00 00	087 00 00
8.	008 00 00	082 00 00	012 00 00	087 00 00
9.	007 00 00	075 00 00	010 00 00	082 00 00
10.	018 00 00	081 00 00	023 30 00	085 00 00
11.	018 00 00	076 00 00	023 00 00	081 00 00
12.	018 00 00	068 00 00	021 00 00	076 00 00
Node 283				
1.	017 30 00	064 30 00	018 45 00	068 00 00
Node 292				
1.	060 30 00	158 00 00	063 00 00	173 00 00
2.	058 15 00	156 00 00	059 00 00	163 00 00
3.	059 00 00	153 00 00	060 30 00	173 00 00
4.	060 30 00	153 00 00	063 00 00	158 00 00
Node 298				
1.	051 00 00	175 00 00	052 00 00	183 00 00
2.	052 00 00	170 00 00	053 00 00	188 00 00
Node 299				
1.	053 00 00	166 00 00	054 00 00	170 00 00
2.	052 00 00	168 00 00	053 00 00	170 00 00
3.	058 15 00	163 00 00	059 00 00	171 00 00
4.	057 00 00	163 00 00	058 00 00	171 00 00
5.	056 00 00	163 00 00	057 00 00	171 00 00
6.	055 00 00	163 00 00	056 00 00	164 00 00
7.	054 00 00	163 00 00	055 00 00	167 00 00
8.	058 00 00	163 00 00	058 15 00	171 00 00
Node 300				
1.	055 00 00	156 00 00	056 00 00	163 00 00
2.	054 00 00	159 00 00	055 00 00	163 00 00
3.	058 00 00	156 00 00	058 15 00	163 00 00
4.	057 00 00	156 00 00	058 00 00	163 00 00
5.	056 00 00	156 00 00	057 00 00	163 00 00
Node 301				
1.	057 00 00	152 00 00	058 00 00	156 00 00
2.	056 00 00	153 00 00	057 00 00	156 00 00
3.	058 00 00	151 00 00	059 00 00	156 00 00
Node 302				
1.	063 00 00	158 00 00	066 15 00	172 00 00
2.	063 00 00	154 00 00	066 15 00	158 00 00
3.	066 15 00	163 00 00	066 45 00	172 00 00
Node 303				
1.	066 15 00	158 00 00	067 00 00	163 00 00
2.	067 00 00	158 00 00	069 00 00	167 00 00
3.	069 00 00	158 00 00	071 00 00	164 00 00
4.	071 00 00	154 00 00	072 00 00	158 00 00
5.	066 15 00	154 00 00	071 00 00	158 00 00
6.	066 45 00	163 00 00	067 00 00	172 00 00
Node 306				
1.	063 00 00	150 00 00	063 30 00	153 00 00
2.	063 30 00	140 37 30	066 15 00	153 00 00
3.	063 00 00	153 00 00	066 15 00	154 00 00
Node 307				
1.	066 15 00	140 37 30	071 00 00	153 00 00
2.	071 00 00	152 00 00	072 00 00	154 00 00
3.	066 15 00	153 00 00	071 00 00	154 00 00
Node 309				
1.	056 00 00	129 45 00	056 15 00	136 00 00
2.	056 15 00	130 15 00	056 37 30	136 00 00
3.	056 37 30	131 15 00	057 00 00	137 00 00
Node 310				
1.	057 00 00	131 15 00	057 30 00	137 00 00
2.	057 30 00	132 15 00	057 45 00	137 00 00
3.	057 45 00	132 30 00	058 00 00	137 00 00
Node 311				
1.	058 00 00	132 45 00	058 15 00	139 00 00
2.	058 15 00	133 00 00	058 30 00	139 00 00
3.	058 30 00	133 15 00	059 00 00	139 00 00
4.	059 00 00	134 00 00	059 07 30	139 00 00

TABLE 18.1.—*Lowest level Helmert block definition areas (continued)*

Node 311 (continued)				
5.	059 07 30	134 00 00	059 15 00	139 00 00
6.	059 15 00	134 30 00	059 22 30	139 00 00
7.	059 22 30	134 30 00	059 30 00	136 45 00
8.	059 30 00	134 45 00	059 45 00	136 45 00
9.	059 45 00	135 15 00	059 52 30	136 15 00
10.	059 22 30	137 45 00	059 37 30	139 00 00
11.	059 37 30	138 15 00	060 00 00	139 00 00
12.	060 00 00	138 45 00	060 30 00	139 00 00
Node 312				
1.	060 30 00	150 00 00	063 00 00	153 00 00
2.	059 00 00	150 00 00	060 30 00	153 00 00
Node 313				
1.	059 00 00	147 00 00	060 30 00	150 00 00
2.	060 30 00	147 00 00	061 15 00	150 00 00
Node 314				
1.	059 00 00	139 00 00	060 30 00	147 00 00
2.	060 30 00	140 37 30	061 15 00	147 00 00
Node 315				
1.	061 15 00	140 37 30	063 30 00	147 00 00
2.	061 15 00	147 00 00	063 30 00	150 00 00
Node 316				
1.	047 00 00	125 00 00	048 00 00	126 00 00
2.	048 00 00	124 00 00	048 45 00	126 00 00
3.	048 00 00	126 00 00	048 07 30	134 07 30
4.	046 00 00	124 00 00	048 00 00	125 00 00
5.	047 00 00	123 00 00	048 00 00	124 00 00
6.	045 00 00	123 00 00	046 00 00	125 00 00
Node 317				
1.	054 00 00	131 00 00	054 22 30	134 00 00
2.	054 22 30	130 00 00	054 45 00	134 00 00
3.	048 07 30	134 00 00	054 07 30	134 07 30
4.	054 45 00	129 45 00	056 00 00	136 00 00
Node 318				
1.	027 00 00	080 00 00	028 00 00	081 00 00
Node 320				
1.	028 00 00	080 00 00	028 30 00	081 00 00
Node 321				
1.	028 30 00	080 00 00	029 00 00	081 00 00

One very important aspect of strategy development involved defining the boundary line between the United States and Canada. This included the boundary between Alaska and Canada as well as between the contiguous United States and Canada. Not only had control stations been placed right on the boundary by the International Boundary Commission but major arcs of triangulation overlapped the political boundary. This meant that a large number of junction stations would be created if the Helmert blocks were divided along the political boundary. The solution involved moving the Helmert block boundary north of the political boundary (fig. 18.4), so that the border surveys were all included in the U.S. block. This "geodetic boundary" had been defined by the Geodetic Survey of Canada in 1978.

The combination of Helmert blocks within the United States was designed from the top down. The first division was made along a path that was roughly the shortest line dividing the country in half. (See red line in fig. 18.5.) This action was taken because the last combination in the strategy was assumed to contain the largest number of unknowns and would take the longest time to solve. As it turned out, the largest set of unknowns was found in the next subdivision of the eastern half of the country. (See orange line in fig. 18.5.) Concern was expressed that these higher level blocks might be too large to process with the computer and software available at the time. Figure 18.5 graphically displays the successive divisions level by level.

The tree resulting from this strategy lacks perfect balance. The most common number of levels is 7, but some dense areas on the east and west coasts contain 10. A major variation occurs in Alaska, where a strategy with six levels is used. This strategy is connected to a seventh-level block in Washington State. The strategy used for Alaska had been developed in 1984 for an earlier test of the Helmert blocking programs. The decision makers decided to keep this strategy intact for the final network adjustment. The trees depicted graphically in figures 18.6 through 18.10 reflect this combined strategy.

Table 18.2 contains statistics for the number of unknowns and the number of observations in each block. The number of inside stations for each lowest level block is also shown. The largest block in terms of numbers of unknowns is block 3, formed by combining block 4 (southeast United States, with 3,209 junction unknowns) and block 5 (northeast United States, with 3,658 junction unknowns). In the combination 2,359 unknowns were common to both blocks, leaving 4,498 unique unknowns in block 3.

18.3 CREATION OF THE APF

The Adjustment Project File (APF) was created and initialized. This file contained information pertinent to the administration of the project, a representation of the strategy and on the state of the adjustment, and locations of the disk and tape files used by the adjustment programs.

A Project Log File was also created and initialized. This log file recorded messages from all computational parts of the adjustment. By the completion of the adjustment, the log file contained more than 6,000 entries.

Other decisions were made at this time concerning the retrieval and processing of data. These retrieval and processing options were also stored in the APF, so that all data base retrievals would be executed with the same set of options. For example, it was necessary to decide which source of astronomic positions, deflections, and geoid heights would be used. A decision was made to use the actual astronomic observations where they existed and the gravimetrically predicted deflections elsewhere. The crustal motion model required a reference epoch from which to calculate corrections to the observations. The date of December 31, 1983, was selected.

Another question requiring resolution was whether to retrieve a complete copy of the horizontal observations from the data base, including observations to reference marks and unrecognized stations. The other option would have been to retrieve only those observations which would be used in the adjustment. Although there was no immediate need for the extra observations, concerns were expressed about analyzing and publishing the data later as well as a proposed change of data base computer. Therefore, the question was resolved by retrieving all horizontal observations into the NAD 83 RESTART files.

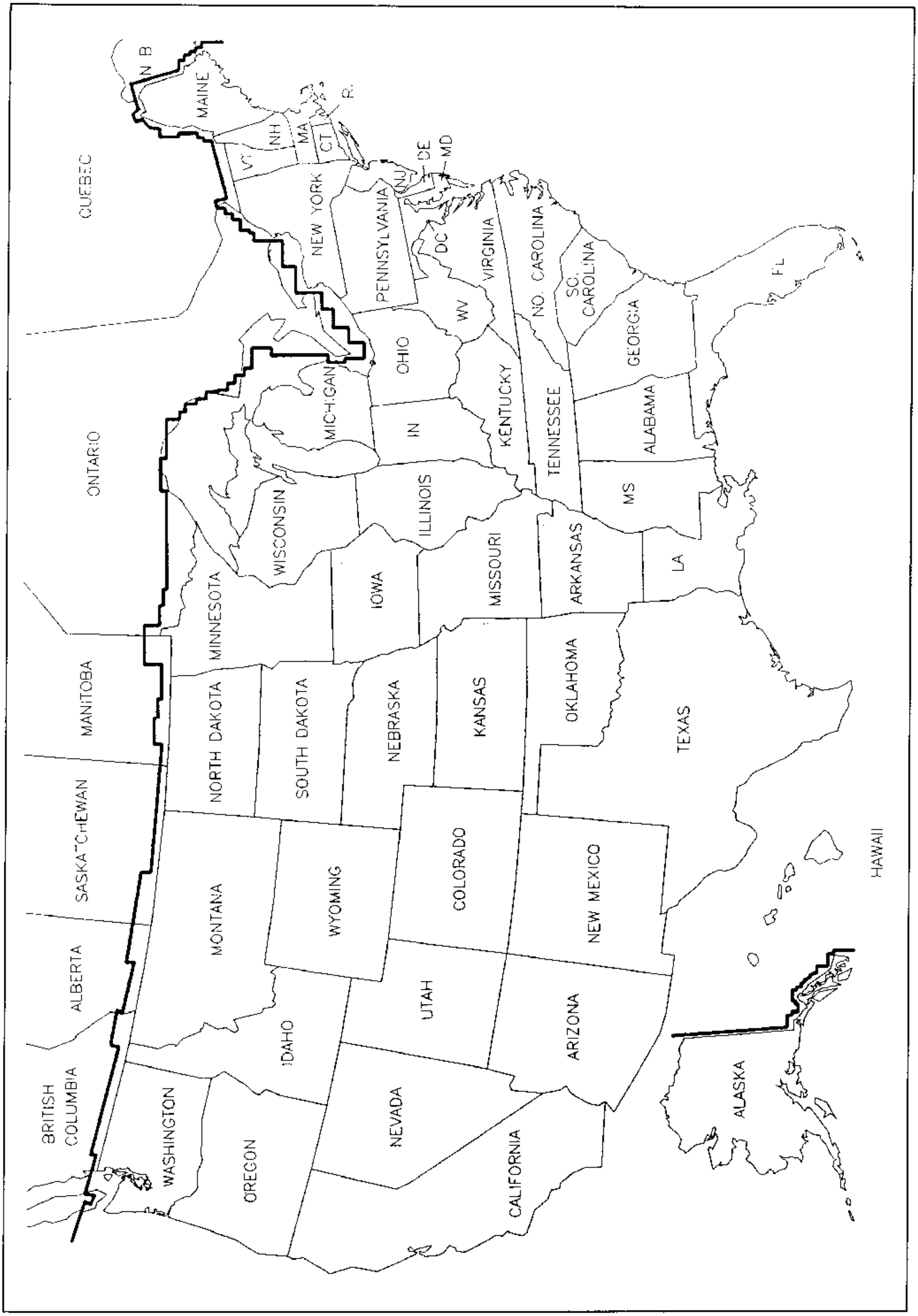

Figure 18.4. United States-Canada geodetic boundary.

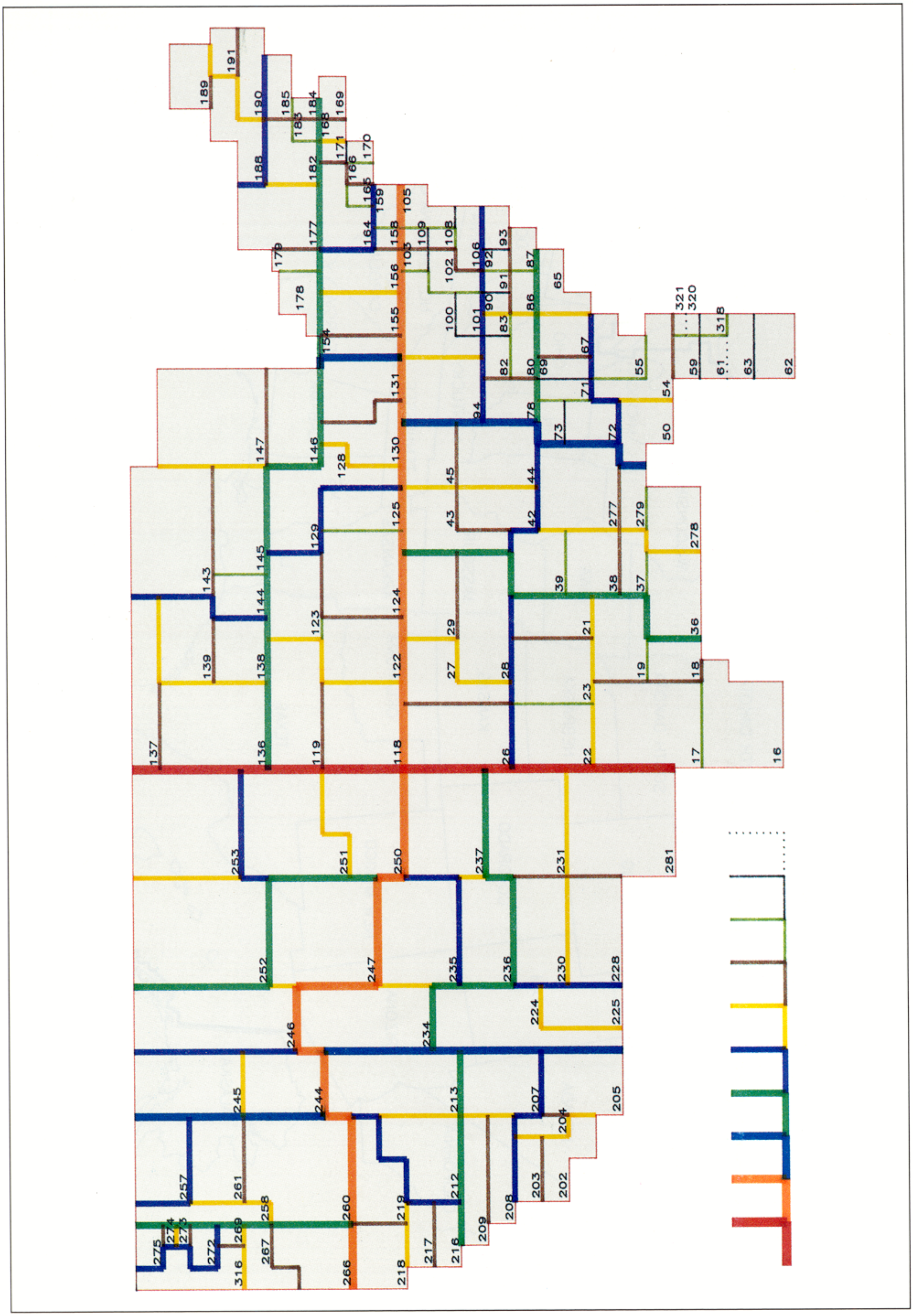

Figure 18.5. Helmert block dividing lines.

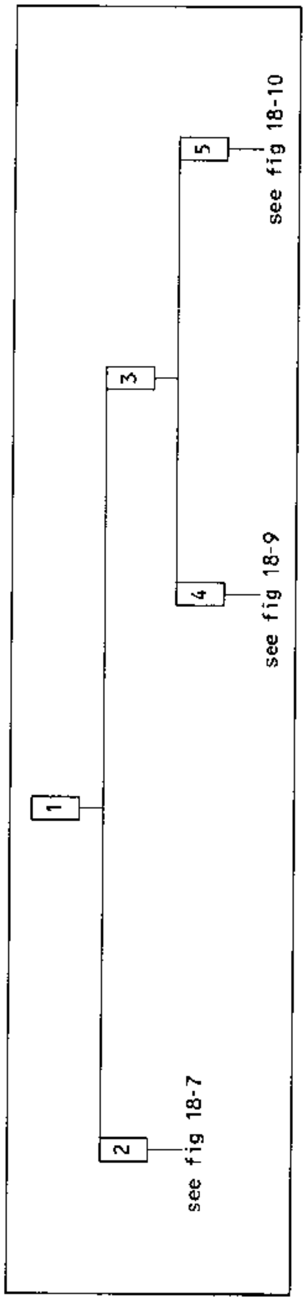

Figure 18.6. Helmert blocking strategy showing highest level terrestrial tree.

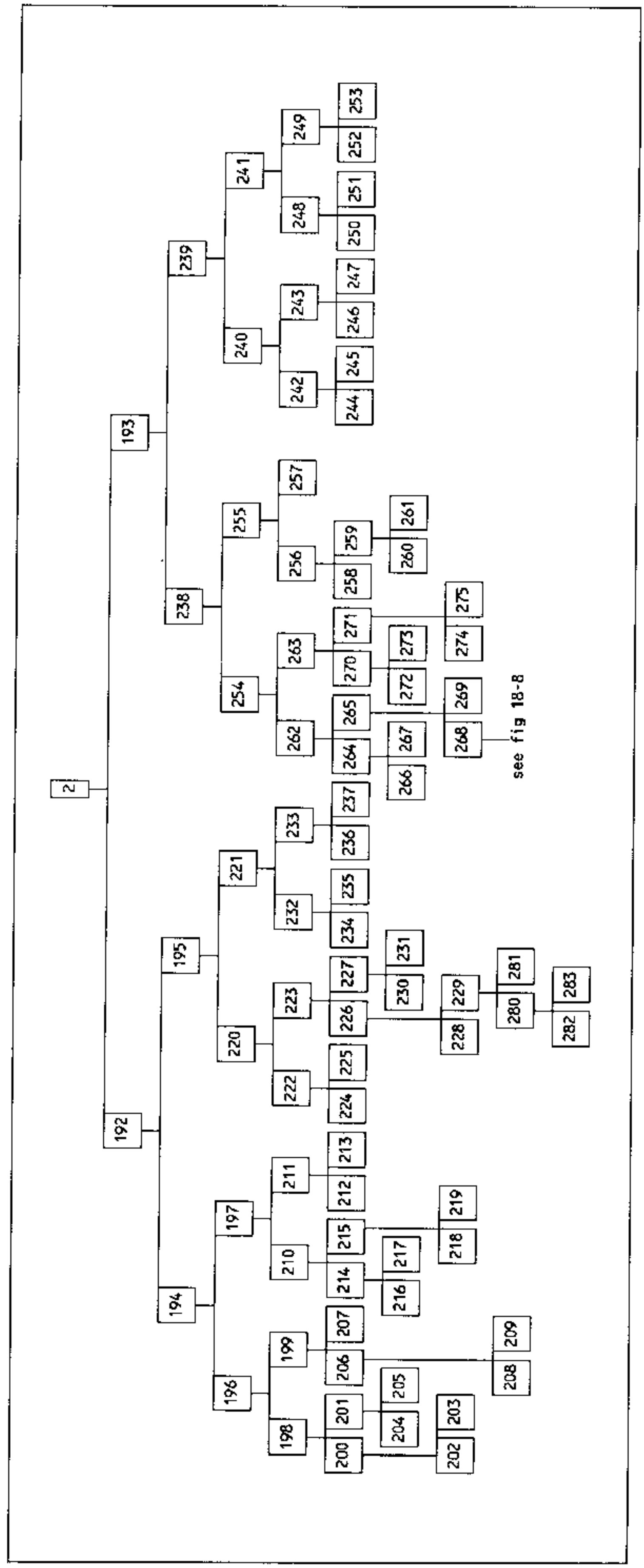

Figure 18.7. Helmert blocking strategy for western United States.

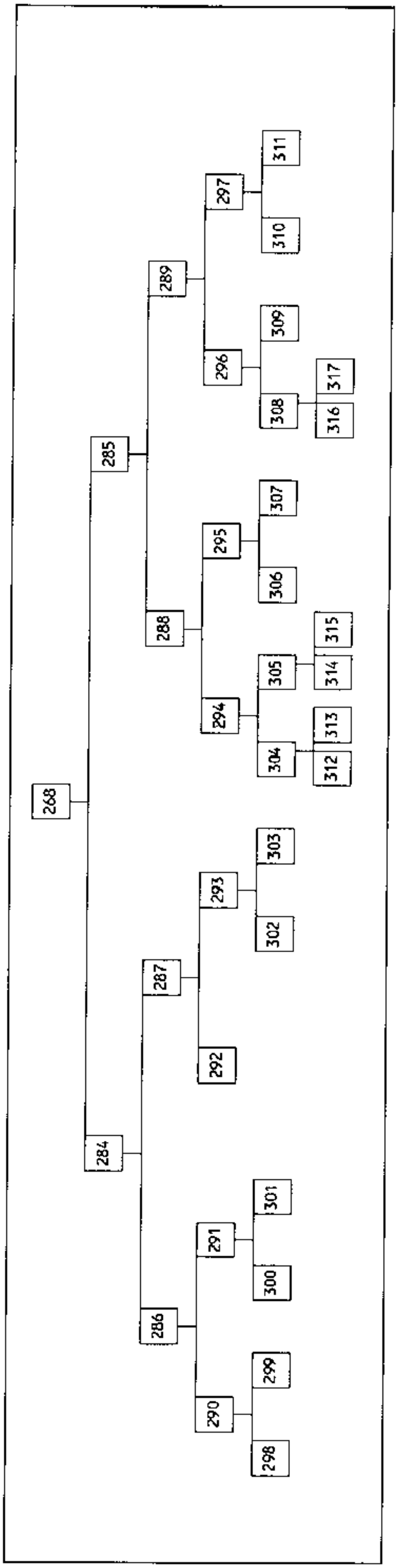

Figure 18.8. Helmert blocking strategy for Alaska.

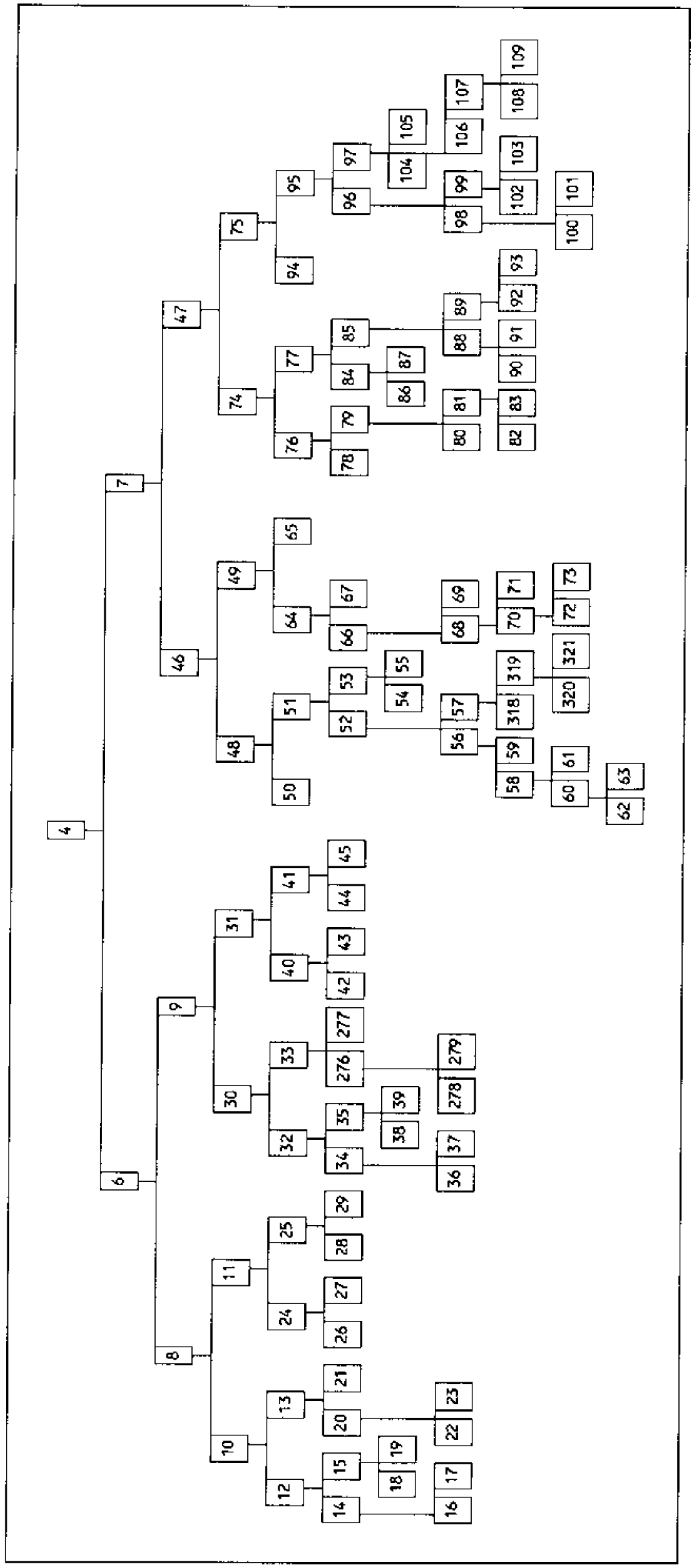

Figure 18.9. Helmert blocking strategy for southeastern United States.

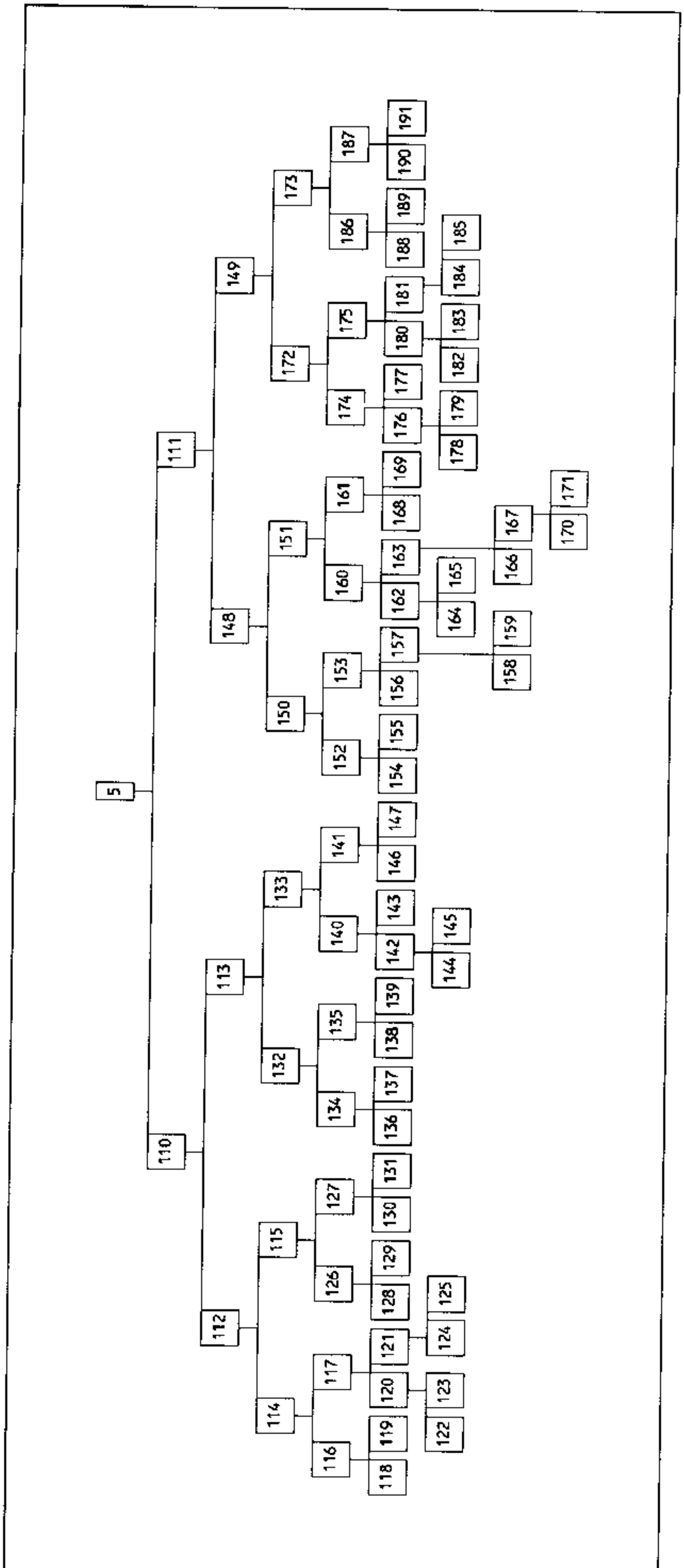

Figure 18.10. Helmert blocking strategy for northeastern United States.

TABLE 18.2.—*NAD 83 Helmert block stations and unknowns*

Block No.	Interior unknowns	Junction unknowns	Cumulative interior unknowns	Cumulative observations	Inside stations
1	1242	2168	894923	1720008	0
2	1576	2529	343633	698474	0
3	2356	2142	550048	1021534	0
4	728	3209	315332	582234	0
5	172	3658	232360	439300	0
6	1296	2551	131393	260347	0
7	776	2132	183211	321887	0
8	370	2898	65777	134279	0
9	320	2249	64320	126068	0
10	438	1813	39622	81360	0
11	506	1862	25785	52919	0
12	322	1399	22064	45582	0
13	714	1305	17120	35778	0
14	805	758	12068	24369	0
15	1066	1309	9656	21215	0
16	5045	448	5045	9932	1593
17	6236	1309	6236	14437	1921
18	4321	1065	4321	10206	1422
19	4269	1482	4269	11009	1389
20	1301	1078	12624	26682	0
21	3782	1160	3482	9096	1196
22	4922	1283	4922	12314	1497
23	6401	1324	6401	14368	1819
24	1798	1432	12541	26720	0
25	1544	1468	12738	26199	0
26	5192	1591	5192	12665	1581
27	5551	2057	5551	14055	1853
28	5721	1620	5721	12956	1798
29	5473	1708	5473	13243	1715
30	772	1684	38879	77455	0
31	316	1232	25121	48613	0
32	212	1993	21915	45472	0
33	629	1249	38879	77455	0
34	1114	1419	10193	20822	0
35	1095	1009	11510	24650	0
36	3428	1309	3428	7962	1350
37	5651	1445	5651	12860	1848
38	6032	1204	6032	13761	1881
39	4383	1135	4383	10889	1411
40	1503	984	11899	23656	0
41	1026	906	12906	24957	0
42	5022	1176	5022	11106	1590
43	5374	1738	5374	12550	1792
44	6448	1222	6448	13911	1962
45	5432	918	5432	11046	1614
46	392	1213	78183	151425	0
47	1004	250	104252	170462	0
48	296	576	47097	93417	0
49	630	1427	30694	58008	0
50	6155	634	6155	12604	1814
51	186	319	40646	80813	0
52	250	197	29961	61482	0
53	669	497	10499	19331	0
54	4677	804	4677	8979	1590
55	5153	553	5153	10352	1795
56	529	393	21270	40972	0
57	251	307	8441	20510	0
58	363	481	15752	33899	0
59	3989	766	3989	7073	1371
60	693	214	11592	24843	0
61	4797	737	4797	9056	1687
62	5896	380	5896	12831	1961
63	5003	743	5003	12012	1647
64	677	1466	25543	48500	0
65	4521	891	4521	9508	1706
66	452	1108	20035	38571	0
67	4831	1303	4831	9929	1883
68	607	956	15589	30746	0
69	3994	804	3994	7825	1581
70	741	742	10936	22431	0
71	4046	1018	4046	8315	1476
72	4901	830	4901	11155	1625
73	5294	914	5294	11276	1752
74	188	2000	52110	76045	0
75	645	2532	53138	94417	0
76	705	1358	18629	29936	0
77	446	1049	31293	46109	0
78	4177	1371	4177	9628	1488
79	729	933	13747	20308	0
80	3630	1234	3630	7381	1457
81	600	841	9388	12927	0
82	4174	845	4174	6490	1482
83	4614	699	4614	6437	1601
84	551	801	10624	17159	0
85	126	1153	20223	28950	0
86	4473	5244	4473	7071	1557
87	5600	674	5600	10088	1942
88	616	607	11843	15481	0
89	592	807	8254	13469	0
90	4431	667	4431	6149	1506
91	6796	758	6796	9332	2296
92	4045	631	4045	6320	1424
93	3617	939	3617	7149	1377
94	5941	1093	5941	11074	1801
95	622	2285	46552	83343	0
96	398	1974	22803	40050	0
97	796	1655	23127	43296	0
98	1163	1256	10883	18548	0
99	1168	1562	11522	21502	0
100	4735	1850	4735	11298	1580
101	4985	711	4985	7253	1715
102	5324	1074	5324	9410	1821
103	5030	1738	5030	12092	1704
104	867	1909	17619	33416	0
105	4712	978	4712	9877	1550
106	5444	1387	5444	11292	1950
107	1074	1808	11308	22124	0
108	3973	1123	3973	8429	1539
109	6261	1898	6261	13695	2246
110	826	2144	105865	195186	0
111	1246	1876	126323	244114	0
112	572	2405	56144	106599	0
113	516	1407	48895	88587	0
114	474	2664	33438	65596	0
115	228	1242	22134	41003	0
116	979	1203	12120	25436	0
117	220	2106	20844	40160	0
118	5739	1217	5739	13287	1690
119	5402	1061	5402	12149	1655
120	835	1537	10980	21301	0
121	972	1031	9644	18859	0
122	5135	1416	5135	11463	1688
123	5010	1006	5010	9838	1763
124	4167	1062	4167	9172	1371
125	4505	1194	4505	9687	1620
126	979	1035	11132	20967	0
127	1243	675	10774	20036	0
128	4288	1198	4288	9161	1494
129	5865	1063	5865	11806	1924
130	4935	1174	4935	9988	1651
131	4596	1081	4596	10048	1538
132	524	1267	22392	41326	0
133	162	1185	25987	47261	0
134	113	1166	11181	21782	0
135	942	1172	10687	19544	0
136	5532	1403	5532	12259	1647
137	4536	1050	4536	9523	1488
138	4198	1106	4198	8243	1607

TABLE 18.2.—*NAD 83 Helmert block stations and unknowns (continued)*

Block No.	Interior unknowns	Junction unknowns	Cumulative interior unknowns	Cumulative observations	Inside stations
139	5547	1218	5547	11301	1849
140	716	1126	16193	28887	0
141	408	387	9632	18374	0
142	702	1762	10289	18155	0
143	5188	1060	5188	10732	1668
144	4903	873	4903	9602	1836
145	4984	1014	4684	8553	1595
146	4725	446	4725	9167	1583
147	4499	458	4499	9207	1403
148	644	2692	67128	134023	0
149	696	1689	57949	110091	0
150	234	2314	26150	52078	0
151	368	1673	40334	81945	0
152	1084	727	10103	19476	0
153	1005	2082	15813	32602	0
154	3882	752	3882	8213	1371
155	5137	1307	5137	11263	1656
156	5413	2073	5413	13231	1805
157	1129	1256	9395	19371	0
158	4199	1583	4199	10550	1493
159	4067	1045	4067	8821	1529
160	1248	1363	29698	60555	0
161	973	1070	10268	21390	0
162	1569	2097	13400	27637	0
163	789	1780	15050	32918	0
164	7128	2360	7128	16821	2568
165	4703	1499	4703	10816	1994
166	4884	1877	4884	13209	1971
167	1070	882	937	19709	0
168	5514	1411	5514	13440	2020
169	3781	906	3781	7950	1583
170	975	758	975	3465	402
171	7332	1490	7332	16244	2754
172	156	2144	35891	71950	0
173	284	964	21362	38141	0
174	456	744	16335	31169	0
175	1124	1736	19400	40781	0
176	459	420	10078	18764	0
177	5801	884	5801	12405	1812
178	4675	581	4675	9559	1589
179	4944	414	4944	9205	1516
180	1627	2084	9576	21553	0
181	1145	2028	8700	19228	0
182	4925	1629	4925	11746	1795
183	3024	2317	3024	9807	1370
184	2989	2427	2989	9371	1580
185	4566	953	4566	9857	1759
186	694	582	10936	19527	0
187	606	737	10142	18614	0
188	5351	1156	5351	10431	1854
189	4891	489	4891	9096	1616
190	4013	1009	4013	9141	1547
191	5523	520	5523	9473	1770
192	650	2709	154328	336387	0
193	540	2990	187729	362087	0
194	464	1423	79494	180063	0
195	1044	2608	74184	156324	0
196	568	991	45802	110868	0
197	632	1415	33228	69195	0
198	590	801	27671	68304	0
199	730	1349	17563	42564	0
200	2245	893	13238	32626	0
201	1654	1139	13843	35678	0
202	3907	1585	3907	9453	1428
203	7086	2006	7086	23173	1870
204	6242	1827	6242	19661	1761
205	5650	1176	5950	16017	1580
206	2049	837	11841	29346	0
207	4992	1359	4992	13218	1478
208	5823	1516	5823	16813	1542
209	3969	1680	3969	12533	1265
210	188	1307	22484	47051	0
211	1385	1405	10112	22144	0
212	4606	1647	4606	11482	1484
213	4121	1283	4121	10662	1202
214	1995	617	12784	27298	0
215	1253	1111	9512	19753	0
216	6345	1528	6345	15719	1784
217	4444	1587	4444	11579	1568
218	3872	885	3872	9490	1163
219	4387	1671	4387	10263	1497
220	406	2229	49599	105043	0
221	280	2508	23541	51281	0
222	1477	1135	12368	26690	0
223	502	1951	36825	78353	0
224	4879	1762	4879	12467	1533
225	6012	1117	6012	14223	1481
226	1054	1311	24794	51922	0
227	1627	1677	11529	26431	0
228	6515	1717	6515	17600	1885
229	419	868	17225	34322	0
230	4419	1894	4419	13902	1282
231	5483	1564	5483	12529	1674
232	1451	1492	11501	27605	0
233	1229	1631	11760	23676	0
234	5554	1645	5554	15801	1723
235	4496	1458	4496	11804	1442
236	4642	1467	4642	10775	1471
237	5889	1467	5889	12901	1830
238	1012	1729	139780	261718	0
239	684	2366	47409	100369	0
240	354	2091	21785	47937	0
241	256	1671	24940	52432	0
242	1336	1133	10096	22419	0
243	1280	1691	11335	25518	0
244	3758	1307	3758	9906	1099
245	5002	1317	5002	12513	1586
246	5011	1529	5011	13264	1551
247	5044	1524	5044	12254	1629
248	1324	1185	12260	26148	0
249	969	1020	12424	26284	0
250	5474	1235	5474	12995	1699
251	5462	1530	5462	13152	1704
252	5766	951	5766	13472	1639
253	5689	1153	5689	12812	1595
254	1278	1866	118855	220915	0
255	779	1920	19913	40803	0
256	932	2068	14482	30279	0
257	4652	982	4652	10524	1437
258	3006	1861	3006	8019	1231
259	1501	1363	10544	22260	0
260	4057	1859	4057	10500	1374
261	4986	1181	4986	11760	1476
262	254	2562	95184	175491	0
263	326	1887	22393	45424	0
264	1494	694	9626	19478	0
265	510	2410	85304	156013	0
266	4172	1288	4172	9738	1328
267	3960	1379	3960	9740	1538
268	274	2078	78887	143657	0
269	5907	1012	5907	12356	2039
270	1140	1793	9049	18995	0
271	1244	814	13018	26429	0
272	3299	1765	3299	8375	1484
273	4610	1248	4610	10620	1609
274	6584	1392	6584	14948	2180
275	5190	932	5190	11481	1663
276	941	858	9734	19397	0

TABLE 18.2.—*NAD 83 Helmert block stations and unknowns (continued)*

Block No.	Interior unknowns	Junction unknowns	Cumulative interior unknowns	Cumulative observations	Inside stations
277	5829	1248	5829	12586	1784
278	3690	973	3690	8349	1274
279	5103	1006	5103	11048	1705
280	185	230	11844	21914	0
281	4962	1077	4962	12408	1426
282	7044	292	7044	12188	2383
283	4615	133	4615	9726	1467
284	44	472	29851	52473	0
285	68	2162	48762	91184	0
286	180	196	16046	27432	0
287	321	392	13761	25041	0
288	190	670	23921	46485	0
289	232	1636	24773	44699	0
290	208	219	7883	13704	0
291	227	340	7983	13728	0
292	5745	513	5745	10595	1903
293	466	378	7695	14446	0
294	508	510	15765	30325	0
295	611	567	7966	16160	0
296	539	1750	16432	30500	0
297	304	355	8109	14199	0
298	4417	155	4417	7672	1559
299	3258	366	3258	6032	1246
300	3366	389	3366	6035	1349
301	4390	220	4390	7693	1553
302	3423	615	3423	6779	1229
303	3806	346	3806	7667	893
304	928	712	7951	15379	0
305	745	834	7306	14946	0
306	3234	807	3234	7768	1025
307	4121	470	4121	8392	1215
308	1050	1848	11277	21915	0
309	4616	795	4616	8585	1669
310	3434	462	3434	6377	1281
311	4371	303	4371	7822	1518
312	2616	895	2616	5642	1019
313	4407	1063	4407	9739	1627
314	3965	594	3965	8054	1341
315	2596	1091	2596	6892	936
316	3588	2241	3588	9201	1417
317	6639	660	6639	12714	2277
318	1435	417	1435	3945	438
319	1672	241	6755	16565	0
320	3283	1181	3283	10426	808
321	1800	999	1800	6139	560

Another parameter to be stored in the APF was an indicator of whether to lock the information in the data base so no changes could be made while NAD 83 was being computed. As it turned out, this feature of the data base was never implemented and this security measure was handled procedurally.

Precautions were taken to ensure that the coordinates of space system stations and the U.S.-Canadian boundary stations were properly identified as special junction points. Table 18.3 lists these special junction points, which were stored in the APF.

TABLE 18.3—*Special junction points by QID/QSN*

1.	01308622l0001	33.	0180974220001	65.	0280924140005	97.	0310893240005	129.	0330912110017
2.	0130891220001	34.	0181554140006	66.	0280974440002	98.	0310893240006	130.	0330951210003
3.	0140832330001	35.	0190751140001	67.	0281001340007	99.	0311002220005	131.	0330983240001
4.	0140873130001	36.	0190812430001	68.	0290892120003	100.	0311024410001	132.	0330991330003
5.	0140892140005	37.	0191551130001	69.	0290952320004	101.	0311104340003	133.	0330993340001
6.	0140901330002	38.	0191551430019	70.	0291014130001	102.	0311164130001	134.	0331001310001
7.	0140903220001	39.	0191552410011	71.	0291041310002	103.	0320804120002	135.	0331012120001
8.	0140911120001	40.	0201553420003	72.	0291043140001	104.	0320981110001	136.	0331014320004
9.	0150833310001	41.	0201561310009	73.	0300814140014	105.	0320981210001	137.	0331031320001
10.	0150861210001	42.	0201561310022	74.	0300814240001	106.	0320992120001	138.	0331033140001
11.	0150903110001	43.	0211021410001	75.	0300834320005	107.	0321001330001	139.	0331033330001
12.	0150912410001	44.	0211024110002	76.	0300854310001	108.	0321013110001	140.	0331041230001
13.	0161694120001	45.	0211572210011	77.	0300864330017	109.	0321024440001	141.	0331062230004
14.	0170644120008	46.	0211573130003	78.	0300921240002	110.	0321042120001	142.	0331062320006
15.	0170644130029	47.	0211582120035	79.	0300934110001	111.	0321051110002	143.	0331063120004
16.	0170644210004	48.	0221593240036	80.	0300972420003	112.	0321052230002	144.	0331064210006
17.	0170644430001	49.	0231011110001	81.	0300973430002	113.	0321062330002	145.	0331103440005
18.	0170891130002	50.	0231044440001	82.	0300973440001	114.	0321062430001	146.	0331161330011
19.	0170892230005	51.	0250801330033	83.	0301014430002	115.	0321072140001	147.	0331161330013
20.	0180643420012	52.	0250801410010	84.	0301033210007	116.	0321092130004	148.	0331174230008
21.	0180643430031	53.	0250971440011	85.	0301034120002	117.	0321114210007	149.	0331181340010
22.	0180652310002	54.	0250981430001	86.	0301034340005	118.	0321144210037	150.	0331181340016
23.	0180652330007	55.	0251012110002	87.	0301034420004	119.	0321164320006	151.	0331181340017
24.	0180652330008	56.	0251044120001	88.	0301041110002	120.	0321164320009	152.	0331181340019
25.	0180653210013	57.	0251084430002	89.	0301041210001	121.	0321171130059	153.	0331193210014
26.	0180653320007	58.	0270804110026	90.	0301041210005	122.	0321171130060	154.	0340782230005
27.	0180663220013	59.	0270824140005	91.	0301041210007	123.	0321171220053	155.	0340821220009
28.	0180663220017	60.	0271012420001	92.	0301041240003	124.	0321171310005	156.	0340881130001
29.	0180663220030	61.	0271043440001	93.	0301042330001	125.	0321171420023	157.	0340944430001
30.	0180663220046	62.	0271072340001	94.	0301043140001	126.	0321184110063	158.	0340951130001
31.	0180672420001	63.	0271093130001	95.	0301112240001	127.	0330823330011	159.	0340952120002
32.	0180882410002	64.	0280924140004	96.	0301152330001	128.	0330862330001	160.	0340973340001

TABLE 18.3—*Special junction points by QID/QSN (continued)*

161.	0340984330003	234.	0370772220005	307.	0401082230001	380.	0420824320008	453.	0440763230001
162.	0340994310002	235.	0370782430001	308.	0401114340007	381.	0420832220006	454.	0440763310002
163.	0341002320005	236.	0370861220003	309.	0401114440001	382.	0421041120002	455.	0440783220001
164.	0341012120001	237.	0371092110001	310.	0401132310003	383.	0421113310001	456.	0441002410007
165.	0341014310002	238.	0371141340001	311.	0401134240002	384.	0421141330003	457.	0441081420002
166.	0341023240004	239.	0371164130003	312.	0401141120002	385.	0421152220001	458.	0441212410003
167.	0341042410001	240.	0371221420149	313.	0401152140001	386.	0421213320003	459.	0441234110004
168.	0341051210001	241.	0371221430053	314.	0401233430002	387.	0430703240002	460.	0450662140001
169.	0341052210002	242.	0371222340002	315.	0401234340003	388.	0430752340013	461.	0450663240001
170.	0341061440007	243.	0371222440002	316.	0410694430042	389.	0430763110002	462.	0450663410001
171.	0341092340001	244.	0371231110001	317.	0410742120004	390.	0430764110001	463.	0450672220035
172.	0341143230002	245.	0380762310006	318.	0410762320001	391.	0430771110001	464.	0450674120001
173.	0341173140002	246.	0380762320014	319.	0410774210001	392.	0430772210002	465.	0450701130002
174.	0341173140004	247.	0380762430012	320.	0410801130002	393.	0430783210001	466.	0450701130003
175.	0341174130003	248.	0380764140002	321.	0410801130004	394.	0430783330024	467.	0450701420015
176.	0341174330005	249.	0380771130029	322.	0410804110002	395.	0430783340004	468.	0450703110001
177.	0341174330010	250.	0380771140088	323.	0410824130001	396.	0430791240001	469.	0450703130001
178.	0341174420001	251.	0380771410029	324.	0410831120001	397.	0430792120022	470.	0450704110001
179.	0341182240010	252.	0380771420016	325.	0410921430002	398.	0430792210010	471.	0450704210012
180.	0341182240012	253.	0380771420042	326.	0410954240003	399.	0430792210011	472.	0450704210023
181.	0341183120006	254.	0380772110026	327.	0410974220003	400.	0430792210124	473.	0450704230018
182.	0341183230022	255.	0380772310007	328.	0411014210001	401.	0430792220003	474.	0450704230021
183.	0341183320018	256.	0380772420008	329.	0411043310015	402.	0430792220011	475.	0450704240001
184.	0341183440008	257.	0380774140010	330.	0411074320001	403.	0430792220184	476.	0450704320001
185.	0341204110003	258.	0380813120015	331.	0411181330001	404.	0430792230001	477.	0450704330001
186.	0350751330007	259.	0380854230005	332.	0411241220004	405.	0430792230002	478.	0450704340001
187.	0350843340006	260.	0380881310003	333.	0420711330014	406.	0430792240001	479.	0450704410001
188.	0350973440001	261.	0381021420003	334.	0420711330018	407.	0430792320001	480.	0450704410002
189.	0350974430009	262.	0381051120007	335.	0420713440002	408.	0430792320002	481.	0450704430001
190.	0350992130007	263.	0381052410001	336.	0420733310020	409.	0430793210001	482.	0450711240001
191.	0351002330002	264.	0381061220003	337.	0420754230001	410.	0430793310001	483.	0450711340001
192.	0351013320001	265.	0381063130001	338.	0420773330002	411.	0430794220001	484.	0450712110002
193.	0351023240002	266.	0381071220004	339.	0420784420002	412.	0430794330001	485.	0450713130001
194.	0351042220003	267.	0381072210001	340.	0420784420004	413.	0430822320002	486.	0450713220001
195.	0351052420002	268.	0381093140001	341.	0420791110003	414.	0430822320003	487.	0450713340001
196.	0351063230001	269.	0381101110009	342.	0420791110004	415.	0430822330002	488.	0450713430001
197.	0351084120002	270.	0381124130001	343.	0420791110022	416.	0430822340001	489.	0450714130001
198.	0351093210001	271.	0381213240019	344.	0420791120001	417.	0430823220001	490.	0450722130001
199.	0351103230002	272.	0381213430006	345.	0420791120002	418.	0430844310006	491.	0450722210001
200.	0351113240019	273.	0381213440004	346.	0420791210001	419.	0430884430001	492.	0450722240001
201.	0351123430003	274.	0381222110005	347.	0420791210002	420.	0430914110004	493.	0450722320013
202.	0351142210012	275.	0381223320025	348.	0420791230001	421.	0430954440001	494.	0450723130001
203.	0351142210013	276.	0381223330017	349.	0420791420001	422.	0431242240001	495.	0450723230018
204.	0351163310010	277.	0390743240022	350.	0420791420002	423.	0440662140001	496.	0450723410001
205.	0351163430003	278.	0390763320022	351.	0420793120001	424.	0440664410001	497.	0450723430001
206.	0351163430011	279.	0390763320041	352.	0420794120001	425.	0440664420001	498.	0450732130001
207.	0351163440002	280.	0390763320063	353.	0420801220001	426.	0440664440044	499.	0450732210001
208.	0351163440007	281.	0390763340007	354.	0420802230006	427.	0440671120010	500.	0450732220030
209.	0351163440008	282.	0390772240055	355.	0420804230001	428.	0440694220001	501.	0450732310001
210.	0351163440009	283.	0390911220001	356.	0420821310001	429.	0440712140001	502.	0450732330027
211.	0351171320002	284.	0390943420003	357.	0420821320001	430.	0440724320001	503.	0450733240001
212.	0351172140002	285.	0390983210001	358.	0420821320002	431.	0440731110004	504.	0450733320009
213.	0351174240011	286.	0391063440001	359.	0420821330001	432.	0440731120011	505.	0450733330005
214.	0360841110004	287.	0391081140001	360.	0420821410001	433.	0440731410022	506.	0450733340001
215.	0360914230003	288.	0391132430001	361.	0420821430002	434.	0440734240001	507.	0450734220001
216.	0360942210011	289.	0391143420008	362.	0420821440025	435.	0440751410001	508.	0450742130001
217.	0361012410007	290.	0391172140001	363.	0420821440026	436.	0440753220002	509.	0450742220001
218.	0361044210003	291.	0391184430007	364.	0420822140001	437.	0440753440009	510.	0450742240001
219.	0361122220001	292.	0391194230013	365.	0420822420001	438.	0440754110001	511.	0450742310001
220.	0361164140002	293.	0391204440009	366.	0420823130002	439.	0440754240002	512.	0450742320001
221.	0361173320003	294.	0391204440013	367.	0420823230002	440.	0440761110001	513.	0450742320007
222.	0361173440001	295.	0391204440018	368.	0420823230004	441.	0440761210001	514.	0450742410001
223.	0361204110006	296.	0391221240001	369.	0420823240001	442.	0440761210002	515.	0450742430001
224.	0361212220002	297.	0391231310004	370.	0420823310001	443.	0440761340001	516.	0450742430002
225.	0361213240013	298.	0391231310005	371.	0420823330001	444.	0440762120048	517.	0450743120001
226.	0361213420005	299.	0391232240019	372.	0420823330002	445.	0440762140001	518.	0450743130001
227.	0361213440019	300.	0391232240020	373.	0420823340001	446.	0440762340004	519.	0450743310001
228.	0361214120014	301.	0400761110002	374.	0420823420001	447.	0440762420001	520.	0450743310002
229.	0361214130001	302.	0400813230003	375.	0420823420005	448.	0440762430001	521.	0450743420001
230.	0361214310029	303.	0400843210001	376.	0420823430006	449.	0440762440001	522.	0450744230001
231.	0361214330025	304.	0400853310001	377.	0420823430021	450.	0440763120001	523.	0450752210001
232.	0361221140005	305.	0400891420002	378.	0420824220002	451.	0440763120002	524.	0450752210002
233.	0370751440039	306.	0400891420006	379.	0420824320007	452.	0440763140001	525.	0450752220001

TABLE 18.3—*Special junction points by QID/QSN (continued)*

526.	0450752340001	599.	0461002320001	672.	0490941110001	745.	0541302130001	818.	0601383210002
527.	0450753220001	600.	0461131110008	673.	0490953220001	746.	0541302420001	819.	0601383430001
528.	0450753330001	601.	0470663440001	674.	0490972230001	747.	0541303240001	820.	0601383430002
529.	0450824410001	602.	0470671240001	675.	0490972240001	748.	0541304330001	821.	0601452440009
530.	0450831410003	603.	0470671320001	676.	0490972310001	749.	0541313320002	822.	0601484430017
531.	0451044110001	604.	0470672210001	677.	0490972420001	750.	0541321240003	823.	0601491110001
532.	0451101320001	605.	0470673440001	678.	0490972430001	751.	0541644330004	824.	0601491120010
533.	0451101340001	606.	0470681310001	679.	0490973130001	752.	0551293110001	825.	0601492340012
534.	0451111210002	607.	0470682410005	680.	0490973220001	753.	0551293340002	826.	0601492420003
535.	0451133310002	608.	0470683140001	681.	0490973310001	754.	0551332410003	827.	0601511320009
536.	0451164330002	609.	0470683210015	682.	0490993130002	755.	0551591140002	828.	0601521320001
537.	0451233410022	610.	0470683440001	683.	0490993240001	756.	0551601440002	829.	0601543310001
538.	0460671110001	611.	0470684310001	684.	0490993330001	757.	0551603110001	830.	0601551110001
539.	0460671110002	612.	0470684330001	685.	0490993430001	758.	0551603120015	831.	0601614420014
540.	0460671310001	613.	0470691230001	686.	0491023330001	759.	0551604230011	832.	0601642320002
541.	0460672220001	614.	0470691240001	687.	0491023410001	760.	0551604420002	833.	0611404110001
542.	0460672410001	615.	0470691330001	688.	0491052310001	761.	0551604440002	834.	0611462310003
543.	0460674110001	616.	0470692410001	689.	0491052320001	762.	0551613340003	835.	0611481430003
544.	0460674320001	617.	0470692430010	690.	0491052440001	763.	0551614210003	836.	0611484340001
545.	0460694340003	618.	0470693120001	691.	0491053310001	764.	0551622220002	837.	0611492430004
546.	0460701210007	619.	0470693210008	692.	0491053410001	765.	0551622330006	838.	0611493120009
547.	0460701310001	620.	0470693240001	693.	0491073310001	766.	0551623210001	839.	0611493230003
548.	0460701320001	621.	0470693240003	694.	0491073440001	767.	0551623240005	840.	0611493310034
549.	0460701320002	622.	0470693330001	695.	0491101320001	768.	0561304240001	841.	0611493310036
550.	0460701330001	623.	0470693420001	696.	0491102340002	769.	0561314240001	842.	0611493420011
551.	0460701410001	624.	0470694230001	697.	0491104310001	770.	0561322440011	843.	0611493420018
552.	0460701430001	625.	0470694240001	698.	0491111420001	771.	0561354120003	844.	0611502240006
553.	0460702310014	626.	0470694430001	699.	0491123140001	772.	0561534310001	845.	0611523310001
554.	0460702310030	627.	0470701120001	700.	0491123340002	773.	0561534420004	846.	0611533320001
555.	0460702320044	628.	0470701320001	701.	0491132140001	774.	0561541420007	847.	0611552430001
556.	0460702340001	629.	0470702230001	702.	0491132330001	775.	0561571340002	848.	0611571420001
557.	0460702420007	630.	0470703240001	703.	0491162130001	776.	0561584110003	849.	0611651320001
558.	0460702440001	631.	0470842330001	704.	0491162330002	777.	0571521420034	850.	0621402330001
559.	0460703110001	632.	0470843110001	705.	0491163120001	778.	0571524110008	851.	0621403130001
560.	0460703130001	633.	0470843220002	706.	0491163310001	779.	0571524140007	852.	0621403210001
561.	0460703220001	634.	0470883220015	707.	0491163420001	780.	0571531410001	853.	0621403310002
562.	0460703320001	635.	0470912140001	708.	0491172340001	781.	0571533220003	854.	0621403320001
563.	0460704120001	636.	0471084130002	709.	0491172430001	782.	0571534110002	855.	0621403410003
564.	0460704230001	637.	0471161310002	710.	0491173230003	783.	0571534140001	856.	0621434410002
565.	0460704310001	638.	0471192310012	711.	0491173340002	784.	0571534330012	857.	0621444320001
566.	0460704420001	639.	0471224220124	712.	0491173410002	785.	0571534430003	858.	0621462230002
567.	0460711130001	640.	0480871410001	713.	0491193110001	786.	0571541210001	859.	0621471410001
568.	0460711230001	641.	0480871420001	714.	0491193130001	787.	0571541240002	860.	0621481210001
569.	0460711430001	642.	0480874110001	715.	0491193130002	788.	0571543120004	861.	0621481410001
570.	0460712130001	643.	0480874420002	716.	0491193210001	789.	0571551310001	862.	0621484320001
571.	0460712310001	644.	0480881240001	717.	0491193230001	790.	0581343120005	863.	0621484430002
572.	0460823420001	645.	0480883110001	718.	0491193240001	791.	0581343120019	864.	0621491340001
573.	0460832430001	646.	0480884240001	719.	0491213110001	792.	0581513430011	865.	0621524110002
574.	0460833340004	647.	0480884320001	720.	0491213120001	793.	0581524220003	866.	0621534330002
575.	0460833440002	648.	0480893140001	721.	0491222220001	794.	0581532230007	867.	0621554110013
576.	0460834120001	649.	0480894430001	722.	0491222230001	795.	0581564240032	868.	0621561130001
577.	0460834240001	650.	0480922440001	723.	0491222430001	796.	0581614410001	869.	0621643440001
578.	0460834440001	651.	0480941310001	724.	0491223230001	797.	0591344340001	870.	0621644220001
579.	0460841210001	652.	0480941310004	725.	0491223310001	798.	0591344340002	871.	0621644230001
580.	0460841230001	653.	0480993330001	726.	0491223320003	799.	0591344430001	872.	0631421240002
581.	0460841240001	654.	0481033340001	727.	0491223330001	800.	0591344430002	873.	0631471440001
582.	0460841320041	655.	0481051420002	728.	0491223340001	801.	0591351120002	874.	0631471440002
583.	0460841320042	656.	0481111130002	729.	0491223430001	802.	0591351210001	875.	0631472230003
584.	0460841330001	657.	0481114110001	730.	0491232210002	803.	0591353210006	876.	0631482130001
585.	0460841420001	658.	0481161310001	731.	0491232210003	804.	0591401240002	877.	0631484430003
586.	0460842110001	659.	0481161310005	732.	0491232220001	805.	0591502410003	878.	0631492240001
587.	0460852430001	660.	0481221140001	733.	0491232220002	806.	0591503140001	879.	0631492340001
588.	0460873440005	661.	0481224310004	734.	0491232240001	807.	0591503140008	880.	0631494120001
589.	0460901210004	662.	0481224410012	735.	0491232440001	808.	0591503340004	881.	0631534340001
590.	0460901330002	663.	0481232410002	736.	0491242210001	809.	0591511330020	882.	0631563220001
591.	0460902140009	664.	0481232410007	737.	0500922240001	810.	0591533440009	883.	0631604410003
592.	0460904130001	665.	0481234120001	738.	0500953240001	811.	0591562140001	884.	0631632220001
593.	0460904310004	666.	0481243140005	739.	0511764110032	812.	0591582140001	885.	0631701340004
594.	0460904420001	667.	0481251120001	740.	0521864420032	813.	0591631120001	886.	0631714130006
595.	0460904420009	668.	0481251130001	741.	0531301410001	814.	0591641110003	887.	0641403330004
596.	0460904440002	669.	0481251210001	742.	0541301230001	815.	0601373440001	888.	0641411210004
597.	0460923430005	670.	0490873330001	743.	0541301320001	816.	0601373440002	889.	0641414240001
598.	0460963220003	671.	0490882330001	744.	0541301330001	817.	0601383210001	890.	0641423340001

TABLE 18.3—*Special junction points by QID/QSN (continued)*

891.	0641432320001	932.	0661541410001	973.	0701433240002	1014.	0611462320029	1055.	0491032230001
892.	0641441220001	933.	0661554140001	974.	0701453320003	1015.	0611451120001	1056.	0491073320001
893.	0641443430001	934.	0661564220001	975.	0701463310001	1016.	0351113240020	1057.	0491012230001
894.	0641444320001	935.	0661623230002	976.	0701473310002	1017.	0391143420007	1058.	0491012330001
895.	0641444410001	936.	0661624120004	977.	0701482430003	1018.	0621463320001	1059.	0481034430001
896.	0641452430001	937.	0671361140001	978.	0701492140002	1019.	0601491320002	1060.	0481034410002
897.	0641453230002	938.	0671383120001	979.	0701493140001	1020.	0601492420006	1061.	0490982210001
898.	0641453310001	939.	0671403430002	980.	0701502130001	1021.	0601492440003	1062.	0490982340001
899.	0641454130001	940.	0671431110001	981.	0701512140001	1022.	0611451310001	1063.	0490983120001
900.	0641464240001	941.	0671461230001	982.	0701512210001	1023.	0611451430001	1064.	0480913220002
901.	0641492310003	942.	0671463210001	983.	0701513120001	1024.	0611454130001	1065.	0480912330001
902.	0641494440002	943.	0671533130001	984.	0701521340001	1025.	0611471420002	1066.	0480884330001
903.	0641511210001	944.	0671573320001	985.	0701544120001	1026.	0621494240001	1067.	0480883410001
904.	0641512430002	945.	0681361120001	986.	0701554330001	1027.	0621502120002	1068.	0641412220003
905.	0641521210001	946.	0681383140001	987.	0701613430001	1028.	0631484320001	1069.	0561321210001
906.	0641651330001	947.	0681402440001	988.	0711543330003	1029.	0641474130011	1070.	0561321310002
907.	0651403210001	948.	0681403440006	989.	0711553230003	1030.	0641481110001	1071.	0541301120001
908.	0651403330001	949.	0681412120001	990.	0711563130016	1031.	0641484130001	1072.	0581351140004
909.	0651403330009	950.	0681484140001	991.	0711563210002	1032.	0641491220001	1073.	0581351410005
910.	0651404140001	951.	0681512140001	992.	0711572320003	1033.	0641492230001	1074.	0561324440001
911.	0651432140001	952.	0681513240003	993.	0521742240008	1034.	0450703110008	1075.	0561324410002
912.	0651432240001	953.	0681514220001	994.	0521742310004	1035.	0311104340004	1076.	0581343210006
913.	0651491430001	954.	0681554140001	995.	0521742220004	1036.	0351113240018	1077.	0581343240006
914.	0651494330001	955.	0681663120001	996.	0511803410012	1037.	0430792220002	1078.	0561323240003
915.	0651501340001	956.	0691383210001	997.	0511803140031	1038.	0521842120014	1079.	0561323130006
916.	0651502140001	957.	0691403110001	998.	0571702340008	1039.	0521842120012	1080.	0541314110002
917.	0651502240001	958.	0691403110002	999.	0571702240004	1040.	0341183440006	1081.	0541321110001
918.	0651502420002	959.	0691403410001	1000.	0571702340006	1041.	0341183440010	1082.	0551321230002
919.	0651503230002	960.	0691403440004	1001.	0571702210001	1042.	0170833440007	1083.	0551321240006
920.	0651503430001	961.	0691404220001	1002.	0561694220008	1043.	0170833440001	1084.	0571334240003
921.	0651674330004	962.	0691404220002	1003.	0571693340001	1044.	0481231240020	1085.	0571334130004
922.	0661361140001	963.	0691404220003	1004.	0180752110004	1045.	0491192330002	1086.	0591351230002
923.	0661401410001	964.	0691404310001	1005.	0180882410001	1046.	0491123340001	1087.	0591351240001
924.	0661402320001	965.	0691404310002	1006.	0231011110002	1047.	0491093340001	1088.	0591351210002
925.	0661404410001	966.	0691404310003	1007.	0341183120026	1048.	0491152230001	1089.	0591351210007
926.	0661441430001	967.	0691411240002	1008.	0390763320021	1049.	0481151140002	1090.	0621404320001
927.	0661451220001	968.	0691421120003	1009.	0491102340001	1050.	0491172330001	1091.	0621411220004
928.	0661451230001	969.	0691424140001	1010.	0491222330001	1051.	0480974440002	1092.	0681412120003
929.	0661464330001	970.	0691444410002	1011.	0511803140022	1052.	0480981110002	1093.	0481232410004
930.	0661493130001	971.	0691503230001	1012.	0521742240018	1053.	0491002330001		
931.	0661514110009	972.	0701433220004	1013.	0591514420001	1054.	0491003330001		

The special unknowns called observation class deck unknowns were defined. Each described a systematic error, such as a scale error, shared by a group or class of observations. All projects were analyzed for common elements, such as same observing organization, same instrument type, and same time epochs. Table 18.4 lists the observation class deck unknowns. The observations sharing an unknown are identified by project identifier (Trav-deck name) and observation type (G, X, T, E, or U), where the observation types are described in Schwarz (1978).

18.4 RETRIEVAL OF FIRST-LEVEL BLOCKS

18.4.1 Terrestrial Survey Observations

The retrieval of the data for the first-level blocks was performed by a data base procedure, controlled by the parameters that had been stored in the APF. This procedure invoked programs that transformed coordinates from NAD 27 to Preliminary NAD 83 (PNAD 83), and computed crustal motion corrections to observations where appropriate. The resulting data set was stored as a RESTART file. This phase of the project began in May 1985 and continued during the formation of the Helmert blocks and forward solution of other blocks.

The RESTART file was familiar to NGS because this file structure had already been created during block validation. This was the format used in loading terrestrial observations into the data base. "Supernumerary" observations, which are observations to reference marks and azimuth marks and are therefore unadjustable, also reside in the RESTART file. These were included for problem solving purposes.

The data base retrieval procedure provided coordinates on PNAD 83. In practice, these were virtual data items. They were actually computed from stored NAD 27 values using models described by Vincenty (1976, 1979). During the first retrievals and preliminary checking of iteration 0, it was discovered that the coordinates of non-monumented intersection stations had not been properly transformed. Furthermore, the results of a 1983 test NAD 83 adjustment in Alaska had not been incorporated into the first retrievals.

TABLE 18.4—*Observation class deck information*

Unknown identifier:
GEODIMETER

Keys:
(CGX)

No area defined

Unknown identifier:
TELLUROMETER

Keys:
(EY)

No area defined

Unknown identifier:
IBCTAPE

Keys:
CRMIBCA1(T)
G14910(T)
G15066(T)
G15656(T)
G16801(TU)
G16938T(T)
G16943(TU)
G16974(TU)
GREATLAK(TU)
IBC16546(T)
IBC16580(TU)
IBC16592(T)
IBC16608(TU)
IBC16944(TU)
LAKOWOOD(T)
LOWNAML(T)
NEWBRUNS(T)
RESUR141(T)
STCRINLN(U)
STCROIXR(TU)
TAA16802(TU)
TAA16933(TU)

No area defined

Unknown identifier:
IBCLIGHT

Keys:
CRMIBCA1(X)
G14910(G)
G15066(G)
G15656(G)
G16938T(X)
G16943(X)
G16974(X)
GREATLAK(X)
IBC16592(X)
LAKOWOOD(X)
NEWBRUNS(X)
RESUR141(X)
TAA16933(X)

No area defined

Unknown identifier:
IBCMICRO

Keys:
G16938T(Y)
G16974(Y)

No area defined

Unknown identifier:
AZLIGHT

Keys:
ASHFORK(X)

Unknown identifier:
AZLIGHT (continued)

Keys:
AZ16322(X)
AZ16323(X)
AZ16340(X)
AZHSWILL(X)
CONGRESS(X)
CORDEJCT(X)
FLAGCAME(X)
FLORMAMM(X)
GILABAJO(X)
GILABUCK(C)
HOLBSNOW(X)
SAFFCLIF(X)
SPRICLIF(X)
STJOHNS(X)
SUNSETCR(G)
TEMPEAPA(X)
TVAPACHE(X)
WHYLUKE(G)
WICKNRIV(X)
WICNBURG(X)
WINSLO(X)

No area defined

Unknown identifier:
CALIGHT

Keys:
ANDEREDD(X)
BAKERSF1(X)
BELLMONT(X)
CADH1533(G)
CADH1543(X)
CADTPOW(X)
CAHSBBO(X)
CAHSBEV(X)
CAHSBOUL(X)
CAHSCBE(X)
CAHSCHIN(X)
CAHSCORC(X)
CAHSGATO(X)
CAHSGIS(G)
CAHSGRAP(G)
CAHSJENN(X)
CAHSLAMO(X)
CAHSLOMP(X)
CAHSMARY(X)
CAHSMINE(X)
CAHSOJAI(X)
CAHSPALM(X)
CAHSPLAC(X)
CAHSPLAT(X)
CAHSQUIN(X)
CAHSRENO(X)
CAHSRIVE(X)
CAHSSISK(X)
CAHSSUSA(G)
CAHSTEHA(X)
CAHSTEME(X)
CAHSVEN(X)
CAHSWASC(X)
CAHSWRIG(X)
CAL1(G)
CAL14716(G)
CAL14953(G)
CAL15115(X)
CAPISTRA(X)
CARPATAS(G)
CONNARVI(X)
DELANO(X)
DIEMETRO(G)

Unknown identifier:
CALIGHT (continued)

Keys:
FRESNOCA(G)
FRESPANO(G)
G14847(G)
G15075(G)
G15102(G)
G15155(G)
G15170(G)
G15181(G)
G15182(G)
G15187(G)
G15229(G)
G15257(X)
G15443(X)
HALLELUJ(C)
JAMBOULE(X)
JUNELAKE(X)
KANESPRS(X)
LEEKETTL(G)
LOSTHILL(X)
MARIN(X)
MOJAVEFJ(X)
MONOBENT(X)
MONTSALI(X)
MOORPARK(X)
OLDSTATI(X)
PRADOPOM(X)
PRAIRIE(G)
REDBEAMO(X)
REDMTNH1(X)
ROBLES(X)
SALIDAJ1(X)
SANGWOOD(X)
SANYSBEL(X)
TEDEVORE(X)
TELEGRAP(X)
TIPPORT(X)
VENTCARP(X)
VICTORVL(X)
VIDALTP(X)
WOODLAND(X)
YUBACITY(X)

No area defined

Unknown identifier:
FLLIGHT

Keys:
BUNNDURB(X)
CHIPLEY(X)
DESTINPW(X)
FALMONTI(X)
G14775(C)
G14877(C)
G15019(C)
LOJASPER(X)
MUSCMOSS(X)
PLANTEAT(C)
STPETERS(GX)
TALAHASE(X)
TALBASMR(X)
TAMPTWIL(X)
VEROSCOT(X)
WILDTRAX(X)

No area defined

Unknown identifier:
FLMICRO

Keys:
BAYCONTY(Y)

Unknown identifier:
FLMICRO (continued)

Keys:
TALBASMR(Y)

No area defined

Unknown identifier:
IDMICRO

Keys:
FRUITBOI(E)
IDHSBLAC(Y)
IDHSMONT(Y)
IDHSPOCA(Y)
IDHSROBS(Y)
IDHSRUPE(Y)
IDHSUTAH(Y)
LOOKPASS(E)
MTHBOIS(E)
MTHJERO(E)

No area defined

Unknown identifier:
ILMICRO

Keys:
CHAMKANK(E)
CHAMPEFF(E)
DEERCR(E)
EFFIHIGH(E)
ILHSBENT(Y)
ILHSCAIR(Y)
ILHSGENE(Y)
ILHSLOUI(Y)
ILHSMANT(Y)
ILHSORIO(Y)
ILHSSPRI(Y)
LITCSTAU(E)
MONTSTLI(Y)
OGLEBENN(E)
ORPEORIA(Y)
SPRIJOLI(E)

No area defined

Unknown identifier:
KYLIGHT

Keys:
KHSPAKL(CG)
KYHS1350(G)
KYHS1358(G)
KYHS1436(G)
UPELIZA(G)

No area defined

Unknown identifier:
KYMICRO

Keys:
COVINGTO(E)
FRANKCAV(E)
KHSPAKL(E)
KYCORBIN(E)
KYHS1225(E)
KYHS1244(E)
KYHS1332(E)
KYHS1339(E)
KYHS1346(E)
KYHS1350(E)
KYHS1358(E)
KYHS1371(E)
KYHS1436(E)

Unknown identifier:
KYMICRO (continued)

Keys:
KYLAKE(E)
KYSTLIN(E)
LOUMID(E)
RICHLEX(E)
RICHLON(E)
UPELIZA(E)

No area defined

Unknown identifier:
LALIGHT

Keys:
BARHAMON(G)
BATONRUG(X)
BOEFRIV(X)
GEISLDH(G)
HAMLDH(G)
HAMMONLA(GX)
HENDERSO(X)
HERBLDH(G)
LAHSARCA(G)
LAHSEGAN(G)
LAHSMIND(G)
LAHSVINT(G)
LAHSWASK(X)
MERMENTA(G)
MONRODHI(G)
MORGANCI(G)
SHREVPTA(X)
SLIDLDH(G)
SURLAFAY(X)
VIBATRUG(CG)
VZACHARY(X)
YOUNGJEA(X)

No area defined

Unknown identifier:
MEMICRO

Keys:
G11736(E)
G11846(Y)
G12003(E)
G12087(Y)

No area defined

Unknown identifier:
MDMICRO

Keys:
ABDELAA(E)
BALTPENN(E)
CIRHYMD(E)
DCFREDMD(E)
MHSHFBLT(E)

No area defined

Unknown identifier:
MNLIGHT

Keys:
BATLAKPE(X)
BEMCASS(X)
BIGFALLS(X)
BLANECAM(X)
DENSAPLE(X)
ERSKINE(G)
FAIRMADE(X)
FOUNTAIN(X)

TABLE 18.4—*Observation class deck information (continued)*

Unknown identifier: MNLIGHT (continued)

Keys:
G14889(G)
G14989MN(G)
G15152(CG)
G15162(G)
G16804(X)
G16935(X)
GLENWOOD(X)
GOODRIDG(X)
GRDULUTH(X)
HADERA8(X)
HASTVSP(G)
HOMERISK(X)
ICFRIDLE(X)
INTERNAT(G)
LAGEORGE(G)
LAKERIVE(X)
LUVERNE(X)
MAPLEGRO(X)
MAPLEWOO(X)
MAYOCLIN(G)
MHDULUTH(X)
MILACA(X)
MINAPLIS(X)
MINNORT1(G)
MINNWAYZ(X)
MNDT2HA(X)
MNDTBJN(X)
MNDTRUSH(X)
MNDTWIND(X)
MNHSBASS(G)
MNHSBENT(X)
MNHSBLOM(X)
MNHSBLOO(G)
MNHSEAGA(X)
MNHSGROV(G)
MNHSINVR(X)
MNHSLITC(X)
MNHSVIRG(X)
MONTHURI(X)
MOOSEDUL(G)
MOOSELKE(G)
MORISTOW(G)
NEWULMD2(X)
NORTHFAI(X)
NORTHHOM(X)
NORWOOD(X)
NOYESB4(X)
NUBROWN(X)
ONAMIAC1(X)
OWATONWI(GX)
PALISADE(X)
PENGILLY(G)
PETESAVA(X)
PIPESTON(X)
REDLAKE(X)
ROCHOWAT(X)
RUSHFORD(X)
SAGINAW(X)
SAUKCNTR(X)
SHAKOPEE(X)
STCLOUD(X)
STFRANCS(X)
STPETER(X)
WABASHA(X)
WASHTNCO(X)
WAVERLY(X)
WESBENTO(X)
WILLMAR(X)
WYKFOUNT(G)

No area defined

Unknown identifier: MNMICRO

Keys:
FERGUSFL(Y)
G12959MN(E)
MINNORT1(E)
MNHSALF(Y)
MNHSALS(Y)
MNHSBENT(Y)
MNHSFAIR(Y)
MNHSJACK(Y)
MNHSJSD(E)
MNHSMNW1(Y)
MNHSPAUL(EY)
MNHSWALL(E)
MOOREHEA(E)
MOOSELKE(E)

No area defined

Unknown identifier: MSMICRO

Keys:
ABERDEEN(E)
BUCAMER(E)
MHSJKGV1(E)
MHSJKGV2(E)
MHSJKGV3(E)
MHSJKGV4(E)
MHSJKGV5(E)
MSLELAND(E)

No area defined

Unknown identifier: NEMICRO

Keys:
NEHSCNP(EY)
NEHSGCO(Y)
NEHSMGI(Y)
NEHSNPO(EY)
NEHSOGB(Y)
NEHSOMAH(Y)

No area defined

Unknown identifier: NMLIGHT

Keys:
AKELANAR(X)
ALBURIOP(G)
BELENBER(X)
BELENLOS(G)
BERNASAN(G)
BERNSANT(G)
BLOOMCOU(X)
JERARANC(X)
LORDSBUR(X)
MONTOGUA(X)
NMHSGAV(X)
NMHSLVSF(X)
RIOPUERR(X)
SANTOBER(X)
SEPARDON(X)
SEQUENCE(X)
TOYATEXA(X)
TRUTCRUC(X)
WATRATON(X)

No area defined

Unknown identifier: NMMICRO

Keys:
AKELANAR(Y)
BLOOMCOU(Y)
GALLUPNO(EY)
JERARANC(Y)
LORDSBUR(Y)
MONTOGUA(Y)
NMHSLVSF(Y)
SANTOBER(Y)
SEPARDON(Y)
SEQUENCE(Y)
SHIPROCK(Y)
TOYATEXA(Y)
WATRATON(Y)

No area defined

Unknown identifier: NCLIGHT

Keys:
ASHBLACK(X)
BENHARNS(G)
BURKECTY(G)
CLEJEUNE(X)
DARENAGS(X)
FAIRBLUF(X)
FORSTONE(G)
FRANKLIN(GX)
G13609(G)
G13611(G)
G13656(G)
G13700(G)
G13701(G)
G13732(G)
G13734(G)
G13750(G)
G13762(X)
G13778(G)
G13779(G)
G13780(G)
G13781(G)
G13782(G)
G13806(G)
G13807(G)
G13907(G)
G14119(G)
G14300(G)
G14428(G)
G14431(G)
G14468(G)
G14540(G)
G14794(G)
G14827(G)
G14863(G)
G14938(G)
G15053(G)
GATESVIL(G)
HAMLETNC(G)
HENDPOLK(X)
HICKORYN(X)
LEECOWGS(X)
LIZARDLI(X)
MARSHVIL(X)
NCG14434(G)
NCG14471(G)
NCG14799(G)
NCG14820(G)
NCG14986(G)
NCG15085(X)
NCG15134(X)
NCG15137(G)

Unknown identifier: NCLIGHT (continued)

Keys:
NCG15146(X)
NCG15298(X)
NCGS1(G)
NCGS2(G)
NCGS3(G)
NCGS4(G)
NCGSAPEX(X)
OLDMARIO(X)
PETTIGRE(X)
PITTSBOR(X)
RALMORA(G)
RANDOLPH(X)
STATESVI(GX)
TAA16724(X)
TAA16725(X)
TAA16785(X)
TAF16784(X)
TAF16786(X)
TAF16787(X)
VAURORA(G)
VGSWAMP(X)
VHILDUR(G)
VICALBEM(X)
VICASHVL(G)
VICBURLG(G)
VICCHARL(G)
VICCLAYT(G)
VICCONCD(G)
VICFAIRT(G)
VICFAYET(G)
VICGASTN(G)
VICGREEN(G)
VICGVILL(G)
VICHENDR(G)
VICHICKY(G)
VICITROY(G)
VICJACKN(G)
VICKERNS(G)
VICKINST(G)
VICLEXT(G)
VICLUMBR(G)
VICMARON(G)
VICNBERN(G)
VICNCHAR(G)
VICOLIVE(G)
VICOSCEO(G)
VICRICHS(G)
VICROCKY(G)
VICSMITH(G)
VICTARBO(G)
VICTHOMS(G)
VICTRYON(G)
VICWADES(G)
VICWILMG(G)
VICWILSN(G)
VICWRAL(G)
VICYADKN(G)
VMOREHED(G)
VMORGAN(G)
VNWWAKE(G)
VRALEIGH(G)
VROCKING(G)
VWOODFIN(G)
WADESBOR(G)
WAKEFRST(X)
WAYNECOU(X)
WILSONPR(X)

No area defined

Unknown identifier: ORMICRO

Keys:
ORE13882(E)
OREABIG(Y)
ORESUNNY(Y)
OREWALF(Y)
VCATHB(Y)

No area defined

Unknown identifier: PAMICRO

Keys:
BREEZEMD(E)
CLARMEAD(E)
EBDUWOR(E)
FALLOHIO(E)
FALLSWAT(Y)
HARDUNA(Y)
HARMD1A(Y)
HARMD2A(Y)
HOLLHOLL(E)
INDEAST(E)
LEWHOPM(E)
MANSFACT(E)
MANSWILL(Y)
MDGORA(Y)
NCENTPA(EY)
NEPHILA(Y)
PHILAREA(Y)
POTTBURG(E)
READHONY(Y)
SCRAKEYS(E)
SEAKWEST(E)
SHEAKERI(E)
STWSCN(EY)
TOWAWILL(Y)
WASHNST(E)
WESTMDA(E)
WORTOHIO(E)

No area defined

Unknown identifier: TNMICRO

Keys:
BSDANTHS(E)
COOKVILL(E)
DIXIECHA(E)
KNOXJELL(E)
KNOXROCK(E)
LAWCOLUM(E)
MEMNASH(E)
NASHALBA(E)
NASHATHS(E)
NASHCHAT(E)
NASHKEN(E)
NASHKY(E)
TENNDY(E)
THSALAWA(E)
THSCFNSH(E)
THSMNASH(E)
THSNBCK(E)
THSTKLCG(E)
TNDENVER(E)
TNHS1267(E)
TNHSBEAN(E)
WARTBURG(E)

No area defined

TABLE 18.4—*Observation class deck information (continued)*

Unknown identifier: VAMICRO

Keys:
ASHACCA(E)
CIRHYVA(E)
CLEARBUC(E)
CRISGLEN(E)
GATBIGA(E)
LYCHHAR(E)
MARSFAIR(E)
PETERNC(E)
RICHNORK(E)
RICHVIC(E)
ROACLIFF(E)
ROALYNA(E)
SWVAHWY(E)
No area defined

Unknown identifier: CDLIGHT

Keys:
CANTRAV1(G)
CANTRAV2(G)
CANTRAV3(G)

Unknown identifier: CDLIGHT (continued)

Keys:
CANTRAV4(G)
CANTRAV6(G)
CANTRAV7(G)
CANTRAV8(G)
CANTRAV9(G)
CANTRV11(G)
CANTRV12(G)
CANTRV13(G)
CANTRV14(G)
CANTRV17(G)
CANTRV18(G)
CANTRV20(G)
CANTRV21(G)
CANTRV22(G)
CANTRV25(G)
No area defined

Unknown identifier: CDMICRO

Keys:
CANTRAV1(E)

Unknown identifier: CDMICRO (continued)

Keys:
CANTRAV2(E)
CANTRAV3(E)
CANTRAV4(E)
CANTRAV5(E)
CANTRAV6(E)
CANTRAV7(E)
CANTRAV8(E)
CANTRAV9(E)
CANTRV10(E)
CANTRV11(E)
CANTRV12(E)
CANTRV13(E)
CANTRV14(E)
CANTRV15(E)
CANTRV16(E)
CANTRV17(E)
CANTRV18(E)
CANTRV21(E)
CANTRV25(E)
CANTRV27(E)
No area defined

Unknown identifier: CDAERO

Keys:
CANTRV19(U)
CANTRV26(U)
No area defined

Unknown identifier: XCDLIGHT

Keys:
CANTRAV1(X)
CANTRAV2(X)
CANTRAV3(X)
CANTRAV4(X)
CANTRAV6(X)
CANTRAV7(X)
CANTRAV8(X)
CANTRAV9(X)
CANTRV11(X)
CANTRV12(X)
CANTRV13(X)
CANTRV14(X)
CANTRV17(X)
CANTRV18(X)
CANTRV20(X)
CANTRV21(X)
CANTRV22(X)
CANTRV25(X)
No area defined

Unknown identifier: YCDMICRO

Keys:
CANTRAV1(Y)
CANTRAV2(Y)
CANTRAV3(Y)
CANTRAV4(Y)
CANTRAV5(Y)
CANTRAV6(Y)
CANTRAV7(Y)
CANTRAV8(Y)
CANTRAV9(Y)
CANTRV10(Y)
CANTRV11(Y)
CANTRV12(Y)
CANTRV13(Y)
CANTRV14(Y)
CANTRV15(Y)
CANTRV16(Y)
CANTRV17(Y)
CANTRV18(Y)
CANTRV21(Y)
CANTRV25(Y)
CANTRV27(Y)
No area defined

The methods used to determine deflections and geoid heights for the United States (including Alaska, Hawaii, and Puerto Rico) went through several different developmental steps. Initially, the deflections and the geoid height were computed at all occupied stations using the procedures described in chapter 16. During the block validation phase, many stations were added to the data base. For new stations in the contiguous 48 United States, the deflections and geoid heights were computed using least squares collocation to interpolate among the abundant gravimetrically determined values. In western Alaska, which contained a small quantity of observed data, Rapp's 180 by 180 spherical harmonic model (Rapp, 1981) was used to predict geoid heights.

During iteration 0, slight inconsistencies in these two methods were identified. When investigations could not identify and resolve these differences, a decision was made to use Rapp's 180 by 180 model to compute geoid heights for the entire NAD 83. This was implemented in two steps. Iteration 0 of the adjustment was by then well advanced for the eastern part of the contiguous United States (east of 101 degrees longitude). These geoid heights were not changed until iteration 1. In the western part of the country, the geoid heights were recalculated prior to the start of iteration 0.

Two other corrections were made to the gravimetric data. The deflection of the vertical in the meridian and the deflection of the vertical in the prime vertical had been inadvertently switched on the added station records computed using the least squares collocation method. These records had to be identified and corrected. The second correction added gravimetric data to those stations which had been missed entirely.

The following additional minor retrieval errors were encountered and corrected: (1) Some of the areas with crustal motion models had not had model parameters computed prior to iteration 0. (2) The data base loading procedure did not initially allow for negative elevations. In correcting this problem, other elevations were truncated to less than 1,000 m. Elevations that were supposed to contain the value zero had been stored as blank fields.

The retrieval procedure also inserted the table of default standard deviations into the RESTART file. This table described the standard deviation to be assigned to observations for which a standard deviation had not been explicitly assigned. These values were as follows:

Code	Observation type	F_1	F_2
1	First-order direction	0.6	0.001
2	Second-order direction	0.7	0.001
3	Third-order direction	1.2	0.001
4	Direction to intersection station	3.0	0.050
A	First-order astronomic azimuth	not specified	
B	Lower-order astronomic azimuth	not specified	
C,G	Electro-optical distance	15.0	1.0
X	Electro-optical, mark-to-mark distance	15.0	1.0
T,U	Taped distance	10.0	0.5
E,Y	Microwave EDM distance	30.0	3.0

Using these factors, the standard deviation of a direction in seconds of arc was the square root of

$$F_1^2 + (206265 * F_2/D)^2$$

where D is the approximate distance between points in meters. The second term accounts for decentering of both the theodolite and the target.

The standard deviation of a distance, in meters, was the square root of

$$(F_1/1000)^2 + (D^*F_2/1000000)^2 + (0.00005^*(h_2 - h_1)/3)^2$$

where h_1 and h_2 are the heights of the two stations.

18.4.2 Space System Observations

Various space systems observations were used to contribute global scale and orientation to the NAD 83 adjustment. These data sets were handled outside the environment of the data base and the APF. The observations were placed in a separate Helmert block created by the Space Systems Observations Adjustment Program, SOAP.

At any given VLBI site, besides the VLBI station, there might be one or more Doppler stations and one or more stations tied to the terrestrial network. The Doppler and VLBI stations could either be tied to the terrestrial network or not, and there could be additional non-tied stations at the site. The VLBI stations are classified as fixed or mobile. Mobile VLBI data were reduced to the ground monument while observations at fixed antennas were referred to the electrical center of the antenna, a point in space. The site at GILMORE CREEK was exceptional in that both fixed and mobile VLBI observations were performed there.

Survey ties were needed to connect the VLBI and Doppler stations to the NGRS (National Geodetic Reference System). GPS observations were provided to complete network ties with VLBI stations in Alaska. Other ties were accomplished using small local network surveys processed through the three-dimensional least squares adjustment program HAVAGO (Vincenty, 1979).

The data collected at each VLBI site were processed by HAVAGO. The program output consists of adjusted positions and observations, including geocentric cartesian coordinates, together with a file containing the ΔX, ΔY, and ΔZ values and the variance-covariance matrix for certain "requested" stations selected by the user. The requested stations were the fewest number of stations that provides ties between the Doppler, VLBI, and the terrestrial network. The output file with the variance covariance matrix transformed into a standard error/correlation coefficient matrix was used for the SOAP input file.

HAVAGO software was first used to process terrestrial survey data (local "table top" surveys) at 22 sites where fixed and mobile VLBI observations had been conducted or were scheduled to be conducted in the near future. Input for HAVAGO was supplied from observations by the Goddard Space Flight Center and the Jet Propulsion Laboratory (JPL). In many instances non-tied stations physically close to each other had similar, or even identical, preliminary positions. Iterations through HAVAGO distinguished the positions. The names and positions of the tied stations were not always identical with those in the terrestrial data base, but exact agreement was not necessary for the adjustment to provide accurate relative position observations and the covariance matrix. No constraints were put on the positions. The file output from HAVAGO was then reconstructed into a format compatible with SOAP. HAVAGO and SOAP had been written at different times and for different purposes. As a result, the input and output formats for each program are not compatible. Rather than modify one or the other program, it was decided to create the SOAP input file by using a text editor to modify the output HAVAGO file.

The SOAP input included the following items which are discussed in detail:

Preliminary NAD geodetic position records. These included station name, station identifier (QID/QSN), and predicted NAD 83 latitude, longitude, elevation, and geoid height values. The input values for four Canadian stations, supplied by the Geodetic Survey of Canada, were added as junction stations. Initially, slight discrepancies existed between two different geoid height models. A program was written to correct geoid heights and these values were updated. Elevation fields were zero filled after the decimal, giving the appearance of being precise, when in fact some elevations had only been scaled from maps. The input values for latitude and longitude were automatically updated with the adjusted values after each iteration.

Preliminary non-NAD geodetic position records. These included the station name and preliminary Cartesian coordinates (X, Y, and Z values) at the VLBI sites, i.e., the adjusted X, Y, and Z values from the HAVAGO output printout. From this source, only the fixed VLBI stations (GOLDSTONE, RICHMOND, OWENS VALLEY, MARYLAND POINT, AND FORT DAVIS) were available for iteration 0. The mobile data became available after iteration 0 was completed, and were included for iteration 1. Values for ONSALA60, NRAO 140, and WETTZELL were also provided. Values at EFLSBERG and CHILBOLTEN were provided by the Goddard Space Flight Center. Values at HAYSTACK and WESTFORD were based on 1972 and 1973 satellite data from JPL measuring the GOLDSTONE-HAYSTACK baseline. Positions for the EFLSBERG and CHILBOLTEN Doppler stations were computed by applying ΔX, ΔY, and ΔZ values to the antenna position. All of these were also included in iteration 0. Positions for KAUAI (Hawaii), KWAJAL26 (Marshall Islands), and KASHIMA (Japan) were included in the final iteration. These stations were necessary because the updated observations available for the final iteration had been combined with them and could not be separated. These values were also updated automatically after each iteration.

Terrestrial survey data. Relative position observations (ΔX, ΔY, and ΔZ coordinates in the form of the position difference between the first and second stations in a group, the first and third, etc.) and the standard error/correlation coefficient matrix from HAVAGO were used in SOAP to tie VLBI and Doppler stations to the network. At POINT REYES, PINYON FLAT, and SANTA PAULA, the VLBI, Doppler, and terrestrial stations are identical. No additional ties were necessary. At GILCREEK, BLACK BUTTE, DEADMAN LAKE, MAMMOTH LAKE, OCOTILLO, YUMA, ELY, HATCREEK, PLATTEVILLE, ONSALA, WETTZELL, WHITEHORSE, ALGONQUIN PARK, YELLOWKNIFE, PENTICTIN, and VANDENBERG, no terrestrial survey data were available, so only their connection to other VLBI sites tied them to the network. The EFLSBERG and CHILBOLTEN observations consisted of the ΔX, ΔY, and ΔZ values along with default matrices.

Fixed VLBI observations. The VLBI data used in iterations 0 and 1 were formatted like terrestrial data. This data set included observations at the following sites: WESTFORD, ONSALA, CHILBOLTEN, MARYLAND POINT, GOLDSTONE, WETTZELL, RICHMOND, and (NRAO). It did not include data at the KAUAI, KWAJAL26, and KASHIMA sites. After the first iteration, more recent (1985) observations became available. It would have been better to have used only these later observations, as they were assumed to be more accurate. However, the 1985 observations did not include all of the original sites, but did include three new sites (mentioned above). A combined group of all original and new stations was selected. Unknown parameters were defined to represent rotations around the X, Y, and Z axes and the scale for these observations.

Mobile VLBI observations (fig. 18.11) (available for iterations 1 and 2). The mobile VLBI observations were included as separate groups for each occupation. The groups were all tied together through the GILCREEK and GOLDSTONE sites. Some of the groups included observations for sites with terrestrial ties. (See Terrestrial survey data, above). Data at the following sites were included: GILCREEK, NOME, VANDENBURG, SANDPOINT, KODIAK, SOURDOUGH, YAKATAGA, WHITEHORSE, ALGONQUIN PARK, YELLOWKNIFE, PENTICTIN, FORT DAVIS, GOLDSTONE, OWENS VALLEY, MAMMOTH LAKES, PINYON FLAT, YUMA, MONUMENT PEAK, BLACK BUTTE, OC-

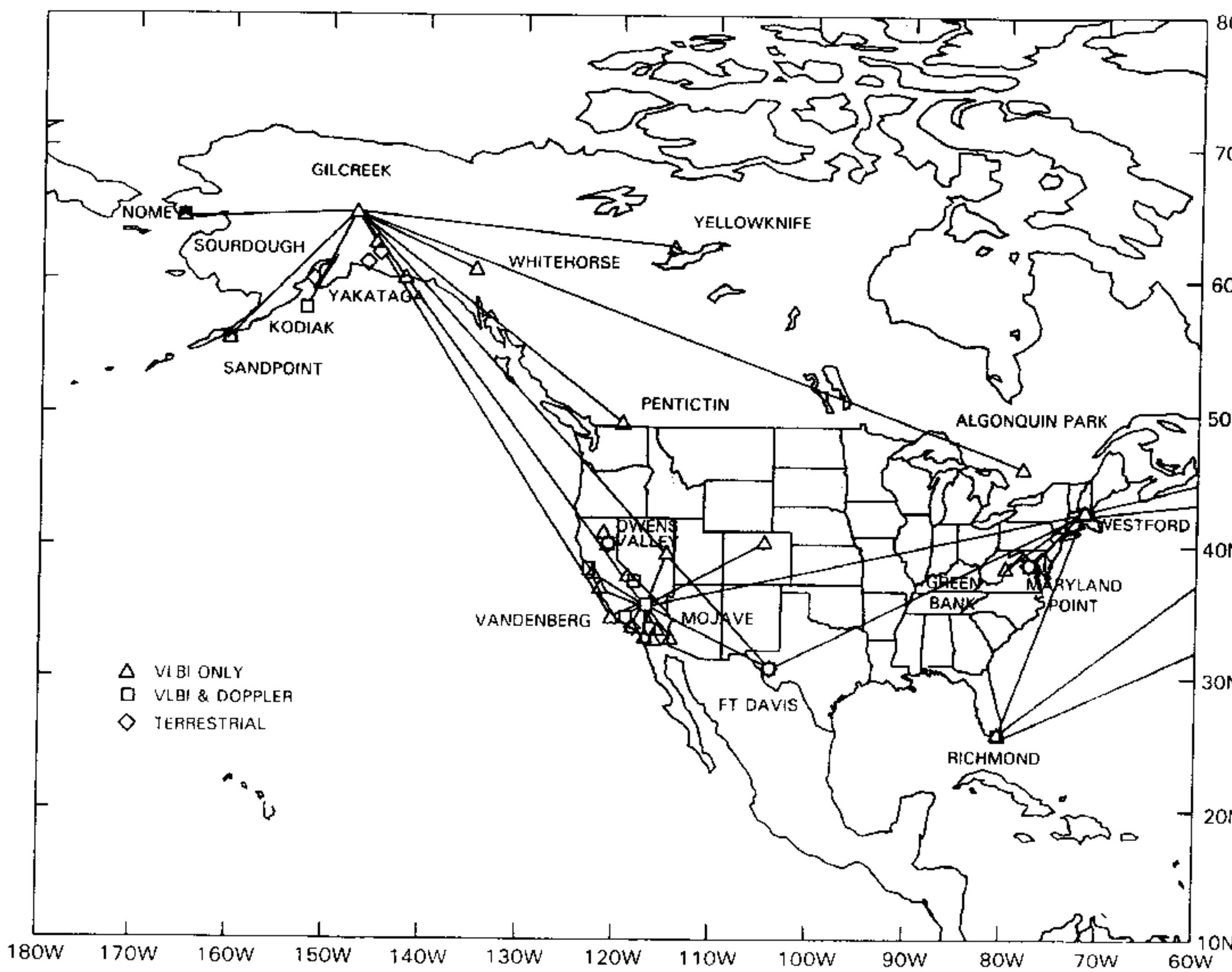

Figure 18.11. Locations of mobile Very Long Baseline Interferometry observatories.

OTILLO, HATCREEK, PALOS VERDES, FORT ORD, PRESIDIO, POINT REYES, ELY, PLATTEVILLE, WESTFORD, and QUINCY. Additional unknown parameters were defined to represent rotations around the *X*, *Y*, and *Z* axes and the scale for each group of observations.

Doppler observations. (See fig. 18.12.) Doppler positions and their covariance matrices had been initially computed on the NSWC 9Z-2 by the point positioning method. For iterations 0 and 1, the coordinates were transformed to PNAD 83 by application of a shift of 4.5 m in *Z*. Unknown parameters were defined to represent the rotation around the *Z* axis and the scale change that would be necessary to bring the Doppler coordinate system into agreement with the final NAD 83 coordinate system. For iteration 2, a scale change of −0.6 parts per million was also applied a priori and the scale change parameter deleted.

GPS observations. Baselines measured by GPS in the GPS Survey Alaska Project (GPS018, July-August 1984, Alaska-Canada NCMN Part I) were processed through program PHASER on the WGS 72 datum to provide network ties for five Alaskan VLBI sites: SANDPOINT, NOME, KODIAK, SOURDOUGH, and CAPE YAKATAGA. They were then transformed into the NAD 83 system. These data were formatted like VLBI and terrestrial observations except that a default covariance matrix (a diagonal matrix whose diagonal value is 0.2) was used.

18.5 CREATION OF HELMERT BLOCKS

18.5.1 Terrestrial Data Blocks

The creation of Helmert blocks for iteration 0 involved formation of the observation equations, normal equations, and the elimination of interior unknowns. These computations were performed by HBNEMO, a modified version of the NEMO program that had been used during block validation. The first solution (iteration 0) was considered to be a final data validation effort. This was a last chance to identify weak stations, keypunching errors, and observational blunders. It was expected that any data problems discovered would involve observations that crossed the boundaries of blocks used for validation.

The HBNEMO program produced two items: (1) large misclosures (computed minus observed terms), and (2) interior (non-junction) stations that appeared to be undetermined, which caused the normal equation coefficient matrix to be singular. In analyzing the

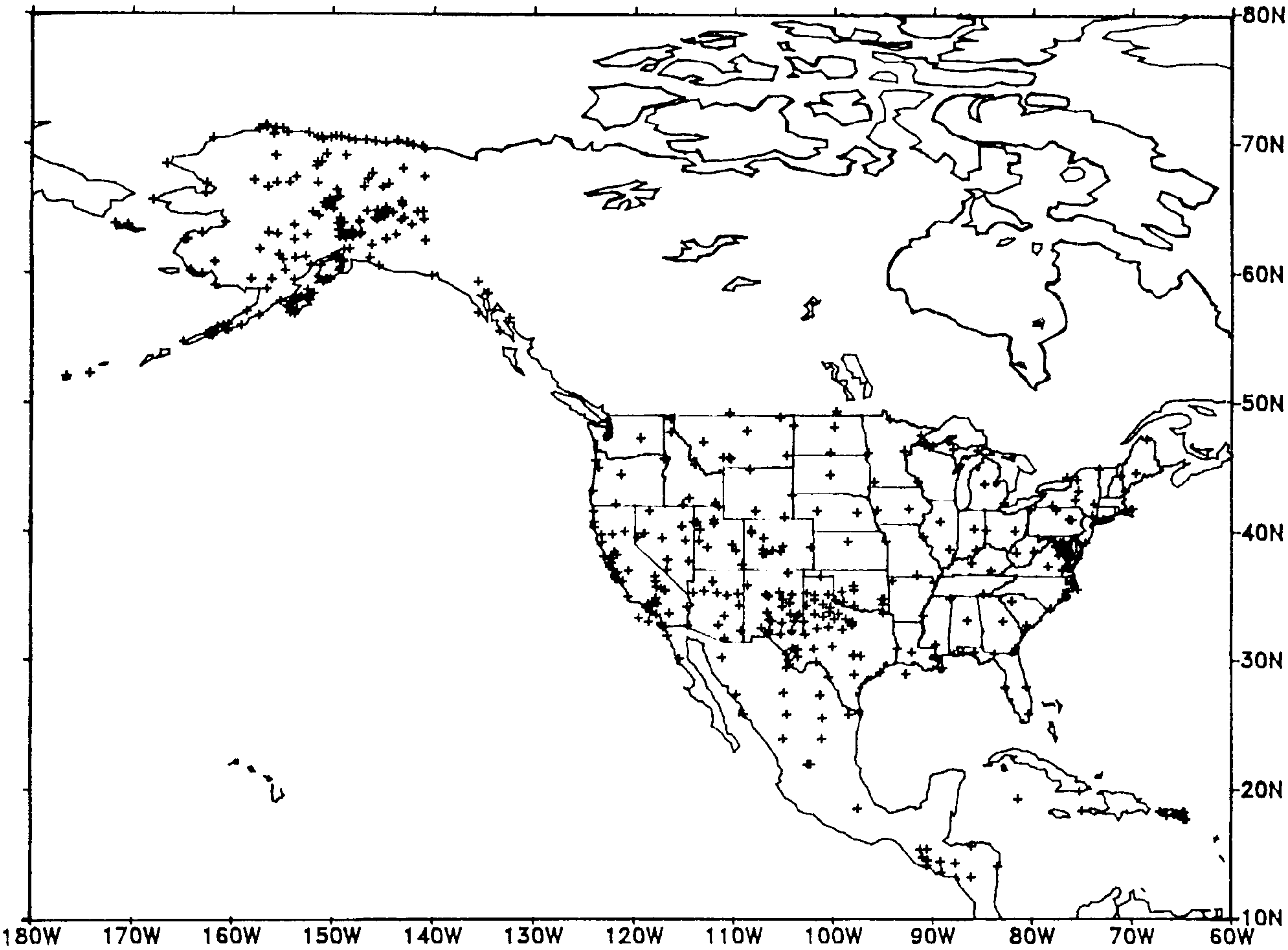

Figure 18.12. Doppler observations for NAD 83 in the United States.

misclosures, the observational input was reverified by checking original hard copy. Checking of preliminary positions was complicated by the fact that this was the first time that PNAD 83 positions had been used. Preliminary positions were checked by calculating positions from the surrounding PNAD 83 positions and the observations. Apparent singularities were corrected by finding misidentified observations or removing questionable stations from the NAD 83 adjustment. At this stage of the computations, the only way to remove a weakly determined station from the adjustment was to reject all of its observations. HBNEMO processed such stations in such a way that the computations could continue with no effect on the other stations in the network.

HBNEMO formed the partial normal equations for each block, eliminated the interior unknowns, and recorded the resulting set of partially reduced normal equations on an output file. This file was then registered with the Helmert block adjustment system. The system of equations was copied to a data set whose name was known to the APF, and the appropriate node of the Strategy was updated to show availability of this block.

18.5.2 Creation of the Space System Helmert Block I—Iteration 0.

The earliest SOAP runs indicated some problems with HAVAGO processing. The covariance elements produced in HAVAGO were recorded to only two decimal places; investigations determined that if the elements were carried out to four decimal places the occurrences of singularities would be eliminated. It was also discovered that the ΔX, ΔY, and ΔZ output of coordinate differences from HAVAGO was being produced in the wrong order. The necessary program changes were made to HAVAGO. The next SOAP run, with all of the above corrections, still resulted in a matrix singularity for the terrestrial data at the GOLDSTONE site. Six of the input correlation values were 0.9999 as a result of rotating weak vertical geometry into an Earth-centered system. Further expansion of the precision of the matrix values, as had been done before, would probably have eliminated the problem. However, the benefits were not judged sufficient to justify the time required to accomplish this. Instead, these values were arbitrarily changed to 0.9998 and the singularity was resolved.

Subsequent SOAP adjustments pointed out data errors and values that had been keyed incorrectly. Once these were corrected, an acceptable adjustment resulted.

Up to this time, no Doppler observations had been added to the SOAP input. All VLBI-related Doppler observations were now retrieved from the data base using a file of station identifiers. It was then discovered that the geodetic data base had not been properly updated with the most current Doppler positions. A paper listing of current positions was obtained. Observations at non-tied stations were selected from the listing because only the stations tied to the terrestrial network had previously been loaded into the data base.

The observations at CHILBOLTEN were reduced for eccentricity. One observation at TIMER (0250801330033) that had been retrieved from the data base was not listed on the printout because it was a DMA observation. Sufficient information had not been provided by DMA, and since there was another valid observation at TIMER, this one was deleted.

Large residuals led to the rejection of Doppler position observations at the following stations:

1 at MCDONALD RM 1 1942	0301041220005
1 at MCDONALD RM 4 1980	non-tied
3 at HAYSTACK OCP NO 3 1975	0420711330014
1 at ARIES RM 1 1976 DOP(51201)	0351163440007
1 at BP ARIES 1 (DOP. 51105)	non-tied

For those Doppler observations rejected during the adjustment, either repeat observations were made or nearby observations substituted. In this way the network was not weakened by the loss of these observations.

An adjustment was made using only VLBI stations with their associated Doppler and VLBI observations. The parameters solved for and their resulting values from this stand-alone unconstrained solution were:

VLBI	X rotation	0.030306
	Y rotation	0.036414
	Z rotation	0.823558
Doppler	scale change	−0.653683

The remaining Doppler observations were retrieved from the data base in SOAP input format. For iteration 0, the VLBI-related Doppler observations were included in a separate group. Some minor but annoying problems were found. For example, an extra digit had been keyed in the Z coordinate for the Doppler observations at station MARS 1963 (0351163440002). These were resolved and corrected in the SOAP input file and in the data base.

A file of geodetic positions of each Doppler point was processed through MODGHT to correct for inconsistencies in geoid height. However, only positions for the west coast were updated at this time, since east coast positions were already at or near the highest level. It was decided that the benefits of correcting the problem did not justify the cost of restarting at the lowest level.

The complete VLBI/Doppler file was then processed by SOAP. This time the program failed because of incorrect standard errors on some additional Doppler observations. The standard errors associated with the Doppler points had apparently not been computed before the data were loaded into the data base. A program for this purpose produced a listing showing the correct standard errors. These were manually corrected in the file, and the data base was also updated.

One additional Doppler observation was rejected due to a high residual. This was at station GEOCEIVER STA 20208 1976 (0351142210013) which had been added with the second group of Doppler observations. All rejected Doppler observations were actually deleted from the Doppler section of the data base and from the SOAP input file.

At this point, observations showing large misclosures were investigated, and the following results obtained:

EDWARDS AFB TRACKING STATION 4 (0341174130003), MOUNT JOAQUIN (0621561130001), and MOOSEHEART MOUNTAIN (0641511210001) all had high misclosures in the "up" direction. As intersection stations, the elevation fields in the RESTART files were blank. However, the Doppler data base elevations were 964.26 m, 916.4 m, and 652.9 m respectively (approximately equivalent to the misclosure value). Since the discrepancy was so large and could affect the outcome of the adjustment, the Doppler observations were removed from the data. At the end of this iteration the elevations in the RESTART files were corrected and the Doppler observations added back in for the next iteration.

WINKLE 1934 (0341051210001) had a scaled elevation in the RESTART file that was found to be in error by 8 m. Since the error was small, the observations were left in, but the RESTART file and SOAP input file were corrected at the beginning of the next iteration.

Stations 0923 NB. TEHUACAN (0180974220001), 1754 ANTONIO (0130862210001), CAL CO 160-A 1963 RM 8 (0280924140004), CAL CO 160-A 1963 RM 7 (0280924140005), CALCO 41-A (0290892120003), BEL 1925 (0551622220002), GOOSE 2 1930 (0561534310001), BAY-COVE POINT 1907 (0581532230007), MASSACRE NORTH BASE USN 1943 (0521864420032), T 41 1955 (0180643430031), MINERS POINT 1908 (0571534140001), DRIFT 1931 (0561534420004), and JOE 1941 (0551603120015) also had large misclosures in the vertical direction. Since verification of elevation was impossible without additional information, no corrections were possible at that time. Many of these elevations were later corrected in iteration 2. (See table 18.9.)

Several stations were found to be misidentified, resulting in large misclosures in the north and/or east directions:

INCORRECT	CORRECT
MILLER 1930 (0430884430001)	MILLER 1930 RM 3(non-tied)
FORT YUKON LOOKOUT TOWER (0661451230001)	FORT YUKON WEST BASE AZ (non-tied)
GINGRICH 1939 (0400891420002)	GINGRICH 1939 RM 4 1969 (0400891420006)
ASTRO PIER 1966 (0390763320022)	SAT TRACK STA 002 1966 (0390763320021)
SAN FERNANDO 1898 (0341183120006)	SAN FERNANDO 1898 RM 3 (0341183120026)
SAGE 1923 (0491102340002)	SAGE 1923 RM 1 (0491102340002)
9171 748 (0231011110001)	0597 GUANGOCHE (0231011110002)

Those stations that were not connected to the terrestrial network were found to be unnecessary and the observations simply deleted from the Doppler data. The others were removed for iteration 0, corrected, and reintroduced at the beginning of iteration 1.

INGRI 1951 (0611651320001) had large misclosures, but no errors could be identified. Since the terrestrial observations at this station had also caused problems in the Helmert block adjustment, and had been deleted there, the Doppler observation was also deleted from the VLBI/Doppler data. FLAGSTAFF NCMN 3 (0351113240019), MALASPINA SW BASE 1892 (0591401240002), STAR 1914 RM 2 1975 (0551591140002), and 6027 ENSENADA NWB (0311164130001) also showed large misclosures in the north or east directions. However, no problems could be found. Since the misclosures were still within reason, the observations were left as they were.

The VLBI and Doppler observations in Hawaii were handled in a separate SOAP input file. The only correction was a change to the MAKAPUU POINT 1872 for a misidentification.

18.6 FORWARD SOLUTION

Most of the time the forward solution was run in an automatic mode, using the DISPATCHER function of the Helmert block adjustment system. If the strategy determined that two Helmert blocks were available for combination and reduction, then the computer processing control was automatically generated to submit the run. (See chapter 15.) Since this was still the first adjustment of the complete data set, apparent singularities in the normal equations were occasionally detected and had to be analyzed. Each such station either had to be removed altogether from the adjustment by removing all associated observations or the determination of the coordinates had to be strengthened by finding and adding new observations. Either option was difficult: the solution had to be restarted at the lowest level with either a new retrieval from the data base, or by editing the block's RESTART file. Then the Helmert block had to be recreated by HBNEMO, and the forward solution rerun along the path to the current computations. In some cases, especially at the upper levels, a decision was made to retain the singularities for iteration 0 and correct the data set at the beginning of iteration 1.

One other problem discovered during the forward solution of iteration 0 was mismatched preliminary values for deflections and geoid heights. The changes discussed in section 18.4.1 were made. For junction points, however, changes were necessary in more than one place. Knowledge of the block boundaries and special junction points was mandatory. In each case, these problems halted the automatic solution while the error was corrected at the lowest level and the forward solution rerun along the affected path.

An area of concern surfaced at the higher levels. The reliability of the computer system being used was severely tested with the combination of the northeast (node 5) and the southeast (node 4) sections of the

United States. This creation of node 3 was within the specifications of the program and within the capability of the computer. However, this step required 8 hours of CPU and 24 hours of wall clock time. Close coordination with the computer operations staff resulted in successful computations for each iteration of the solution.

The iteration 0 forward solution was performed in parallel with the retrieval of the RESTART files and the creation of the Helmert blocks.

18.7 HIGHEST LEVEL HELMERT BLOCK

The highest level Helmert block in the Strategy (block I) was reached on August 31, 1985. At this point, 894,923 unknowns had been eliminated from the system of equations, leaving a set of 2,168 equations for the remaining junction unknowns. These unknowns were the coordinates of special junction points, junction points on the U.S.-Canadian border, and the global parameters.

Before proceeding, a stand-alone adjustment of this block was performed. In this mode the software performs a free adjustment by fixing any parameters that appear to be indeterminant. The solution obtained is the one that would be derived if these parameters were constrained to their preliminary values.

This was the first adjustment of the entire U.S. geodetic network. The variance of unit weight for the stand- alone solution was 4,000,000!!! Even though the software provided for the analysis of partial solutions, the project team had not stopped the forward solution along the way because no serious data problems had arisen.

To isolate the cause of the huge variance, the team returned to the fourth- and fifth-level Helmert blocks. (See fig. 18.13.) Stand-alone solutions were performed for each of these. The problem was found in the Alaska block. It was finally discovered that on a single direction observation, the field which should have contained the degrees of arc had been blanked out, resulting in a residual of approximately 30 degrees of arc. The correction of this single data transfer blunder brought the variance down to 14. This value was still unacceptably large.

Looking at the other regional values (fig. 18.14), large variances were also noted in block 149 (New England) and block 8 (Texas). In analyzing these blocks, the team found **two** additional direction observations that crossed the block validation boundaries and were in error. Removing these observations brought the variance of unit weight to 4 for the first solution. This was considered to be acceptable and the adjustment continued.

The task remained to combine this block with the blocks from Canada and the space system observations. By design, the automatic combination of blocks stopped at this point. The remaining combinations were initiated explicitly by the project manager and team, since analysis and interaction were required at each step. Table 18.5 describes the blocks defined at this level and table 18.6 provides a statistical analysis.

The typical set of computations that were made at any of the highest levels, without regard to investigation runs, started with the creation of the space system Helmert blocks, Doppler, and VLBI. The blocks for (1) Hawaiian Doppler and (2) special observations from DMA, including Doppler observations on Swan Island, were created separately. A special Doppler block containing just VLBI and Doppler observations at the Santa Paula VLBI site of the National Crustal Motion Network (NCMN) was also created. Thus began the series of combinations shown in figure 18.15.

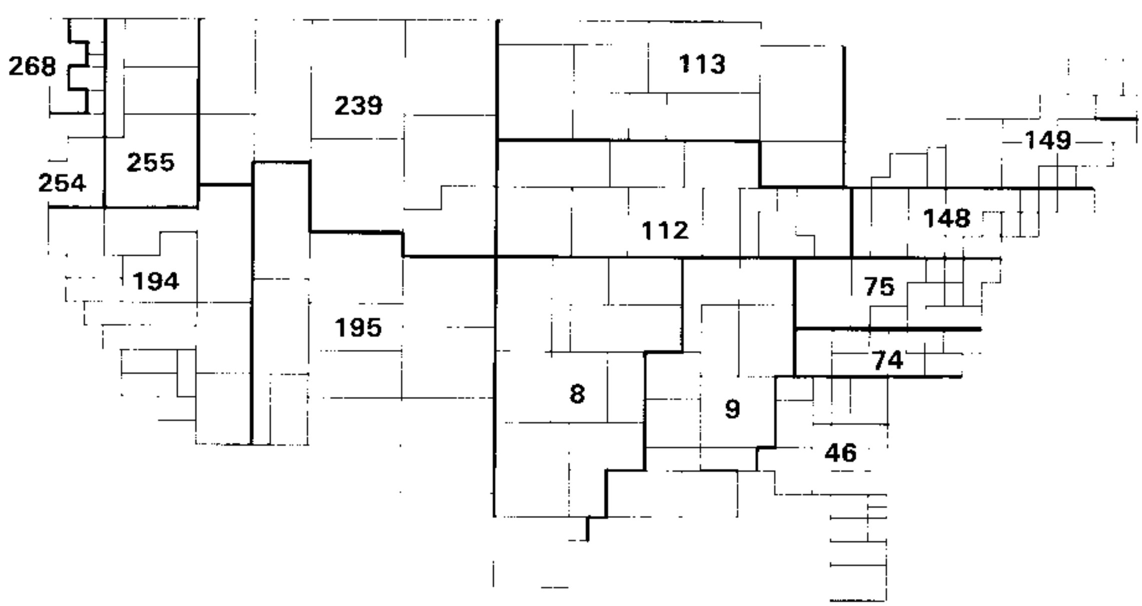

Figure 18.13. Fourth- and fifth-level Helmert blocks.

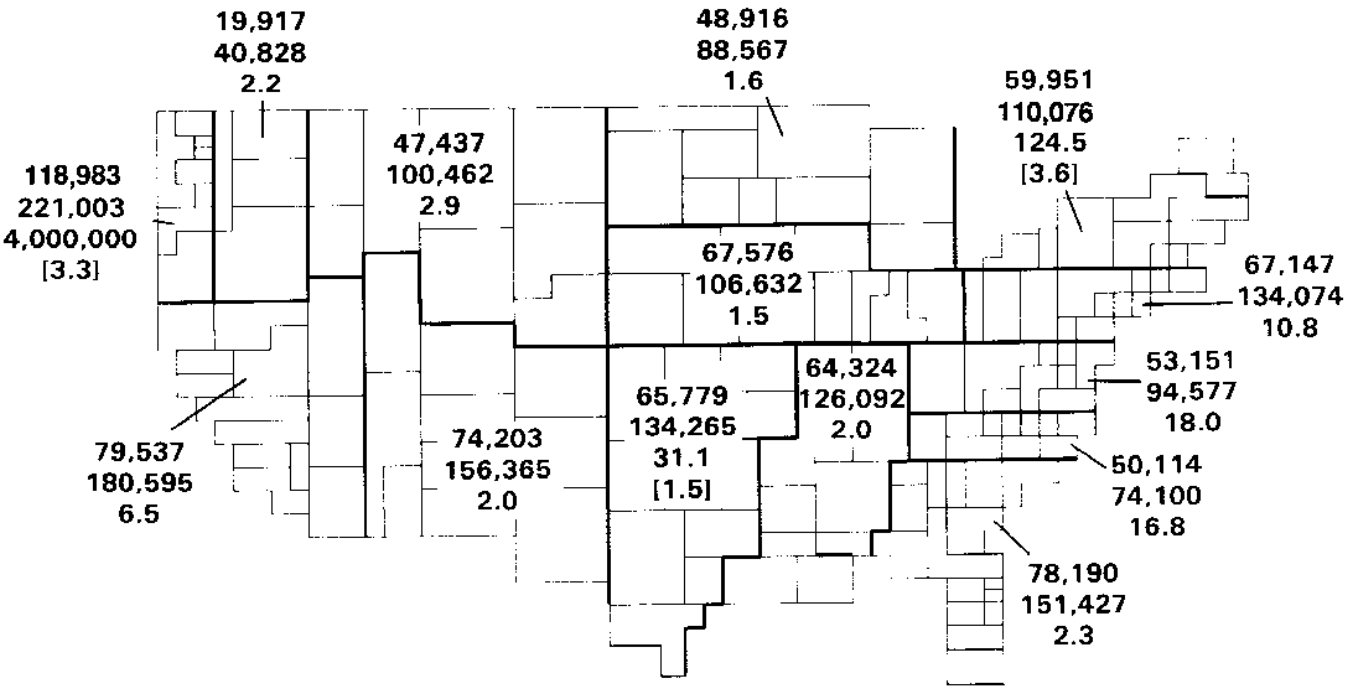

Figure 18.14. Variances in intermediate-level Helmert blocks. Each set of numbers represents the following: first number—unknowns, second number—observations, third number—variance, and fourth number—corrected variance.

TABLE 18.5.—*Highest level Helmert blocks*

900	Solution of highest level
901	Block containing all observations and unknowns
902	Constraint equations to equivalence mobile VLBI parameters
903	Combination of U.S. plus Canadian data
904	Special Helmert block containing Santa Paula VLBI site
905	All U.S. data except Santa Paula
906	Canadian data
907	Reduction to Can.-U.S. junctions, Santa Paula, DMA, and Swan data
908	Special DMA Doppler observations and Swan Island Doppler
909	Hawaiian and U.S. terrestrial, Doppler, VLBI
910	Special junction points considered interior at this level
911	Hawaiian terrestrial, Doppler, VLBI
912	Contiguous U.S. and Alaskan Doppler and VLBI
913	Contiguous U.S. and Alaskan Doppler
914	VLBI
915	Hawaiian terrestrial, Hawaiian Doppler
916	Hawaiian Doppler
801	Highest level Hawaiian terrestrial
802	Lower level Hawaiian terrestrial
803	Lower level Hawaiian terrestrial

A unique situation arose at the highest level for iteration 0. The U.S. terrestrial data, fixed VLBI data, and Doppler data were ready and available for adjustment in October 1985. The Canadian data (block 906) and the mobile VLBI data (part of block 914) were not available and not expected to be ready until at least December 1985, possibly later. A decision had to be made whether to wait for these data or to continue iteration 0 without them. The basic factors in the decision were that iteration 0 still involved the final cleansing of the data and that the back solution and the next forward solution could be processed by the

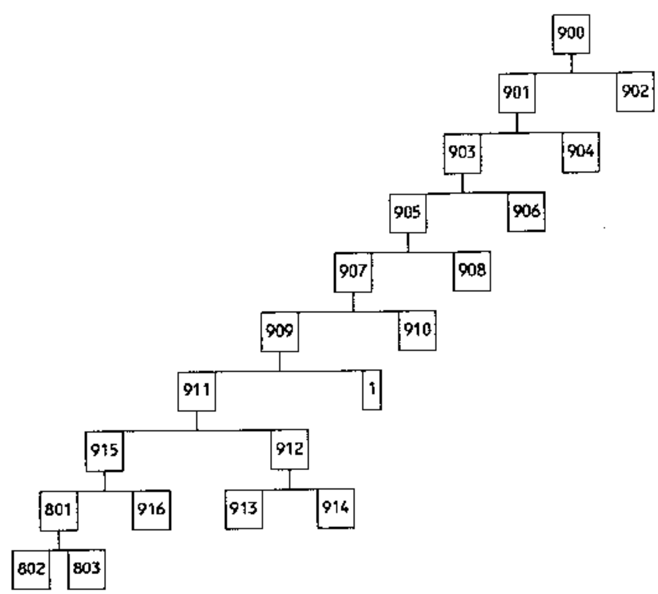

Figure 18.15. Helmert blocking strategy with highest level combinations.

time the other data sets were ready. The computations were continued.

Now the entire data set was ready to be solved. The last step included the addition of constraints that would make all VLBI rotations and scale parameters equal. The equations had been formed assuming that each session of data had a unique set of rotation and scale parameters. After investigations and test solutions, it was decided that all of the parameters should be reduced to a single set for all sessions.

At the very top level (block 900) all unknowns became interior. At this point 897,861 unknowns had been eliminated and none was left! This solution was considered final for iteration 0.

TABLE 18.6.—*Analysis of highest level Helmert block stations and unknowns*

Block No.	Interior unknowns	Junction unknowns	Cumulative interior unknowns	Cumulative observations	Inside stations
801	133	24	9015	18828	0
802	3878	71	3878	8161	1208
803	5004	96	5004	10667	1438
900	1114	0	928735	1785772	0
901	0	1114	927621	1785670	0
902	0	0	0	102	0
903	0	1112	927621	1785667	0
904	0	4	0	3	0
905	0	1068	905905	1741320	0
906	21716	953	21716	44347	7454
907	1967	1059	905905	1741296	0
908	0	25	0	24	0
909	0	3026	903938	1741296	0
910	0	0	0	0	0
911	0	1964	9015	21288	0
912	0	1929	0	2427	0
913	0	1621	0	1872	1
914	0	372	0	555	49
915	0	35	9015	18861	0
916	0	31	0	33	0

18.8 BACK SOLUTION

After completion of the highest level solution, the results were substituted back down the strategy tree to the lowest level blocks. There was no interaction with this process. All computations were initiated automatically by the DISPATCHER. Unfortunately, a few forward solution storage tapes were found to be unreadable and had to be recreated. The entire back solution took just 1 week in the middle of October 1985.

18.9 ANALYSIS OF RESULTS—ITERATION 0 (FIRST LINEARIZATION)

The lowest level results were available by October 20, 1985. Analysis and investigations required during the Helmert blocking computations varied widely in time and effort needed from one area of the country to another as well as from one phase of computations to another. Only a few people had been involved during the highest level and back solutions. Now the entire staff of more than 40 was reassigned to the project to perform the analysis of the results for iteration 0.

Three solutions were obtained during iteration 0. The results of the first solution for the classical terrestrial observations alone were:

Total observations	1,721,143
Total unknowns	897,218
Variance of unit weight	4.69
Degrees of freedom	823,925

The results of a combined terrestrial, Doppler, and fixed VLBI solution were:

Total observations	1,723,198
Total unknowns	897,861
Variance of unit weight	4.74
Degrees of freedom	825,337

Because the Canadian data had not been available, the results that were considered final had been obtained by constraining the Canadian boundary junction positions. These results were:

Total observations	1,724,008
Total unknowns	897,861
Variance of unit weight	6.06
Degrees of freedom	826,147

The final values for the space system parameters were:

Doppler *Z* rotation	−0.744 arc second
Doppler scale	−0.65 ppm
VLBI *X* rotation	−0.080 arc second
VLBI *Y* rotation	−0.080 arc second
VLBI *Z* rotation	+ 0.076 arc second

The Doppler *Z* coordinates (in NSWC 9Z-2) had been translated +4.5 m a priori.

The analysis at the end of iteration 0 examined the larger residuals as a means of detecting any remaining blunders. Since the results of iteration 0 did not produce final coordinates, care was taken to determine if relative position shifts were small. Otherwise, the existence of a large residual might only be a reflection of the nonlinear terms in the observation equation rather than a real indicator of a possible blunder. To aid in determining whether the residuals were accurate, least squares adjustments were run on the blocks in an isolated setting as well as in the combined simultaneous mode. These residuals were compared to the residuals after iteration 0. Main differences were found along the block boundary where the stand-alone adjustment could not contain all information from the neighboring block. As long as consistency was maintained, the residuals could be considered real and were analyzed. One other aid was to analyze the relative position shift versus the linear error. Since the shifts between stations were expected to be reduced by at least one order of magnitude on subsequent iterations, the residuals were analyzed when the relative shift vector was less than 10 times the linear error. The relationship was as follows:

relative shift $= [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}$

linear error $=$ sin (standard error) * distance of observation

The same guidance that had been used in block validation was again followed in determining when to reject an observation, when to change the standard error associated with an observation, and when to unreject an observation. For this iteration, 9,379 observations had normalized residuals larger than 3.5. Of these, 1,671 were rejected and 2,457 were downweighted. A total of 1,044 previously rejected observations were readmitted.

18.10 ITERATION 1 (SECOND LINEARIZATION AND SOLUTION)

The subsequent iterations of NAD 83 were smoother in both observation analysis and computer computations. By this time all of the observations had been reviewed on an individual basis, combined in small groups, and finally used in a simultaneous solution. The team was satisfied that all blunders had been detected and removed. The data base had been completely validated. All of the computer programs had been used together.

As mentioned in previous sections, some of the data inconsistencies discovered during iteration 0 were corrected at the beginning of iteration 1. Geoid heights and deflections of the vertical were corrected. Rejection and standard error changes were made.

The solution computations and analysis for the entire iteration 1 required only 4 months, lasting from December 1985 to March 1986. (Iteration 0 had taken 6 months.) Several factors played a role in this. First, more computer facilities were available at the end of December. Multiple parts of the strategy tree were started simultaneously. Second, the review of the misclosures went more quickly, since the primary task involved verification of previous decisions rather than new investigations. Lastly, the analysis of singularities was quicker because of the removal of observations to and from points that had been deleted from the computation. A Googe number of -10.00 for the latitude and longitude unknowns of these stations denoted a total singularity. This indicated that all observations had been successfully removed.

During the forward solution, stand-alone adjustments were completed and compared to the area solutions from iteration 0. As shown in figure 18.16, most areas on the east coast improved significantly. The variance of unit weight in block 148 (Pennsylvania) decreased from 10.8 to 2.1, in block 75 (Virginia) the decrease was from 18.0 to 3.4, and for block 74 (North Carolina) the decrease was from 16.8 to 1.8. Upon further study of the crustal motion models, the variance of unit weight for block 194 (California) decreased from 6.5 to 2.5.

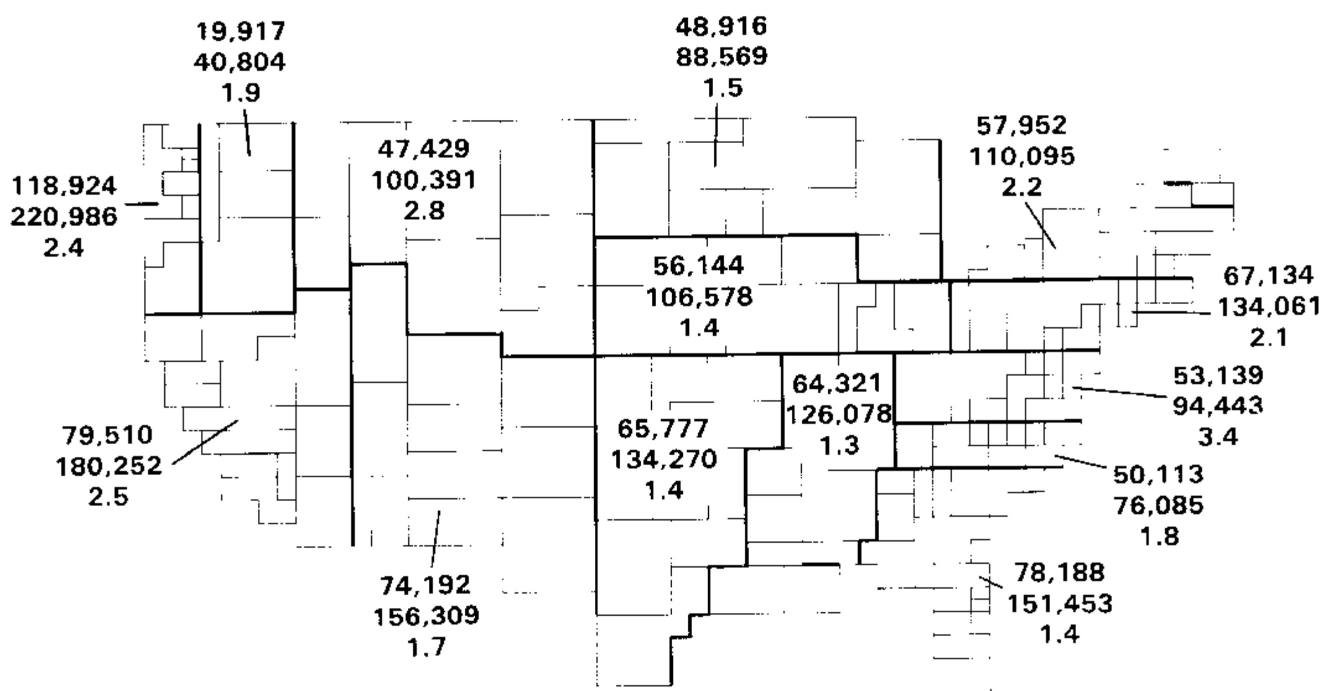

Figure 18.16. Variances in intermediate-level blocks for iteration 1. Each set of numbers represents the following: first number—unknowns, second number—observations, and third number—variance.

For the remaining iterations, the space data were split into two files. One file contained all VLBI stations including the terrestrial and GPS observations associated with them. This set also contained the Doppler observations at the non-tied stations. The other file contained all Doppler observations at stations that were tied to the terrestrial network. Mobile VLBI data were added to fixed VLBI data along with terrestrial data at sites PALOS VERDES, QUINCY, PRESIDIO, PASADENA, MONUMENT PEAK, and FORT ORD. The GPS ties at KODIAK, NOME, SOURDOUGH, CAPE YAKATAGA, and SANDPOINT were also included.

The initial iteration 1 solutions for the Doppler Z rotation parameter differed by 0.3 arc second from the solution obtained in iteration 0. This was considered to be unacceptable. The only reason for this difference could be the mobile VLBI observations which were added at iteration 1. Numerous solutions were run to investigate which mobile VLBI baseline was causing the problem. To isolate the inconsistency, it was necessary to remove the mobile VLBI observation groups one at a time.

After performing several test runs, the SANTA PAULA site was isolated as the source of the problem. The cause might have been a weak connection to VANDENBURG. Because of the location of the VANDENBURG station and the consistently poor weather, the position of the antenna was considered minimally acceptable. This group was deleted, along with the SANTA PAULA terrestrial group. This eliminated a tie to the DEADMAN LAKE site, but since the connection was so poor to begin with (this is the only observation) and since coverage throughout California is already dense, the decision did not cause a problem.

The observations at the PEARBLOSSOM site also appeared to cause problems, as evidenced by large residuals. Therefore, this group was also deleted.

Additional Doppler observations also became available for this iteration. Doppler observations at PEARBLOSSOM NCMN <7254>, HARVARD RM 3 1979, and KODIAK MON <7278>(three observations) were identified in a file of observations at Doppler stations that had not been matched to terrestrial stations. These were added to the non-tied group. An observation at QUINCY 7221 NCMN (0391204440018), also identified in the same file, was added. Two observations each at BOB 1925 (0521742240018) and BAKER EAST BASE 1945 (0511803140022) in Alaska were observed by a private contractor, Itech, Ltd., to provide needed control in the Aleutian chain. These observations were also added. Because SATELLITE TRI STA 111 1965 (0341173140004) had erroneously been deleted or omitted from the file in the previous iteration, it was added at this time. Also, the positions and observations found to be misidentified in iteration 0 were corrected and added. ELY AIRPORT 1954 (0391143420008) had two distinct positions in the data base, a readjusted position and the original position which had never been deleted. Since the position in the Doppler file was the original one, it was corrected. (The data base had already been corrected during block validation when the adjustability flag was set.) The Doppler observations at the following stations also had large residuals and were deleted: 4189 CONSEJO (0180882410001), 1106 A TURBIAS EC (0170891130002), 4089 BT-1 (CAYO) (0170892230005), PILOT 1926 (0351172140002), and SPEEDY GSC 1967 RM2 1975 (0481232410002).

The reduced normal equations from the Canadian network were available for this iteration and were included.

The three final solutions obtained during iteration 1 are outlined as follows.

The results of the classical terrestrial solution are:

Total observations	1,720,374
Total unknowns	897,131
Variance of unit weight	2.00
Degrees of freedom	823,243

The results of a combined terrestrial, Doppler, and VLBI solution were:

Total observations	1,741,668
Total unknowns	906,968
Variance of unit weight	2.01
Degrees of freedom	834,700

The combined U. S. and Canadian results were:

Total observations	1,786,037
Total unknowns	926,448
Variance of unit weight	2.01
Degrees of freedom	857,589

The final values for the space system parameters were:

Doppler Z rotation	-0.721 arc second
Doppler scale	-0.53 ppm
VLBI X rotation	$+0.030$ arc second
VLBI Y rotation	$+0.030$ arc second
VLBI Z rotation	$+0.110$ arc second

The Doppler Z coordinates (in NSWC 9Z-2) had been translated $+4.5$ m a priori.

Some singularities remained for the final solution of iteration 1. St. George Island and St. Paul Island in Alaska did not have Doppler control. One station on each island was held fixed. A Canadian junction station, KINGSVILLE USLS, did not have enough observations to it and so could not be positioned. The FLAGSTAFF NCMN 2 station required a longitude constraint. Errors in the HAVAGO program had reduced the tie to terrestrial data. Lastly, the SANTA PAULA NCMN station required a longitude constraint. The site at this station was thought to be connected to the rest of the network only through the VLBI vector. Weakness in the vector and the weights justified constraining the longitude for this iteration.

After the back solution was completed, only 2,046 observations were found to have normalized residuals greater than 3.5. Because there were far fewer such observations, this analysis proceeded more quickly than it had for iteration 0. Of these observations, 549 were rejected and 1,065 were downweighted. A total of 319 previously rejected observations were readmitted.

The adjusted coordinates for the junction points were sent to the Geodetic Survey of Canada, so that a separate back solution could be carried out for the Canadian network.

18.11 ITERATION 2 (THIRD LINEARIZATION AND SOLUTION)

The last iteration began in April 1986. Its completion on July 31, 1986, was recognized as the official completion date of NAD 83. Data errors which had caused HAVAGO to fail when the terrestrial data at the FLAGSTAFF site were processed were corrected and the resulting data added to the file. DMA Doppler observations were added for the Alaska Islands where singularities had occurred during iteration 1. Observations were added to the U. S. Lake Survey station KINGSVILLE to resolve the singularity.

After iteration 1, fixed VLBI observations from 1985 were added and a test adjustment was run. A major blunder was evident at the GILCREEK site. GILCREEK is unique in that both mobile and fixed observations were measured there. It was found that when the GILCREEK observations and position were added to the mobile VLBI data for iteration 1, the position used was that at the antenna, while the data had been reduced to the monument. The adjustment had shifted the position to the monument. Now the observation being added was a fixed VLBI observation to the antenna. The preliminary position had to be changed back to the antenna.

An additional misidentification of a Doppler station was found and corrected. MARK NW COR BLDG 1970 (0311104340004) should have been HOPKINS 1970 (0311104340003).

After processing had already begun for the last iteration, it was discovered that terrestrial observations for a Doppler station, TRANET 747 USAF 1973 (0411043310010), had not been included. The benefit of adding the observations at that time did not justify the cost of returning to the lowest level and starting again.

At the highest level, numerous investigations were undertaken to resolve the different solutions possible when using different observations. The various investigative data sets included Doppler observations at station BALDY, Doppler observations at station SANTA PAULA, DMA Doppler observations on St. Paul and St. George Islands, AK, and simulated Doppler observations on Swan Island and Buldir Island. Table 18.7 shows the 17 investigations and their results.

TABLE 18.7.—*Iteration 2—Highest level test runs*

Observations Included	Parameters Included	Variance	Doppler *Z* Rotation	VLBI Rotation	Doppler Scale	Singularities	Comments
Terrestrial[1] Doppler[2] VLBI Hawaii terrestrial Hawaii Doppler	VLBI *X,Y,Z* rotations Doppler *Z* rotation Doppler scale	1.837	−0.72 (HI=6.58)	+0.12	−0.53	Santa Paula[5] Swan Island Buldir Island St. Paul Island St. George Island Canada junctions HI Doppler scale	Hawaii (HI) Doppler Group not equivalenced to U.S. Doppler Group
Terrestrial[1] Doppler[2] VLBI Hawaii terrestrial Hawaii Doppler	Same as above	1.837	−0.71	+0.12	−0.53	All of the above except HI Doppler scale	Hawaii Doppler equivalenced to U.S. Doppler
Terrestrial[1] Doppler[2] VLBI Canada Simulated Doppler[8] All Hawaii	VLBI *X,Y,Z* rotations VLBI scale Doppler *Z* rotation	1.830	−0.44	+0.39	—	None	Doppler observations at Baldy corrected. Simulated Doppler stations have SE=0.8. Doppler scale transformed −0.6 ppm
Terrestrial[1] Doppler[2] VLBI Simulated Doppler All Hawaii	Same as above	1.837	−0.40	+0.44	—	Canada junctions	Same as above without Canada
Terrestrial[1] Doppler[2] VLBI All Hawaii Simulated Doppler[8]	Same as above	1.837	−0.40	+0.44	—	Canada junctions	Incorrect Baldy observations used

TABLE 18.7.—*Iteration 2—Highest level test runs (continued)*

Observations Included	Parameters Included	Variance	Doppler Z Rotation	VLBI Rotation	Doppler Scale	Singularities	Comments
Terrestrial[1] Doppler[2] VLBI All Hawaii Simulated Doppler[8]	VLBI *X,Y,Z* rotations Doppler *Z* rotation Doppler scale	1.837	−0.40	+0.44	−0.53	Canada junctions	Correct Baldy obs. Doppler not transformed. Scale on Doppler not on VLBI
Terrestrial[1] Doppler[2] VLBI All Hawaii Simulated Doppler[8]	VLBI *X,Y,Z* rotations VLBI scale Doppler *Z* rotation	1.836	−0.72	+0.12	—	Santa Paula[5] Swan Island Buldir Island Canada junctions	Correct Baldy Obs. Additional Doppler Reg & SE changes SE 9.9 on simulated Doppler deck.
Same as above with Canada	Same as above	1.829	−0.73	+0.10	—	Santa Paula[5] Swan Island Buldir Island	
Terrestrial[1] Doppler[2] Simulated Doppler[8] Canada All Hawaii	Same as above	1.829	−0.46	+0.38	—	None	SE reduced to 1.1 m in simulated deck
Terrestrial[1] Doppler[3] VLBI Simulated Doppler[8] All Hawaii (no Canada)	Same as above	1.836	−0.41	+0.42	—	Canadian junctions	Simulated Santa Paula observations removed SE on actual Santa Paula observations reduced from 0.8 to 0.4
Terrestrial[1] Doppler[3] VLBI All Hawaii (No Canada or simulated Doppler)	Same as above	1.836	−0.41	+0.42	—	Swan Island Buldir Island St. Paul Island St. George Island Canadian junctions	SE on real Santa Paula observation=0.4 No simulated or bad observations
Terrestrial[1] All Hawaii Doppler[4] VLBI	Same as above	1.836	−0.41	+0.42	—	Santa Paula[6] Swan Island Buldir Island St. Paul Island St. George Island Canadian junctions	Delete real Santa Paula Doppler observation
Add deck with poor Doppler on St. George and St. Paul Islands to above	Same as above	1.836	−0.41	+0.42	—	Santa Paula[6] Swan Island Buldir Island Canadian junctions	
Add simulated Dopplers on Buldir Island	Same as above	1.836	−0.41	+0.43	—	Santa Paula[6] Swan Island Canadian junctions	
Add simulated Doppler on Swan Is.	Same as above	1.836	−0.41	+0.43	—	Santa Paula[6] Canadian junctions	
Add Canadian deck	Same as above	1.829	−0.46	+0.38	—	Santa Paula[6]	
Add Santa Paula Doppler SE=0.4	Same as above	1.829	−0.46	+0.38	—	None	This solution used
Change Santa Paula	Same as above	1.829	−0.70	+0.14	—	Santa Paula[7]	

[1] Terrestrial deck contains all classical horizontal observations in the continental U.S., Alaska, Puerto Rico, and Central America.
[2] Doppler deck containing observations at Santa Paula with standard error SE=0.8.
[3] Doppler Deck containing observations at Santa Paula with standard error SE=0.4.
[4] No Observations at Santa Paula in the Doppler deck.
[5] Googe Numbers = $X = -0.35$; $Y = -0.01$.
[6] Googe Numbers = $X = -7.38$; $Y = -6.40$.
[7] Googe Numbers = $X = -0.06$; $Y = -0.16$.
[8] The "simulated" Doppler deck contains poor DMA Doppler observations on St. Paul and St. George Islands, AK, along with "simulated" Doppler observations on Swan Island and Buldir Island; and an additional simulated Doppler observation in the Santa Paula network. The Santa Paula observation was subsequently removed (as indicated in comments sections).

The results of these investigations were divided into two groups. The first group had a Doppler Z rotation of approximately -0.72 and a VLBI Z rotations of about $+0.12$. The second set of rotations approximated -0.46 and $+0.38$ respectively. The first group always had the VLBI site of SANTA PAULA as singular. This observational set used a terrestrial network that was not connected to any other terrestrial stations, a space system table top survey to tie the terrestrial stations to the Doppler and VLBI stations, a Doppler observation, and a VLBI observation. These observations together should have been sufficient to solve the network at SANTA PAULA; however, depending on the weight placed on the Doppler observation, two very different answers were obtained. For the solution to have no singularities, the weight on the Doppler observations at this station needed to be tightened from the average of 0.8 m to 0.4 m. (See fig. 18.17.)

The sixteenth trial solution was selected for NAD 83. Thus the final statistics are:

Total observations	1,785,772
Total unknowns	928,735
Variance of unit weight	1.829
Degrees of freedom	857,037

The final values for the space system parameters were:

Doppler Z rotation	-0.449 arc second
VLBI X rotation	$+0.022$ arc second
VLBI Y rotation	$+0.026$ arc second
VLBI Z rotation	$+0.375$ arc second
VLBI scale	-0.075 ppm

Doppler Z observations were translated $+4.5$ m and scaled by -0.6 ppm a priori. The Doppler scale unknown was eliminated and a VLBI scale unknown added. Investigations in the VLBI field had lead to a definition of the BIH meridian which was adopted for NAD 83. Since the mathematical model of the adjustment did not allow for the solving of the astronomical meridian separate from the BIH, the parameters were solved as follows: a Canadian Doppler Z rotation of -0.443 arc second and a U.S. Doppler Z rotation of -0.455 arc second. These values were averaged to the joint result of -0.449 arc second. The above definition

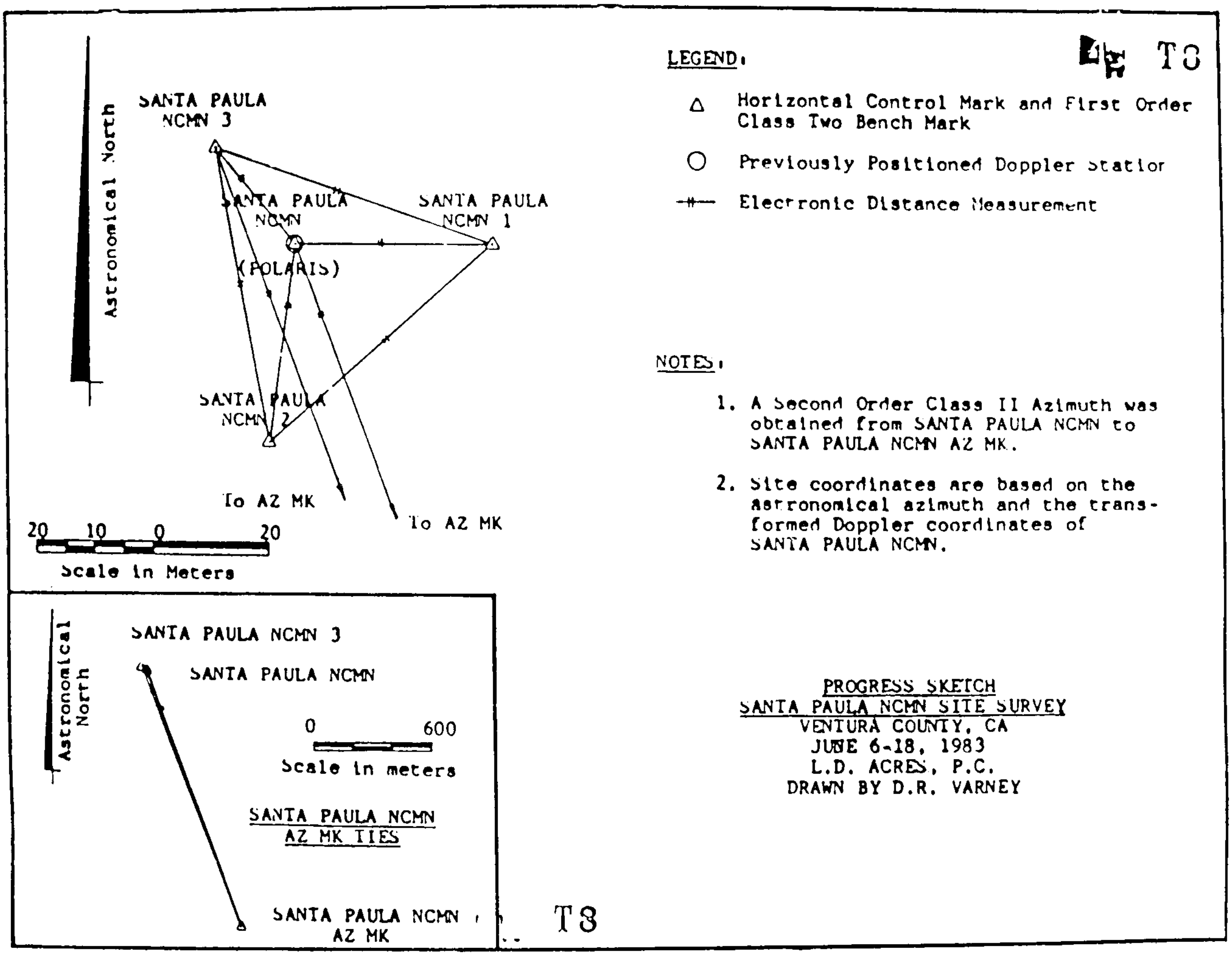

Figure 18.17. Santa Paula site sketch.

of the BIH was a −0.814 arc second rotation for Doppler to the BIH meridian. To obtain the required final value, a further −0.365 arc second was applied to all longitudes. Figure 18.18 shows the relationships between NAD 83 space parameters.

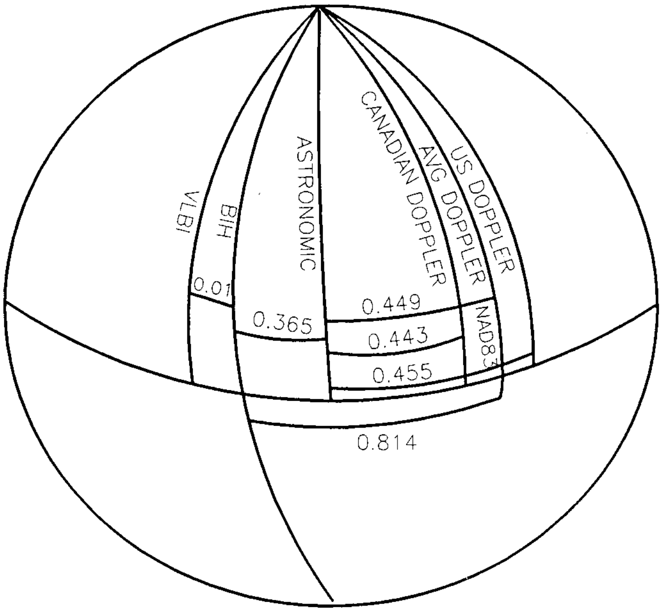

Figure 18.18. Orientation and scale relationships.

In the final NAD 83 solution, 455 normalized residuals were greater than 3.5.

18.12 SPECIAL PARAMETERS AND UNKNOWNS

The space system parameters were discussed in the previous sections on iterations 0, 1, and 2. In addition to these special parameters, there were special scale unknowns for systematic errors in groups of distance observations, as described in Creation of the APF. Chapter 6 discusses the reasons for these unknowns. Table 18.8 lists the final values for the observation class deck parameters.

TABLE 18.8.—*Special parameter values*

Unknown Identifier	Final value	Standard error
AZLIGHT	−1.36718 ppm	0.245 ppm
CALIGHT	−0.78012 ppm	0.100 ppm
CDAERO	−3.95600 ppm	0.401 ppm
FLLIGHT	2.89880 ppm	0.399 ppm
FLMICRO	−0.18795 ppm	2.430 ppm
GEODIMETER	−0.23688 ppm	0.049 ppm
IBCLIGHT	−0.59974 ppm	0.768 ppm
IBCMICRO	3.10641 ppm	0.683 ppm
IBCTAPE	−14.32133 ppm	1.153 ppm
IDMICRO	−1.10761 ppm	0.883 ppm
ILMICRO	−8.88897 ppm	0.441 ppm
KYLIGHT	14.55963 ppm	5.400 ppm
KYMICRO	5.10219 ppm	0.570 ppm
LALIGHT	2.31350 ppm	0.449 ppm

TABLE 18.8.—*Special parameter values (continued)*

Unknown Identifier	Final value	Standard error
MDMICRO	0.63379 ppm	1.293 ppm
MEMICRO	−18.64216 ppm	1.486 ppm
MNLIGHT	−1.41113 ppm	0.195 ppm
MNMICRO	−1.08156 ppm	0.548 ppm
MSMICRO	11.37948 ppm	1.214 ppm
NCLIGHT	0.15428 ppm	0.217 ppm
NEMICRO	3.32683 ppm	0.484 ppm
NMLIGHT	−1.43044 ppm	0.312 ppm
NMMICRO	3.53219 ppm	0.420 ppm
ORMICRO	9.89767 ppm	1.371 ppm
PAMICRO	5.34388 ppm	0.252 ppm
TELLUROMETER	1.54100 ppm	0.080 ppm
TNMICRO	2.99565 ppm	0.287 ppm
VAMICRO	1.54079 ppm	0.281 ppm
XCDLIGHT	−0.86389 ppm	0.097 ppm
YCDMICRO	2.33160 ppm	0.095 ppm
VLBI *X* ROTATION	0.022 second	0.006 second
VLBI *Y* ROTATION	0.026 second	0.004 second
VLBI *Z* ROTATION	0.375 second	0.044 second
VLBI SCALE	−0.07889 ppm	0.014 ppm
DOPPLER Z ROT	−0.455 second	0.043 second

The a priori transformations for Doppler positions were:

Translation *X*: .... 0.000 m
Translation *Y*: .... 0.000 m
Translation *Z*: .... 4.500 m
Scale change: .... −0.600 ppm

The height-controlled mathematical model was unique in that each space system station was associated with two heights. The first height was the elevation needed for the classical reduction of observations. The second was the up coordinate in the rectangular coordinate system of the three-dimensional space system. The shifts at each of these stations were analyzed at the conclusion of the NAD 83 adjustment. Some elevations which had been scaled from maps were given a more accurate elevation. (See table 18.9.) To use the NAD 83 coordinates in a three-dimensional mode, the corrections to all of the space system points must be taken into account.

TABLE 18.9.—*Elevation changes to space system*

Name	Old elevation (m)	New elevation (m)
CAL CO 160-A 1963 RM 8	0	27
CAL CO 160-A 1963 RM 7	0	27
CALCO 41-A	0	10
PENTHENY 1919	98	100
T 41 1955	450	460
SAT TRACK STA 104 1973	7	5
HENRY 1934	1225	1223
FLAT TOP 1934	2019	2020
RESERVE 1933	26	29
BLACK POINT 2 1933	247	246
RENFROE 1934 RM 5	151	149
MIAMI 1938	1225	1227
WINDING STAIR 1919	725	722
BUFFALO 1952	661	650
SIGNAL 1952	439	437
GASOLINE 1964	762	761

TABLE 18.9.—*Elevation changes to space system (continued)*

Name	Old elevation (m)	New elevation (m)
WINKLE 1934	1694	1699
SAN FERNANDO 1898 RM 3	1142	1139
ST ELMORE 1934	295	296
CASTRO SLOPE 1932	826	827
MOLERA 1932	22	23
POINT NO POINT 2 1934	0	2
TUCKERMAN 1934	1	0
CEDAR POINT 2 1934	2	1
THOMAS 1961	144	143
GREEN 1954	1271	1267
BCTS NO 3 1966	42	40
ACADEMY HILL 2 NYSS 1934	114	110
MOUND 1942	236	238
MCM 91 1939	276	274
FINLAND 1952	594	593
GURA 1907	5	3
JOE 1941	1	15
VOLEAST 1941	93	84
BEL 1925	0	11
NOL 1923	23	10
FORT WRANGELL N B 2 1916	1	3
GOOSE 2 1930	38	48
DRIFT 1931	186	201
NOF 2 1967	27	15
POV 1908	6	10
WIDE 1931	6	5
MINERS POINT 1908	46	57
TOPE 1929	12	18
CLEFT 1908	15	13
EDDIE 1959	1	7

TABLE 18.9.—*Elevation changes to space system (continued)*

Name	Old elevation (m)	New elevation (m)
AIRPORT 1959	1	8
BAY-COVE POINT 1907	1	9
TREE 1927	8	10
LOOK 1930	2	5
RATION	10	14
GLOBE B I E USE 1961	67	69
FAREWELL ET USGS	454	456
SAVOOGNA 1951 RM 1	51	50
LAKE 1945	2	4
GEO STA 20197 1973	9	7
NAN 1947	16	14
TESTCELL 1949	3	1
FAIR 1965	143	142

18.13 CONVERGENCE TESTS

At each iteration of the adjustment the magnitude of the corrections to station coordinates was examined. As can be seen from figure 18.19, the iteration 0 mean vector shifts and their standard deviations were high and relative accuracy varied from area to area. The mean absolute vector shift was 4.0 m with a standard deviation of 7.2. The direction of the vectors was not computed for this iteration.

The next iteration, shown in figure 18.20, produced much smaller shifts. Shifts were larger where the iter-

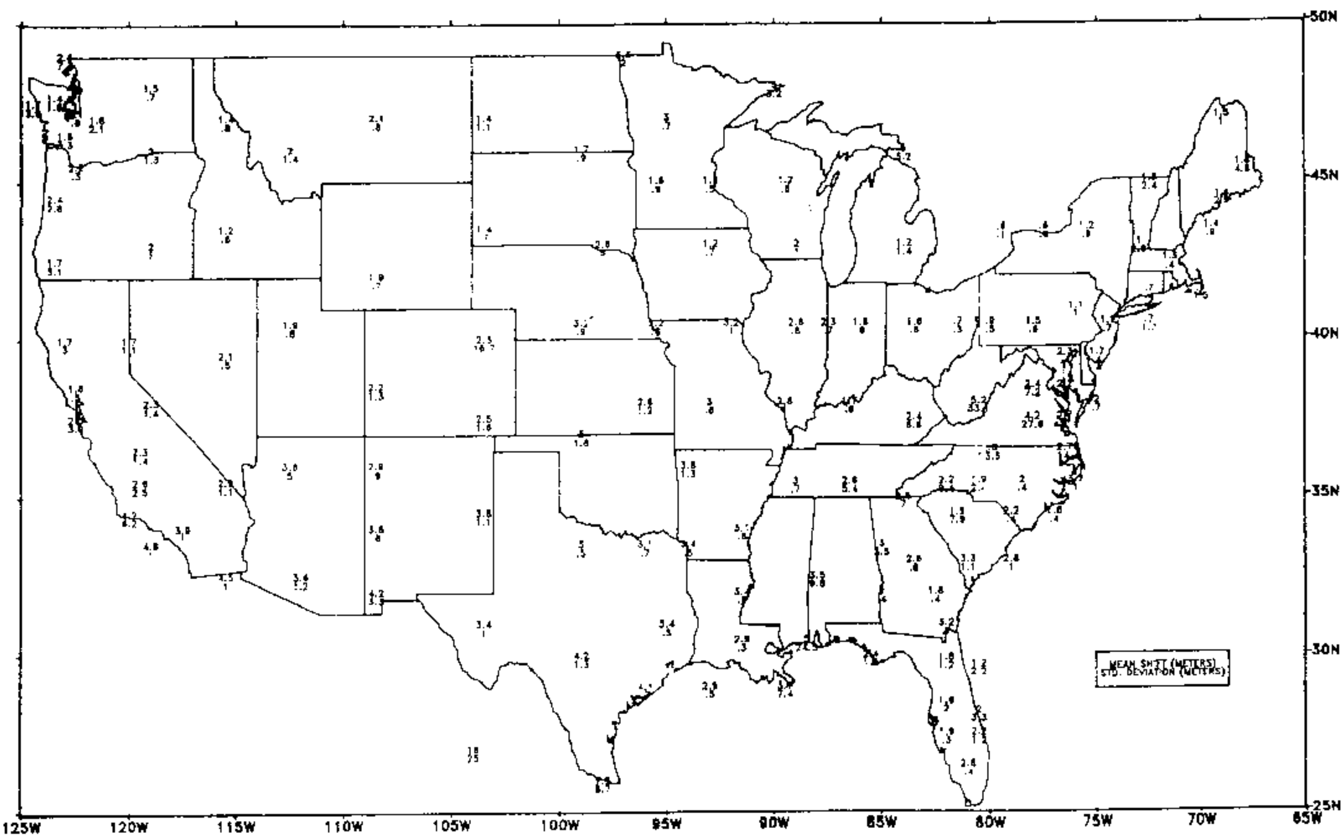

Figure 18.19. Mean shifts from iteration 0.

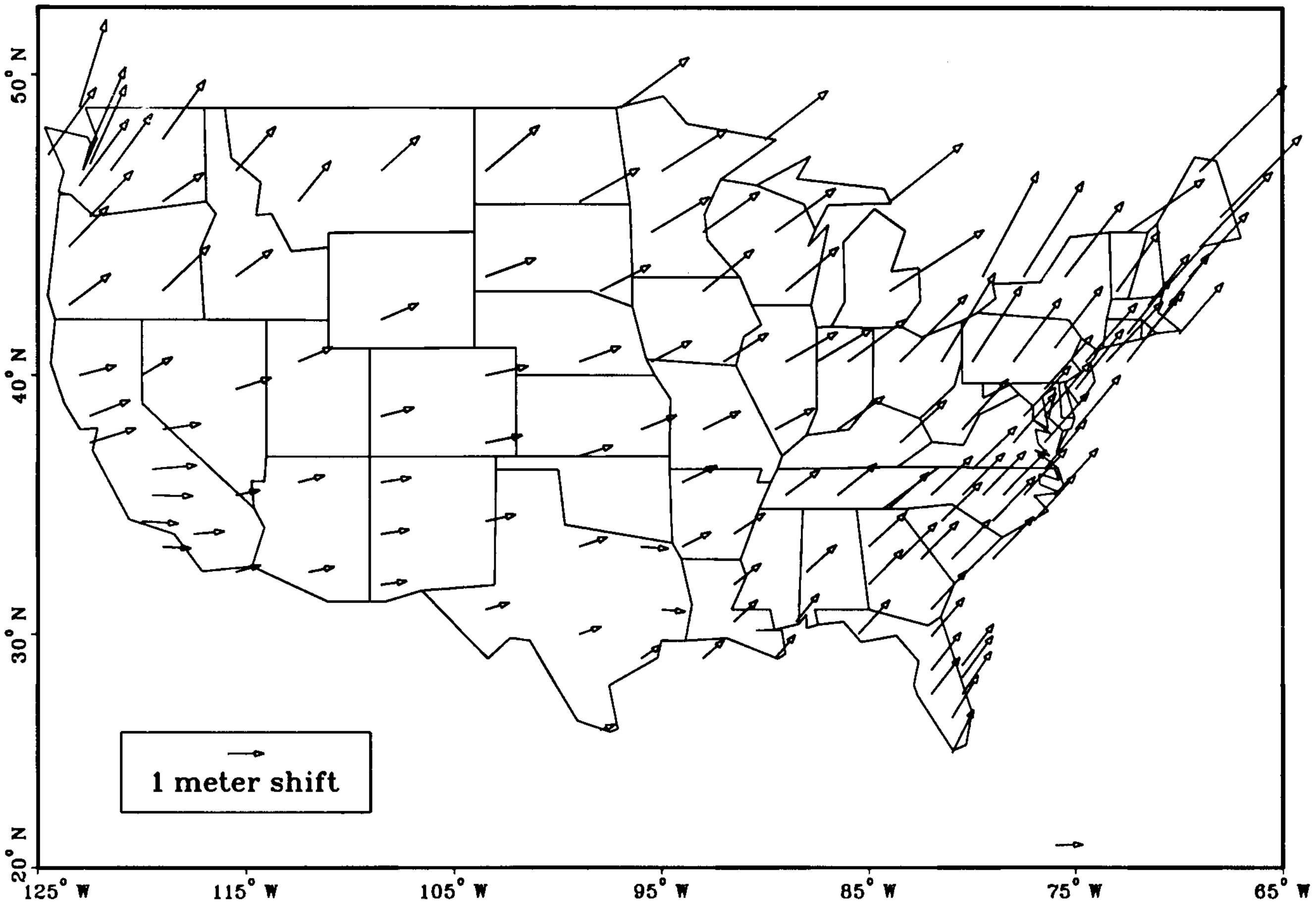

Figure 18.20. Average latitude and longitude shifts from iteration 1.

ation 0 area variance of unit weights was high, necessitating more corrections (i.e., along the east coast). Larger shifts also were computed along the U.S.-Canada boundary where positions had been constrained for iteration 0. The average vector shift was 1.78 m with a standard deviation of 1.43. The vectors were in a northeasterly direction.

A convergence criterion became necessary. The adopted definition stated that all relative shifts in the primary network should be 1:100,000 or smaller. Any other large shifts would be analyzed in accordance with the accuracy of that part of the network.

The criteria for each large shift included: 1) nature of the station, 2) shift relative to nearby stations, 3) whether the relative shift was larger than the predicted uncertainty for the observing technique(s) and geometry, and 4) whether the shift was a drift or an oscillation.

When the shifts for iteration 1 were analyzed, the average shifts per block indicated that the criterion of 1:100,000 had been met. However, relative shifts between stations failed to meet stated criteria in several areas. For example, figure 18.21 depicts an area in New York along the U.S.-Canada boundary showing relative shifts of 1:10,000, 1:32,000, 1:70,000, and 1:100,000. There were quite a few areas along the U.S.-Canada boundary like this. Figure 18.22 shows an area in Arkansas that had unacceptably large relative shifts between close stations. These and other similar situations appeared in sufficient quantity to warrant another iteration.

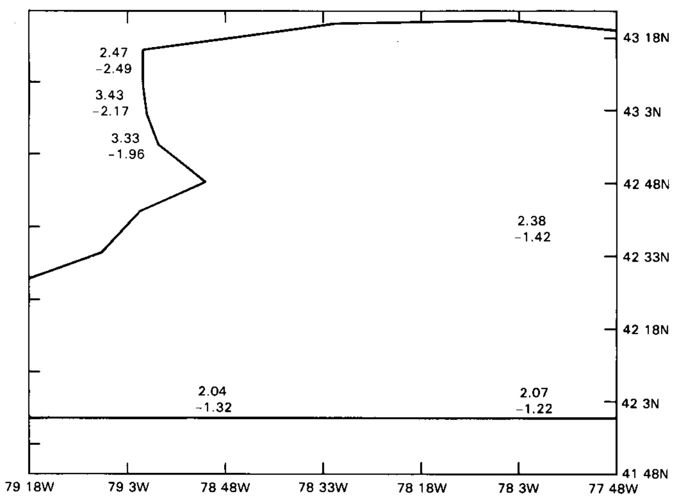

Figure 18.21. Block 178. Individual station shifts in meters from iteration 1.

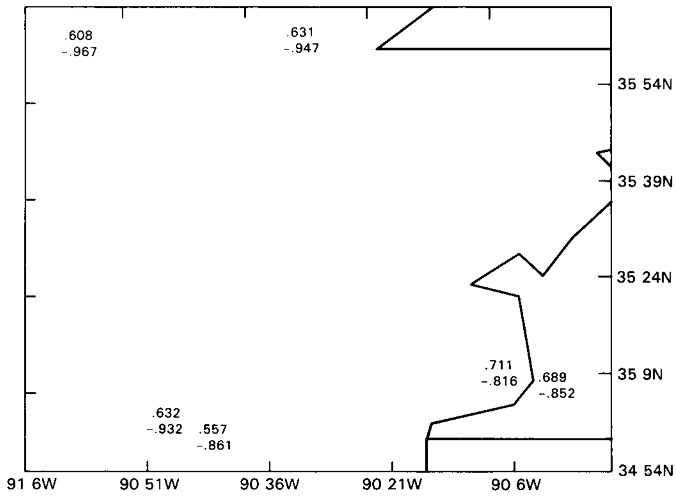

Figure 18.22. Block 43. Individual station shifts in meters from iteration 1.

The last iteration for NAD 83 reduced the absolute position shifts to the sub-decimeter level. (See fig. 18.23.) The average vector shift was 0.08 m with a standard deviation of 0.07. The analysis of individual station position shifts had become acceptable to the criteria for the order and nature of the stations and the geometry of the network.

18.14 REFERENCES

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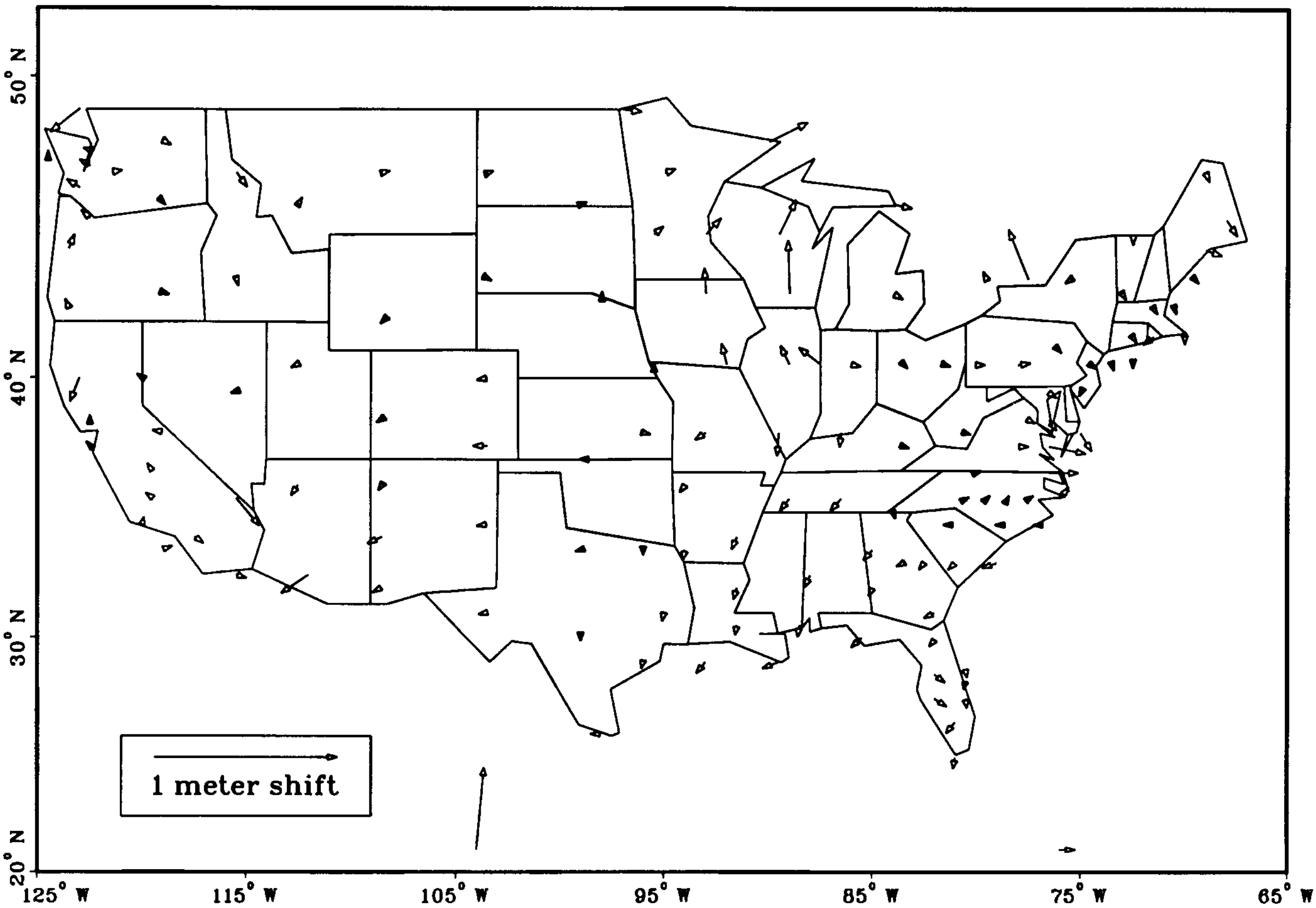

Figure 18.23. Average latitude and longitude shifts from iteration 1.

19. ACCURACY ANALYSIS

Richard A. Snay

19.1 INTRODUCTION

The North American geodetic community undertook the NAD 83 project to provide a more accurate horizontal reference system for supporting modern surveying and mapping activities. With the help of high-powered computers and sophisticated computational techniques, new horizontal coordinates were rigorously determined for more than 250,000 geodetic stations. The accuracy characterizing these new coordinates constitutes the topic of this analysis.

Just as position may be considered in either the absolute or the relative sense, so also can positional accuracy be considered. The term *absolute positional accuracy* is used to characterize the error in the latitude and longitude coordinates of a station relative to certain defined parameters of the reference system. It is important to realize that absolute positional accuracy is still relative; it is only meaningful within the context of the given reference system. On the other hand, *relative positional accuracy* is used to characterize the error in the coordinates of one station relative to the coordinates of another station. Relative positional accuracy is conventionally expressed in terms of distance accuracy and orientation accuracy.

Geodetic stations may be categorized into control points and landmarks. A control point is an accurately positioned station whose ground location is identified with special monumentation (often a brass marker). Landmarks include such structures as radio towers, church steeples, and water tanks. Not all stations in the geodetic reference system are positioned with the same accuracy. To differentiate among accuracy levels, NGS has assigned each control point an *order* in accordance with accuracy standards established by the Federal Geodetic Control Committee (FGCC, 1984). Standards exist for first-, second-, and third-order control points. Of these, first-order control points have the greatest accuracy, and third order, the least. Landmarks are usually positioned to less than third-order accuracy. Identified here as fourth-order stations, landmarks have been positioned for the convenience of surveyors in orienting low-order surveys.

In this chapter, positional accuracy is explored from four perspectives. First, NAD 83 coordinates are compared with coordinates derived from recent Global Positioning System (GPS) surveys. This comparison has led to the formulation of empirical rules that suitably quantify distance accuracy and orientation accuracy when interstation distances range between 10 km and 100 km. For example, for first-order stations in the 48 conterminous states, the empirical rule for distance accuracy was found to be

$$e = 0.008\ K^{0.7}. \qquad (19.1)$$

Here the root mean square (rms) error in distance (e, measured in meters) is characterized as a function of interstation distance (K, measured in kilometers). Second, the residuals of the various observations as obtained from the NAD 83 adjustment are examined. These residuals identify some local and regional problems with NAD 83 coordinates. The residuals also indicate that third-order observations in coastal areas may have been overweighted. Third, the covariance matrix of the adjusted coordinates is analyzed as it pertains to Alaskan stations. Covariance matrix elements have yet to be computed for the remainder of the United States. This covariance analysis reveals that absolute positional accuracies are similar in magnitude for first-, second-, and third-order stations, and that relative positional accuracies for Alaska are demonstrably poorer than those for the 48 conterminous United States. Fourth, various error sources are investigated. In particular, those errors associated with the adopted values for deflections of the vertical, for crustal movements, and for station heights are considered. These adopted values were held fixed in the NAD 83 adjustment. Also the error associated with leveling a theodolite (the instrument used for measuring directions and azimuths) is considered. Section 19.9 summarizes the analysis.

19.2 NAD 83 VERSUS GPS

During the mid-1980s, NGS adopted GPS technology to position new geodetic stations relative to existing stations. By 1988, NGS had added more than 30 GPS surveys. (See fig. 19.1.) Only one of these GPS surveys, however, was performed soon enough to include its observations in the NAD 83 adjustment. The observations from the remaining GPS surveys, therefore, provide an independent standard for gauging the relative positional accuracy of NAD 83 coordinates. These GPS observations have an accuracy of about one part-per-million (ppm) and, hence, provide an excellent standard.

For each of several selected GPS surveys, a minimally constrained adjustment of the corresponding data was performed by holding fixed the NAD 83 coordinates of one previously existing station in the survey. Also, the ellipsoidal height of this station was held fixed to an adopted value. The adjustment produced three-dimensional coordinates (latitude, longitude, and ellipsoidal height) for all stations in the GPS survey. For the pre-existing stations, the differences between their GPS-derived horizontal coordinates and their NAD 83 coordinates were then plotted as vectors onto maps, one map for each GPS survey. Figure 19.2 shows a sample of four such maps. In each map, a star identifies the "fixed" station. The circles at other stations represent tolerances for the plotted vectors. These tolerances correspond in value to Federal Geodetic Control Committee (1984) standards for distance accuracies. For a first-order station, the circle's radius equals 1:100,000 of the distance from the station to the fixed station. For second- and third-order stations, the circle's radius corresponds to 1:50,000 and 1:10,000, respectively. The maps in figure 19.2 illustrate that differences between GPS and NAD 83 coordinates for relative position are significantly less than FGCC standards most of the time.

The agreement between GPS and NAD 83 was explored further to quantify relative positional accuracy for the NAD 83 coordinates. A horizontal difference vector between GPS-derived coordinates and NAD 83 coordinates, such as the vectors depicted in figure 19.2, was computed for each pair of NAD stations in each GPS survey, even for pairs that do not include the fixed station. The *collinear* component of such a vector (the component parallel to the line connecting the two stations) quantifies the distance error over the interstation line. The *transverse* component of the vector (the component perpendicular to the line) quantifies the orientation error. For this sample of vectors, both components tend to increase in size as a function of interstation distance. The relation is approximated with the equation

$$e = aK^b. \tag{19.2}$$

Here e denotes the rms value of the vector component in meters, and K denotes interstation distance in kilometers. The parameters a and b are quantities whose values depend upon whether e refers to the collinear or the transverse component and also upon line classification. (A line is assigned the order of the least accurate station connected by it.) Table 19.1 lists suitable values for a and b for first-, second-, and third-order lines. The graphs in figure 19.3 illustrate how the rms errors predicted with eq. 19.2 compare with the actual rms errors obtained from the available sample of horizontal difference vectors. The a and b values were chosen so that predicted rms errors generally exceed actual rms errors. Also in selecting a and b, greater emphasis was placed in matching rms errors when interstation distances range between 10 and 100 km. The form of eq. 19.2 and the values for a and b were chosen mainly for empirical reasons as opposed to theoretical reasons. These choices, however, were influenced by the report, *North American Datum*, prepared by the National Academy of Sciences/National Academy of Engineering (1971: p. 24).

TABLE 19.1.—*Values for parameters in eq. 19.2*

Line order	Collinear component	Transverse component
First	$a = 0.008$ $b = 0.7$	$a = 0.020$ $b = 0.5$
Second	$a = 0.010$ $b = 0.7$	$a = 0.025$ $b = 0.5$
Third	$a = 0.010$ $b = 0.7$	$a = 0.030$ $b = 0.5$

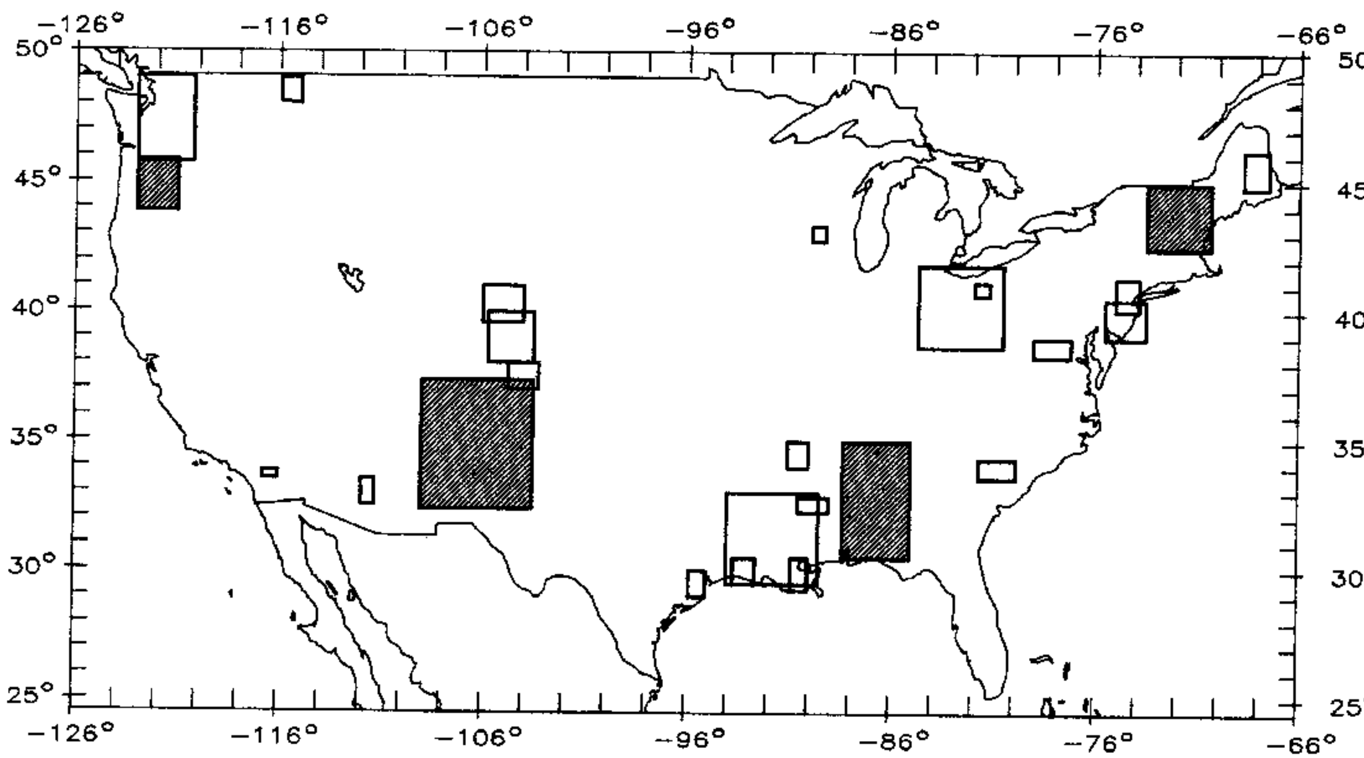

Figure 19.1. Rectangles locate the GPS surveys used to evaluate NAD 83 accuracy. The shaded rectangles locate the GPS surveys for which vector differences are plotted in figure 19.2.

An expression in the form of eq. 19.2 was also employed by Simmons (1950) to characterize distance accuracies for first-order lines in the NAD 27 reference system. Simmons' values for a (= 0.059) and b (= 0.667) were determined empirically to approximate the misclosures of several large loops in the then-existing first-order geodetic network. According to Joseph F. Dracup, former chief of the NGS Horizontal Network Branch (personal communication, 1989), Simmons used the formula to represent the 2-sigma error level. The values in table 19.1, on the other hand, represent the rms error between GPS and NAD 83, and hence these tabulated values approximate the 1-sigma level. Taking this difference into account, Simmons' values yield error estimates more than 3 times greater than the values given in table 19.1 for NAD 83 first-order distances (a = 0.008, b = 0.7). This result reflects a large improvement in the relative accuracies for the new NAD 83 coordinates. The improved relative accuracies for NAD 83 may be attributed to several reasons. Electronic distance measurement technology became operational in the mid-1950s, and as a result, more than 80 percent of all distance measurements have been observed since this time. About 75 percent of the 4,470 astronomic azimuth observations have been performed since 1960. The highly precise (~1 ppm) Transcontinental Traverse surveys were conducted in the 1960s and 1970s to provide a new framework for the National Geodetic Reference System. Space-related technologies for geodesy became operational in the 1970s, and more than 600 Doppler positions and over 100 Very Long Baseline Interferometry (VLBI) baselines have since been established in the United States. Finally, in using the best available predictions for deflections, geoid heights, and crustal movements, the NAD 83 coordinates were obtained with greater scientific rigor.

It is important to realize that eq. 19.2 provides only a statistical measure of quality. Deviations from eq. 19.2 due to local and regional conditions may be expected. Short lines (K < 10 km) exhibit large deviations because their accuracies depend highly on local network geometry and on their relative closeness to measured distances and azimuths. Moreover, the GPS versus NAD 83 comparisons, upon which eq. 19.2 is based, contained relatively few lines under 10 km in length. Consequently, the user should not rely on eq. 19.2 for characterizing the accuracy of such short lines. Also, as to be discussed in section 19.5, eq. 19.2 should not be used for Alaska, where the network geometry is considerably weaker than that found in the 48 conterminous states.

According to eq. 19.2, relative positional error grows nonlinearly as a function of K. In particular, b < 1.0. For simplicity, however, relative positional error has often been expressed as a linear function of K. This simplification corresponds to the assumption that b = 1.0, and it enables relative positional error to be expressed as a ratio not depending on K, for example, 1:100,000. Figure 19.4 illustrates the inadequacy of using linear relations. Because of its nonlinear dependence on K, relative positional accuracy cannot be rigorously quantified without specifying K. Moreover, although eq. 19.2 represents relative positional accuracy better than linear relations, even this expression becomes inaccurate for representing accuracy at large interstation distances (K > 100 km). That is, eq. 19.2 inaccurately predicts that relative positional error will continually grow as interstation distance increases. In actuality, positional error (both absolute and relative) is bounded in size due to the presence of Doppler and VLBI observations.

Figure 19.4 also demonstrates that, for first-order lines exceeding 10 km in length, the rms collinear error is less than 4 ppm (1:250,000). Consequently, based on statistical considerations, the collinear component of the difference between GPS and NAD 83 coordinates for these lines should meet first-order FGCC standards (1:100,000) about 99 percent of the time. Table 19.2 shows that this actually is the case for the available sample of vector differences. Moreover, table 19.2 gives appropriate statistics for both the collinear and the transverse components and for first-, second-, and third-order lines. A surprising result is that the accuracies of second- and third-order lines in the sample also exceed the first-order FGCC standard a large percentage of the time! Thus the accuracies of these second- and third-order lines greatly exceed the second- and third-order FGCC standards (1:50,000 and 1:10,000, respectively). Such good accuracy is not necessarily intrinsic to the quality of third-order observations, but it is obtained by integrating these observations into a network whose framework is based on more accurate measurements. Consequently, the good results given in table 19.2 do not correspond to all third-order lines. In particular, these results generally do not correspond to third-order lines under 10 km in length.

TABLE 19.2.—*Distribution for vector differences between GPS and NAD 83 coordinates*

Line order	Line length (km)	Sample size	Magnitude of collinear component			Magnitude of transverse component		
			0-5 ppm (percent)	5-10 ppm (percent)	10+ ppm (percent)	0-5 ppm (percent)	5-10 ppm (percent)	10+ ppm (percent)
First	5 - 50	125	84.2	14.4	1.4	78.3	16.7	5.0
Second	5 - 50	167	81.4	13.8	4.8	73.7	19.8	6.6
Third	5 - 50	98	64.4	26.0	10.5	71.4	18.7	9.9
First	50 - 500	357	99.1	0.9	0.0	98.4	1.6	0.0
Second	50 - 500	655	97.3	2.2	0.4	98.3	1.0	0.7
Third	50 - 500	276	94.9	5.1	0.0	95.7	4.3	0.0

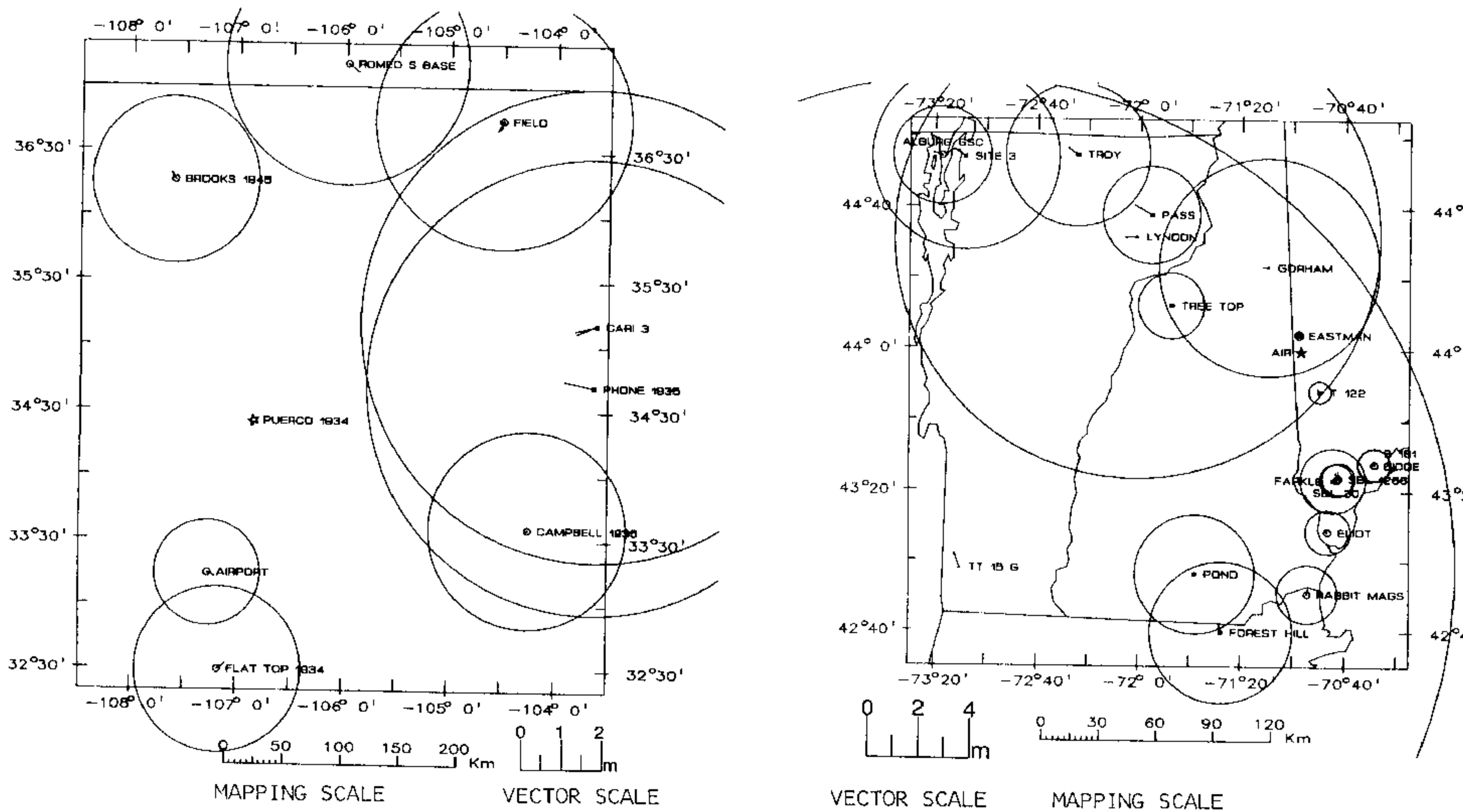

Figure 19.2. Vectors represent GPS minus NAD 83 coordinates. These vectors are relative to the "fixed" station identified with a star. Circles represent tolerances for the vectors. A circle's radius corresponds to 1:100,000 at first-order stations (small circles), 1:50,000 at second-order stations (rectangles), and 1:10,000 at third-order stations (triangles).

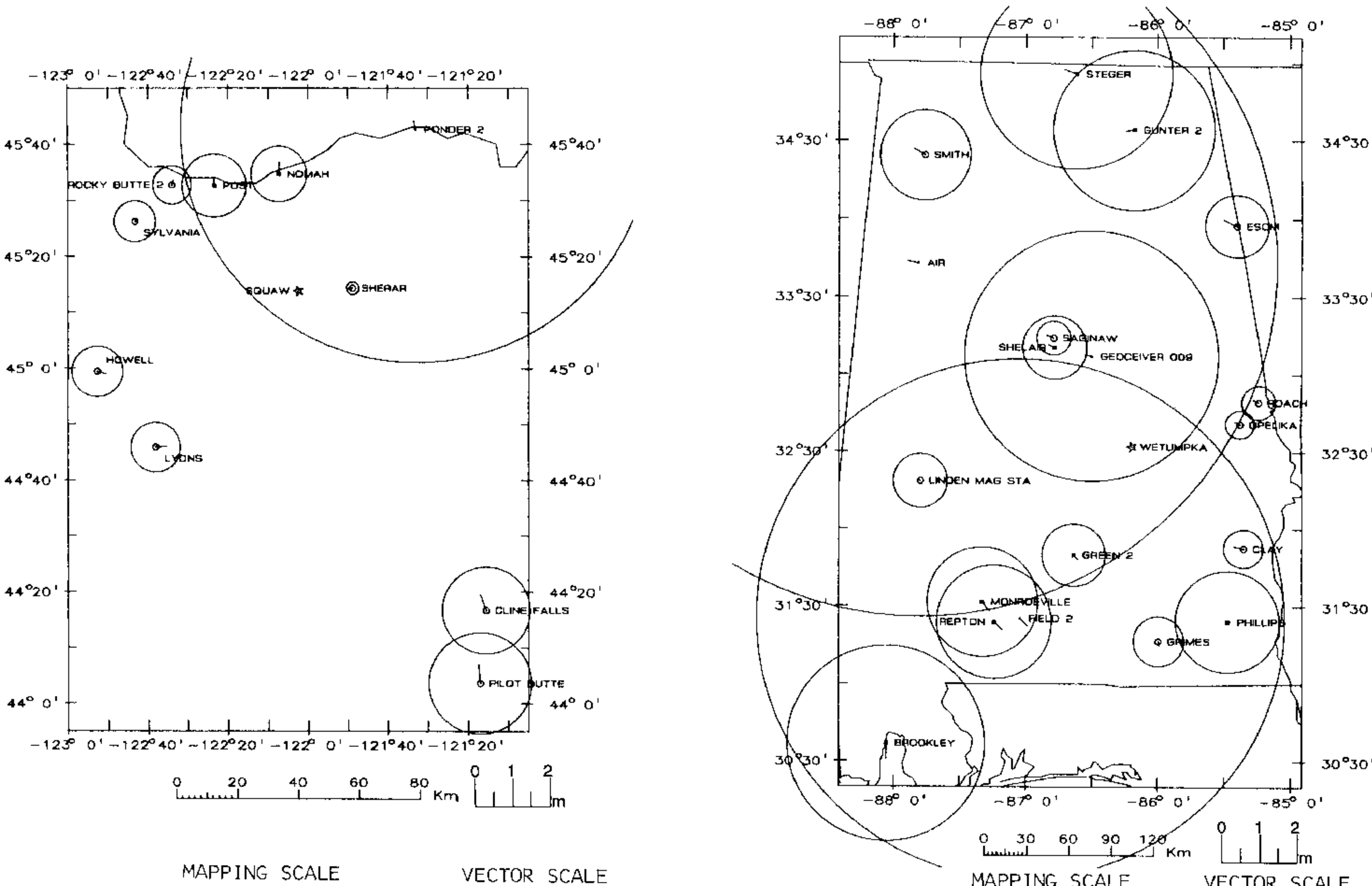

Figure 19.2. Vectors represent GPS minus NAD 83 coordinates. These vectors are relative to the "fixed" station identified with a star. Circles represent tolerances for the vectors. A circle's radius corresponds to 1:100,000 at first-order stations (small circles), 1:50,000 at second-order stations (rectangles), and 1:10,000 at third-order stations (triangles) (continued).

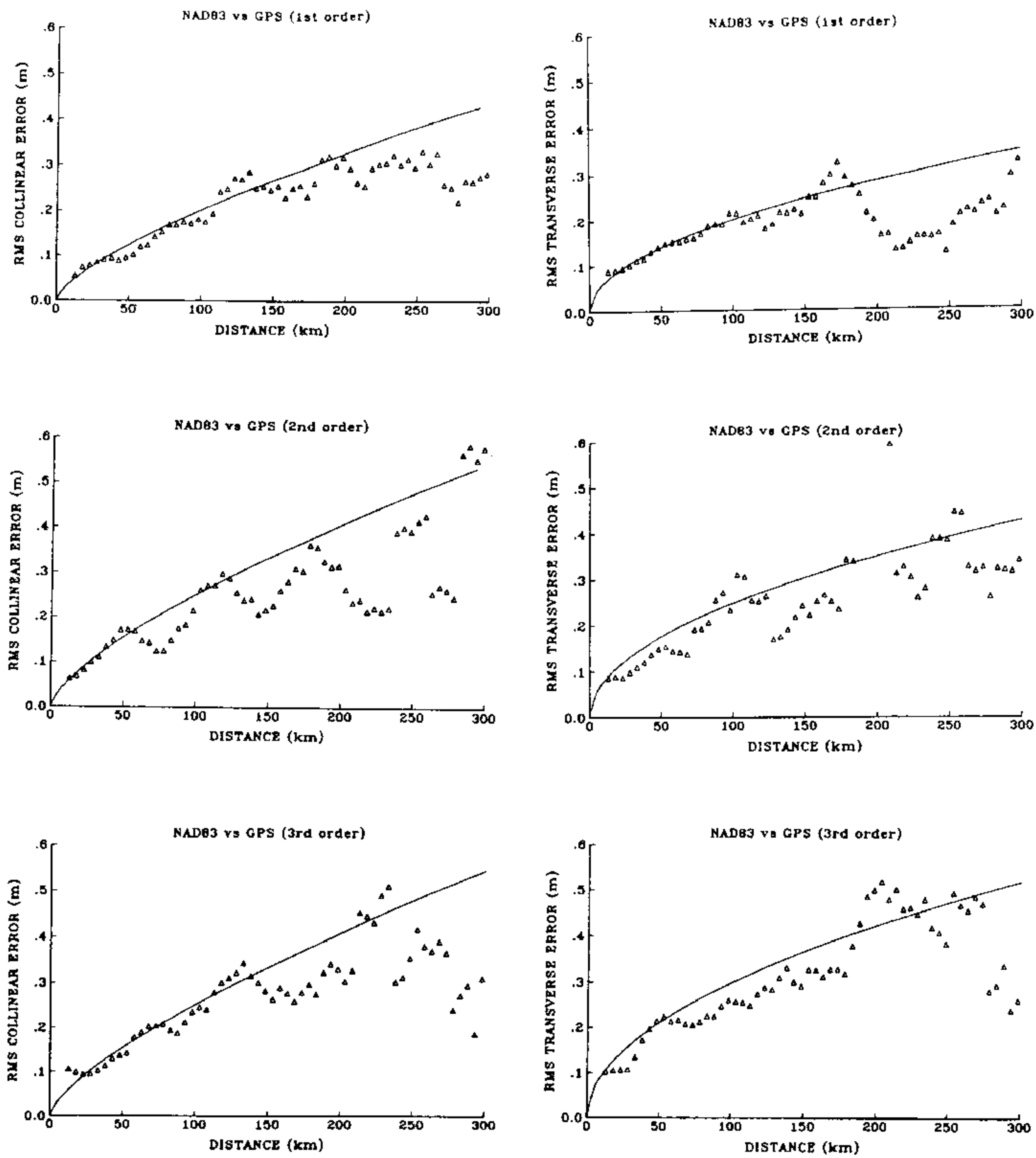

Figure 19.3. Components of the GPS-NAD 83 difference vectors plotted as a function of interstation distance. Each triangle represents the rms value computed from all vectors whose corresponding interstation distances fall within a 25-kilometer window centered on the triangle. Curves represent rms values as given by eq. 19.2.

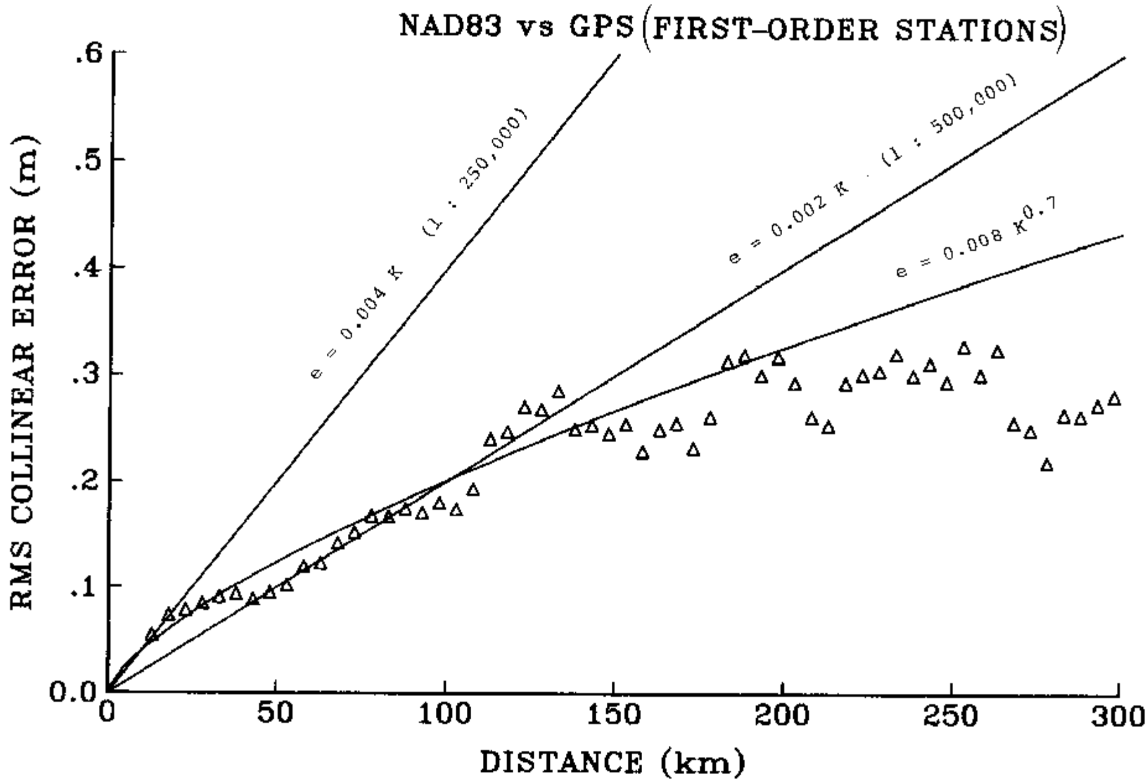

Figure 19.4. Collinear component of the difference vector plotted as in figure 19.3 but with additional curves to represent expressions for which error depends linearly on interstation distance.

19.3 DOPPLER RESIDUALS

For NAD 83 the U.S. data set contained 1,541,090 direction observations, 188,629 distance observations, 4,470 astronomic azimuth observations, 666 Doppler (point positioning) observations, 112 VLBI (relative positioning) observations, and 5 GPS (relative positioning) observations. The residuals for these observations were examined to evaluate NAD 83 quality. Doppler residuals are discussed in this section, and direction, azimuth, and distance residuals in the next section.

A Doppler observation corresponds to a measurement of three-dimensional position in the NSWC 9Z-2 geodetic reference system. The relationship between NSWC 9Z-2 and NAD 83 is defined by a seven-parameter transformation (three translations, three rotations, and a scale change). Doppler residuals were computed by subtracting the transformed observations from the adjusted NAD 83 coordinates of the corresponding stations. (For Doppler stations, all three positional coordinates were treated as unknown parameters in the NAD 83 adjustment.) Figure 19.5 shows the horizontal projections of these three-dimensional Doppler residual vectors. The existence of a few large residual vectors and of some regional trends among residual vectors indicates that some local mending of the NAD 83 coordinates may be in order. Suspiciously large residual vectors occur at stations CHILLIGAN and TOPE in Alaska and at station SELIGMAN in Arizona. Regional trends occur in northern California (eastward trend), in southwestern Colorado (southwestward trend), along the Gulf Coast (northward trend), and in northern Wisconsin (southward trend). Rectifying these problems may not be as straightforward as discarding a few suspicious observations. The situation may need careful analysis. New observations should be brought to bear on the problem whenever possible. To illustrate this point, consider the situation at station CHILLIGAN where the residual vector is oriented oppositely to the residual vectors at neighboring stations. One possibility is that the CHILLIGAN observation is simply a blunder that should be discarded. Another and stronger possibility, however, is that this large residual vector is caused by a faulty model for the ground motion associated with the 1964 Prince William Sound earthquake. Snay et al. (1987: fig. E.1) comment that the modeled movements in this area are highly suspect because the postearthquake survey has poor network geometry. Because the prequake surveys have adequate network geometry, the crustal motion model could be assessed (and rectified, if necessary) with a new GPS survey in the area.

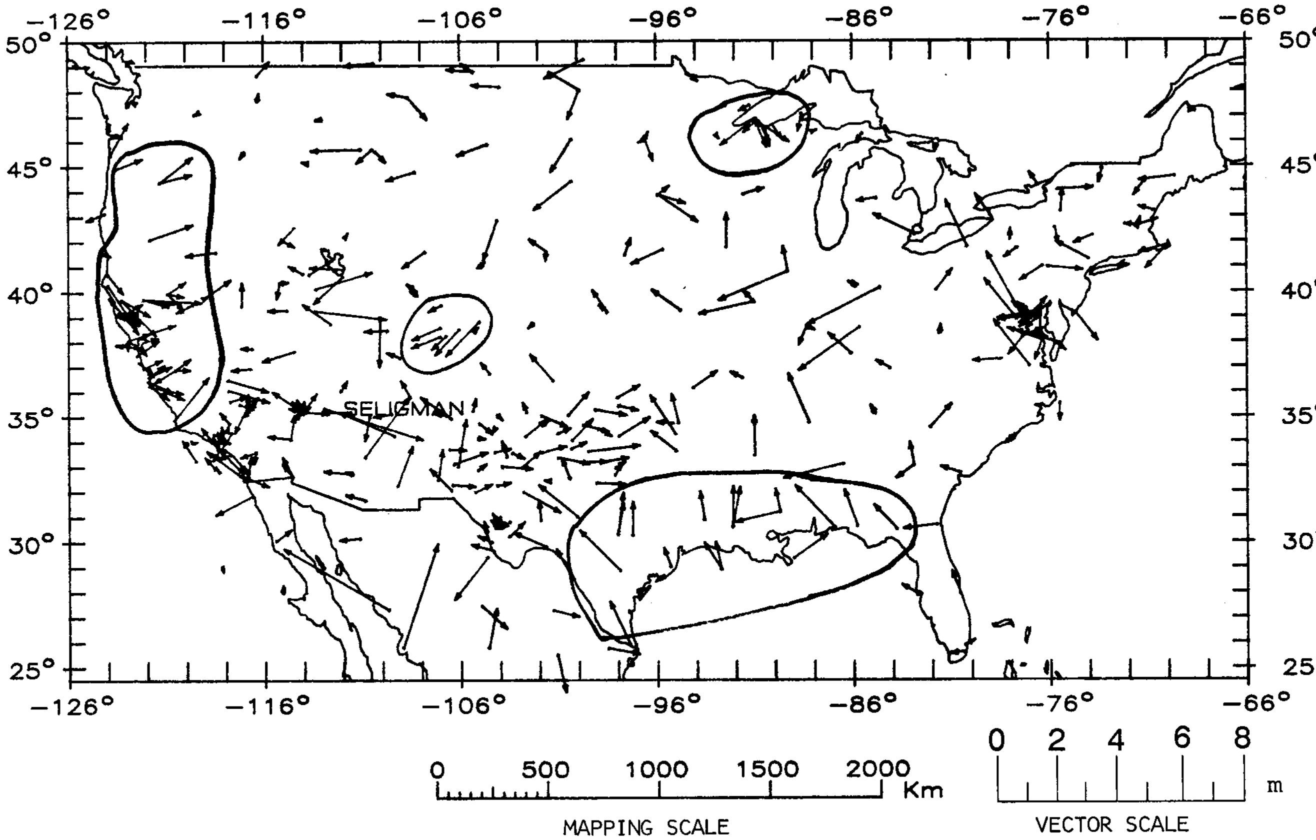

Figure 19.5A. Horizontal projections of Doppler residual vectors (NAD 83 coordinates minus transformed Doppler observations) for conterminous United States. The loops encircle areas where regional trends occur among the residuals.

Although figure 19.5 indicates that a few local problems may exist, NAD 83 coordinates agree well with Doppler observations for the most part. The north-south components of Doppler residual vectors have an rms value of 0.591 m, and the east-west components have an rms value of 0.744 m. For Doppler observations, a priori standard errors were assigned by individual components when the vector is expressed in the local horizon reference system; that is, a standard error was assigned for the north-south component, one for the east-west component, and one for the vertical component. The assigned values for these standard errors depended primarily on the number of satellite passes that were tracked during the observing session. Covariances or correlations between components were assumed to equal zero. Each component of the Doppler residual vector was subsequently divided by the a priori standard error assigned to that component of the observation, thus producing a "normalized" residual vector. (A statistic is customarily normalized by dividing its value by the value of its standard error. For this analysis, however, the residual is divided by the a priori standard error of the observation, and not by the standard error of the residual.) The rms values of these normalized residual vectors were then computed, component by component, except for the vertical component for which residuals were mostly zero. For the north-south component, the rms value equals 1.32, and for the east-west component, 1.40. Before discussing the significance of these unitless quantities, it is instructive to consider the relationship between the rms value and another statistical measure of data quality.

Let A represent a subset of the observations involved in an adjustment. The rms of the normalized residuals of A is computed by the formula

$$\mathrm{R_A} = \left[\sum_{i=1}^{n_A} r_i^2/n_A\right]^{0.5} \qquad (19.3)$$

where n_A denotes the number of observations in A and r_i (for $i = 1,2,\ldots,n_A$) denotes a normalized residual.

Consider now the statistic

$$\mathrm{S_A} = \left[\sum_{i=1}^{n_A} r_i^2/q_A\right]^{0.5} \qquad (19.4)$$

where q_A measures the number of redundant observations in A. If the observations in A are mutually independent then q_A may be computed by the formula

$$q_A = \sum_{i=1}^{n_A} (\sigma_{v_i}/\sigma_{b_i})^2 \qquad (19.5)$$

where σ_{v_i} denotes the standard error of the i-th residual and σ_{b_i} denotes the a priori standard error of the i-th observation. [See Milbert (1985) for computing q_A

when the observations in A are correlated.] If the a priori variances of the observations in A differ from their true variances by a common factor, then Horn et al. (1975) show that S_A^2 provides an "Almost Unbiased Estimate" of this variance factor. Indeed, S_A^2 provides an unbiased estimate of this variance factor if (1) observational errors have a Gaussian distribution with zero mean, (2) correct a priori standard errors were assigned to all observations, and (3) the mathematical nature of the observations was properly parameterized for the adjustment. Under these assumptions S_A^2 has an expected value of 1.00. Consequently, if S_A deviates significantly from 1.00, then one may suspect the existence of blunders, incorrectly assigned standard errors, or systematic errors (for example, refraction and crustal motion). However, S_A is usually expensive to compute, whereby R_A is computed as an economical substitute. Because

$$R_A = S_A \, (q_A/n_A)^{0.5} \tag{19.6}$$

and because $q_A < n_A$, it follows that R_A has an expected value less than 1.00. Hence, such problems may still be suspected if R_A is significantly greater than 1.00. More specifically, a problem may be suspected (at the 0.01 significance level) for the 666 Doppler observations if $R_A > 1.08$. (For this computation, it was assumed that the distribution for S_A^2 could be adequately approximated by a chi-squared-over-degrees-of-freedom distribution with 500 degrees of freedom.) R_A exceeds this critical value for both horizontal components: $R_A = 1.32$ for the north-south component and $R_A = 1.40$ for the east-west component. Figure 19.5 indicates that these high rms values are partially caused by observational blunders and/or systematic errors. Overly optimistic a priori standard errors may also have been assigned to the Doppler observations.

19.4 DIRECTION, AZIMUTH, AND DISTANCE RESIDUALS

For each of the 161 first-level Helmert blocks, a sample of first-order direction observations was selected and the rms of the normalized residuals was then computed for this sample. Here, as with the Doppler observations, a residual is normalized by dividing it by the a priori standard error of the cor-

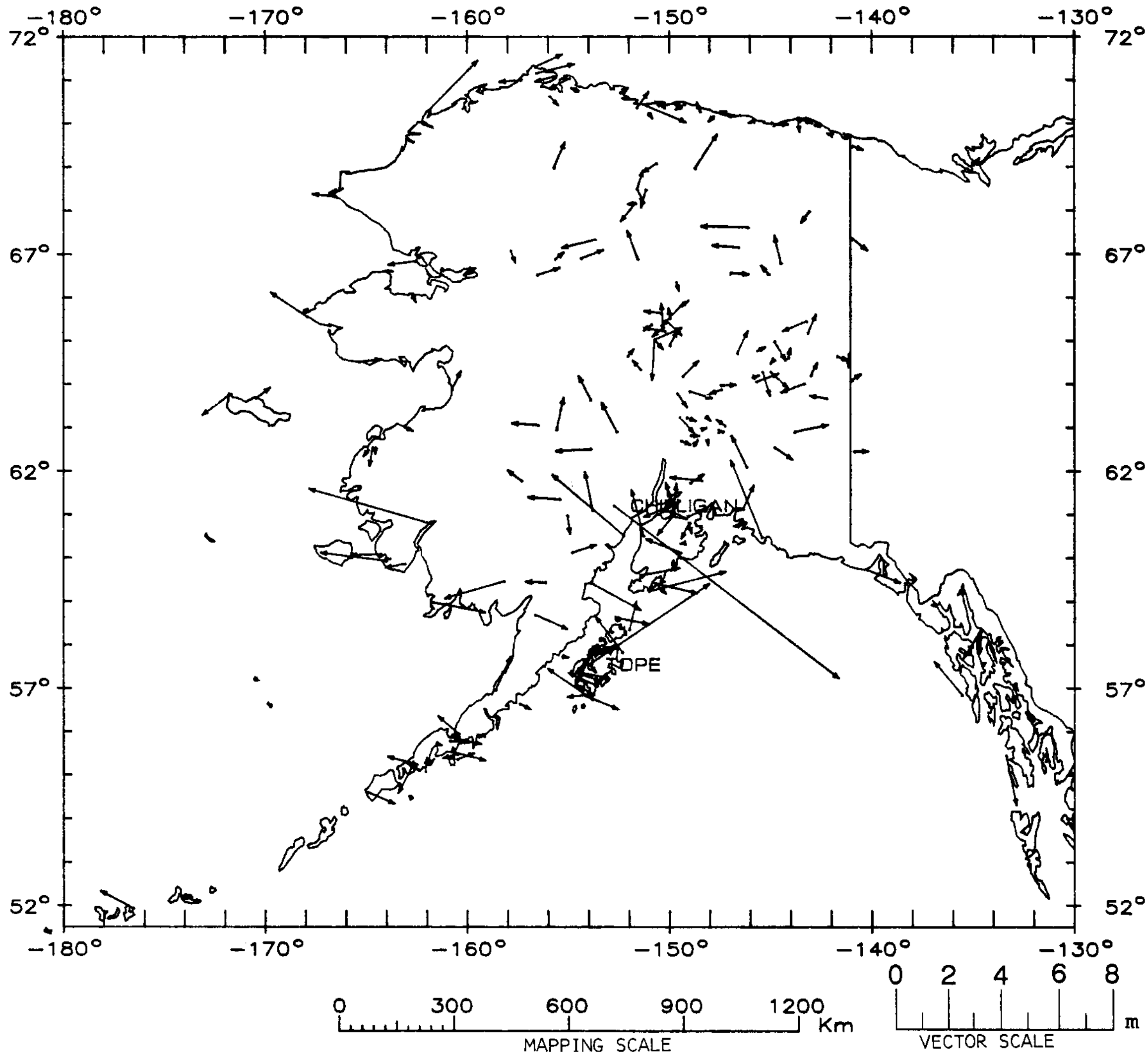

Figure 19.5B. Horizontal projections of Doppler residual vectors (NAD 83 coordinates minus transformed Doppler observations) for Alaska.

responding observation. Figure 19.6A summarizes results of these rms computations. Similarly, samples were selected for second-order directions, third-order directions, fourth-order directions (directions to landmarks), first-order astronomic azimuths, taped distances, lightwave distances, and microwave distances. Figures 19.6B through 19.6H summarize the results of the rms computations for these samples. The rms normalized residual for each data type was also computed from samples representing the complete U.S. data set. Table 19.3 lists these rms values.

TABLE 19.3.—*The rms values for normalized residuals from samples representing complete U.S. data set*

Observational type	Sample size	a priori std. error[1]	rms
Doppler (north-south)	624	0.46 meter	1.32
Doppler (east-west)	624	0.58 meter	1.40
First-order direction	186,912	0.6 arc second	0.97
Second-order direction	199,143	0.7 arc second	0.91
Third-order direction	122,247	1.2 arc seconds	0.97
Fourth-order direction	126,933	3.0 arc seconds	0.79
First-order azimuth	2,118	1.4 arc seconds	1.02
Taped distances	10,209	10 mm + 1.0 ppm	0.77
Lightwave distances	33,617	15 mm + 1.0 ppm	0.57
Microwave distances	2,870	30 mm + 3.0 ppm	1.09
Combined	685,287		0.90

[1] Nominal value

In figure 19.6A tinted areas identify Helmert blocks whose sample contains at least 200 first-order direction observations. Warmer colors indicate areas where the higher rms values occur. In analyzing figure 19.6A, not much significance should be placed on the rms values of individual Helmert blocks as they may correspond to statistical anomalies. Instead the reader should look for regional trends. One such trend is that rms values are relatively high along the eastern front of the Rocky Mountains. Indeed, the rms normalized residual for first-order directions in this region is 8.7 percent higher than the rms normalized residual for all first-order directions. In mountainous areas such as this, direction residuals are more sensitive to the errors in the deflection of the vertical and errors in leveling the theodolite. The effect of both error sources grows in proportion to the slope of the observed line. The higher residuals found in the mountains indicate that the standard errors assigned to direction observations should depend on line slope. Such was not the case for the NAD 83 adjustment. Deflections and theodolite tilt as error sources are discussed further in section 19.6.

A second trend exhibited in figure 19.6A is that rms values are relatively high along the Pacific coast. The mountainous terrain near this coast might in itself explain those high rms values, but the Pacific coast is also the region of greatest horizontal crustal movements in the conterminous United States. Consequently, crustal motion has to be considered as a supplemental cause for these high rms values. The motion in California was modeled (chapter 17), and using this model the geodetic observations were "temporally homogenized"; that is, observed values were replaced by values that the model predicts as if the observations had been performed on December 31, 1983. The relatively high rms values, therefore, manifest the error level in these modified observations. The problem here is not necessarily that predicted crustal movements are poor in quality, but that the standard errors of the revised observations did not reflect the uncertainties associated with the crustal motion models. Crustal motion as an error source is discussed further in section 19.7.

A third trend exhibited in figure 19.6A is that rms values are relatively high in the vicinity of New York City. This area is not especially mountainous, nor is it significantly deformed by recent crustal movements. [The latter, however, is the topic of some controversy between Zoback et al. (1985) and Snay (1986).] One can only speculate as to the cause of the New York trend. The cause may be that the observations for one or more surveys in this area were weighted incorrectly. Another possibility is that the New York trend may be an artifact of using the rms value (R_A) as opposed to the statistic S_A of eq. 19.4 for intercomparing residuals among Helmert blocks. According to eq. 19.6, the two statistics are related by the quantity $(q_A/n_A)^{0.5}$ which will subsequently be referred to as the redundancy factor. To the extent that this redundancy factor remains uniform from sample to sample, then R_A would be just as effective as S_A for intercomparing samples of residuals. However, near New York City, because of a high population density and because of a relatively long geodetic history, one may expect a relatively large number of geodetic observations per mark. (The first geodetic surveys in the United States were conducted here during the early part of the 19th century.) Consequently, the redundancy factor and R_A should be higher near New York City than they are for other U.S. localities. Had the statistic S_A been used instead of R_A, then the residuals near New York City might not have seemed higher than those in other areas, but the New York residuals (and, hence, the residulas in other areas) would still be extremely high. This result follows because S_A is always greater than R_A and because $R_A > 1.05$ for several Helmert blocks near New York City.

Figure 19.6B summarizes the rms computations for second-order direction observations. A trend for higher residuals near the eastern front of the Rocky Mountains is present, but this trend is not as pronounced as that for first-order directions. The rms normalized residual for second-order directions in this region is 4.5 percent higher than the rms normalized residual for all second-order directions. Recall that for first-order normalized residuals, an 8.7 percent increase was found. This difference may be partly explained by the fact that second-order directions were assigned higher a priori standard errors than first-order directions (0.7 arc second versus 0.6 arc second). Section 19.6 shows some related computations. Nevertheless, the trend among second-order residuals further supports the case for assigning standard errors to direction observations as a function of line slope.

Figure 19.6B also exhibits a trend of relatively high rms values along the Pacific coast as well as along much of the Atlantic coast including the vicinity of New York City. Crustal movements can explain the Pacific trend. Relatively high redundancy factors can explain the trend along the densely populated Atlantic coast where the number of observations per mark is relatively high. However, the trend along both coasts may also be related to the assignment of overly optimistic standard errors to third-order directions. This possibility is discussed in the following paragraph.

Figure 19.6C summarizes the rms computations for third-order directions. The plot reveals that rms values are systematically high in coastal areas (Pacific, Atlantic, and Gulf coasts) and that they are systematically low inland. This pattern may be explained, in part, by the use of R_A. That is, the existence of relatively more observations per mark in the densely populated coastal areas produces relatively higher redundancy factors there. However, even if S_A had been used, coastal residuals would be high because, as before, S_A is always greater than R_A and because $R_A > 1.05$ for most coastal Helmert blocks. A more plausible explanation for the coastal trend is that third-order directions there are not as accurate as those inland. Most third-order surveys in coastal areas are performed in support of hydrographic charting; most third-order surveys in the interior of the country are performed in support of topographic mapping. Figure 19.6C may thus be interpreted to suggest that the surveys supporting topographic mapping are performed more accurately than those supporting hydrographic charting although both groups of observations are assigned equivalent standard errors. The possibility that third-order directions in coastal areas have been assigned overly optimistic standard errors provides a tentative explanation for the relatively high rms values found among second-order direction residuals in these same coastal areas. That is, by overweighting the third-order directions, their errors map more readily onto the residuals for the higher order directions. This effect is more manifest among second-order direction residuals, as opposed to first-order direction residuals, because third-order surveys are connected to the geodetic reference network primarily through the second-order subnetwork.

Figure 19.6D summarizes the rms computations for fourth-order directions. These rms values are relatively low compared to those for the higher order directions. Fourth-order directions sight on landmarks such as radio towers. Hence, these observations were assigned pessimistic standard errors so that their observational errors would have relatively little influence on the positional coordinates computed for the higher order stations whose locations are more precisely defined by geodetic monuments. As with second- and third-order direction residuals, the rms values for fourth-order direction residuals are relatively higher in coastal areas. It is uncertain whether this trend reflects that coastal fourth-order directions are less accurate than their inland counterpart or whether this trend is a result of overweighting the coastal third-order directions.

Figure 19.6E summarizes the rms computations for first-order astronomic azimuth observations. Unlike the plots for directions (figs. 19.6A through 19.6D) in which a tinted block represents a sample containing at least 200 observations, a tinted block in figure 19.6E represents a sample containing at least 10 observations. Because of this smaller sample size, these rms values exhibit greater block-to-block variation. In figure 19.6E, rms values are relatively higher along the Canadian border and in southern California. The trend along the Canadian border may indicate a systematic difference between network orientation as defined by these observations and network orientation as defined by Canadian observations. The adjustment, however, included a special parameter which should have accounted for such a systematic difference. Hence, other possibilities seem more likely. Roelofs (1950) theorized that the uncertainty for astronomic azimuth observations should increase with latitude. Carter et al. (1978) tested this theory and found that an increase in standard error with latitude is consistent with their test data, but that the rate of increase is negligible relative to other large systematic errors in the data, in particular, observer bias. According to an experiment described by Carter et al. (1978), observer bias can average as high as 1.5 arc seconds. Thus observer bias in itself can account for the high rms values along the Canadian border if an observer with a bad "personal equation" performed a significant percentage of the observations in that region. The high rms values in southern California are attributed to crustal motion.

The rms values were also calculated for three categories of distance observations: taped measurements, lightwave measurements, and microwave measurements. For taped distances, rms values were computed for samples containing at least 30 observations. (See fig. 19.6F.) More than half of these samples have an rms value less than 0.85. Furthermore, the rms values show no regional trends. For lightwave distances, rms values were computed for samples containing at least 50 observations. (See fig. 19.6G.) Almost all of these samples have an rms value less than 0.75. Again, these rms values exhibit no regional trends. For microwave distances, rms values were computed for samples containing at least 30 observations. (See fig. 19.6H.) Only 22 samples qualify, and rms values are on the high side but vary greatly. The rms computations for the three distance categories thus indicate that assigned standard errors for lightwave observations were too large relative to those for taped observations and that assigned standard errors for microwave distances were too small relative to those for taped distances. (There are many more lightwave measurements throughout the country than figure 19.6G indicates. The statistics for the corresponding residuals, however, were not compiled as part of the NAD 83 project.)

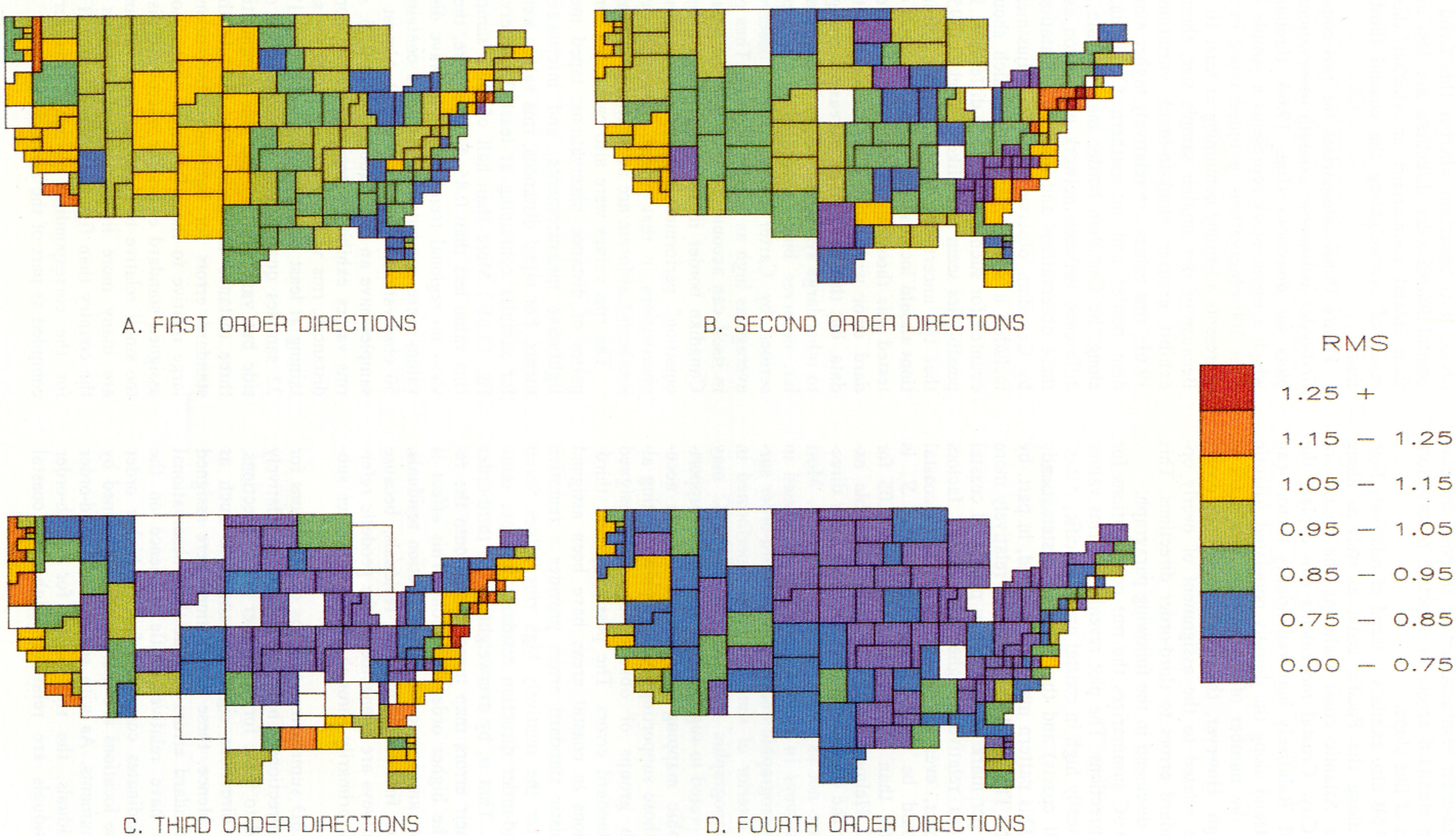

Figure 19.6. The rms values of normalized residuals for various samples of observations. Untinted areas identify Helmert blocks for which the corresponding sample contained an insufficient number of observations. For directions, the sample needed to contain 200 observations; for azimuth, 10 observations; for lightwave distances, 50 observations; and for taped and microwave distances, 30 observations.

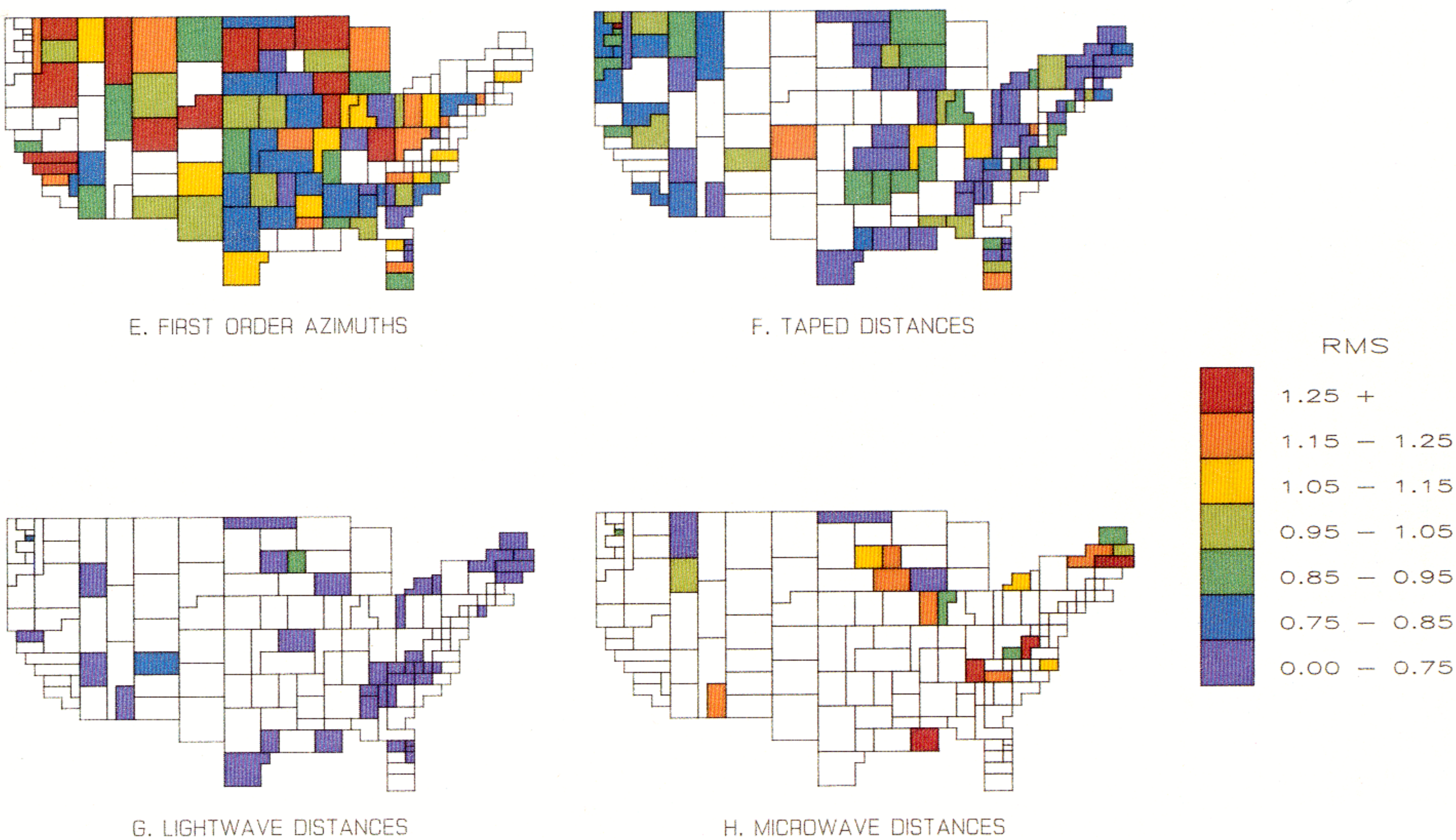

Figure 19.6. The rms values of normalized residuals for various samples of observations. Untinted areas identify Helmert blocks for which the corresponding sample contained an insufficient number of observations. For directions, the sample needed to contain 200 observations; for azimuth, 10 observations; for lightwave distances, 50 observations; and for taped and microwave distances, 30 observations (continued).

In the preceding paragraphs, rms values were explicitly or implicitly compared among different Helmert blocks and also among different data types. These comparisons prompted certain conclusions about the existence of systematic error and/or about the appropriateness of assigned standard errors. These conclusions, unfortunately, must be qualified by caveats about observational redundancy. These qualifications could have been avoided if, instead of the rms values (R_A), the statistic S_A of eq. 19.4 had been computed for the various data samples. The preferred S_A statistic, however, was computed for the entire U.S. data set after each iteration of the adjustment: for iteration 0, $S_A = 2.18$; for iteration 1, $S_A = 1.42$; and for iteration 2, $S_A = 1.35$. By comparison $R_A = 0.90$ for the U.S. data set. (See table 19.3.) Several refinements from iteration to iteration are responsible for reducing S_A. The most important refinements were corrections for blunders. Chapter 18 documents many of the corrective actions that were taken between the different iterations.

Despite the decrease in S_A, the final value ($S_A = 1.35$) is excessive. Indeed, with the data corresponding to more than 800,000 degrees of freedom, S_A should lie within a 99-percent confidence interval bounded by 0.99 and 1.01. Many factors contribute to this inflated final value for S_A, but the dominant factors must be associated with direction observations as they comprise about 99 percent of the total data set. Hence, in view of the preceding discussion, the high S_A value is partly attributed to: (a) overly optimistic standard errors on the third-order directions in coastal areas, (b) overly optimistic standard errors on direction observations over steeply inclined lines, and (c) the effect of errors in predicted crustal movements.

The fact that a high value for S_A was obtained for the NAD 83 adjustment does not necessarily imply that the derived station positions are significantly biased. Indeed, the GPS comparisons (sec. 19.2) and the Doppler residuals (sec. 19.3) suggest that adjusted positions are excellent except in a few localities. Of the three identified problems, only errors in the crustal motion predictions may be expected to bias positions systematically. The problem with overly optimistic standard errors on third-order directions in coastal areas and on steeply inclined lines should affect positions mostly in a random manner.

19.5 FORMAL ERROR STATISTICS

In addition to providing a means to estimate station coordinates and other unknown parameters, the adjustment procedure provides a capability to assess the accuracy of these estimates. This accuracy information comes in the form of a symmetric matrix whose i,j-element gives the covariance between the estimates for the i-th and j-th parameters as referred to an adopted numbering scheme. This covariance matrix enables the user to compute the a posteriori standard error for any quantity that is mathematically related to the unknown parameters; for example, for the latitude of an NAD 83 station, or for the adjusted distance between two NAD 83 stations. Such computed error statistics depend in quality upon several assumptions (the absence of systematic errors and blunders, the distribution of observational errors being Gaussian, and the applicability of the equations for linear error propagation). Hence, these so-called "formal" error statistics provide only a conditional assessment of positional accuracy. A more objective assessment is obtained by comparing the NAD 83 coordinates with accurate, independently derived coordinates, such as the NAD 83-versus-GPS comparison discussed in section 19.2. However, GPS data exist only for a small subset of the NAD 83 stations, and so that comparison provides only an overview of network accuracy. The covariance matrix, on the other hand, can identify local variations in network accuracy.

Numerical values for the elements in the covariance matrix may be obtained by inverting the coefficient matrix for the normal equations and then scaling this inverted matrix by the variance of unit weight for the adjustment (S_A^2 where A represents the complete NAD 83 data set). The matrix inversion operation, however, is expensive when many unknown parameters are involved. As parameter estimates may be obtained without inverting this matrix, the covariance matrix is commonly not computed or it is computed only in part. The latter is the case for the NAD 83 adjustment. Only that part of the covariance matrix corresponding to the Alaskan stations was computed. All elements of the reduced normal equations matrix, however, were saved on magnetic tapes so that additional parts of the covariance matrix may be computed in the future. With the availability of the covariance matrix for Alaska, absolute and relative positional accuracies were examined for stations in the Helmert block referred to here as HB300. (See fig. 19.7 for location.)

The terms, *absolute position* and *absolute positional accuracy*, are misleading. Positional coordinates, such as latitude and longitude, are not absolute; they are relative to certain defined quantities. For NAD 83 these defined quantities locate, orient, and scale a three-dimensional Cartesian coordinate system, and they specify the size and shape of an ellipsoid. (See chapter 11.) The NAD 83 covariance matrix was computed as if these defined quantities were errorless. This is an acceptable practice since the quantities are "defined." Confusion may arise, however, because these defined quantities were chosen so as to approximate physically meaningful quantities. For example, the NAD 83 origin approximates the Earth's center of mass. One may mistakenly think that the NAD 83 covariance matrix incorporates the uncertainties associated with such approximations. It does not. Whether or not it should is debatable. On one hand, it may be argued that the true uncertainties of these approximations are unknown. On the other hand, it may be argued that some estimates for the uncertainties always exist, even if they are purely subjective. The NAD 83 origin essentially corresponds to the origin defined by the North American Doppler data as transformed so that the NAD 83 origin approximates the BTS 84 origin (Bureau International de l'Heure, 1985). If standard errors for latitude and longitude relative to the Earth's center of mass were desired (instead of relative to the defined NAD 83 origin), then an uncertainty would have to be assigned for how well the NAD 83 origin approximates the BTS 84 origin and how well the BTS 84 origin approximates the Earth's center of mass. Similar logic applies to the other defined quantities.

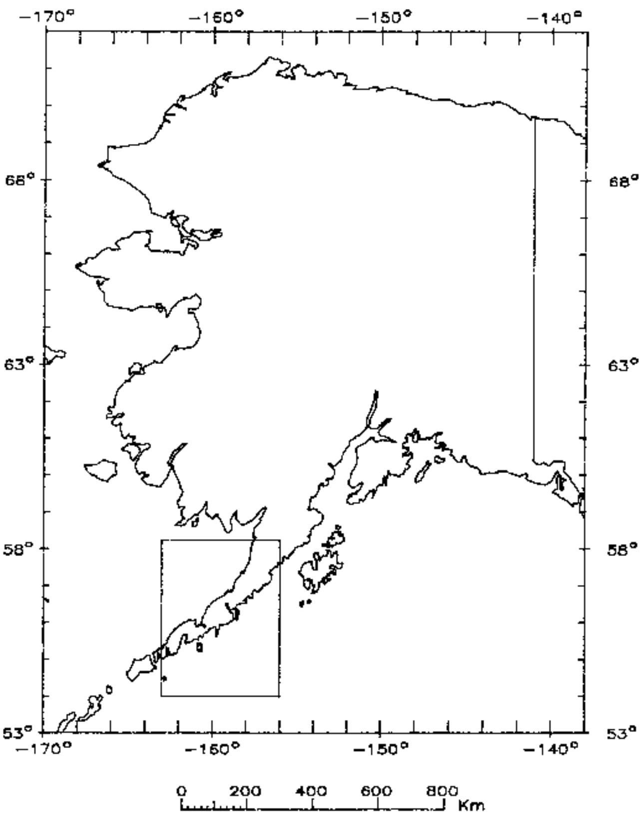

Figure 19.7. Location for the Helmert block referred to as HB300.

Moreover, the NAD 83 covariance matrix was computed as though the origin of longitude were defined in terms of the North American observations for astronomic longitude; that is, the corresponding Z-rotation parameter for these data was fixed at 0.0 arc second in the adjustment. However, a post-adjustment modification of the derived station coordinates was enforced—all geodetic longitudes were shifted by 0.365 arc second. (See sec. 11.4.) This modification served to redefine the NAD 83 longitude origin in terms of the North American Doppler data as transformed to make the NAD 83 longitude origin approximate the BTS 84 longitude origin. That is, the final NAD 83 coordinates were computed as if the Z-rotation of the Doppler data was held fixed, and not the Z-rotation of the astronomic longitude data. The NAD 83 covariance matrix, however, was not modified accordingly.

The histograms in figure 19.8 portray distributions for latitude and longitude standard errors corresponding to the stations in HB300—a pair of histograms for each classification of stations. These standard errors need to be interpreted in light of the previous discussion. In particular, the fact that longitude standard errors are about three times larger than latitude standard errors simply reflects the fact that the Doppler Z-rotation was estimated using the NAD 83 data set whereas the Doppler X- and Y-rotations and the Doppler Z-translation (being defined quantities) were treated as if without error. The NAD 83 estimate for the Doppler Z-rotation has a standard error of 0.0435 arc second which corresponds to 0.75 m in longitude at the center of HB300. This 0.75 m uncertainty is included in the computed longitude standard errors. If the NAD 83 covariance matrix were to be modified to take into account that the Doppler Z-rotation, in fact, became a defined quantity subsequent to the adjustment, then this 0.75 m uncertainty would not be included in the longitude standard errors, and thus, longitude standard errors would have magnitudes similar to those for latitude standard errors. This result emphasizes the care that must be exercised in interpreting absolute positional accuracies.

Despite the cited shortcomings, the latitude and longitude standard errors derived from the NAD 83 covariance matrix do convey some valuable information. In particular, they may be used to compare the absolute positional uncertainty of one station relative to that of another station. As such, the histograms in figure 19.8 indicate that absolute positional accuracy is essentially independent of station classification, except for fourth-order stations. This result (that first-, second-, and third-order stations have similar positional accuracies) also follows by inspecting the rms values for latitude and longitude standard errors. For the stations in HB300, the rms standard error in latitude equals 0.309 m for first-order stations, 0.349 m for second-order, 0.396 m for third-order, and 1.574 m for fourth-order. The rms standard error in longitude equals 1.055 m for first-order stations, 1.075 m for second-order, 1.113 m for third-order, and 2.339 m for fourth-order stations.

Although the NAD 83 covariance matrix produces absolute positional accuracies that require careful interpretation, its relative positional accuracies may be taken more at face value. As in section 19.2, the relative positional accuracies between two stations defines a vector whose horizontal projection may be resolved into a collinear component and a transverse component. The collinear component corresponds to distance accuracy, the transverse to orientation accuracy. Figure 19.9 plots collinear standard error as a function of interstation distance as computed from the NAD 83 covariance matrix for a sample of first-order station pairs in HB300. Figure 19.9 also displays the curve that corresponds to eq. 19.2 for approximating the rms collinear error in first-order lines. Recall that this curve is based on a comparison between NAD 83 and GPS coordinates. Similarly, Figure 19.10 plots transverse standard errors for the same sample of station pairs as well as the corresponding curve for rms transverse error as given by eq. 19.2 for first-order lines. Figures 19.9 and 19.10 demonstrate that the

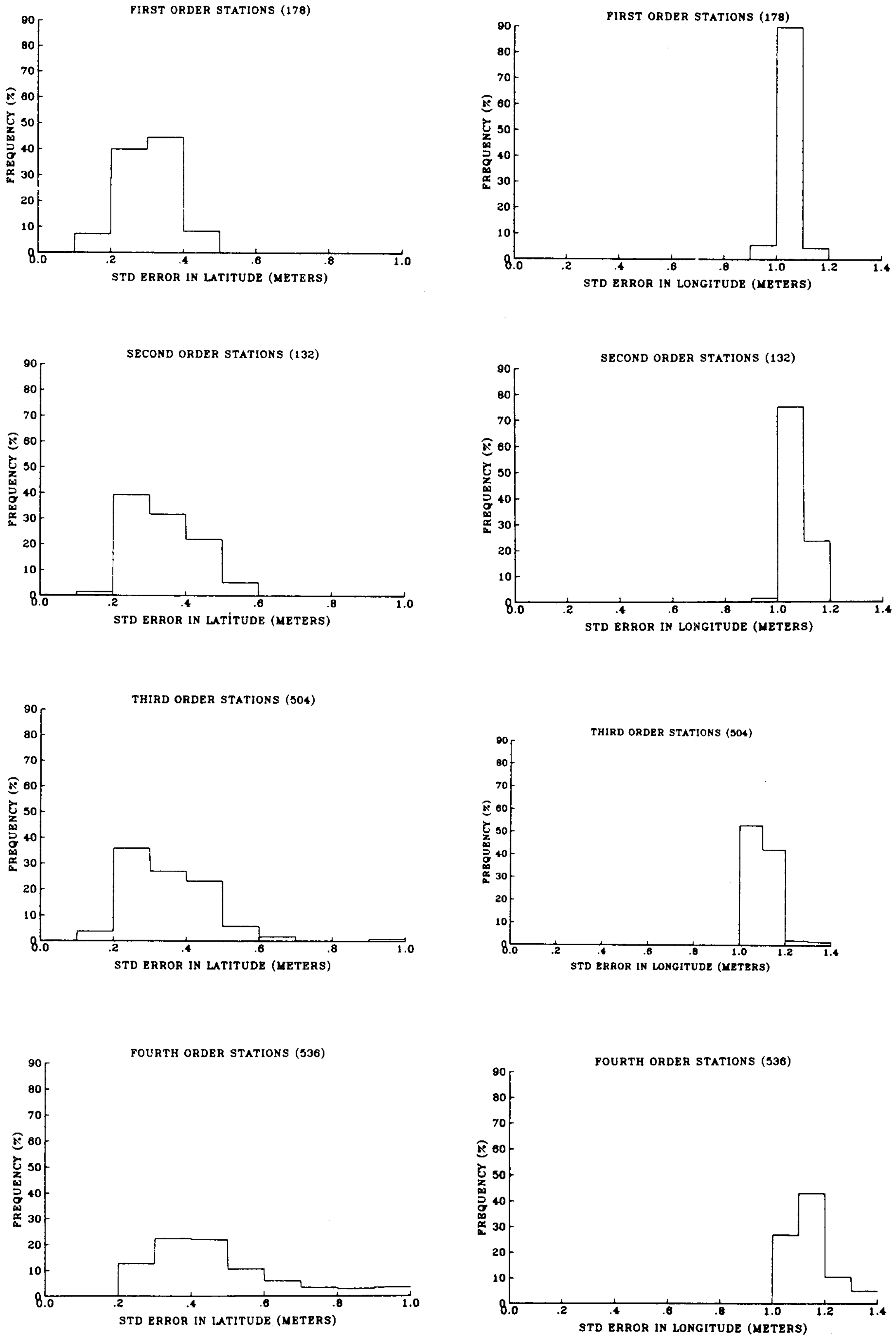

Figure 19.8. Distribution for latitude and longitude standard errors for the stations located in HB300. Numbers in parentheses denote the number of stations in the respective categories.

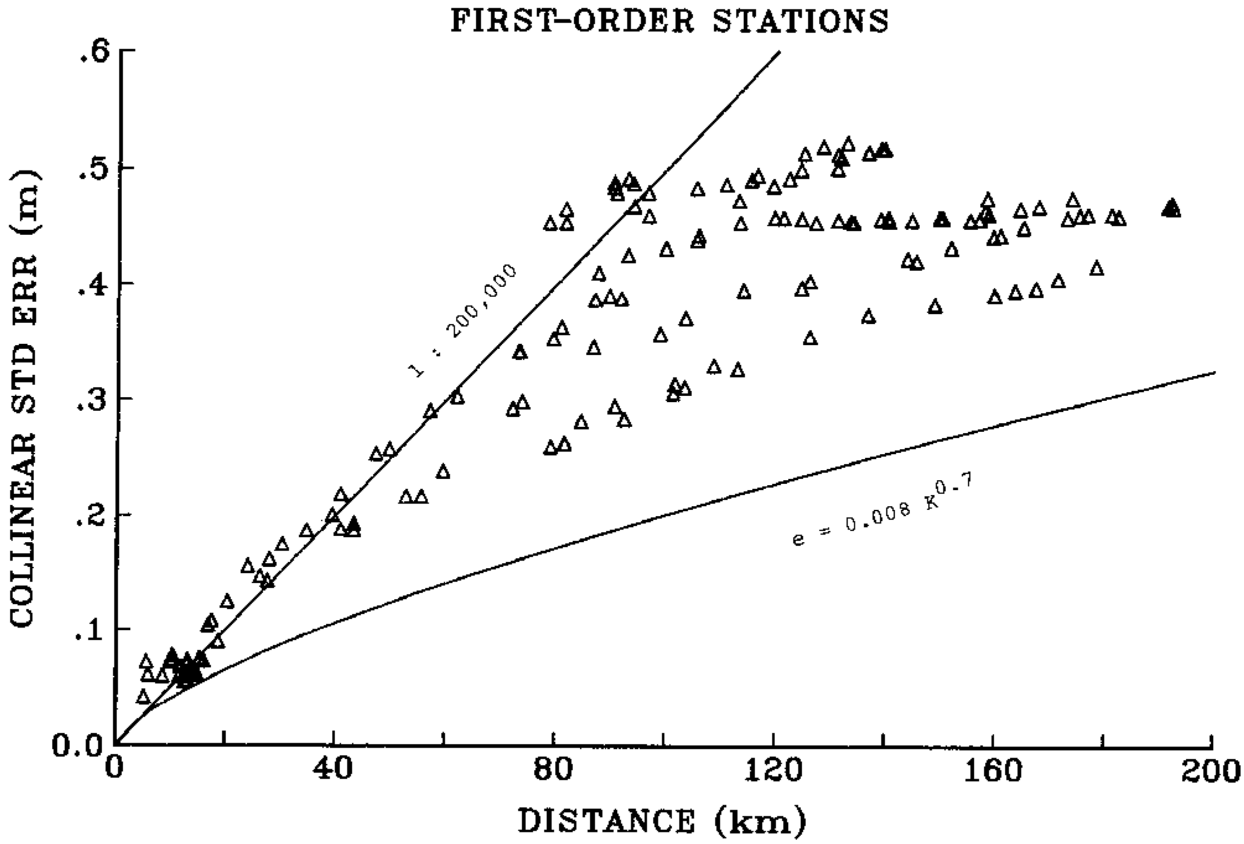

Figure 19.9. Triangles represent standard errors for the collinear component of relative position for a sample of first-order station pairs in HB300. These standard errors were computed using the NAD 83 covariance matrix. The curve represents the empirical formula (eq. 19.2) that characterizes the rms collinear error of first-order lines. This formula was derived from a comparison of NAD 83 and GPS coordinates (sec. 19.2). The straight line, representing a relative accuracy of 1:200,000, is included for reference.

standard errors derived from the covariance matrix generally overestimate the values from the curves. This result is curious. Because standard errors from the covariance matrix neglect systematic errors and blunders, these standard errors would be expected to underestimate the more objective error estimates given by the curves. The fact that they overestimate indicates that relative positions for HB300 are considerably less accurate than those for other parts of the United States. The statistics in table 19.4 corroborate this supposition. This table compares the station distribution by order for HB300 with that for the State of Florida. Not only does Florida have about three times as many stations per unit area, but 56 percent of the Florida stations are either first or second order. By comparison only 23 percent of the stations in HB300 are first or second order. More importantly, no first- or second-order triangulation/trilateration surveys have been performed in HB300 since 1960. Thus this part of Alaska does not contain many electronic distance measurements. Dense distributions of these accurate distance measurements strengthen the geodetic network throughout the majority of the country.

TABLE 19.4—*Station distribution for Florida as compared to that for a Helmert block in Alaska*

	Florida	HB300
First-order stations	1,900	178
Second-order stations	5,200	132
Third-order stations	2,600	504
Fourth-order stations	2,900	536
Total number of stations	12,600	1,350
Land area	152,000 km^2	48,000 km^2
Stations per 1,000 km^2	83	28

The discrepancy between the standard errors computed from the covariance matrix for HB300 and the curves corresponding to eq. 19.2 emphasizes the importance of computing more of the covariance matrix. The national geodetic reference network is inhomogeneous, and formulas such as eq. 19.2 apply only on the average. With the full covariance matrix, the standard errors of relative position would be available for every station pair. The cost of computing the full covariance matrix, however, is prohibitive. A more practical goal would be to compute only certain covariance matrix values; for example, those needed to compute relative position standard errors for the observed lines. With better knowledge of these standard errors, users of the National Geodetic Reference System could better decide what stations to employ in their activities. Also with this information, users would be able to evaluate any new observations among NAD 83 stations more definitively than is possible using either the FGCC standards or empirical formulas such as eq. 19.2.

19.6 DEFLECTIONS AND THEODOLITE TILT AS ERROR SOURCES

As mentioned in section 19.4, the rms normalized residual for first-order directions near the eastern front of the Rocky Mountains is 8.7 percent higher than the rms normalized residual for all first-order directions. Similarly, the rms normalized residual for second-order directions in this region is 4.5 percent higher than the rms normalized residual for all second-order directions. In such mountainous terrain, the orientation of the theodolite's axis relative to the orientation of the local normal to the ellipsoid takes on greater significance. If the error in the determination of the space angle between these two orientations has magnitude d in the azimuth α_d, then this error produces an error in a direction observation whose value e is approximated by the equation

$$e = (d)(\Delta h/S)\sin(\alpha_o - \alpha_d) \tag{19.7}$$

where Δh denotes the height difference between the theodolite and the target, S denotes the horizontal distance between the theodolite and the target, and α_o denotes the azimuth of the observed line. Hence, direction residuals will scale in proportion to line slope, $\Delta h/S$.

Consider now the case for which the 1-sigma uncertainty in d is 6.0 arc seconds in a relatively mountainous region where $\Delta h/S$ has an rms value of 0.05. Then

$$\sigma_e \approx \sigma_d \,\mathrm{rms}(\Delta h/S)\,[(\int_0^{2\pi} \sin^2\theta\, d\theta)/2\pi]^{0.5}$$

$$\approx (6.0)(0.05)(2)^{-0.5} = 0.212 \text{ arc second.} \tag{19.8}$$

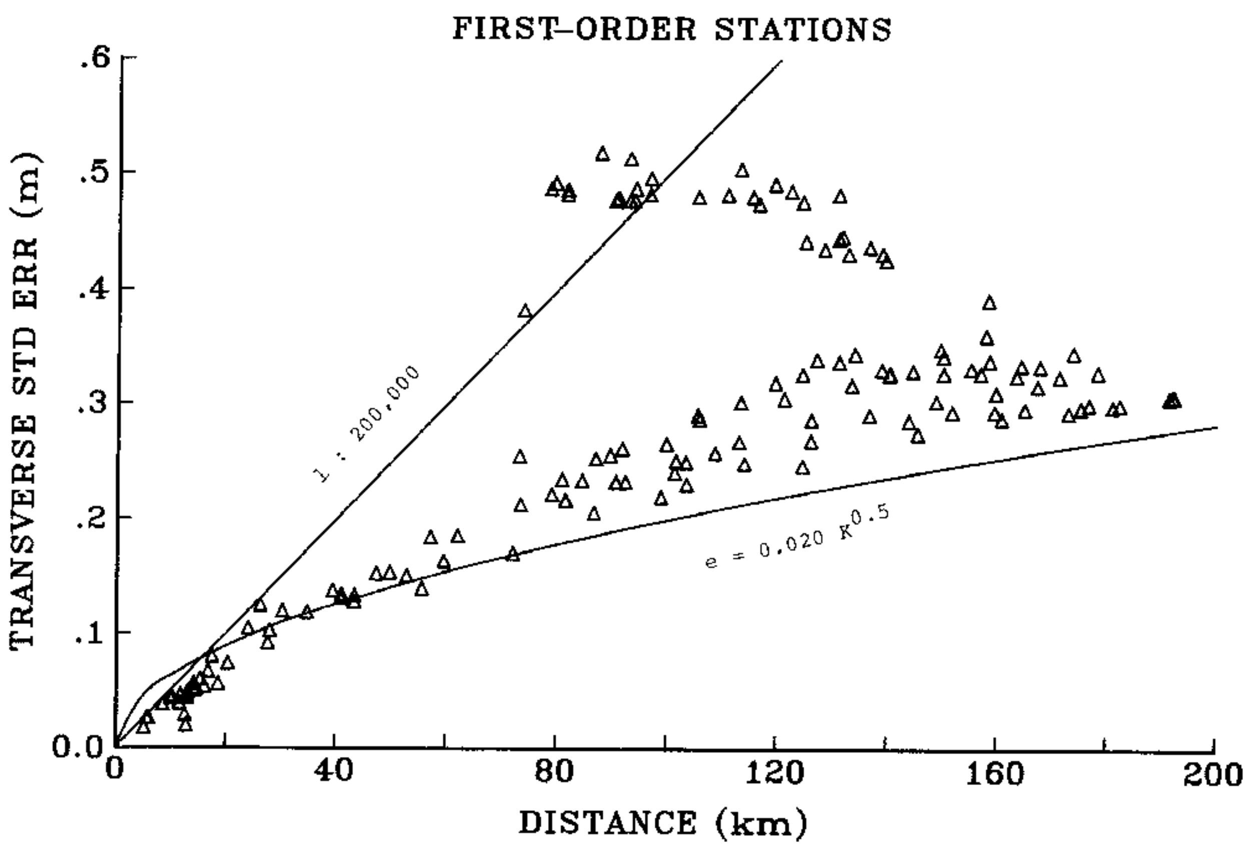

Figure 19.10. Triangles represent standard errors for the transverse component of relative position for a sample of first-order station pairs in HB300. The curve represents the empirical formula (eq. 19.2) that characterizes the rms transverse error of first-order lines. This formula was derived from a comparison of NAD 83 and GPS coordinates (sec. 19.2). The straight line, representing a relative accuracy of 1:200,000, is included for reference.

Under such conditions, the standard error assigned to first-order directions should be increased (on the average) from 0.6 arc second to

$$\sigma = [(0.6)^2 + (0.212)^2]^{0.5} = 0.636 \text{ arc second.} \quad (19.9)$$

Similarly, the standard error assigned to second-order directions should be increased (on the average) from 0.7 arc second to

$$\sigma = [(0.7)^2 + (0.212)^2]^{0.5} = 0.731 \text{ arc second.} \quad (19.10)$$

These new standard errors represent a 6.1 percent increase for first-order observations and a 4.5 percent increase for second-order observations. These calculations suggest that the higher residuals near the eastern Rocky Mountain front may be explained in large part by an rms error on the order of 6.0 arc seconds in the determination of the angle between the theodolite's axis and the local normal to the ellipsoid. This error depends on how accurately the deflections of the vertical have been determined and how well the theodolites have been leveled. These two error sources are considered in the following paragraphs.

The deflection of the vertical at a point represents the angle between the gravity vector and the normal to the ellipsoid at this point. This angle is quantified by the equations

$$\begin{aligned} \xi &= \Phi - \phi \\ \eta &= (\Lambda - \lambda) \cos \phi \,. \end{aligned} \quad (19.11)$$

Here Φ and Λ denote the point's astronomic latitude and longitude; ϕ and λ denote the point's geodetic latitude and longitude. Fury (1984) predicted values for ξ and η at about 180,000 control points (chapter 16). Fury's technique employed numerical integration applied to existing gravity data. A least squares collocation technique was employed to predict deflections at the 13,000 control points that were added to the data base after Fury's computation were complete. The accuracy of these predictions were assessed by comparing them with "observed" deflections at 10 stations in New York State. (See table 19.5.) That is, predicted deflections were compared with the deflections obtained by substituting observed values for ξ and η into eq. 19.11. The astronomic coordinates for the 10 stations were observed only after the corresponding deflections had been predicted. The deflection predicted via numerical integration differs from the "observed" deflections by an rms value of 0.67 arc second in ξ and 0.70 arc second in η. The deflections predicted via collocation differ from the "observed" deflections by an rms value of 2.81 arc seconds in ξ and 1.77 arc seconds in η. The goal for the NAD 83 project was to predict deflections with an rms accuracy of 1.0 arc seconds (Schwarz, 1978). This comparison indicates that the numerical integration predictions meet this goal, at least for this part of New York. The relative inaccuracy of the collocation predictions is of little consequence to this analysis because accurate deflections are needed only at the control points from which directions and azimuths were observed and because Fury's predictions were used at all but a few percent of these control points.

In a more comprehensive test, Fury (1984) also found that deflections predicted by numerical integration are approximately accurate to 1.0 arc second. At each of some 3,115 astronomic stations, Fury estimated yet a different deflection value by interpolating his predicted deflection from neighboring stations. These interpolated deflections differ from the "observed" deflections by an rms value of 1.33 arc seconds in ξ and 1.15 arc seconds in η. Because each interpolated deflection was obtained without using the predicted deflection at the station in question, these interpolated deflections may be expected to be somewhat less accurate than 1.0 arc second even if the predicted deflections have this accuracy. Predicted deflections were not used directly in the comparison because the astronomic data had been used in the prediction process in such a way that predicted deflections agree exactly with "observed" deflections in most cases.

TABLE 19.5.—*Comparison of deflections of the vertical*

			ξ (arc seconds)[1]			η (arc seconds)[1]		
Station	Lat. (deg)	Long. (deg)	Obs.	Integration	Collocation	Obs.	Integration	Collocation
HAWES 1942	42.8	74.1	0.9	0.81	−3.4	−0.4	−0.06	−0.4
COLUMBIA 1940	42.9	74.9	3.0	4.30	4.8	1.8	2.46	4.8
BRIGHT 1942	42.4	74.8	7.5	8.14	8.4	1.5	2.58	5.1
RIPLEY HILL 1882	42.8	76.1	4.4	4.71	1.4	2.6	2.81	2.4
OTSELIC 1942	42.6	75.8	1.6	1.18	−0.8	2.1	2.83	2.0
CHASE 1942	44.8	75.1	1.6	2.45	0.1	5.4	5.34	6.0
FOWLER 1942	44.3	75.4	0.9	2.00	−1.1	7.4	7.12	8.8
CEMETARY 1929	42.8	77.6	7.6	7.87	3.8	−2.3	−1.88	−1.2
HORNELL 1935	42.3	77.6	8.4	8.38	5.6	−4.2	−3.34	−2.2
WILLIAMS 1939	42.6	77.3	8.2	7.94	4.6	−1.2	0.08	0.1

[1] Deflections expressed in NAD 27 reference system.

Although the preceding tests indicate that the predicted deflections are accurate to 1.0 arc second, one caveat exists. These tests involved deflections referred to the NAD 27 reference system. Predicted deflections have since been transformed to the NAD 83 reference system for the adjustment. Frank (1987) suspects that this transformation was improperly implemented. After independently transforming a subset of the deflections, Frank found that his transformed deflections differ from the adopted transformed deflections by a mean of 0.08 arc second in ξ and 0.46 arc second in η. In spite of this discrepancy, the predicted deflections actually used in the NAD 83 adjustment should be accurate to about 1.5 arc seconds in each component. Hence the accuracy of the total deflection should be about $(1.5)(2)^{0.5} \approx 2.0$ arc seconds. This value is considerably less than the 6.0 arc second error hypothetically needed to explain the relatively high direction residuals found near the eastern Rocky Mountain front.

Another error source whose effect grows in proportion to $\Delta h/S$ is associated with leveling the theodolite. Using well calibrated level vials, the observer orients the vertical axis of the theodolite parallel to the local direction of gravity. Adopted field procedures (Gossett, 1959) stipulate that for lines of sight having an inclination in excess of 2 degrees from the horizon (that is, for lines where $\Delta h/S$ exceeds 0.035 in magnitude), then the observer should maintain the orientation of the theodolite's axis to within one division on the level vial. If this orientation is maintained throughout the observing session, then the maximum tilt of the theodolite's axis would be about 3.5 arc seconds in any given direction because one division corresponds to less than 7.0 arc seconds and because the theodolite is rotated full circle. Consequently, the maximum value for the total tilt is $(3.5)(2)^{0.5} \approx 5.0$ arc seconds. This maximum tilt is still less than the 6.0 arc second rms error hypothetically needed to explain the relatively high direction residuals found near the eastern Rocky Mountain front.

In light of the arguments forwarded in the previous paragraphs, one can only speculate about what has caused the higher residuals found near the eastern Rocky Mountain front. Perhaps several observers failed to follow the adopted theodolite leveling procedures, or perhaps some yet identified error source has significantly affected direction observations over steep lines (for example, lateral refraction as the line of sight passes through different atmospheric layers). Neither of these speculations were investigated for this report. Nevertheless, the higher residuals exist, and whatever their cause, these residuals suggest that standard errors on direction observations should have been assigned as a function of line slope for the NAD 83 adjustment.

19.7 CRUSTAL MOTION AS AN ERROR SOURCE

Horizontal crustal motion introduces a source of systematic error. The largest movements occur in Alaska and California. Hence, Snay et al. (1987) developed the so-called REDEAM models to characterize these movements as well as those occurring in parts of Nevada and Hawaii. (See also chapter 17.) Geodetic observations in the modeled regions were processed with these REDEAM models to replace observed values with estimates of what these values would be had the observations been performed on December 31, 1983. It is these updated observations that were entered into the NAD 83 adjustment. Thus, just as with the deflection values, the crustal motion parameters were estimated in an isolated process, and then they were held fixed in the adjustment. Moreover, as is the case for deflections, the uncertainty associated with the crustal motion parameters was not carried forward into the adjustment. As a result, errors in the crustal motion models have mapped into the residuals of the observations, and thus these errors have inflated the standard error of unit weight. High rms values for the normalized residuals in California (figs. 19.6A - 19.6E) indicate that this is the case. Furthermore, Doppler residuals (fig. 19.5) indicate that errors in the crustal motion models could have biased the estimated positional coordinates for stations in Alaska and in California.

In the case of Alaska, it has already been stated that a large Doppler residual at station CHILLIGAN corroborates the suspicion of a problem with the REDEAM model for the 1964 earthquake. Figure 19.11 shows the corresponding earthquake displacement field. This displacement field was derived from a comparison between pre- and post-earthquake surveys, and this field served as the basis for formulating the analytical expressions that constitute the REDEAM model for Alaska. In figure 19.11, the displacement vectors for stations located west of the 151°W meridian of longitude are oriented contrary to geophysical expectation. This problem is thought to be caused by a blunder in the post-earthquake survey. Such a blunder has yet to be identified, however, due to the poor network geometry of this survey. [See Parkin (1969: fig. 3).]

In the case of California, Doppler residuals have approximately the same magnitude as those elsewhere in the United States, but these residuals are systematically oriented eastward throughout a large part of that state (fig. 19.5A). However, because similar trends among the Doppler residuals also occur in nondeforming U.S. regions (fig. 19.5A), Doppler residuals can not in themselves be used to assess the accuracy of the REDEAM models for California. Hence, results from repeated VLBI measurements spanning the 1982.7-1987.2 interval will be used here for such an assessment. From these VLBI measurements, Clark et al. (1987) estimated the secular horizontal velocities of several California stations relative to the station designated as MOJAVE. (See fig. 19.12.) Clark's velocities are used here to update the REDEAM models. The discrepancy between these updated models and the original models is then used to infer that the original models contribute less than 2.2 ppm in rms scale error and less than 0.90 arc second in rms orientation error to the relative positional coordinates of California stations.

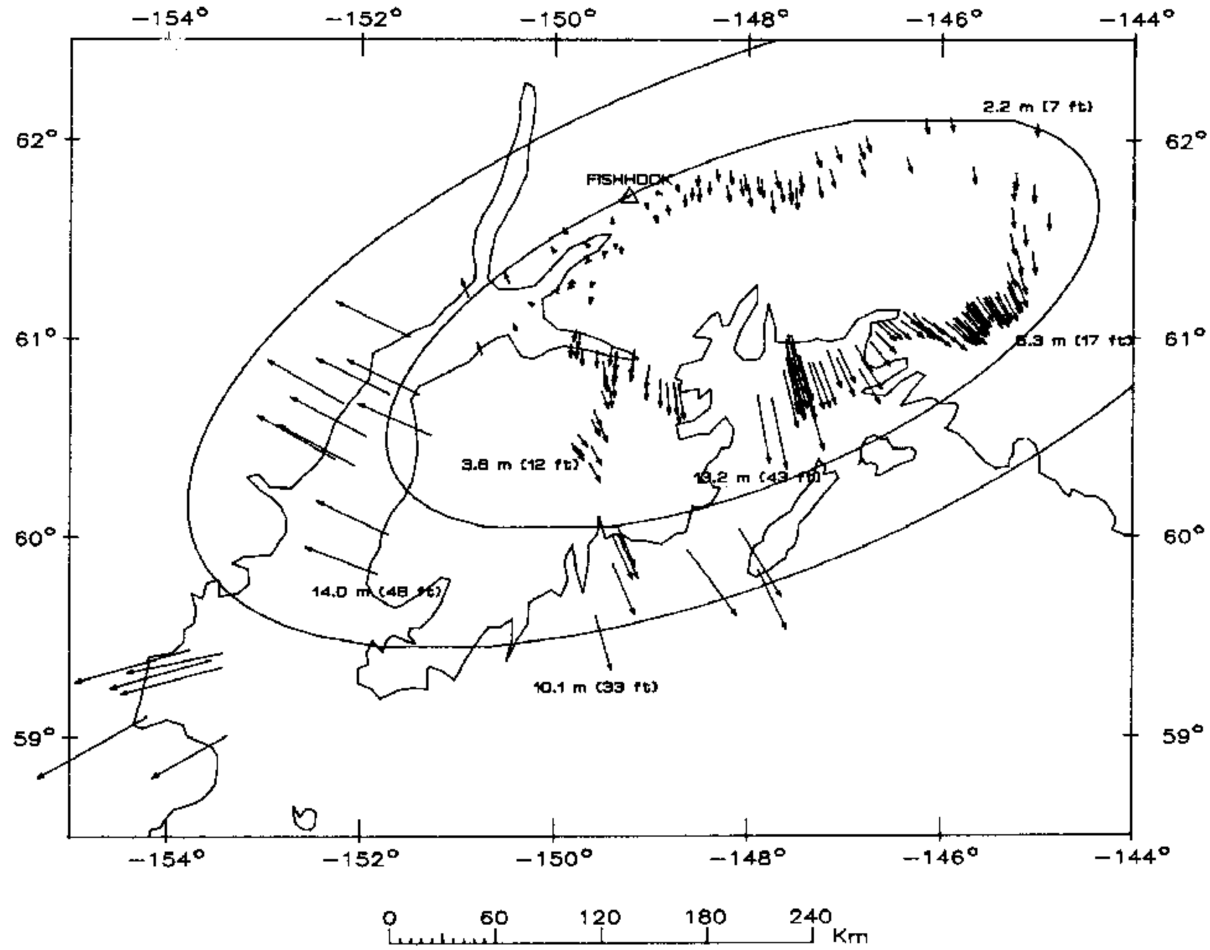

Figure 19.11. Horizontal displacement vectors for the 1964 Prince William Sound, AK, earthquake as derived by differencing pre- and post-earthquake positions. Vectors are relative to the assumption of no motion at station FISHHOOK. Vectors west of the 151°W meridian of longitude are highly suspect because of poor network geometry. The REDEAM model consists of polynomial expressions that approximate these vectors as a function of latitude and longitude. Between the ellipses, the polynomials are scaled from their full value (inside the interior ellipse) to zero (outside the exterior ellipse) to dampen polynomial growth.

Actually, the motion in California was originally characterized by 16 distinct REDEAM models, one for each of 16 mutually disjoint regions. These regions, in combination span the state. (See fig. 19.13.)

The updated model was created by introducing 64 "correction" parameters; that is, 4 parameters for each of the 16 regions. These parameters serve to make the individual models more consistent among themselves as well as more consistent with the repeated Very Long Baseline Interferometry (VLBI) data. Let P denote a point in the i-th region, and let $u_i(P)$ and $v_i(P)$ denote the northern and western components of the secular horizontal velocity at P as given by the REDEAM model for the i-th region. Then the horizontal velocity components for the updated model, denoted $\bar{u}_i(P)$ and $\bar{v}_i(P)$ are given by the equations

$$\bar{u}_i(P) = u_i(P) + \mu_i + \rho_i x_i + \omega_i y_i$$
$$\bar{v}_i(P) = v_i(P) + \gamma_i - \omega_i x_i + \rho_i y_i \qquad (19.12)$$

Here μ_i, γ_i, ρ_i, and ω_i represent the four correction parameters for the i-th region. Also, x_i (positive north) and y_i (positive west) represent the coordinates of P (in meters) in a planar coordinate system for the i-th region. The origin of this coordinate system corresponds to the point to which velocities are referenced in the original models.

Values for the 64 correction parameters were estimated via the least squares procedure. Four types of quasi-observations were involved. The first two types have the form

$$\bar{u}_i(P) = \bar{u}_j(P) + e_1$$
$$\bar{v}_i(P) = \bar{v}_j(P) + e_2 \qquad (19.13)$$

for P located on the common boundary between the i-th and j-th regions. Here e_1 and e_2 represent residuals. The second two types have the form

$$\bar{u}_i(P) = u_C(P) + e_3$$
$$\bar{v}_i(P) = v_C(P) + e_4 \qquad (19.14)$$

for P corresponding to a VLBI station with $u_C(P)$ and $v_C(P)$ denoting the velocity at P derived by Clark et al. (1987). Again e_3 and e_4 represent residuals. Figure 19.13 identifies the points for which the first two quasi-observation types were formulated. Each such quasi-observation was assigned a standard error of 3 mm/yr. Figure 19.12 identifies the VLBI points for which the last two quasi-observations types were formulated. Each such quasi-observation was assigned a standard error of 10 mm/yr except the quasi-observations for MOJAVE. The MOJAVE quasi-observations were assigned a standard error of 0.1 mm/yr each.

Table 19.6 gives the estimated values for all 64 parameters. Figure 19.12 and table 19.7 compare the updated REDEAM velocities with Clark's velocities. In this latter table, velocities are given in polar coordinates as opposed to the rectilinear u,v-coordinates. The least squares adjustment generated the following statistics:

$$\text{rms}(e_1) = 3.4 \text{ mm/yr.}$$
$$\text{rms}(e_2) = 3.3 \text{ mm/yr.}$$
$$\text{rms}(e_3) = 7.0 \text{ mm/yr.}$$
$$\text{rms}(e_4) = 7.9 \text{ mm/yr.}$$

The parameters μ_i and γ_i bear no relationship to the accuracy of the original REDEAM models. These parameters serve only to reference the updated REDEAM velocities relative to MOJAVE instead of to the 16 individual origins associated with the original models. The value of ρ_i may be viewed as the correction necessary to modify the average extensional rate (that is, the rate of scale change) predicted by the original REDEAM model for the i-th region. The 16 ρ_i values have an rms value of 0.087 ppm/yr. Because 90 percent of the distance observations have been performed during the last 25 years, errors in the original REDEAM models should have produced rms scale errors less than 2.2 ppm (= 0.087*25) in the relative positional coordinates of California stations. The value of ω_i may be viewed as the correction necessary to

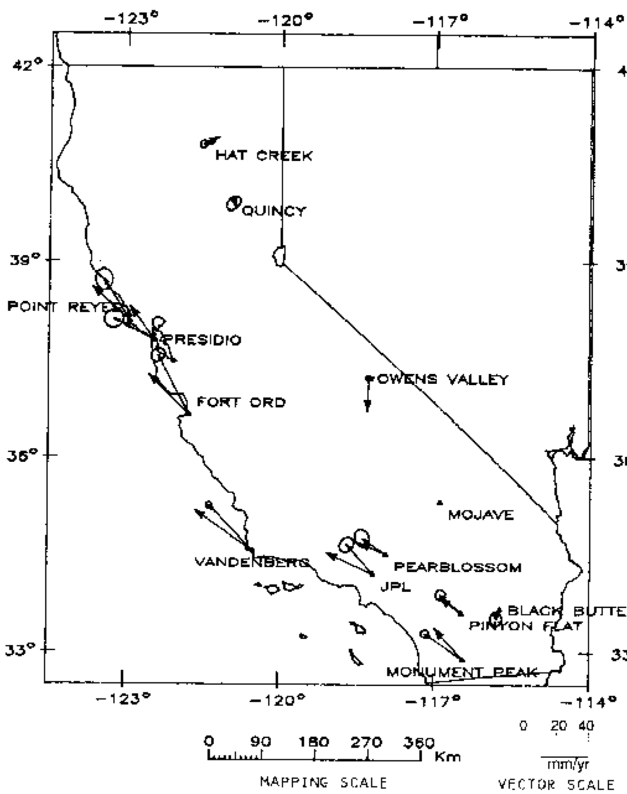

Figure 19.12. Horizontal velocities at selected California sites. Vectors are relative to the assumption of no motion at station MOJAVE. Vectors with smaller arrowheads and 95-percent confidence regions were derived by Clark et al. (1987) from repeated VLBI observations. Vectors with larger arrowheads represent the updated REDEAM models.

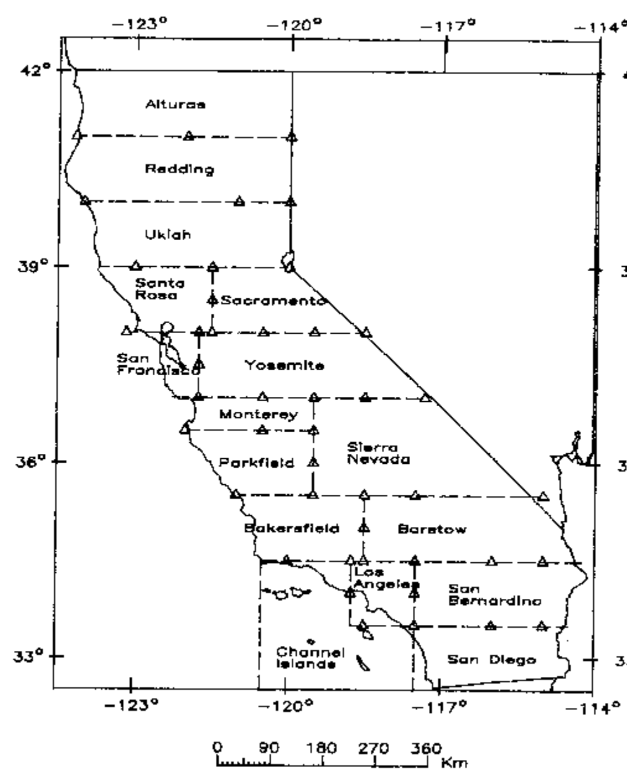

Figure 19.13. The triangles locate the points at which observations were introduced so that the updated REDEAM models would yield more consistent velocities along the boundaries of adjacent regions.

modify the average rotational rate predicted by the original REDEAM model for the i-th region. The 16 ω_i values have an rms of 0.036 arc second/yr (0.18 μrad/yr). Because 75 percent of the azimuth observations have been performed during the last 25 years, errors in the original REDEAM models should have produced rms orientation errors less than 0.90 arc second = (0.036*25) in the relative positional coordinates of the California stations.

19.8 STATION HEIGHTS AS AN ERROR SOURCE

Station heights were held fixed for NAD 83 except where Doppler, VLBI, or GPS data were observed. Therefore, as with deflection errors and crustal motion errors, height errors have been mapped wrongfully into residuals on the observations. Height errors would primarily affect distance observations, but the corresponding residuals seem low relative to the standard errors assigned to the observations. In particular, for lightwave distance measurements, which comprise about 65 percent of all the distance measurements, the normalized residuals have relatively low rms values. (See fig. 19.6G.) These relatively low normalized residuals resulted in part because NGS included a slope-dependent term in its formula for assigning standard errors to electronic distance observations; that is, the assigned standard error σ_t was calculated by the equation

$$\sigma_t = [\sigma_o^2 + (\Delta h/S)^2 \, \sigma_{\Delta h}^2]^{0.5} \qquad (19.15)$$

where σ_o represents the 1-sigma observational uncertainty and $\sigma_{\Delta h}$ the 1-sigma uncertainty in the inter-station height difference. For NAD 83 the value $\sigma_{\Delta h}$ = 0.00005 $S/3$ was assumed. So unlike the case for deflection error and crustal motion error, the uncertainty due to station height error was incorporated into the total uncertainty assigned to electronic distance observations.

For NAD 83, a station's ellipsoidal height was computed as the sum of its orthometric height and its geoid height. Orthometric heights were derived from leveling and vertical angle data, or in the absence of such measurements, orthometric heights were scaled from topographic maps. All geoid heights were computed using the spherical harmonic expansion model of the Earth's gravitational potential identified as Rapp 1978 (Rapp, 1979). The quality of the Rapp 1978 geoid heights is discussed in the following paragraphs.

The Rapp 1978 spherical harmonic expansion includes all terms through degree and order 180. More recently, Rapp and Cruz (1986) developed the OSU86F spherical harmonic expansion which includes all terms through degree and order 360. The difference in geoid heights between these two representations was calculated for an area of dramatic topographic relief in Colorado. Figure 19.14 illustrates the Rapp 1978 geoid heights for this area, and figure 19.15 depicts the difference between Rapp 1978 and OSU86F geoid heights for this same area. According to figure 19.15, the largest discrepancy between Rapp 1978 and OSU86F in the relative geoid height for station pairs in this area is about 1.4 m.

Another comparison indicates that the error in the Rapp 1978 geoid heights may even exceed 1.4 m in some locations. For the stations identified in figure 19.15, Zilkoski and Hothem (1989) compared relative geoid heights as obtained by four different methods. In addition to Rapp 1978 and OSU86F, Zilkoski and Hothem considered the relative geoid heights obtained by numerically integrating existing gravity data (Fury, 1984) and those obtained by differencing ellipsoidal heights (derived from GPS data) with orthometric

TABLE 19.6.—*Correction parameters for updating the California REDEAM models. (Numbers in parentheses correspond to 1-sigma formal errors.)*

Region	μ mm/yr	γ mm/yr	ρ ppm/yr	ω μrad/yr
San Diego	5.8 (3.9)	−2.9 (3.9)	0.099 (0.022)	−0.068 (0.022)
San Bernardino	2.6 (2.4)	0.6 (2.4)	0.084 (0.016)	−0.058 (0.016)
Barstow	1.2 (0.7)	−0.8 (0.7)	0.061 (0.012)	−0.017 (0.012)
Channel Islands	22.1 (2.6)	33.0 (2.6)	0.081 (0.017)	0.068 (0.017)
Los Angeles	14.9 (2.1)	25.0 (2.1)	0.103 (0.018)	0.056 (0.018)
Bakersfield	16.2 (2.4)	30.1 (2.4)	0.020 (0.014)	−0.149 (0.014)
Sierra Nevada	−29.8 (2.4)	2.8 (2.4)	0.027 (0.010)	−0.124 (0.010)
Parkfield	−4.7 (2.9)	16.6 (2.9)	0.085 (0.016)	−0.079 (0.016)
Monterey	23.0 (3.6)	25.3 (3.6)	0.057 (0.015)	−0.217 (0.015)
Yosemite	−30.8 (3.6)	19.3 (3.6)	0.032 (0.011)	−0.059 (0.011)
San Francisco	−1.6 (3.7)	10.4 (3.7)	0.118 (0.024)	0.146 (0.024)
Sacramento	−11.4 (3.7)	4.8 (3.7)	−0.001 (0.015)	−0.544 (0.015)
Santa Rosa	6.1 (4.4)	7.2 (4.4)	0.152 (0.020)	−0.157 (0.020)
Ukiah	−10.9 (4.1)	5.0 (4.1)	0.033 (0.020)	−0.133 (0.020)
Redding	20.2 (8.4)	15.3 (8.4)	−0.180 (0.024)	0.204 (0.024)
Alturas	10.4 (7.6)	−10.0 (7.6)	0.032 (0.028)	0.110 (0.028)

TABLE 19.7.—*Comparison of horizontal velocities at selected California sites*

	Velocities from VLBI[1]		REDEAM velocities		
	Rate (mm/yr)	Azimuth (deg)	Rate (mm/yr)	Azimuth (deg)	Velocity discrepancy (mm/yr)
MOJAVE	0.0	—	0.0	—	—
HATCREEK	2.0	255	9.5	62	11.4
QUINCY	2.9	178	5.8	163	3.1
PT. REYES	31.2	327	32.3	311	8.9
PRESIDIO	29.6	297	26.5	324	13.4
FORT ORD	43.0	332	36.6	314	13.8
VANDENBERG	41.3	316	46.0	305	9.6
JPL	26.0	319	33.2	295	14.0
PEARBLOSSOM	18.7	305	19.1	294	3.5
MONUMENT PEAK	30.0	305	27.3	318	7.2
PINYON FLAT	18.1	312	17.1	307	1.9
BLACK BUTTE	6.1	199	1.1	38	7.2
OWENS VALLEY	0.5	286	21.3	183	21.4

[1] Velocities derived by Clark et al. (1987) from data spanning the 1982.8-1987.2 interval.

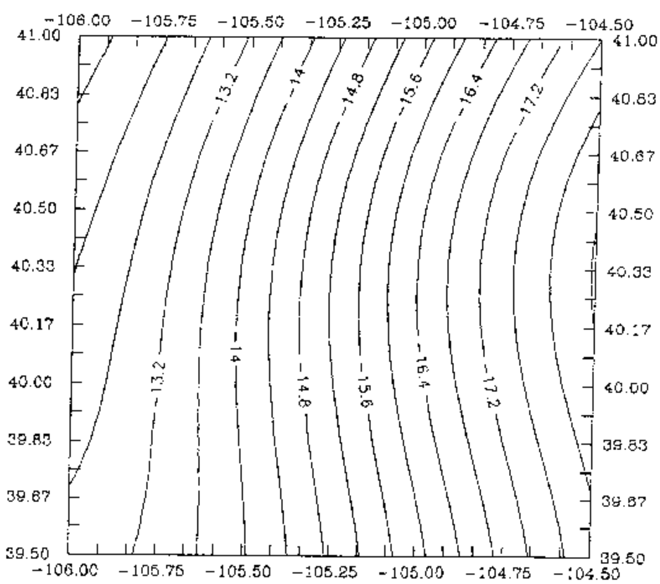

Figure 19.14. Geoid heights predicted with the Rapp 1978 spherical harmonic expansion.

heights (derived from leveling observations). Their results indicate that the error in the relative geoid heights predicted by Rapp 1978 can be as large as 2 m between stations only 70 km apart. (See fig. 19.16.) However, even this error, which represents a near extreme case for the United States, would not seriously distort NAD 83 station coordinates. To demonstrate this assertion, note that the horizontal distance S between two stations may be approximated by the equation

$$S \approx (L^2 - \Delta h^2)^{0.5} \qquad (19.16)$$

where L denotes the interstation chord distance and Δh denotes the interstation ellipsoidal height difference. (In this approximation, the Earth's surface is considered planar.) Consequently,

$$dS \approx -(\Delta h/S)\, d\Delta h. \qquad (19.17)$$

Thus for $d\Delta h = 2$ m, $S = 70$ km, and $\Delta h = 1{,}000$ m, the horizontal distance error (dS) would be about 0.029 m in magnitude. The corresponding relative dis-

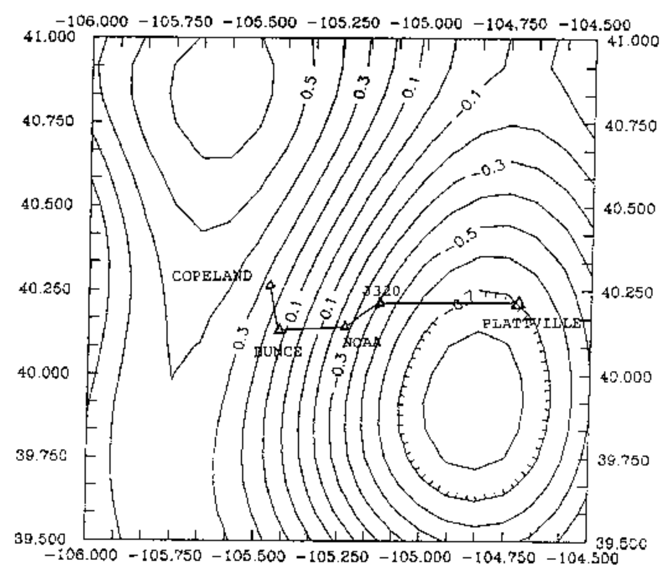

Figure 19.15. Contours represent OSU86F geoid heights minus Rapp 1978 geoid heights. Triangles locate the stations at which Zilkoski and Hothem (1989) compared different estimates for relative geoid height.

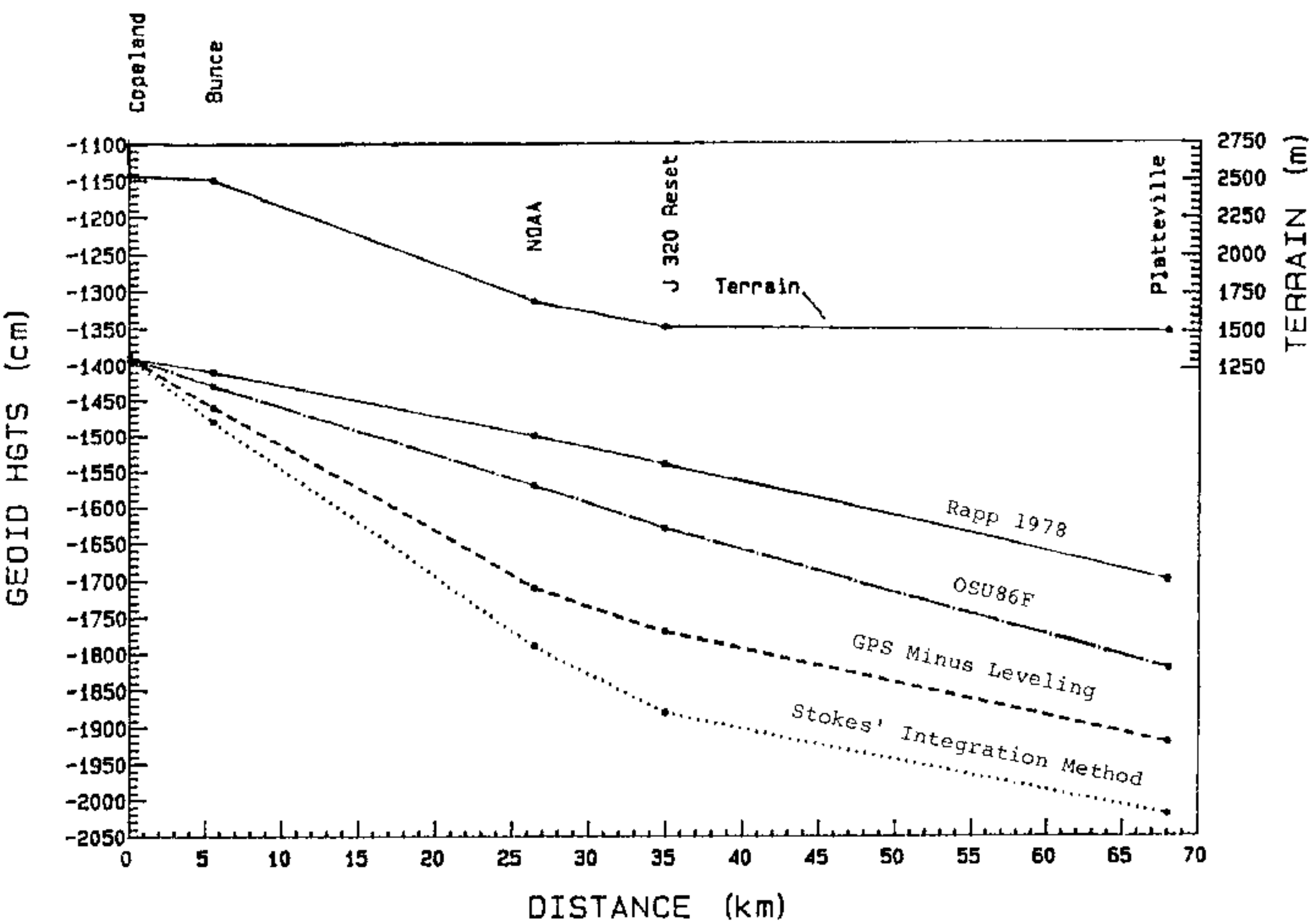

Figure 19.16.—Different estimates for the relative geoid height of several stations in Colorado (after Zilkoski and Hothem, 1989). The geoid height for station COPELAND was arbitrarily set to −1,400 cm.

tance error (dS/S) would be only about 0.4 ppm (= 0.029/70,000). It is thus concluded that errors associated with Rapp 1978 geoid heights did not significantly distort NAD 83 coordinates.

19.9 SUMMARY

In this chapter, the quality of the new NAD 83 reference system was considered from four perspectives: (1) NAD 83 coordinates were compared against GPS-derived coordinates, (2) trends among the residuals were inspected, (3) formal error statistics for absolute and relative positioning were generated with the covariance matrix, and (4) various error sources were investigated. To document this analysis has, hence, required a rather lengthy chapter which is summarized here by identifying the major results discussed in the individual sections.

NAD 83 Versus GPS (sec. 19.2)

- For first-order lines with lengths between 10 km and 100 km, the rms discrepancy between GPS distances and NAD 83 distances may be approximated by the empirical rule $e = 0.008\ K^{0.7}$.
- For first-order lines with lengths between 10 km and 100 km, the rms discrepancy between GPS azimuths and NAD 83 azimuths may be approximated by the empirical rule $e = 0.020\ K^{0.5}$.
- For second- and third-order lines with lengths between 10 km and 100 km, the rms discrepancies between GPS and NAD 83 distances and azimuths are less than 50 percent larger than the corresponding discrepancies for first-order lines.

Doppler Residuals (sec. 19.3)

- A few large (> 3 m) Doppler residuals exist.
- Doppler residuals exhibit some systematic trends in a few geographic regions.
- The north-south components of the Doppler residuals have an rms of 0.59 m; the east-west components have an rms of 0.74 m.

Direction, Azimuth, and Distance Residuals (sec. 19.4)

- The rms normalized residual for first-order directions near the eastern front of the Rocky Mountains is 8.7 percent higher than the rms normalized residual for all first-order directions. Similarly, the rms

normalized residual for second-order directions in this region is 4.5 percent higher than the rms normalized residual of all second-order directions.
- Relatively high normalized residuals occur along the west coast because the a priori standard errors assigned to the corresponding observations do not incorporate the uncertainties associated with the crustal motion models that were used to revise these observations.
- The normalized residuals for third-order direction observations are relatively high in all coastal areas, and they are relatively low inland. This pattern reflects a possible difference in quality between third-order surveys supporting hydrographic charting and third-order surveys supporting topographic mapping.
- The normalized residuals for fourth-order direction observations are low relative to the normalized residuals of higher order direction observations.
- The normalized residuals for first-order azimuth observations are relatively high in the more northern regions of the conterminous United States.
- Normalized residuals for lightwave distance observations are low relative to those for taped distances. Normalized residuals for microwave distance observations are high relative to those for taped distances.
- Although the standard error of unit weight for the NAD 83 adjustment is high at 1.35, the derived positions are thought not to be significantly biased except in a few locations.

Formal Error Statistics (sec. 19.5)

- The covariance matrix for adjusted parameters was computed for Alaska only.
- Absolute positional accuracies (latitude and longitude standard errors) obtained from the covariance matrix must be interpreted with caution.
- Absolute positional accuracies differ very little among first-, second-, and third-order stations.
- Relative positional accuracies are much weaker for Alaska than they are for most other parts of the United States.

Deflections and Theodolite Tilt as Error Sources (sec. 19.6)

- The relatively high residuals found near the eastern front of the Rocky Mountains may be explained in large part by an rms error on the order of 6.0 arc seconds in the determination of the angle between the theodolite's axis and the local normal to the ellipsoid.
- Predicted deflections of the vertical, as used for NAD 83, have a 1-sigma accuracy of about 2.0 arc seconds (that is, 1.5 arc seconds in each component). Theodolite tilts in mountainous regions are less than 5.0 arc seconds, if adopted field procedures were sustained.
- Standard errors on direction observations should have been assigned as a function of line slope for the NAD 83 adjustment.

Crustal Motion as an Error Source (sec. 19.7)

- NAD 83 coordinates may be biased by a few meters in the western part of the region affected by the 1964 Alaska earthquake.
- In California, errors in the crustal motion models contribute less than 2.2 ppm in rms scale error and less than 0.90 arc second in rms orientation error.

Station Heights as an Error Source (sec. 19.8)

- The standard errors assigned to distance observations were formulated so as to account for the uncertainty in station heights as well as to account for observational error.
- The NAD 83 adjustment employed Rapp 1978 geoid heights. Errors in these geoid heights would distort no distance derived from NAD 83 coordinates by more than 0.4 ppm.

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20. DISSEMINATION OF NAD 83 DATA

John F. Spencer, Jr.
Steven A. Vogel
William W. Wallace

20.1 INTRODUCTION

The task of data dissemination started as soon as the formal numerical task of the NAD 83 adjustment was completed. The National Geodetic Information Branch (NGIB) had been preparing for this task for several years. NAD 83 coordinates were distributed to subscribers of the NOAA Geodetic Control Automatic Mailing Service in March 1987. NGIB began publishing NAD 83 data sheets, including new state plane coordinates, station descriptions, and other related data elements in October 1987. These published data are available in quadrangle format, identified as NAD 83 horizontal geodetic control data quads. The first publication in this format was available for the region bordering the Gulf of Alaska in the Cold Bay area.

This chapter details the various aspects of disseminating NAD 83 data, including data availability and applications as they are now being assimilated into the surveying and mapping community.

20.2 GEODETIC DATA PUBLICATION: A HISTORICAL PERSPECTIVE

The National Geodetic Survey and its predecessors have distributed horizontal control data for more than 150 years. The results of the 1927 adjustment were first published in bound *Special Publications* by states (Dracup, 1976). These included volumes for Arizona, Arkansas, California, Colorado, Kansas, Louisiana, Michigan, Minnesota, Missouri, New Mexico, North Carolina, Oklahoma, Oregon, South Carolina, Tennessee, Texas, Utah, and Wyoming. In the late 1930s, lithoprint data were introduced in loose-leaf form, apparently to make updating easier. This involved separate listings for geographic positions, plane coordinates, and descriptions. Figures 20.1, 20.2, and 20.3 show examples of these formats, called "lithos." Some advantages to this format over the previous bound state volumes included the availability of all geodetic

Department of Commerce
Form 26 B

GEOGRAPHIC POSITIONS Rev 1/72 — *Accession No. of Computation:* 65444 — 296

Locality Berlin to Pocomoke City — *North American 1927 Datum* Second —order Triangulation. State Maryland

STATION	LATITUDE AND LONGITUDE	SECONDS IN METERS	AZIMUTH	BACK AZIMUTH	TO STATION	DISTANCE: Logarithm (meters)	Meters	Feet
Pocomoke City, north	38 04 13353	411.7	219 20 50.5	39 23 22.5	Parson	3.976040	9463.2	31,047
transmission tower, 1942 (d.)	75 34 47886	1167.2	255 42 36.3	75 47 26.6	Klej	4.073073	11832.4	38,820
			290 19 34.5	110 20 39.0	Holt	3.434394	2718.9	8,920
Pocomoke City, south	38 04 06168	190.2	218 00 21.2	38 02 50.4	Parson	3.980848	9568.6	31,393
transmission tower, 1942 (d.)	75 34 43447	1059.0	254 32 13.6	74 37 01.1	Klej	4.071319	11784.7	38,664
			286 29 40.9	106 30 42.6	Holt	3.405890	2546.2	8,354
Pocomoke City, municipal	38 04 10074	310.6	211 58 48.5	32 00 45.9	Parson	3.941789	8745.6	28,693
standpipe, 1932 (d.) r. 1942	75 33 51713	1260.5	253 20 59.1	73 25 14.8	Klej	4.022819	10539.5	34,578
			305 33 31.9	125 34 01.8	Holt	3.161593	1450.8	4,760
Pocomoke City, Mason	38 04 13195	406.8	217 28 38.7	37 31 00.9	Parson	3.965072	9227.3	30,273
Canning Co, tank, 1942 (d.)	75 34 32053	781.3	255 12 46.8	75 17 27.3	Klej	4.059187	11460.1	37,599
			293 28 33.9	113 29 28.6	Holt	3.372704	2358.9	7,739
Green Hill Lookout	38 07 06096	188.0	349 14 07.2	169 14 16.1	Smullen	3.273208	1875.9	6,155
Tower, 1942 (d.)	75 40 29.237	712.2	71 42 46.6	251 38 18.5	Noble	4.047424	11153.8	36,594
			206 56 35.6	26 57 16.8	Mitchell	3.554594	3585.9	11,765

[1] No check on this position. Abbreviations used: d.—described; m.—marked; n.—not; r.—recovered; l.—lost; p.—probably. (Examples: n. d.—not described; p. l.—probably lost.)

USCOMM—NOAA—ASHEVILLE

Figure 20.1. Sample data sheet of geographic positions on old format.

DEPARTMENT OF COMMERCE
U.S. COAST AND GEODETIC SURVEY

PLANE COORDINATES Revised: 11/5/42; 4/24/44 1/58 15

Delaware-Pennsylvania Circular Boundary

State Maryland Locality James River, Va. to Washington, D.C. North American 1927 datum. Projection Lambert Zone

Station	x Coordinate / y Coordinate	Plane azimuth	Mark	Station	x Coordinate / y Coordinate	Plane azimuth	Mark
	Feet				Feet	° ′ ″	
⊕ Boundary Monument No.4½,	1,155,984			Owens, (Va.), 1934	782,491.86	28 13 41 *	Azimuth Mark,
(Del.-Pa.), 1892 (n.d.)	711,021			(d.m.)	160,795.31		R.M. No. 1.
⊕ Boundary Monument No.5,	1,157,698			Cash, (Va.), 1934	734,135.02	264 45 07 *	Azimuth Mark,
(Del.-Pa.), 1892 (n.d.)	713,028			(d.m.)	173,584.71		R.M. No. 1.
⊕ Boundary Monument No.5½,	1,159,497			McDaniels, (Va.),1934	765,300.44	117 12 55 *	Azimuth Mark,
(Del.-Pa.), 1892 (n.d.)	714,959			(d.m.)	181,428.45		R.M. No. 3.
⊕ Boundary Monument No.6,	1,161,376			Brent Point, (Va.),	710,691.54	99 07 25 *	Azimuth Mark,
(Del.-Pa.), 1892 (n.d.)	716,810			1934 (d.m.)	205,509.38		R.M. No. 2.
⊕ Boundary Monument No.6½,	1,163,337			Diggs, 1934	758,058.96	152 58 47 *	Azimuth Mark,
(Del.-Pa.), 1892 (n.d.)	718,580			(d.m.)	201,681.88		R.M. No. 3.
⊕ Boundary Monument No.7,	1,165,371			Reservoir, (Va.), 1934	709,075.27		
(Del.-Pa.), 1892 (n.d.)	720,262			(d.m.)	252,187.40		
⊕ Boundary Monument No.7½,	1,167,476			Hilltop, 1934	764,056.49	266 39 38 *	Azimuth Mark,
(Del.-Pa.), 1892 (n.d.)	721,856			(d.m.)	237,931.36		R.M. No. 1.
⊕ Boundary Monument No.8,	1,169,647			Wig, (Va.), 1934	755,002.46	106 14 14 *	Azimuth Mark,
(Del.-Pa.), 1892 (n.d.)	723,357			(d.m.)	302,710.16		R.M. No. 2.
⊕ Boundary Monument No.8½,	1,171,882			Pomfret, 1934	790,935.76	204 27 06 *	Azimuth Mark,
(Del.-Pa.), 1892 (n.d.)	724,762			(d.m.)	271,010.70		R.M. No. 1.
⊕ Boundary Monument No.9,	1,174,174			Lacey, (Va.), 1934	754,074.00	141 50 08 *	Azimuth Mark,
(Del.-Pa.), 1892 (n.d.)	726,070			(d.m.)	328,622.99	308 16 57 *	R.M. No. 3 (Reset)

⊕ These stations are not to be used as basis for further triangulation.

(1941)

Figure 20.2. Sample of state plane coordinates on old format.

azimuths and lengths, and presentation of the data in a more orderly fashion with regard to the actual route of the survey and the area covered. Unfortunately, these advantages were more than offset by the numerous sheets of paper required to compile the information for a single station.

During the period from the mid-1930s to the mid-1950s, the number of stations published by the Coast and Geodetic Survey (C&GS) increased by a factor of 10. Since the cost of reprinting and publishing "lithos" was high, publishing and updating new data proceeded slowly.

In 1957 the Coast and Geodetic Survey (C&GS) reorganized its data publications into smaller groupings. A 30-minute by 30-minute quadrangle format was chosen. At the same time, the geographic positions, plane coordinates, and descriptions, which had been published as separate booklets under the litho format, were combined into a single data sheet. The new quad format contained a single sheet for each geodetic control station. As part of the process of creating a single data sheet, all of the data for a station except the description were put into computer-readable format. The earliest version of this key entry process created "9-cards," so called because the data for each station were contained on nine 80-character punch cards. Data from the "9-cards" were automatically printed on the right side of the new data sheet, and the station description was typed or pasted on the left side. (See fig. 20.4.)

The conversion to the quadrangle format was undertaken as a long-term project, to be accomplished systematically over a period of years. This task proceeded slowly but steadily until the early 1970s, when planning for the NAD 83 readjustment began. One of the first tasks of the new adjustment was to form an inventory of NGS' data holdings, and the files of published stations were the most reliable guide to that information. It was clear at the outset that all activities associated with the new adjustment would be performed by computer, and that a data base of these data holdings would be built to support the many computational and data processing activities associated with the new adjustment. NGS therefore recognized a requirement to place all the coordinates in machine-readable form. Since the conversion to quadrangle format was only partially complete at this time, this new requirement provided an impetus to accelerate and complete the conversion.

At the same time, NGS decided to perform key entry for the descriptions and for the supporting data accompanying other publications as well as for the coordinates. Furthermore, the publication data would also be carried in the single NGS data base. This

UNITED STATES COAST AND GEODETIC SURVEY
Descriptions of Triangulation Stations

Hudson River, Part V, Staatsburg to Malden, New York

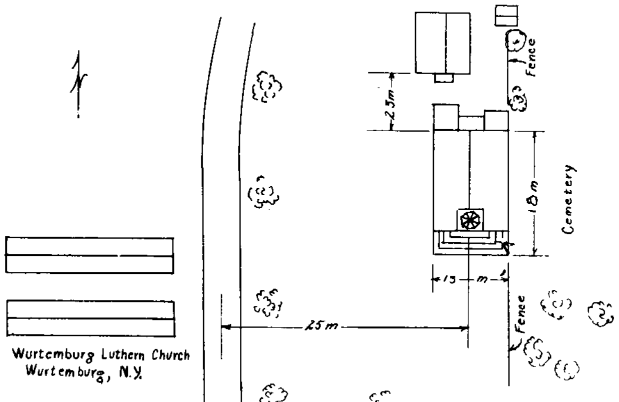
TRAVER (Dutchess County, New York,E.B.,1857;C.A.E.,1933)--Station searched for; unable to recover same. The hill on which this station is located has been under cultivation in recent years and the station was probably destroyed. New station established. (No description received.)

****

WURTEMBURG PRESBYTERIAN CHURCH (Dutchess County, New York, E.B.,1857;C.A.E.,1933)--As there is only one church in Wurtemburg, New York, which is the Wurtemburg Lutheran Church, it is believed that the description and sketch accompanying the records is of the spire located in 1857 and listed in the data as Wurtemburg Presbyterian Church. No original description.

****

WURTEMBURG LUTHERAN CHURCH (Dutchess County, New York,C.A.E., 1933)--This is the only church in Wurtemburg, New York. The station is a cupola on top of the church. The church is white with blue blinds and a shingle roof. There are two gold balls and a wind vane on top of the cupola. Distance and direction not computed nor observed upon.
Height of signal above station mark - 15 meters.

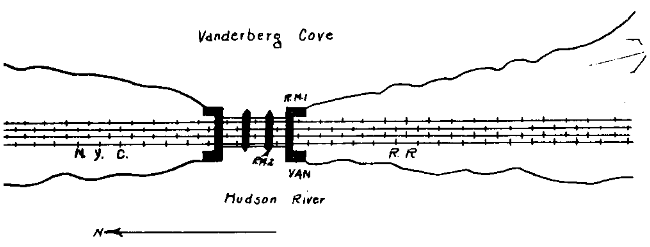


****

VAN (Dutchess County, New York,C.A.E.,1933)--This station is located on the E shore of the Hudson River on the W end of the S abutment to the railroad bridge over the stream connecting Vanderberg Cove with the Hudson River. The station is marked by a standard disk cemented in a drill hole in the top of the abutment 2 feet from the W edge of the abutment and about 2 feet from the N edge of the abutment. The abutment has a large crack in it.
Reference mark No.1 is a standard disk in the E end of the same abutment and reference mark No.2 is a similar disk in the W end of the S pier of the bridge. Unable to occupy reference marks.

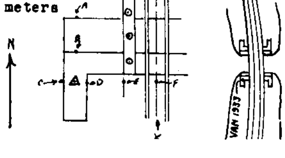
OBJECT	DISTANCE	DIRECTION
CAVE 2	meters	0°00'00".0
Esopus Meadows Lt. Az. mk.		15 42 45.0
R.M.No.1	17.58	209 07 19.9
R.M.No.2	11.69	126 41 22.5
R.M.No.1 to R.M.No.2	19.76	

Height of signal above station mark - 3.0 meters.
Height of telescope above station mark - 2.0 meters.



VAN (Dutchess County, New York,C.A.E.,1933;R.C.B.,1935)--Recovered in good condition as described.
Additional measurements were taken, as per sketch, to assist in locating station on the aerial photographs.

Distances to Station
A - 2.0 meters
B - 0.9 meter
C - 0.8 meter
D - 0.1 meter
E - 1.2 meters
F - 2.8 meters



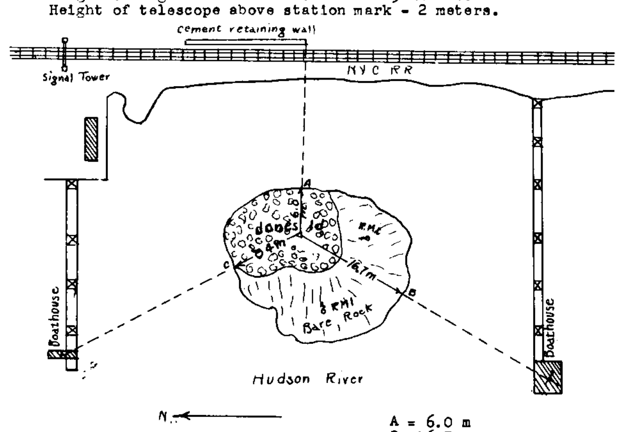
JONES ISLAND (Dutchess County,New York,E.B.,1857;C.A.E., 1933)--Not recovered. The description of this old station gives a sketch and states that the station consists of a pole secured with an iron cone. It is believed that this was done by E. Blunt in 1857. The entire top of the island was spaded up but no iron cone could be found. A new station named "JONES ISLAND 2, 1933" was established on the top of the island.

****

JONES ISLAND 2 (Dutchess County,New York,C.A.E.,1933)--This station is located on the highest part of Jones Island in about the same place as old station JONES ISLAND 1857 which could not be recovered. This island consists of a large solid rock with earth on the top located on the E side of the Hudson River near the NW end of Vanderberg Cove. The station is marked by a standard disk stamped "JONES ISLAND 2, 1933" cemented in a drill hole in a large rock buried flush with the surface of the ground on the highest part of the island.
Reference marks No.1 and 2 are standard disks stamped "JONES ISLAND 2, 1933" cemented in drill holes in the bare rock ledge on the island.

OBJECT	DISTANCE	DIRECTION
SUNOCO	meters	0°00'00".0
Esopus Meadows L.H.Az.Mk.		271 06 33.1
R.M.No.1	8.00	287 56 09.4
R.M.No.2	9.94	226 46 45.1
R.M.No.1 to R.M.No.2	9.25	

Height of signal above station mark - 3 meters.
Height of telescope above station mark - 2 meters.

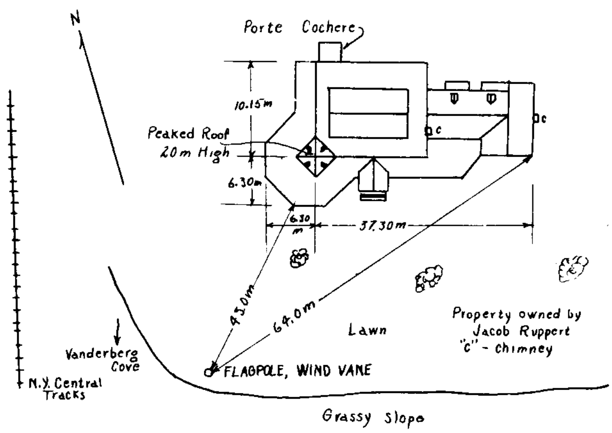


A = 6.0 m
B = 16.7 m
C = 8.4 m

JONES ISLAND 2 (Dutchess County,New York,C.A.E.,1933;R.C.B., 1935)--Recovered in good condition as described. Description is adequate.

****

FLAGPOLE, WIND VANE (Dutchess County,New York,C.A.E.,1933)--This station consists of a flagpole with wind vane on top located on the lawn in front of Jacob Ruppert's home just N of Vanderberg Cove, near Jones Island. It is about 2 miles S of Rhinecliff, New York. The house is white and green with a slate roof with some red slate. The house has a sharp pointed 4-sided roof. Sketch gives further details.
Height of signal above station mark - 25 meters.



FLAGPOLE, WIND VANE (Dutchess County,New York,C.A.E.,1933; R.C.B.,1935)--The flagpole has broken off at the base (about 1 year ago) and only the concrete base now remains. Mr. Ruppert stated that a new flagpole would be placed in exactly the same location in the near future. Station is lost.

Figure 20.3. Sample description of triangulation stations on old format.

JULY 1976
U.S. DEPARTMENT OF COMMERCE
ENVIRONMENTAL SCIENCE SERVICES ADMINISTRATION
COAST AND GEODETIC SURVEY

HORIZONTAL CONTROL DATA

by the
Coast and Geodetic Survey
NORTH AMERICAN 1927 DATUM

FLORIDA VOL II Page 1021
QUAD 300861 STATION
AIA-FLA
LATITUDE 30° 30′ TO 31° 00′
LONGITUDE 86° 00′ TO 86° 30′
DIAGRAM NH 16-5 PENSACOLA

NOAA FORM 76-39
(12-70)
FORMERLY C&GS FORM 525

U.S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL GEODETIC NETWORKS

DESCRIPTION OF TRIANGULATION STATION

NAME OF STATION: CAIN 2 STATE: Florida COUNTY: Okaloosa
NEAREST TOWN: Niceville QUADRANGLE NO. 300861
CHIEF OF PARTY: C.L. Novak YEAR: 1975 DESCRIBED BY: R.P. Konrady

NOTE.* HEIGHT OF TELESCOPE ABOVE STATION MARK 1 METERS.† HEIGHT OF LIGHT ABOVE STATION MARK 1 METERS.

NOTE.*	SURFACE-STATION MARK, UNDERGROUND-STATION MARK				
1b					
7a					
	DISTANCES AND DIRECTIONS TO AZIMUTH MARK, REFERENCE MARKS AND PROMINENT OBJECTS WHICH CAN BE SEEN FROM THE GROUND AT THE STATION				
	OBJECT	BEARING	DISTANCE FEET	DISTANCE METERS	DIRECTIONS ° ′ ″
	Eglin Field Auxiliary Field 2 Water Tank 1942 (finial)	SW	Approx. 1.5 miles		00 00 00.0
11b	RM 3	NE	37.71	11.494	175 31 54
11b	AZ MK	NE	Approx. 0.3 mile		184 46 23.9
11b	RM 4	SW	37.19	11.335	355 31 28
	RM 3 to RM 4		74.89	22.827	

Detailed description:

Station is about 14.5 miles southeast of Crestview, 8.5 miles northeast of Niceville, 1.75 miles northeast of Auxiliary Field 2 and on State Highway right-of-way property.

To reach the station from the junction of State Highways 85 and 20 in Niceville, go east on State Highway 20 for 0.8 mile to the junction of State Highway 285 on left. Turn left and go northeast on State Highway 285 for 5.8 miles to a crossroad. Continue northeast on State Highway 285 for 1.65 miles to the top of rise of hill and the station on left.

Station mark, a standard disk stamped CAIN 2 1975, is set in the top of a 12-inch cylindrical concrete monument that is set flush the the ground surface. It is 194 feet northwest of power line pole EAA 44, 86 feet northwest of the center of State Highway 285, 2 feet northeast of a metal witness post and 2 feet southwest of a metal witness post. The underground mark is set in the top of an irregular mass of concrete 3.6 feet below the ground surface and stamped CAIN 2 1975.

Reference mark 3, a standard disk stamped CAIN 2 NO 3 1975, is set in the top of a 12-inch cylindrical concrete monument that is set flush with the ground surface. It is 206 feet northwest of power line pole EAA 44, 88 feet northwest of the center of State Highway 285, and 1.5 feet west of a metal witness post.

Reference mark 4, a standard disk stamped CAIN 2 NO 4 1975, is set in the top of a 12-inch cylindrical concrete monument that is set flush with the ground surface. It is 189 feet northwest of power line pole EAA 44, 84 feet northwest of the center of State Highway 285 and 2.3 feet northeast of a metal witness post.

Azimuth mark, a standard disk stamped CAIN 2 AZ 1975, is set in the top of a 12-inch cylindrical concrete monument that is projecting 3-inches above the ground surface. It is 77.5 feet southeast of the center of State Highway 285, 73 feet southwest of power line pole EAA 49, 1 foot northwest of the right-of-way line and 6-inches northwest of a metal witness post.

To reach the azimuth mark from the station, go northeast on State Highway 285 for 0.3 mile to the azimuth mark on right.

*Refers to notes in manuals of triangulation and state publications of triangulation. †Direction-angle measured clockwise, referred to initial station.
†To nearest meter only, when no trigonometric leveling is being done.

ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: CAIN 2
STATE: Florida YEAR: 1975 First ORDER
LOCALITY
SOURCE: G-10823 FIELD SKETCH
No observation check on this position

	° ′ ″			
GEODETIC LATITUDE	30 35 56.38236	ELEVATION	56	METERS
GEODETIC LONGITUDE	86 25 59.65788		scaled	FEET

STATE COORDINATES (feet)

STATE & ZONE	CODE	X	Y	θ (OR Δ α) ANGLE* ° ′ ″
FL N	0903	1,391,762.40	586,654.68	- 0 58 17

*PLANE AZIMUTH HAS BEEN COMPUTED BY THE θ (OR Δ α) FORMULA NEGLECTING THE SECOND TERM

TO STATION OR OBJECT	GEODETIC AZIMUTH (From south)	PLANE AZIMUTH* (From south)	CODE
CAIN 2 1975 AZ MK	208° 28′ 08.″7	209° 26′ 26″	0903

Position determined by traverse from station CAIN 1950

Figure 20.4. Example of horizontal control data provided to requesters until the late 1970s.

decision opened the way for complete automation of the publication process. It also provided the tools by which NGS could develop a consistent view of its data holdings.

In 1973, the "9-cards" were condensed to a "3-card" format. The remaining 3-cards needed to complete the conversion to quads were keypunched and verified by in-house personnel during 1973 and 1974. When this task was finished, this file contained more than 700,000 80-character records.

In order to match all data files for a station—3-cards, descriptions, and observations—a unique identifier called a QID/QSN (quad identifier/quad stations number) was established. The QID identified the 30-minute quad, and the QSN was the station number within that quad. The unique identifier not only assured that the matching process would be valid, but also provided the basis for an automated publication system in the future.

The description files contained an estimated 10 million 80-character records. The task of keying these data would require more than 50 staff years of effort (Wallace, 1979). This figure did not include time for data preparation, coding, and post-keying validation and editing. The key entry of the station description information was perceived and planned as a project which could be more appropriately accomplished by contract than by in-house personnel. As early as 1974, pilot projects with the Veterans Administration and the Federal Prison Industries determined that keying the descriptions under contract was feasible and practicable.

In 1975, the project to key station descriptions received a boost in the form of unexpected funding. In that year, the U.S. Congress provided funds to assist economically depressed cities under the Title X program. This program would fund various government projects as long as the work was carried out in one of the specified cities. NGS applied for and received $700,000 under this program for keying station descriptions. The initial contract was with Steele Data Processing Corporation in Detroit, Michigan. More than one-half of the station descriptions and recovery notes in the historical files were keyed under this contract.

The contractor used key-to-disk equipment for this operation and produced a final magnetic tape only when all reformatting and validation were completed. An error rate of 0.3 percent or less was required. This 1-year contract provided NGS with more than 500 magnetic tapes, each holding approximately 10,000 records which would form the horizontal station description portion of the NGS data base.

Over the next 3 years (1978-80), several contracts provided the means to complete the keying of historical horizontal descriptive data. Although these data were relatively error-free, further processing was necessary to make them conform with the standards prescribed in *Input Formats and Specifications of the National Geodetic Survey Data Base*, informally known as the "Blue Book" (Pfeifer, 1980; revised FGCC, 1989). This process, called "extraction," is discussed in section 20.4. The processing of new descriptions and recovery information is an ongoing procedure. The goal is to complete the extraction of data from existing records and update the NGS data base without accumulating any backlog.

By digitizing station descriptions, NGS produced a complete sheet by an automated process for the first time. This was done by combining information from the position and description files in the data base. Figure 20.5 shows the resultant data sheet. During the early 1980s, preceding the final stages of the NAD 83 adjustment, this was the horizontal geodetic data product disseminated to users.

20.3 GEODETIC NETWORK DIAGRAMS

Project sketches provide important documentation and are always included in the final report of a survey project (Spencer and Collom, 1980). These sketches are developed during the planning and execution phases of the survey project. After completion of a project, they provide a graphical depiction of the locations of the geodetic control marks. By showing the observations, they also provide the user with an indication of how strongly each control station is tied to other stations in the network.

Several methods have been used to provide a consolidated representation of all geodetic control in a given geographical area. Prior to 1927, the networks were shown only for coastal areas using nautical chart bases published by C&GS. They were also shown on annual status maps depicting the entire U.S. horizontal network on a single sheet.

After the 1927 adjustment, densification surveys were performed on a large scale by various Federal, state, and local governments and private organizations, greatly increasing the total number of control points. The geodetic control diagram was introduced to assist users by showing all geodetic control for a given area. There have been several series of these diagrams. The major series, which evolved in the late 1950s, provides horizontal and vertical control network information overlaid on U.S. Geological Survey (USGS) 1:250,000 base maps. Because it provides an index of geodetic control over a large geographic area, this series has been very popular with users of geodetic data.

Another series of geodetic control diagrams exists for coastal regions. It depicts horizontal control on the nautical chart bases of the National Ocean Service (formerly the U.S. Coast and Geodetic Survey). These chart bases are typically at much larger scales than the 1:250,000 USGS bases. Scales of 1:40,000 and 1:80,000 are common. These larger scales are advantageously used, since the geodetic control networks tend to be denser and more congested in coastal areas. Another series covering Alaska uses much smaller scale (1:500,000) aeronautical chart bases produced by the National Ocean Service to portray control in less congested areas (Spencer and Nussear, 1986).

Geodetic control diagrams have been published by manual cartographic processes. The revision of a diagram by manual means typically requires 2 to 3

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US DEPARTMENT OF COMMERCE - NOAA                                                     PAGE 011
NOS - NATIONAL GEODETIC SURVEY          HORIZONTAL CONTROL DATA          QUAD N26080111  QSN 0007
ROCKVILLE MD 20852 - JUL 1980          NORTH AMERICAN DATUM 1927         CONTROL DIAGRAM NG 17-5
                                     PROJECT ACCESSION NUMBER 14831      FL-PALM BEACH COUNTY

HORIZONTAL CONTROL STATION:  PANOS
GEODETIC POSITION DATA----------------------    STATION INFORMATION------------------------------------------
                    DEG MIN SECONDS                 TYPE:  1ST-ORDER TRIANGULATION
          LATITUDE:     26 54 25.49203N             OBSERVATIONS BY NATIONAL GEODETIC SURVEY IN 1970
          LONGITUDE:    80 04 03.96078W             ADJUSTED BY NATIONAL GEODETIC SURVEY IN 1979
          AZIMUTH 1:   343 16 53.4  FROM SOUTH      AZIMUTH REFERENCE OBJECT 1:   GOLF 2 1970

          ELEVATION:        2.4  METERS             ELEVATION DETERMINED BY TRIGONOMETRIC LEVELING
                            7.9 FEET
          GEOID HEIGHT:     7.0 METERS

STATE PLANE AND UNIVERSAL TRANSVERSE MERCATOR COORDINATE SYSTEMS---------------------------------------------
              X           Y        X/EASTING    Y/NORTHING   POINT SCALE  CONVERGENCE  GRID AZIMUTH 1  GRID AZIMUTH 2
 GRID  ZONE   FEET        FEET     METERS       METERS       FACTOR       DEG MIN SEC  DEG MIN SEC     DEG MIN SEC
FL-TM  E    803816.04   936473.43  245003.618   285437.672   1.00004700   +0 25 18.9   342 51 34.5*
 UTM   17                          592571.725  2976317.703   0.99970579   +0 25 18.9   162 51 34.5*
 UTM   18                          -3611.639   2986073.223   1.00273260   -2 17 53.6   165 34 47.0*

                                                         *CAUTION - ARC-TO-CHORD CORRECTION ASSUMED ZERO

STATION DESCRIPTION--------------------------------------------------------------------------------------
ORGANIZATIONS MARK:  NATIONAL GEODETIC SURVEY                                HEIGHT OF TELESCOPE:  23.7 METERS
YEAR DESCRIBED:  1970    CHIEF OF PARTY:  CLH    REACHED BY:  CAR            PACK TIME:  00 HRS 00 MIN

CODE-- MARK-------- TYPE OF MARK---------------- SETTING/LANDMARK TYPE----------------------------- MAGNETIC PROPERTY
       SURFACE       TRIANG STA DISK      SET INTO THE TOP OF A ROUND CONCRETE MONUMENT
D04    UNDERGROUND   SURVEY DISK          SET INTO THE TOP OF AN IRREGULAR MASS OF CONCRETE        UNKNOWN
D08    REFERENCE     SURVEY DISK          SET INTO THE TOP OF A ROUND CONCRETE MONUMENT            UNKNOWN
L13    REFERENCE     LANDMARK             LIGHTHOUSE

  CODE------ REFERENCE OBJECT------- HEADING------------- DISTANCE------------ DIRECTION------- MAGNETIC PROPERTY
  D08    GOLF 2                                                               000 00 00.0      UNKNOWN
  D08    PANOS RM 2                  NW            77.01 FEET  23.470 MTRS    169 42 02        UNKNOWN
  L13    JUPITER INLET LIGHTHOUSE    N     ESTIM APPROX 4 MI                  179 11 49.2      UNKNOWN
  D08    PANOS RM 1                  SE            82.08 FEET  25.019 MTRS    348 35 19        UNKNOWN
           PANOS RM 1
         TO PANOS RM 2                            158.95 FEET

STATION IS LOCATED ABOUT 4 MILES SOUTH OF JUPITER INLET COLONY AND 2 MILES NORTH OF JUNO BEACH ON THE WEST RIGHT-OF-WAY
OF U.S. HIGHWAY 1.

TO REACH FROM THE JUNCTION OF U.S. HIGHWAYS 1 AND A1A AT THE NORTH END OF BRIDGE OVER JUPITER INLET, GO SOUTH ON U.S.
HIGHWAY 1 FOR 3.3 MILES TO STATION ON RIGHT AS DESCRIBED.

STATION MARK, A STANDARD DISK STAMPED PANOS 1970, IS SET IN THE TOP OF A ROUND CONCRETE POST WHICH IS FLUSH WITH THE
SURFACE OF THE GROUND.  THE MARK IS 138 FEET WEST OF A CONCRETE POWER LINE POLE NO. 1908, 36 FEET WEST OF THE CENTER
LINE OF THE SOUTH BOUND LANE OF U.S. HIGHWAY 1 AND 4.7 FEET SOUTH OF A METAL WITNESS POST.

                                     (CONTINUED ON NEXT PAGE)
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Figure 20.5. Example of first computer-generated horizontal control data sheet.

months. In the mid-1970s many of the cartographic resources used to revise these diagrams were diverted to other projects supporting the NAD 83 adjustment. As a result, many diagrams were allowed to become out of date by 15 years or more years.

At the same time planning began on the production of geodetic control diagrams by computer-assisted cartography. It was known that both station positions and observations were being put into machine-readable form for the NAD 83 adjustment. These were exactly the data that were needed for the geodetic control diagrams. Furthermore, these data would be available through the NGS data base. Hardware and software were still needed for the drafting tasks.

In 1984 NGS arranged to share an Intergraph computer-aided drafting system with the Nautical Charting Division of the Office of Charting and Geodetic Services. Specialized software was still needed to manage the data and to implement those functions which were particular to the geodetic control diagram series. This software was completed soon after the NAD 83 adjustment, giving NGS the capacity to produce more than 100 updated geodetic control diagrams per year. Unfortunately, at the time of this report, budgetary constraints forced suspension of the production process, and the cartographic staff was transferred to other duties.

Because all data are in machine-readable form, several different products become technically feasible. For example, an overlay registered to the USGS base could be furnished on Mylar to fit over the user's own base map. Alternatively, NGS could furnish data and programs suitable for execution on a personal computer, so that each user could select the appropriate options and have just the portion of the network of immediate interest drawn on the computer screen. NGS will continue to consider the format of a new standard diagram product. In the meantime, until existing inventories are depleted, requests for the old geodetic control diagrams will be filled.

20.4 PUBLICATION OF NAD 83 DATA

NGS began publishing NAD 83 data in two formats. First, listings of final adjusted coordinate data (fig. 20.6) were published in blocks of 1 degree of latitude by 2 degrees of longitude, corresponding to areas depicted on the geodetic control diagrams. These listings include other numerical data associated with each station, such as plane coordinates and elevations.

Next, NGS began the publication of NAD 83 data sheets. Today's sheets include comprehensive recovery information, a complete description of the station's location, and the adjusted coordinate data. Data sheets are grouped into 7½-minute quadrangles corresponding to the U.S. Geological Survey 1:24,000 scale map series. The production of this second publication format is more complex, requiring that station description information be updated, corrected, and merged with NAD 83 coordinate data.

Although the adjustment has been completed, validating and entering the computer-readable station descriptions into the NGS data base, a task begun in 1978, are still continuing. This process is extremely labor-intensive, often necessitating extensive manual searches to uncover discrepancies, duplicates, or voids in the information record for a given station.

The extraction process involves reviewing and correcting the station description and recovery information. The data elements include the text and codes used to describe the following: the surface and underground markers used for a control point; how the mark is set in the ground; magnetic properties of the station mark which may aid in locating it; the mode of transportation used to reach the station, and any "pack time" (the time required to carry equipment from the last point of transportation) to the station; distinguishing features of a man-made landmark used as a survey point; and the condition of the station when it was recovered for use in various survey projects. In addition, azimuths embedded in the descriptive text are converted to compass bearings during the extraction process. An extensive set of standards for these data elements are contained in the NGS Blue Book (Pfeifer, 1980).

The persons verifying this descriptive information use software programs to help identify apparent duplicate, missing, or incorrect information. In the case of man-made landmarks, a single survey point may be described in different surveys as a water tower, water tank, standpipe tank, or elevated tank. Here, the description checking program may indicate mismatched information, but it remains for human minds to discern whether this information is in fact mismatched, or whether it refers to a single point which has been given different designations in different survey projects. After verifying the apparent errors uncovered by the checking software, the station description information is visually reviewed and edited for any remaining errors and updated with recent recovery information before being released for final publication.

Each data sheet of the NAD 83 horizontal geodetic control data quads contains the following: latitude and longitude, the designations of the 7½-minute quadrangle in which the station is located and the geodetic control diagram on which is it shown, the position classification (order of accuracy), the accession number of the project in which the station's position is determined, the surveying method used to determine the position, the elevation, state plane coordinates, distances and directions to reference objects, azimuths to azimuth marks, a complete description of the station's locations, and comprehensive station recovery information.

In addition to the above information, the NAD 83 data sheet contains the following elements useful for a large variety of geodetic data applications: geoid height value, deflection of the vertical at the station, the latitude and longitude shift at the station in seconds (NAD 27 minus NAD 83), point position accuracy referenced to the center of mass of the Earth (when available), Universal Transverse Mercator

NOAA - NOS - C&GS
NATIONAL GEODETIC SURVEY
May 1989

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Geodetic Control Diagram

WASHINGTON

NAD 83 GEODETIC AND STATE PLANE COORDINATES

QIDQSN	STATION NAME	LATITUDE (NORTH)	LONGITUDE (WEST)	NORTHING METERS*	EASTING METERS*	ZONE	CONVERGENCE	SCALE FACTOR	ELEV. (M)	GEOID HGT (±1 M)	POS QUAL
380764430043	EDGE	38 50 9.17488	076 52 34.09965	129795.014	410753.854	MD	0 04 39.86	0.9999501	81.00	- 32.6	2
380764110037	EDGE 1933	38 57 3.38819	076 33 34.71733	142652.269	438170.781	MD	0 16 34.98	0.9999507	1 SC	- 32.6	3
380771130013	EDSALL	38 48 8.81025	077 09 48.45125	2126916.134	3616091.543	VA N	0 50 2.97	0.9999536	75.54	- 32.5	3
380761130066	EDWARD MSFC 1909	38 51 43.22050	076 10 36.22145	133009.830	471450.847	MD	0 31 0.17	0.9999499	1 SC	- 33.4	3
380763130040	EDWARDS	38 22 28.75870	076 44 36.87725	78621.214	422406.896	MD	0 09 39.38	0.9999879	43.10	- 33.5	1
380772430003	EDWARDS 1934	38 19 55.70158	077 23 49.33528	2074445.272	3596431.237	VA N	0 41 18.16	0.9999607	75 SC	- 32.6	1
380763420024	EEDLING 1908	38 19 20.07985	076 51 25.01207	72781.833	412509.395	MD	0 05 23.22	0.9999962	1 SC	33.3	3
	EEDLING 1908	38 19 20.07985	076 51 25.01207	2074053.337	3643666.354	VA N	1 01 31.65	0.9999616	1 SC	- 33.3	3
380762210037	EGG 1901	38 08 28.98974	076 00 32.30033	53167.914	486878.037	MD	0 37 19.21	1.0000314	0 SC	- 35.7	3
380761230062	ELEANOR MSFC 1910	38 31 54.30163	076 12 54.36167	96321.924	468435.842	MD	0 29 33.47	0.9999678	2 SC	- 34.4	3
380771110287	ELECTRIC USE 1932	38 54 6.82827	077 03 44.00493	137117.694	394602.629	MD	-0 02 20.59	0.9999500	1 SC	- 32.4	3
	ELECTRIC USE 1932	38 54 6.82827	077 03 44.00493	2138087.319	3624711.205	VA N	0 53 50.43	0.9999607	1 SC	32.4	3
380771420011	ELEVEN OAKS 1957	38 50 14.18070	077 18 31.62796	2130607.855	3603419.090	VA N	0 44 36.45	0.9999557	141.7	32.5	2
380771410012	ELKINS PB AND PP 1930	38 59 13.99019	077 15 15.29883	2147314.727	3607927.641	VA N	0 46 38.98	0.9999692	93 SC	- 32.4	2
	ELKINS PB AND PP 1930	38 59 13.99019	077 15 15.29883	146618.119	377972.395	MD	-0 09 34.47	0.9999517	93 SC	- 32.4	2
380774320001	ELKWOOD 1942	38 30 46.22450	077 49 13.70758	2094143.003	3559262.878	VA N	0 25 26.77	0.9999500	107.1	- 32.4	2
380762120006	ELLIOTT 1910	38 18 33.19637	076 01 1.67276	71789.195	485962.230	MD	0 37 0.77	0.9999984	2 SC	- 35.1	3
380762120005	ELLIOTT CHURCH	38 18 33.58146	076 00 7.25201	71815.413	487284.178	MD	0 37 34.93	0.9999984		- 35.1	4
380771120054	ELLSWORTH	38 48 30.09787	077 04 9.04424	2127695.890	3624269.778	VA N	0 53 34.80	0.9999539	0 SC	- 32.5	3
	ELLSWORTH	38 48 30.09787	077 04 9.04424	126734.940	393991.442	MD	-0 02 36.31	0.9999505	0 SC	- 32.5	3
380761130085	ELMORE MSFC 1909	38 46 0.29781	076 13 46.23590	122396.154	466959.332	MD	0 29 0.91	0.9999517	1 SC	- 33.0	3
380761410059	END	38 59 37.92853	076 17 8.36449	147567.730	461882.718	MD	0 26 54.05	0.9999520		- 32.4	4
380772120014	END 1918	38 19 51.50635	077 01 41.77724	2074768.047	3628671.860	VA N	0 55 6.72	0.9999608	4 SC	- 33.1	3
	END 1918	38 19 51.50635	077 01 41.77724	73741.417	397528.063	MD	-0 01 3.88	0.9999948	4 SC	- 33.1	3
380772120014	END 1918	38 19 51.50635	077 01 41.77724	1222708.214	3628688.373	VA S	0 53 35.62	1.0000868	4 SC	- 33.1	3
380764440124	END USE 1931	38 52 33.46311	076 58 32.89642	134237.172	402099.513	MD	0 00 54.67	0.9999498	10.29	- 32.4	3
	END USE 1931	38 52 33.46311	076 58 32.89642	2135329.652	3632254.228	VA N	0 57 4.60	0.9999585	10.29	- 32.4	3
380772430004	ENNIS 1934	38 16 38.86333	077 28 51.36945	2068291.790	3589163.743	VA N	0 38 9.66	0.9999659	74 SC	- 32.5	1
380772430005	ENNIS 2	38 16 39.20258	077 28 51.17810	2068302.301	3589168.277	VA N	0 38 9.78	0.9999659	74.01	- 32.5	2
380762330032	ENOUGH MSFC 1908	38 07 0.92131	076 23 48.87137	50155.529	452888.345	MD	0 22 42.67	1.0000369	2 SC	- 34.7	3
	ENOUGH MSFC 1908	38 07 0.92131	076 23 48.87137	1199998.125	3684416.455	VA S	1 16 35.11	1.0000309	2 SC	- 34.7	3
380762330032	ENOUGH MSFC 1908	38 07 0.92131	076 23 48.87137	2052089.792	3684407.358	VA N	1 18 45.28	0.9999863	2 SC	- 34.7	3
380771110246	ERICSSON PB AND PP 1932	38 53 11.95319	077 03 0.71887	135424.950	395644.671	MD	-0 01 53.43	0.9999499	3 SC	- 32.4	3
	ERICSSON PB AND PP 1932	38 53 11.95319	077 03 0.71887	2136411.813	3625780.781	VA N	0 54 17.45	0.9999594	3 SC	- 32.4	3
380764410003	ESSEX 1958	38 57 26.07255	076 51 57.85383	143268.213	411608.235	MD	0 05 2.61	0.9999509	58 SC	- 32.4	2
380771110016	ETHAN ALLEN PB PP 1934	38 55 33.54953	077 06 59.84553	2140688.649	3619952.661	VA N	0 51 48.20	0.9999628	71.9	- 32.4	1
	ETHAN ALLEN PB PP 1934	38 55 33.54953	077 06 59.84553	139796.420	389887.292	MD	-0 04 23.51	0.9999502	71.9	- 32.4	1
380761230055	ETTA MSFC 1910	38 32 2.55975	076 11 55.16457	96589.003	469867.301	MD	0 30 10.62	0.9999675	0 SC	- 34.4	3
380774430003	EVERGREEN ERDL 1959	38 47 24.93734	077 59 18.43964	2124842.033	3544441.470	VA N	0 19 9.35	0.9999529	199.0	- 32.6	2
380771110290	F B 1913	38 53 1.81415	077 02 49.42744	135112.163	395916.633	MD	-0 01 46.34	0.9999499	1 SC	- 32.4	3
	F B 1913	38 53 1.81415	077 02 49.42744	2136103.509	3626057.821	VA N	0 54 24.49	0.9999592	1 SC	- 32.4	3
380771120201	F USE 1913	38 52 14.16880	077 01 6.96725	133642.110	398385.724	MD	-0 00 42.03	0.9999499	4 SC	- 32.4	3
	F USE 1913	38 52 14.16880	077 01 6.96725	2134673.986	3628550.626	VA N	0 55 28.44	0.9999581	4 SC	- 32.4	3
380771120056	F USE 1924	38 48 39.50213	077 02 24.43604	2128025.569	3626788.689	VA N	0 54 40.09	0.9999541	3 SC	- 32.5	2
	F USE 1924	38 48 39.50213	077 02 24.43604	127023.411	396515.396	MD	-0 01 30.65	0.9999505	3 SC	- 32.5	2

* For conversion of meters to U.S. Survey Feet multiply the meters by 39.37/12.0 which is 3.28083333333 to 12 significant figures

* For conversion of meters to International Feet multiply the meters by 100.0/30.48 which is 3.28083989501 to 12 significant figures

Figure 20.6. Geodetic and state plane coordinates on NAD 83.

DEPARTMENT OF COMMERCE
NOAA - NOS - C&GS

NATIONAL GEODETIC SURVEY
PUBLISHED: MARCH 1989

NORTH AMERICAN DATUM 1983
HORIZONTAL CONTROL DATA

GEODETIC DATA

STATION NAME: ELKWOOD

STATE: VIRGINIA
COUNTY: CULPEPER

GEODETIC DEG MIN SEC

LATITUDE: 38 30 46.22450 N
LONGITUDE: 077 49 13.70758 W

CLASSIFICATION: SECOND ORDER

QUAD: N38077432
CONTROL DIAGRAM: NJ 18-4
WASHINGTON
USGS QUAD SHEET: REMINGTON
PROJECT ACCESSION NUMBER: G17289

		STD. ERROR	
GEODETIC AZIMUTH:	NONE		AZIMUTH MARK: NONE
ELEVATION: (ABOVE GEOID)	107.1 METERS	±0.3 METER	ELEVATION DETERMINED BY TRIGONOMETRIC LEVELING AND REFERRED TO NGVD 29.
GEOID HEIGHT:	-32.4 METERS	±1 METER	GEOID HEIGHT IS BASED ON RAPP'S OSU 78 MODEL
DEFLECTION OF THE VERTICAL:			
MERIDIAN	+0.8 SEC	±0.4 SEC	
PRIME VERTICAL	+6.0 SEC	±0.9 SEC	

THE HORIZONTAL COORDINATES WERE ESTABLISHED BY CLASSICAL GEODETIC METHODS AND ADJUSTED BY THE NATIONAL GEODETIC SURVEY IN JULY 1986. EARLIEST OBSERVATIONS WERE MADE BY THE COAST AND GEODETIC SURVEY IN 1942.

SHIFT AT THIS STATION IN SECONDS (NAD 27 MINUS NAD 83): LATITUDE -0.44996 LONGITUDE +1.02354

STATE PLANE AND UNIVERSAL TRANSVERSE MERCATOR COORDINATES

GRID	ZONE	NORTHING METERS	EASTING METERS	POINT SCALE FACTOR	CONVERGENCE DEG MIN SEC	GRID AZIMUTH TO MARK (FROM NORTH)* DEG MIN SEC
SPC	VA N	2094143.003	3559262.878	0.99995004	+ 0 25 26.8	
UTM	18	4266488.949	254081.784	1.00034482	- 1 45 25.8	

*ARC-TO-CHORD CORRECTION NOT APPLIED

STATION MARKS AND REFERENCE OBJECTS

CODE	REFERENCE OBJECT	HEADING	DISTANCE	DIRECTION
	ELKWOOD AZ MK	SW	APPROX. 0.45 MI	000 00 00.0
	ELKWOOD AZ MK 2	SW	APPROX. 0.45 MI	000 37 23.5
	ELKWOOD RM 2	NW	28.685 METERS	122 05 36
	ELKWOOD RM 1	NE	126.99 FEET	199 09 08
	REMINGTON WARRENTON TNG CEN	E	APPROX. 2.0 MI	235 33 31.0

THE SURFACE STATION MARK IS A SURVEY DISK SET INTO THE TOP OF A SQUARE CONCRETE MONUMENT.
THE UNDERGROUND STATION MARK IS A SURVEY DISK SET INTO THE TOP OF AN IRREGULAR MASS OF CONCRETE.

(CONTINUED ON NEXT PAGE)

1

Figure 20.7. Sample data sheet for NAD 83.

CONTINUED FROM PREVIOUS PAGE

STATION NAME: ELKWOOD STATE: VIRGINIA

STATION MARK HISTORY

YEAR RECOVERED OR DESCRIBED	CONDITION OF MARK	RECOVERED OR DESCRIBED BY (CHIEF OF PARTY)
1942*	STATION ESTABLISHED	COAST AND GEODETIC SURVEY (PLB)
1958*	GOOD	COAST AND GEODETIC SURVEY (LFV)
1964*	GOOD	COAST AND GEODETIC SURVEY (JCB)
1965*	GOOD	COAST AND GEODETIC SURVEY (ELH)
1971*	GOOD	NATIONAL GEODETIC SURVEY (LFS)

*SEE PUBLISHED TEXT

STATION DESCRIPTION

DESCRIBED BY THE COAST AND GEODETIC SURVEY IN 1942 (PLB).

STATION IS ABOUT 1.75 MILES E OF ELKWOOD RAILROAD STATION, AND ABOUT 0.75 MILE S BY W FROM REMINGTON RAILROAD STATION, ON LAND OWNED BY MR. H.K. PORTIS. STATION IS 155 FEET SW OF THE SW CORNER OF HOUSE, 15 FEET NW OF THE NW CORNER OF THE CHICKEN HOUSE. IT IS STAMPED ELKWOOD 1942, AND PROJECTS ABOUT 4 INCHES.

SURFACE, UNDERGROUND, REFERENCE, AND AZIMUTH MARKS ARE STANDARD BRONZE DISKS SET IN CONCRETE, AS DESCRIBED IN NOTES 1A, 7A, AND 11A.

REFERENCE MARK NO. 1 IS 30 FEET S OF THE SW CORNER OF THE HOUSE AND 30 FEET NW OF THE NW CORNER OF A SMALL BARN ABOUT 1 FOOT S OF THE PICKET FENCE AROUND THE HOUSE. IT IS STAMPED ELKWOOD NO. 1 1942, AND PROJECTS ABOUT 5 INCHES.

REFERENCE MARK NO. 2 IS ON THE W EDGE OF THE FIELD 10 FEET W OF THE CENTER WIRE OF THE THREE WIRE HIGH TENSION TRANSMISSION LINE, 99 FEET S OF POLE NO. SB R 68. IT IS STAMPED ELKWOOD NO. 2 1942, AND PROJECTS ABOUT 6 INCHES.

AZIMUTH MARK IS 0.4 MILE NE OF THE JUNCTION OF COUNTY ROADS 673 AND 674 ON THE W SIDE OF COUNTY ROAD 673, 17 PACES W OF THE CENTER LINE OF THE ROAD.

TO REACH FROM BRANDY RAILROAD STATION, WHICH IS ON U.S. HIGHWAYS 29 AND 15, GO EASTERLY ON U.S. HIGHWAY 15 AND 29 FOR 2.5 MILES TO ELKWOOD POST OFFICE ON THE LEFT AND RAILROAD STATION ON THE RIGHT, LEAVE U.S. HIGHWAYS 15 AND 29, TURN RIGHT ACROSS RAILROAD TRACKS, THEN LEFT AND FOLLOW COUNTY ROAD 674 E AND SE FOR 1.7 MILES TO CROSSROADS, THEN TURN LEFT, AND GO NE ON GRAVEL COUNTY ROAD 673 FOR 0.8 MILE TO MR. PORTIS HOUSE ON THE RIGHT AND WOODEN GATE ON THE LEFT, TURN LEFT THROUGH THE GATE AND FOLLOW FARM ROAD NORTHERLY FOR 0.2 MILE TO A YELLOW FARMHOUSE, THEN CONTINUE PAST THE FARMHOUSE FOR ABOUT 50 YARDS TO STATION.

HEIGHT OF LIGHT ABOVE STATION MARK - 27 METERS.

STATION RECOVERY

REPORTED BY THE COAST AND GEODETIC SURVEY IN 1958 (LFV).

STATION, AZIMUTH AND REFERENCE MARKS RECOVERED IN GOOD CONDITION. ANGLE BETWEEN REFERENCE MARKS AND DISTANCES TO REFERENCE MARKS CHECKED AND VERIFIED. DESCRIPTION ADEQUATE AND CORRECT EXCEPT FOR THE FOLLOWING.

STATION IS 45.5 FEET NORTHWEST OF NORTHWEST CORNER OF A CHICKEN HOUSE AND 16 FEET EAST OF TWIN 8 INCH CEDAR TREES.

AZIMUTH MARK IS 74 FEET SOUTHWEST OF FENCE CORNER ON EAST SIDE OF COUNTY ROAD 673, 17 FEET WEST OF C/L OF ROAD, ABOUT 3 FEET HIGHER THAN ROAD, AND SET IN AN OLD FENCE LINE.

(CONTINUED ON NEXT PAGE)

Figure 20.7. Sample data sheet for NAD 83 (continued).

coordinates, and standard errors associated with many of the values listed above. NAD 83 data also include, for each 7½-minute quadrangle, an explanation of the terms and codes that are used. Figure 20.7 shows a sample data sheet.

20.5 PUBLICATION PRIORITY FOR NAD 83 DATA SHEETS

Publication priority depends on several factors, including the degree of economic development and the level of field survey activity in a given area. Data for Alaska were published first, based on these priorities and the needs of a large-scale mapping program in the state. Data for Florida were recently published. Publication of data for the Gulf Coast States is now underway. Figure 20.8 shows the publication schedule by geographic areas. Data sheets based on NAD 83 will be prepared for all of the United States over a 5-year period (Spencer and Bishop, 1986).

20.6 PUBLICATION TECHNOLOGIES

What are the considerations for distributing such a large volume of data to a large and diverse population of users? To answer this question, consider the following: Since the last general adjustment of 1927, the size of the horizontal network within the National Geodetic Reference System has increased more than tenfold. Originally NAD 27 contained 25,000 points. It has gradually increased by extension and densification to a 270,000-station network.

Based on samplings of computer-generated data sheets published in the 7½-minute quad format, similar to figure 20.7, an average of 2-¼ pages of published data is required for each station. Therefore, one set of the NAD 83 published results would comprise approximately 600,000 pages, enough to fill 17 file cabinets. For NAD 27 data, an average of 200 copies was printed for each data sheet. Assuming the same demand for NAD 83 data sheets leads to a requirement for 120 million pages, occupying nearly 10 miles of storage space. Fortunately, the demand for paper copy is expected to lessen in the future, to be replaced by demand for data on magnetic tape and floppy diskettes.

The increased cost of printing and handling these data led NGS to investigate alternative publication methods, including micrographics and laser printing technologies. NGS anticipates the following benefits by applying these methods in the future (Spencer, 1988).

Computer-generated micrographics (reformatted digital data produced on microform) significantly reduce output requirements and associated material costs. The largest cost savings are realized by reduced physical storage and data handling. For example, to store one copy of the entire NAD 83 results, only half of one cabinet, instead of the previously mentioned 17 cabinets, would be needed.

With high-speed laser printers, it is possible to print individual data sheets and quadrangles on demand. This eliminates the need for mass printing, storage, and data handling (including manual file maintenance and data retirement). NGS expects to produce most of its future paper products by this method.

A new technology which may be appropriate for publication of geodetic data is the CD-ROM (compact disk, read only memory). A single diskette would hold the geodetic data for a large area of the country (Spencer and Bishop, 1986).

Without computer technology the NAD 83 adjustment would have been impossible, as well as many new information processes. The primary advantage of computer technology is flexibility, which in this case provides diversified products and customized services to the user (Spencer, 1979).

20.7 USERS OF NAD 83 DATA

In the last few decades the users of horizontal control data have become more diverse. In addition to the direct user in the surveying and mapping communities, many users of surveying and mapping services now have an interest in horizontal geodetic data. Included are those involved in endeavors such as satellite tracking and data collection, urban and regional planning, natural resource development and management, environmental hazard reduction programs, land information systems, and nationwide transportation, navigation, and communication systems. Scientific users have become interested in the use of geodetic control for the positioning of precise satellite tracking systems and for the analysis of horizontal crustal motion.

As the number and types of horizontal control users have increased, so have their accuracy needs. Many users now require relative positions accurate to 1:100,000 where 1:25,000 was once satisfactory. Furthermore, many now want to know the accuracy of relative and absolute positions.

The NAD 83 horizontal geodetic control data sheets contain several data elements that were not previously available. These include the geoid height and deflection of the vertical at each station, its standard error, the accuracy of the position referenced to the center of mass of the Earth (when available), and the accuracies of adjusted azimuths and distances to nearby points. These new data elements are provided for the use of a new class of users, composed of those who have the requirements, the understanding, and the proper tools to make use of these data elements.

20.8 NOAA AUTOMATIC MAILING SERVICE

A subscriber to NOAA's Automatic Mailing Service (AMS) purchases the latest geodetic data for a specified area and then automatically receives a notice of availability concerning revised or new data for that particular area (Spencer and Horn, 1981). Subscribers also automatically receive information flyers announcing other geodetic products. This service eliminates the need for users to check periodically with NGS to

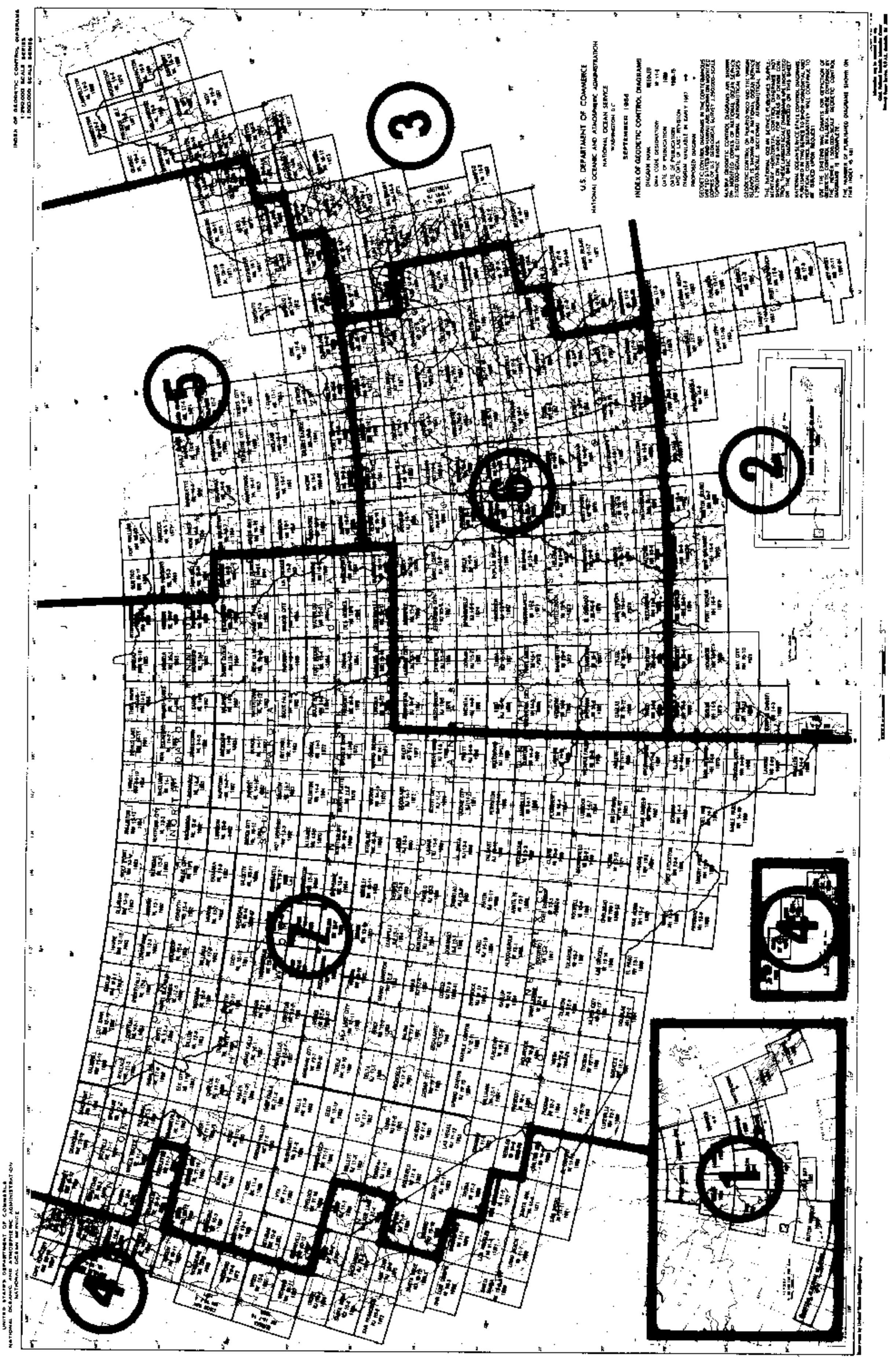

Figure 20.8. Publication priority for NAD 83 data by geographic area.

NATIONAL OCEAN SERVICE
CHARTING AND GEODETIC SERVICES
NATIONAL GEODETIC SURVEY

AUTOMATIC MAILING SERVICE

The National Geodetic Survey (NGS) announces an improved mailing service to users of geodetic data and related products. NGS notifies users of newly published geodetic data in areas they specify and issues flyers announcing other geodetic products. To receive notificaton of products, please complete the information below and mail to the address on the back of this flyer.

Name ____________________

Address ____________________

____

Area Code and Telephone Number ____________________

Indicate categories for which you wish to receive notification:

_______ Horizontal Geodetic Data: Coordinate Lists, Horizontal Control Quads, and Survey Project Data

_______ Vertical Geodetic Data: Vertical Control Quads, and Survey Project Data

_______ Gravity Data

_______ Geodetic Control Diagrams

_______ Calibration Base Lines

_______ Software and Digital Data

_______ Publications

Indicate your area of interest for these data categories by providing a detailed description (e.g., states, counties, 30' quadrangle area, or latitude and longitude boundaries) or by attaching a small map.

The procedure for ordering NGS products is provided on the back of this flyer. Products may be ordered at any time once NGS notifies users of their availability.

Figure 20.9. Automatic Mailing Service agreement used for purchasing geodetic data.

NATIONAL OCEAN SERVICE
CHARTING AND GEODETIC SERVICES
NATIONAL GEODETIC SURVEY

Information Flyer 86-13

DOCUMENTS AVAILABLE ON NAD 83

NOAA's National Geodetic Survey (NGS) has completed the final computation of a North American horizontal geodetic datum, designated the North American Datum of 1983 (NAD 83). This achievement, based on the adjustment of a quarter of a million points, provides a unified, consistent network of latitude and longitude values for the entire North American continent. This network is used by regional planners, engineers, surveyors, navigators, geophysicists, and a variety of other professionals who depend on accurate and reliable horizontal reference data.

NGS has numerous publications describing various aspects and applications of NAD 83. A sampling of these is listed below. To order any of these publications, complete the requested information and mail with payment to:

National Geodetic Information Branch
N/CG174, Rockwall Building, Room 24
National Geodetic Survey, NOAA
Rockville, Maryland 20852
Telephone: 1-301-443-8631

Prepayment is required. Make check or money order payable to: NOAA, National Geodetic Survey. Payment may also be made by VISA, American Express, or MasterCard. For orders sent outside the United States, a 25% surcharge must be added to the prices listed below to cover additional postage.

( ) NOAA Completes North American Datum Readjustment, by Vogel, S.A. 1986. 4 pp. $1.00. No. of copies ordered ________.

( ) The New Horizontal Control Datum for North America: NAD 83, by Vogel, S.A. 1986. 34 pp. $2.00. No. of copies ordered ________.

( ) Impact of North American Datum of 1983, by Wade, E.B. 1986. 14 pp. $1.00. No. of copies ordered ________.

( ) NAD 83 Publication, by Spencer, J.F. and Bishop, W.R. 1986. 8 pp. $1.00. No. of copies ordered ________.

( ) Alaska Test of the Helmert Blocking Phase of the North American Datum, by Vorhauer, M.L., and Wade, E.B. 1985. 10 pp. $1.00. No. of copies ordered ________.

( ) The North American Datum of 1983; Collection of Papers Describing the Planning and Implementation of the Readjustment of the North American Horizontal Network, 1983. 48 pp. $2.50. No. of copies ordered ________.

( ) Datum Transformation from NAD 27 to NAD 83, by Wade, E.B., and Doyle, D.R. 1987. 9 pp. $1.00. No. of copies ordered ________.

(continued)

Figure 20.10. NGS flyer describing NAD 83 publications (first page).

determine whether updated survey information in the region of interest is available and then to place a separate order for the data. There is no charge for AMS membership. Subscribers pay only for data requested. This service provides the primary means by which the NAD 83 results are being distributed to users and cooperative affiliates.

Users subscribe to the AMS by completing the appropriate NOAA form. (See fig. 20.9.) Copies of the form are available from NGS at the following address:

National Geodetic Information Branch
N/CG174, Rockwall Building, Room 24
National Geodetic Survey, NOAA
Rockville, Maryland 20852
Telephone 1-301-443-8631

20.9 AVAILABILITY OF NAD 83 DATA PRODUCTS

In addition to the two formats of NAD 83 data mentioned previously, NGS provides software for use with NAD 83 data and publications describing various aspects of the adjustment. NGS also sponsors workshops on interpreting and usings the data. Figure 20.10 depicts an information flyer which lists publications available on NAD 83.

20.10 REFERENCES

Dracup, Joseph F., 1976: "National Geodetic Survey Data: Availability, Explanation, and Application." *NOAA Technical Memorandum* NOS NGS-5 (out of print).

Pfeifer, Ludvik, 1980, rev. NGS, 1989: *Input Formats and Specifications of the National Geodetic Survey Data Base*, Volume I: "Horizontal Control Data." National Geodetic Information Branch, NOAA, Rockville, MD 20852, 250 pp.

Spencer, J. F., 1979: "The New Adjustment of the North American Datum, Article No. 16: Publication and Distribution of Adjustment Results." *Bulletin of the American Congress on Surveying and Mapping*, No. 66, pp. 31-32.

Spencer, J. F., 1988: "Availability of NAD 83 Products." *1988 ACSM-ASPRS Annual Convention Technical Papers*, Vol. 1, Surveying, pp. 195-205.

Spencer, J. F. and Bishop, W. R., 1986: "NAD 83 Publication." *Proceedings of the ASPRS-ACSM Fall Convention*, pp. 380-387.

Spencer, J. F. and Collom, J. M., 1980: "The New Adjustment of the North American Horizontal Datum, Article No. 21: Geodetic Network Diagrams." *Bulletin of the American Congress on Surveying and Mapping*, No. 71, pp. 31-33.

Spencer, J. F. and Horn, N. B., 1981: "The New Adjustment of the North American Horizontal Datum, Article No. 25: Automatic Mailing Service." *Bulletin of the American Congress on Surveying and Mapping*, No. 75, p. 50.

Spencer, J. F. and Nussear, R. K., 1986: "Geodetic Control Diagrams." *1986 ASPRS-ACSM Fall Convention Technical Papers*, pp. 170-180.

Wallace, William W., 1979: "The New Adjustment of the North American Horizontal Datum, Article 14: Digitization of Station Descriptions." *Bulletin of the American Congress on Surveying and Mapping*, No. 64, p. 19.

21. USER PARTICIPATION AND IMPACT

James E. Stem

21.1 INTRODUCTION

The direct users of the National Geodetic Reference System (NGRS) fall into three general categories. The primary user is the geodetic surveyor/engineer who relies on NGRS not only for project scale and orientation but overall confirmation as to the correctness of the survey. Secondary users rely on the surveys of primary users to produce a multitude of cartographic products. Tertiary users are the many and varied organizations actively coding land use information with coordinate information for applications within automated mapping and data base systems.

In addition, other users indirectly benefit from and support the system, although they do not directly use the data products. This group includes teachers of surveying and geodesy, researchers, consultants, and vendors of software products.

NGS attempted to involve interested users and supporters in planning and implementing NAD 83. A policy, initiated in 1975, publicized the solicitation of geodetic survey observations from public and private agencies for inclusion in the new adjustment. A 1977 *Federal Register* notice announced the plan to develop the State Plane Coordinate System of 1983 (SPCS 83), a plan requiring user participation. And in 1981, initial technical guidance was developed that described a methodology for performing the transformation of coordinates from NAD 27 to NAD 83. These three programs were designed to facilitate user familiarity and acceptance.

Over the entire decade of the project the interest of users and supporters was maintained as NGS geodesists authored NOAA reports on the new adjustment and wrote articles for professional geodetic and surveying journals. From the beginning of the project, it was clear that NAD 83 would impact all who used coordinate information.

21.2 USER PARTICIPATION

21.2.1 Project and Data Submission

As discussed in chapter 6, in the early 1970s the NGS Horizontal Branch prepared new surveys for digitizing in the TRAV10 format. TRAV10 was selected as the format into which all geodetic data were placed for the adjustment. At the time NGS policy dictated that the Horizontal Network Branch would process geodetic surveys of agencies that adhered to requirements set forth by NOAA. Some of these requirements were documented, while others were verbal. NGS did not require that the observational data be digitized. Paper-copy field records were accepted in any format and then digitized by NGS. This policy, however, proved to be costly and was discontinued in 1975.

Two factors affected NGS' decision to accept only digital data for contributed projects. First, the volume of contributed projects was increasing and, second, available resources were limited and needed to be directed to NAD 83. Recognizing the merit to be derived from these project submissions, NGS wanted to encourage others to perform and submit even more surveys. Because funds were not appropriated for keying, NGS could no longer perform the task in-house. Believing that the analysis, adjustment, and publication of received data would more than compensate a contributing agency for the cost of placing the survey data in computer-readable form, NGS decided this responsibility belonged to the contributor.

In 1975 NGS released the first draft of *Input Formats and Specifications of the NGS Data Base*. An updated draft, titled *NOAA Manual* NOS NGS 2, was prepared in 1978 for in-house use. This version was not officially released to the public. However, when the manual was again revised in 1980, it was officially released as a three-volume Federal Geodetic Control Committee (FGCC) publication. Volume I was titled "Horizontal Control Data" (Pfeifer, 1980). Later the same year volume II, "Vertical Control Data" (Pfeifer and Morrison, 1980) was released, and in 1985, volume III, "Gravity Control Data" (Dewhurst, 1985) was published.

In 1989 the FGCC published a major revision of volume I (Federal Geodetic Control Committee, 1989). This latest issuance includes appropriate references to NAD 83, provisions for submitting Global Position System surveys, and a new station description format that is applicable to all control points regardless of the methodology used to position the station.

To facilitate updates, the three-volume FGCC manual is published in a blue loose-leaf notebook and is unofficially called the "Bluebook." The terms and conditions for using the Bluebook were documented in a policy statement titled "Policy of the National Ocean Service with Regard to the Incorporation of Geodetic Data of other Organizations in the National Geodetic Data Base." The policy addressed the following subjects: format, accuracy, monumentation, field records, project reports, and reconnaissance review. By addendum to this policy, contributing agencies were kept informed of the date after which submitted projects could not be included within the simultaneous adjustment of NAD 83. Throughout the period of the new adjustment the policy statement, which accompanies each Blue Book, has remained the document describing the conditions under which NGS would analyze, adjust, and publish surveys of other organizations, and the datum on which this would be done.

NGS input formats and specifications were designed to make the National Geodetic Survey data base the repository of geodetic surveys that have been connected to NGRS. At the time these formats were conceived, it was recognized that Federal agencies housed primary sources of surveys that needed to be included in the NAD 83 project. Therefore, NGS assisted FGCC agencies to prepare their data for submission. Between 1974 and 1981 a total of 1,071 projects containing 83,243 stations were completed for inclusion in NAD 83, of which two-thirds were contributed in Bluebook format by public agencies. A detailed list follows.

Organization	No. of stations	Projects
National Geodetic Survey, NOS	32,767	293
Atlantic Marine Center, NOS	1,352	52
Pacific Marine Center, NOS	2,696	97
U.S. Geological Survey	14,914	255
State highway departments	10,221	231
Other state organizations	4,650	33
Defense Mapping Agency	731	8
International Boundary Commission	7,052	20
Bureau of Land Management	763	29
Other organizations	8,097	53
Total	83,243	1,071

Public agencies viewed the use of the Bluebook as an opportunity to obtain consistency between their survey projects and NGRS. This consistency was most easily obtained if the NGS data base contained their survey observations.

Consequently, surveys were received from state agencies—generally departments of transportation or departments of natural resources—from regional utility companies, and from local governments—often departments of public works. Although some private firms submitted projects directly to NGS, generally they were under contract from a public agency. Although most projects were submitted as a unit of work as observed in the field, some submissions were a composite of many projects, especially from agencies submitting surveys from their archives. Use of the Bluebook provided many public agencies with their first chance to prepare for NAD 83.

21.2.2 Development of the State Plane Coordinate System of 1983

NGS realized that the geodetic positions of all stations would change as a result of the redefinition and the readjustment. These changes presented an opportunity to readdress the SPCS. Prior to NAD 83, the SPCS had been a system of map projections, projecting the ellipsoid of NAD 27 onto a plane. NGS had to decide whether to select either the identical map projection system or a different one to derive NAD 83 plane coordinates. The new system would be identified as the State Plane Coordinate System of 1983 (SPCS 83), and the existing system renamed the "State Plane Coordinate System of 1927" (Stem, 1989).

Several alternatives were considered for SPCS 83. Some advocated retaining the design of the existing SPCS by retaining the projection types, boundaries, and defining constants. Others believed that a system based on a single projection type should be adopted. The single projection proponents contended that the present SPCS was cumbersome, since three projection types involving 127 zones were employed.

The single projection concept was evaluated with respect to the following criteria: ease of understanding, ease of computation, and ease of implementation. Initially it appeared that the Universal Transverse Mercator (UTM) system would be the best solution because the grid had long been established, to some extent was being used, and the basic formulas were identical in all situations. However, on further examination, the UTM 6-degree zone widths presented several problems that might impede its overall acceptance by the surveying profession.

For example, to accommodate the wider zone width, a grid scale factor of 1:2,500 exists on the central meridian, while a grid scale factor of 1:1,250 exists at zone boundaries. Similar grid scale factors on SPCS 27 rarely exceeded 1:10,000. In addition, the "arc-to-chord" correction term which converts observed geodetic angles to grid angles is larger with UTM. Finally, the UTM zone definitions did not coincide with state or county boundaries.

These problems were not viewed as critical, but most surveyors and engineers considered the existing SPCS 27 the simpler system and found UTM unacceptable, primarily because of its rapidly changing grid scale factors.

NGS also evaluated the transverse Mercator projection with zones of 2 degrees in width. This grid met the primary conditions of a single national system. By reducing the zone width, the grid scale factor and the arc-to-chord correction would be no worse than in SPCS 27. The major disadvantage of the 2-degree transverse Mercator grid was that the zones being defined by meridians rarely fell along state and county boundaries. The 2-degree grid could be modified to accommodate zones following county lines, but several of the larger counties would require two zones. The average number of zones per state was not decreased by this approach.

Three dominant factors emerged for retaining the SPCS 27 design. The SPCS had been accepted by legislative action in 37 states, the grids had been in use for more than 40 years, and most surveyors and engineers were familiar with the definitions and procedures for their use. Except for academic considerations, SPCS 27 was fundamentally sound. With the availability of electronic calculators and computers, little merit was found in reducing the number of zones or the number of projection types employed. There was merit in minimizing the number of changes to SPCS legislation. For these reasons, NGS decided to retain the basic design of SPCS 27 in SPCS 83, and to publish UTM coordinates for those users who preferred that system. Both grids are now fully supported by NGS for surveying and mapping.

The decision that NGS would publish NAD 83 coordinates in an SPCS 83 system, designed similar to SPCS 27, was first published in the *Federal Register* on March 24, 1977 (FR Doc.77-8847). The notice declared, "[SPCS] ...will consist of the same projections and defining parameters as published in USC&GS *Special Publication* 235 (1974 revision) [Mitchell and Simmons, 1945, rev. 1974] and legally adopted in 35 states, except for the following changes:

1. The grid will be marked on the ground using the 1983 NAD.
2. Distances from the origin will be expressed in meters and fractions thereof. One additional decimal place should be used for the metric expression of a value previously expressed in feet.
3. The arbitrary numeric constant presently assigned to the origin will be unchanged but will be considered as meters instead of feet, except for the following: If a state elects to have a different constant(s) assigned to the origin so that the 1983 NAD plane coordinates will appear significantly different from the 1927 NAD positions, when considering the overall system, then the National Geodetic Survey will consider changing the origin constant. If the state so elects, it must amend its legislation to accommodate this change.
4. Michigan's Transverse Mercator system will be eliminated in favor of the legislatively approved Lambert system.
5. Projection equations will be programmed such that the maximum computing error of a coordinate will never exceed 0.1 mm when computing the coordinate of a point within the zone boundaries.

From April 1978 through January 1979, NGS solicited comments on this policy by canvassing board members of the National Council of Engineering Examiners, all individual land surveyor members of each board, the secretary of each section and affiliate of the American Congress on Surveying and Mapping (ACSM), and state and local public agencies. As of August 1988, the 1978-79 solicitation and articles on the subject had produced committees or liaison contacts in 43 states. Through these contacts, NGS presented the options to be considered in zone delineation of SPCS 83, and options in adopting the defining mathematical constants for each zone.

Although most states left unchanged the list of counties that comprised a zone, three states—South Carolina, Montana, and Nebraska—elected to have a single zone cover the entire state, replacing what had been several zones on SPCS 27. In these states the grid scale factor correction to distances now exceeds 1:10,000, and the arc-to-chord correction to azimuths and angles may become significant. A zone definition change also occurred in New Mexico, due to creation of a new county, and in California where zone 7 of the SPCS 27 was incorporated into zone 5 of SPCS 83.

Several states chose to modify one or more of the defining constants of their zones. Some of these changes increase the magnitude of the grid scale factor and arc-to-chord correction terms. All grid origins were changed because they are defined in meters within SPCS 83 and new values were adopted. This new grid origin was selected by the states based on the following criteria:

- Keep the number of digits in the coordinate to a minimum.
- Create a new range for easting and/or northing in meters on NAD 83 that would not overlap the range of *X* and/or *Y* in feet on the existing NAD 27. If an overlap could not be avoided, the location of the band of overlap (i.e., where the range of *X* and/or *Y* on the 1927 datum intersects the range on the 1983 datum) could be positioned anywhere through the selection of an appropriate grid origin.
- Select different grid origins (either in northing or easting) for each zone so the coordinate user could determine the zone from the magnitude of the coordinate. This usually required the easting origin to be the smallest in the easternmost zone to avoid easting values close in magnitude for points near boundaries of adjacent transverse Mercator zones. It required the northing origin of the northernmost zone to be the smallest for adjacent Lambert zones for the same reason.
- Create different orders of magnitude for northing and easting to reduce the possibility of transposition errors.

The grid origin selection influenced only the appearance of the coordinate system, but not its accuracy or usefulness.

Prior to the beginning of the new adjustment, 37 states had passed acts creating an SPCS, the first in 1935. As of August 1988, 42 states had enacted SPCS 27 legislation, most recently Illinois, New Hampshire, North Dakota, South Carolina, and West Virginia. Of these five, only Illinois did not simultaneously include the definition of SPCS 83 within its SPCS 27 legislative authority. In addition, as of August 1988, 26 states had also enacted legislation approving SPCS 83.

In almost all states, the SPCS 83 legislation was prepared and pursued by the states' societies of professional land surveyors. In a few states the departments of transportation initiated the legislation. In about half the states, two submissions to the state legislature were required. Many state societies are still actively pursuing SPCS 83 legislative approval.

For many professionals, especially those outside the surveying and mapping community, the discussion of SPCS 83 was their first introduction to NAD 83. In society meetings, legislative committees, and state legislature sessions, the justification articulated for SPCS 83 remained the same as for NAD 83. Plans for implementing NAD 83 sometimes developed as a by-product of the review process by state agencies. Because this task was delegated to the states, a significant portion of NGS users became aware of NAD 83 through the SPCS 83 design process.

21.2.3 Geographic Coordinate Transformations

The final datum shifts appear in figures 21.1 through 21.8. These shifts are similar to those predicted in 1979 (Vincenty, 1979), but show greater detail. The major portion of the datum shift is due to the change in shape, origin, and orientation of the reference ellipsoid. However, the small local wiggles in the contour lines represent distortions which were present in NAD 27.

The contour maps contain all the necessary information, but cannot be read with sufficient resolution to satisfy the need of primary users. There still remained the question of how the datum shift should be computed (or approximated) for any particular purpose.

The greatest interest in this question naturally occurred as the project neared completion and when NAD 83 values were actually disseminated. For much of the NGS user community, the answer to this question was the extent of their interest in NAD 83. This group encompassed many secondary and tertiary users of the horizontal portion of the NGRS, and most did not perform geodetic surveys. The issue of transforming coordinates between datums was the only interaction between NGS and this sector of NGS users.

Three general approaches to datum transformations were described in the policy statement of 1980. The approach selected depended on the accuracy requirement of the conversion, geographical coverage, and the amount of supporting data and resources available. NGS provided consultation to assist users in selecting the appropriate approach to conversion. As part of the technology transfer process, NGS held 1-day workshops on datum transformations. In its most popular year, 1987, 20 workshops were held.

21.2.3.1 Transformation Using Original Data

The first and most accurate approach to transformation required availability of the original observations from which the 1927 coordinates were derived. This approach required readjusting traverses and surveys to obtain agreement with NAD 83 constraints. Readjustment of the user's project could be performed either by the surveyor or by NGS. If NGS performed the readjustment, as it preferred, submission of the project in Bluebook format was required.

This cooperative program served a twofold purpose. First, it channeled projects to NGS for inclusion in the readjustment and, upon completion of the project, it provided a mechanism to update surveys not originally submitted for the new adjustment.

21.2.3.2 LEFTI

The second approach promulgated by NGS was a similarity transformation. Although more sophisticated techniques were considered, NGS believed a similarity transformation using four parameters was adequate in almost all cases. Accordingly, NGS provided advice on the applicability of similarity transformations and developed software, program LEFTI (Vincenty, 1987), to perform the four-parameter similarity transformation.

The user either purchased the source code for LEFTI or submitted digitized data for processing by NGS. The input format for LEFTI was incorporated in the transformation policy statement. The user supplied a digitized file of coordinates to be transformed from the local system and the NAD 83 values for at least four of those points. LEFTI performed a least squares solution for the rotation angle between the coordinate systems, a single scale factor between coordinate systems, and the translations of the X and Y coordinates. A measure of the validity of the transformation was obtained from examination of the residuals. LEFTI augmented the similarity transformation as transformed points were additionally translated based on the residuals of nearby stations obtained from the least squares solution.

21.2.3.3 Simplified Transformation

The third conversion approach promulgated by NGS and described in the 1980 transformation policy statement was the simplified transformation. By this method, an average coordinate shift determined from points common to both datums could be applied. The datum shifts for an area could be obtained from tables that specify the datum shift at the corners of 7½ minute quadrangles (fig. 21.9) or from published station information. The tables were prepared for the U.S. Geological Survey 7½ minute quadrangle sheets. NGS did not recommend interpolation within these tables, since the variation in the datum shift with position was not smooth. In addition, these tables were obtained using LEFTI and believed to be accurate to approximately 1 m.

21.2.3.4 NADCON

Dewhurst (1990) provided a different approach to the transformation problem, one with a significant improvement in accuracy over other methods, as well as increased simplicity. This method, referred to as NADCON (North American Datum CONversion), relied upon a simultaneous model of the shift values for a large region, such as the conterminous United States, in order to obtain estimates on a regularly spaced grid. From these estimates, local modeling using a low-order polynomial (equivalent to bilinear interpolation) can be used to obtain shift "correctors" applicable to NAD 27 referenced coordinates. In addition, through the application of successive iteration, it was possible to perform a transformation in either direction, from NAD 27 to NAD 83 or vice versa. Consistency between results was accomplished through the use of low residual tolerances (e.g., the convergence criteria) within the NADCON FORTRAN code. Thus it was possible to obtain unique coordinates in either datum, transform the results, and obtain the original values once again.

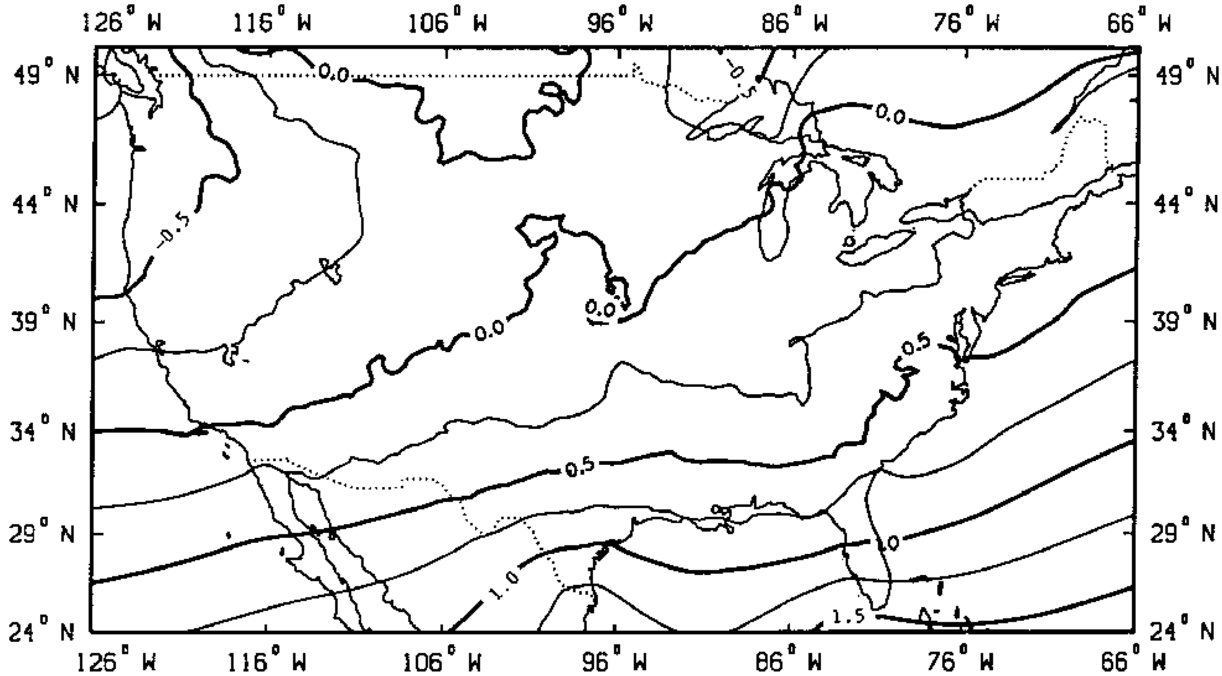

Figure 21.1. Latitude datum shift in the conterminous United States
in seconds of arc (NAD 83 minus NAD 27).

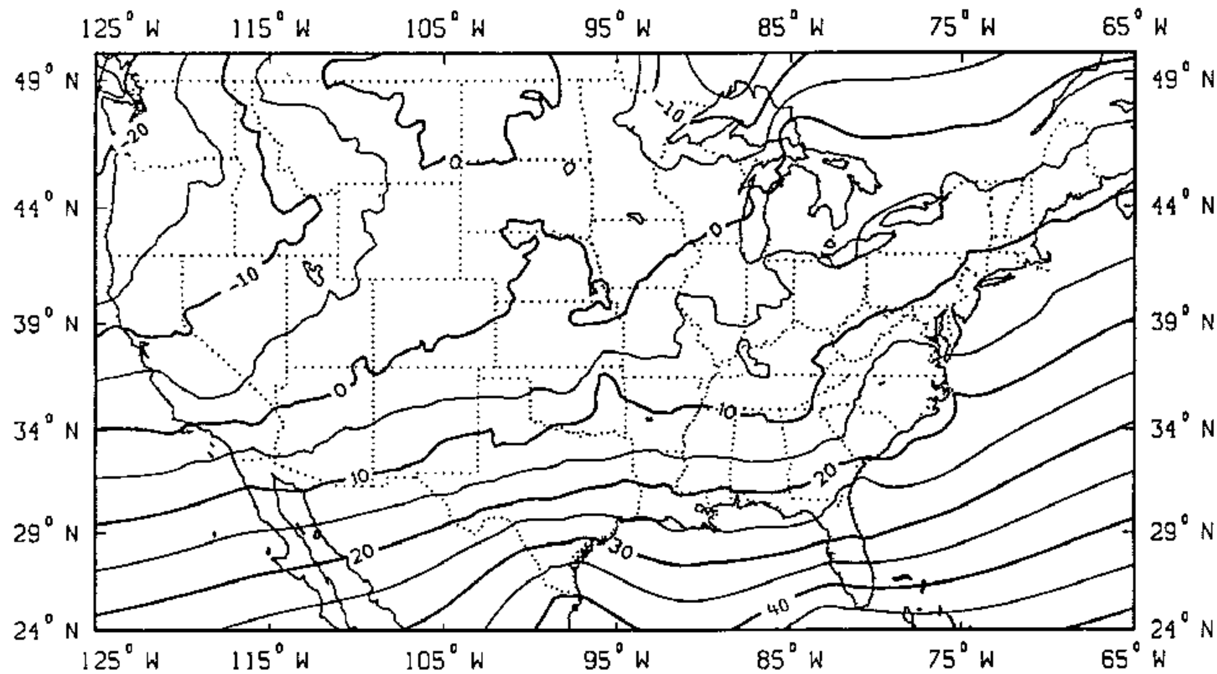

Figure 21.2. Latitude datum shift in the conterminous United States in meters (NAD 83 minus NAD 27).

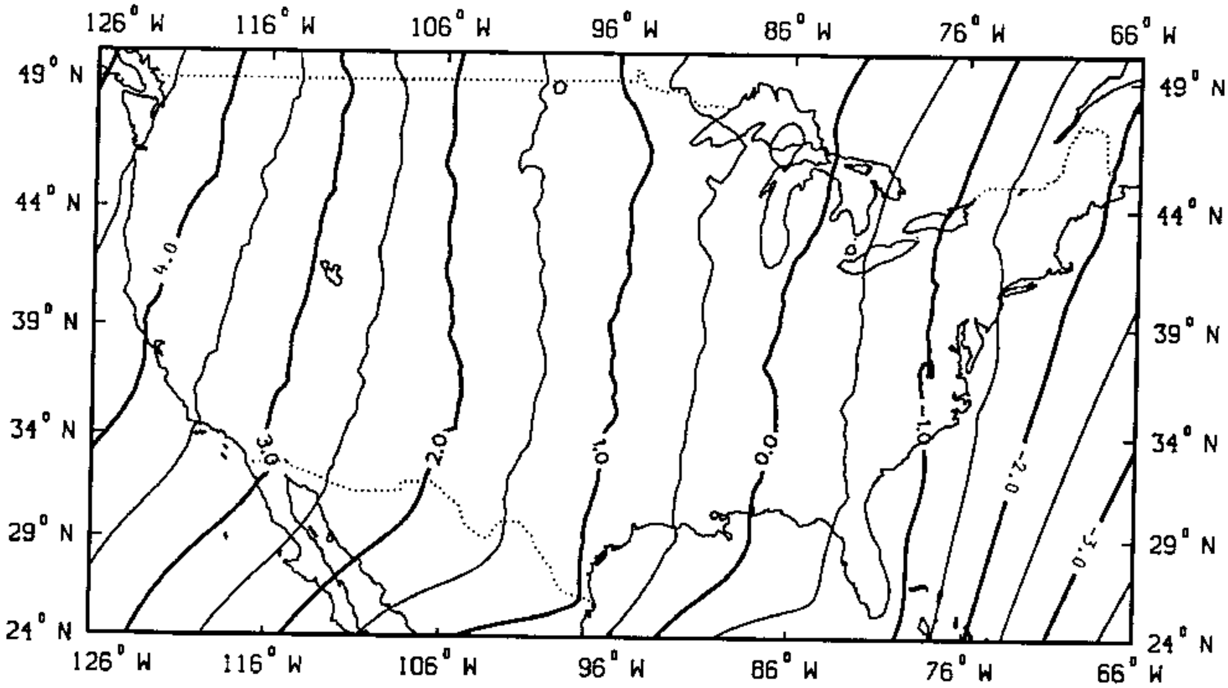

Figure 21.3. Longitude datum shift in the conterminous United States in seconds of arc (NAD 83 minus NAD 27).

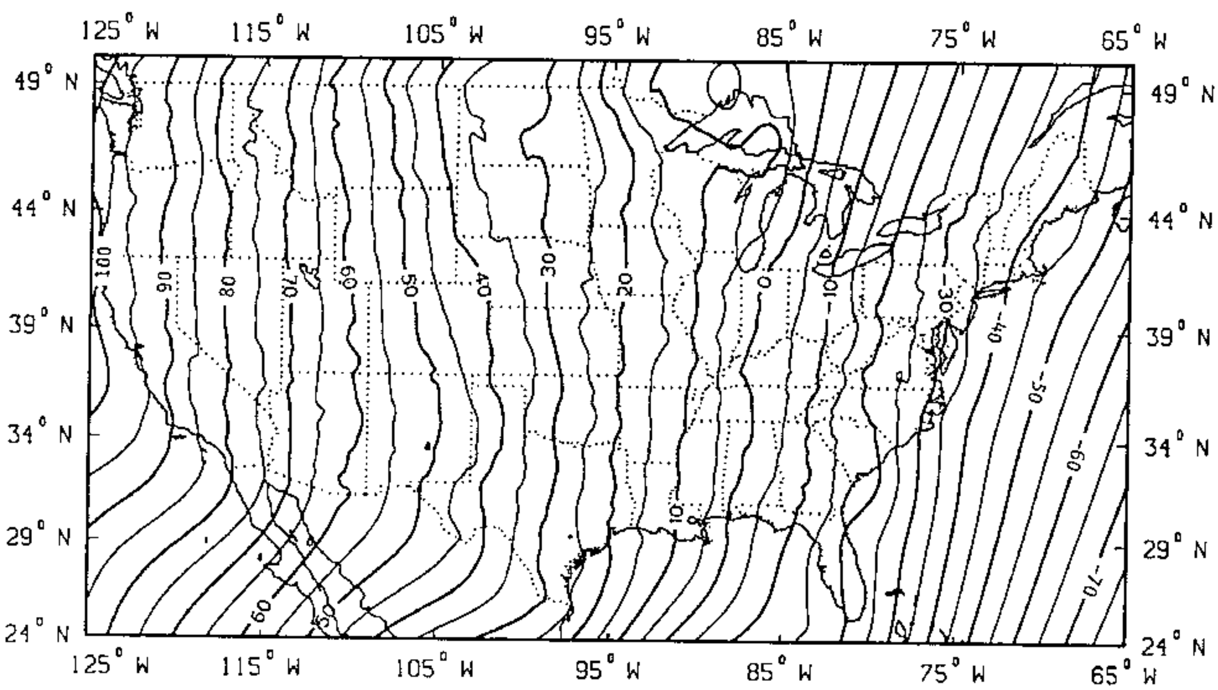

Figure 21.4. Longitude datum shift in the conterminous United States in meters (NAD 83 minus NAD 27).

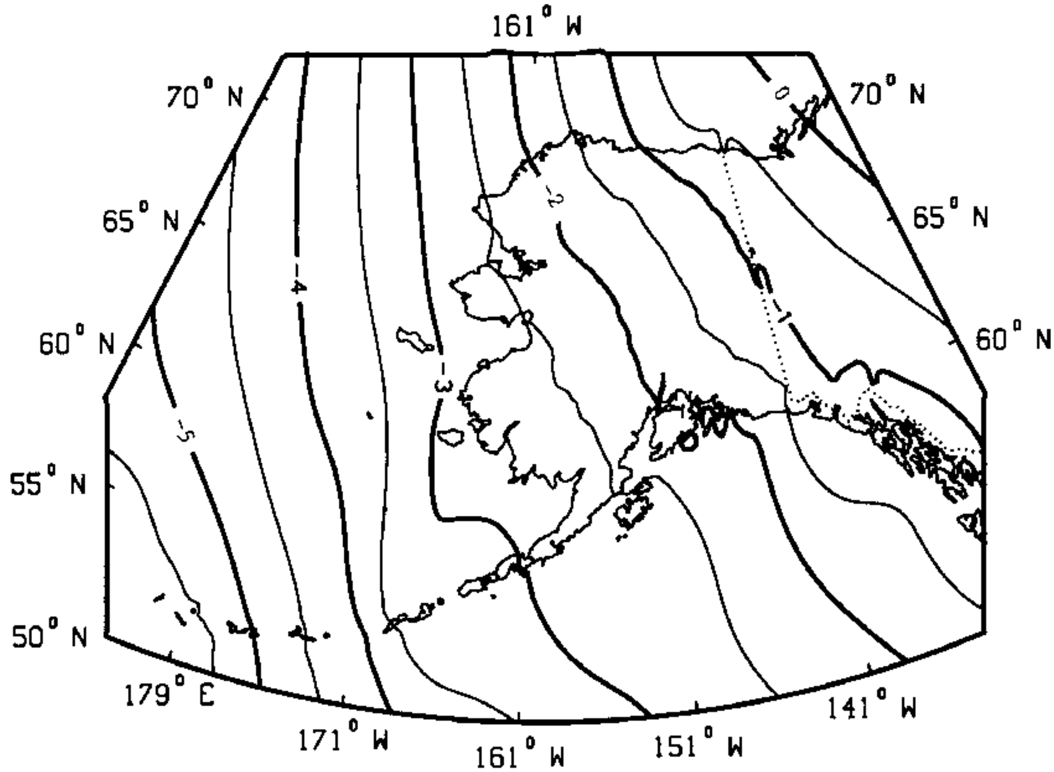

Figure 21.5. Latitude datum shift in Alaska in seconds of arc (NAD 83 minus NAD 27).

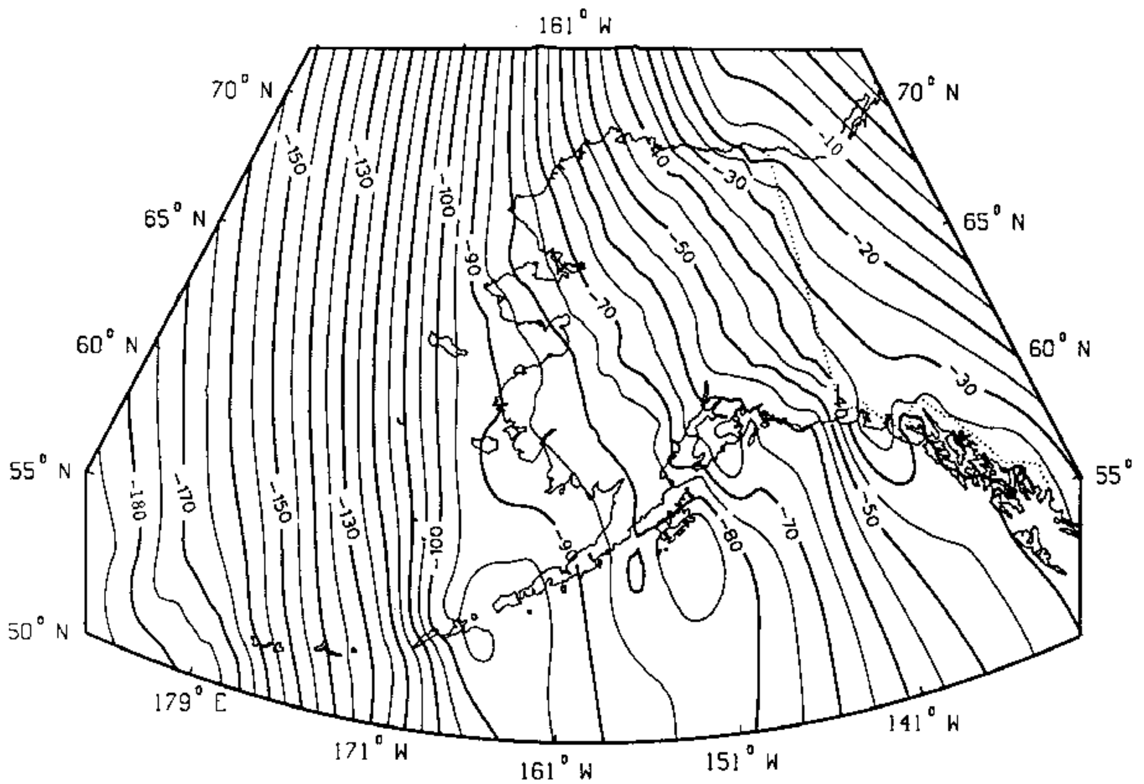

Figure 21.6. Latitude datum shift in Alaska in meters (NAD 83 minus NAD 27).

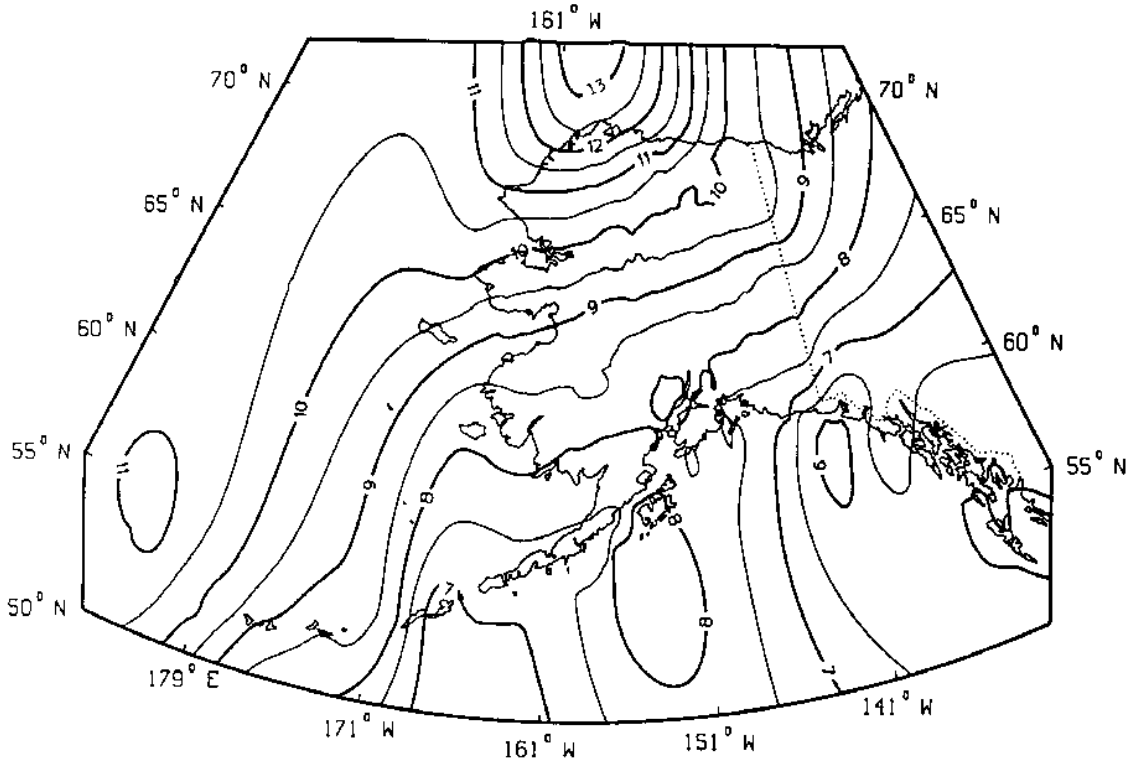

Figure 21.7. Longitude datum shift in Alaska in seconds of arc (NAD 83 minus NAD 27).

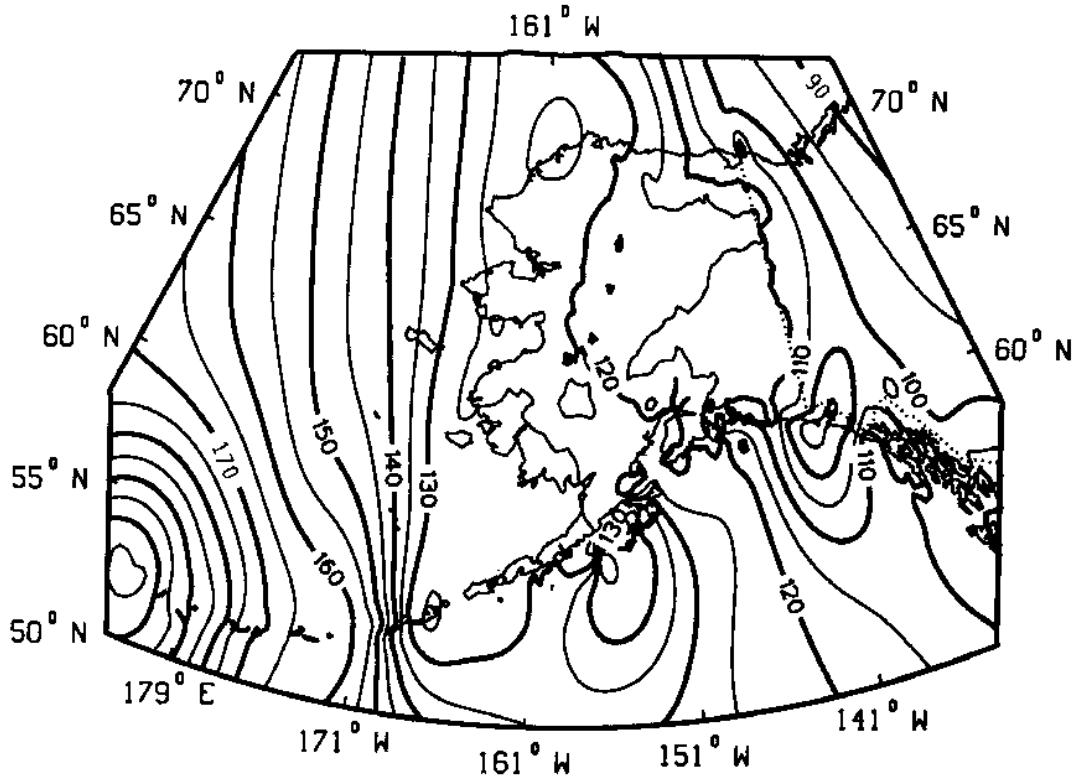

Figure 21.8. Longitude datum shift in Alaska in meters (NAD 83 minus NAD 27).

NAD 27 LATITUDE 26°30′00″

NAD 27 Longitude			NAD 83 Latitude			NAD 83 Longitude			Difference (meters)	
Deg	Min	Sec	Deg	Min	Sec	Deg	Min	Sec	Lat	Long
80	00	00	26	30	01.26	79	59	59.15	38.74	23.58
80	07	30	26	30	01.26	80	07	29.16	38.79	23.32
80	15	00	26	30	01.26	80	14	59.17	38.82	22.98
80	22	30	26	30	01.26	80	22	29.18	38.87	22.68
80	30	00	26	30	01.26	80	29	59.19	38.89	22.35
80	37	30	26	30	01.26	80	37	29.20	38.93	22.02
80	45	00	26	30	01.26	80	44	59.22	38.95	21.68
80	52	30	26	30	01.27	80	52	29.23	38.97	21.35
81	00	00	26	30	01.27	80	59	59.24	39.00	21.04
81	07	30	26	30	01.27	81	07	29.25	39.04	20.75
81	15	00	26	30	01.27	81	14	59.26	39.10	20.43
81	22	30	26	30	01.27	81	22	29.27	39.14	20.13
81	30	00	26	30	01.27	81	29	59.28	39.20	19.86
81	37	30	26	30	01.28	81	37	29.29	39.28	19.64
81	45	00	26	30	01.28	81	44	59.30	39.46	19.34
81	52	30	26	30	01.28	81	52	29.32	39.38	18.94

Figure 21.9.—Example of listing of NAD 83 datum shift at 7½ minute quadrange corners (U.S. Geological Survey, 1989: vol. A, p. A-11.)

NADCON employs the minimization of global curvature (Briggs, 1974) to model the actual shift values. This method, although new to geodesy, has been employed in geophysics and engineering in the past. The most common geophysical usage is in the modeling of potential field data (e.g., gravity and magnetic observations) or in the presentation of discrete data of any sort in the form of contour maps or 3-D "wire" diagrams. Engineering applications include the prediction of deformation within thin homogeneous plates.

The NADCON method, based upon a set of biharmonic partial differential equations whose solutions are cubic splines, guarantees continuity and smoothness. For example, figures 21.1 through 21.8 were directly derived from the NADCON-produced grids and display a high level of smoothness, with no edge discontinuity due to boundary conditions. Grid spacing is a function of point-distribution and desired accuracy. Thus, NADCON employs various grid spacings, depending upon region. A target accuracy was better than 1 m at the 67-percent confidence level. This modeling technique permitted the development of a very small-scale (large areal extent) surface of shift values, based upon the consideration of all appropriate and verified observations (usually first- and second-order control). Shift values on the grid, once obtained, could be held invariant, thus creating stability among various user communities when the same interpolation technique is employed. The accuracy, typically less than 15 cm, permits the utilization of NADCON-transformed results in a wide variety of applications, including very large-scale mapping to National Map Accuracy Standards (U.S. Bureau of the Budget, 1941, rev. 1947). NADCON, ratified by the Federal Geodetic Control Committee (American Congress on Surveying and Mapping, 1990: p. 16), now provides the Nation with a simple, standardized, and accurate method for datum transformation.

21.3 IMPACT UPON USERS

The official notification of a new North American Datum appeared in the *Federal Register*. In 1977, the first announcement (FR Doc. 77-8847) provided information that the plane coordinate values of the SPCS were to be replaced by NAD 83. This notice initiated the SPCS 83 design process via state liaisons. (See sec. 21.2.1.)

A *Federal Register* notice in 1979 emphasized the following: both geographic and plane coordinates would be changed by NAD 83, completion was projected for 1983-84, and an additional 12 months would be required to disseminate the information (FR Doc. 79-20169). But the primary function of the notice is revealed in its first paragraph:

> This document serves as official notification of the establishment of a new Datum to which the geographic and plane coordinate values for the National Network of Horizontal Geodetic Control will be referenced. The new Datum shall be known as the North American Datum of 1983 and may be referred to as NAD of 1983, 1983 NAD, or NAD 83.

In 1989 NGS placed another notice in the *Federal Register* (FR Doc. 89-14076) announcing the official completion of the project. The summary statement from this notice read:

> The Office of Charting and Geodetic Services (C&GS), National Geodetic Survey Division, has completed the redefinition and readjustment of the North American Datum of 1927 (NAD 27), creating the North American Datum of 1983 (NAD 83). The interagency Federal Geodetic Control Committee (FGCC) affirmed NAD 83 is the official civilian horizontal datum for U.S. surveying and mapping activities performed or financed by the Federal Government. Furthermore, to the extent practicable, legally allowable and feasible, all Federal agencies using or producing coordinate information should provide for an orderly transition from NAD 27 to NAD 83.

This notice was affirmed by the 10 FGCC-member agencies.

NAD 83 serves as the response to a researched and documented requirement for an upgraded horizontal reference system (National Research Council, 1971). It provides the solution for improved relative accuracies between control stations.

Public awareness of the new adjustment was an important factor in acceptance by the surveying and engineering communities. This was due to the effort NGS placed on a newly established long-range educational program. The publications program in support of NAD 83 produced 125 serialized *NOAA Technical Reports*, *NOAA Technical Memorandums*, *NOAA Manuals*, and nonserialized reports. To reach a wider audience, a series of 27 NGS-authored articles was published in the *ACSM Bulletin* (Journal of the American Congress of Surveying and Mapping) under the title, "New Adjustment of the North American Datum." This program, in addition to numerous presentations by NGS personnel at professional meetings and the creation of several workshops, prepared users for the impact of NAD 83. This educational effort also contributed to user participation in the programs described in the preceding section.

21.3.1 Impact on Field Surveying and Engineering Users—Primary NGRS Users

Surveyors and engineers have traditionally been the primary users of the NGRS. Based on the datum defined by NGS, they provide the underlying geometric data required for the production of plats, maps, charts, and drawings. These knowledgeable professionals understand the requirement for numerous datums. Many participated in the NAD program by submitting data for adjustment and inclusion in the NAD 83. Many also participated in the design of their local SPCS 83. They were the first to address the issue of conversion from NAD 27 to NAD 83. Many understood the significance of NAD 83 and the fact that the new datum would cause minimal disruption to their work. New surveys referenced to NAD 83 required only minor technical changes to procedures and software.

However, for a larger number of surveyors and engineers, their first real interest in NAD 83 surfaced when NAD 83 coordinates were received in their office or when a client requested NAD 83 values. While many were able to implement the required technical changes immediately, others were less knowledgeable. The most common mistake was an attempt to use NAD 83 coordinates in software in which NAD 27 ellipsoid constants were imbedded. For many of these individuals, a certain mystique surrounded NAD 83 and they approached the new datum with caution.

21.3.2 Cartographic Impact—Secondary NGRS Users

Cartographers and other professionals who are responsible for the preparation of graphic and digital cartographic products comprise another group affected by NAD 83. For them, preparation of new products on NAD 83 did not present new technical problems, but instead raised concerns about user acceptance and understanding.

Updating existing NAD 27 cartographic products to NAD 83 could be addressed in several ways. To consider these options with respect to graphic products, one may look at the four options considered by the U.S. Geological Survey (USGS), as they are representative of the possibilities available (Jones and Needham, 1985).

As the Nation's largest civilian mapping agency, USGS was faced with transforming map series of various scales to NAD 83. The most profound impact involved two series identified as 15-minute and 7½ minute quadrangles, as these formats were adopted after the completion of NAD 27. Of the 70,000 maps stocked by USGS, 50,000 fall into these series. Annually, 7 million map copies are distributed. These series, cast on either a polyconic projection or the projection of the State Plane Coordinate System of 1927, show the lines of NAD 27 latitude and longitude. Most also show rectangular grid systems based on SPCS 27 and UTM 27. The result is a complex grid and graticule pattern.

Option 1 proposed the retention of the projection on which the map was cast, including grids and graticule, but at the same time showing NAD 83 map sheet corners as crosses and describing the components of the shift between the two datums in the map margin. Although this was the least costly option, it unfortunately perpetuated an outdated datum.

Option 2 adjusted the map detail cartographically to NAD 83 and recast the graticule to retain the 7½ minute or 15-minute divisions of a degree. NAD 27 map sheet corners would be shown as crosses, and grid and graticules would be based on 1983 systems. This approach was considered more expensive than option 1.

Option 3 recompiled the maps to conform to NAD 83 control stations, with grid and graticules based on 1983 systems. Map sheet detail would be compiled based on NAD 83 and sheet format based on NAD 83 divisions. This option was the most costly.

Option 4 would recast the map projection and grids on NAD 83 and SPCS 83 to fit existing mapped area. The bounding meridian and parallel lines would be in the same location as on the NAD 27 map, but would be labeled with NAD 83 values.

The advantages and disadvantages of the four options depend on the status of the mapping program. Since the complete coverage of the United States by the USGS 7½ minute series was recently accomplished, transforming to NAD 83 will begin during the forthcoming revision program and, simultaneously, these maps are being digitized for the National Digital Cartographic Data Base. Thus, as the revision program progresses, the map data being revised will change from graphic to digital. Initially, older maps and those that have serious deficiencies will be transformed to NAD 83 by replacement mapping, option 3, and large projects of maps needing updating will be transformed using option 2. However, most of the maps in the 7½ minute series will probably be transformed to NAD 83 later, after being converted to a digital form. Datum change effort is much less for the smaller scale maps due to less maps being involved and the datum shift values being nearly negligible because of the small scale.

One state agency is also known to have adopted option 4. The Maryland Department of Assessment and Taxation replaced the SPCS 27 grid with the SPCS 83 grid on its statewide 1 inch = 400 feet map series. Maryland's application of option 4 differs from that of USGS in that a graticule of NAD 27 latitude and longitude did not exist to update to NAD 83. For each of the integral 2,000-foot divisions of SPCS 27, an SPCS 83 coordinate value was computed using

NGS transformation software LEFTI. The grid lines on each of the map sheets were then manually corrected.

Transformation of digital map products to NAD 83 presents different problems. There are two steps for planimetric map detail:

- Change the coordinate values for the map detail points by applying a constant shift or by altering the transformation parameters in the data file header. When a more accurate transformation is required, compute new coordinate values for each point in the file using bilinear interpolation.
- Because of the coordinate shift, the block of map detail will not fit the NAD 83 map outline. The detail will go overedge on two sides and leave gaps on the other sides. Data from adjacent files will be mosaicked and the whole file repartitioned to fit the new outline.

The complete transformation of Digital Elevation Models (DEM) presents an additional step. After following the steps given above, the posts will no longer be whole units, such as seconds of arc of latitude and longitude or whole meter units of UTM. Since repositioning the posts to whole coordinates requires resampling, a technique that degrades the elevation data, this step may never be done.

The logistics of actually implementing the NAD 83 conversion impacts users of digital map products. Facility data bases maintained by many utility companies have files structured with respect to SPCS 27 and facilities coded with SPCS 27 coordinates. These dynamic data bases can suffer only minimal disruption. Under such conditions, the process of mosaicking, repartitioning, and datum transformations would create logistical hardships.

21.3.3 Impact upon Producers and Users of Geocoded Information—Tertiary NGRS Users

It has been estimated that 95 percent of all land information is spatially located, either implicitly or explicitly, absolutely or relatively. The nature of the spatial connections to NGRS is generally unknown, but during the last decade strides have been made to remedy that situation. The proliferation of Land Information Systems/Geographic Information Systems (LIS/GIS) has emphasized the requirement that land information be connected to a single reference system, and geodetic or plane coordinates of NAD 27 have frequently been used. Consequently, a significant number of organizations are faced with the task of transforming to NAD 83 information previously referenced to NAD 27.

LIS/GIS represent not only the more recently developed automated systems, but also manually constructed and maintained files of land information operated by many public and private entities. Many of these files are as simple as a list of coordinates representing the locations of a single attribute. The files are the responsibility of a diverse cross section of disciplines, most of them far removed from surveying and mapping. Consequently, addressing the NAD 27 to NAD 83 conversion to such a broad spectrum of people and applications presents a challenge.

In many LIS/GIS the role of the property map was elevated to serve as a spatial base on which all other information was merged. Unfortunately, most property maps were compiled from uncontrolled or partially controlled aerial photo-mosaics. Primary points of reference are features such as fences and road centerlines, which have not been tied to the NGRS. When the opportunity exists to check positional accuracy of such property maps, errors of several hundred feet are not uncommon.

In many other LIS/GIS, the 7½ minute quadrangle of the USGS was the spatial base on which land information was merged. Generally features on these quadrangles have been plotted to an accuracy of 30 to 100 feet of their position with respect to the NGRS, so these systems begin with this error. Sometimes property maps have been developed using the USGS quadrangle sheet as a basis for positional control.

Clearly, geocoded land information exists with various accuracies necessitating different approaches to conversion. Typically more accuracy was assumed than existed, and this fact often influenced how the issue of datum conversion was addressed. Admission of less accurate positions coded on the land information permits more simple conversion methodology and application of that methodology. Hence, the questions asked of a geodetic data user faced with a conversion problem were: On which datum are the existing coordinates? On which datum do I want the new coordinates? Are there any constraints on the size of the geographical area to be converted at one time? How many points are common to both datums? What is the distribution of the common points? How accurate are existing coordinates? How much positional uncertainty can be introduced by the conversion? The answers to these questions determine the appropriate transformation methodology recommended by NGS. The choice is between a rigorous affine transformation by software such as LEFTI or an average shift. In either case the partitioning scheme requires analysis.

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22. RELATION OF NAD 83 TO WGS 84

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This chapter addresses the differences between the North American Datum of 1983 and the World Geodetic System of 1984 (WGS 84) of the U.S. Defense Mapping Agency (DMA). Both NAD 83 and WGS 84 were defined (in words) to be geocentric, and oriented as the BIH Terrestrial System. In principle, the three-dimensional coordinates of a single physical point should therefore be the same in both systems; in practice, small differences are sometimes found. The original intent was that both systems would also use the Geodetic Reference System of 1980 (GRS 80) as a reference ellipsoid. As it happened, the WGS 84 ellipsoid differs very slightly from GRS 80.

22.1 THE CONCEPT OF A GEODETIC DATUM

To understand the sources and importance of these differences, it is necessary to take a close look at the concept of a datum and at how the coordinates in a datum are actually computed. The concept of a horizontal geodetic datum actually involves several ideas. A definition almost always begins with some form of specification of a reference surface. This involves the specification of the dimension of a reference ellipsoid, as well as quantities which determine the origin and orientation of the ellipsoid with respect to the Earth. (See, for instance, National Geodetic Survey, 1986.)

22.1.1 A Datum as a Coordinate System

A three-dimensional Cartesian coordinate system is associated with every geodetic datum. This coordinate system must be fixed in the physical earth. This specification of the origin and orientation of the coordinate system can be expressed in several ways. With local horizontal datums, these quantities were fixed by specifying the geodetic coordinates of an initial point and at least one azimuth. With the use of satellite geodesy, the origin and orientation of the coordinate system are determined (usually overdetermined) by specifying the three-dimensional coordinates of a number of points. A coordinate system can also be specified by describing the relationship between it and another coordinate system. This is the case with NAD 83 and WGS 84. Both are defined (in words) in terms of their relationship to the NWSC 9Z-2 coordinate system. Both transformations are attempts to realize the BIH Terrestrial System (BTS). The two transformations are exactly the same because DMA and NGS coordinated their efforts in this regard. Thus, the NAD 83 and WGS 84 coordinate systems are identical.

22.1.2 A Datum as Ellipsoid

The WGS 84 ellipsoid differs very slightly from the GRS 80 ellipsoid which was used for NAD 83. The differences can be seen in tables 22.1 and 22.2. These differences arise because DMA used the normalized form of the coefficient of the second zonal harmonic of the gravity field as a fundamental constant, while GRS 80 had used the unnormalized form. Furthermore, the normalized value used by DMA was obtained by using the mathematical relationship

$$\bar{C}_{2,0} = -J_2/(5)^{1/2}$$

and rounding the result to eight significant figures (Defense Mapping Agency, 1987). Thus quantities depending directly on the form factor, such as the flattening, generally differ after the eighth significant digit, while linear quantities, such as the semiminor axis, generally differ after the tenth significant digit. These differences, while small, can cause confusion among users who attempt to compare computations in the two systems. Most analysts agree that these differences will be of no significance for practical applications.

TABLE 22.1.—*Defining (fundamental) parameters*

Parameter	Notation	Units	Ellipsoid	
			GRS 80	WGS 84
Semimajor axis	a	m	6378137	6378137
Angular velocity of the Earth	ω	rad s^{-1}	7292115 x 10^{-11}	7292115 x 10^{-11}
Gravitational constant	GM	$m^3\ s^{-2}$	3986005 x 10^8	3986005 x 10^8
Dynamic form factor				
unnormalized form	J_2		108263 x 10^{-8}	
normalized form	$\bar{C}_{2,0}$			−484.16685 x 10^{-6}

TABLE 22.2.—*Derived geometrical constants*

Parameter	Notation	Units	Ellipsoid	
			GRS 80	WGS 84
Semiminor axis	b	m	6356752.3141	6356752.3142
Eccentricity squared	e^2		0.00669438002290	0.00669437999013
Flattening	f		0.00335281068118	0.00335281066474
Reciprocal flattening	f^{-1}		298.257222101	298.257223563
Polar radius of curvature	c	m	6399593.6259	6399593.6258

22.1.3 A Datum as Coordinates

The specification of a reference surface defines a datum only in an idealized sense. This specification is usually supplemented by a second definition which states that a horizontal geodetic datum is composed of the adopted horizontal coordinates of a set of physical points in that datum. This is the operational definition. It is from this second definition—the adopted coordinates—that we actually determine the origin and orientation of a datum. In this sense, the first definition is more a statement of intention than a statement of reality.

There are other qualities connoted by the concept of a datum. The idea that there are adopted coordinates implies that a datum is stable—the coordinates seldom change. Furthermore, a datum must be *extensible*—there must be some way of computing the coordinates of new points. Often there are preferred or expected ways to determine these new coordinates. For instance, it is expected that new NAD 83 points will be established by running new horizontal surveys using theodolites and distance measuring equipment. It is also expected that if one uses Global Positioning System (GPS) observations in the single point positioning mode, together with a satellite ephemeris given in the WGS 84 coordinate system, then the resulting coordinates will also be in WGS 84.

The idea of extending a datum by adding new points implies that there are some *fundamental* points from which the process is begun. By definition, these are the points that participate in the initial network adjustment, irrespective of accuracy or order. All of the points that participated in the NAD 83 adjustment are thus fundamental points of that datum. New points that will be added are not. In most geodetic datums, the distinction between fundamental and non-fundamental points has been lost. Typically a new point surveyed to first-order accuracy and adjusted into the network has been treated as equal in usefulness to a fundamental first-order point, and superior to a fundamental second-order point. This common, but incorrect, practice has often misled users as to the accuracy of a point's coordinates.

Some physical points are fundamental to both NAD 83 and WGS 84. The coordinates of these points in the two systems may differ because the two adjustments which produced the coordinates of the two sets of fundamental points were based on two different sets of observations. For instance, a Doppler survey may have been performed at a point by either DMA or NGS, and the data may have been exchanged, so that both agencies had exactly the same data set. Furthermore, the two agencies agreed on all the details of data processing, so that both agencies determined the same set of Doppler-derived three-dimensional coordinates. Even further, the agencies agreed exactly on how to transform the Doppler-derived NWSC 9Z-2 coordinates into the BIH Terrestrial System. However, in the NAD 83 adjustment these coordinates received corrections due to interactions with other observations (mostly classical triangulation and traverses), while no such corrections were made in the determination of the WGS 84 coordinates. These corrections can amount to a meter or more. However, both adjustments are still thought to be valid. The differences of coordinates are thought to be simply the effect of small random measurement errors in the two sets of observations. Even though differences as large as several meters are found occasionally, the expected value of these differences is zero.

Other physical points are derived, rather than fundamental. For these points, coordinates in the two datums may differ for two reasons:

1. The two coordinate determinations are based on different fundamental points.
2. The observations used to extend the datum may differ.

The method of labeling the datum for derived points is mainly a matter of convention. The actual physical observations (such as angles or distances) are themselves independent of any datum. When a new point is surveyed for the purpose of determining its coordinates, the survey must be tied to one or more old points. If the coordinates of the old point in the NAD 83 system are used in the computations, the coordinates of the new point are also said to be in NAD 83. Similarly, if the coordinates of the old point in WGS 84 are used, the coordinates of the new point are said to be in WGS 84.

22.2 USING NAD 83 AND WGS 84 POINTS

NAD 83 and WGS 84 should be thought of as geographically overlapping datums (in the sense of datum as adopted coordinates). There will be points with coordinates in both datums. The action to take when confronted with two sets of coordinates for a single point is up to the user. If neither position determination contains a blunder, then the differences of coordinates should be small. In fact, the expected size of these differences can be computed from the uncertainties of the two determinations. If the differences are smaller than the accuracy required, then the user may select either determination (or some combination of the two).

"Small" differences must be properly understood here. The actual difference between coordinates may quite possibly be a meter or more. Although this might be disturbing to some, this is actually the magnitude of the uncertainty of the differences that would be computed from the uncertainties of the two coordinate determinations. It reflects the fact that the two coordinate determinations are independent and uncorrelated.

22.2.1 Mixing Coordinates

Surveyors are familiar with the limitations imposed when mixing the results of two independent surveys (or two datums) in a single positioning problem. Within a single survey, the relative coordinates of nearby points are much more accurate than the coordinates of either. This is not the case if the two sets of coordinates come from different surveys.

Suppose that within a local area there is both an NAD 83 point and a WGS 84 point. Suppose also that a survey is run to determine the distance between the points. The measured distance could differ from the value computed from the coordinates by a meter or more. Some might find this difference to be disturbing, but it is only a reflection of the fact that the variance of relative coordinates from two different surveys is much larger than the variance of the relative coordinates of two points from the same survey.

We thus say that the most common reason that we find differences between the NAD 83 and the WGS 84 coordinates of a point is that we are dealing with two independent determinations of the same thing. Both determinations are affected by the small statistical variations which are inherent in any measurement process. Each has its own associated standard deviation, but each is valid in its own way. The user may chose either, but must be careful about mixing coordinates.

22.2.2 Area of Validity

Some investigators have suggested that a difference between NAD 83 and WGS 84 is that NAD 83 is valid only within North America, while WGS 84 is valid worldwide. This is incorrect. If one has an accurate method of extending NAD 83 outside of North America, then there is no reason not to do so, nor is there any reason to think that the resulting coordinates would differ from WGS 84 coordinates. In fact, as part of the NAD 83 adjustment, Doppler observations were used to extend the datum outside of the contiguous survey networks to isolated areas such as Greenland, Puerto Rico, and Hawaii.

22.2.3 Extending the Datum Offshore

The case of a ship navigating offshore is of particular interest to the hydrographic and bathymetric surveying activities of the National Ocean Service. If the ship navigates with a radio navigation system using shore-based transmitters, and if the coordinates of the transmitters are known in NAD 83 coordinates, then the navigated position will also be in NAD 83. The ship may also navigate with a satellite-based system which yields coordinates in the WGS 84 system. We expect both navigation systems to provide the same coordinates at each instant of time; but due to unavoidable measurement errors we may find small differences. The existence of such differences should not be interpreted to mean that there is a difference in the two datums. Unless there is some reason to suspect that one or the other navigation system is producing serious errors, the differences between the coordinates produced by the two systems should be attributed simply to measurement error. The navigator may choose to use either set of coordinates. Only the navigator with extraordinarily demanding accuracy requirements will need to worry about computing some combination of the two sets of coordinates.

22.2.4 Computational Differences

There are some differences between NAD 83 and WGS 84 which may arise because of approximations made in a particular method of computing coordinates. For most applications, the effect of these approximations is considerably smaller than the effect of observational errors. These differences are important only if one is testing the accuracy of a set of equations or a method of computing coordinates.

One such set of approximations concerns the different ellipsoids used for NAD 83 and WGS 84. This difference has no effect on the three-dimensional coordinates of a point computed by satellite surveying. If such a set of three-dimensional Cartesian coordinates is converted to latitude and longitude using the two coordinate systems, there would be no difference in the longitudes, and the latitude difference would be

$$\phi'' \approx f \sin 2\phi / \sin 1''$$

which reaches a maximum value of 0.000003 second of arc (or 0.0001 meter) at a latitude of 45 degrees. It is assumed that most users will ignore this very small difference.

Another approximation concerns the datum shifts computed for map sheets. The National Geodetic Survey has computed a latitude and longitude shift for every map sheet published by the U.S. Geological Survey. These pairs of numbers were computed by meaning the actual shifts from NAD 27 to NAD 83 at all points falling on the map sheet. These mean shifts are then assumed to be correct for the entire map sheet. Thus a very small error, amounting to the difference between the actual datum shift and the mean datum shift for the map sheet, is committed at each point. This error is everywhere much smaller than the observational errors committed when coordinates are scaled from maps.

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