

## 7. A REFINEMENT OF THE COMBINATION EQUATIONS FOR EVAPORATION

P. C. D. MILLY

*U.S. Geological Survey, Geophysical Fluid Dynamics Laboratory/NOAA, P.O. Box, 308, Princeton,  
NJ 08542, U.S.A.*

**Abstract.** Most combination equations for evaporation rely on a linear expansion of the saturation vapor-pressure curve around the air temperature. Because the temperature at the surface may differ from this temperature by several degrees, and because the saturation vapor-pressure curve is nonlinear, this approximation leads to a certain degree of error in those evaporation equations. It is possible, however, to introduce higher-order polynomial approximations for the saturation vapor-pressure curve and to derive a family of explicit equations for evaporation, having any desired degree of accuracy. Under the linear approximation, the new family of equations for evaporation reduces, in particular cases, to the combination equations of H. L. Penman (Natural evaporation from open water, bare soil and grass, *Proc. R. Soc. London, Ser. A* **193**, 120-145, 1948) and of subsequent workers. Comparison of the linear and quadratic approximations leads to a simple approximate expression for the error associated with the linear case. Equations based on the conventional linear approximation consistently underestimate evaporation, sometimes by a substantial amount.

### Introduction

The 'combination method' for estimation of evaporation refers to the simultaneous solution of the equations of surface energy balance and turbulent transport of heat and water vapor, sometimes with allowance for internal plant resistance to transpiration. This approach was initiated by Penman (1948), who used several approximations to arrive at an explicit 'combination equation' for the evaporation rate as a function of several easily measured quantities. The empirical components and some of the physical limitations of Penman's original equation were later removed by him and a series of other investigators (Penman and Schofield, 1951; Businger, 1956; Tanner and Pelton, 1960; Monteith, 1965; Van Bavel, 1966; Thom, 1972; Thom and Oliver, 1972). Brutsaert (1982) provides a sound overview.

One approximation (in fact the critical step), introduced by Penman to obtain an explicit evaporation equation, has persisted to this day. To eliminate the effective surface temperature from the system of equations, he introduced a linear relation between temperature and saturation vapor pressure, with the slope  $\Delta$  of the relation determined at the air temperature. An exact solution would instead use the chord-slope evaluated between the air temperature and the surface temperature. The latter is not known *a priori*, so the approximation is necessary to avoid iteration.

Some investigators have made estimates of the error induced by ignoring the temperature difference between air and underlying surface in applications of Penman's combination equation. Tanner and Pelton (1960) used data for Madison, Wisconsin, to show that the relative error in evaporation rate was less than one-tenth of the relative error in  $\Delta$  for March through September. Temperature

measurements made for 8 days above and at an alfalfa-brome surface suggested that the relative error of  $\Delta$  averaged about 10%, so they concluded that relative error in computed evaporation rate would be negligible.

Van Bavel (1966) considered three widely differing climatic situations, and leaf-air temperature differences reported in the literature, in an assessment of the error induced by ignoring the temperature difference. He acknowledged that the analysis was artificial, because the temperature differences were specified independently of the other climatic conditions. He concluded that the approximation was an adequate working assumption, because the errors were less than instrumental errors, "and the utility of an iterative solution to the problem" was then 'questionable'.

Slatyer and McIlroy (1961, Chapter 3, p. 70) stated that the approximation led to as much as 10% error in calculated evaporation rates, citing unpublished data of McIlroy.

Paw U and Gao (1988) considered hypothetical situations in which the conventional linearization of the saturation vapor pressure curve led to relative errors in evaporation as high as 10% or more. Almost all the errors were negative.

The possibility of iterating for a mathematically exact solution of the combination problem has been recognized. This is the approach taken by Budyko (1951), and several others. However, this approach is not universally applied, despite the obvious benefits in accuracy. Undoubtedly the operational simplicity of a closed-form, explicit equation for evaporation is one of the main reasons for this, though this advantage is probably more apparent than real; the iteration procedure itself is computationally trivial. Additionally, there is a certain theoretical appeal to an expression that directly reveals the dependence of the evaporation rate upon the various related variables. It has also been argued (somewhat unconvincingly) that if the measurement errors are as large as the errors introduced by the mathematical approximation, then there is no reason to improve the computations.

Paw U and Gao (1988) have recently presented a method that has higher accuracy than the usual linearization approach and does not require iteration. They represented the saturation vapor-pressure curve using a second-order Taylor series, and arrived at a quadratic equation for the evaporation rate, with coefficients dependent on the various atmospheric and surface factors. Taking their work one step further, one can obtain an explicit, second-order equation for evaporation. Paw U and Gao also presented a quartic solution procedure corresponding to a fourth-order approximation to the saturation vapor pressure curve. However, the explicit evaporation equation that could result from this method is extremely complex and would have little theoretical appeal for purposes other than computation.

The current work was performed independently of that of Paw U and Gao (1988). It is generally similar in thrust, but results in a family of evaporation equations that are considerably simpler in form, while yielding any desired degree of accuracy.

This paper provides a new solution of the coupled system of equations governing the turbulent transfer of heat and water vapor from the land surface, the energy balance of the surface, and stomatal control of transpiration. The saturation vapor-pressure relation is not linearized in the solution procedure, but is instead represented by a higher-order Taylor series. The solution is compared to previous solutions, and some general conclusions are made regarding the systematic error in the classical first-order solutions.

### Derivation

The development presented here is conceptually similar to those of Monteith (1965) and Thom (1972). It is assumed that the absorption and emission of radiation and the vaporization of water occur only at leaf surfaces, and that all such sites of energy exchange are characterized at any time by a single temperature. Collectively, these sites are termed the 'surface' in this paper. The development also applies to a free surface of water, as in the case considered by Penman (1948), and to a moist soil surface, if the stomatal resistance is taken to be zero.

The statement of energy conservation at the surface is

$$R_n - G = LE + H, \quad (1)$$

or

$$R_n - G = LE(1 + Bo), \quad (2)$$

in which  $R_n$  is the net radiation,  $G$  is the rate of heat storage beneath the surface (i.e., in the vegetation and soil),  $H$ , is the sensible heat flux into the atmosphere,  $L$  is the latent heat of vaporization of water,  $E$  is the evaporation rate, and  $Bo$  is the Bowen ratio (the ratio of sensible to latent heat flux).

Resistance formulations for  $H$  and  $LE$  are of the form

$$H = \rho c_p (T_l - T_a) / r_{ah}, \quad (3)$$

$$LE = \rho c_p (e_l - e_a) / \gamma r_{av}, \quad (4)$$

in which the subscripts  $a$  and  $l$  refer to conditions at an arbitrary level in the atmosphere above the surface and at a point in the air immediately adjacent to a leaf (or other active surface),  $\rho$  is the density and  $c_p$  the specific heat at constant pressure of the air,  $T$  is temperature,  $e$  is vapor pressure,  $r_{ah}$  is the aerodynamic resistance to heat transport,  $r_{av}$  is the aerodynamic resistance to vapor transport, and  $\gamma$  is the psychrometric constant, given by

$$\gamma = c_p p / 0.622L, \quad (5)$$

in which  $p$  is the air pressure.

To relate the surface temperature to the surface vapor pressure, the usual stomatal resistance parameterization is used,

$$LE = \rho c_p (e_{st} - e_l) / \gamma r_{st}, \quad (6)$$

in which  $e_{st}$  is the vapor pressure inside the plant stomata and  $r_{st}$  is the bulk stomatal resistance. In addition, there may be a vapor pressure deficit  $d_{st}$  within the stomatal cavities,

$$e_{st} = e^*(T_l) - d_{st}, \quad (7)$$

in which  $e^*(T)$  is the saturation vapor pressure of water vapor. It is convenient to combine (4), (6), and (7) to obtain

$$e_l - e_a = [e^*(T_l) - e_a - d_{st}] r_{aw} / (r_{st} + r_{aw}). \quad (8)$$

If the quantities  $R_n - G$ ,  $r_{aw}$ ,  $r_{ah}$ ,  $r_{st}$ , and  $d_{st}$  are all known or otherwise modeled in terms of the other variables, then the coupled system of Equations (1), (3), (4) and (8) can be solved for  $T_l$ ,  $e_l$ ,  $H$ , and  $LE$ . Because of the nonlinearity of  $e^*(T)$ , this solution cannot be performed exactly except by iterative techniques. In the search for convenient solutions of various simplified versions of this set of equations, previous investigators have linearized  $e^*(T)$ , by expanding it in a first-order Taylor series, usually around the (easily measured) air temperature  $T_a$ . To improve upon this approximation, a higher-order expansion is introduced here.

Let us define the dewpoint function  $T^*(e)$  as the inverse of the function  $e^*(T)$ . We expand  $T^*(e)$  in a Taylor series around the state of the air at the measurement level, and evaluate  $T_l$  using that expansion,

$$T_l = T_a + \sum_{m=1}^{\infty} \frac{1}{m!} \left[ \frac{d^m T^*}{de^m} \right]_a [e^*(T_l) - e^*(T_a)]^m. \quad (9)$$

The first derivative may be expressed in terms of conventional notation as the reciprocal of  $\Delta_a$ , which is the slope of the saturation vapor pressure curve, evaluated using the air temperature at measurement level. The function  $e^*(T)$  is approximately exponential, so it will be convenient to write

$$\left[ \frac{d^m T^*}{de^m} \right]_a = (-1)^{m-1} (m-1)! \beta_m / \Delta_a [e^*(T_a)]^{m-1}. \quad (10)$$

By definition,  $\beta_1$  is unity. The motivation for the form of (10) is the fact that, for  $e^*(T)$  exactly exponential, all other  $\beta_m$  would also be unity. In fact, the  $\beta_m$  ( $m = 2, 3, \dots$ ) depart somewhat from unity and vary slightly with temperature; values computed on the basis of Lowe's (1977) polynomial for  $e^*(T)$ , together with some basic calculus, are given in Table I. (The accuracy of the higher-order derivatives based on this polynomial is suspect). Substitution of (10) into (9) yields

$$T_l - T_a = \Delta_a^{-1} [e^*(T_l) - e^*(T_a)] \sum_{m=1}^{\infty} \frac{\beta_m (-1)^{m-1}}{m} \times \left[ \frac{e^*(T_l) - e^*(T_a)}{e^*(T_a)} \right]^{m-1} \quad (11)$$

TABLE I  
Values of  $B_m(T_a)$

Air temperature (deg C)	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$
0	0.889	0.844	0.817	1.077
5	0.884	0.837	0.808	1.059
10	0.882	0.835	0.806	1.053
15	0.879	0.830	0.801	1.043
20	0.875	0.826	0.796	1.033
25	0.872	0.822	0.791	1.023
30	0.869	0.817	0.787	1.013
35	0.867	0.814	0.782	1.005
40	0.864	0.810	0.788	0.997

When (11) and (8) are substituted into (3) and (4), the resulting Bowen ratio is

$$Bo = (\gamma^*/\Delta_a) \left\{ \frac{[e^*(T_l) - e^*(T_a)]}{[e^*(T_l) - e_a - d_{st}]} \right\} \sum_{m=1}^{\infty} \frac{\beta_m (-1)^{m-1}}{m} \times \left[ \frac{[e^*(T_l) - e^*(T_a)]}{e^*(T_a)} \right]^{m-1}, \quad (12)$$

in which we have introduced the new quantity

$$\gamma^* = \gamma(r_{av} + r_{st})/r_{ah}. \quad (13)$$

We next introduce two dimensionless parameters,

$$\eta = \frac{e^*(T_l) - e_a - d_{st}}{d_a - d_{st}} = \frac{(r_{av} + r_{st})\gamma LE}{\rho c_p (d_a - d_{st})}, \quad (14)$$

in which  $d_a$  is the vapor pressure deficit of the air, and

$$\sigma = (d_a - d_{st})/e^*(T_a). \quad (15)$$

Algebraic manipulation of (2), (12), (14), and (15) then leads to

$$\frac{\gamma(r_{av} + r_{st})(R_n - G)}{\rho c_p (d_a - d_{st})} = \eta + (\gamma^*/\Delta_a)(\eta - 1) \times \sum_{m=1}^{\infty} \frac{\beta_m (-1)^{m-1}}{m} (\eta - 1)^{m-1} \sigma^{m-1}. \quad (16)$$

At this point we seek a solution of (16) for  $\eta$  as a power series in the parameter  $\sigma$ ,

$$\eta = 1 + \sum_{i=0}^{\infty} \eta_i \sigma^i. \quad (17)$$

Substitution of (17) into (16) and subsequent collection of like powers of  $\sigma$  lead to the following relations:

$$\eta_0 = \frac{\gamma(R_n - G)(r_{av} + r_{st}) - \rho c_p(d_a - d_{st})}{\rho c_p(d_a - d_{st})[1 + \gamma^*/\Delta_a]}, \quad (18)$$

$$\eta_1 = \frac{\eta_0^2}{(1 + \Delta_a/\gamma^*)} [\beta_2/2], \quad (19)$$

$$\eta_2 = \frac{\eta_0^3}{(1 + \Delta_a/\gamma^*)^2} [(-\beta_3/3)(\Delta_a/\gamma^*) + (\beta_2^2/2 - \beta_3/3)], \quad (20)$$

$$\eta_3 = \frac{\eta_0^4}{(1 + \Delta_a/\gamma^*)^3} [(\beta_4/4)(\Delta_a/\gamma^*)^2 + (\beta_4/2 - 5\beta_2\beta_3/6)(\Delta_a/\gamma^*) + (5\beta_2^3/8 - 5\beta_2\beta_3/6 + \beta_4/4)], \quad (21)$$

$$\eta_4 = \frac{\eta_0^5}{(1 + \Delta_a/\gamma^*)^4} [(-\beta_5/5)(\Delta_a/\gamma^*)^3 + (3\beta_2\beta_4 + \beta_3^2/3 - 3\beta_5/5)(\Delta_a/\gamma^*)^2 + (3\beta_2\beta_4/2 - 7\beta_2^2\beta_3/4 + 2\beta_3^2/3 - 3\beta_5/5)(\Delta_a/\gamma^*) + (3\beta_2\beta_4/4 + 7\beta_2^4/8 - 7\beta_2^2\beta_3/4 + \beta_3^2/3 - \beta_5/5)]. \quad (22)$$

These yield the evaporation rate as

$$\begin{aligned} E = \frac{\Delta_a(R_n - G)/L}{\Delta_a + \gamma^*} & \left\{ 1 + A + [\beta_2/2] \frac{(1 - A\Delta_a/\gamma^*)^2}{A(1 + \Delta_a/\gamma^*)^2} \sigma \right. \\ & + [(-\beta_3/3)(\Delta_a/\gamma^*) + (\beta_2^2/2 - \beta_3/3)] \frac{(1 - A\Delta_a/\gamma^*)^3}{A^2(1 + \Delta_a/\gamma^*)^4} \sigma^2 \\ & + [(\beta_4/4)(\Delta_a/\gamma^*)^2 + (\beta_4/2 - 5\beta_2\beta_3/6)(\Delta_a/\gamma^*) \\ & + (5\beta_2^3/8 - 5\beta_2\beta_3/6 + \beta_4/4)] \frac{(1 - A\Delta_a/\gamma^*)^4}{A^3(1 + \Delta_a/\gamma^*)^6} \sigma^3 \\ & + [(-\beta_5/5)(\Delta_a/\gamma^*)^3 + (3\beta_2\beta_4/4 + \beta_3^2/3 - 3\beta_5/5)(\Delta_a/\gamma^*)^2 \\ & + (3\beta_2\beta_4/2 - 7\beta_2^2\beta_3/4 + 2\beta_3^2/3 - 3\beta_5/5)(\Delta_a/\gamma^*) \\ & + (3\beta_2\beta_4/4 + 7\beta_2^4/8 - 7\beta_2^2\beta_3/4 + \beta_3^2/3 - \beta_5/5)] \\ & \left. \times \frac{(1 - A\Delta_a/\gamma^*)^5}{A^4(1 + \Delta_a/\gamma^*)^8} \sigma^4 + O\left[\frac{(1 - A\Delta_a/\gamma^*)^6}{A^5(1 + \Delta_a/\gamma^*)^6} \sigma^5\right] \right\}, \quad (23) \end{aligned}$$

in which  $A$  is given by

$$A = \frac{\rho c_p(d_a - d_{st})}{\Delta_a(R_n - G)r_{ah}}. \quad (24)$$

The quantity  $A$  may be recognized as the ratio between the two terms in Penman's (1948) combination equation, i.e., the ratio of the wind term to the radiation term.

Equation (23) is, in fact, a family of combination equations. It may be truncated to any order in  $\sigma$ , yielding equations of varying degrees of accuracy. In principle, the coefficients of even higher order could be derived, but their practical significance is questionable. If we truncate the expansion to first order in  $\sigma$  and approximate  $\beta_2$  by a constant, (23) becomes

$$E = \frac{\Delta_a(R_n - G)/L}{\Delta_a + \gamma^*} \left\{ 1 + A + 0.44 \frac{(1 - A\Delta_a/\gamma^*)^2}{A(1 + \Delta_a/\gamma^*)^2} \sigma \right\}. \quad (25)$$

The conditions for convergence of the series (23) may be readily established by examination of the ratios of successive terms. For reasonable combinations of  $A$ ,  $\Delta_a/\gamma^*$ , and  $\sigma$ , it appears that (23) will indeed converge. This is certainly consistent with the general experience that the linear approach usually provides a good approximation.

### Relation to Previous Combination Equations

Equations (23) and (25) both encompass several other previously published combination equations. If  $\beta_m (m > 1)$  is set to zero, corresponding to linearization of  $e^*(T)$ , and if  $d_{st}$  is set to zero, they reduce to the expression of Thom (1972),

$$E = \frac{\Delta_a(R_n - G)/L}{\Delta_a + \gamma^*} + \frac{\rho c_p d_a / r_{ah} L}{\Delta_a + \gamma^*}. \quad (26)$$

Thom did not specify a temperature for the evaluation of  $\Delta$ .

With the additional assumption that  $r_{aw}$  and  $r_{ah}$  are identical, the combination equation reduces to one that is formally equivalent to that of Monteith (1965, p. 210). (In fact, Monteith reduced the truncation error in the first-order form of (9) by expanding not around air temperature, but around a more central temperature. A strict reading of his paper suggests that  $\Delta$  is to be evaluated at the mean of the surface temperature and the wet-bulb temperature of the air. In any case, the whole point of combination equations is to eliminate the need for knowledge of the surface temperature, so, in practice,  $\Delta_a$  is used.)

Penman's (1948) form of the equation results when the stomatal resistance is also ignored,

$$E = \frac{\Delta_a(R_n - G)/L}{\Delta_a + \gamma} + \frac{\rho c_p d_a / r_{ah} L}{\Delta_a + \gamma}. \quad (27)$$

Penman clearly intended that (27) should be used with  $\Delta_a$ . Monteith (1980) notes that  $\Delta$  should be evaluated at a temperature 'between'  $T_a$  and  $T_l$ , but that "in practice,  $\Delta$  must be evaluated at  $T_a$ ". He further notes that the resultant error in computed  $E$  is usually small.

If the vapor pressure deficit at the surface is retained and the stomatal resistance is neglected, an equation similar to that of Slatyer and McIlroy (1961, Ch. 3, p. 61) is obtained.

$$E = \frac{\Delta(R_n - G)/L}{\Delta + \gamma} + \frac{\rho c_p d_a - d_{st})/r_{ah}L}{\Delta + \gamma}. \quad (28)$$

(In this case,  $d_{st}$  simply represents the vapor pressure deficit of the air at the surface.) Slatyer and McIlroy recognized the importance of using a more representative temperature than  $T_a$  to evaluate  $\Delta$ , and they suggested using an average of the wet-bulb temperatures of the air at and above the surface. The disadvantage of such an approach, of course, lies in the necessity of measuring air properties at the surface.

In a recent analysis, Paw U and Gao (1988) presented a quadratic equation for the latent heat flux, correct to second order, under the assumptions of Monteith cited earlier. An explicit formula for evaporation in terms of the controlling variables was not given. After various manipulations, it can be shown that the method of Paw U and Gao, applied to the slightly more general problem considered in this paper, leads eventually to

$$E = (R_n - G)/L \left\{ 1 + \frac{A\gamma^*(1 + \Delta_a/\gamma^*)}{\beta_2 \sigma \Delta_a} \times \left[ 1 - \left[ 1 + \frac{2\beta_2 \sigma \Delta_a (1 - A\Delta_a/\gamma^*)}{A\gamma^*(1 + \Delta_a/\gamma^*)^2} \right]^{1/2} \right] \right\}. \quad (29)$$

It is not immediately apparent how (29) compares to (23) or (25), nor is it obvious how (29) differs from the conventional combination equations. However, if we make the assumption (consistent with the quadratic approximation) that the innermost term in  $\sigma$  is small, it can be shown, still to first order in  $\sigma$ , that (29) reduces to (25). Both (25) and (29) result from the application of the second-order form of (9). The results differ because (29) comes from an exact solution of the quadratic equation, whereas (25) comes from an approximate perturbative solution. However, both formulae have error on the order of  $\sigma^2$ , since the accuracy of both is limited by the second-order expansion of  $e^*(T)$ .

Paw U and Gao (1988) also showed how a fourth-order expansion of  $e^*(T)$  yields a quartic equation for  $E$  that can be solved by the usual algorithm for such equations. In principle, this permits the development of an explicit equation for evaporation rate as a function of the controlling variables. Presumably the equation would be equivalent to (23) through the third order in  $\sigma$ , but I have not attempted to establish this equivalence.

#### Comments on the Error in First-Order Combination Equations

A simple assessment of the error in conventional first-order combination equations can be obtained from (25), the last term in braces approximating the magnitude of



the error. The relative error  $\varepsilon$  in evaporation rate computed from the first-order approximation is

$$\varepsilon = -\frac{0.44(1 - A\Delta_a/\gamma^*)^2\sigma}{A(1 + A)(1 + \Delta_a/\gamma^*)^2} \quad (30)$$

Two important results are immediately apparent. First, the error is always non-positive, so the first-order equation can underestimate, but cannot overestimate, the evaporation rate. Second, the error goes to zero when the ratio  $A$  of the 'wind term' to the 'radiation term' is equal to  $\gamma^*/\Delta_a$ . It can be shown that this condition is met if and only if the latent heat flux is identical to the available energy  $R_n - G$ . This happens when there is no sensible heat flux, hence when the surface and air temperatures are identical. In that case, there is of course no need for a higher-order expansion of the saturation vapor pressure curve.

Some numerical examples of the relative error associated with the linear approximation may also be obtained. Taking the typical case  $A = 0.3$ , and using  $\sigma = 0.5$ , we find relative errors of  $-0.07$ ,  $-0.01$ , and  $-0.001$  when  $\Delta_a/\gamma^*$  takes the values 1, 2, and 4, corresponding to negligible  $r_{st}$  and air temperatures of about 6, 18, and 32 deg C respectively. For  $r_{st}$  equal in size to the aerodynamic resistances, the errors at these same air temperatures rise to  $-0.18$ ,  $-0.07$ , and  $-0.01$  respectively.

The adequacy of the quadratic approximation and the convergence of (23) may be established for these same examples by computing the ratio of the  $\sigma^2$ -term in (23) to the  $\sigma$ -term. For negligible  $r_{st}$  these ratios are  $-0.11$ ,  $-0.07$ , and  $0.03$ , and for  $r_{st}$  equal to the aerodynamic resistances the ratios are  $-0.04$ ,  $-0.11$ , and  $-0.07$ , at air temperatures of 6, 18, and 32 deg C respectively.

### Discussion

We may distinguish between the 'combination method', which is the joint solution of (1), (3), (4) and (8), or some similar set of equations, by some means, and the 'combination equations', which are specific solutions of such sets of equations for evaporation, obtained by introducing some mathematical approximation for  $e^*(T)$ . For maximum computational accuracy, the combination method may be applied in conjunction with iteration. Very high accuracy can also be obtained by use of one of the higher order combination equations (23) or (25), or by application of Paw U and Gao's (1988) quartic solution; possibly these are also useful as benchmarks for testing of iterative methods.

For a theoretical interpretation of the functional dependence of evaporation on the controlling variables, a simple evaporation equation is desirable. The new evaporation equations presented here, particularly (25), have a direct relation to previous first-order equations, and clearly show the dependence of the higher-order terms on atmospheric conditions.

### Acknowledgements

Dara Entekhabi, of the Massachusetts Institute of Technology, and Phillippe Martin, of the National Center for Atmospheric Research, both called my attention to the relevant work of Paw U after the preparation of an earlier version of this paper. Tom Delworth and Syukuro Manabe, at the Geophysical Fluid Dynamics Laboratory of the National Oceanic and Atmospheric Administration, also provided helpful reviews.

### References

- Brutsaert, W. H.: 1982, *Evaporation into the Atmosphere*, Kluwer Acad. Publ. Dordrecht, Holland, pp. 110–112, 215–218, 223–224.
- Budyko, M. I.: 1951, 'On the Influence of Reclamation Measures Upon Potential Evapotranspiration', (in Russian). *Izv. Akad. Nauk. SSSR Ser. Geogr.*, No. 1, 16–35.
- Businger, J. A.: 1956, 'Some Remarks on Penman's Equations for Evapotranspiration', *Neth. J. Agr. Sci.* **44**, 77–80.
- Lowc, P. R.: 1977, 'An Approximating Polynomial for the Computation of Saturation Vapor Pressure', *J. Appl. Met.* **16**, 100–103.
- Monteith, J. L.: 1965, 'Evaporation and Environment', in *The State and Movement of Water in Living Organisms, 19th Symp. Soc. Exp. Biol.*, pp. 205–234.
- Paw, U and Gao, W.: 1988, 'Applications of Solutions to Non-Linear Energy Budget Equations', *Agr. For. Met.* **43**, 121–145.
- Penman, H. L.: 1948, 'Natural Evaporation from Open Water, Bare Soil and Grass', *Proc. R. Soc. London, Ser. A*, **193**, 120–145.
- Penman, H. L. and Schofield, R. K.: 1951, 'Some Physical Aspects of Assimilation and Transpiration', in *Carbon Dioxide Fixation and Photosynthesis, 5th Symp. Soc. Exp. Biol.*, pp. 115–129.
- Slatyer, R. O. and McIlroy, I. C.: 1961, *Practical Microclimatology*, CSIRO, Melbourne.
- Tanner, C. B. and Pelton, W. L.: 1960, 'Potential Evapotranspiration Estimates by the Approximate Energy Balance Method of Penman', *J. Geophys. Res.* **65**(10), 3391–3413.
- Thom, A. S.: 1972, 'Momentum, Mass and Heat Exchange of Vegetation', *Quart. J. R. Met. Soc.* **98**, 124–134.
- Thom, A. S. and Oliver, H. R.: 1977, 'On Penman's Equation for Estimating Regional Evaporation', *Quart. J. R. Met. Soc.* **103**, 345–357.
- Van Bravel, C. H. M.: 1966, 'Potential Evaporation: The Combination Concept and its Experimental Verification', *Water Resour. Res.* **2**(3), 455–467.