Reply

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1. Introduction

In my 1982 paper (henceforth referred to as F82), I was perhaps too conservative in describing the range over which the scale-dependent damping rate parameterization is valid, and Dr. Zhu's comments have lead me to reexamine the question more carefully. In what follows, I shall present some of the results of this investigation.

It is important at the outset to understand that the key question is the validity of Zhu's Eq. (9), which is just Eqs. (6b) and (9) of my 1982 paper. Once one is convinced of its applicability in a given circumstance, the rest is simple mathematics. In this connection, it is crucial to realize that any imaginary part of the wavenumber n must be included in the amplitude $A(z)$. It therefore follows that when the real and imaginary parts of n are equal, the wave and envelope scales will be of the same order, and caution is indicated. This was explicitly addressed in Appendix (D) of F82 [especially Table (3)], where it was shown that for waves with exponential envelopes, the WKB approximation begins to fail when the envelope and wave scales become similar. The ratio of these two scales is of course controlled by the parameter which Zhu calls γ, and thus as long as it is small, the WKB approximation should be good. It was this which motivated condition (6a) in F82.

The preceding statement is somewhat vague, and obviously leads one to wonder just how far the parameterization can be pushed before it fails. Clearly, the only definitive way to answer this question is to solve the wave equation using the full nonlocal radiation operator, and to compare this with the solution obtained using the scale-dependent damping parameterization. I have actually carried this out for a number of cases, and describe the results below.

2. Equations

The "exact" wave equation is the usual one for a vertically propagating wave in the absence of shear (cf. Zhu's Eq. 10):

$$\frac{d^2\chi}{dz^2}+n^2\chi=\frac{i}{\omega}e^{-z/2H}\int_0^\infty e^{z'/2H}K(z,z')\times\left(\frac{d^2}{dz'^2}-\frac{1}{4H^2}\right)\chi(z')dz'. \quad (1)$$

In the above, χ is the reduced vertical velocity, i.e., $w = \chi e^{z/2H}$, and n the vertical wavenumber, which Zhu calls n_0.

In the calculations which follow, it was necessary to use high vertical resolution; typically, Δz is 0.25–0.5 km. For this reason, the radiation operator is the Curtis matrix for the CO_2 15-micron band, computed using the approximate expression

$$A(p,p')=C\log[1+b|p^2-p'^2|^{1/2}],$$
$$(p,p'\text{ in mb}, C=0.163\,\text{cm}^{-1}, b=27.22\,\text{mb}^{-1}),$$

for the absorption. The scale-dependent damping rates calculated from this agree with the more accurate ones in F82 to within about 30%, most of this error being due to the neglect of temperature-variation of line intensities. This expression has the great advantage (relative to the precomputed functions used in F82) of allowing one to refine the vertical resolution very easily. Since it was used for both the exact and parameterized calculations, any shortcomings in its accuracy are unimportant.

Given the Curtis-matrix form of the radiation operator, it is a simple matter to calculate the scale-dependent damping rates

$$\tau^{-1}(n,z)=-\int_0^\infty K(z,z')\cos n(z-z')dz'$$

at each height z for use in the parameterized wave calculations:

$$\frac{d^2\chi}{dz^2}+n^2\chi=\frac{-i}{\omega\tau}\left(\frac{d^2\chi}{dz^2}-\frac{\chi}{4H^2}\right). \quad (2)$$

In the above, the value used for n, the vertical wavenumber, was adjusted to take into account the existance of the damping.

3. Results

Equation (1) was solved iteratively for a variety of parameter settings, and compared with the solution of the parameterized Eq. (2). In addition, solutions were also computed using the *scale-independent* Newtonian cooling approximation. This was done largely to convince nonbelievers of the need to take scale-dependence into account. In a comparison such as this one, only the relative sizes of the various fields are of interest; for this reason, the somewhat artificial boundary condition $w(0) = 1.0$ was used in all cases. At 60 km, a radiation condition was imposed. The vertical resolution was 0.5 km for most cases, but results were checked by decreasing it to 0.25 km for cases with vertical wavenumber of 1.0 km^{-1}.

Rather than the wave fields themselves, we shall look at the Eliassen–Palm flux divergence, which is often the most useful wave-related quantity. In fact, we have chosen to display the wave-driven mean flow acceleration a as a function of height:

$$a(z) = e^{z/H}(e^{-z/H}\overline{u'w'})_z.$$

The several cases shown have been chosen to cover a part of parameter space of particular importance for the tropical stratosphere. The "classical" Kelvin and mixed-gravity–Rossby waves described by Wallace (1973) have vertical wavelengths between 6 and 12 km, and periods from 4 to 20 days. When one takes into account the effect of the local mean stratospheric winds, Doppler-shifted periods as long as 60 days may be encountered, with wavelengths as short as a few kilometers. We therefore exhibit results for waves with periods of 6.3 days, 31 days, and a few examples with periods of 63 days.

Figure (1a) shows the comparison for $T = 6.3$ days, $n = 1.0\ \text{km}^{-1}$. The panel on the left of the figure shows the value $\gamma = 1/\omega\tau$, which is a measure of the strength of the radiative damping. In this case, it is seen that the agreement between the exact and parameterized results is very good. In view of the smallness of γ, and the fact that $nH \gg 1$, this is perhaps not surprising. Notice also that the scale-independent damping gives disastrous results at most heights.

Figure (1b) illustrates the results for $T = 6.3$ days, and $n = 0.5\ \text{km}^{-1}$; by virtue of the longer wavelength, the radiative damping is now weaker than in the previous case, while the scale-separation assumption is less well satisfied. Again, however, the parameterization does an excellent job, whereas the Newtonian cooling results are very poor. Figure (1c) shows the same comparison for $T = 6.3$ days, and $n = 0.25\ \text{km}^{-1}$. In this case too, the agreement is still good, although $nH \sim 2$. We observe that the Newtonian cooling approximation for this case, although less good than the scale-dependent parameterization, is better than in the previous two examples, due to the relatively long vertical wavelength here. In all of these cases, $\gamma \ll 1$, so that assumption (6a) of F82 is well satisfied.

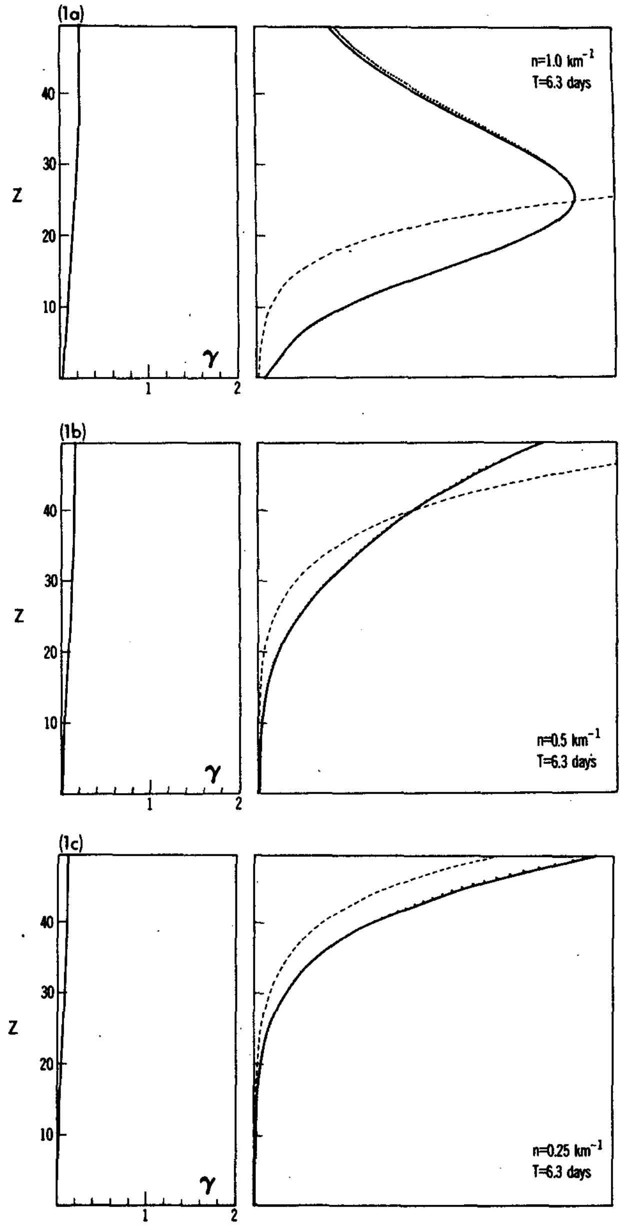

FIG. 1. Left panel: the damping parameter $\gamma = 1/\omega\tau$ as a function of height; Right panel: the wave-induced acceleration $e^{z/H}(e^{-z/H}\overline{u'w'})_z$ as a function of height. Dotted line: exact results; solid line: results using scale-dependent parameterization; dashed line: results using scale-independent parameterization. The scale of the right-hand panels is arbitrary. All cases have a wave period of 6.3 days.

We turn now to the situation when $T = 31$ days, so that the effect of damping on the wave amplitude is much more pronounced. This is evident from the fact that the EP flux divergence curves peak lower than in the corresponding cases shown previously.

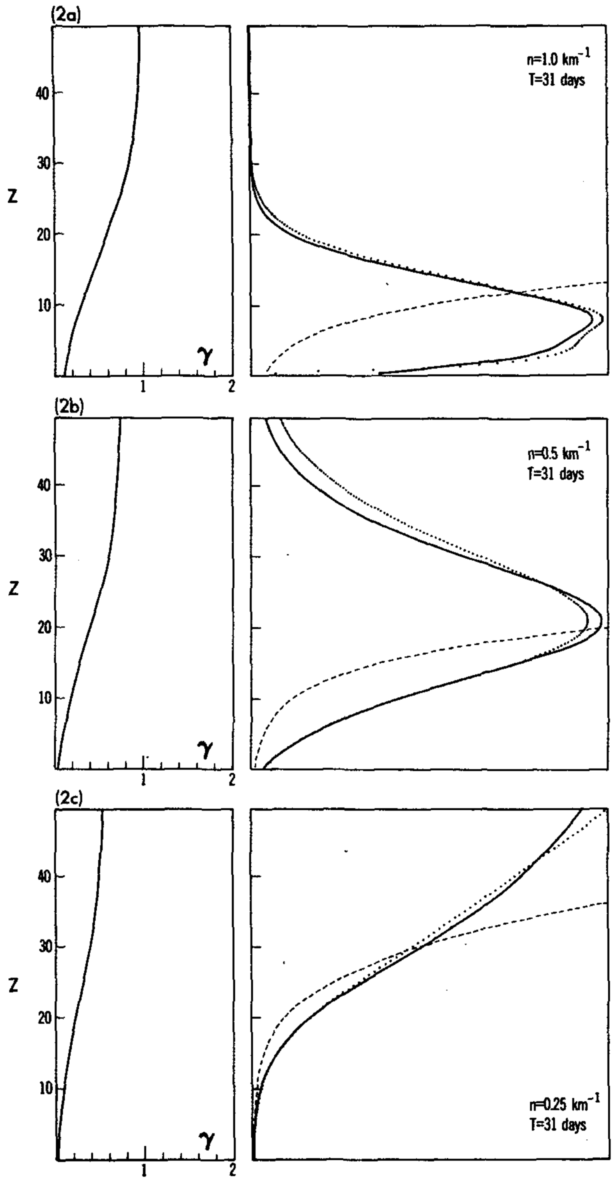

FIG. 2. As in Fig. 1, but with wave period of 31 days.

Considering first the case where $n = 1.0$ km^{-1} (Fig. 2a), we see that the parameterized results still agree well with the exact, although there are now noticeable errors at all heights. We should remind the reader that neither the "exact" nor the parameterized results have any physical validity near the ground, or indeed in the troposphere, since only the effects of CO_2 are included in the radiative scheme.

The $n = 0.5$ and $n = 0.25$ cases (Figs. 2b and 2c) also show good agreement between the exact and parameterized results, although there are noticeable errors in both cases above 25 km, where $\gamma \sim 1$.

Finally, Figs. (3a–c) show the situation when $T = 63$ days for $n = 1.0$, 0.5 and 0.25 km^{-1}. As before, the results for 1.0 and 0.5 are reasonably good, while for $n = 0.25$, serious (but perhaps not disastrous) errors exist above 30 km. The scatter in the points representing the exact results below 2 km is due to the somewhat arbitrary assumptions made about the behavior of the thermal structure near the ground, and does not affect the results in the stratosphere.

4. Conclusions

Returning finally to the points raised by Zhu, the situation is as follows: in general, the scale-dependent

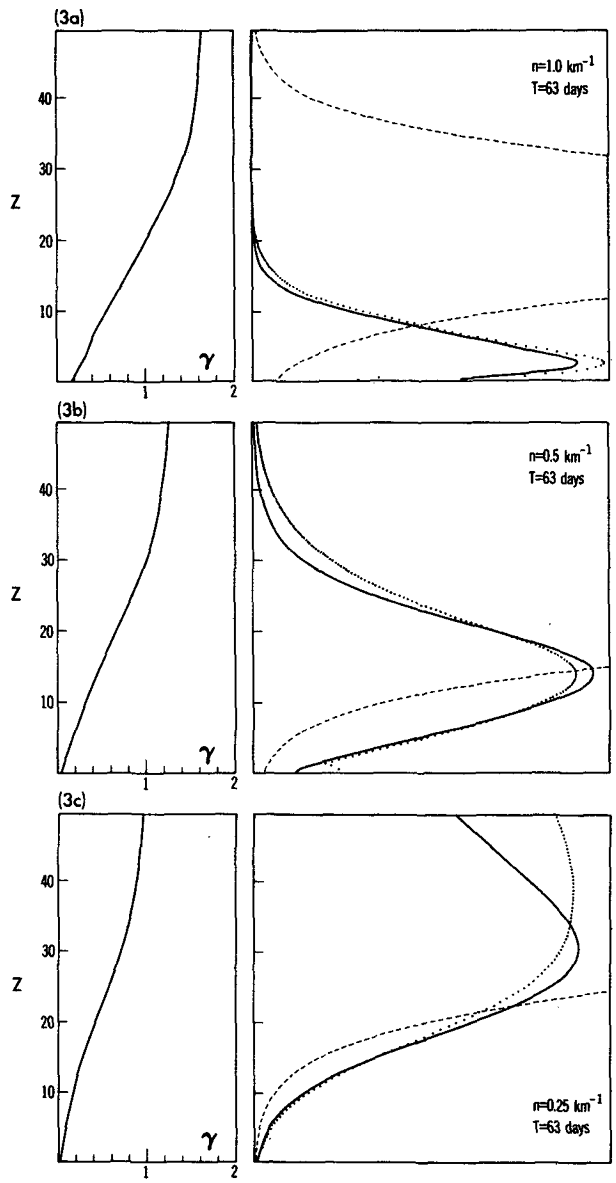

FIG. 3. As in Fig. 1, but with wave period of 63 days.

damping parameterization is remarkably good as long as $nH \gg 1$, although there are noticeable errors whenever the damping parameter γ exceeds 0.5. The existence of these is due to the violation of assumption (6a) of F82, as discussed in the introduction. That the errors are not larger seems to be due to the fact that γ is usually small in the lower atmosphere; at those heights where $\gamma \sim 1$, the wave has already decayed to a small fraction of its original amplitude, so that errors are not important. On the other hand, for $n = 0.25$ km^{-1}, the case in which $\gamma \sim 1$ does show significant errors, counseling some caution in the use of the parameterization. Thus, only when assumptions (6a) and (6b) of F82 are *both* violated does the parameterization seem to fail seriously.

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